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Testing Dynamical Reduction Models at the Gran Sasso Underground Laboratory

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CENTRO FERMI

MUSEO STORICO DELLA FISICA E CENTRO STUDI E RICERCHE ENRICO FERMI

Is quantum theory exact?

The quest for the spin-statistics connection violation and related items

2-5 July 2018 *Laboratori Nazionali di Frascati INFN*
Europe/Rome timezone

Measurement problem

The linear nature of QM allows superposition of macro-object states → *Von Neumann measurement scheme* (A. Bassi, G. C. Ghirardi Phys. Rep 379 257 (2003))

If we assume the theory is complete .. two possible ways out

- Two dynamical principles: a) evolution governed by Schrödinger equation (unitary, linear) b) measurement process governed by WPR (stochastic, nonlinear). But .. where does quantum and classical behaviours split?

- Dynamical Reduction Models: non linear *and* stochastic modification of the Hamiltonian dynamics:

QMSL - particles experience spontaneous localizations around appropriate positions, at random times according to a Poisson distribution with $\lambda = 10^{-16}\ s^{-1}$.
(Ghirardi, Rimini, and Weber, Phys. Rev. D 34, 470 (1986); ibid. 36, 3287 (1987); Found. Phys. 18, 1 (1988))

CSL - stochastic and nonlinear terms in the Schrödinger equation induce diffusion process for the state vector → reduction.

CSL model

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} H\,dt + \sqrt{\lambda}\int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t) dW_t(\mathbf{x}) - \frac{\lambda}{2}\int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t)^2 dt \right] |\psi_t\rangle$$

System's Hamiltonian

NEW COLLAPSE TERMS → **New Physics**

$N(\mathbf{x}) = a^\dagger(\mathbf{x}) a(\mathbf{x})$ particle density operator — **choice of the preferred basis**

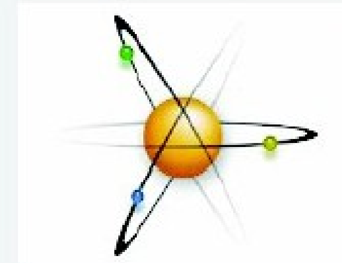
$\langle N(\mathbf{x})\rangle_t = \langle \psi_t | N(\mathbf{x}) | \psi_t \rangle$ — **nonlinearity**

$W_t(\mathbf{x}) =$ noise $\quad \mathbb{E}[W_t(\mathbf{x})] = 0, \quad \mathbb{E}[W_t(\mathbf{x}) W_s(\mathbf{y})] = \delta(t-s) e^{-(\alpha/4)(\mathbf{x}-\mathbf{y})^2}$ — **stochasticity**

$\lambda =$ collapse strength $\qquad r_C = 1/\sqrt{\alpha} =$ correlation length — **two parameters**

Which values for λ and r_c?

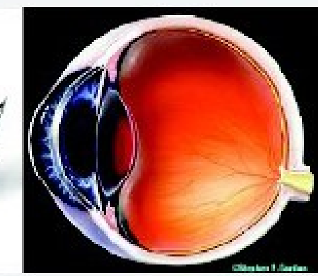
Microscopic world
(few particles)



$$\lambda \sim 10^{-8\pm 2}\mathrm{s}^{-1}$$

QUANTUM – CLASSICAL TRANSITION (Adler - 2007)

Mesoscopic world
Latent image formation
+
perception in the eye
(~ 10^4 - 10^5 particles)



S.L. Adler, JPA 40, 2935 (2007)

A. Bassi, D.A. Deckert & L. Ferialdi, EPL 92, 50006 (2010)

$$\lambda \sim 10^{-17}\mathrm{s}^{-1}$$

QUANTUM – CLASSICAL TRANSITION (GRW - 1986)

Macroscopic world
(> 10^{13} particles)

G.C. Ghirardi, A. Rimini and T. Weber, PRD 34, 470 (1986)

$$r_C = 1/\sqrt{\alpha} \sim 10^{-5}\mathrm{cm}$$

Increasing size of the system

Diosi – Penrose collapse model

- Wave function collapse induced by gravity:

When a system is in a quantum superposition of two different positions then a corresponding superposition of two
different space-times is generated, the superposition of the two bumps in space-time associated to
the two mass distributions.

Superpositions of different geometries would be suppressed.

- The more massive the superposition, the faster it is suppressed.

The model characteristic parameter:

R_0 - size of the wave function defining the mass distribution

… spontaneous photon emission

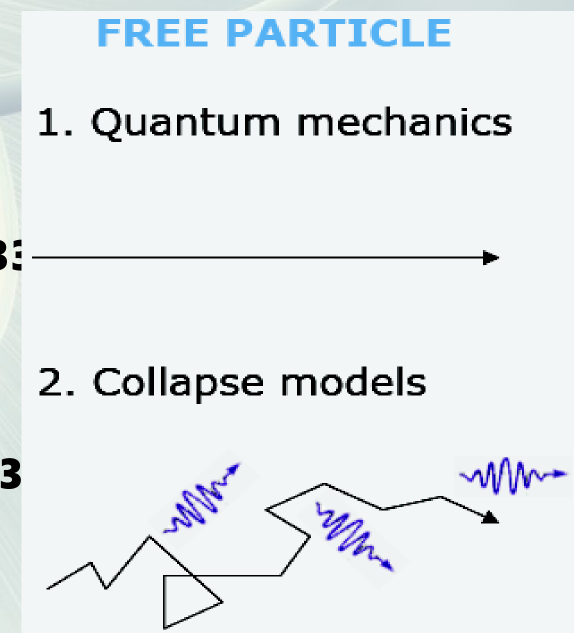
Besides collapsing the state vector to the position basis in non relativistic QM the interaction with the stochastic field increases the expectation value of particle's energy

↓

implies for a charged particle energy radiation (not present in standard QM)

1) **test of collapse models (ex. Karolyhazy model, collapse is induced by fluctuations in space-time → unreasonable amount of radiation in the X-ray range).**

2) **provides constraints on the parameters of the CSL model**



Q. Fu, Phys. Rev. A 56, 1806 (1997)
S. L. Adler and F. M. Ramazanoglu, J. Phys. A40, 13
J. Phys. A42, 109801 (2009)
S. L. Adler, A. Bassi and S. Donadi,
J. Phys. A46, 245304 (2013)
S. Donadi, D. A. Deckert and A. Bassi, Annals of Physics 3

First limit from Ge detector measurement

Q. Fu, Phys. Rev. A 56, 1806 (1997) → upper limit on λ comparing with the radiation measured with isolated slab of Ge (raw data not background subtracted)
H. S. Miley, et al., Phys. Rev. Lett. 65, 3092 (1990)

Energy (keV)	Expt. upper bound (counts/keV/kg/day)	Theory (counts/keV/kg/day)
11	0.049	0.071
101	0.031	0.0073
201	0.030	0.0037
301	0.024	0.0028
401	0.017	0.0019
501	0.014	0.0015

TABLE I. Experimental upper bounds and theoretical predictions of the spontaneous radiation by free electrons in Ge for a range of photon energy values.

