



Dissipative and colored collapse models from the non-interferometric perspective

Laboratori Nazionali di Frascati

2nd - 5th July 2018

Matteo Carlesso (University of Trieste & INFN)

Outlook

- Continuous Spontaneous Localization (CSL) model
 - Connection between theory and non-interferometric experiments
- Colored CSL model
 - Intro
 - Theory and Experimental bounds
- Dissipative CSL model
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Continuous Spontaneous Localization (CSL) model

P. Pearle, *Phys. Rev. A* 39, 2277 (1989). G.C. Ghirardi, P. Pearle and A. Rimini, *Phys. Rev. A* 42, 78 (1990)

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} \hat{H} dt + \frac{\sqrt{\gamma}}{m_0} \int d\mathbf{y} \left(\hat{M}(\mathbf{y}) - \langle \hat{M}(\mathbf{y}) \rangle \right) dW_t(\mathbf{y}) + \right. \\ \left. - \frac{\gamma}{2m_0^2} \int d\mathbf{y} \left(\hat{M}(\mathbf{y}) - \langle \hat{M}(\mathbf{y}) \rangle \right)^2 dt \right] |\psi_t\rangle$$

$$\hat{M}(\mathbf{y}) = \frac{m}{(2\pi\hbar)^3} \sum_{\alpha} \int d\mathbf{Q} e^{\frac{i}{\hbar} \mathbf{Q} \cdot (\hat{\mathbf{x}}_{\alpha} - \mathbf{y})} e^{-\frac{r_C^2}{2\hbar^2} \mathbf{Q}^2}$$

Localization operator: in position

$$\mathbb{E}[dW_t(\mathbf{x})dW_s(\mathbf{y})] = \delta(\mathbf{x} - \mathbf{y})dt$$

Noise : uncorrelated in space and time

Two parameters:

$$\lambda = \frac{\gamma}{(4\pi r_C^2)^{3/2}} = \text{collapse rate}$$

$$r_C = \text{localization resolution}$$

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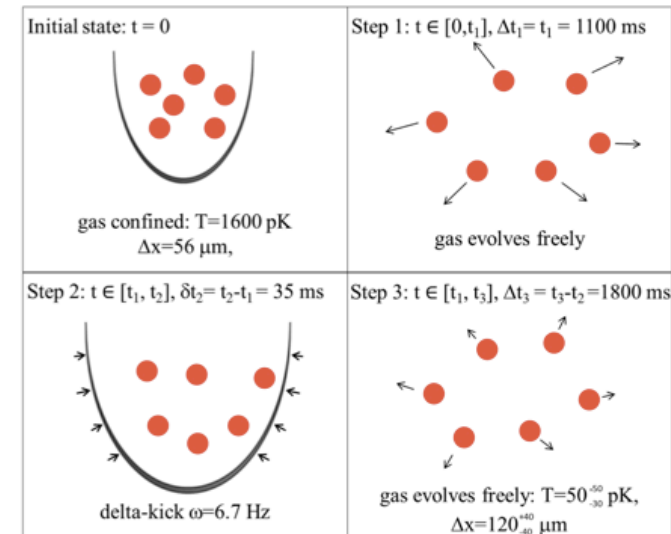
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Cold Atoms

Bilardello *et al.*, *Physica A* 462, 764 (2016).



