Quantum formulation of the Einstein Equivalence Principle

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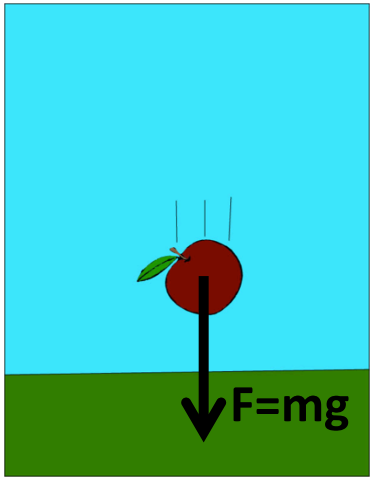
Nature Physics (2018) *in press*

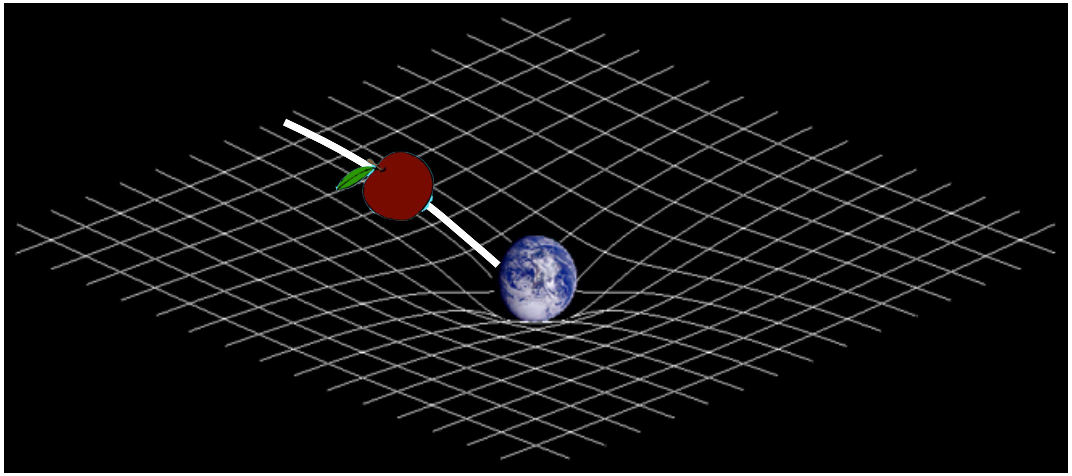
Outline

- Preliminaries:
  - Why study equivalence principle?
  - Mass-energy of a composite quantum particle?
- Framework: relativistic composite particles
- Quantum formulation of the Einstein Equivalence Principle
- Q-EEP: conceptual aspects and experimental tests

Why study validity of EEP

General relativity: free fall is inertial motion





$$S = -mc^2 \int \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\,dt$$

action (dynamics)

metric (geometry)

What are the conditions for this geometric picture of gravity?

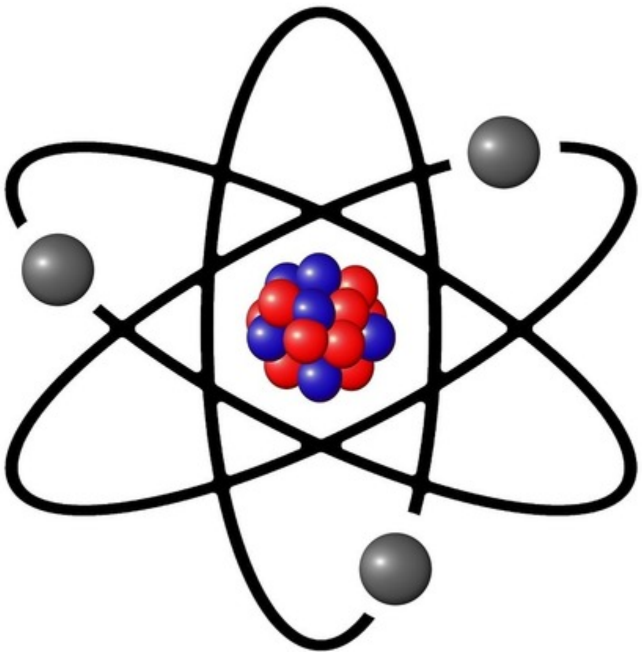
Mass-energy equivalence

^{16}O binding energy ≈ ***0.8%***

elephant binding energy ≈ 80g

^{56}Fe: biding energy ≈ ***1%***

Mass-energy equivalence



Static part (effective mass parameter)

Dynamical part – drives internal evolution

$$\hat{M} = m\hat{\mathbb{1}}_{int} + \frac{\hat{H}_{int}}{c^2}$$

masses of p,n,e ≈900MeV,
binding energies ≈9MeV

Ionisation energy ≈10eV
Hyperfine transitions ≈10^{-5}eV

Relativistic composite quantum particles

N bosonic fields in curved space-time

mass matrix

$$S = \int d^4x \sqrt{-g} \left(\sum_J g^{\mu\nu} \partial_\mu \varphi_J \partial_\nu \varphi_J + \sum_{J,K} \boxed{M^2_{JK}} c^2 \varphi_J \varphi_K \right)$$

Euler-Lagrange Eq.: $\left(g^{\mu\nu} \nabla_\mu \nabla_\nu - \hat{M}^2 c^2 \right) \Phi = 0. \quad \Phi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_N \end{pmatrix}$