Comparison with the lower energy bin, due to the *non-relativistic constraint* of the CSL model

$$\frac{d\Gamma(E)}{dE} = c\,\frac{e^2\lambda}{4\pi^2 r_C^2 m^2 E} = (4)\cdot(8.29\;10^{24})\cdot(8.64\;10^{4})\frac{e^2\lambda}{4\pi^2 r_C^2 m^2 E} \le \left.\frac{d\Gamma(E)}{dE}\right|_{ex}$$

4 valence electrons are considered
BE ~ 10 eV « energy of emitted γ ~ 11 keV
***quasi-free* electrons**

(Atoms / Kg) in Ge

1 day

S. L. Adler, F. M. Ramazanoglu, J. Phys. A40 (2007), 13395
J. Mullin, P. Pearle, Phys. Rev. A90 (2014), 052119

$\lambda < 2 \times 10^{-16}\ s^{-1}$ non-mass proportional
$\lambda < 8 \times 10^{-10}\ s^{-1}$ mass proportional

Improvement from IGEX data

***ADVANTAGES*:**

- IGEX low-activity Ge based experiment dedicated to the $\beta\beta0\nu$ decay research. (C. E. Aalseth et al., IGEX collaboration Phys. Rev. C 59, 2108 (1999))

- exposure of 80 *kg day* in the energy range: ΔE = (4 – 49) *keV* « m_e = 512 keV (A. Morales et al., IGEX collaboration Phys. Lett. B 532, 8-14 (2002)) → possibility to perform a fit,

***DISADVANTAGE*:**

- no simulation of the known background sources is available . . .

ASSUMPTION 1 - the upper limit on λ corresponds to the case in which all the measured X-ray emission would be produced by spontaneous emission processes

ASSUMPTION 2 - the detector efficiency in ΔE is one, muon veto and pulse shape analysis un-efficiencies are small above 4keV.

Increasing the number of emitting electrons

Consider the 30 outermost electrons emitting *quasi free* → we are confined to the experimental range: ΔE = (14 – 49) keV <u>fit is not more reliable</u> ...

$$\frac{d\Gamma(E)}{dE} = \frac{\alpha(\lambda)}{E}.$$

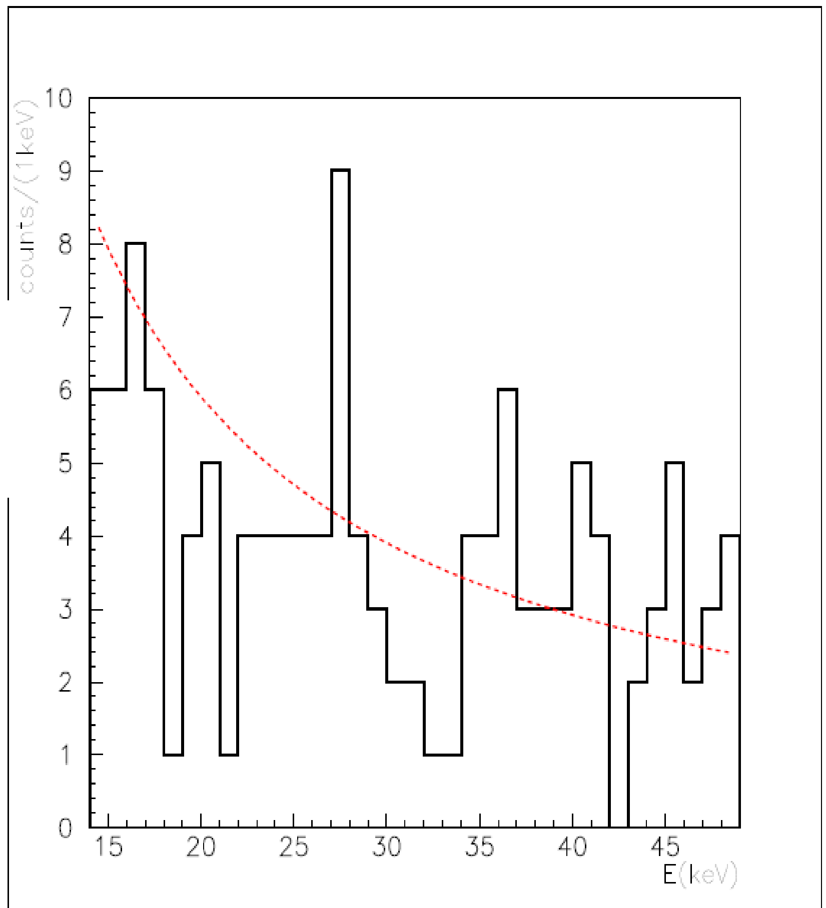


Figure 1. Fit of the X-ray emission spectrum measured by the IGEX experiment [14,15], using the theoretical fit function Equation (7). The black line corresponds to the experimental distribution; the red dashed line represents the fit. See the text for more details.

Increasing the number of emitting electrons

Consider the 30 outermost electrons emitting *quasi free* → we are confined to the experimental range: ΔE = (14 – 49) <u>fit is not more reliable</u> ...

let's extract the p. d. f. of λ:

experimental ingredient

$$G(y_i|P,\Lambda_i) = \frac{\Lambda_i^{y_i} e^{-\Lambda_i}}{y_i!}$$

$$y = \sum_{i=1}^{n} y_i \quad , \quad \Lambda = \sum_{i=1}^{n} \Lambda_i$$

theoretical ingredient

$$\Lambda(\lambda) = y_s + 1 = \sum_{i=1}^{n} c \frac{e^2 \lambda}{4\pi^2 r_C^2 m^2 E_i} + 1 = \sum_{i=1}^{n} \frac{\alpha(\lambda)}{E_i} + 1$$

Bayesian probability inversion

$$G'(\lambda|G(y|P,\Lambda)) \propto \left(\sum_{i=1}^{n} \frac{\alpha(\lambda)}{E_i} + 1\right)^y e^{-\left(\sum_{i=1}^{n} \frac{\alpha(\lambda)}{E_i} + 1\right)}$$

