Higher Order Quantum Computation

Alessandro Bisio

Quantum Foundations Workshop
Is Quantum Theory exact?

July 3nd 2018
Laboratori Nazionali di Frascati

In collaboration with:

Paolo Perinotti

Supported by:

JOHN TEMPLETON FOUNDATION

Outline

Axiomatic approach to state transformations

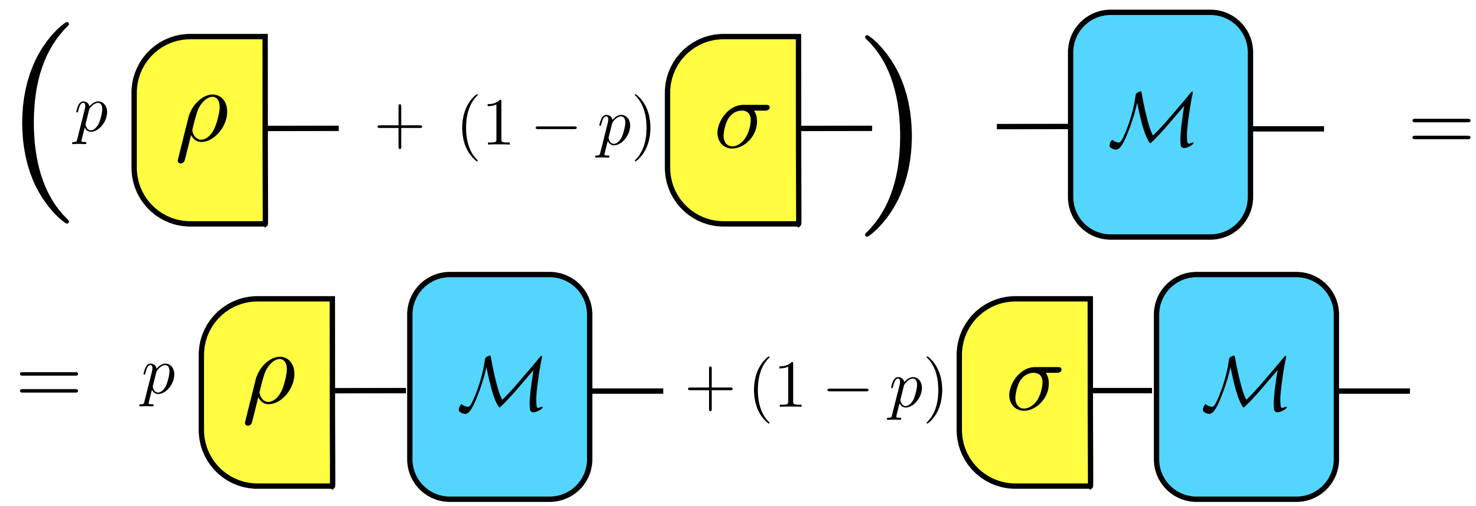
Processing quantum transformations: Quantum Supermaps

Axiomatic theory of Higher Order Quantum Computation

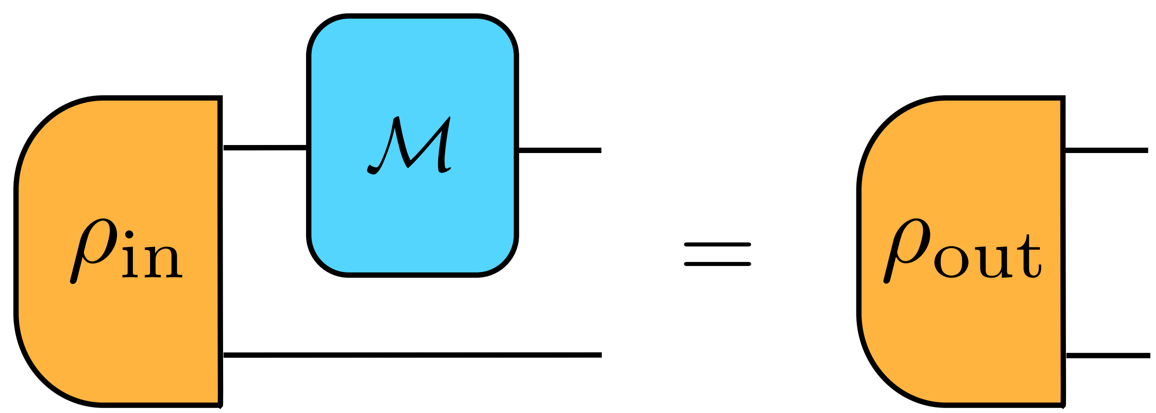
Conclusions

Admissibility conditions (transformations)