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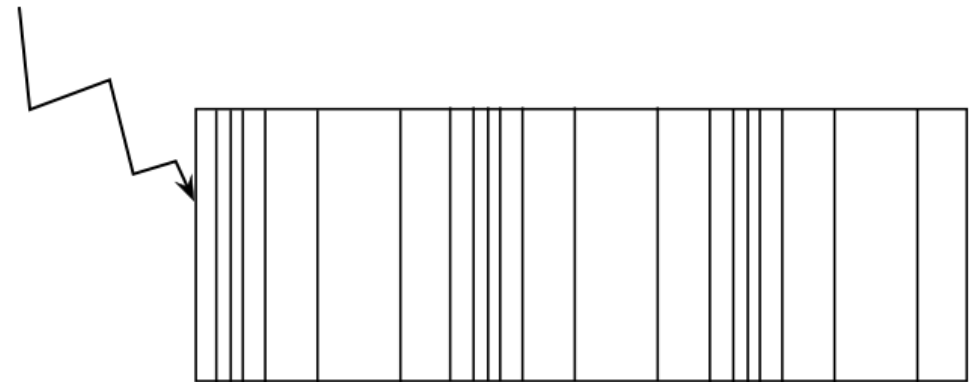
Phonon excitations

Adler *et al.*, *Phys. Rev. A* 97, 052119 (2018),
Bahrami *et al.*, *Phys. Rev. A* 97, 052118 (2018).

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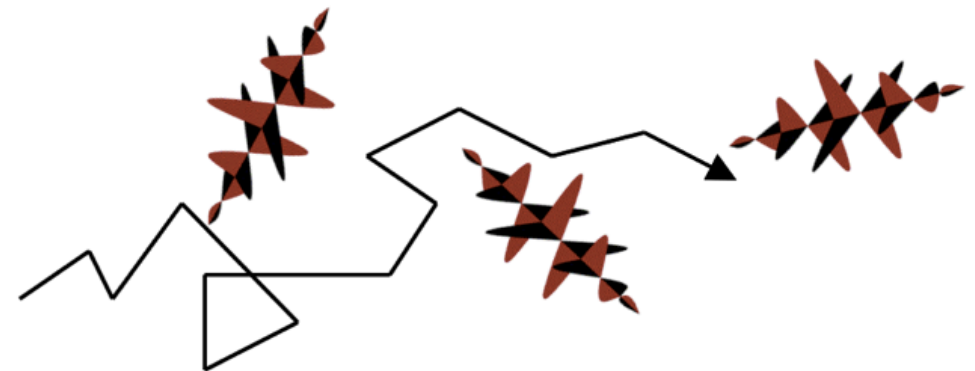
X-ray emission

Piscicchia *et al.*, *Entropy* **19** (2017).

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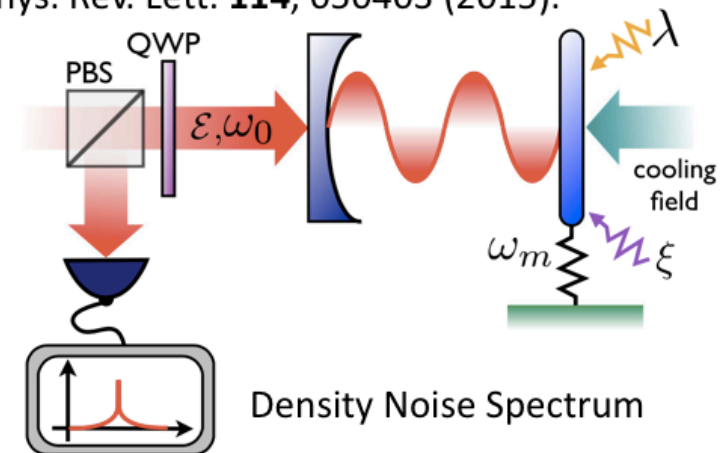
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Optomechanical systems

