No-hypersignaling principle

Michele Dall'Arno, Sarah Brandsen, AT, Francesco Buscemi, Vlatko Vedral

No-hypersignaling principle, Phys. Rev. Lett. 119, 020401 (2017)

Alessandro Tosini, QUIT group, Pavia University

2-5 July 2018 Laboratori Nazionali di Frascati INFN, Italy

Scenario

Understanding the mathematical structure of quantum theory

→ derivation from "intuitive" principles (e.g. informational)

→ separate quantum theory from other possible theories

Scenario

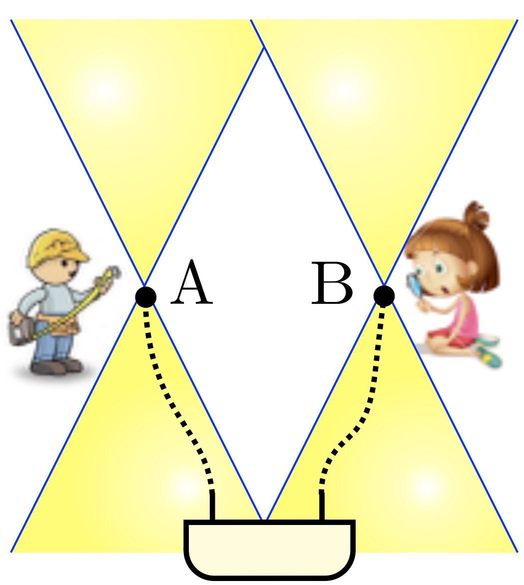
Understanding the mathematical structure of quantum theory

derivation from “intuitive” principles (e.g. informational)

separate quantum theory from other possible theories

Typical case

Characterise quantum space-like correlations (Bell-like)



Scenario

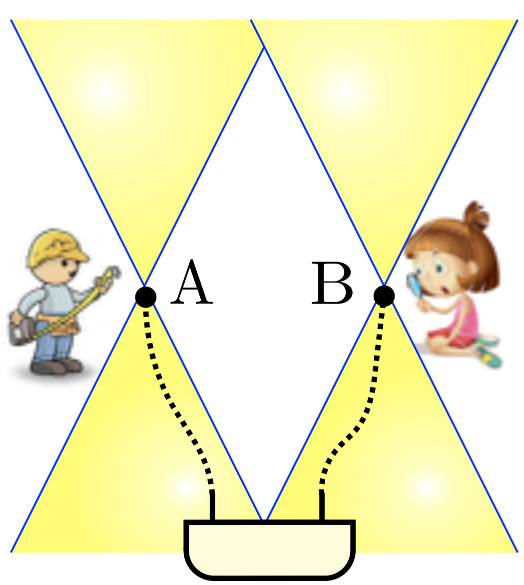
Understanding the mathematical structure of quantum theory

→ derivation from "intuitive" principles (e.g. informational)

→ separate quantum theory from other possible theories

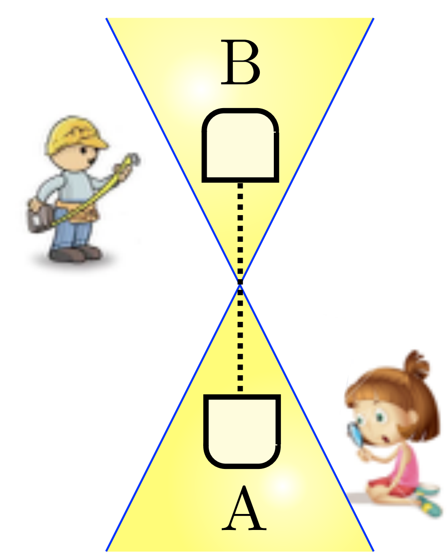
Typical case

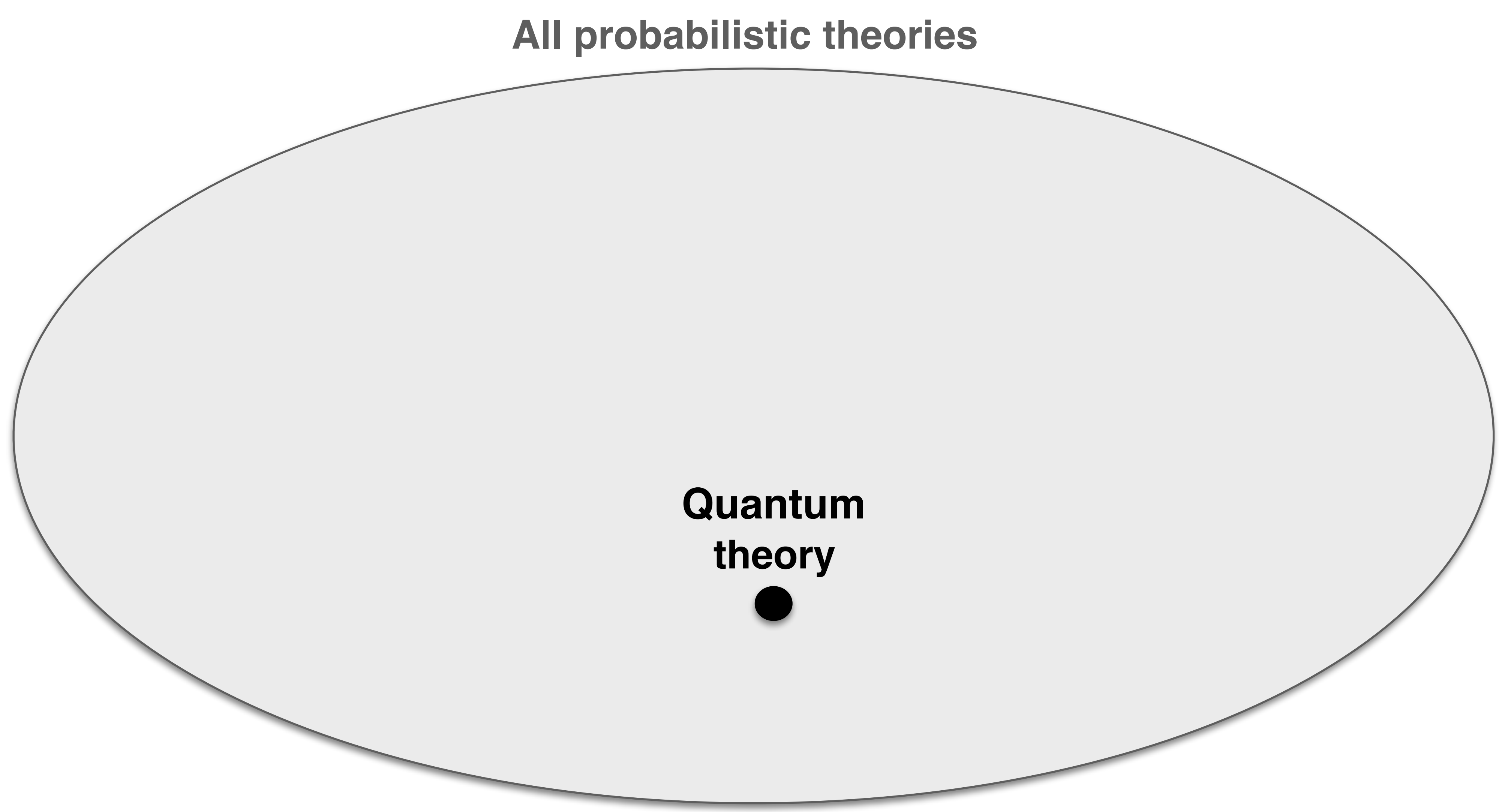
Characterise quantum space-like correlations (Bell-like)



In this talk

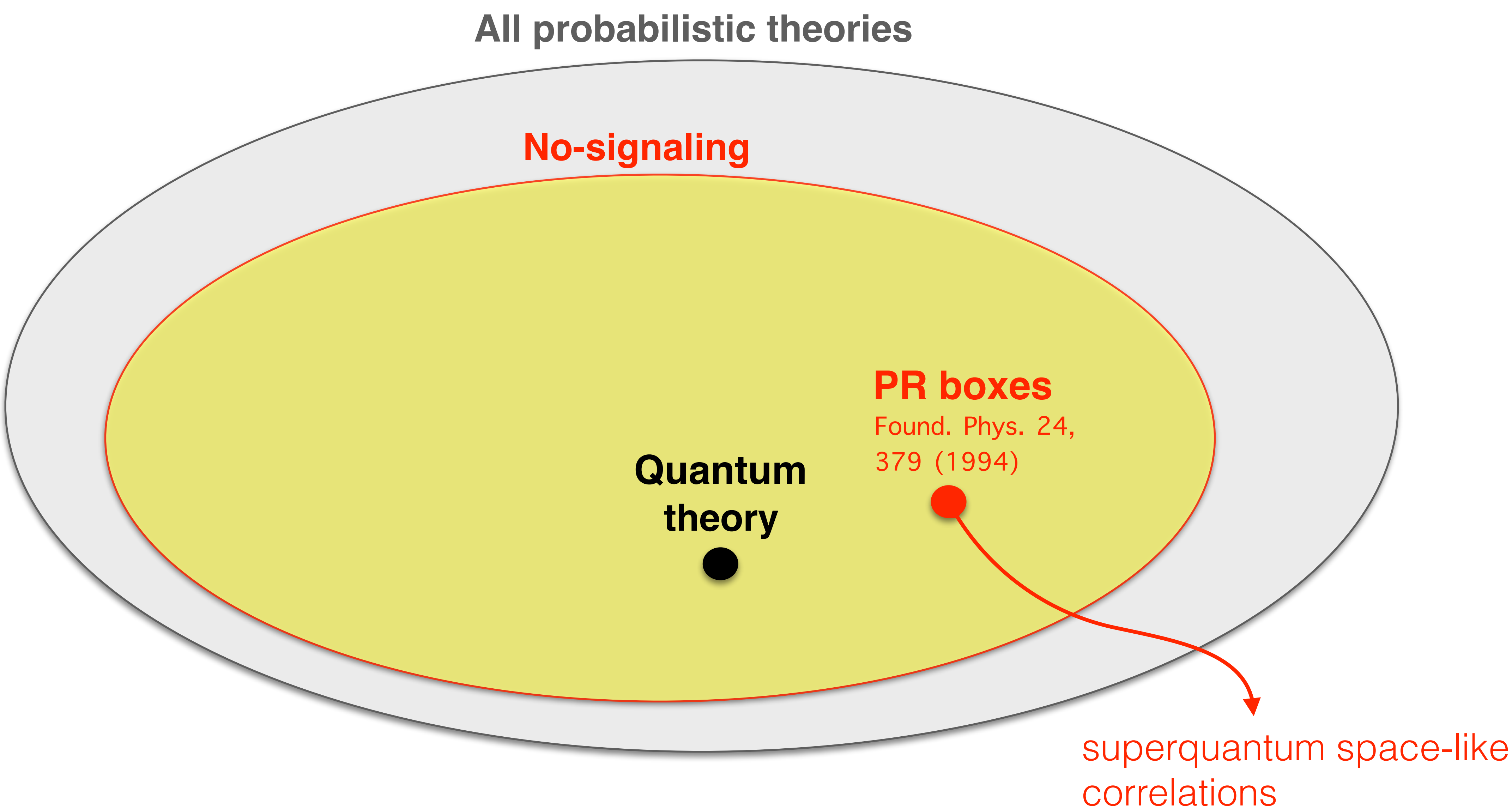
Characterise quantum time-like correlations