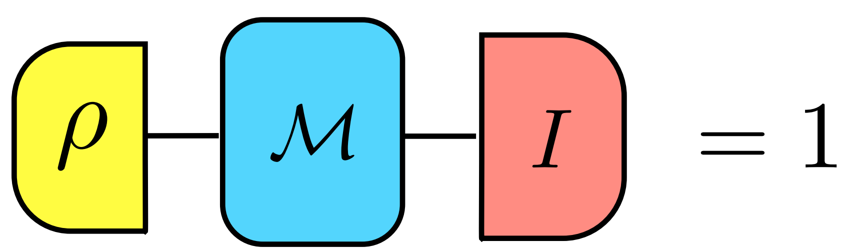
linearity



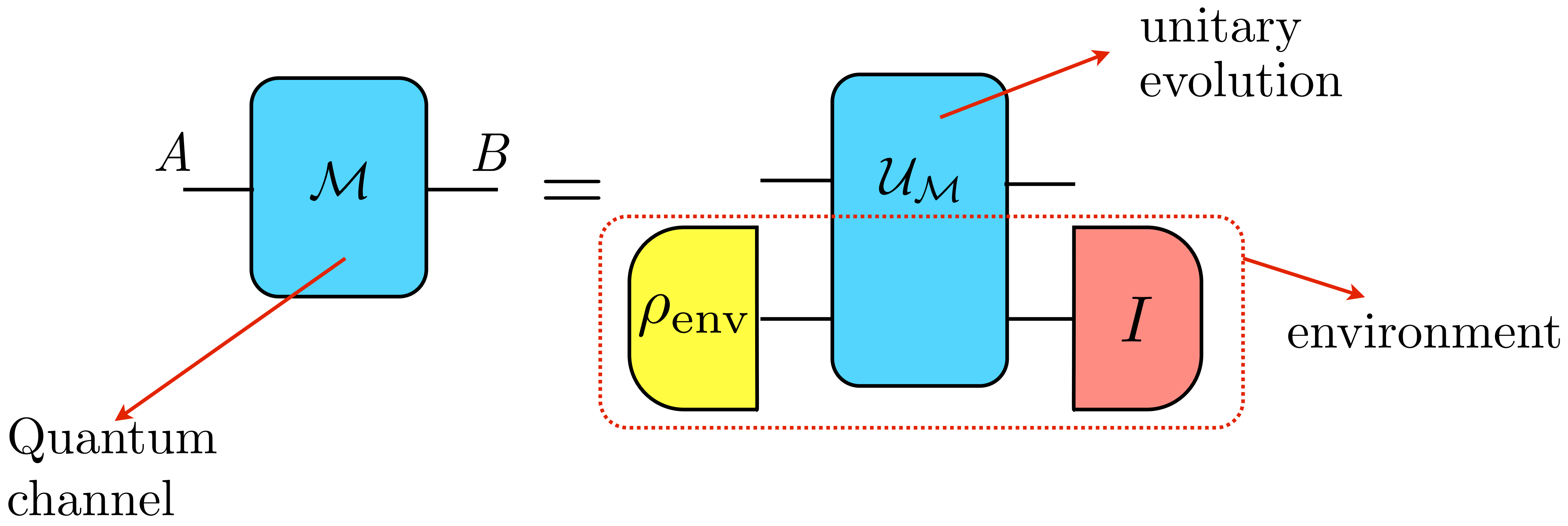
complete positivity



normalisation

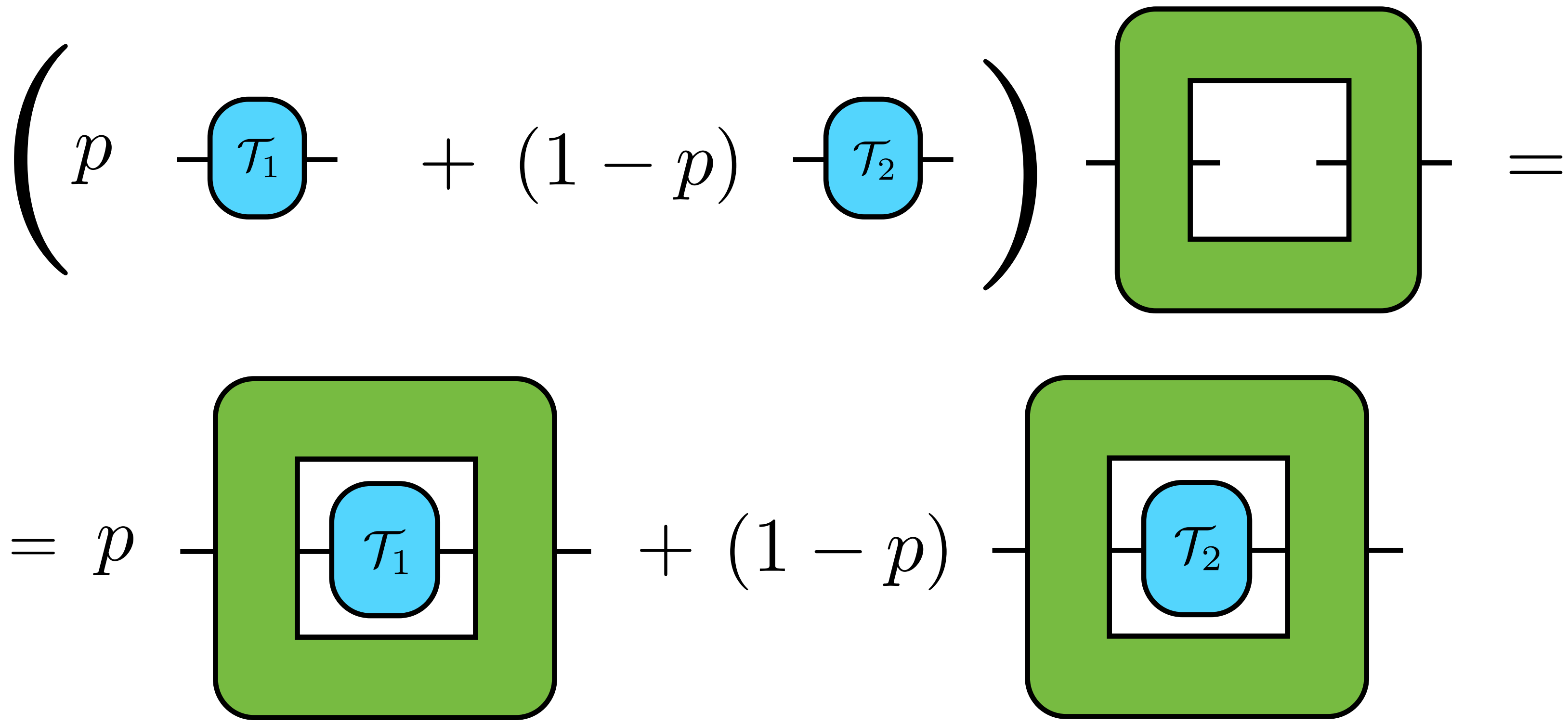


Realisation theorem (transformations)

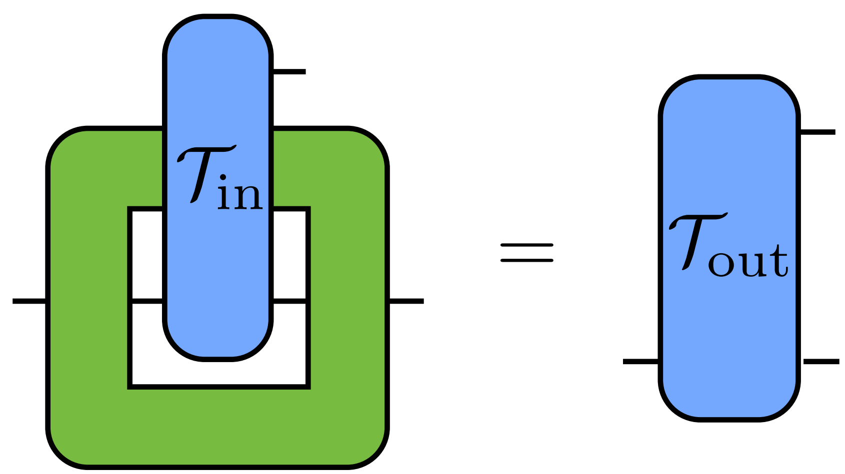


Admissibility conditions (supermaps)

linearity

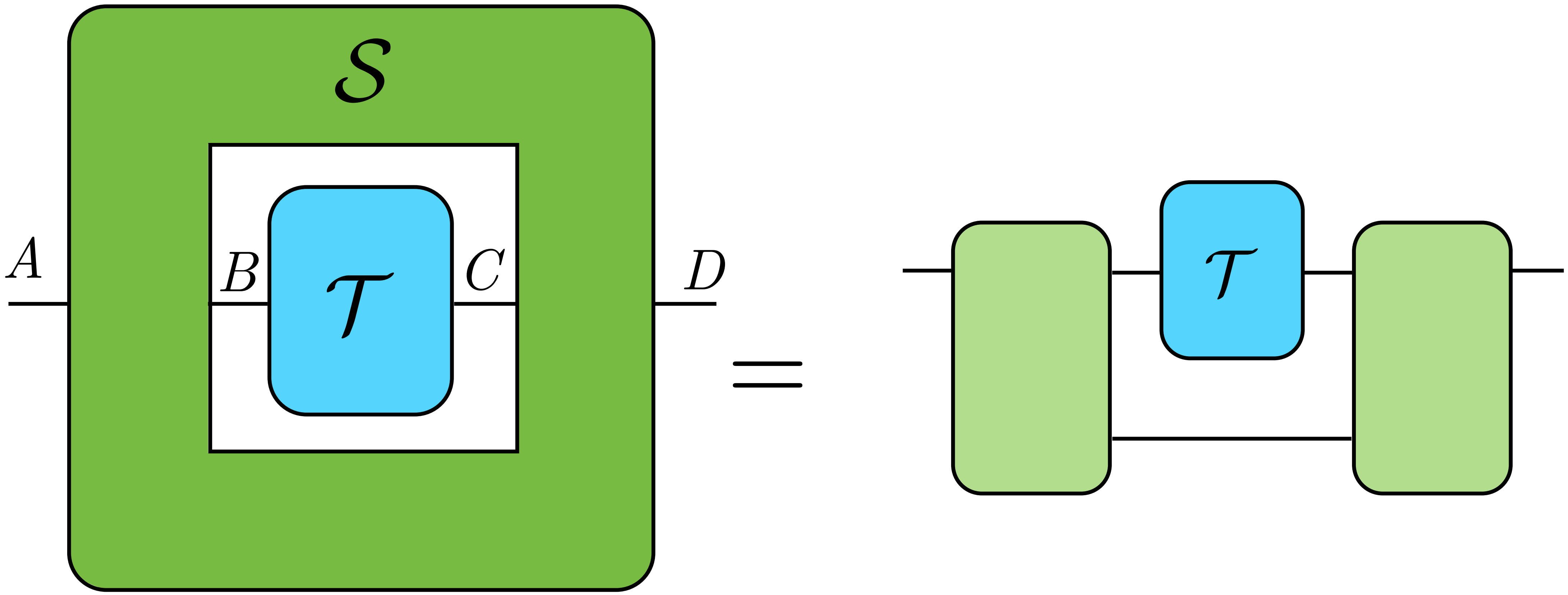


complete positivity



normalisation — channels into channels

Realisation theorem (Supermaps)



deterministic
supermap

quantum
circuit

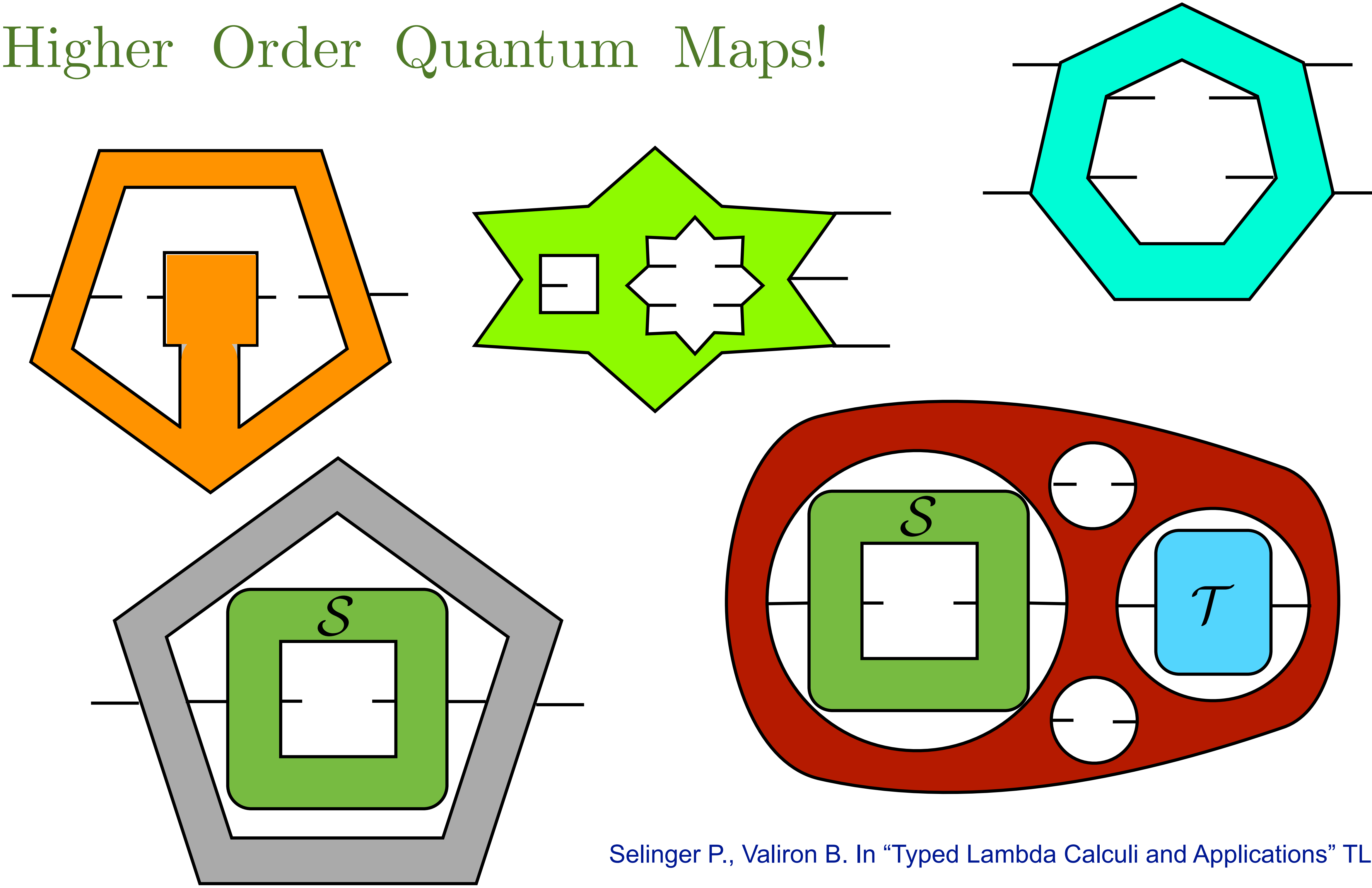
AB, G. Chiribella, G. M. D'Ariano P. Perinotti Phys. Rev. A 83, 022325 (2011)

G. Chiribella, G. M. D'Ariano P. Perinotti Phys. Rev. A 80, 022339 (2009)

Transforming supermaps?