Diagonalization: $(g^{\mu\nu} \nabla_\mu \nabla_\nu - m_a^2 c^2) \phi_a = 0$

Ansatz for the Klein-Gordon field: $\phi_a = e^{i(c^2 S_0(x) + S_1(x) + c^{-2} S_2(x) \dots)}$

Schrödinger Eq: $\hat{H} = \sqrt{-g_{00} \left(c^2 \hat{P}_i \hat{P}^i + \hat{M}^2 c^4 \right)}$

$$\hat{H} = \hat{M} c^2 + \frac{\hat{p}^2}{2\hat{M}} + \boxed{\hat{M}} \Phi(\hat{x}) \qquad \hat{M} = m \hat{\mathbb{1}}_{int} + \frac{\hat{H}_{int}}{c^2}$$

C. Kiefer and T. P. Singh, Phys. Rev. D **44,** 1067 (1991), C. Anastopoulos, B. L. Hu, New J. Phys **16** (2014), M.Z. *Quantum Systems Under Gravitational Time Dilation* (Springer Int. Publ, 2017).

Composite quantum systems in general relativity

H more compactly:

$$H = H_{cm} + H_{int}\left(1 + \frac{\Phi(x)}{c^2} - \frac{p^2}{2m^2c^2}\right)$$

Interaction between c.m. and internal d.o.fs

$$H_{cm} = mc^2 + \frac{p^2}{2m} + m\Phi(x) + \ \ldots$$

Composite quantum systems in general relativity

$$H = H_{cm} + H_{int}\left(1 + \frac{\Phi(x)}{c^2} - \frac{p^2}{2m^2c^2}\right)$$

Gravitational time dilation

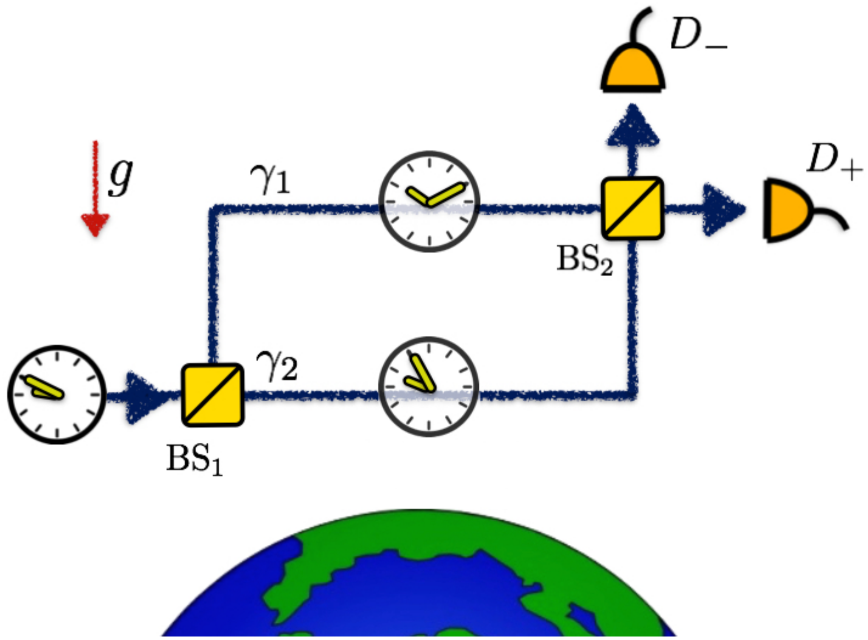
Special relativistic time dilation

$$H_{int} = \hbar\omega \longrightarrow \hbar\omega\left(1 + \frac{gh}{c^2}\right) \quad \hbar\omega\left(1 - \frac{p^2}{2(mc)^2}\right)$$

Time dilation ⇔ mass-energy equivalence

What new effects are predicted by this Hamiltonian?

Quantum Twin Paradox



„Single twin“ in a superposition of being older-and-younger than himself...