All probabilistic theories
Quantum
theory

Constraints on spacelike correlations

no-signaling

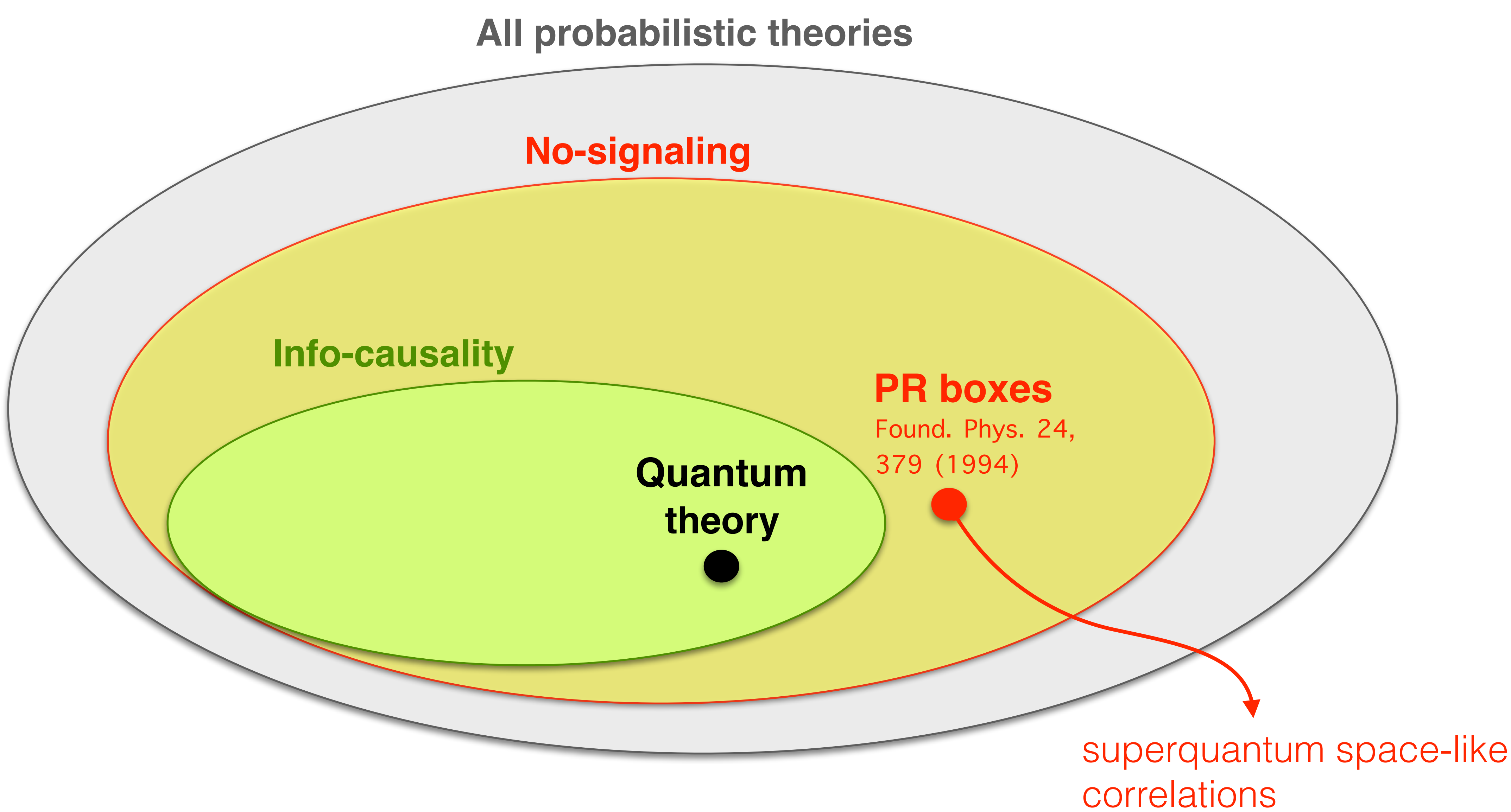


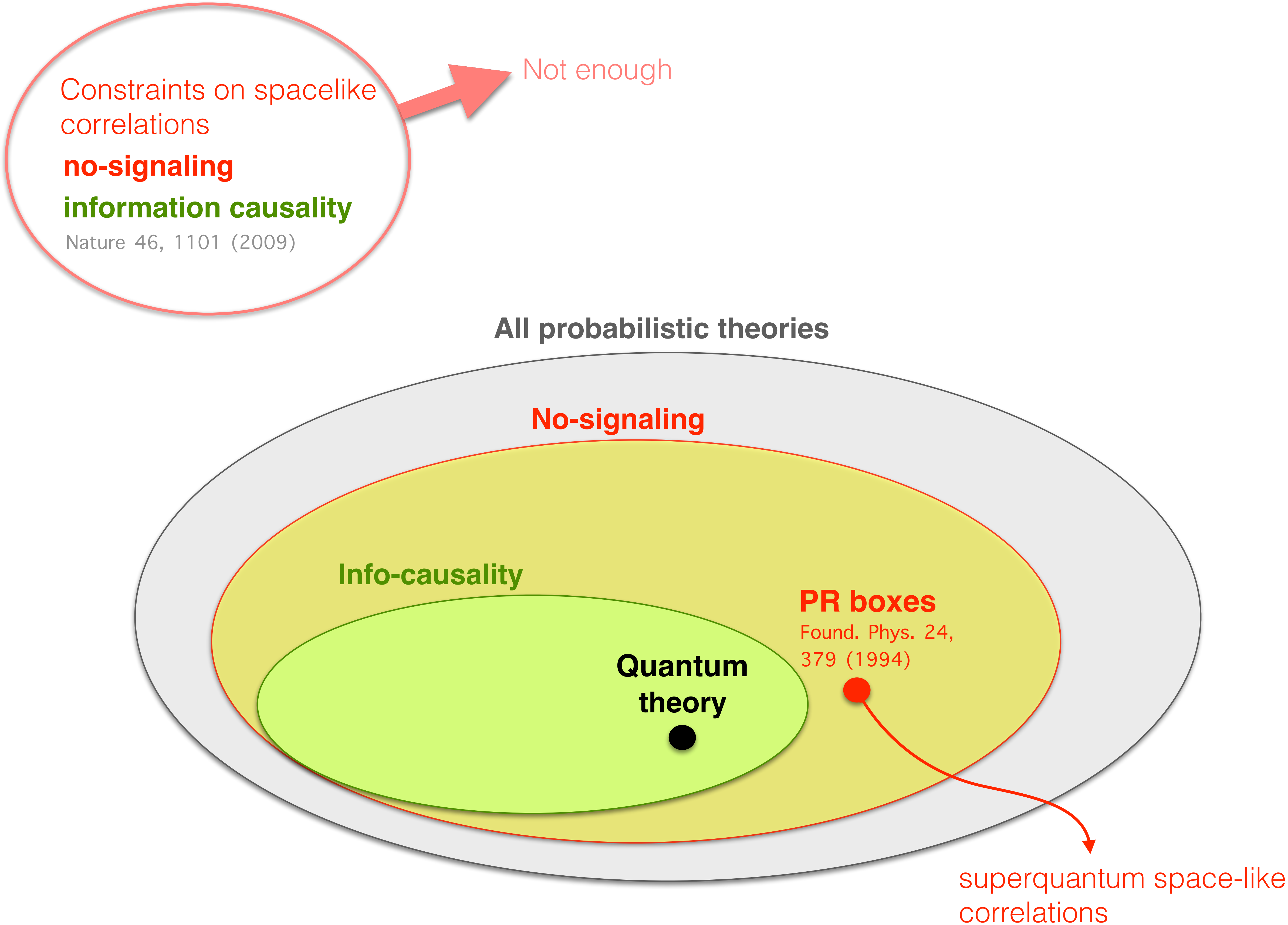
Constraints on spacelike correlations

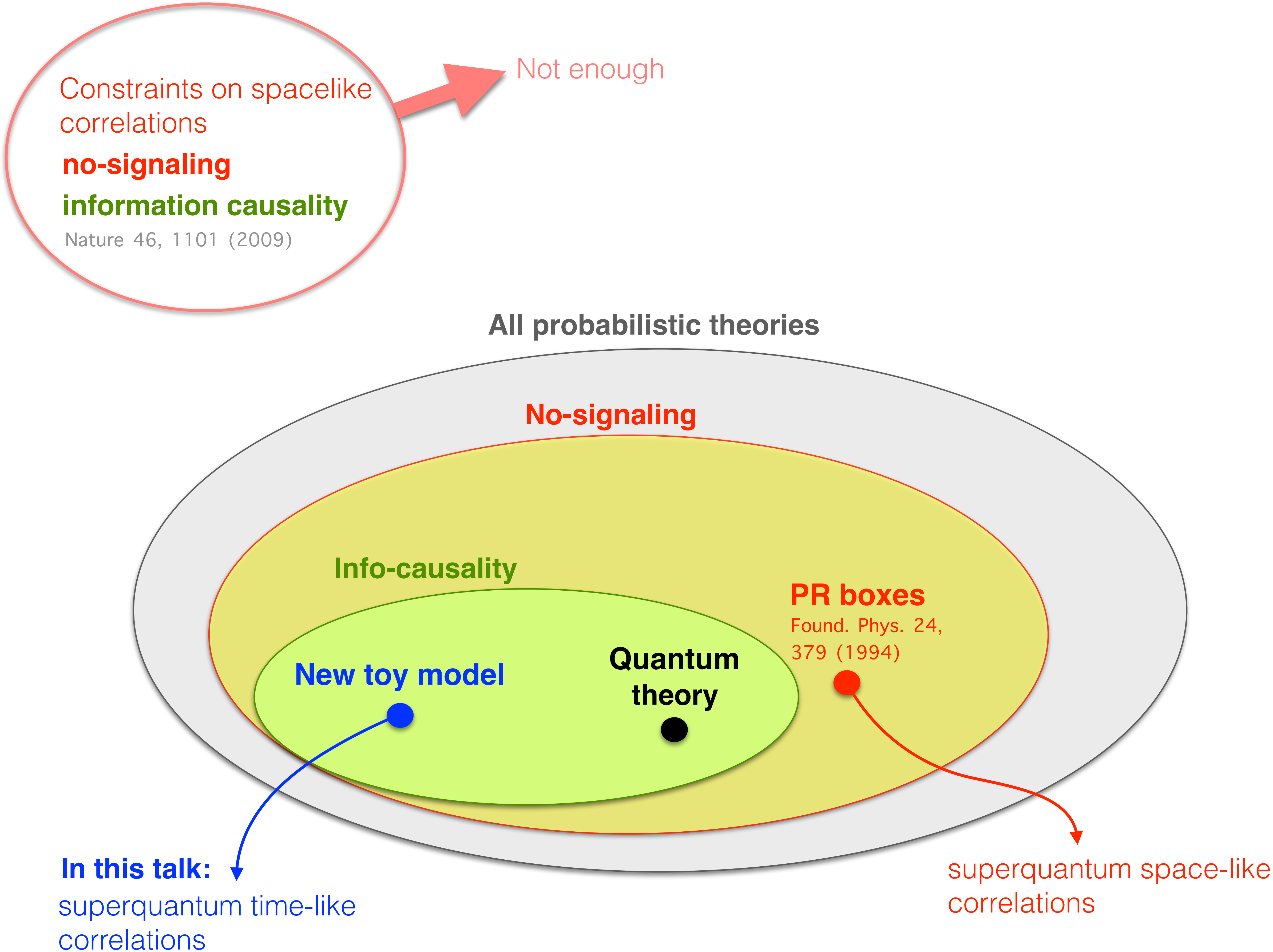
no-signaling

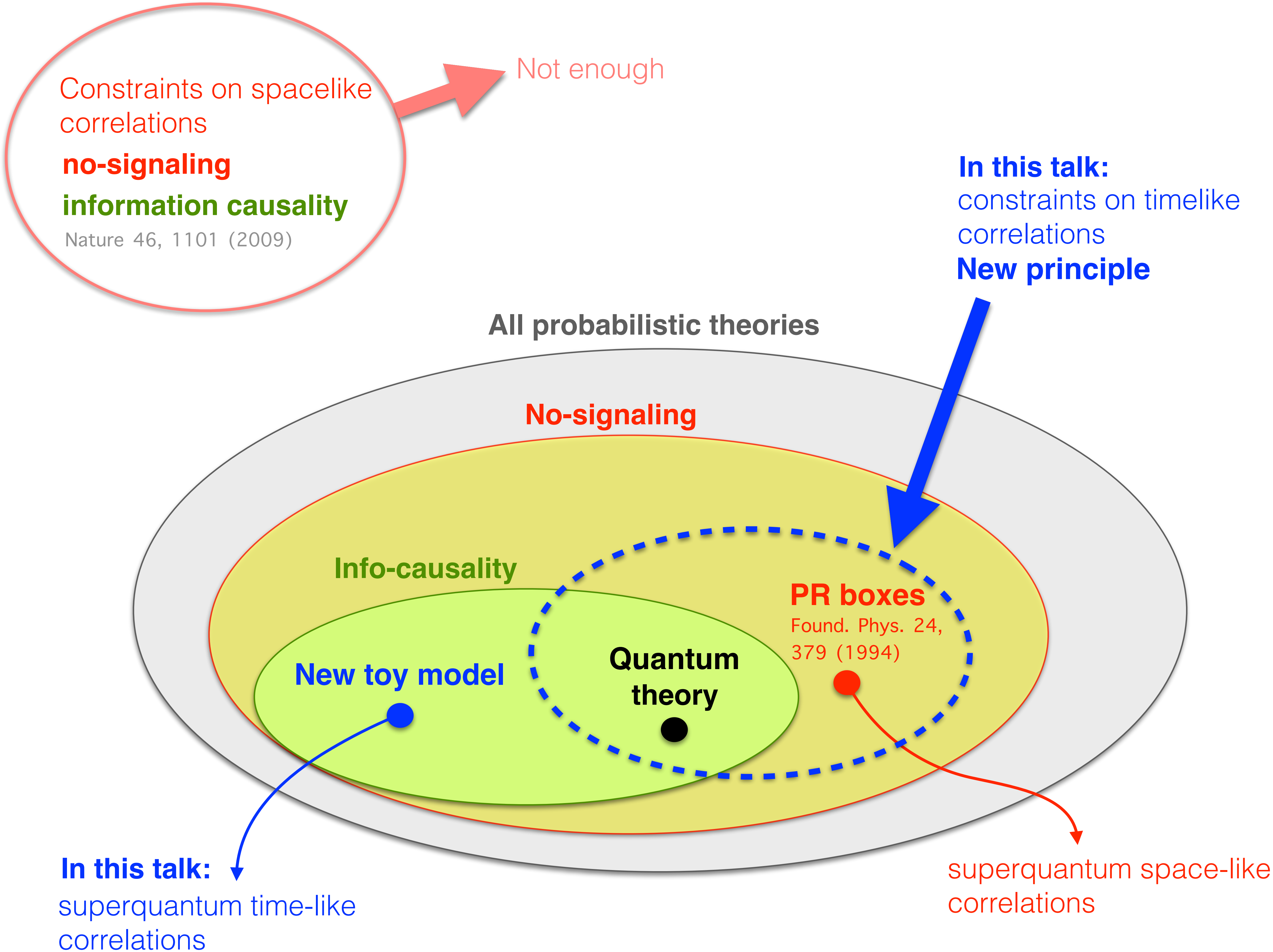
information causality

Nature 46, 1101 (2009)




Constraints on spacelike correlations
no-signaling
information causality
Nature 46, 1101 (2009)
Not enough
All probabilistic theories
No-signaling
Info-causality
PR boxes
Found. Phys. 24, 379 (1994)
Quantum theory
superquantum space-like correlations


Constraints on spacelike correlations
no-signaling
information causality
Nature 46, 1101 (2009)
Not enough
All probabilistic theories
No-signaling
Info-causality
PR boxes
Found. Phys. 24, 379 (1994)
New toy model
Quantum theory
In this talk:
superquantum time-like correlations
superquantum space-like correlations


Constraints on spacelike correlations
no-signaling
information causality
Nature 46, 1101 (2009)
Not enough
In this talk:
constraints on timelike correlations
New principle
All probabilistic theories
No-signaling
Info-causality
PR boxes
Found. Phys. 24, 379 (1994)
New toy model
Quantum theory
In this talk:
superquantum time-like correlations
superquantum space-like correlations

Outline

1. **Operational notion of system dimension**

   set of all possible "input-output correlations" it allows

2. **Separation principle: No-hypersignaling**

   describes how this dimension behaves under system composition in the quantum case

3. **No-hypersignaling violation: superquantum toy model**

   Find a toy model that *outperforms QT in a communication game*

Operational probabilistic theories

Hardy, L. quant-ph/0101012 (2001)

CDP, Phys. Rev. A 84, 012311 (2011)

Operational probabilistic theories

Hardy, L. quant-ph/0101012 (2001)

CDP, Phys. Rev. A 84, 012311 (2011)

Systems:

Operational probabilistic theories

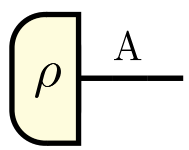
Hardy, L. quant-ph/0101012 (2001)

CDP, Phys. Rev. A 84, 012311 (2011)

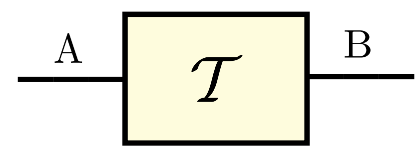
Systems:

Events:

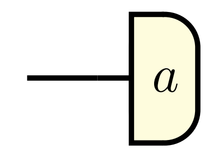
preparation event
state



transformation event
maps



observation event
measurement



Operational probabilistic theories

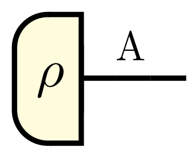
Hardy, L. quant-ph/0101012 (2001)

CDP, Phys. Rev. A 84, 012311 (2011)

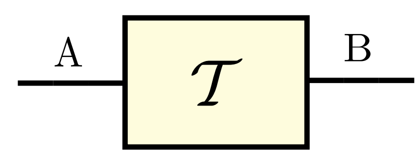
Systems:

Events:

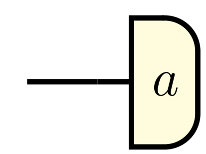
preparation event
state



transformation event
maps

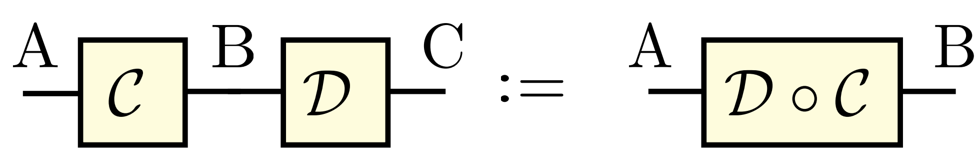


observation event
measurement

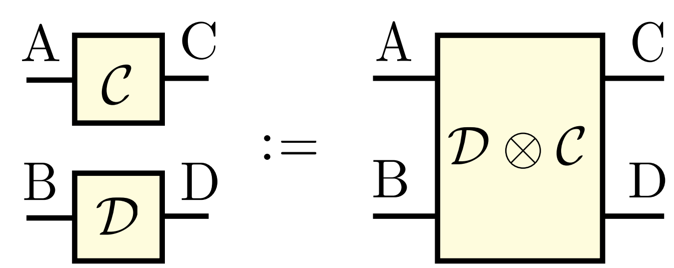


Composition:

sequential



parallel



Operational probabilistic theories

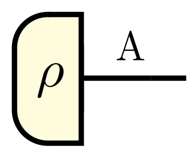
Hardy, L. quant-ph/0101012 (2001)

CDP, Phys. Rev. A 84, 012311 (2011)

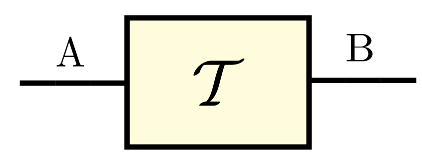
Systems:

Events:

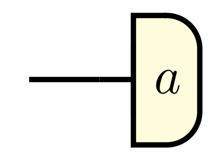
preparation event
state



transformation event
maps

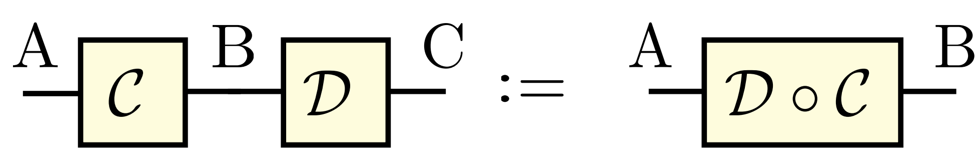


observation event
measurement

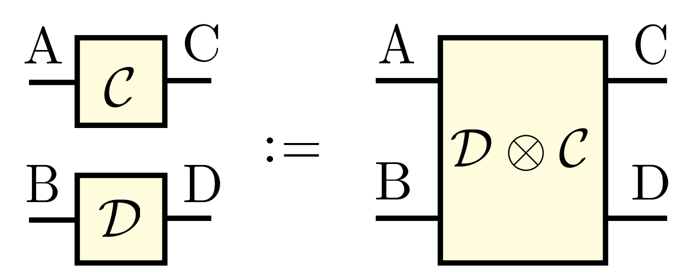


Composition:

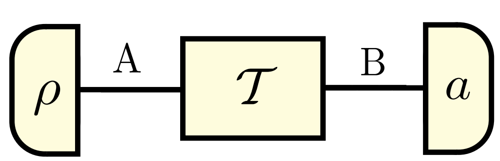
sequential



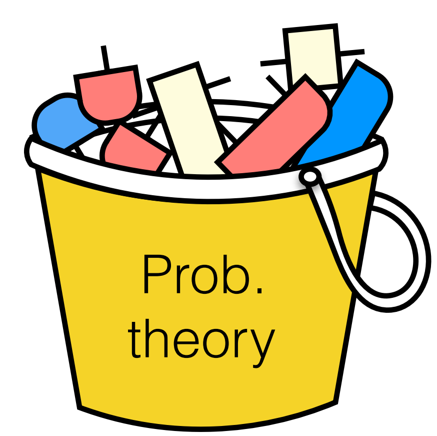
parallel



Probabilistic structure:



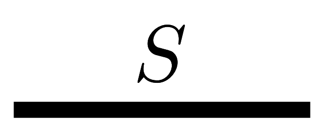
$$\mathrm{Prob}(\rho, \mathcal{T}, a)$$



Operational notion of system dimension

Let S be a system of a generic probabilistic theory

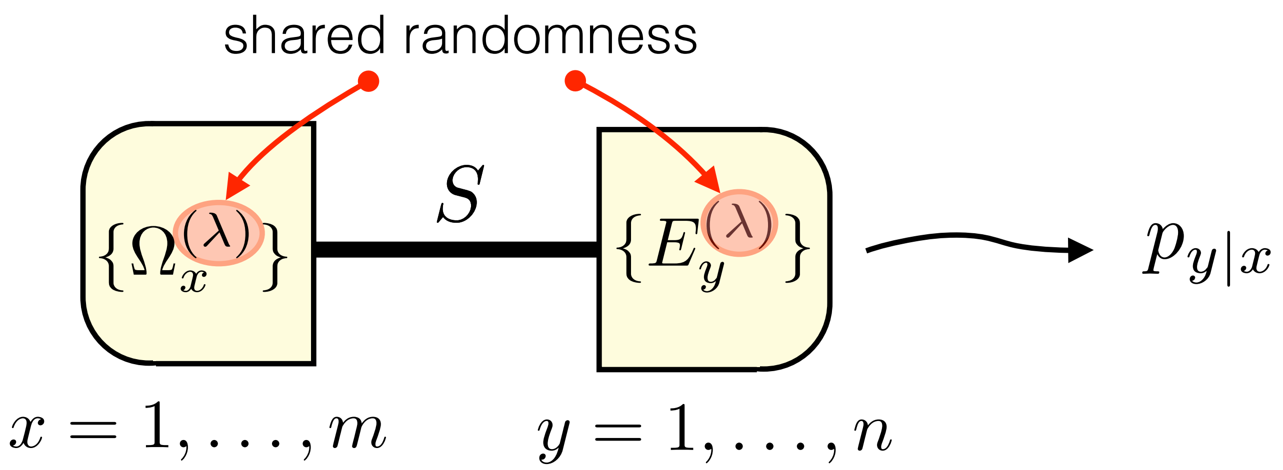
How much information can practically be transmitted via S ?



Operational notion of system dimension

Let S be a system of a generic probabilistic theory

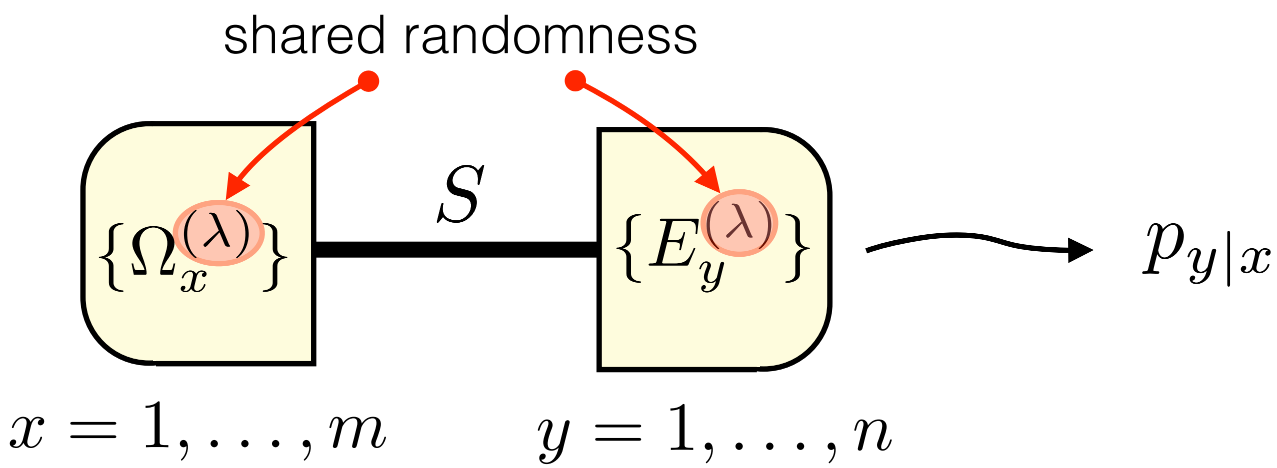
How much information can practically be transmitted via S ?