Transforming supermaps?

Higher Order Quantum Maps!



Selinger P., Valiron B. In “Typed Lambda Calculi and Applications” TLCA (2005).

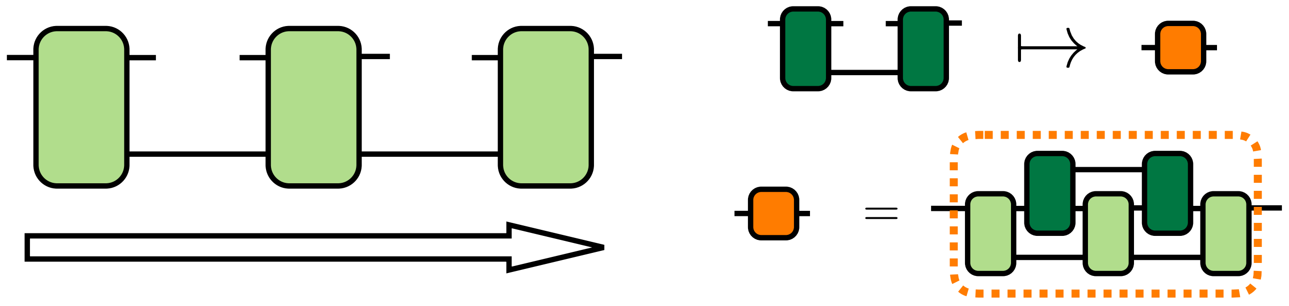
Pagani, P. Selinger, B. Valiron, POPL, ACM (2014).

Higher Order Quantum Maps

Why?

Causal structures

Quantum circuits are higher order maps

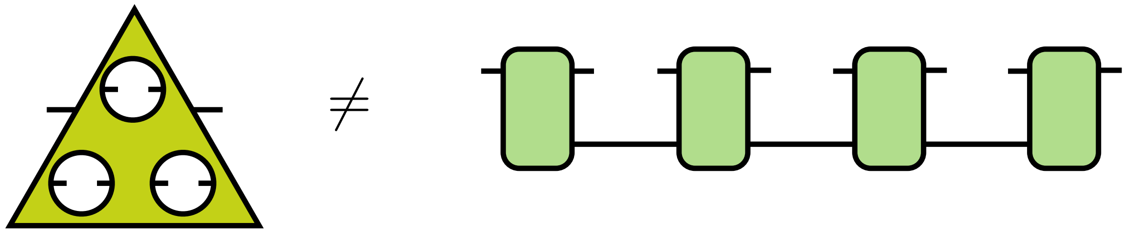


Well defined causal structure

G. Chiribella, G. M. D'Ariano P. Perinotti
Phys. Rev. A 80, 022339 (2009)

AB, G. Chiribella, G. M. D'Ariano P. Perinotti
Acta Physica Slovaca 61, No.3, 273-390 (2011)

Not every higher order map
is a quantum circuit!



Indefinite causal structure

G. Chiribella, G. M. D'Ariano P. Perinotti, B. Valiron,
Phys. Rev. A 88, 022318 (2013)

O. Oreshkov, F. Costa, C. Brukner,
Nature Commun. 3 1092 (2012)

C. Branciard et al.,
New J. Phys. 18 013008 (2016)

Causal
Inequalities

G. Rubino et al., Science
Advances 3 e1602589 (2017)

Experiment

Higher Order Quantum Maps

Why?

Causal structures

Computational advantage

Polynomial speedup with
quantum-controlled order of gates

M. Araújo et al.Phys. Rev. Lett. 113 250402 (2014)

Indefinite causal order
can enhance channel capacity

D. Ebler, S. Salek and G. Chiribella Phys. Rev. Lett. 120, 120502 (2018)

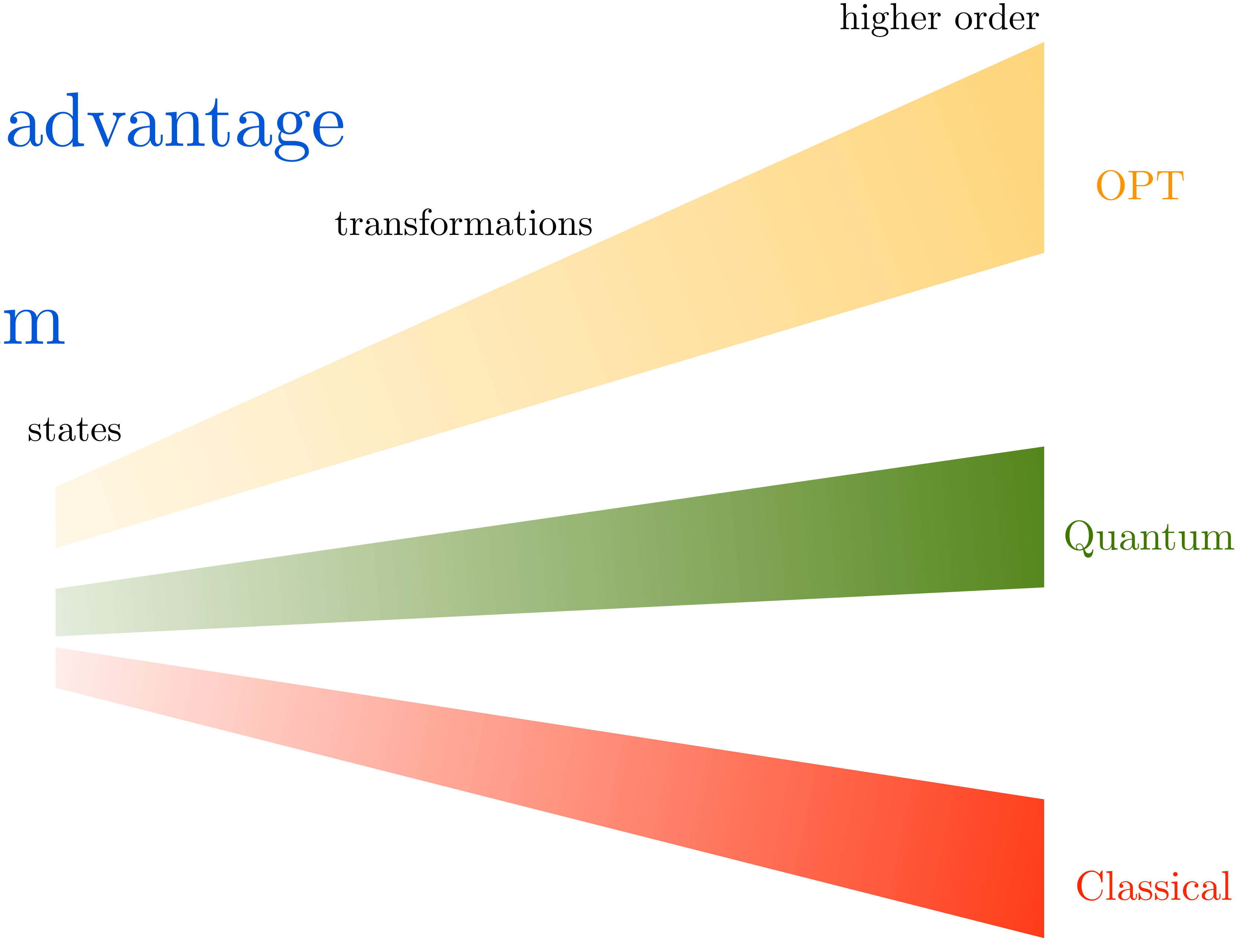
Higher Order Quantum Maps

Why?

Causal structures

Computational advantage

Beyond Quantum



Higher Order Quantum Maps

Why?

Causal structures

Computational advantage

Beyond Quantum

How?

Types

$A, B, \ldots$ elementary types ⟷ quantum systems

I is the elementary type of the trivial (one dimensional) quantum system

Recursive definition

if x, y are types, then $(x \to y)$ is also a type.

A type is a string like

$((A \to B) \to ((C \to D) \to E))) \to H$ (outermost parenthesis are omitted)

$A \to B$ quantum operation (states to states)

$A \to I$ effects

$(A_1 \to B_1) \to (A_2 \to B_2)$ supermaps (quantum operation to quantum operations)

What is admissible?

How can we generalise complete positivity?

Def: Extension by an elementary type

x is a type, E is an elementary type. The extension of x by E, denoted by $x \parallel E$, is recursively defined as follows:

$$A \parallel E := AE,$$

$$(x \to y) \parallel E := (x \to y \parallel E).$$

Example:

$$((A \to B) \to (C \to D)) \parallel E = ((A \to B) \to (C \to D) \parallel E) =$$
$$= ((A \to B) \to (C \to D \parallel E)) = ((A \to B) \to (C \to DE))$$

Admissibility conditions

Def: admissible elementary event (aka quantum state)

Let A be a quantum system $\mathcal{L}(\mathcal{H}_A)$. An admissible elementary event of A is an operator $\rho \in \mathcal{L}(\mathcal{H}_A)$ such that:

$\rho \geqslant 0$, $\mathrm{Tr}[\rho] \leqslant 1$. If $\mathrm{Tr}[\rho] = 1$ then ρ is a deterministic event.