<u>Clock interferometry</u>

- *M.Z., F.Costa, I.Pikovski, Č.Brukner. Nat. commun.* **2**, *505 (2011)*

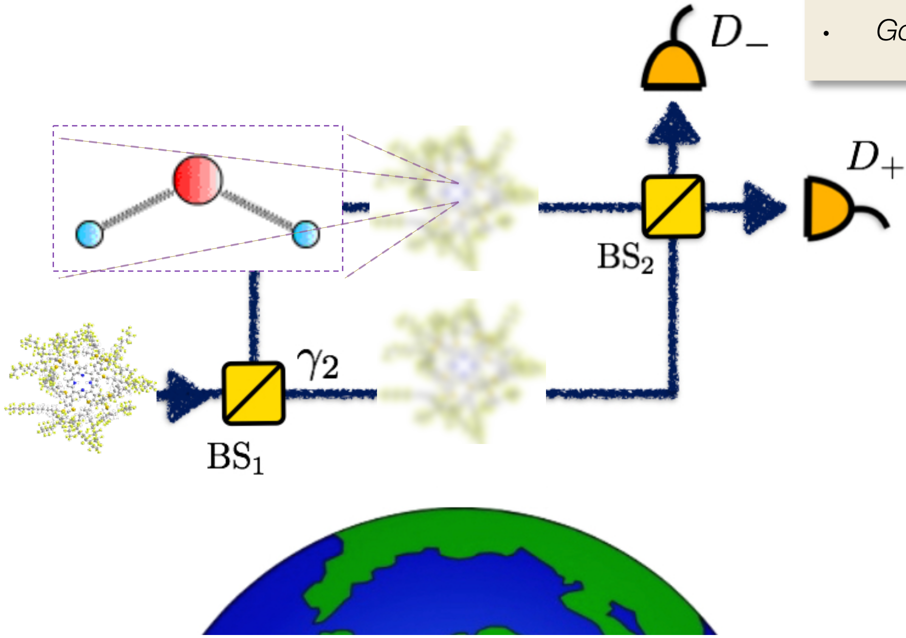
<u>Photons a clocks (Shapiro delay)</u>

- *M.Z., F.Costa, I Pikovski, T.Ralph, Č.Brukner. Class. Quan. Grav.* **29**, *224010 (2012)*
- *A. Brodutch et al. PR D* **91**, *064041 (2015)*

<u>Electron as s clock</u>

- *P. Bushev, J. H. Cole, D. Sholokhov, N. Kukharchyk, M.Z. NJP* **18** 093050 (2016)

Decoherence from time dilation



Decoherence from time dilation

- *Pikovski, M.Z., Costa, Brukner. Nat Phys 11 668—672 (2015)*
- *M.Z., Pikovski, Costa, Brukner JPCS 723 012044 (2016)*
- *Pikovski, M.Z., Costa,Brukner. NJP 19 025011 (2017)*

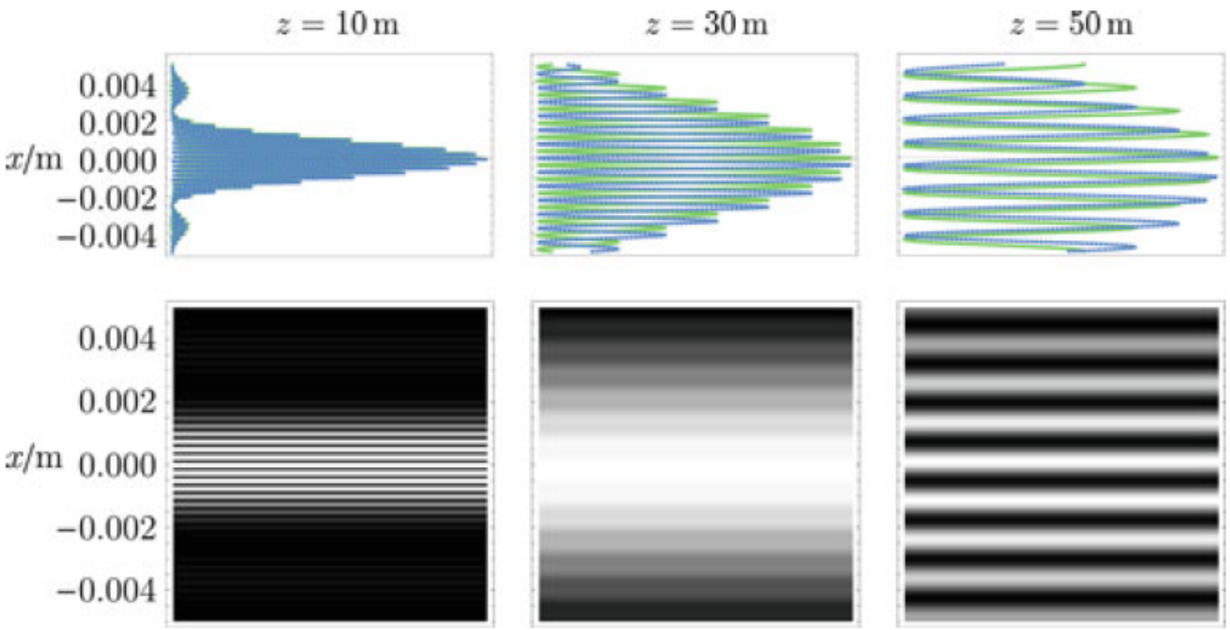
see also

- *Gooding, Unruh PRD 90(4) 044071 (2014)*
- *Gooding, Unruh, Found.Physics 1-13 (2015)*

How does interference pattern fall?

Dispersion from mass-energy equiv.