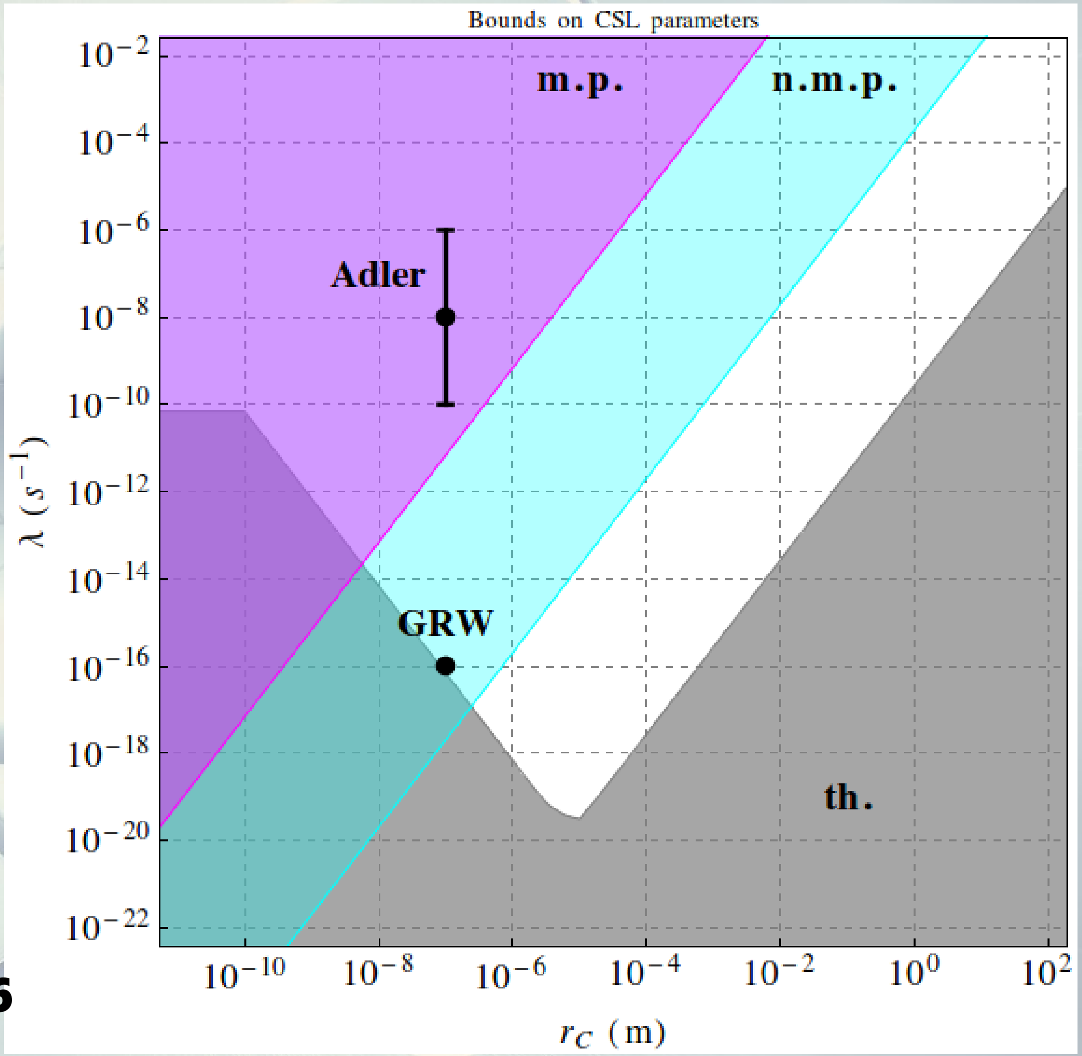
Upper limit on λ : $\int_0^{\lambda_0} G'(\lambda|G(y|P,\Lambda))\, d\lambda$

Further increasing the number of emitting electrons

$$\lambda \leq 6.8 \cdot 10^{-12} s^{-1} \quad \text{mass prop.,}$$

$$\lambda \leq 2.0 \cdot 10^{-18} s^{-1} \quad \text{non-mass prop..}$$

With probability 95%



K. Piscicchia et al., Entropy 2017, 19(7), 319http://www.mdpi.com/1099-4300/19/7/319

th. gray bound:

- M. Carlesso, A. Bassi, P. Falferi and A. Vinante, Phys. Rev. D 94, (2016) 124036

- M. Toroš and A. Bassi, https://arxiv.org/pdf/1601.03672.pdf

Applying the method to a dedicated experiment

…

unfolding the BKG contribution from known emission processes.

The setup

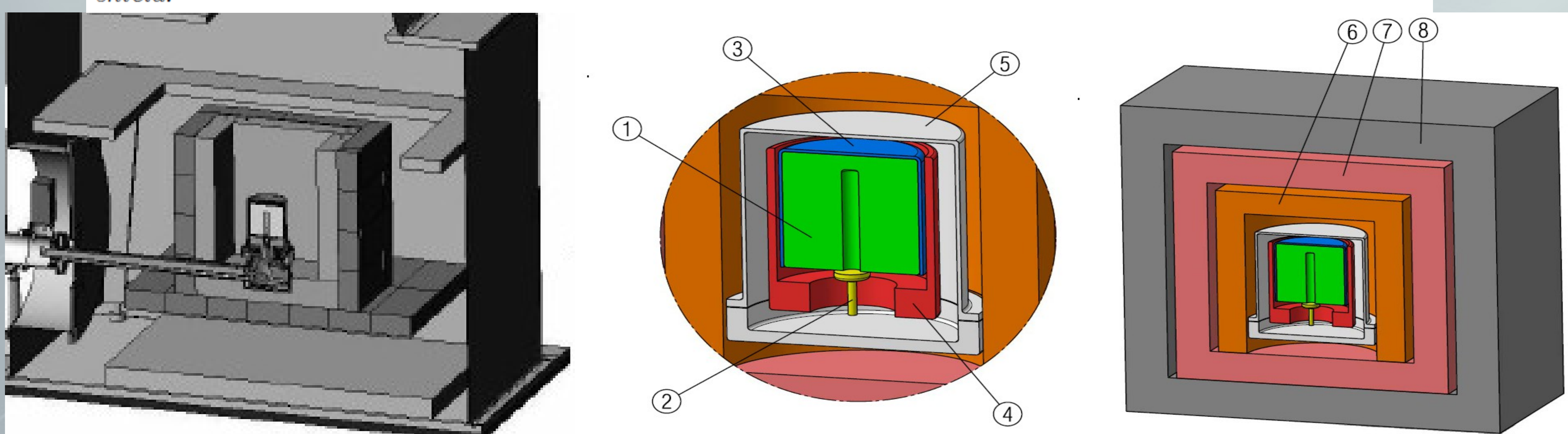
High purity Ge detector measurement:

- **active Ge detector surrounded by complex electrolytic Cu + Pb shielding**

- **10B-polyethylene plates reduce the neutron flux towards the detector**

- **shield + cryostat enclosed in air tight steel housing flushed with nitrogen to avoid contact with external air (and thus radon).**

FIG. 1: *Schematic representation of the experimental setup: 1 - Ge crystal, 2 - Electric contact, 3 - Plastic isolator, 4 - Copper cup, 5 - Copper end-cup, 6 - Copper block + plate, 7 Inner Copper shield, 8 - Lead shield.*



p. d. f. of λ
theoretical information

Goal: obtain the probability distribution function PDF(λ) of the collapse rate parameter given:

- the **theoretical information**

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University and INFN of Trieste

$$\frac{d\Gamma}{dE} = \left\{ \left(N_p^2 + N_e \right) \cdot \left(m\, n\, T \right) \right\} \frac{\lambda \hbar e^2}{4\pi^2 \epsilon_0 c^3 m_N^2 r_c^2 E}$$

Rate of spontaneously emitted photons as a consequence of *p* and *e* interaction with the stochastic field,

(depending on λ)

as a function of *E*

(mass of the emitting material • number of atoms per unit mass • total acquisition time)

p. d. f. of λ
theoretical information

Goal: obtain the probability distribution function PDF(λ) of the collapse rate parameter given:

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$$\frac{d\Gamma}{dE} = \left\{ \left(N_p^2 + N_e \right) \cdot \left(m\, n\, T \right) \right\} \frac{\lambda \hbar e^2}{4\pi^2 \epsilon_0 c^3 m_N^2 r_c^2 E}$$