$\mathsf{T}(A)$ is the set of admissible elementary events of A. (sub-normalised states)

$\mathsf{T}_1(A)$ is the set of deterministic events of A. (normalised states)

If A, B are a quantum systems, we denote with AB the parallel composition of the two system $\mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B) =: \mathcal{L}(\mathcal{H}_{AB})$

Admissibility conditions

Recursive definition

Def: admissible event of type $x \to y$

An admissible event of type $x \to y$ is a linear map $\mathcal{M}$, such that, for any E:

1) $[\mathcal{M} \otimes \mathcal{I}_E](\mathrm{T}(x \parallel E)) \subseteq \mathrm{T}(y \parallel E)$

Admissible to admissible

2) $\exists \{\mathcal{N}_i\}\ i = 1, \ldots, n$ such that:

2a) $[\mathcal{N}_i \otimes \mathcal{I}_E](\mathrm{T}(x \parallel E)) \subseteq \mathrm{T}(y \parallel E)$

2b) $[(\mathcal{M} + \sum_i \mathcal{N}_i) \otimes \mathcal{I}_E](\mathrm{T}(x \parallel E)) \subseteq \mathrm{T}(y \parallel E)$

2c) $[(\mathcal{M} + \sum_i \mathcal{N}_i) \otimes \mathcal{I}_E](\mathrm{T}_1(x \parallel E)) \subseteq \mathrm{T}_1(y \parallel E)$

If $[\mathcal{M} \otimes \mathcal{I}_E](\mathrm{T}_1(x \parallel E)) \subseteq \mathrm{T}_1(y \parallel E)$ is deterministic.

Deterministic to deterministic

$\mathrm{T}(x)$ is the set of admissible events of type x.

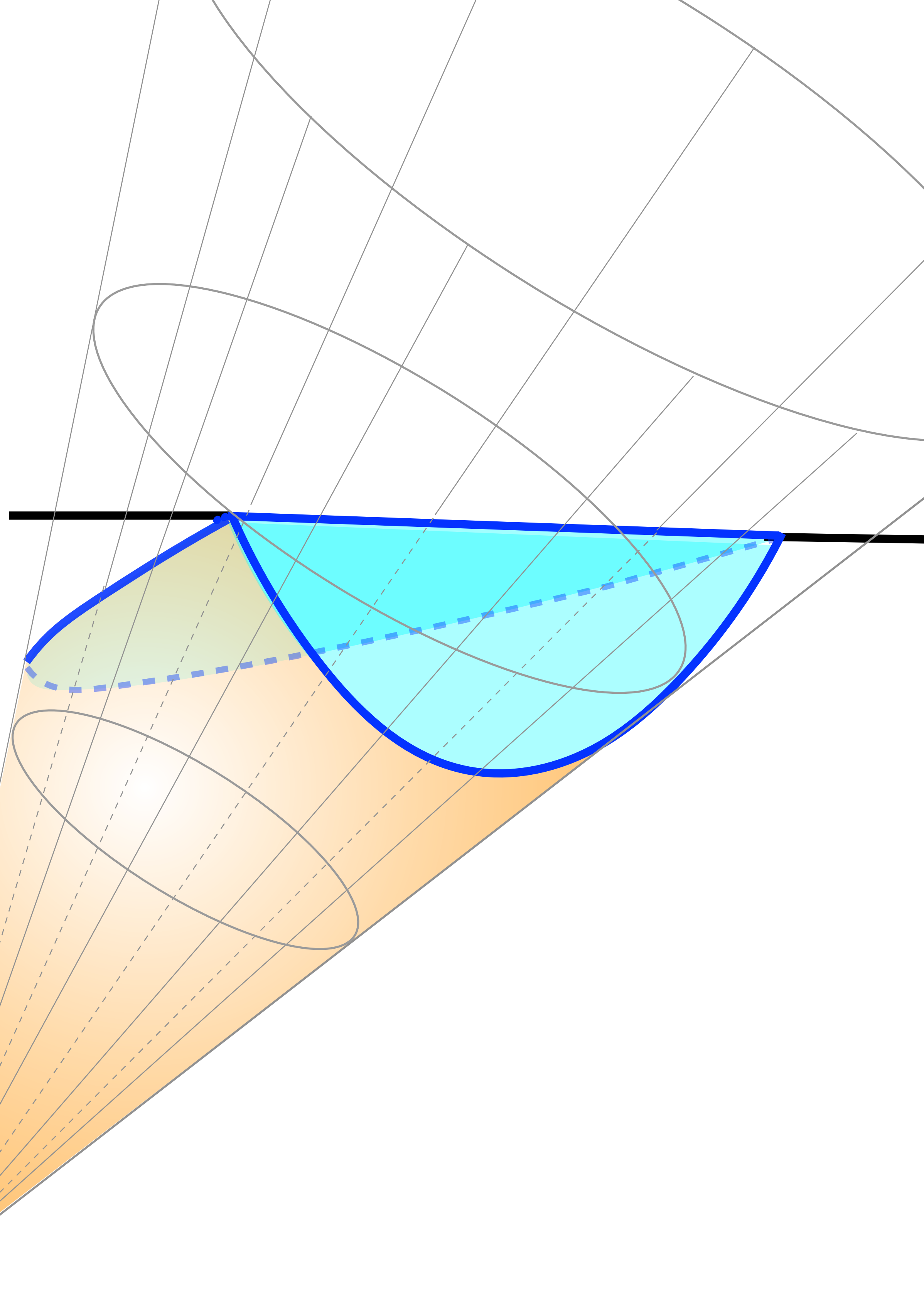
$\mathrm{T}_1(x)$ is the set of deterministic events of type x.

Results

Complete positivity is derived

$$\mathcal{M} \text{ admissible } \overset{*}{\Longleftrightarrow} \mathscr{C}(\mathcal{M}) \geqslant 0$$

Choi operator



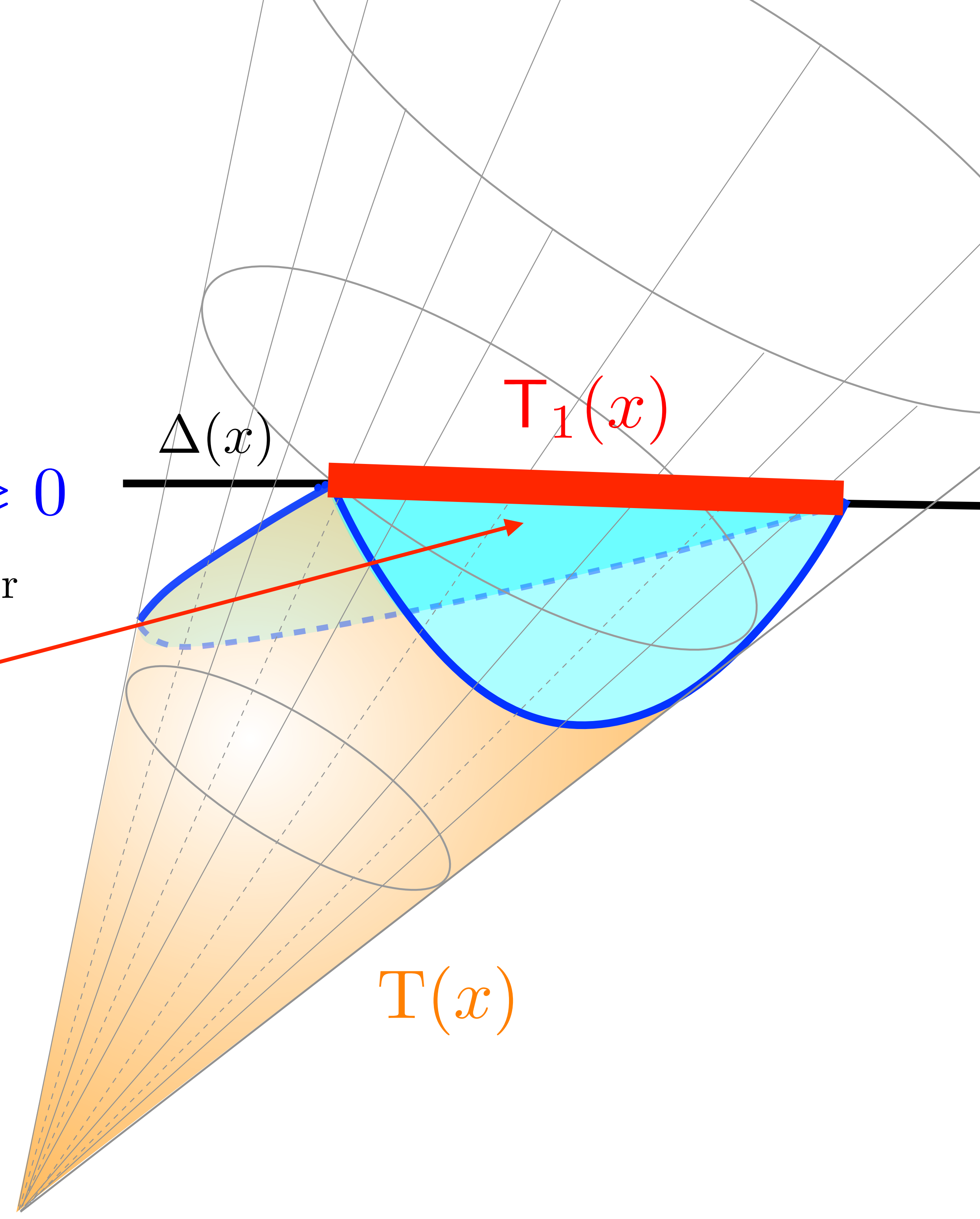
Results

Complete positivity is derived

$\mathcal{M}$ admissible $\overset{*}{\Longleftrightarrow}$ $\mathscr{C}(\mathcal{M}) \geqslant 0$