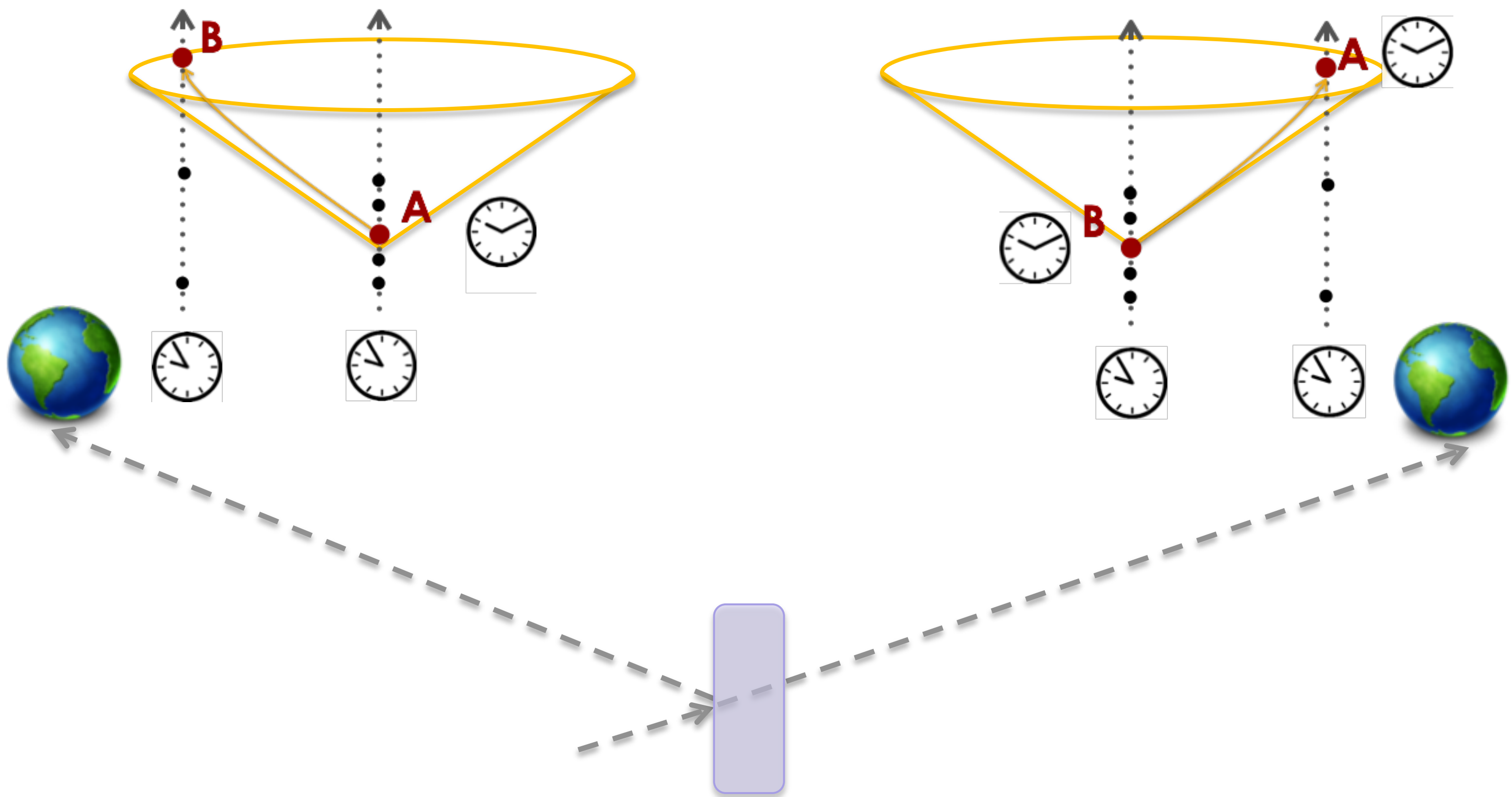
- *B. Pang, F. Y. Khalili, Y. Chen On universal decoherence under gravity: a perspective through the Equivalence Principle. arxiv:1603.01984:*
- *Patrick J. Orlando, Felix A. Pollock and Kavan Modi, How Does Interference Fall?, Lectures on General Quantum Correlations and their Applications, Quantum Science and Technology*



Plots from: Patrick J. Orlando, Felix A. Pollock and Kavan Modi,
How Does Interference Fall?,
in *Lectures on General Quantum Correlations and their Applications, Quantum Science and Technology*

Quantum causal structures

- *M.Z., F.Costa, I.Pikovski, Č.Brukner. Entanglement of temporal order from quantum theory and gravity.*
- *E. Castro Ruiz, F. Giacomini, Č.Brukner, Measuring time with physical clock. arxiv: 1507.01955*



Entanglement between temporal order of time-like separated events

What about the Einstein Equivalence Principle?

The Einstein Equivalence Principle (EEP)

Weak Equivalence Principle (WEP):

Test bodies fall with the same acceleration

Local Lorentz Invariance (LLI):

In a local freely falling frame, physics (non-gravitational) is independent of frame's velocity

Local Position Invariance (LPI):

In a local freely falling frame, physics (non-gravitational) is independent of frame's location

quantum Hamiltonian

$$\hat{H}^Q = mc^2 + \frac{\hat{p}^2}{2m} + m\Phi(\hat{x}) + \hat{H}_{int} - \hat{H}_{int}\frac{\hat{p}^2}{2m^2c^2} + \hat{H}_{int}\frac{\Phi(\hat{x})}{c^2}$$

classical Hamiltonian

$$H^C = mc^2 + \frac{p^2}{2m} + m\Phi(x) + E - E\frac{p^2}{2m^2c^2} + E\frac{\Phi(x)}{c^2}$$

---

quantum test model violating EEP

$$\hat{H}^Q_{test} = m_r c^2 + \frac{\hat{p}^2}{2m_i} + m_g\Phi(\hat{x}) + \hat{H}_{int,r} - \hat{H}_{int,i}\frac{\hat{p}^2}{2m_i^2c^2} + \hat{H}_{int,g}\frac{\Phi(\hat{x})}{c^2}$$

classical test model violating EEP

$$H^C_{test} = m_r c^2 + \frac{p^2}{2m_i} + m_g\Phi(x) + E_r - E_i\frac{p^2}{2m_i^2c^2} + E_g\frac{\Phi(x)}{c^2}$$