Operational notion of system dimension

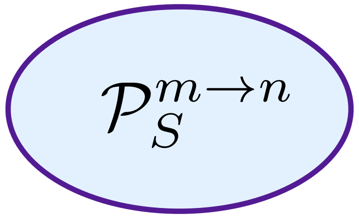
Let S be a system of a generic probabilistic theory

How much information can practically be transmitted via S ?



Set of all m-input n-output cond. prob. dis. via S

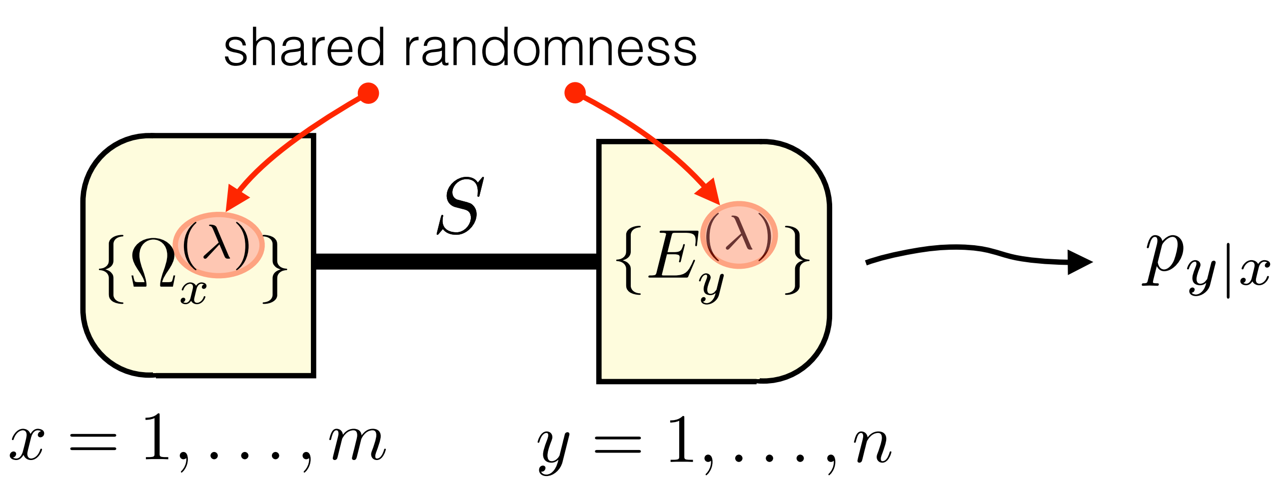
convex set



Operational notion of system dimension

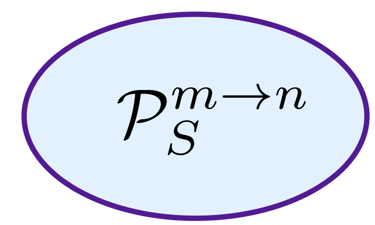
Let S be a system of a generic probabilistic theory

How much information can practically be transmitted via S ?



Set of all m-input n-output cond. prob. dis. via S

convex set



Recent results "gen. of Holevo bound"

$$\mathcal{P}_{Q_d}^{m \to n} = \mathcal{P}_d^{m \to n} \quad \forall d, m, n$$

Q_d: d-level quantum system

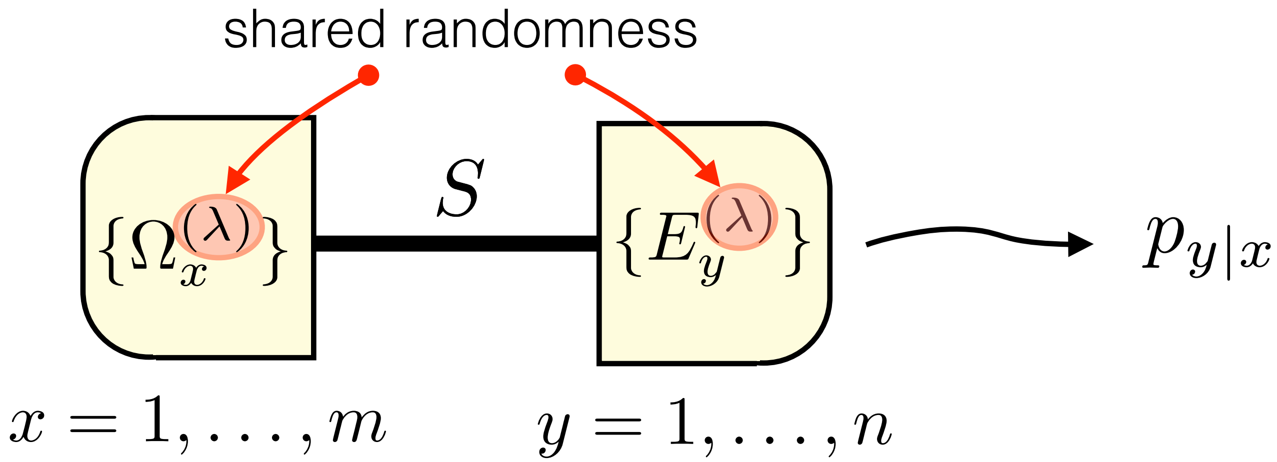
d: d-level classical system

P.E. Frenkel, M. Weiner,
Commun. Math. Phys. 340, 563 (2015)

Operational notion of system dimension

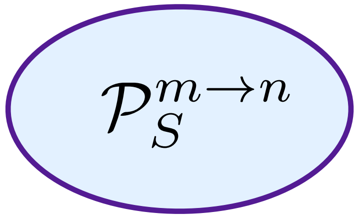
Let S be a system of a generic probabilistic theory

How much information can practically be transmitted via S ?



Set of all m-input n-output cond. prob. dis. via S

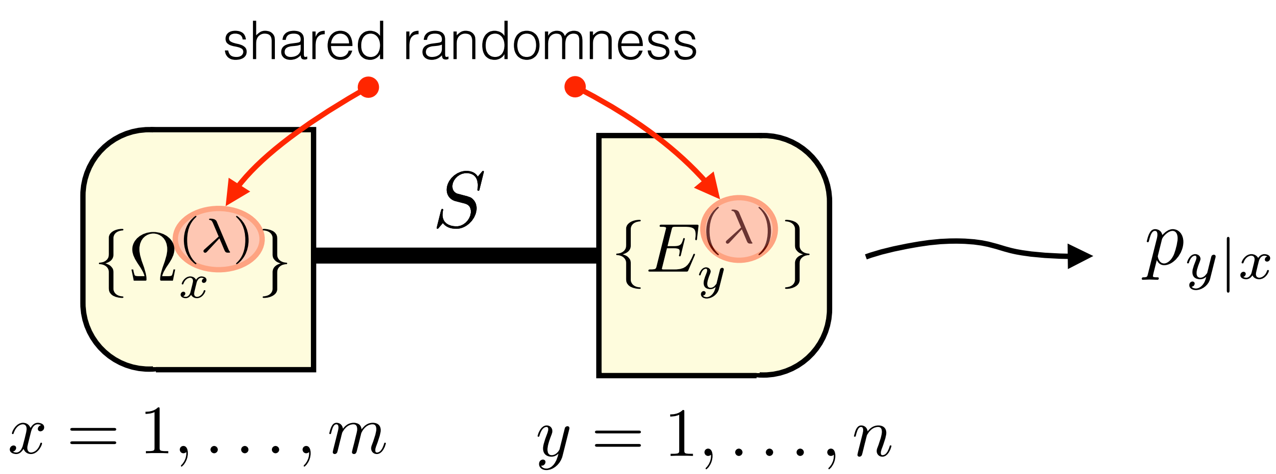
convex set



Operational notion of system dimension

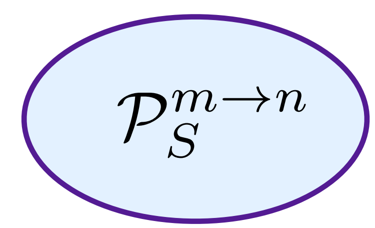
Let S be a system of a generic probabilistic theory

How much information can practically be transmitted via S ?



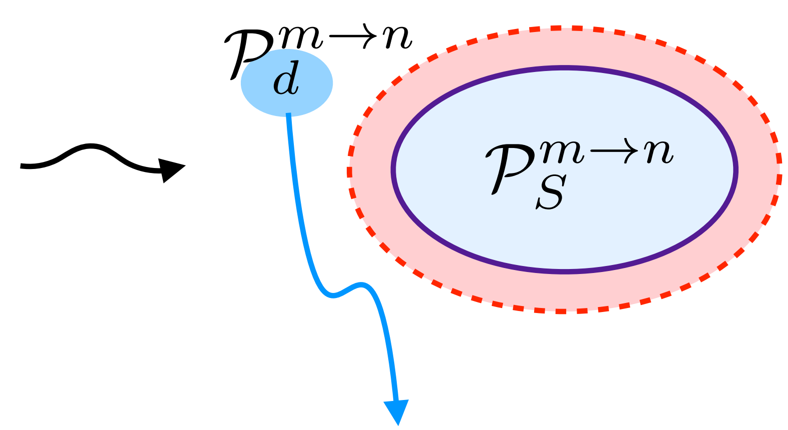
Set of all m-input n-output cond. prob. dis. via S

convex set



DEFINITION. **Signaling Dimension** of system S:

the *smallest* integer d s.t.



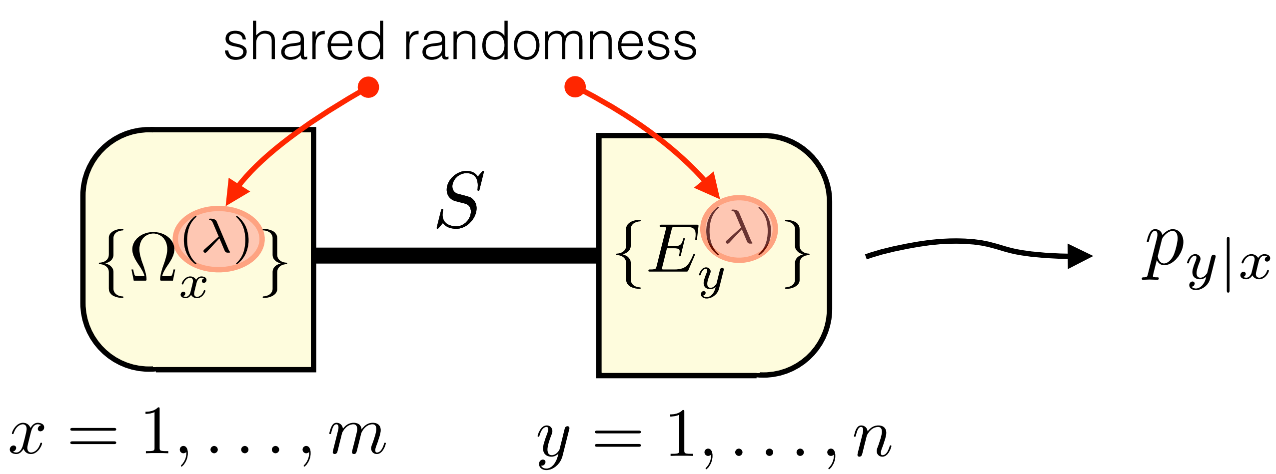
$\forall m, n \in \mathbb{N}$

d-level classical system

Operational notion of system dimension

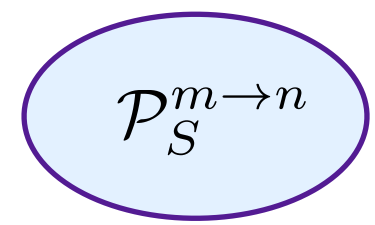
Let *S* be a system of a generic probabilistic theory

How much information can practically be transmitted via *S* ?



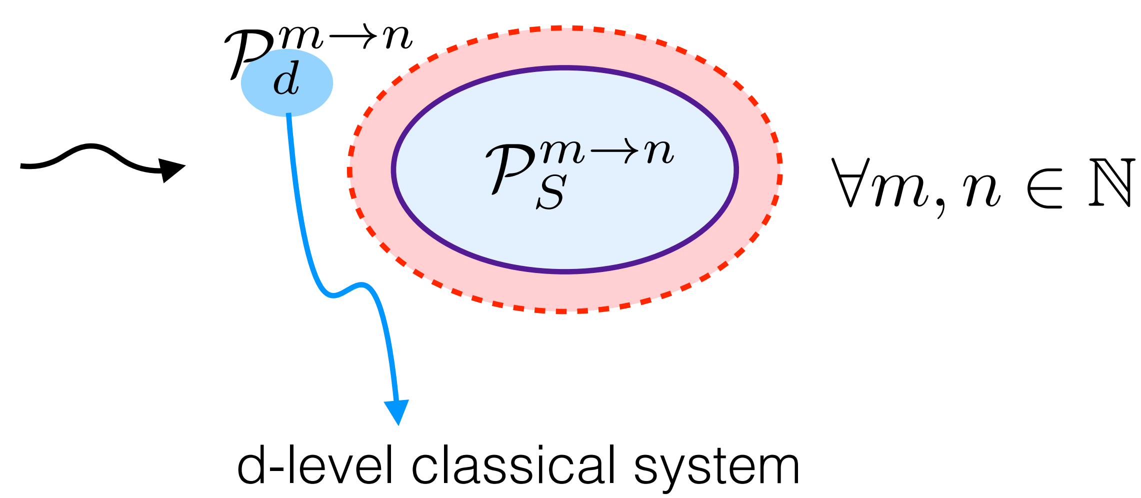
Set of all *m*-input *n*-output cond. prob. dis. via *S*

convex set



DEFINITION. **Signaling Dimension** of system S:

the *smallest* integer *d* s.t.



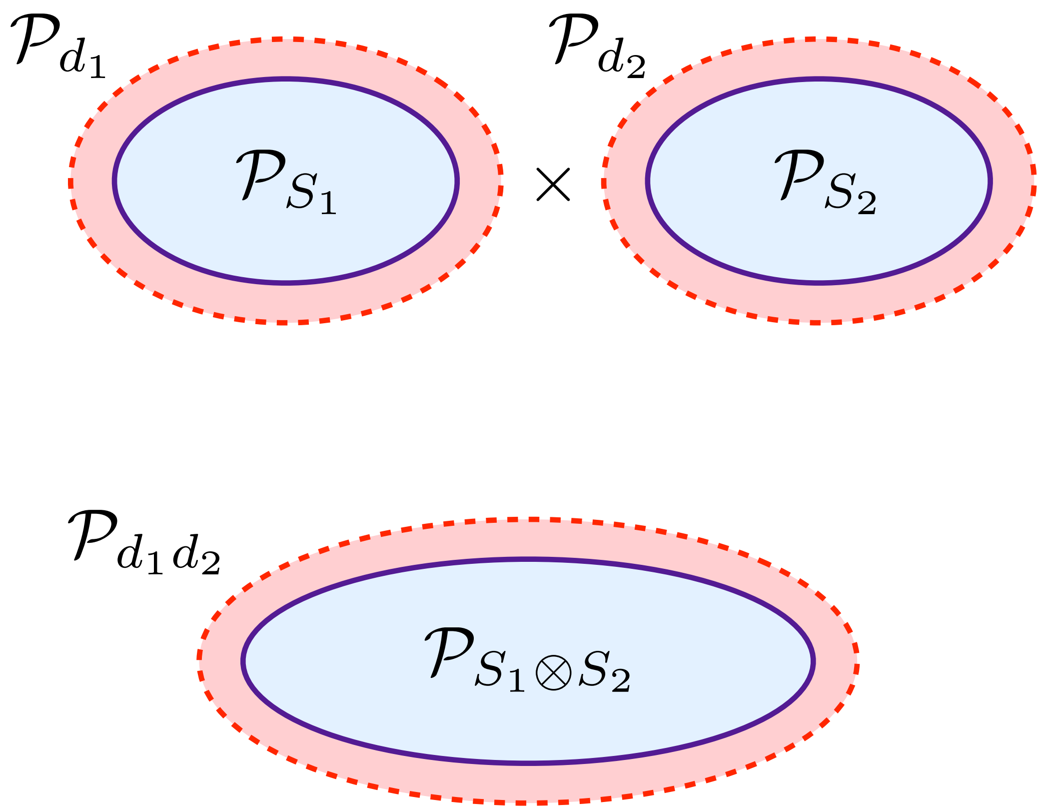
Signaling dimension of S denoted $\kappa(S)$

Separation principle: No-hypersignaling

Intuitive: Any input-output correlation that can be obtained by transmitting a composite system should also be obtainable by independently transmitting its constituents.

Separation principle: No-hypersignaling

Intuitive: Any input-output correlation that can be obtained by transmitting a composite system should also be obtainable by independently transmitting its constituents.