Choi operator

Characterisation of deterministic events



Results

Complete positivity is derived

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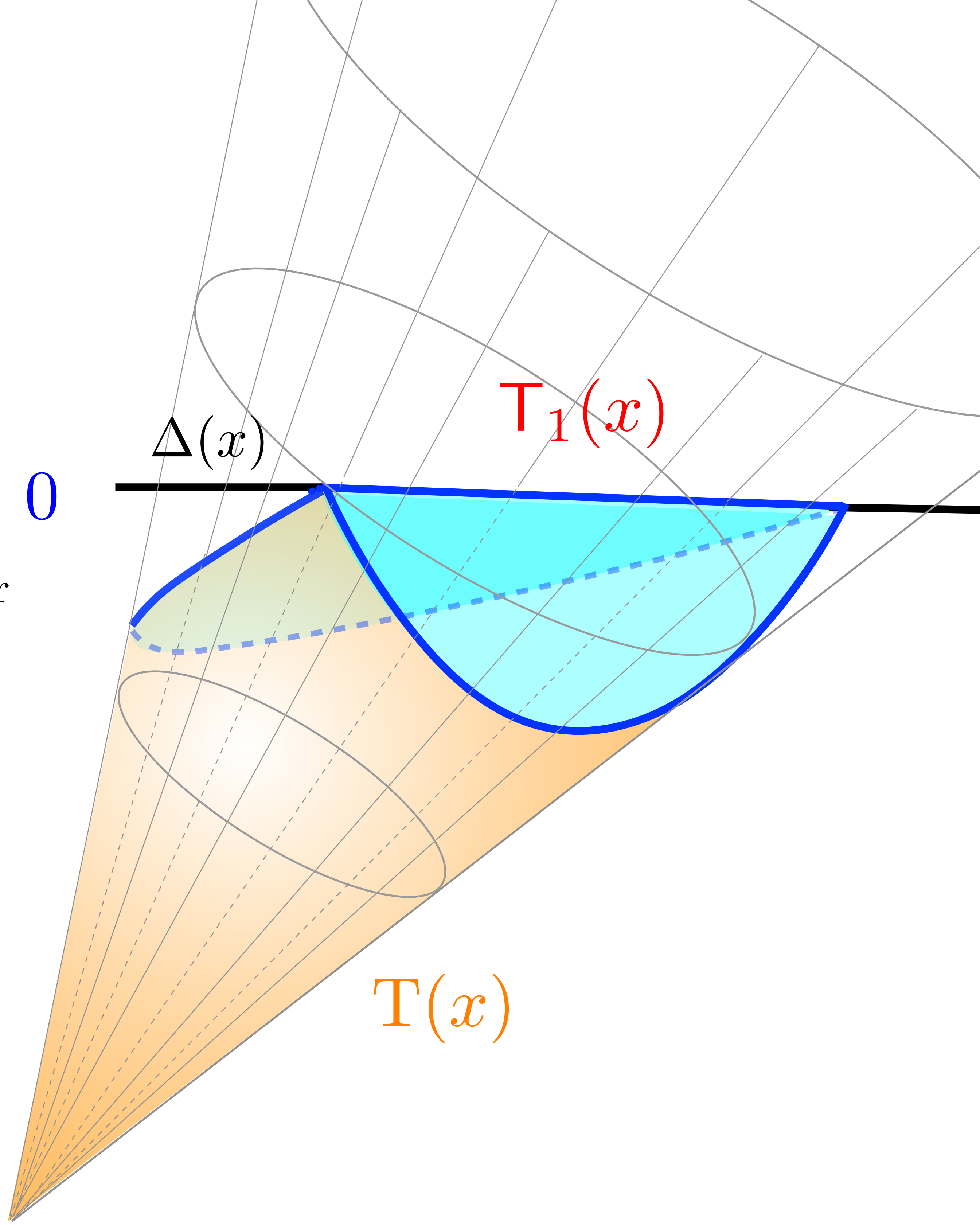
Choi operator

Characterisation of deterministic events

Ready for general operational probabilistic theories



Typing rules $x \equiv y \iff \mathsf{T}(x) = \mathsf{T}(y)$

Functionals

$$(x \to I) \to I \equiv x$$

Tensor product $x \tilde{\otimes} y := (x \to (y \to I)) \to I$

commutativity $x \tilde{\otimes} y \equiv y \tilde{\otimes} x$

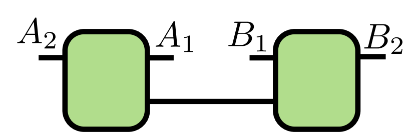
$(A \to B) \tilde{\otimes} (C \to D)$ non-signalling channels

associativity $(x \tilde{\otimes} y) \tilde{\otimes} z \equiv x \tilde{\otimes} (y \tilde{\otimes} z)$

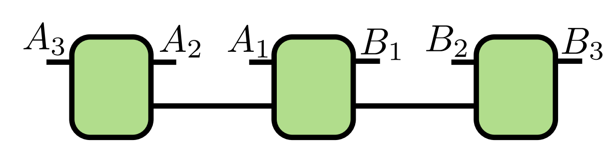
uncurrying $(x \tilde{\otimes} y) \to z \equiv x \to (y \to z)$ $g_x(y) = f(x, y)$

Causally order Quantum Networks (combs)

$((A_2 \to A_1) \to B_1) \to B_2 \equiv (A_1 \to B_1) \to (A_2 \to B_2)$



$((((A_3 \to A_2) \to A_1) \to B_1) \to B_2) \to B_3 \equiv ((A_1 \to B_1) \to (A_2 \to B_2)) \to (A_3 \to B_3)$

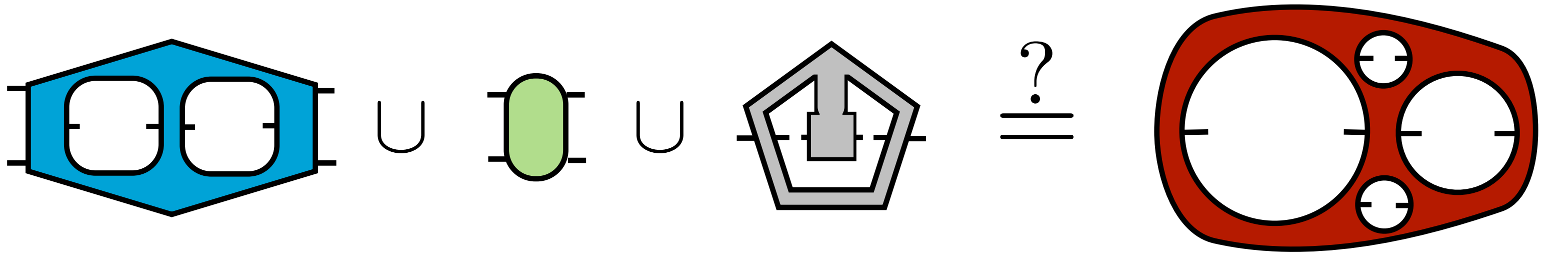


etc.

Future perspectives

Complete set of typing rules

Universal set



Causal structure

Implementation: admissible vs realizable

General operational probabilistic theories

Type equivalence

$$x \equiv y \iff \mathsf{T}_1(x) = \mathsf{T}_1(y)$$

Tensor product

$$x \,\tilde{\otimes}\, y := (x \to (y \to I)) \to I$$

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Tensor product

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Examples:

$$A \mathbin{\tilde{\otimes}} B = AB$$

Type equivalence

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Tensor product

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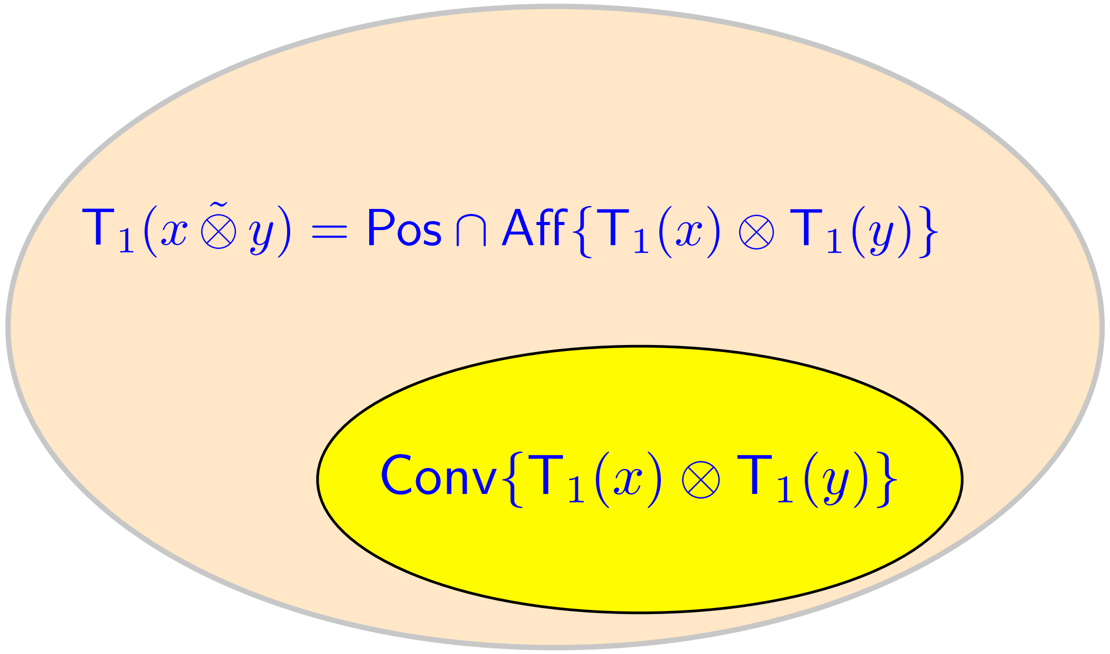
Examples:

$$A \mathbin{\tilde{\otimes}} B = AB$$

$$(A \to B) \mathbin{\tilde{\otimes}} (C \to D)$$

non-signalling channels



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Tensor product

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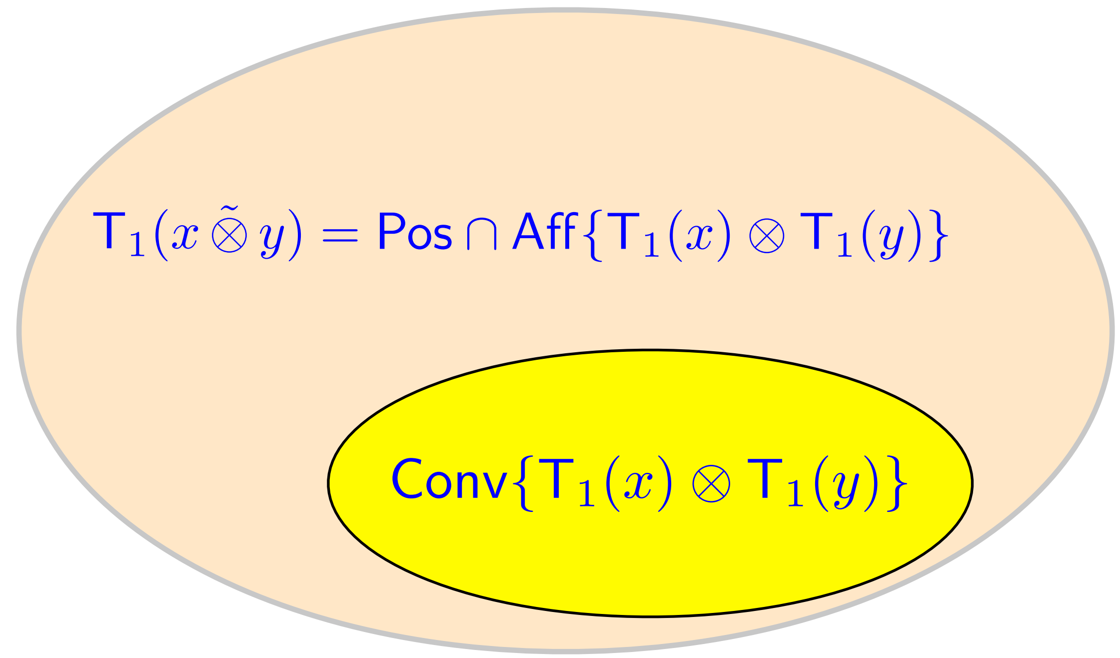
Examples:

$$A \tilde{\otimes} B = AB$$

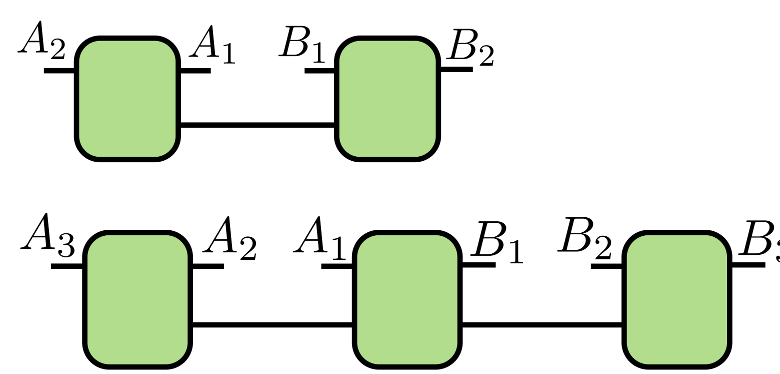


$$(A \to B) \tilde{\otimes} (C \to D)$$ non-signalling channels

$$((A \to B) \tilde{\otimes} (C \to D)) \to I$$ process matrices

$$(A_1 \to B_1) \to (A_2 \to B_2)$$ supermaps

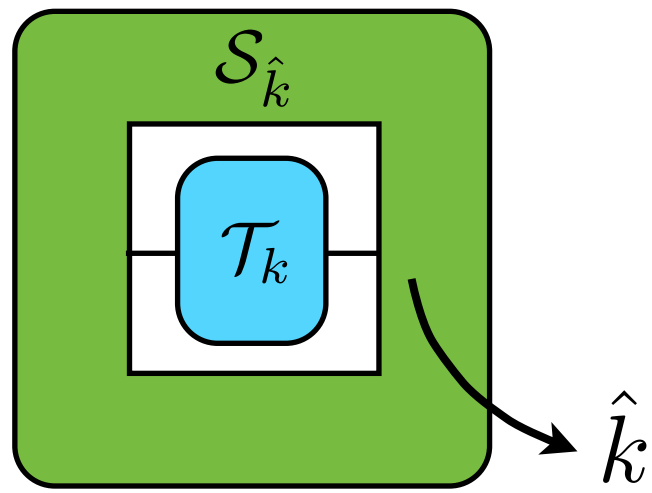
$$((A_1 \to B_1) \to (A_2 \to B_2)) \to (A_3 \to B_3)$$



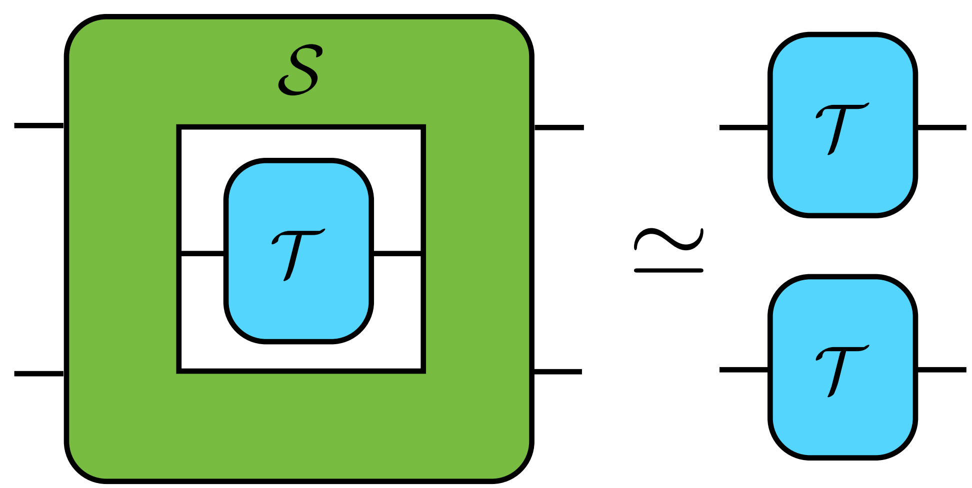
causally ordered quantum networks (combs)

Transformations as carriers of information

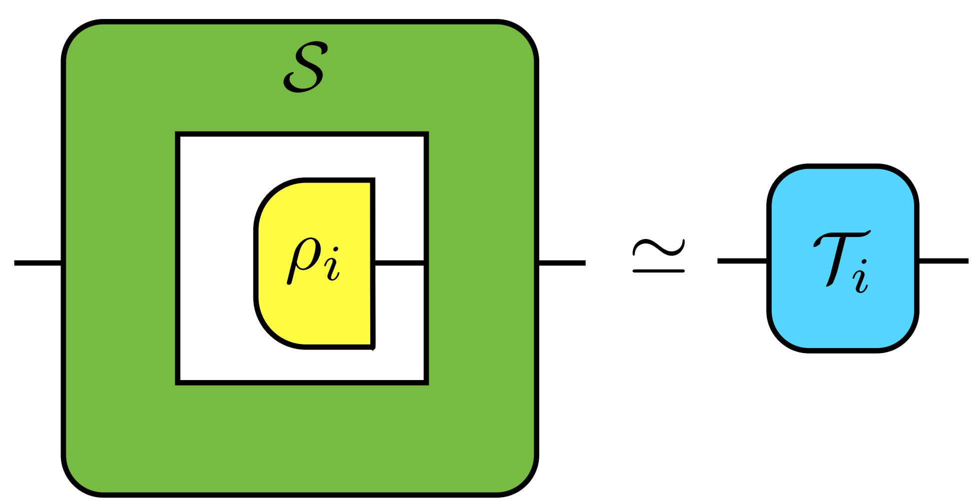
channel estimation



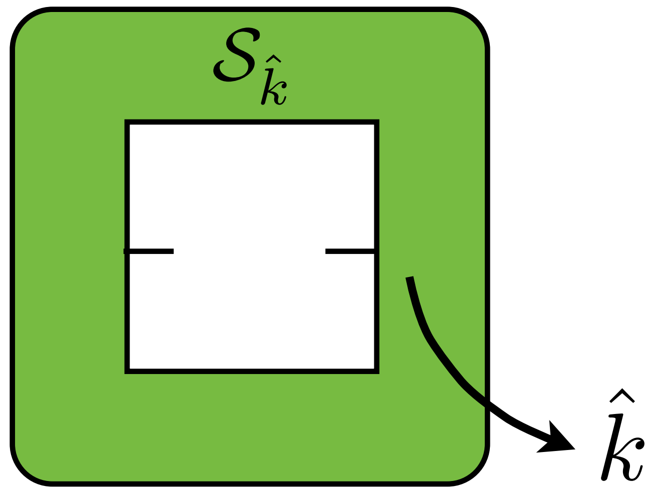
cloning



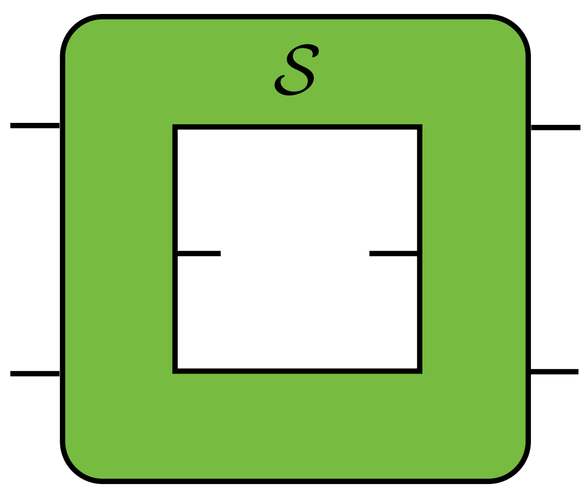
programmable channels



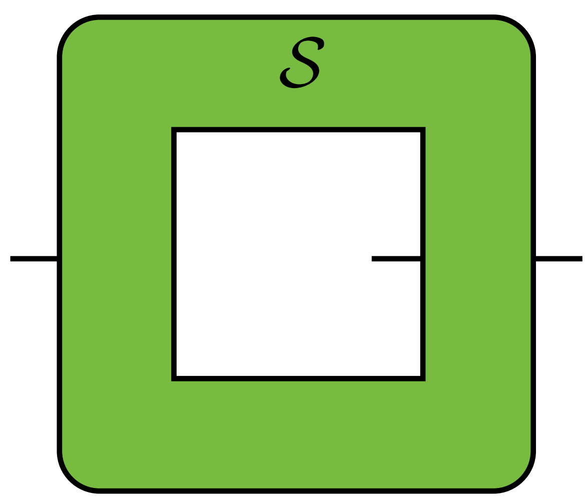
Transformations as carriers of information