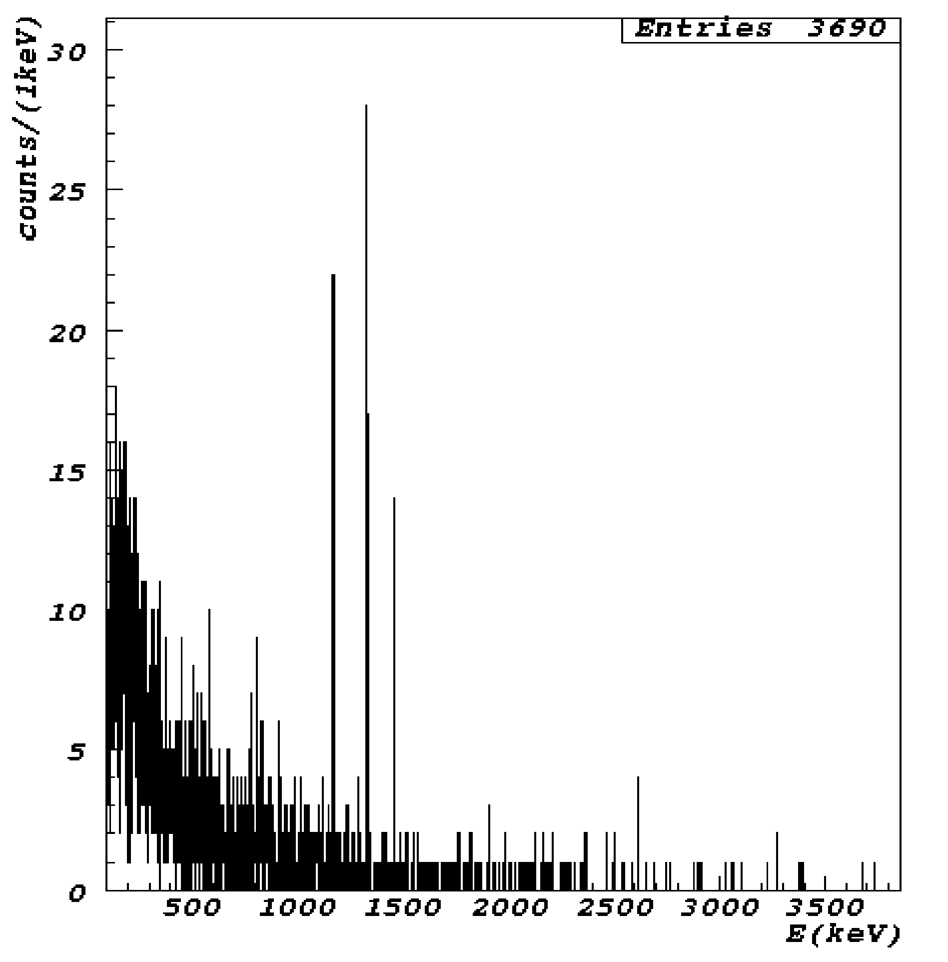
Provided that the wavelength of the emitted photon:

- **is greater then the nuclear dimensions → protons contribute coherently**
- **is smaller then the lower electronic orbit → protons and electrons emit independently**
- **electrons and protons can be considered as non-relativistic.**

p. d. f. of λ
experimental information

Goal: obtain the probability distribution function PDF(λ) of the collapse rate parameter given:

- **the experimental information**



low background environment of the LNGS (INFN)

low activity Ge detectors. (three months data taking with 2kg germanium active mass)

protons emission is considered in ΔE=(1000-3800)keV.

For lower energies residual cosmic rays and Compton in the outer lead shield complex MC staff.

p. d. f. of λ
experimental information

Goal: obtain the probability distribution function PDF(λ) of the collapse rate parameter given:

- the experimental information

total number of counts in the selected energy range:

$$f(z_c) = \frac{\Lambda_c^{z_c} e^{-\Lambda_c}}{z_c!}$$

from MC of the detector →

from theory weighted by detector efficiency →

- z_b = number of counts due to background,
- z_s = number of counts due to signal,
- $z_c = z_b + z_s$; $z_s \sim P_{\Lambda_s}$; $z_b \sim P_{\Lambda_b}$,

$$f(\lambda|\mathrm{ex},\mathrm{th}) = \frac{(\Lambda_s(\lambda) + \Lambda_b)^{z_c} \cdot e^{-(\Lambda_s(\lambda)+\Lambda_b)}}{z_c!} \qquad \lambda < 10^{-6}\mathrm{s}^{-1}$$

Advantages .. - possibility to extract unambiguous limits corresponding to the probability level you prefer,

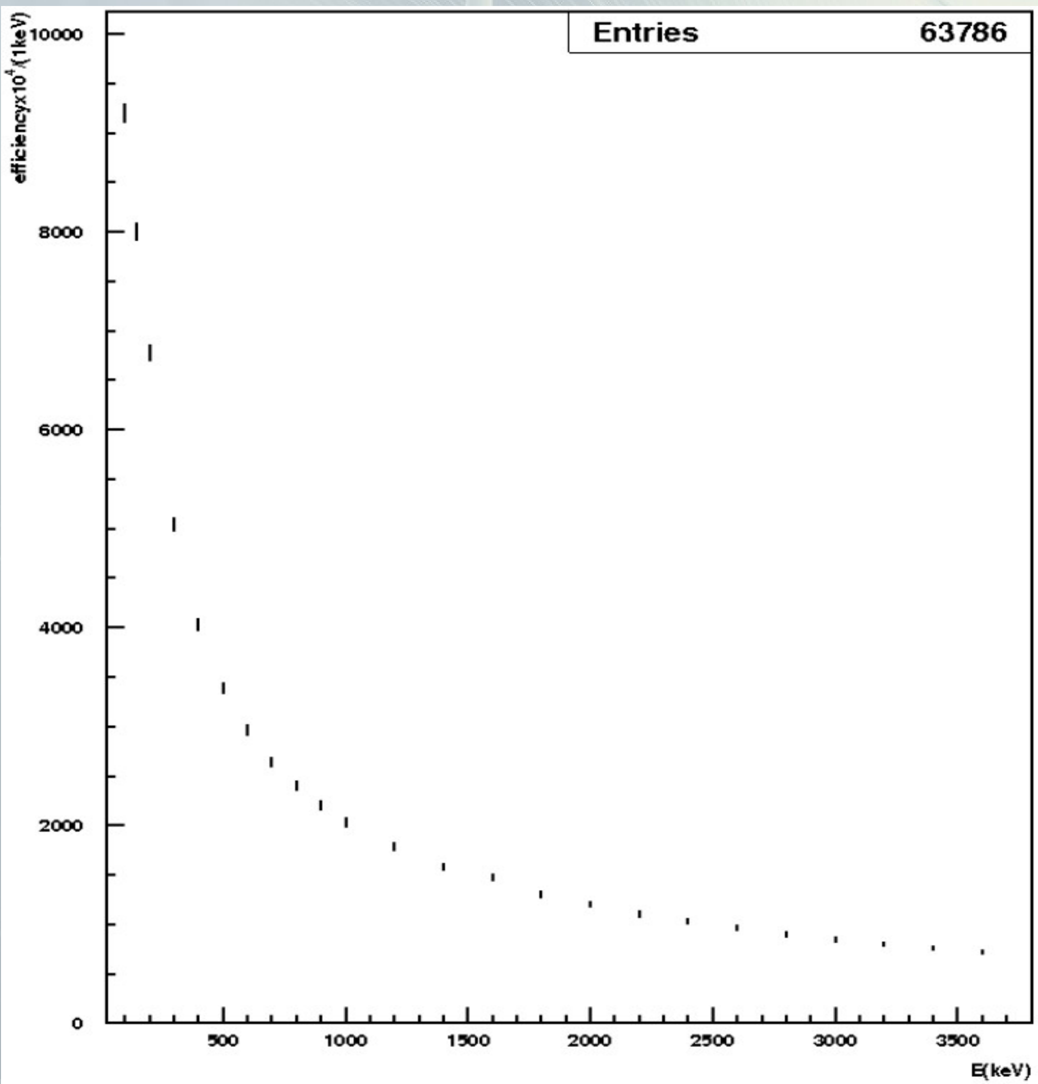
- $f(\lambda)$ can be updated with all the experimental information at your disposal by updating the likelihood,