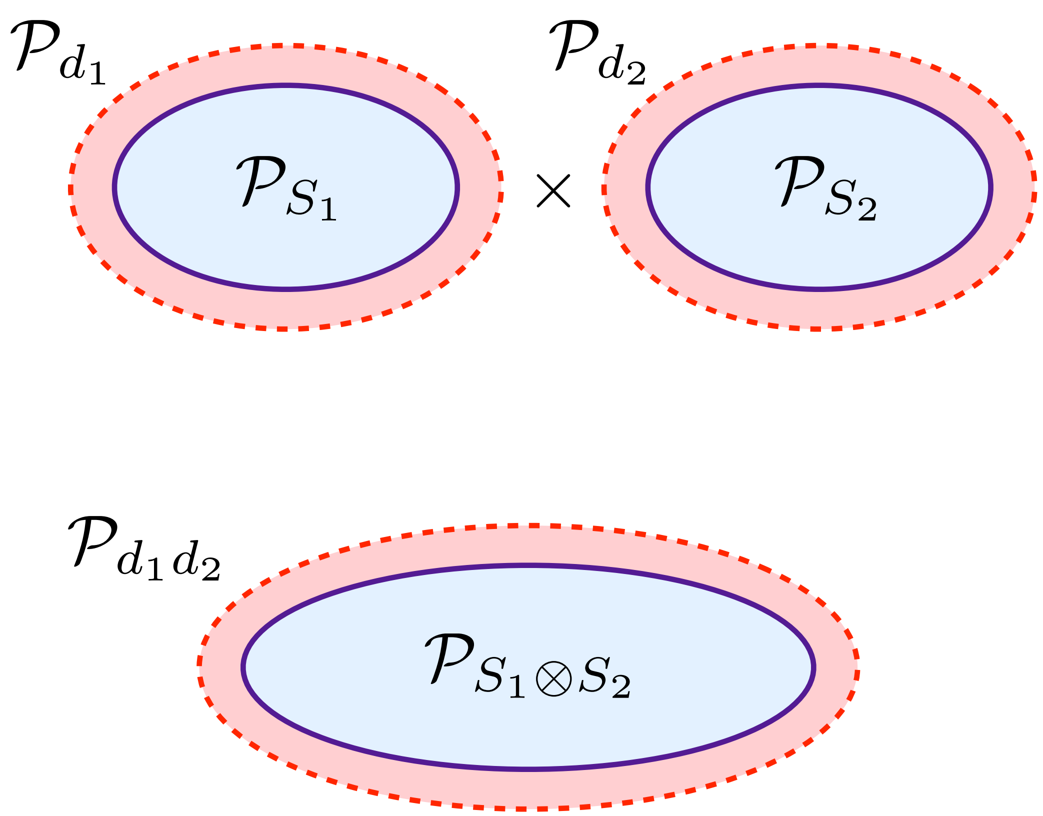


This is the quantum behaviour

Separation principle: No-hypersignaling

Intuitive: Any input-output correlation that can be obtained by transmitting a composite system should also be obtainable by independently transmitting its constituents.

Formal: For any set of systems $\{S_k\}$ with signalling dimensions $\kappa(S_k)$ $\Rightarrow \quad \kappa(\otimes_k S_k) \leq \prod_k \kappa(S_k)$

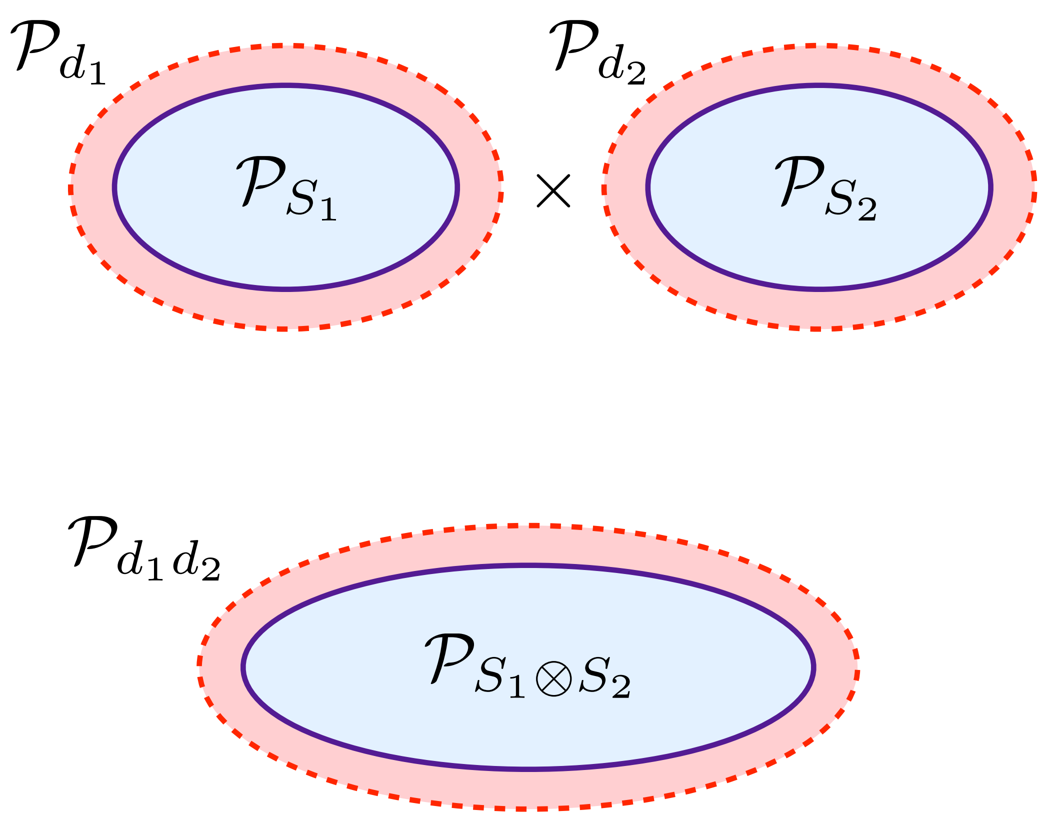


This is the quantum behaviour

Separation principle: No-hypersignaling

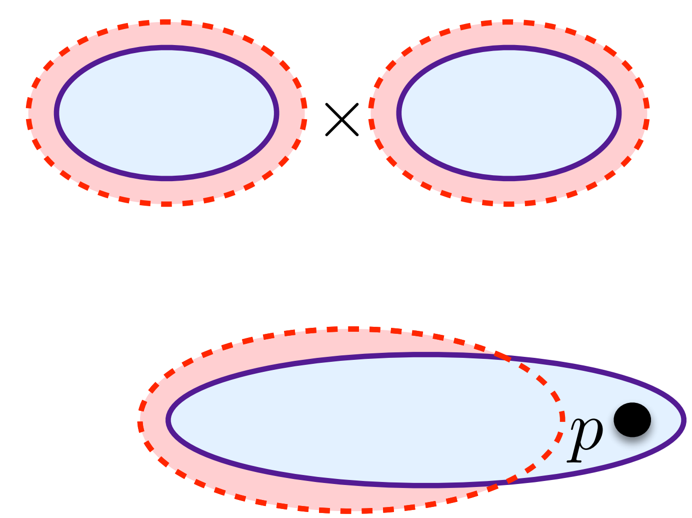
Intuitive: Any input-output correlation that can be obtained by transmitting a composite system should also be obtainable by independently transmitting its constituents.

Formal: For any set of systems $\{S_k\}$ with signalling dimensions $\kappa(S_k)$ $\Rightarrow$ $\kappa(\otimes_k S_k) \leq \prod_k \kappa(S_k)$



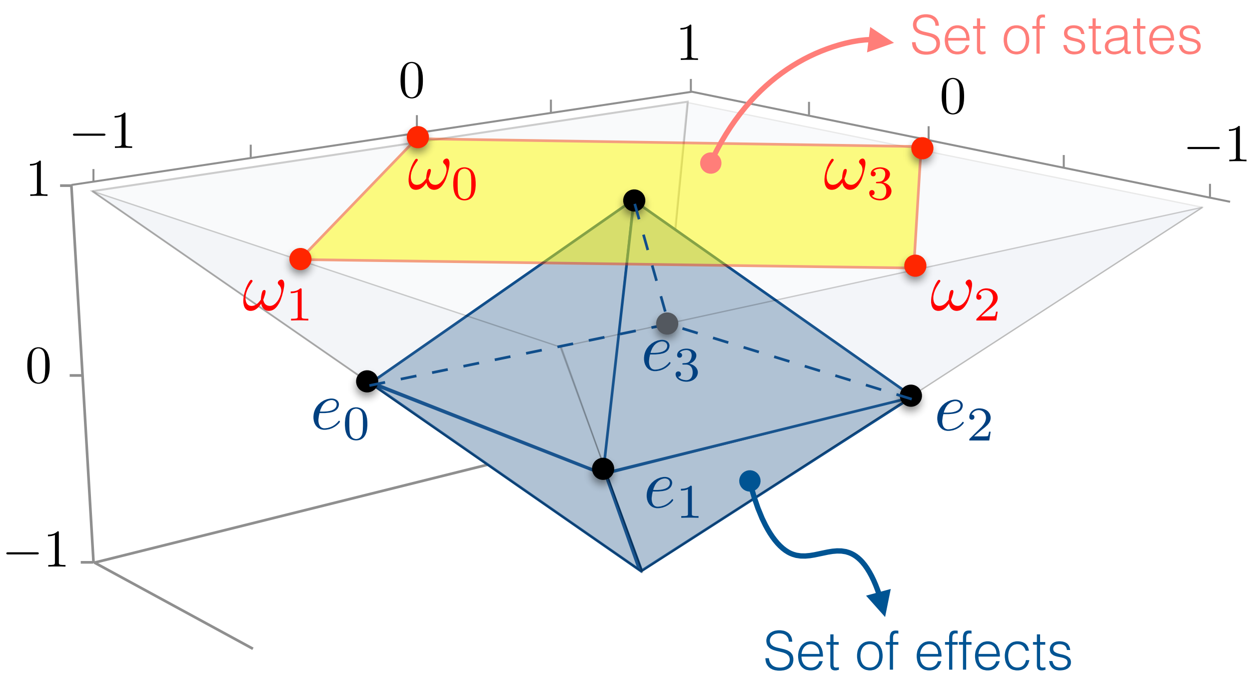
This is the quantum behaviour

no-hypersignaling violation



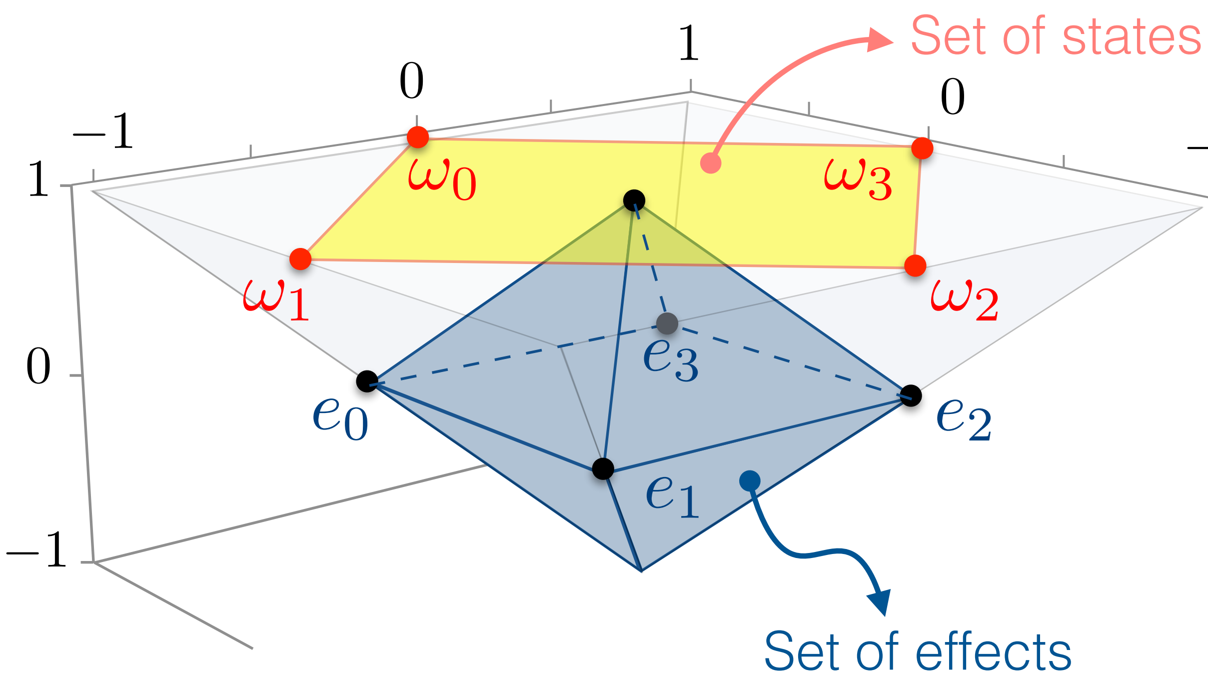
A class of toy models

Elementary system

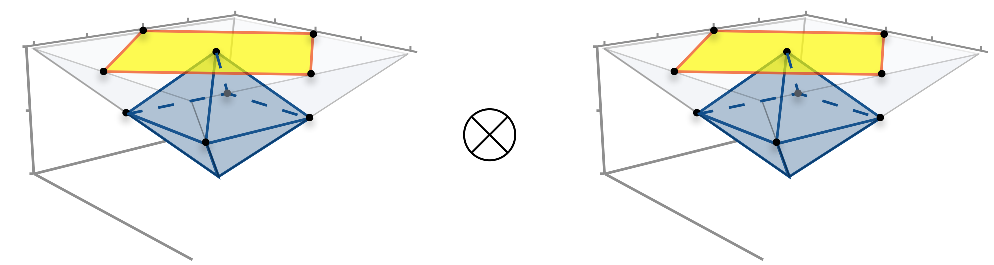


A class of toy models

Elementary system



Composite systems



24 possible bipartite states and effects

states Ω_k

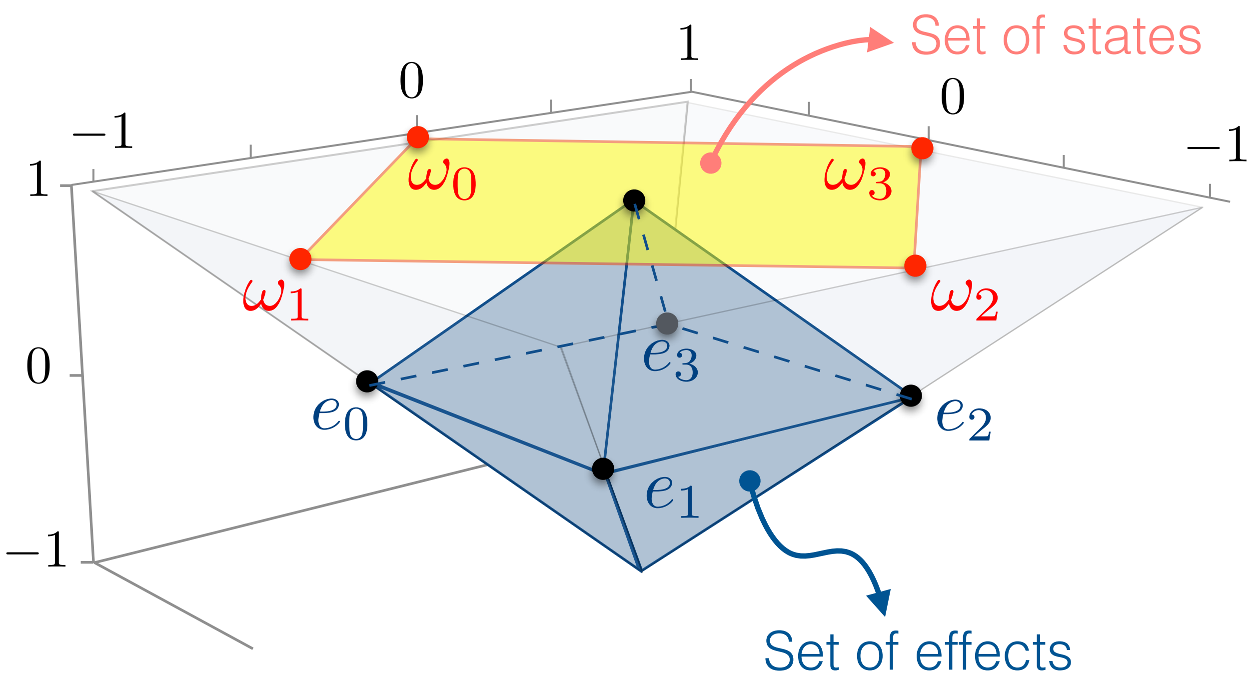
effects E_k

16 factorized
$\omega_i \otimes \omega_j$
+ 8 non-local

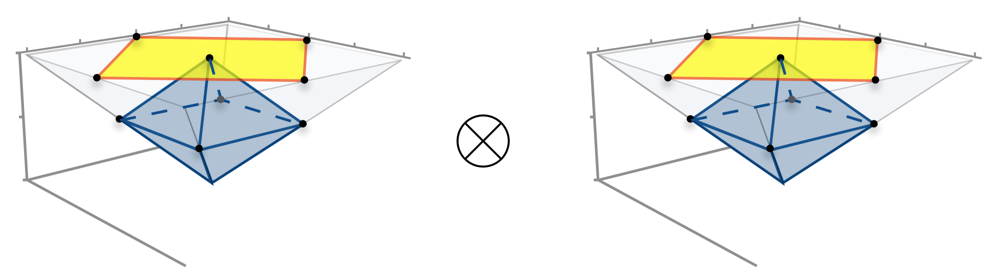
16 factorized
$e_i \otimes e_j$
+ 8 non-local

A class of toy models

Elementary system



Composite systems



24 possible bipartite states and effects

states Ω_k

effects E_k

16 factorized
$\omega_i \otimes \omega_j$
+ 8 non-local

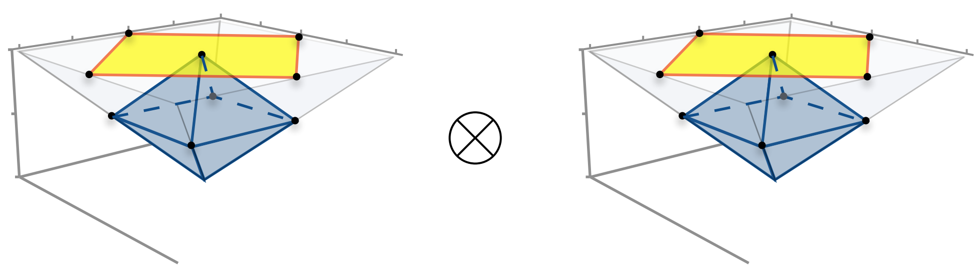
16 factorized
$e_i \otimes e_j$
+ 8 non-local

trade-off states/effects

Not all states and effects
are compatible

A. J. Short, J. Barrett, J. Phys. 12, 033034 (2010)

Full classification



states Ω_k

16 factorized

$\omega_i \otimes \omega_j$

+ 8 non-local

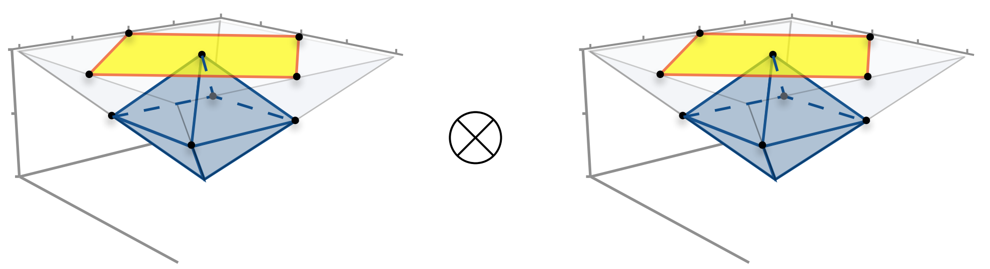
effects E_k

16 factorized

$e_i \otimes e_j$

+ 8 non-local

Full classification



states Ω_k

16 factorized
$\omega_i \otimes \omega_j$
+ 8 non-local

effects E_k

16 factorized
$e_i \otimes e_j$
+ 8 non-local

All the admissible choices

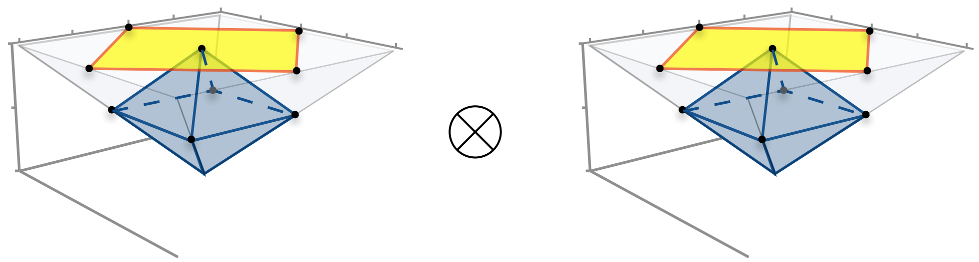
1. all the **8** non-local states
   only (factorized) effects

PR-Model: The well known PR-boxes

superquatum
space-like corr.