quantum channel → classical outcome

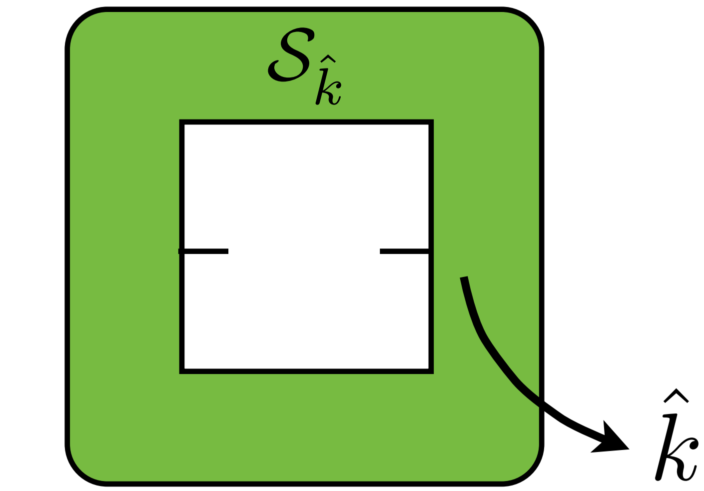


quantum channel → quantum channel

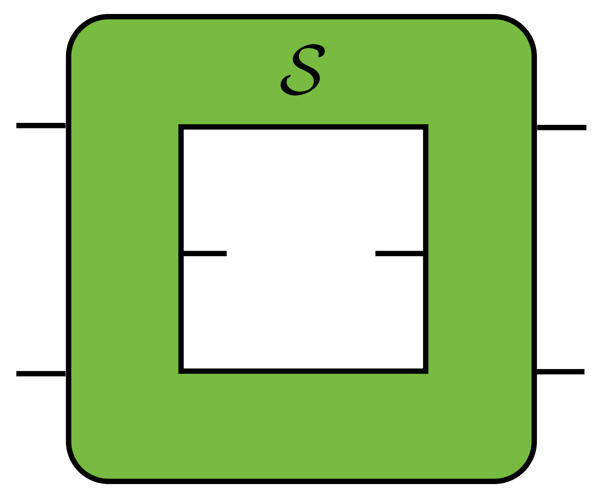


quantum state → quantum channel

Transformations as carriers of information



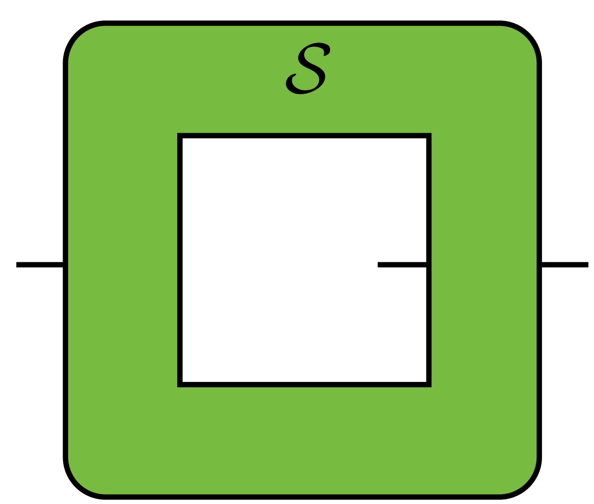
quantum channel → classical outcome



quantum channel → quantum channel

transformations of transformations:

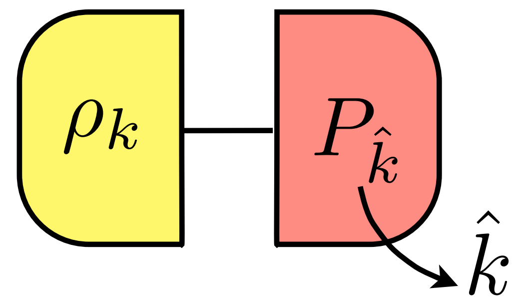
Quantum “Supermaps”



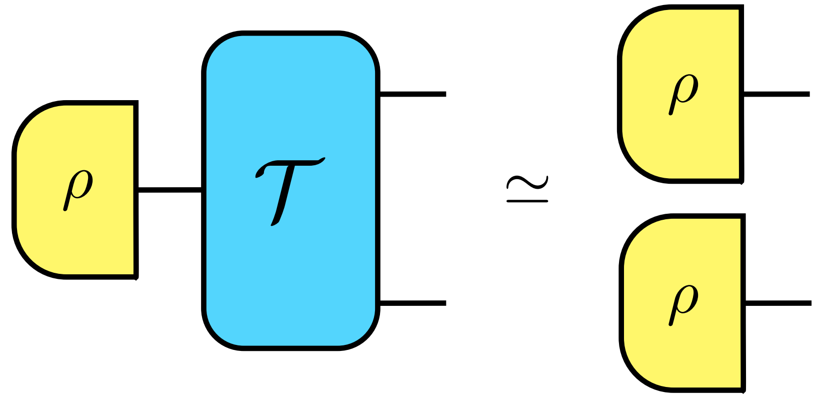
quantum state → quantum channel

State transformations

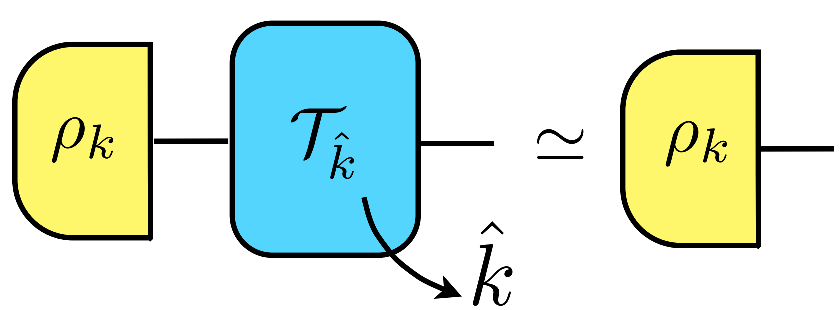
state estimation



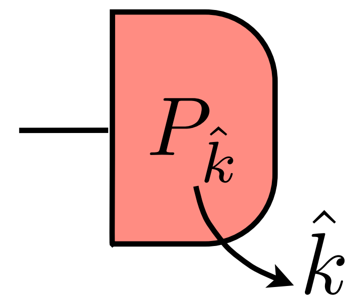
quantum cloning



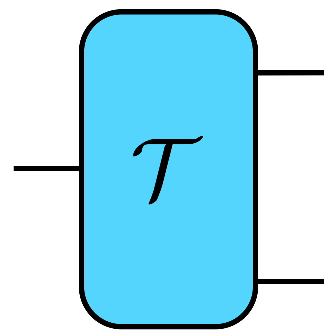
information
disturbance
tradeoff



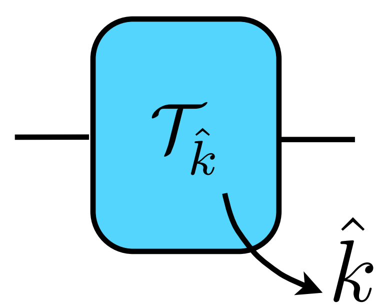
State transformations



quantum state → classical outcome

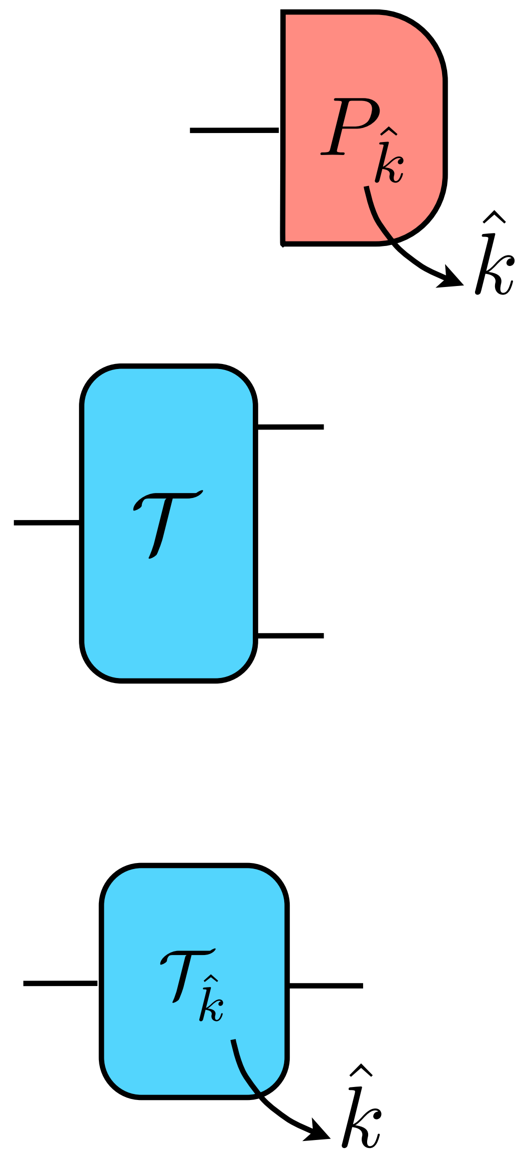


quantum state → quantum state



quantum state → quantum state + classical outcome

State transformations



quantum state ⇒ classical outcome

quantum state ⇒ quantum state

The most general state transformation?

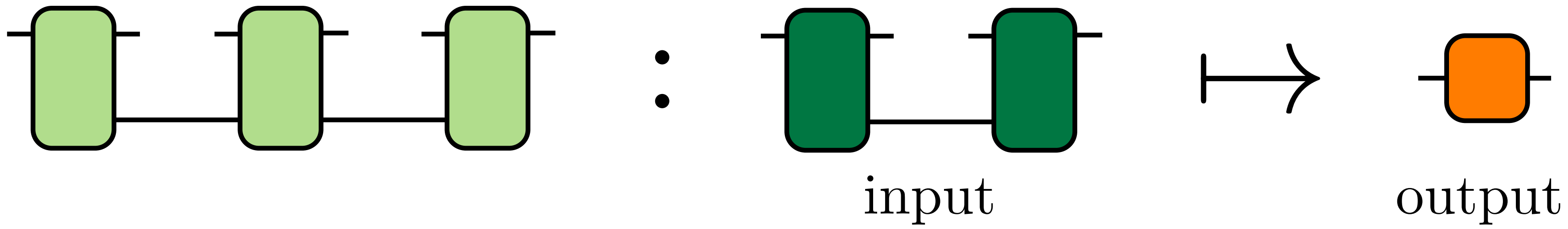
quantum state ⇒ quantum state + classical outcome

Higher Order Quantum Maps

Why?