- competing or future models can be simply implemented

Expected spontaneous emission signal

Each material spontaneously emits with different ***masses , densities*** **and ε(*E*)**

(depending on the material and the geometry of the detector)



Simulated detection eciency for γs produced inside the Germanium detector, multiplied by 10^4

Photon detection efficiencies obtained by means of MC simulations, ganerating γs in the range (E1 – E2) (25 points for each material).

The detector components have been put into a validated MC code
(MaGe, Boswell et al., 2011)
Based on the GEANT4 software library
(Agostinelli et al., 2003)

Expected spontaneous emission signal

Expected signal is obtained by weighting for the detection efficiencies

efficiency distributions fitted to obtain the efficiency functions:

$$\epsilon_i(E) = \sum_{j=0}^{ci} \xi_{ij} E^j$$

to obtain the signal predicted by theory & processed by the detector

$$z_s(\lambda) = \sum_i \int_{E_1}^{E_2} \left.\frac{d\Gamma}{dE}\right|_i \epsilon_i(E)\, dE =$$

$$= \sum_i \int_{E_1}^{E_2} N_{pi}^2\, \alpha_i\, \beta\, \frac{\lambda}{E} \sum_{j=0}^{ci} \xi_{ij} E^j\, dE$$

with:

$$\alpha_i = m_i n_i T,$$

$$\beta = \frac{\hbar e^2}{4\pi^2 \epsilon_0 c^3 m_N^2 r_c^2}$$

Expected BKG

radionuclides decay simulation accounts for:

- emission probabilities & decay scheme of each radionuclide
- photons propagation and interactions inside the materials of the detector
- detection efficiency,

Considered contributions:

- Co60 from the inner Copper
- Co60 from the Copper block + plate
- Co58 from the Copper block + plate
- K40 from Bronze
- Ra226 from Bronze
- Bi214 from Bronze
- Pb214 from Bronze
- Bi212 from Bronze
- Pb212 from Bronze
- Tl208 from Bronze
- Ra226 from Poliethylene
- Bi214 from Poliethylene
- Pb214 from Poliethylene

Expected BKG

radionuclides decay simulation accounts for:

- **emission probabilities & decay scheme of each radionuclide**
- **photons propagation and interactions inside the materials of the detector**
- **detection efficiency,**

Considered contributions:

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- Ra226 from Bronze
- Bi214 from Bronze
- Pb214 from Bronze
- Bi212 from Bronze
- Pb212 from Bronze
- Tl208 from Bronze
- Ra226 from Poliethylene
- Bi214 from Poliethylene
- Pb214 from Poliethylene

measured activities

$$z_{b,ij} = \frac{m_i \, A_{ij} \, T \, N_{rec,ij}}{N_{ik}}$$

detected MC γs

 # simulated events

Expected number of background counts

$$\Lambda_b = z_b + 1$$

Presently we can describe 88% of the measured spectrum

Upper limit for the collapse rate parameter λ

- From the p.d.f we obtain the cumulative distribution function:

$$F(\lambda) = \frac{\int_0^{\lambda} f(\lambda|\mathrm{ex},\mathrm{th})\mathrm{d}\lambda}{\int_0^{\infty} f(\lambda|\mathrm{ex},\mathrm{th})\mathrm{d}\lambda} = \frac{\int_0^{\lambda} \frac{1}{z_c!}(a\lambda + \Lambda_b + 1)^{z_c} e^{-(a\lambda+\Lambda_b+1)}\mathrm{d}\lambda}{\int_0^{\infty} \frac{1}{z_c!}(a\lambda + \Lambda_b + 1)^{z_c} e^{-(a\lambda+\Lambda_b+1)}\mathrm{d}\lambda}$$

which we express in terms of upper incomplete gamma functions

$$F(\lambda) = 1 - \frac{\Gamma(z_c + 1, a\lambda + 1 + \Lambda_b)}{\Gamma(z_c + 1, 1 + \Lambda_b)}$$

- put the measured z_c and the calculated $\Lambda_s(\lambda) = a\lambda + 1$, Λ_b in the cumulative distribution function

Preliminary

extract the limit at the desired probability level ...

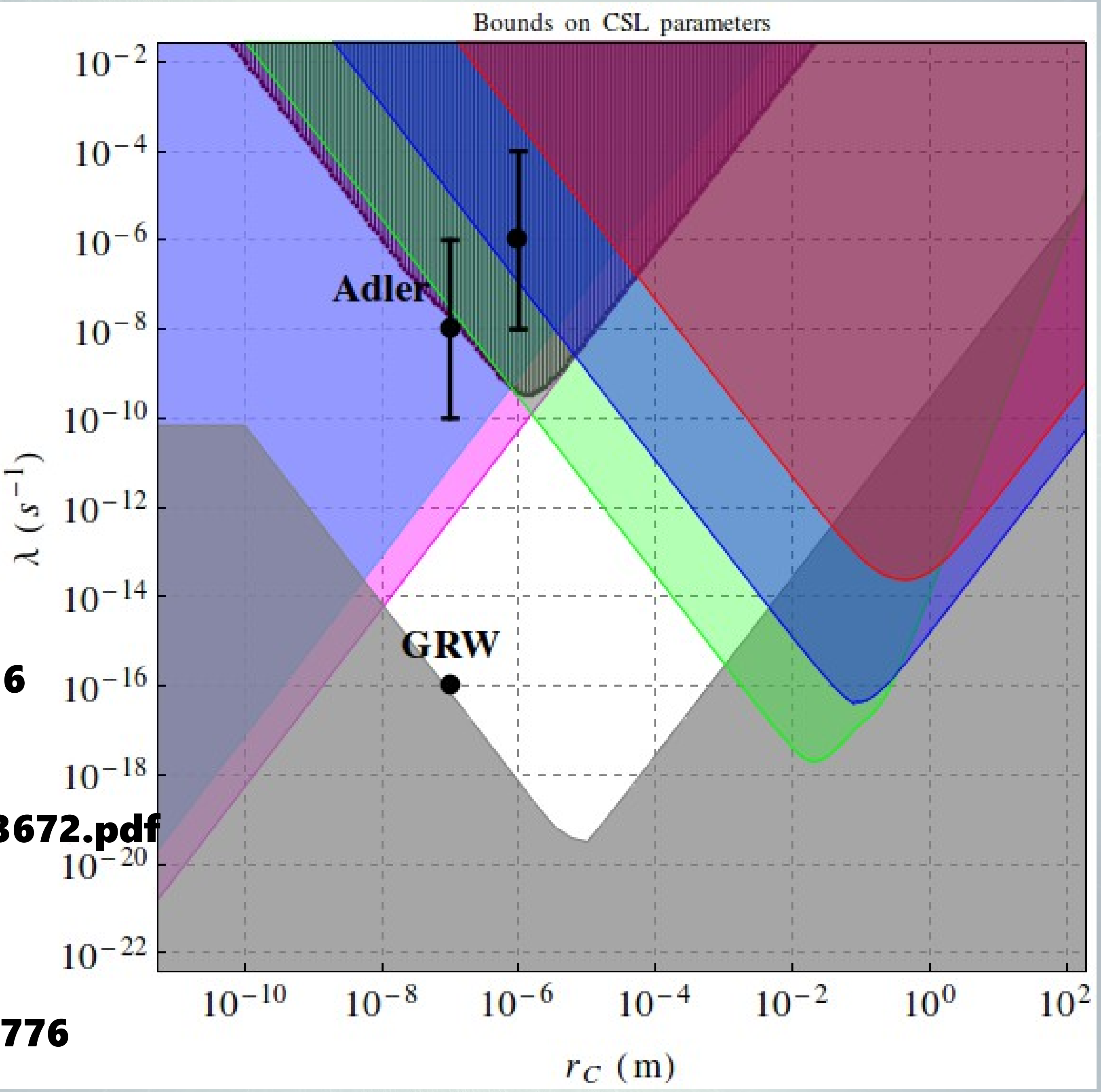
$\lambda < 5{,}2 \cdot 10^{-13}$ with a probability of 95%