Full classification

states Ω_k effects E_k



16 factorized
$\omega_i \otimes \omega_j$
+ 8 non-local

16 factorized
$e_i \otimes e_j$
+ 8 non-local

All the admissible choices

1. all the **8** non-local states
   only (factorized) effects

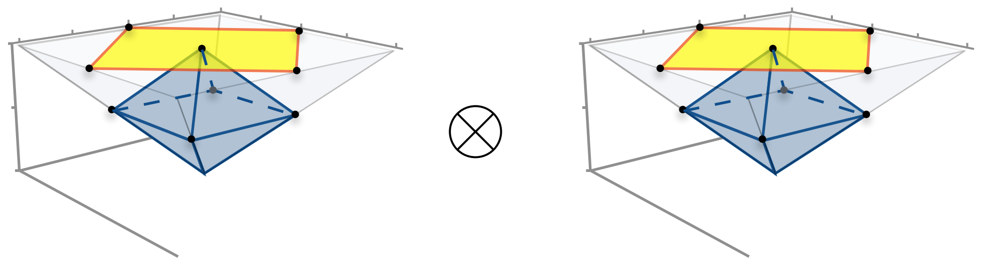
PR-Model: The well known PR-boxes

superquatum
space-like corr.

2. only factorized states
   all the **8** non-local effects

HS-Model: "Dual of the PR-boxes"

Full classification



states Ω_k effects E_k

16 factorized
$\omega_i \otimes \omega_j$
+ 8 non-local

16 factorized
$e_i \otimes e_j$
+ 8 non-local

All the admissible choices

1. all the **8** non-local states
   only (factorized) effects

PR-Model: The well known PR-boxes

superquatum
space-like corr.

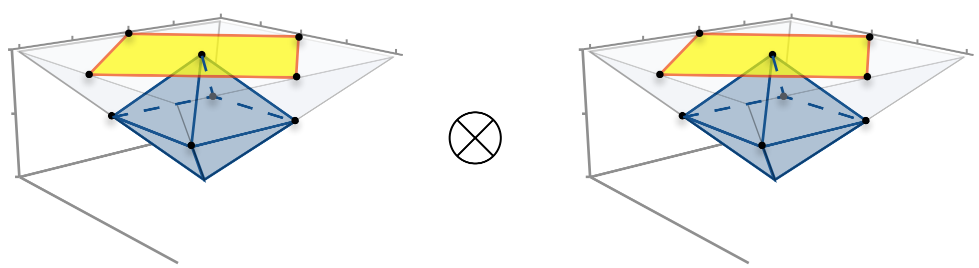
2. only factorized states
   all the **8** non-local effects

HS-Model: “Dual of the PR-boxes”

3. **2** non-local states
   2 non-local states

Hybrid-Models

Full classification



states Ω_k

16 factorized

$\omega_i \otimes \omega_j$

+ 8 non-local

effects E_k

16 factorized

$e_i \otimes e_j$

+ 8 non-local

All the admissible choices

1. all the **8** non-local states
   only (factorized) effects

PR-Model: The well known PR-boxes — superquatum space-like corr.

2. only factorized states
   all the **8** non-local effects

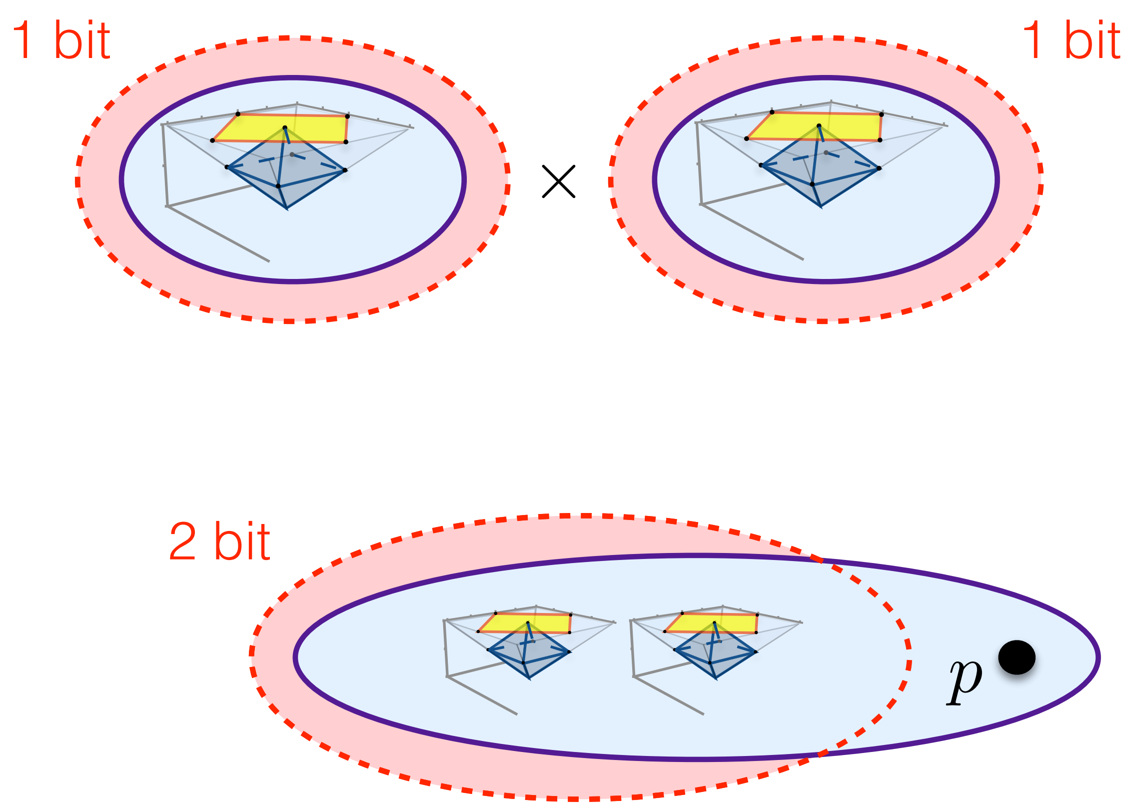
HS-Model: "Dual of the PR-boxes" — superquatum time-like corr.

3. **2** non-local states
   2 non-local states

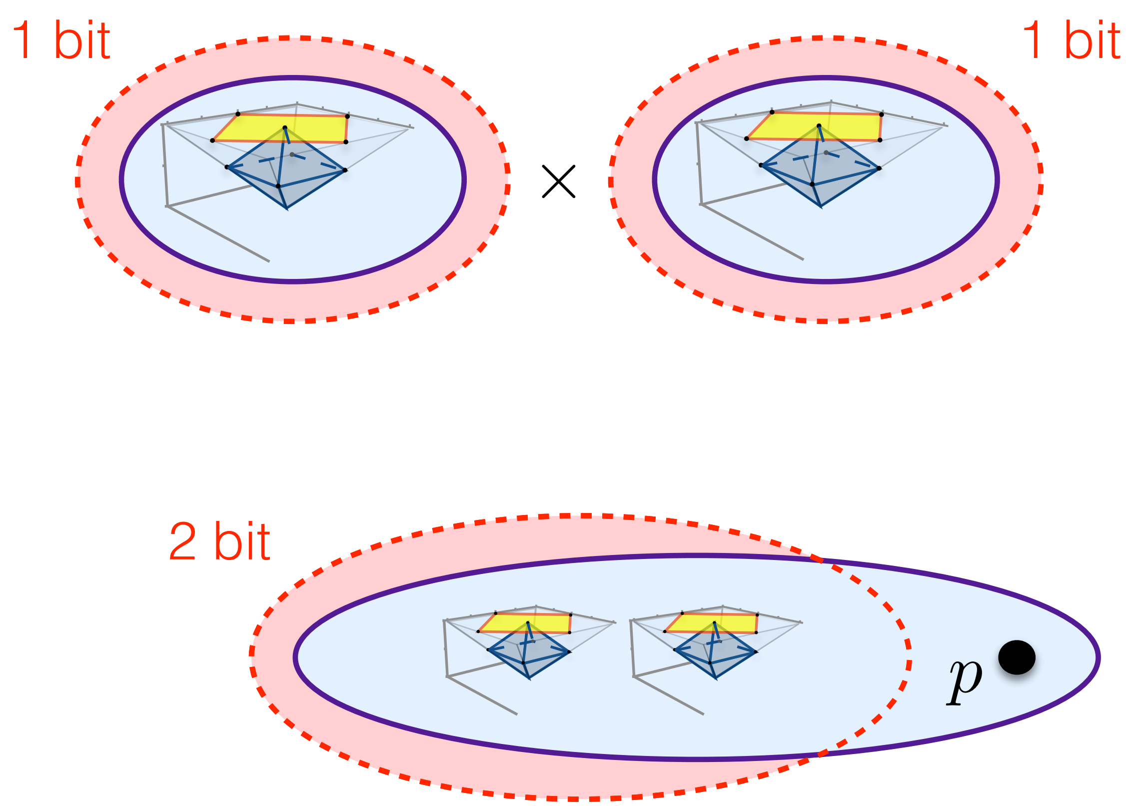
Hybrid-Models

no-hypersignaling violation

No-hypersignaling violation



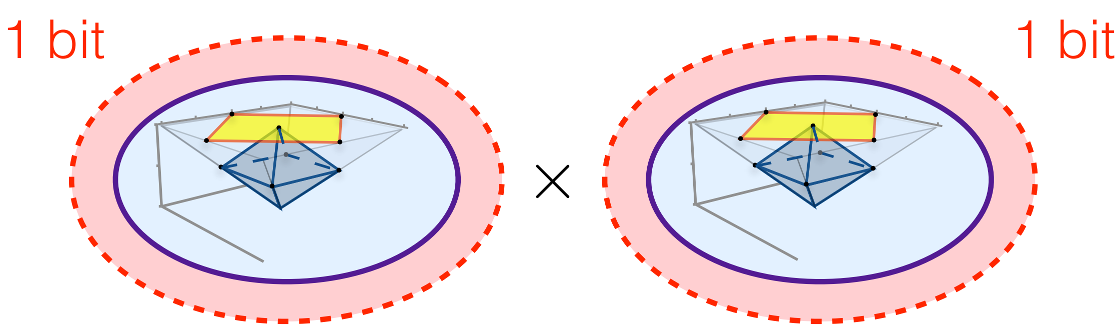
No-hypersignaling violation



Theorem:

If a theory violates no-hypersignaling then the violation occurs for POVMs with extremal effects

No-hypersignaling violation



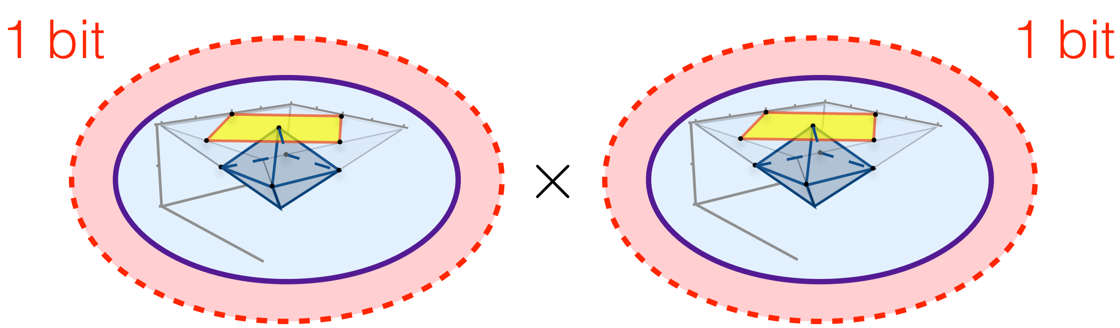
Theorem:

If a theory violates no-hypersignaling then the violation occurs for POVMs with extremal effects

Characterization of all extremal POVMS

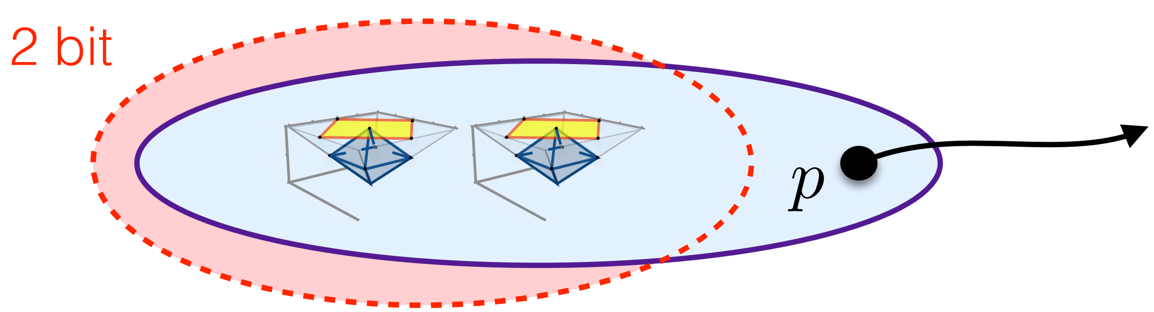
M	#	E_0	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	E_{17}	E_{18}	E_{19}	E_{20}	E_{21}	E_{22}	E_{23}
0	2																	$\frac{1}{2}$		$\frac{1}{2}$					
1	4	1/4		1/4						1/4		1/4													
2	4	1/4		1/4							1/4		1/4												
3	6	1/8	1/8									1/8	1/8							1/4					1/4
4	6	1/8					1/8					1/8					1/8					1/4			1/4
5	6	1/6										1/6							1/6	1/6		1/6			1/6
6	7	1/8	1/8					1/8		1/8		1/8					1/8								1/4
7	[8]	[illegible]	1/12			1/12						1/6					1/12			1/6		1/6			1/6
8	[8]	[illegible]	1/12					1/6		1/12	1/12						1/6	1/6							1/6
[9]	[illegible]	1/6	1/12					1/12			1/12		1/6			1/12				1/6					1/6
[illegible]	8	1/8					1/8						1/8			1/8				1/8	1/8	1/8			1/8
11	9	1/12	1/12			1/12		1/12			1/12	1/12					1/6					1/6			1/6
12	9	1/16	1/16			1/16		1/8			1/8						3/16	1/8				1/8			1/8
13	9	1/12	1/12			1/12			1/12			1/12	1/12		1/12	1/12				1/3					
14	9	1/10		1/10			1/10						1/5		1/10					1/10	1/10			1/10	1/10
M	#	E_0	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	E_{17}	E_{18}	E_{19}	E_{20}	E_{21}	E_{22}	E_{23}

No-hypersignaling violation



Theorem:

If a theory violates no-hypersignaling then the violation occurs for POVMs with extremal effects