Causal structures

Quantum circuits are higher order maps



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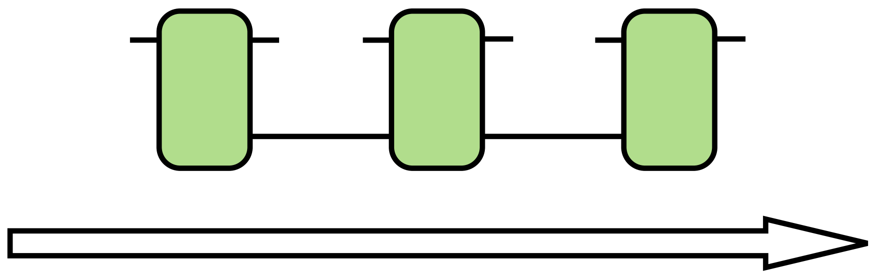

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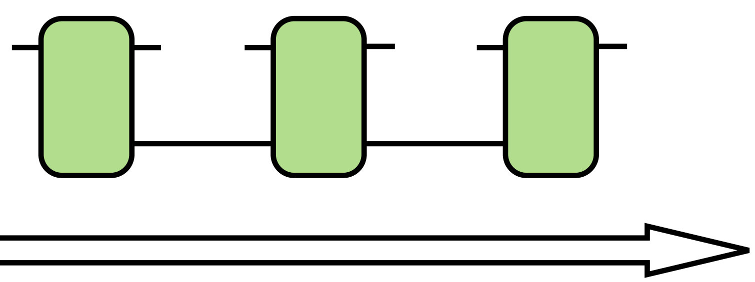

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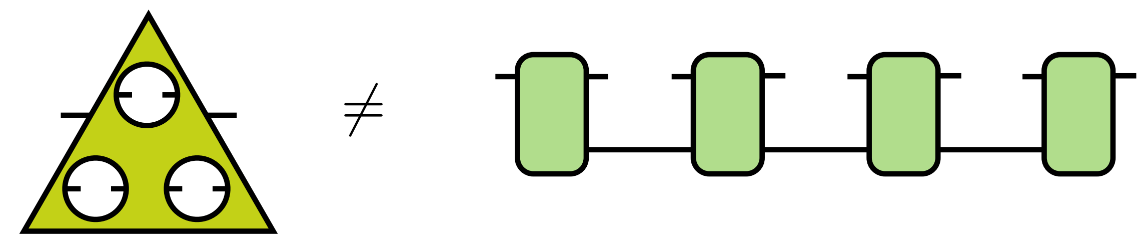
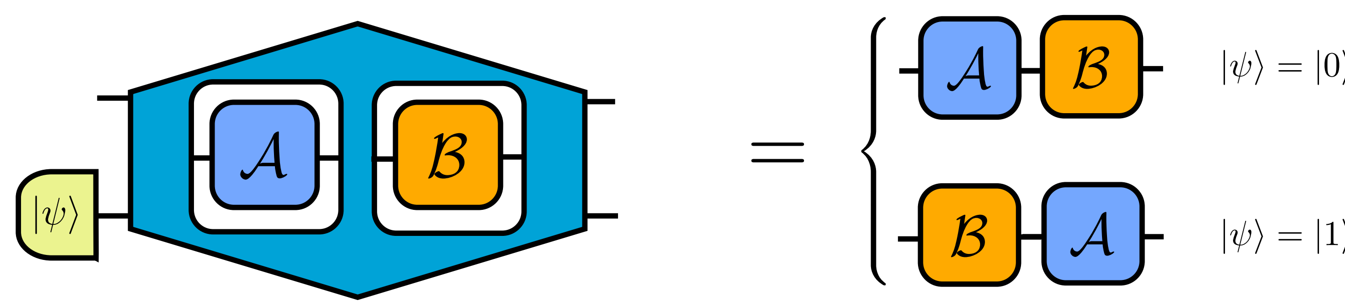
Not every higher order map
is a quantum circuit!



Quantum switch



G. Chiribella, G. M. D'Ariano P. Perinotti, B. Valiron,
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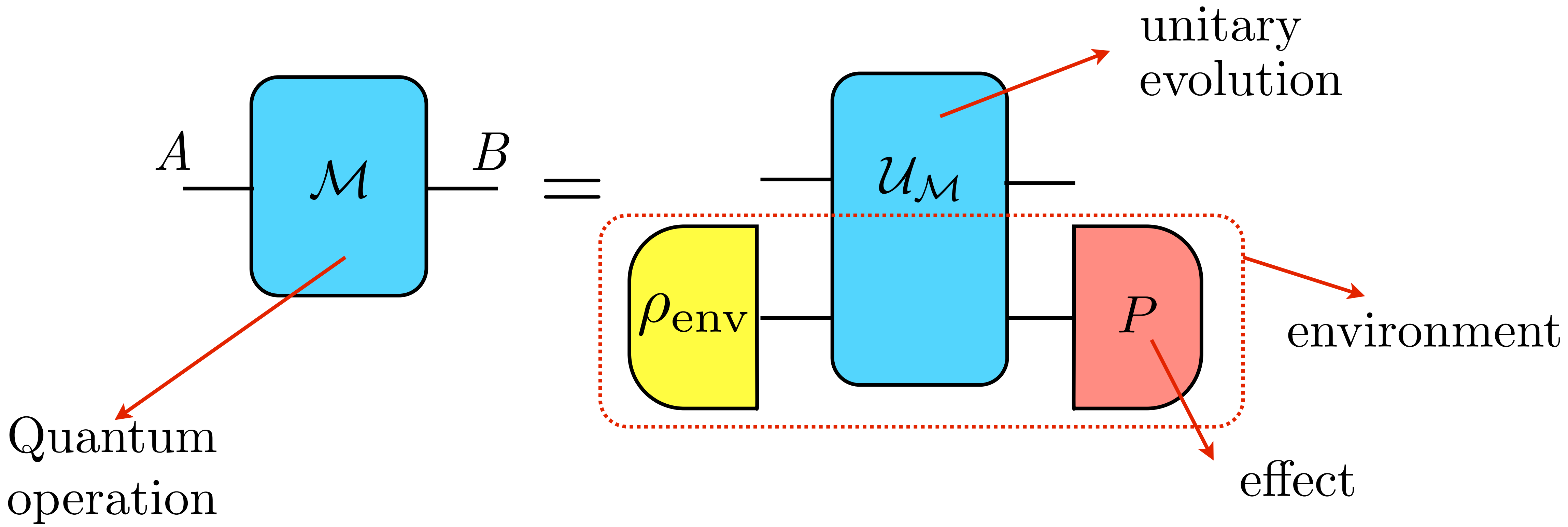
Realisation theorem (probabilistic)



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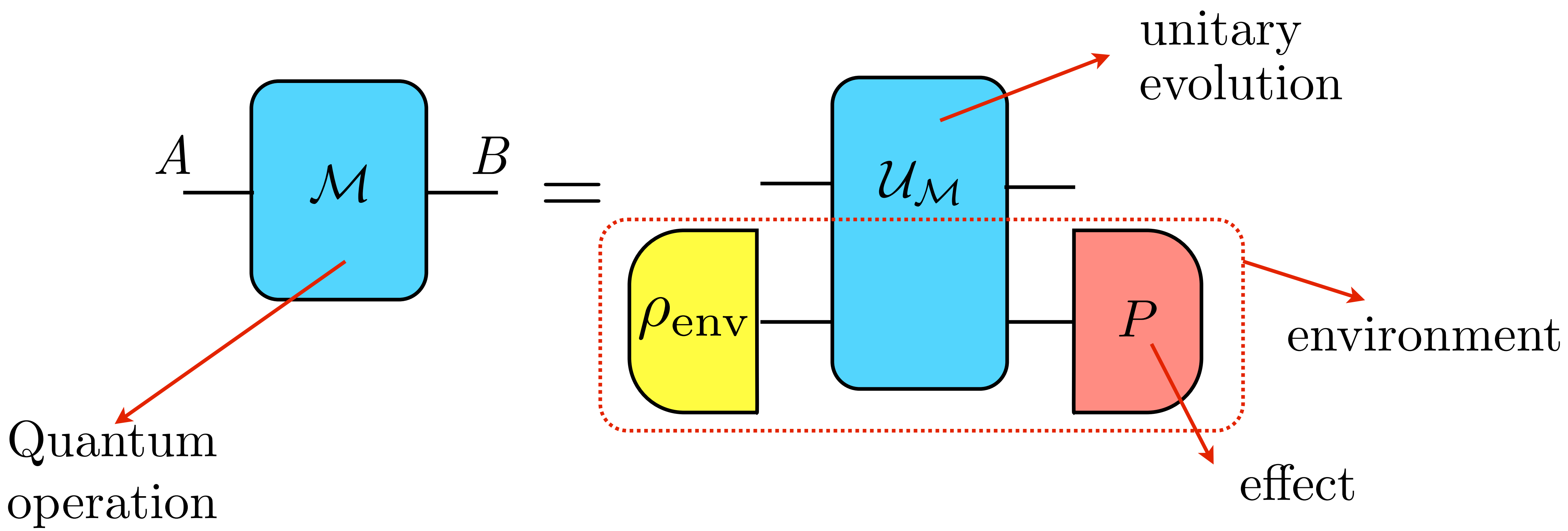

Deeper understanding of the probabilistic structure of Quantum Mechanics.