Quantum vs Classical Equivalence Principle

	CLASSICAL	QUANTUM
	$M_\alpha := m_\alpha + \frac{E_\alpha}{c^2} \quad \alpha = r, i, g$	$\hat{M}_\alpha := m_\alpha + \frac{\hat{H}_{int,\alpha}}{c^2} \quad \alpha = r, i, g$
WEP (universality of free fall)	$M_i = M_g$	$\hat{M}_i = \hat{M}_g$
LLI (universality of SR time dilation)	$E_r = E_i$	$\hat{H}_{int,r} = \hat{H}_{int,i}$
LPI (universality of grav. time dilation)	$E_r = E_g$	$\hat{H}_{int,r} = \hat{H}_{int,g}$

# of parameters for N-level system: 2N-1 < 2N^2-1

- classical conditions – special case $[\hat{H}_{int,\alpha}, \hat{H}_{int,\beta}] = 0$
- conceptual & qualitative difference between EEP in class. and quant. theories
- the difference comes due to the internal degrees of freedom

Validity of EEP in QM requires independent experimental verification

„Quantum" tests of the EEP?

tests of the **Classical** EEP:
violation of the EEP can be explained by H^{C}_{test}

compatible with $[\hat{M}_\alpha, \hat{M}_\beta] = 0$ although $\hat{M}_r \neq \hat{M}_i \neq \hat{M}_g$

Quantum vs classical WEP

$$\hat{H}^{C}_{test} = \frac{\hat{p}^2}{2M_i} + M_i \underbrace{\left(M_g M_i^{-1}\right) g}_{a} \hat{z}$$

eigenstates free fall with different gravitational accelerations

$$M_g M_i^{-1} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}$$

classical WEP: $r_1 = r_2 = 1$

$$\hat{H}^{Q}_{test} = \frac{\hat{p}^2}{2\hat{M}_i} + \hat{M}_i \underbrace{\left(\hat{M}_g \hat{M}_i^{-1}\right) g}_{\hat{a}} \hat{z}$$

states free fall with different accelerations in superposition

$$\hat{M}_g \hat{M}_i^{-1} = \begin{pmatrix} r_1 & r \\ r^* & r_2 \end{pmatrix}$$

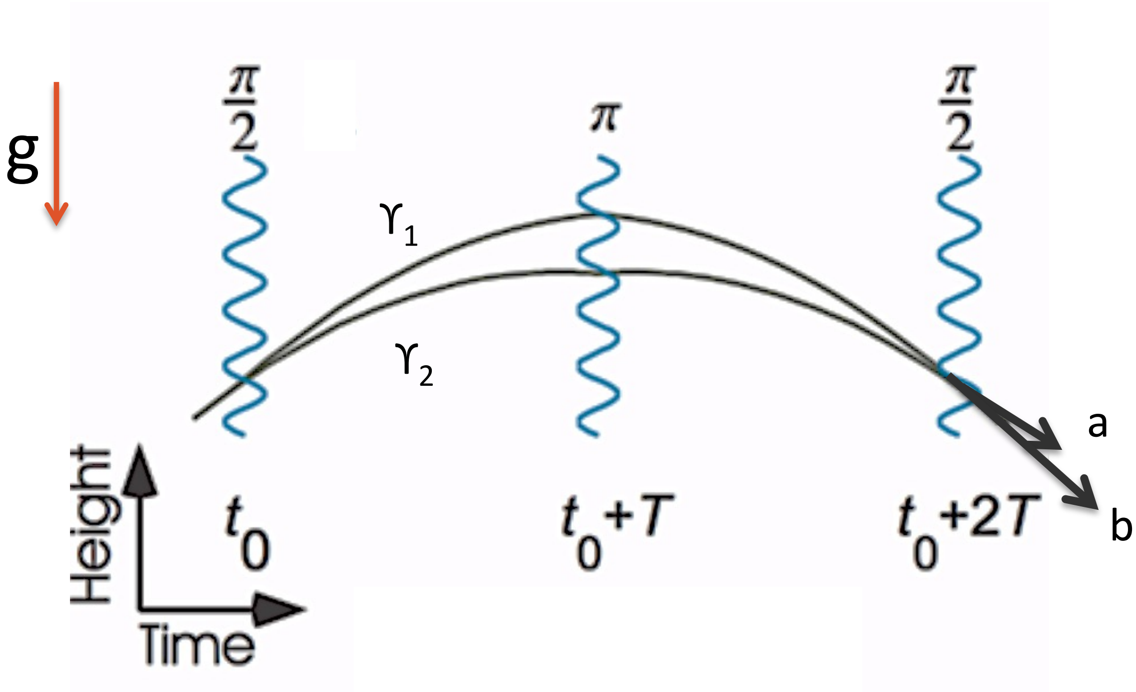
quantum WEP: ... and r=0

Eötvös ratio:

$$\eta_{A-B} = 2\frac{|a_A - a_B|}{|a_A + a_B|}$$

A, B= diff. states/substances

Quantum interferometry tests of WEP



- T=150 ms → 2π = 10^{-6} g
- Sensitivity 10^{-9} g/shot

$$|\Psi_a\rangle \propto \left(|\gamma_1\rangle + e^{i\Delta\Phi}|\gamma_2\rangle\right) \qquad \Delta\Phi = k_e g T^2$$

$$N_a = \frac{N}{2}(1 + \cos[\Delta\Phi])$$

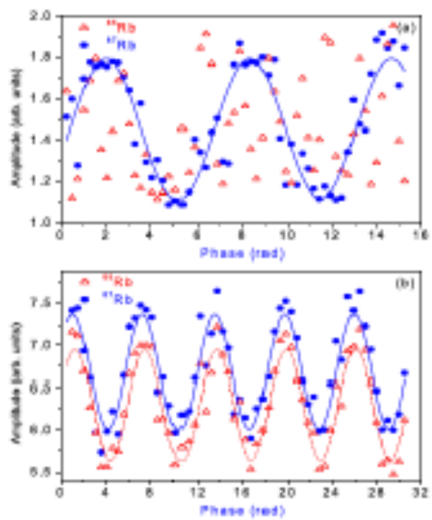
Atom interferometers can measure gravitational acceleration