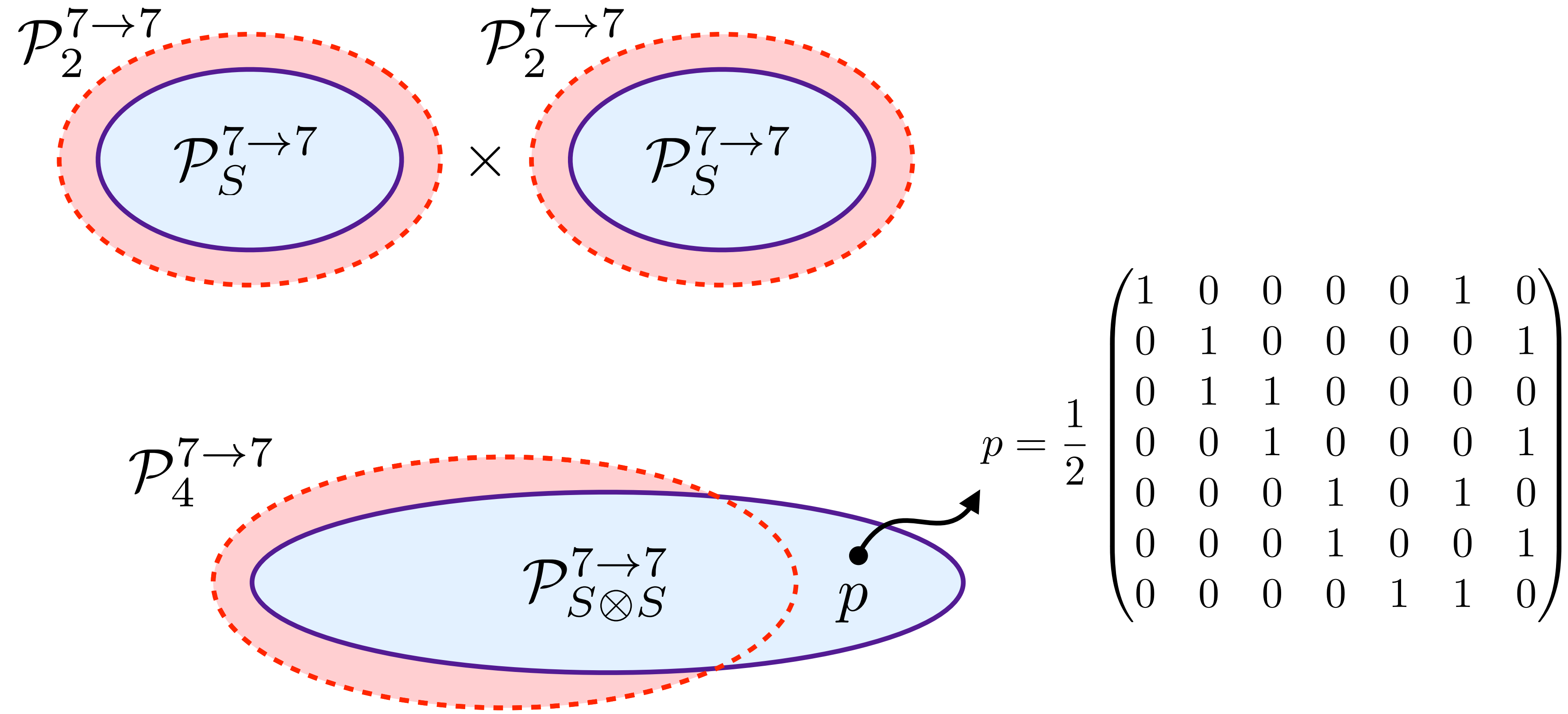


$$p = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

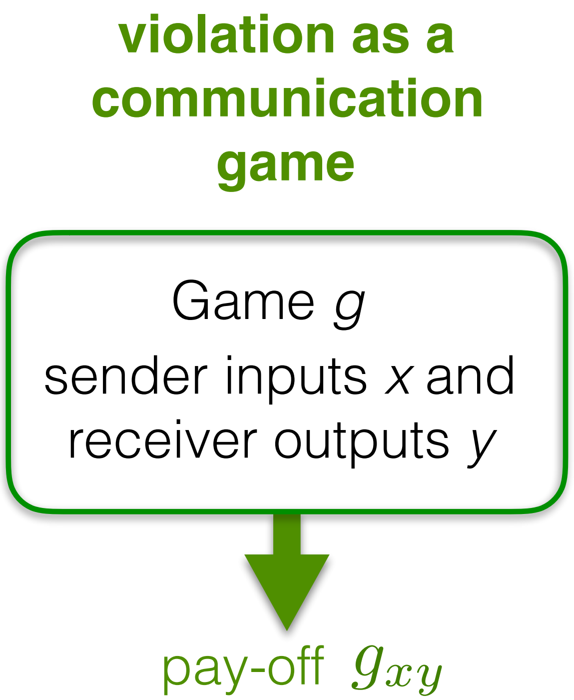
it's his fault

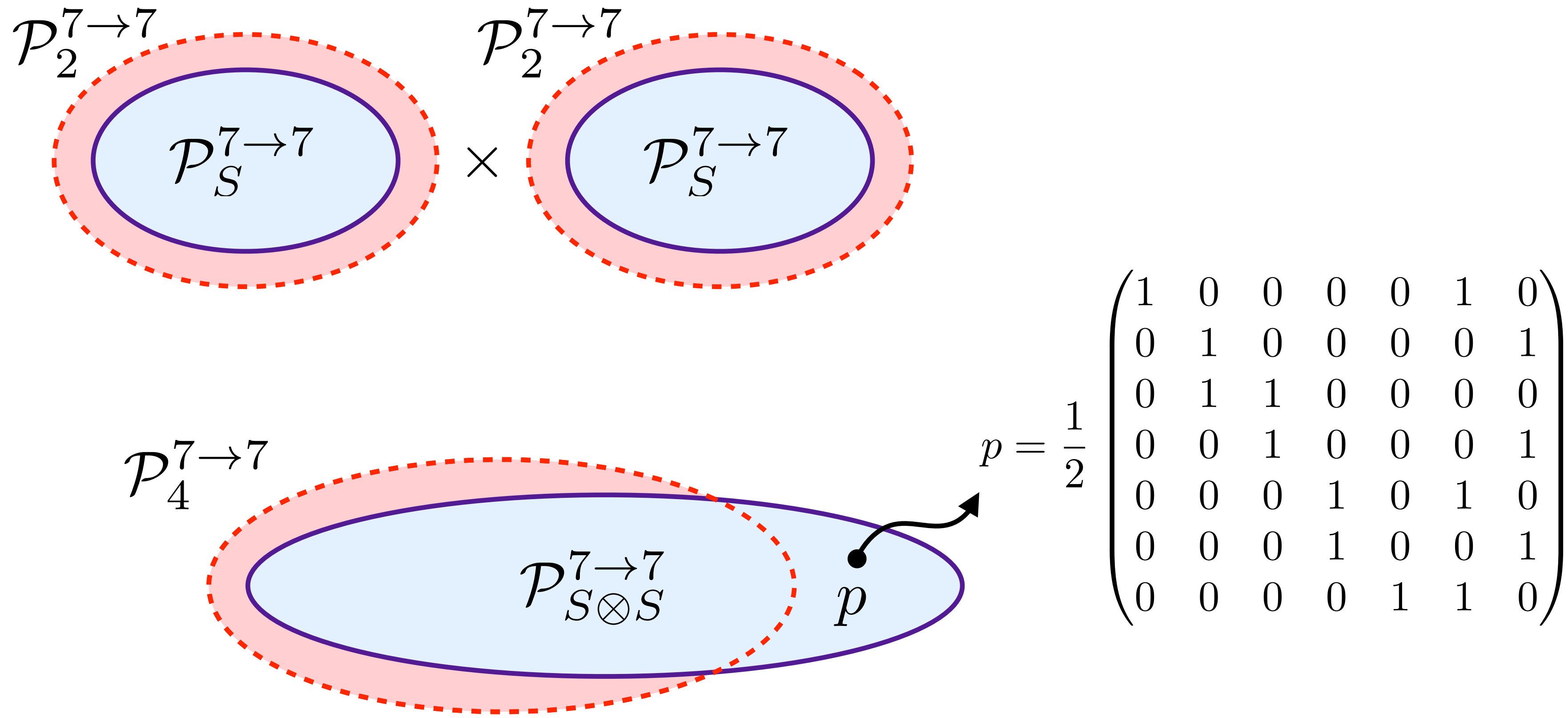
POVM $\left\{\frac{1}{8}E_0, \frac{1}{8}E_1, \frac{1}{8}E_6, \frac{1}{8}E_8, \frac{1}{8}E_{10}, \frac{1}{8}E_{15}, \frac{1}{4}E_{23}\right\}$

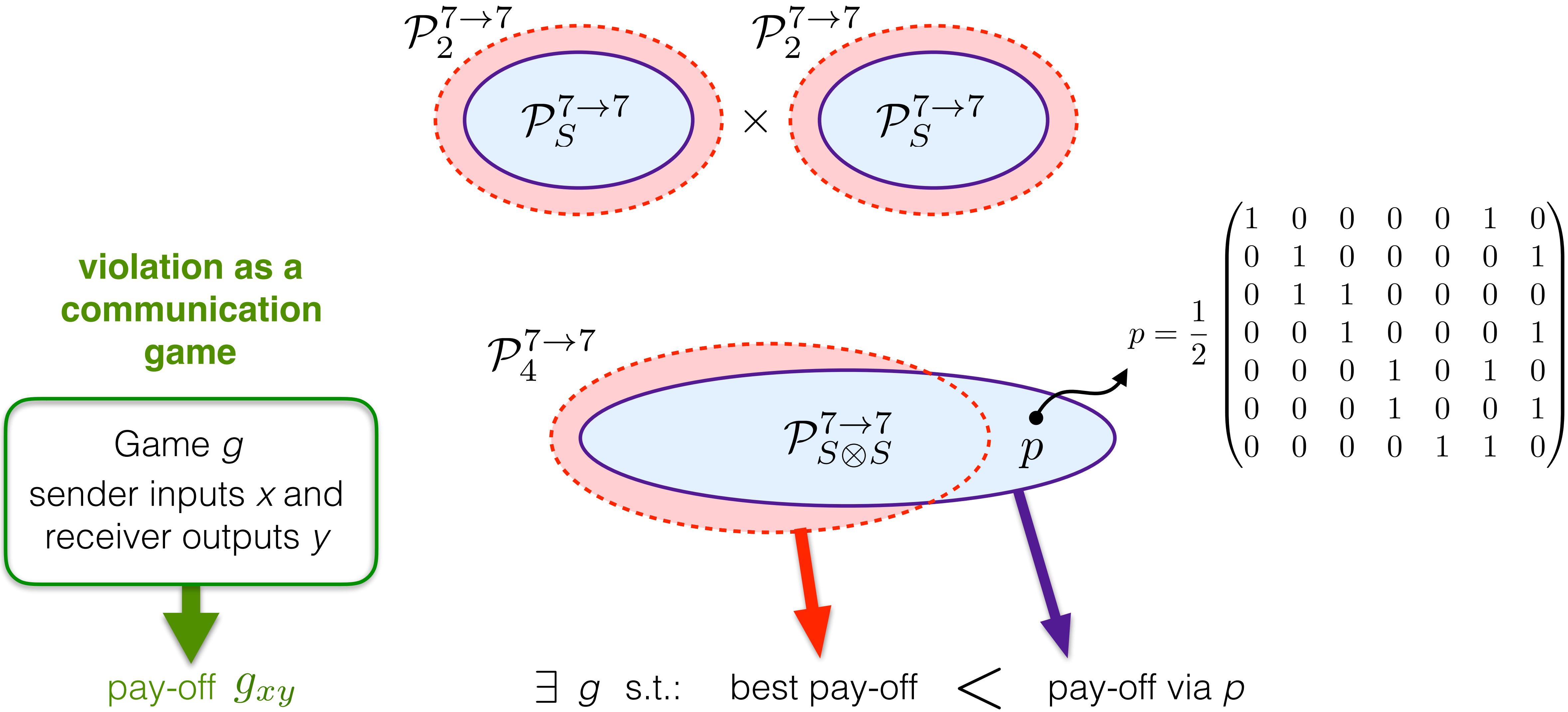
Ensamble $\{\Omega_0, \Omega_2, \Omega_6, \Omega_7, \Omega_{12}, \Omega_{13}, \Omega_{15}\}$



$\mathcal{P}_2^{7\to7}$
$\mathcal{P}_S^{7\to7}$
$\times$
$\mathcal{P}_2^{7\to7}$
$\mathcal{P}_S^{7\to7}$
$\mathcal{P}_4^{7\to7}$
$\mathcal{P}_{S\otimes S}^{7\to7}$
p
$p = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$

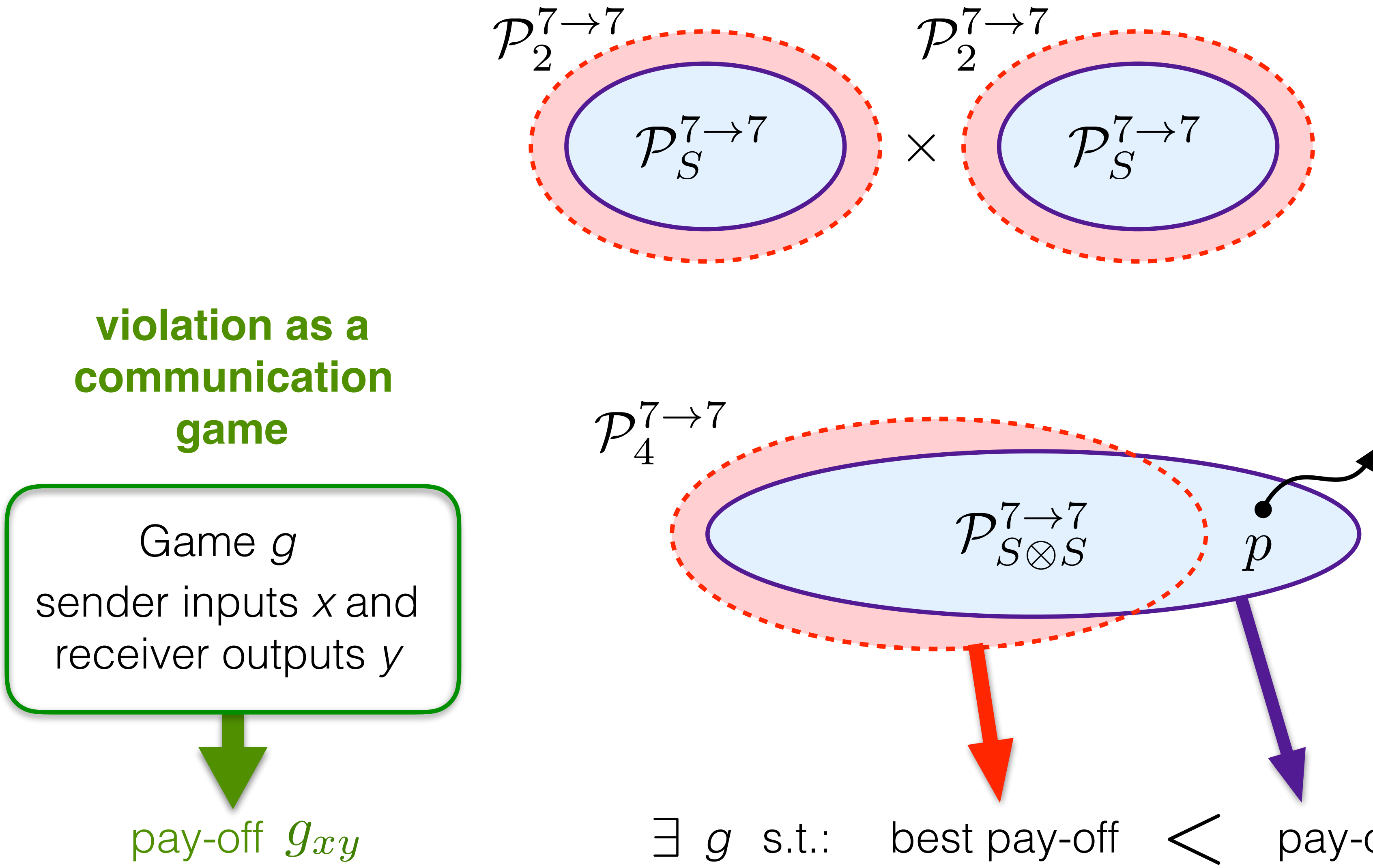

violation as a communication game
Game g
sender inputs x and receiver outputs y
pay-off g_{xy}


$\mathcal{P}_2^{7\to 7}$
$\mathcal{P}_S^{7\to 7}$
$\times$
$\mathcal{P}_2^{7\to 7}$
$\mathcal{P}_S^{7\to 7}$
$\mathcal{P}_4^{7\to 7}$
$\mathcal{P}_{S\otimes S}^{7\to 7}$
p
$p = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$



$\mathcal{P}_2^{7\to7}$
$\mathcal{P}_S^{7\to7}$
$\times$
$\mathcal{P}_2^{7\to7}$
$\mathcal{P}_S^{7\to7}$
violation as a communication game
Game g
sender inputs x and receiver outputs y
pay-off g_{xy}
$\mathcal{P}_4^{7\to7}$
$\mathcal{P}_{S\otimes S}^{7\to7}$
p
$p = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$
$\exists\ g$ s.t.: best pay-off $<$ pay-off via p

violation as a communication game



$$p = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$\exists$ g s.t.: best pay-off $<$ pay-off via p

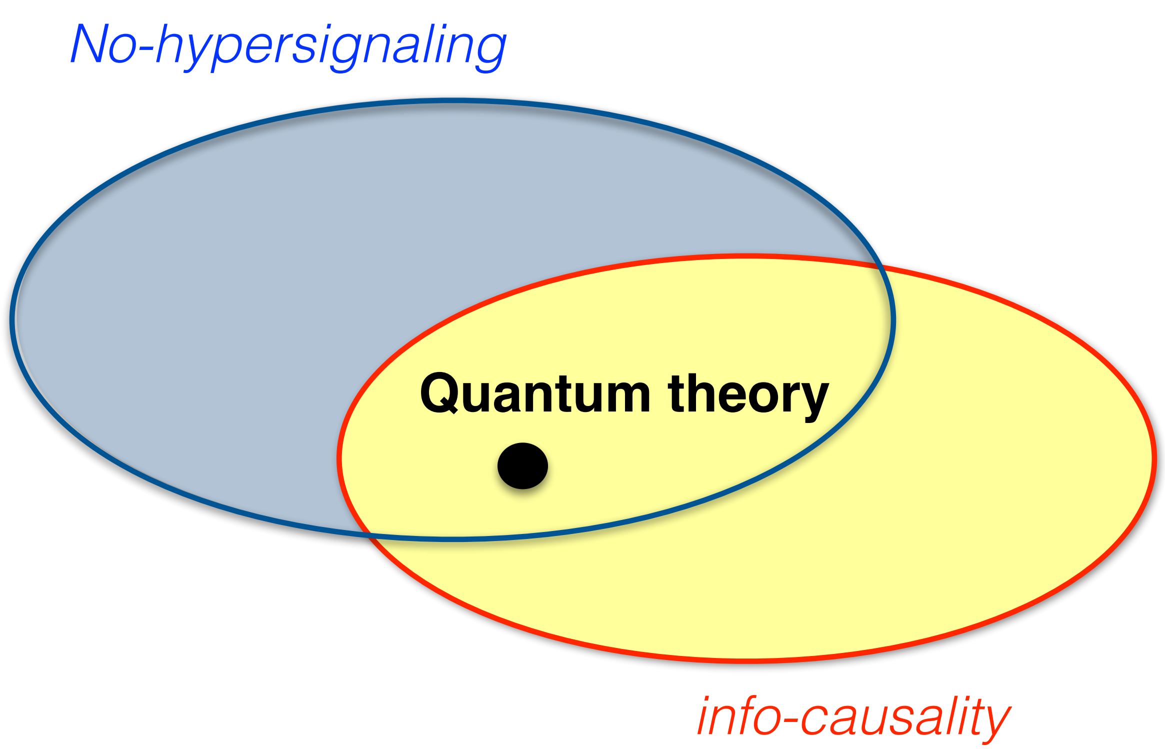
this game is ok!

$$\max_{q \in \mathcal{P}_4^{7\to 7}} \sum_{xy} g_{xy} q_{xy} < \sum_{xy} g_{xy} p_{xy}$$

$$\frac{10}{21} \qquad \frac{1}{2}$$

$$g = \frac{1}{21}\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 \end{pmatrix}$$