Bahrami *et al.*, *Phys. Rev. Lett.* **112**, 210404 (2014),
Nimmrichter *et al.*, *Phys. Rev. Lett.* **113**, 020405 (2014),
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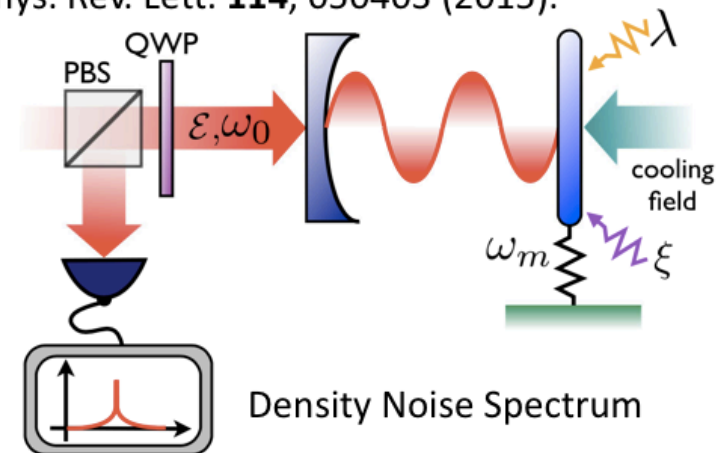
Linear stochastic unravelling

$$d|\psi_t\rangle = -\frac{i}{\hbar} \left(\hat{H} + \hat{V}_{\text{CSL}} \right) dt |\psi_t\rangle$$

$$\hat{V}_{\text{CSL}} = -\frac{\hbar\sqrt{\lambda}}{\pi^{3/4}r_C^{3/2}m_0} \int d\mathbf{y} \hat{M}(\mathbf{y})w(\mathbf{y}, t)$$

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Non-interferometric tests – theory

$$\frac{d}{dt}\hat{x}(t) = \frac{\hat{p}(t)}{M}$$

$$\frac{d}{dt}\hat{p}(t) = -M\omega_0^2\hat{x}(t) - \gamma\hat{p}(t) + \xi(t) + F_{\text{CSL}}(t)$$

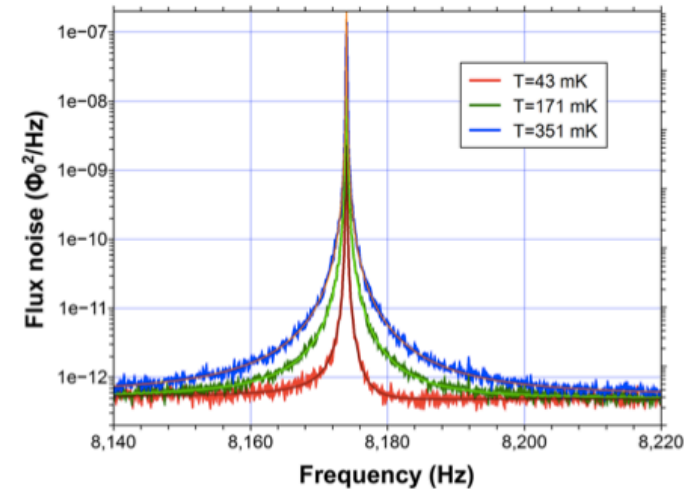
Density Noise Spectrum

$$S_{xx}(\omega) = \frac{1}{4\pi} \int d\Omega \langle \{\tilde{x}(\omega), \tilde{x}(\Omega)\} \rangle$$

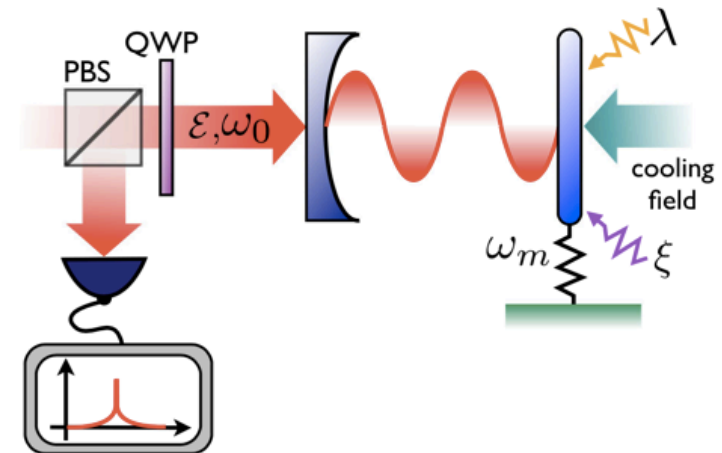
$$= \frac{1}{m^2} \frac{2m\gamma k_B T + S_{FF}(\omega)}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

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Vinante *et al.*, Phys. Rev. Lett. **119**, 110401 (2017).