Tests of WEP thus far

→ tests of classical WEP using quantum sensors

Different species/isotopes

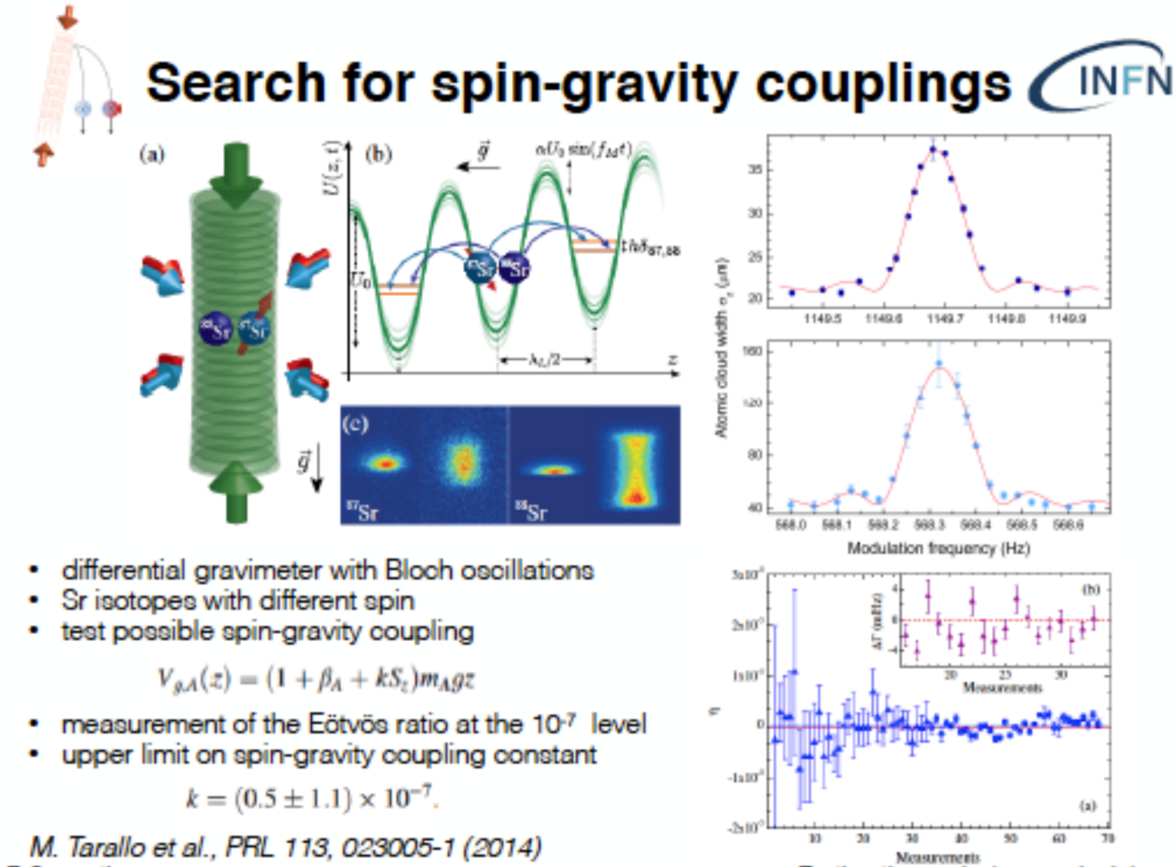
- Tests already achieved (at $10^{-7} \div 10^{-8}$ level)
  - ^{87}Rb-^{85}Rb [*A. Bonnin et al., PRA 88, 043615 (2013)*]
  - K-Rb [*D. Schlippert et al., PRL 112, 203002 (2014)*]
  - ^{87}Sr-^{88}Sr [*M. G. Tarallo et al., PRL 113, 023005 (2014)*]
  - ^{87}Rb-^{85}Rb [*L. Zhou et al., PRL 115, 013004 (2015)*]
- Many other planned/ongoing



Slide courtesy: *Fiodor Sorrentino* (*INFN Sezione di Genova, Italy)*

Tests of WEP thus far

→ tests of classical WEP using quantum sensors



Slide courtesy: *Fiodor Sorrentino* (*INFN Sezione di Genova, Italy)*

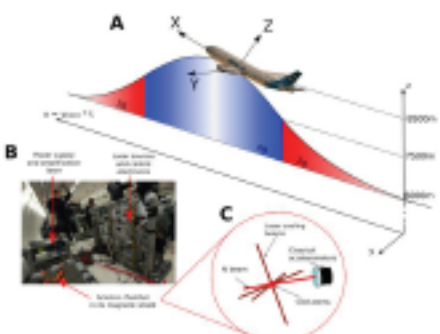
Tests of WEP thus far

→ tests of classical WEP using quantum sensors

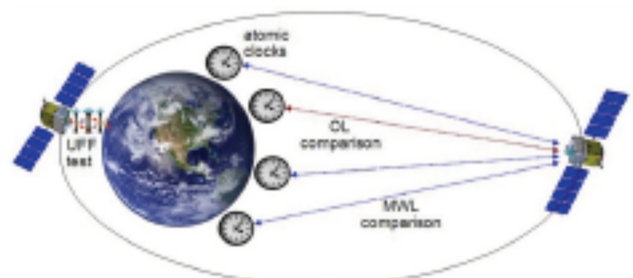
Tests in microgravity

INFN

- Airborne: ICE (CNES)
- Sounding rocket: QUANTUS (DLR)