Gain factor ~ 13

Upper limit for the collapse rate parameter λ

$\lambda < 5{,}3 \cdot 10^{-13}$ with a probability of 95%

Preliminary



See also

- M. Carlesso, A. Bassi, P. Falferi and A. Vinante, Phys. Rev. D 94, (2016) 124036
- M. Toroš and A. Bassi, https://arxiv.org/pdf/1601.03672.pdf
- Nanomechanical Cantilever Vinante, Mezzena, Falferi, Carlesso, Bassi, ArXiv 1611.09776

Upper limit on the parameter R_0

- **The total number of expected counts has now a non-linar dependence on the parameter R_0 :**

$$\Lambda_c = z_s + z_b + 1 = \frac{a}{R^3} + z_b + 1,$$

- **Again we apply probability inversion to obtain the searched pdf(R_0):**

$$\tilde{p}\left(R_0|p(z_c|\Lambda_c)\right) = \frac{p(z_c|\Lambda_c)\cdot\tilde{p}_0(R_0)}{\int_D p(z_c|\Lambda_c)\cdot\tilde{p}_0(R_0)dR_0}, \quad \tilde{p}_0(R_0) = \begin{cases} 0, & \text{if } R_0 < 10^{-14} = R_0^{min} \\ \text{const}, & \text{otherwise} \end{cases}$$

prior imposed by the model

$$\tilde{p}\left(R_0|p(z_c|\Lambda_c)\right) = \frac{\left(\frac{a}{R_0^3} + z_b + 1\right)^{z_c}\cdot e^{-\left(\frac{a}{R_0^3}+z_b+1\right)}\cdot\theta(R_0 - R_0^{min})}{\int_D \left(\frac{a}{R_0^3} + z_b + 1\right)^{z_c}\cdot e^{-\left(\frac{a}{R_0^3}+z_b+1\right)}\cdot\theta(R_0 - R_0^{min})dR_0}.$$

- **From which the cumulative pdf assumes the expression:**

$$\tilde{P}(\bar{R}_0) = \frac{\sum_{k=0}^{z_c}\binom{z_c}{k}b^{z_c-k}\int_{\frac{a}{\bar{R}_0^3}}^{\frac{a}{(R_0^{min})^3}} y^{k-\frac{4}{3}}e^{-y}dy}{\sum_{k=0}^{z_c}\binom{z_c}{k}b^{z_c-k}\int_0^{\frac{a}{(R_0^{min})^3}} y^{k-\frac{4}{3}}e^{-y}dy},$$

Upper limit on the parameter R_0

- **Cumulative pdf to be handled with care :**

$$\tilde{P}(\bar{R}_0) = \frac{\sum_{k=0}^{z_c} \binom{z_c}{k} b^{z_c-k} \int_{\frac{a}{\bar{R}_0^3}}^{\frac{a}{(R_0^{min})^3}} y^{k-\frac{4}{3}} e^{-y} dy}{\sum_{k=0}^{z_c} \binom{z_c}{k} b^{z_c-k} \int_{0}^{\frac{a}{(R_0^{min})^3}} y^{k-\frac{4}{3}} e^{-y} dy},$$

$$\tilde{P}(\bar{R}_0) = \frac{\sum_{k=0}^{z_c} \binom{z_c}{k} b^{z_c-k} \left[\Gamma\left(k-\frac{1}{3}, \frac{a}{\bar{R}_0^3}\right) - \Gamma\left(k-\frac{1}{3}, \frac{a}{(R_0^{min})^3}\right)\right]}{\sum_{k=0}^{z_c} \binom{z_c}{k} b^{z_c-k} \left[\Gamma\left(k-\frac{1}{3}\right) - \Gamma\left(k-\frac{1}{3}, \frac{a}{(R_0^{min})^3}\right)\right]}.$$

$$\tilde{P}(\bar{R}_0) = \frac{\sum_{k=0}^{z_c} \binom{z_c}{k} b^{z_c-k} \left[\sum_{n=0}^{\infty} \frac{-1^n}{n!} \frac{\left(\frac{a}{(R_0^{min})}^{n+k-1/3}\right) - \left(\frac{a}{(\bar{R}_0)}^{n+k-1/3}\right)}{n+k-1/3}\right]}{\sum_{k=0}^{z_c} \binom{z_c}{k} b^{z_c-k} \left[\sum_{n=0}^{\infty} \frac{-1^n}{n!} \frac{\left(\frac{a}{(R_0^{min})}^{n+k-1/3}\right) - \left(\frac{a}{(\bar{R}_0)}^{n+k-1/3}\right)}{n+k-1/3}\right]}.$$