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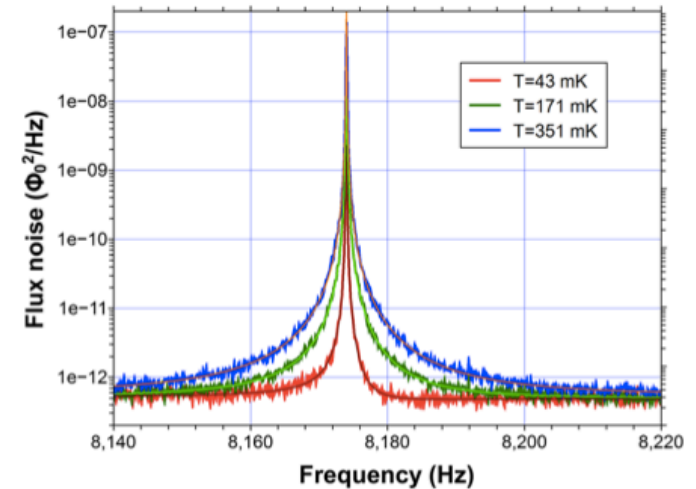
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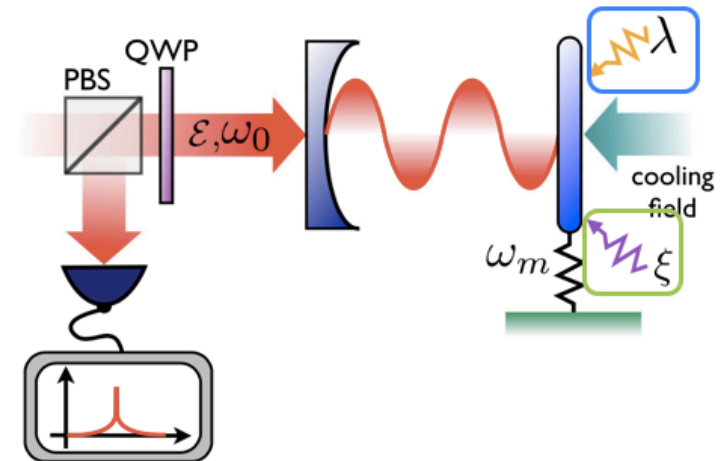
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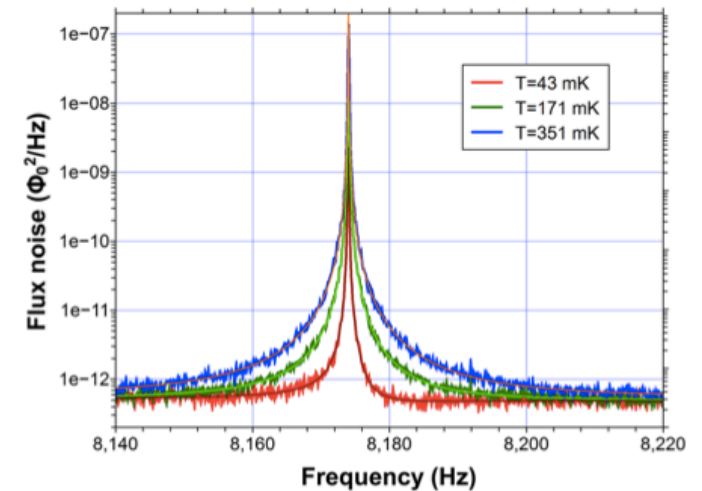
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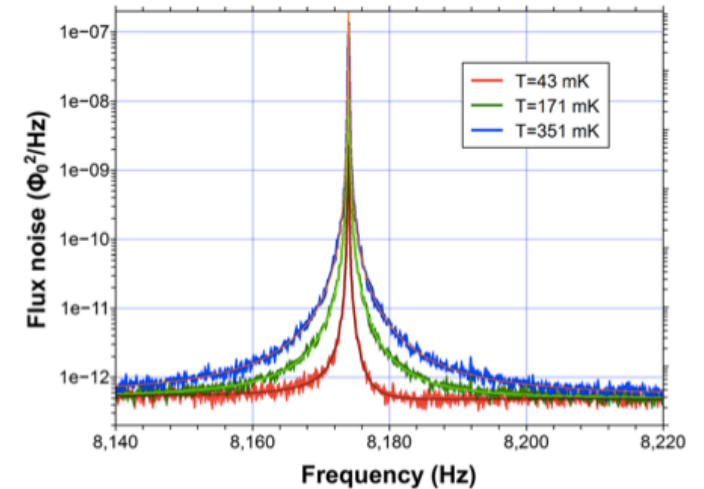
$$\frac{d}{dt}\hat{L}_x(t) = -I\omega_\phi^2\hat{\phi}(t) - \frac{D_\phi}{I}\hat{L}_x(t) + \hat{\xi}_R(t) + \tau_{\text{CSL}}^x(t)$$