- STE-QUEST mission proposal
  - Primary Goal: UFF test and red shift measurement
  - Observables: Clock redshift measurements;
    - Differential acceleration measurements of freely falling atoms
  - Instruments:
    - A microwave clock based on laser cooled rubidium atoms;
    - A differential atom interferometer operating on the two rubidium isotopes;
    - Time and frequency transfer links in the microwave and optical domain for space-to-ground comparisons of clocks.
  - Orbit: Highly elliptical orbit around the Earth
  - Type: M-class mission



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Tests of WEP thus far

So far experiments only tested classical WEP,
at most using quantum-based setups!

Can we test quantum formulation of WEP?

Quantum formulation and proposals for quantum tests of EEP

- *M.Z., Č.Brukner Nat Phys (2018) in press*
- *Orlalndo et al. A Simple Test of the Equivalence Principle(s) for Quantum Superpositions. arXiv: 1511.02943*
- *Zhang, M. Weak values obtained from Einstein equivalence principle, arXiv:1601.06077*

G. Rosi, G. D'Amico, L. Cacciapuoti, F. Sorrentino, M. Prevedelli, M. Zych, Č. Brukner & G.M. Tino *Quantum test of the equivalence principle for atoms in coherent superposition of internal energy states*,
Nature Communications **8**, 15529 (2017)

Sensitivity to accelerations

$$\hat{H}^Q_{test} = \frac{\hat{p}^2}{2\hat{M}_i} + \hat{M}_i \left(\hat{M}_g \hat{M}_i^{-1}\right) g\hat{z} \qquad \hat{M}_g \hat{M}_i^{-1} = \begin{pmatrix} r_1 & r \\ r^* & r_2 \end{pmatrix} \qquad r = |r| e^{i\phi_r}$$

$$|1\rangle = |F=1, m_F = 0\rangle \qquad \left|\langle 1|\hat{U}|1\rangle\right|^2 \approx \frac{1}{2}\left[1 - \cos\left(k_{\text{eff}} g T^2 r_1\right)\right]$$

$gr_1 = a_1$

$$|2\rangle = |F=2, m_F = 0\rangle \qquad \left|\langle 2|\hat{U}|2\rangle\right|^2 \approx \frac{1}{2}\left[1 - \cos\left(k_{\text{eff}} g T^2 r_2\right)\right]$$

$gr_2 = a_2$

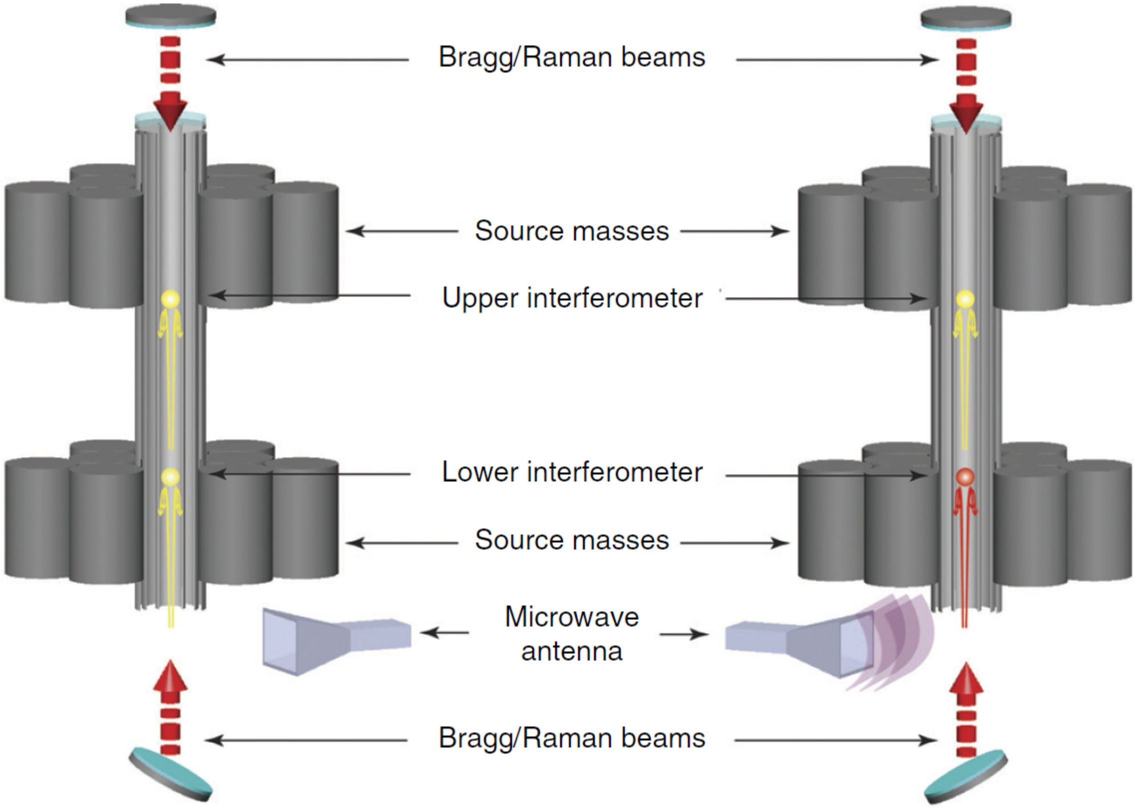
$$|s\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + e^{i\gamma}|2\rangle\right) \qquad \left|\langle 1|\hat{U}|s\rangle\right|^2 + \left|\langle 2|\hat{U}|s\rangle\right|^2$$