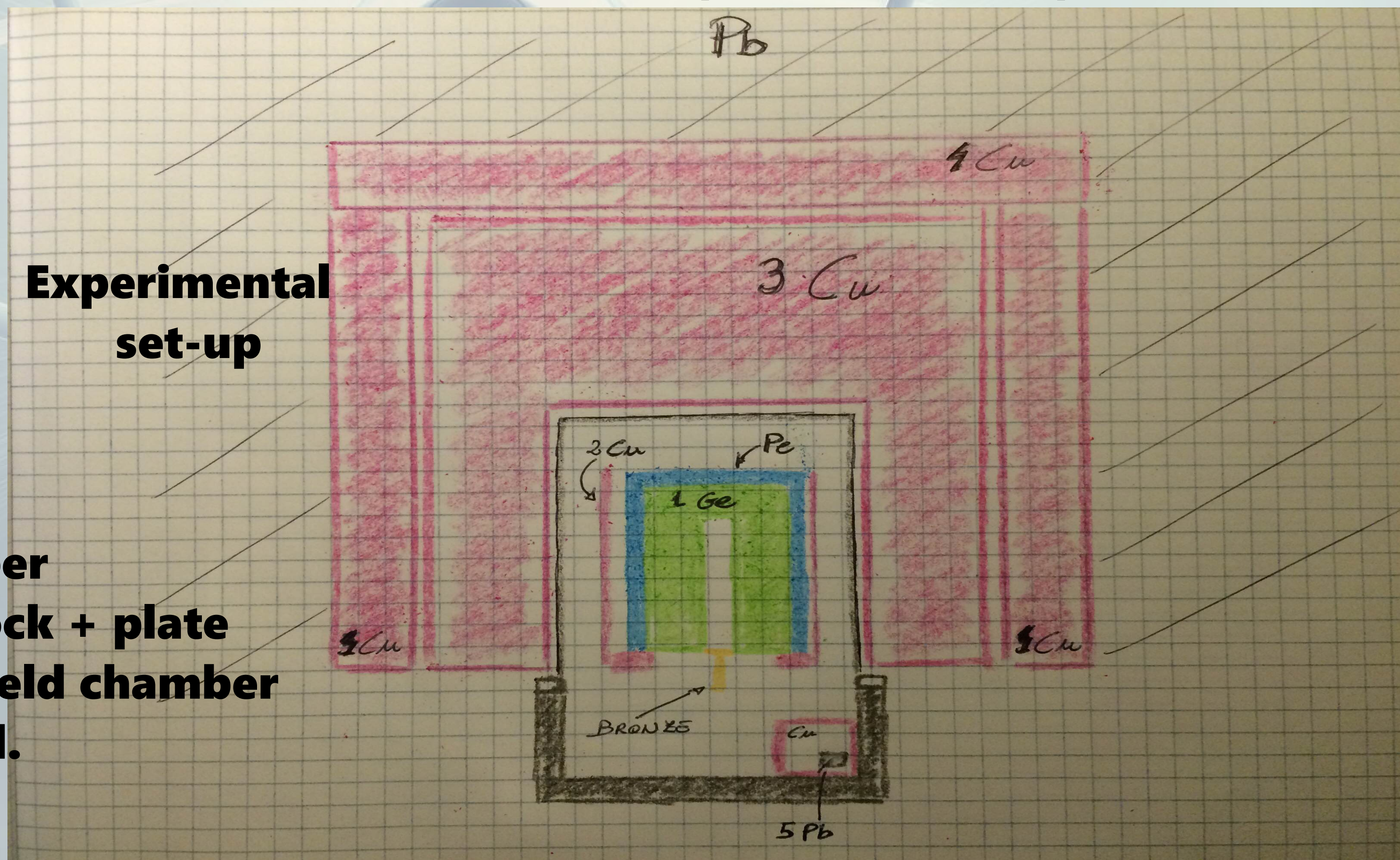
- **<u>work in progress,</u> expected limit : $R_0 > 10^{-11} - 10^{-10}$**


Thanks
:)

The setup

High purity Ge detector measurement collaboration with M. Laubenstein @ LNGS (INFN):

- **active Ge detector surrounded by complex electrolytic Cu + Pb shielding**
- **polyethylene plates reduce the neutron flux towards the detector**
- **shield + cryostat enclosed in air tight steel housing flushed with nitrogen to avoid contact with external air (and thus radon).**



Experimental set-up

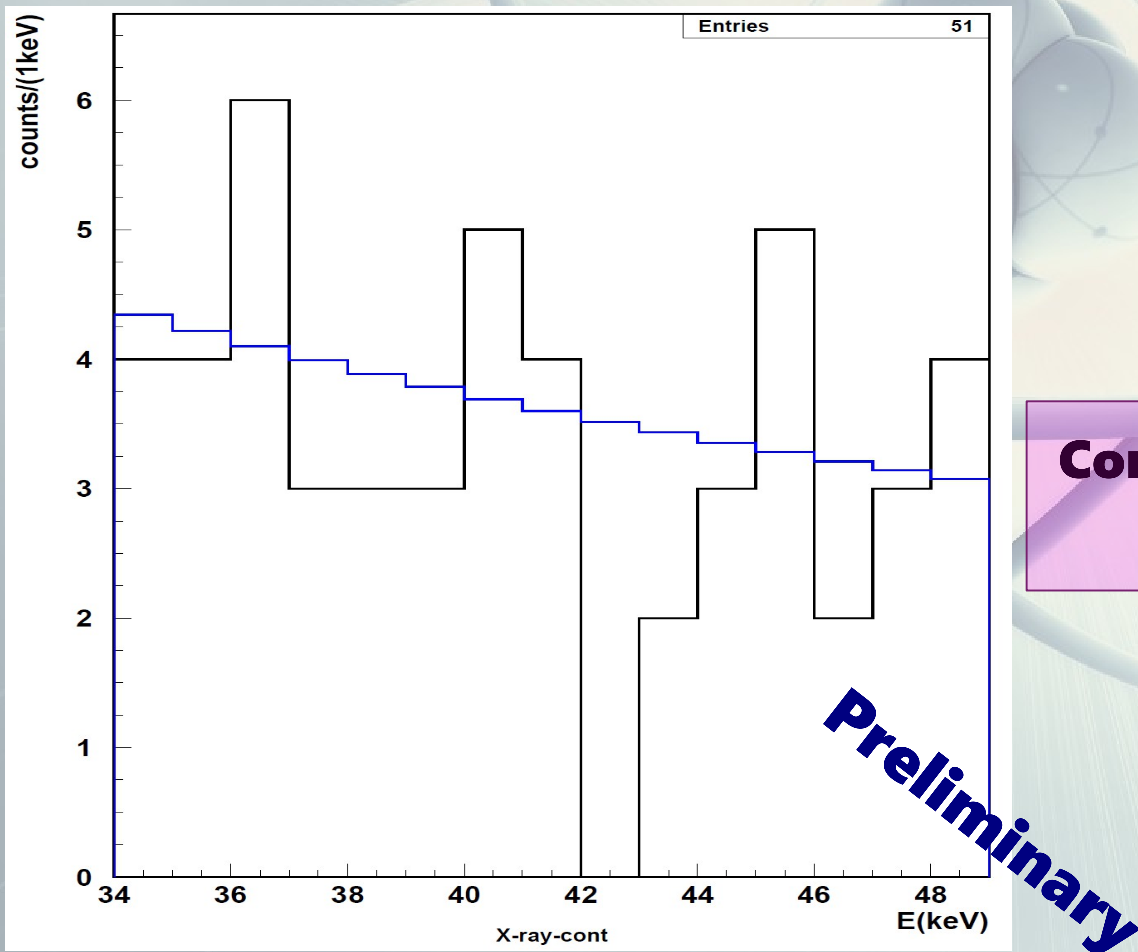
1 = Ge crystal
2 = inner Copper
3 = Copper block + plate
4 = Copper shield chamber
5 = Lead shield.

Spontaneous emission including nuclear protons

The interval ΔE = (35 – 49) *keV* of the IGEX measured X-ray spectrum was fitted assuming the predicted energy dependence:

$$\frac{d\Gamma_k}{dk} = \frac{\alpha(\lambda)}{k}$$

Bayesian fit with $\alpha(\lambda)$ free parameter.