Outlook *Is no-hypersignaling trivial?*



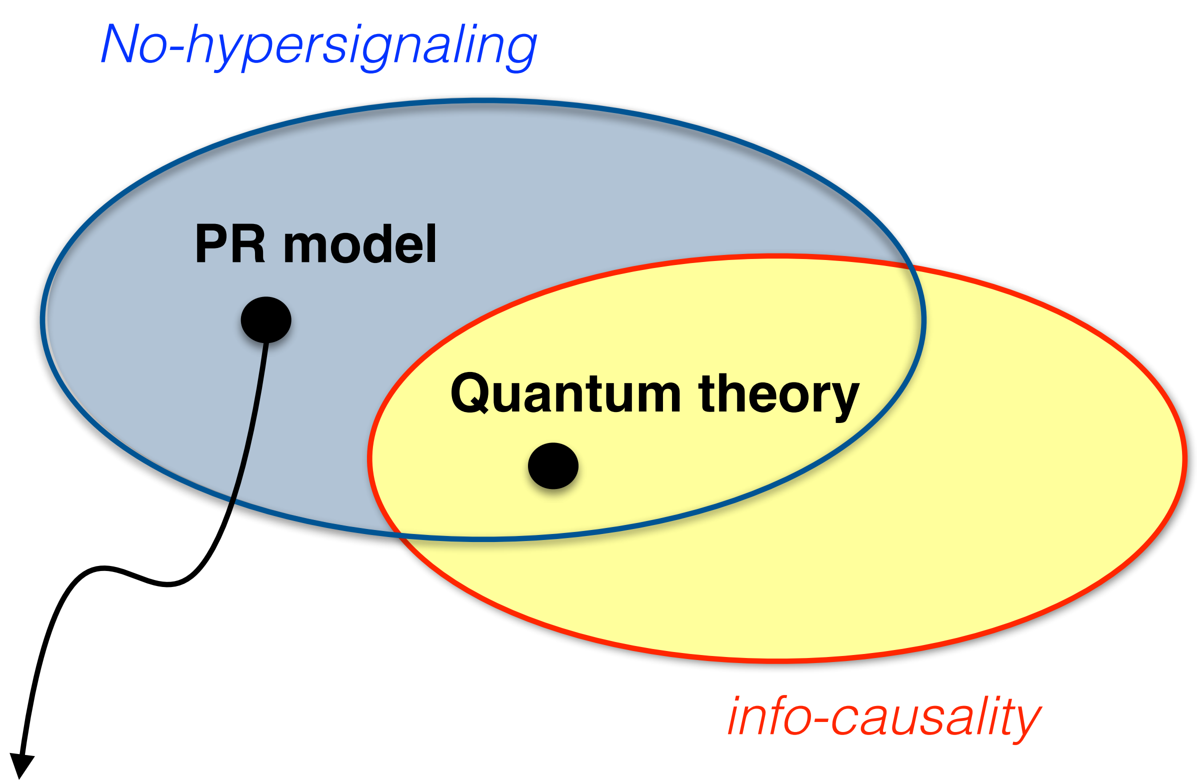
NOTICE: violation of info-causality

$\Rightarrow$ $\exists$ entangles state

violation of no-hypersignaling

$\Rightarrow$ $\exists$ entangled measure

Outlook *Is no-hypersignaling trivial?*



PR model:

- info-causality ✗
- no-hypersignaling ✓

  (only separable measurements)

NOTICE: violation of info-causality

$\Rightarrow$ $\exists$ entangles state

violation of no-hypersignaling

$\Rightarrow$ $\exists$ entangled measure

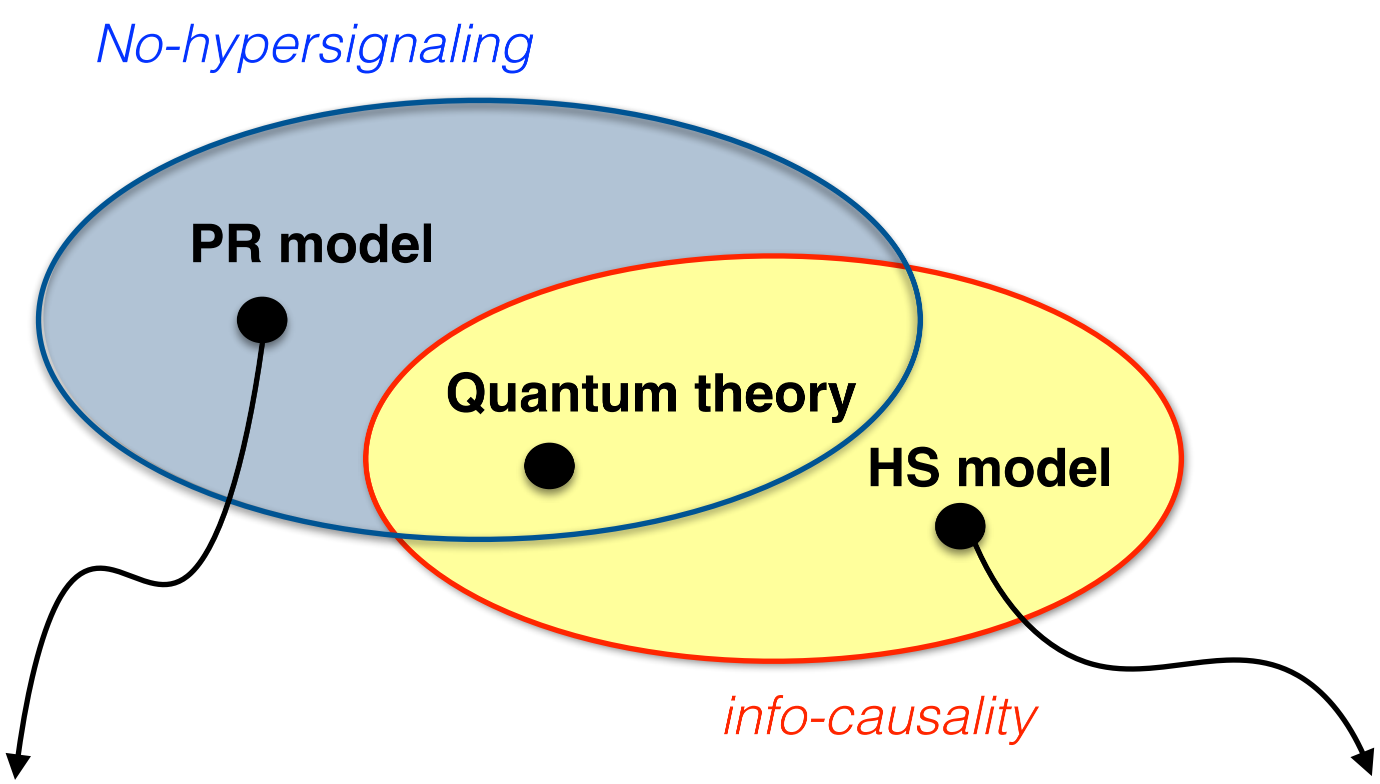
Outlook *Is no-hypersignaling trivial?*

NOTICE: violation of info-causality

$\Rightarrow$ $\exists$ entangles state

violation of no-hypersignaling

$\Rightarrow$ $\exists$ entangled measure



PR model:

- info-causality ✗
- no-hypersignaling ✓
  (only separable measurements)

HS model:

- info-casuality ✓
  (only separable states)
- no-hypersignaling ✗

violation of no-hypersignaling

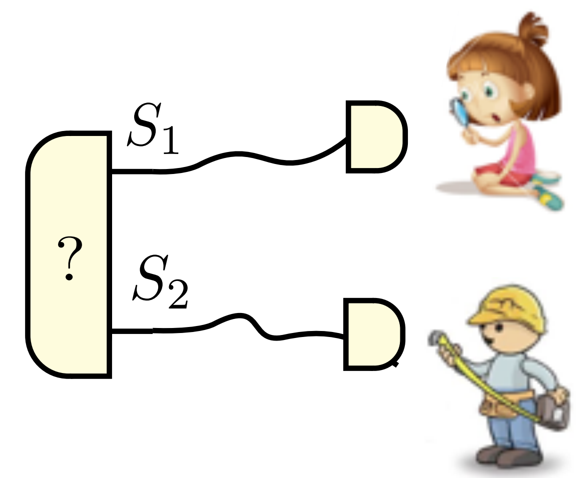
$\kappa(S)$ signaling dimension

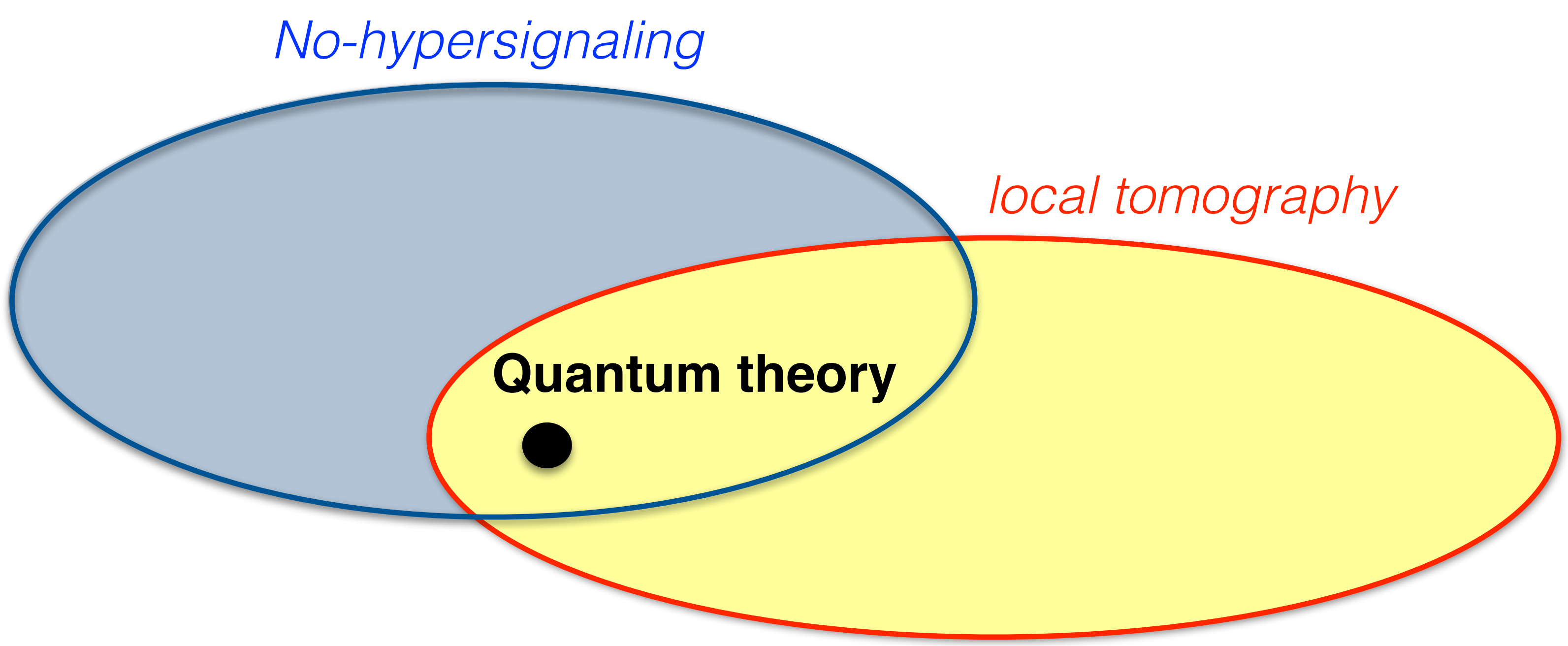
$$\kappa(S_1 \otimes S_2) > \kappa(S_1)\kappa(S_2)$$

violation of local tomography

$D(S)$ lineal dimension

$$D(S_1 S_2) > D(S_1)D(S_2)$$





violation of no-hypersignaling

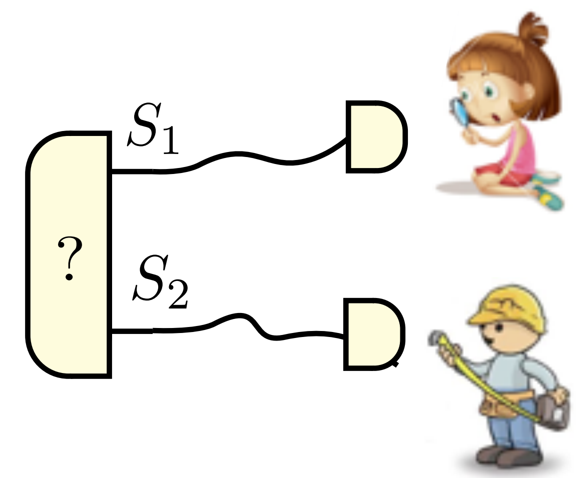
$\kappa(S)$ signaling dimension

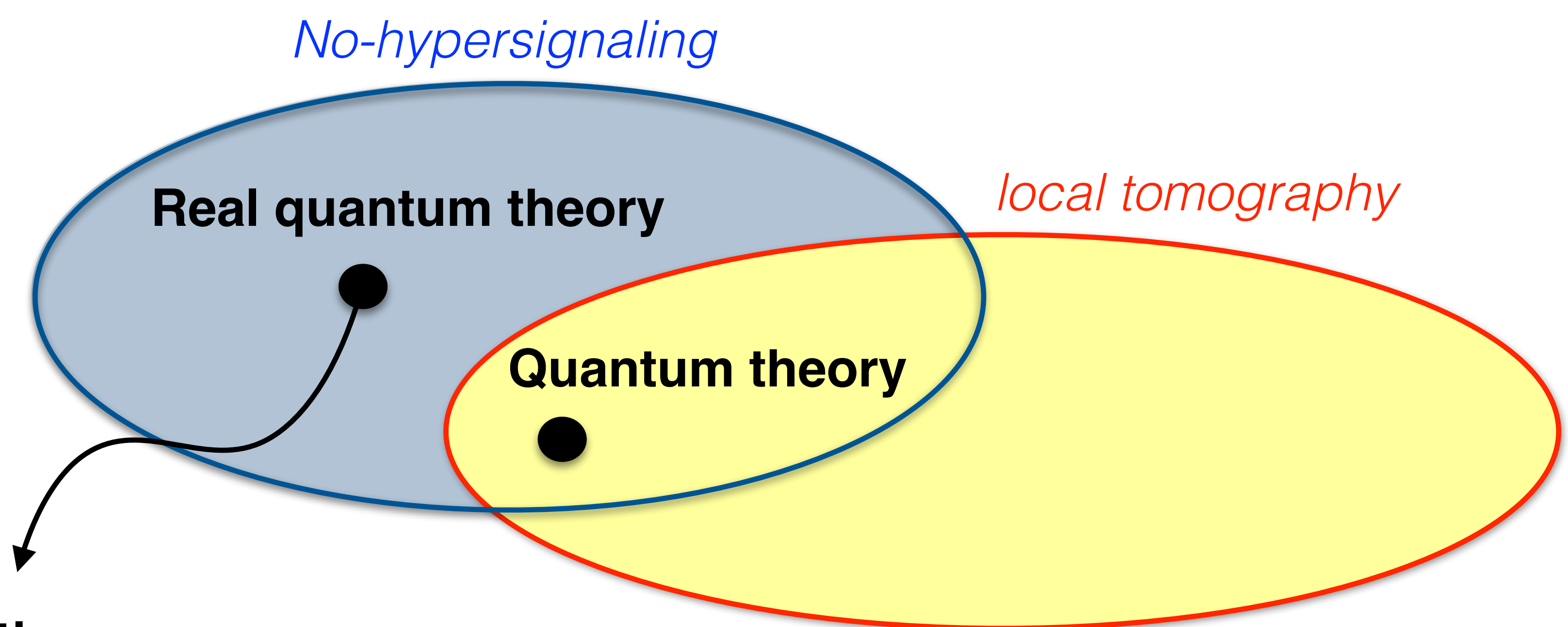
$$\kappa(S_1 \otimes S_2) > \kappa(S_1)\kappa(S_2)$$