$$\approx \frac{1}{2}\left[1 - \cos\left(k_{\text{eff}} g T^2 \left(\frac{r_1 + r_2}{2} + |r| \cos(\gamma + \varphi_r)\right)\right)\right]$$

$gx = a_s$

- classical WEP violation (due to diagonal elements $r_{1,2}$):
  → differential acceleration prop. to $r_1 - r_2$.
- quantum WEP violation (due to r):
  → phase noise in the acceleration measurements due to random phase $\gamma >> 2\pi$

Testing quantum WEP



Upper interferometer always has state F=1

RF pulse preparers the superposition,
phase γ differs from run to run

Bragg gradiometer to compare free fall accelerations of atoms

- prepared in pure hyperfine states F = 1, F = 2 (left)
- prepared in a superposition of the hyperfine states F=1 and F=2 (right)

Results

Three possible configurations:

→ two classical WEP tests: 1—1 , 1—2

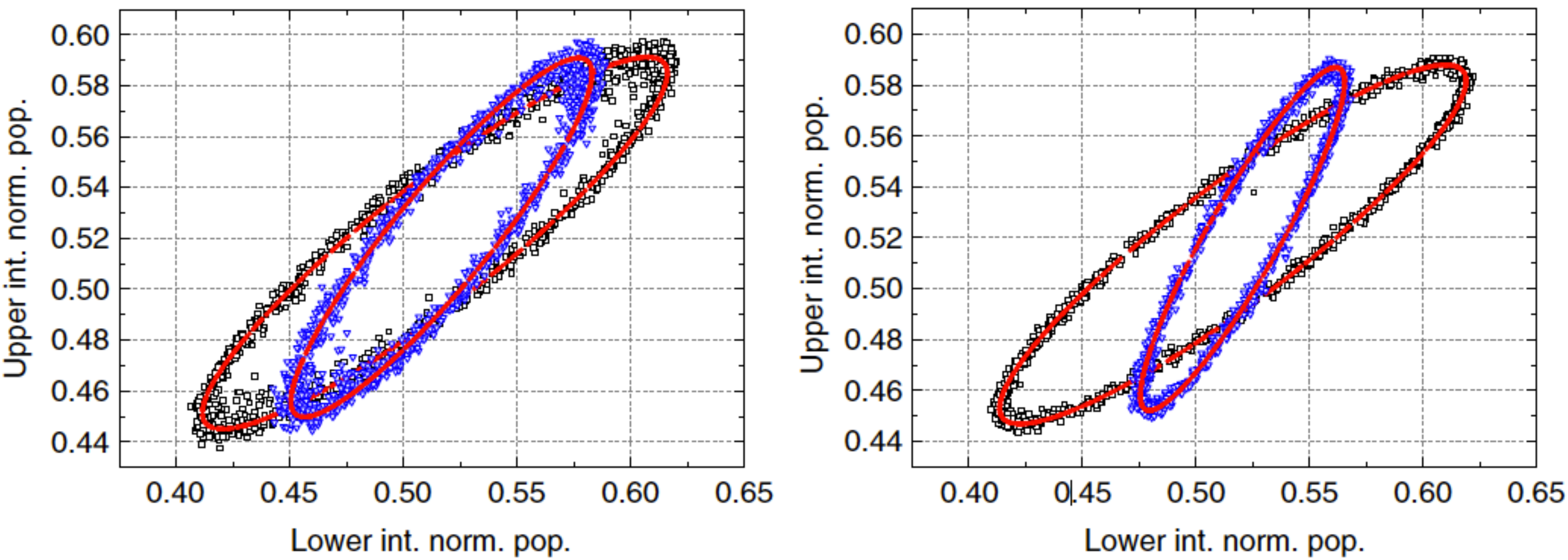
→ one quantum WEP test : 1—s

$\eta_{1-2} \leq 10^{-9}$

$\eta_{1-s} \leq 10^{-9}$ $|r| \leq 5 \cdot 10^{-8}$

black ellipse: 1—1
blue ellipse: 1—s

black: 1—1
blue: 1—2



Conclusion and Outlook

- composite systems subject to GR – novel possibilities for experiments and conceptual insights into QM+GR

- verifying EEP in QM compared to CM:
  – more parameters to test,
  – different experiments needed

- quantum WEP has been tested for the first time,

in the future:

- increase energy gap e.g. using Sr in atom interferometer*,
  → optical energy gap (1.8 eV) instead of hyperfine (28 μeV)

  **Hu, Poli, Salvi, Tino arXiv:1708.05116v1 (to appear in PRL)*

- increase momentum transfer for larger signal

  *($\Delta\Phi = k_e g T^2$, k_e=n*k where n= no. of photon recoils)*