Fit result:

$\alpha(\lambda) = 148 \pm 21$

χ^2/ n.d.f. = 0.8

Corresponding to the limit on the spontaneous emission rate:

$\lambda < 2.7 \times 10^{-13}\ s^{-1}$

Mass-proportional

3 O. M. improovement

Spontaneous emission including nuclear protons

When the emission of nuclear protons is also considered, the spontaneous emission rate is:

A. Bassi & S. Donadi

$$\frac{d\Gamma_k}{dk} = \left(N_P^2 + N_e\right) \frac{e^2 \lambda}{4\pi^2 a^2 m_N^2 k}$$

provided that the emitted photon wavelength λ_{ph} satisfies the following conditions:

1) $\lambda_{ph} > 10^{-15}$ m (nuclear dimension) → protons contribute coherently

2) $\lambda_{ph} <$ (electronic orbit radius) → electrons and protons emit independently → NO cancellation

We consider in the calculation the 30 outermost electrons (down to 2s orbit) $r_e = 4 \times 10^{-10}$ m and take only the measured rate for $k > 35$ keV

Moreover BE_{2s} = 1.4 keV « k_{min} → electrons can be considered as *quasi-free*

Probability distribution function of λ
experimental information

Goal: obtain the probability distribution function PDF(λ) of the collapse rate parameter given:

- the experimental information

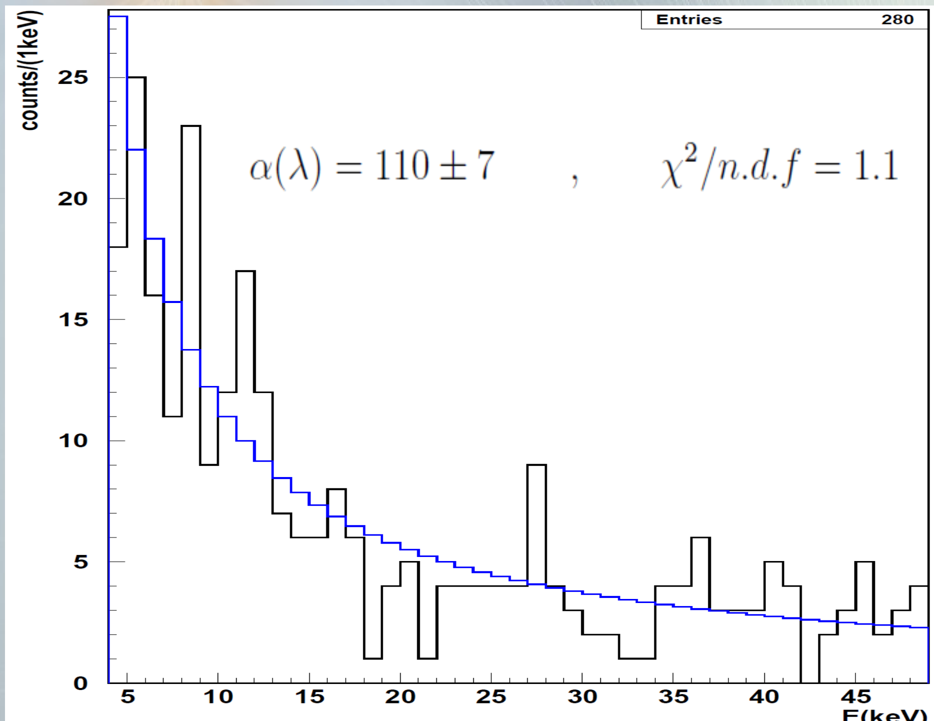
total number of counts in the selected energy range:

from MC of the detector →
- z_b = number of counts due to background,

from theory weighted by detector efficiency →
- z_s = number of counts due to signal,
- $z_c = z_b + z_s$; $z_s \sim P_{\Lambda_s}$; $z_b \sim P_{\Lambda_b}$,

$$f(z_c|P_{\lambda s}, P_{\lambda b}) = \sum_{z_s, z_b} \delta_{z_c, z_s+z_b}\, f(z_s|P_{\lambda s})\, f(z_b|P_{\lambda b}) = \frac{(\Lambda_s + \Lambda_b)^{z_s+z_b}\, e^{(\Lambda_s+\Lambda_b)}}{z_c!}$$

Improvement from IGEX data



Spectrum fitted with energy dependence:

$$\frac{d\Gamma_k}{dk} = \frac{\alpha(\lambda)}{k}$$

bin contents are treated with Poisson statistics.

Taking the 22 outer electrons (down to the 3s orbit BE_{3s} = 180.1 eV) in the calculation

(assume $r_c = 10^{-7}$ m) ...

2 O. M. improovement

$\lambda < 2.5 \times 10^{-18}\ s^{-1}$	**$\lambda < 8.5 \times 10^{-12}\ s^{-1}$**
No mass-proportional	mass-proportional

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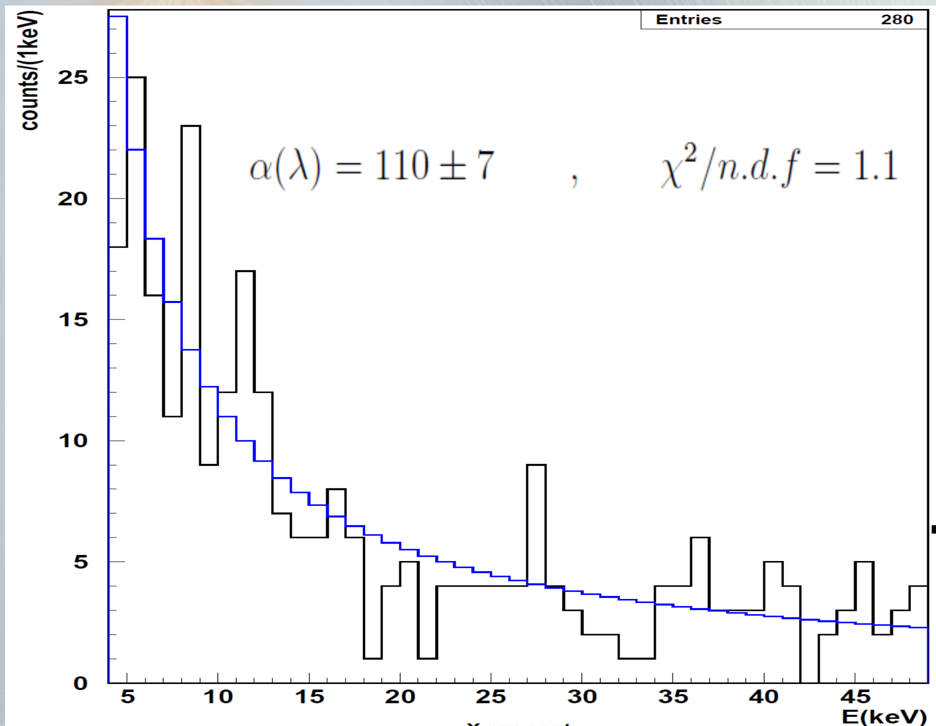
Improvement from IGEX data



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2 O. M. improovement

$\lambda < 2.5 \times 10^{-18}$ s^{-1} No mass-proportional	**$\lambda < 8.5 \times 10^{-12}$ s^{-1}** mass-proportional

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- **No mass-proportional model excluded (for white noise, $r_c = 10^{-7}$ m)**
- **Adler's value excluded even in the mass-proportional case (for white noise, $r_c = 10^{-7}$ m)**