violation of local tomography

$D(S)$ lineal dimension

$$D(S_1 S_2) > D(S_1)D(S_2)$$





Real quantum theory

- local tomography ✗

  L. Hardy and W. K. Wootters, FOOP 42, 45(2012)

- no-hypersignaling ✓

  it is a superselection of quantum theory

  G. M. D'Ariano, F. Manessi, P. Perinotti and AT, IJMPA (2014)

violation of no-hypersignaling

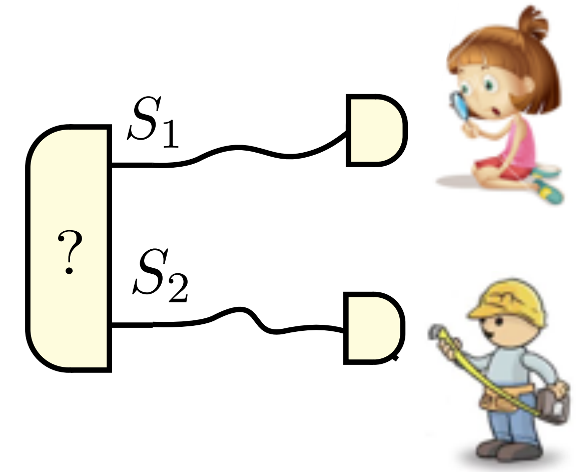
$\kappa(S)$ signaling dimension

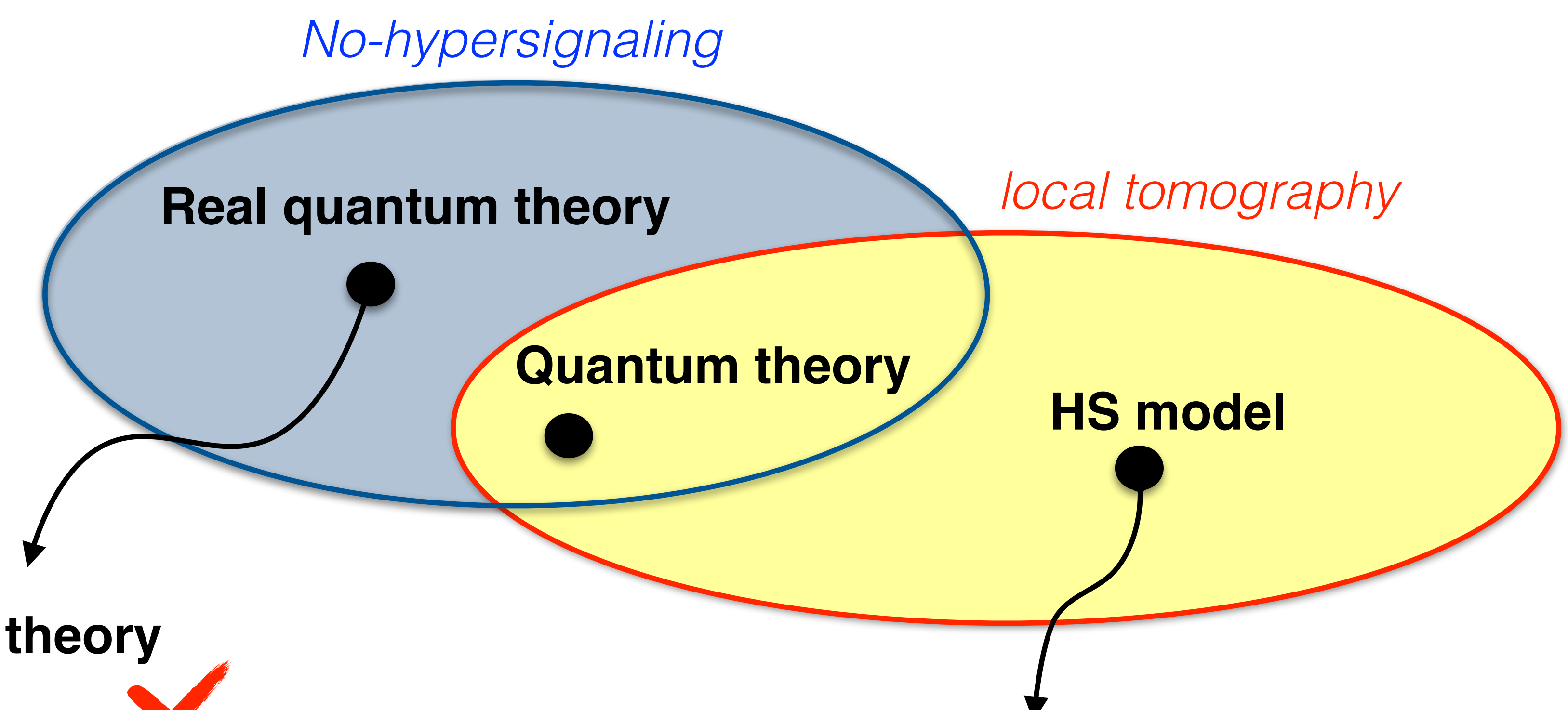
$$\kappa(S_1 \otimes S_2) > \kappa(S_1)\kappa(S_2)$$

violation of local tomography

$D(S)$ lineal dimension

$$D(S_1 S_2) > D(S_1)D(S_2)$$





Real quantum theory

- local tomography ✗

  L. Hardy and W. K. Wootters, FOOP 42, 45(2012)

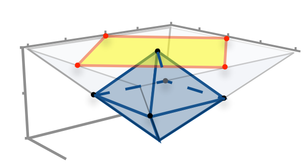
- no-hypersignaling ✓

  it is a superselection of quantum theory

  G. M. D'Ariano, F. Manessi, P. Perinotti and AT, IJMPA (2014)

HS model

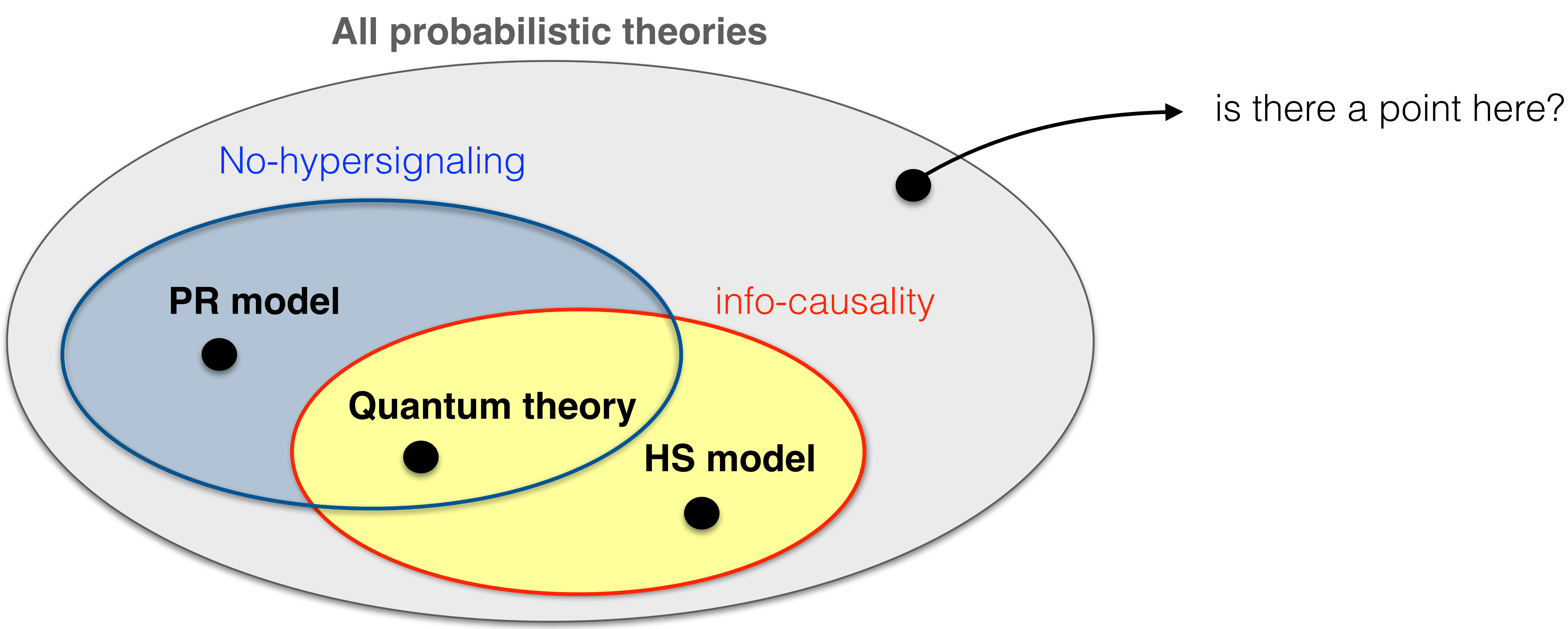
- local tomography ✓



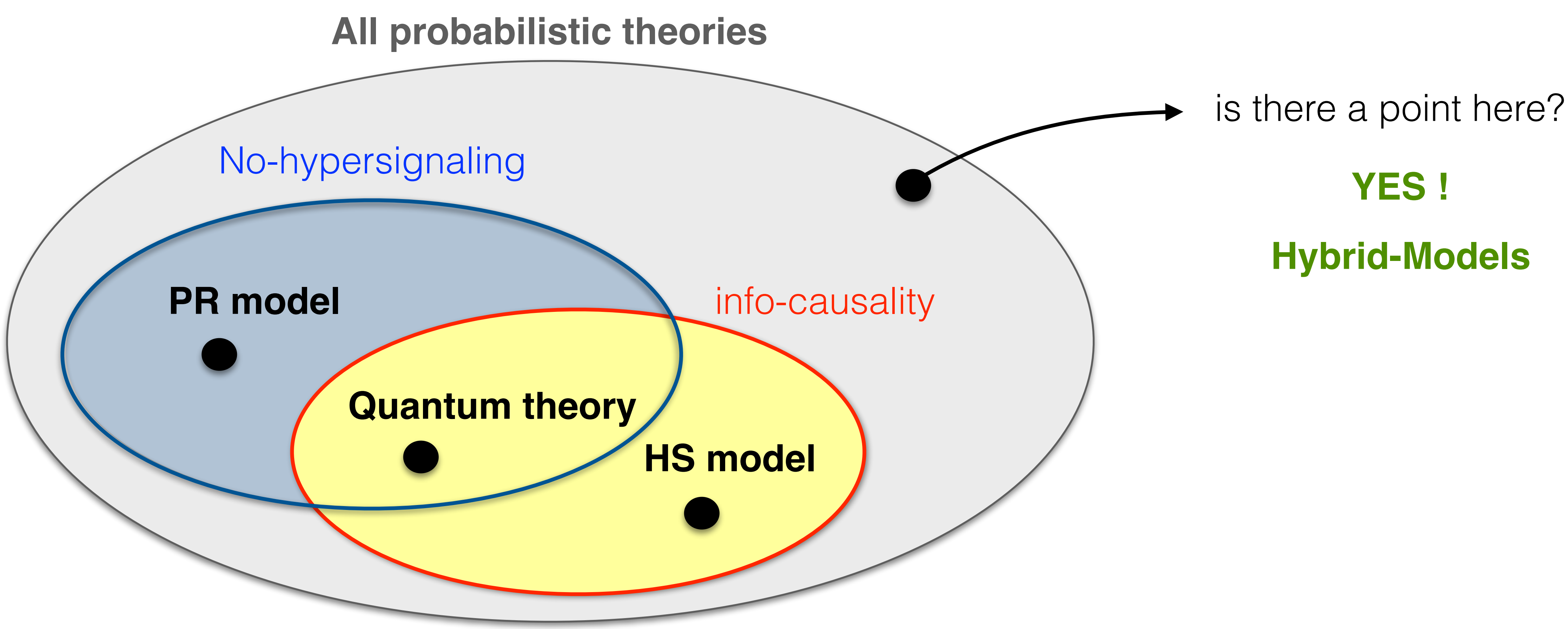
$$D(S) = 3$$
$$D(SS) = D^2(S) = 9$$

- no-hypersignaling ✗

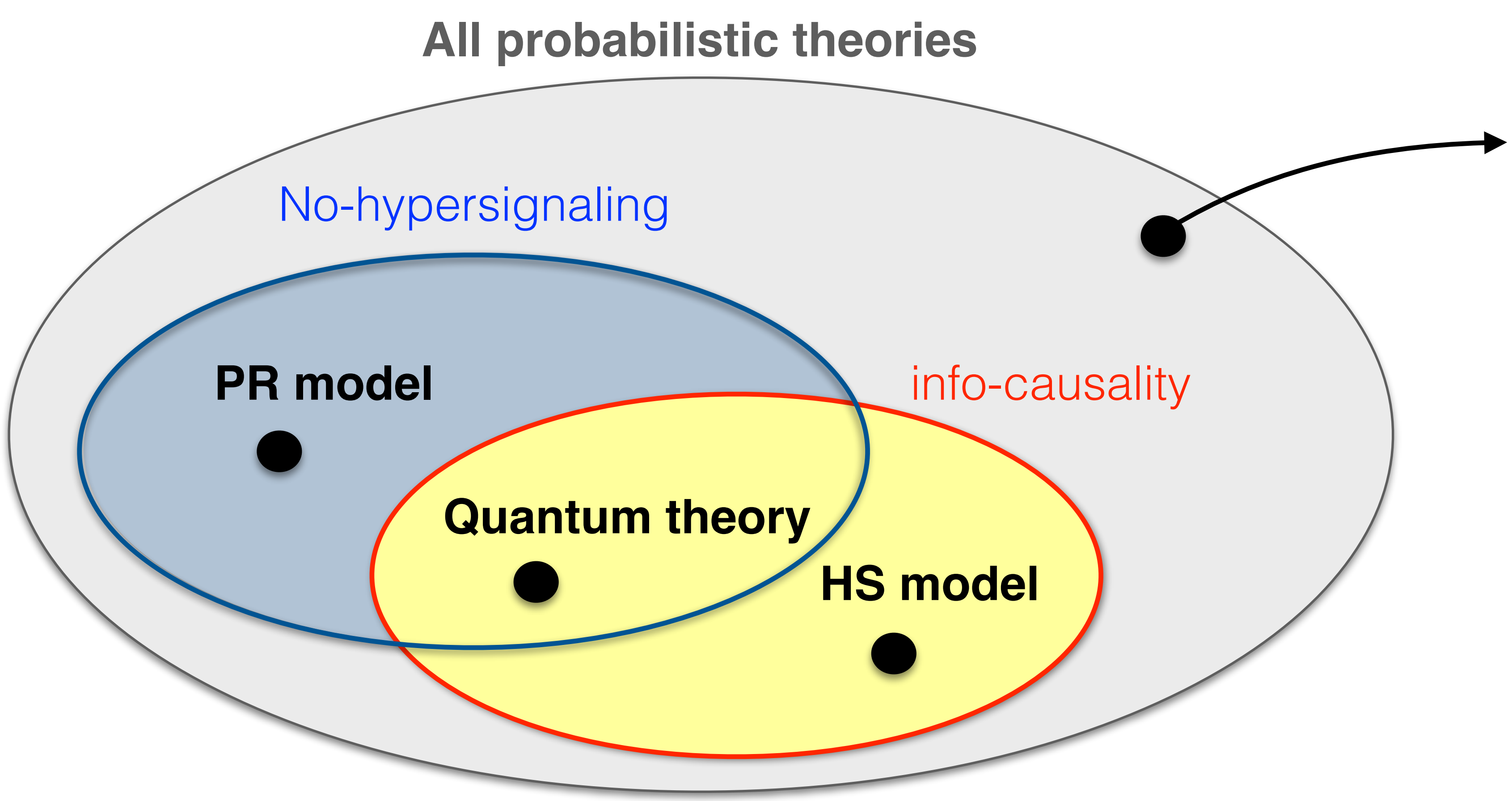
Can a theory have both superquantum space-time and time-like correlations



Can a theory have both superquantum space-time and time-like correlations



Can a theory have both superquantum space-time and time-like correlations

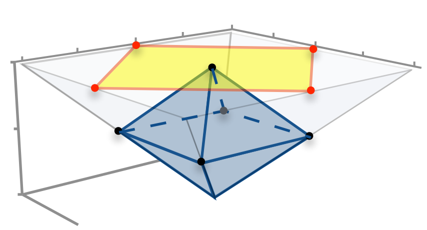


is there a point here?

YES !

Hybrid-Models

Theories compatible
with
elementary system

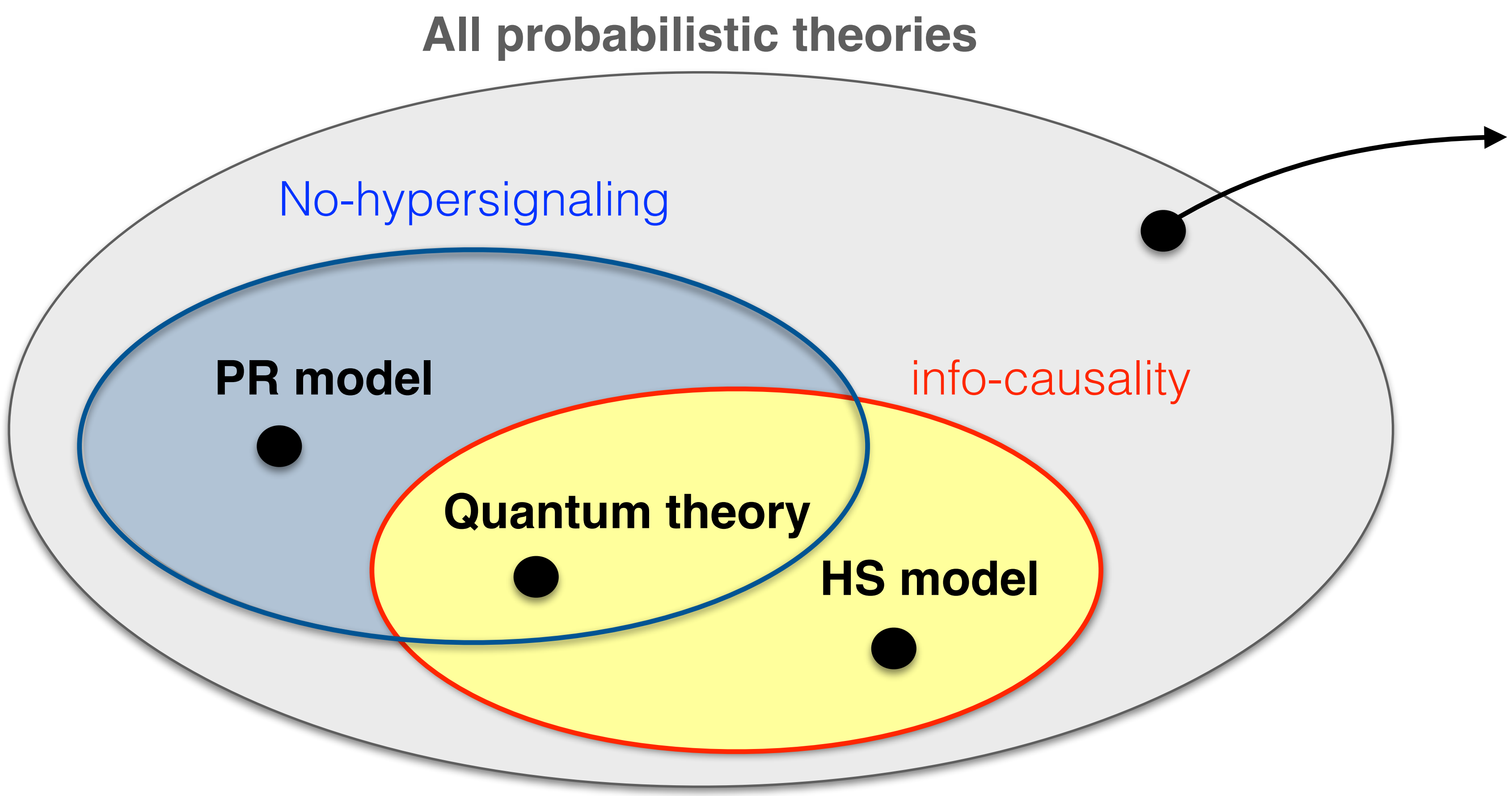


PR-Model only factorized effects

HS-Model only factorized states

Hybrid-Models 2 entangled states and 2 entangled effects

Can a theory have both superquantum space-time and time-like correlations

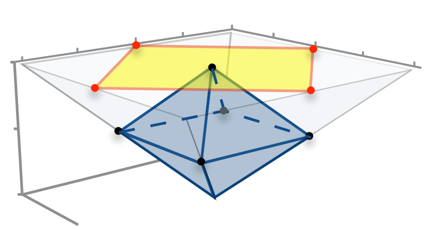


is there a point here?

YES !

Hybrid-Models

Theories compatible
with
elementary system



PR-Model only factorized effects

HS-Model only factorized states

Hybrid-Models 2 entangled states and 2 entangled effects

Thank you!