Density Noise Spectrum

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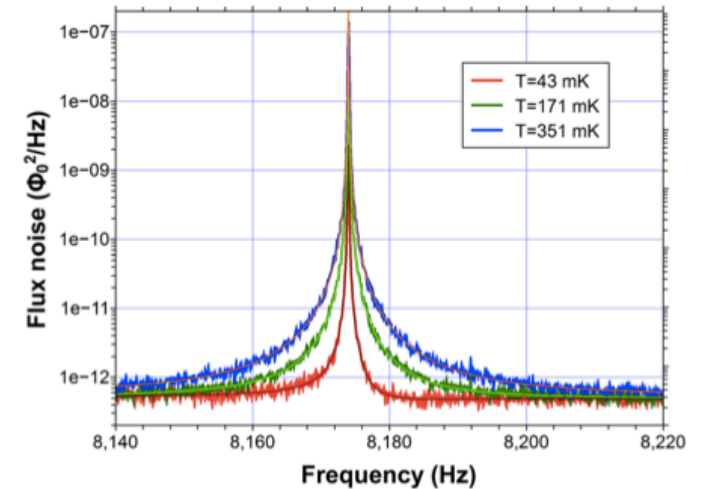
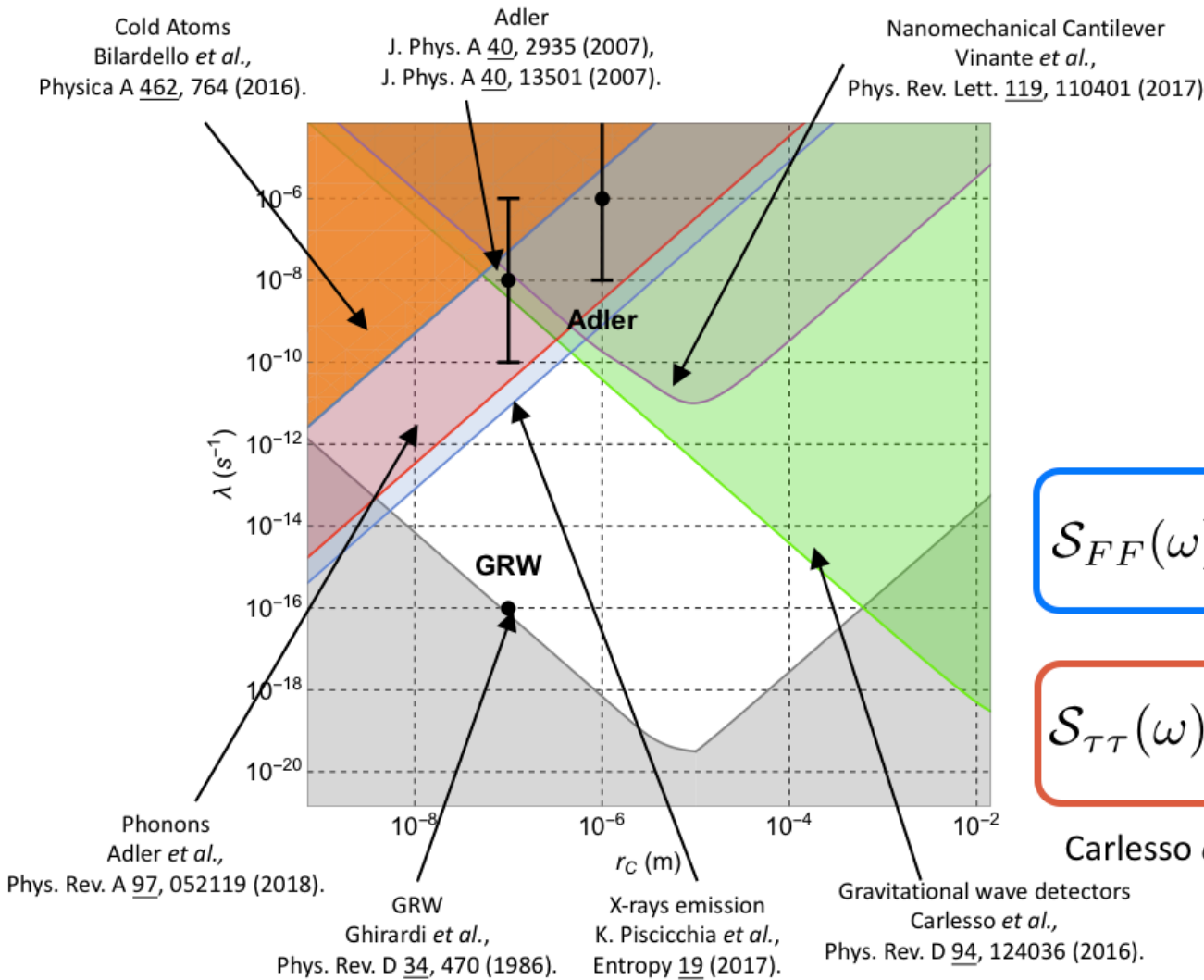


$$S_{FF}(\omega) = \frac{1}{4\pi} \int d\Omega \langle \{\tilde{F}(\omega), \tilde{F}(\Omega)\} \rangle$$

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Carlesso *et al.*, ArXiv 1708.04812 (2017)

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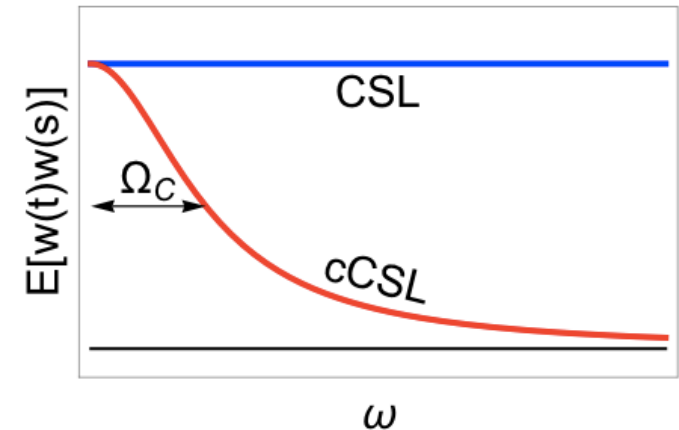
CSL – weak points of the model

Two weak points:

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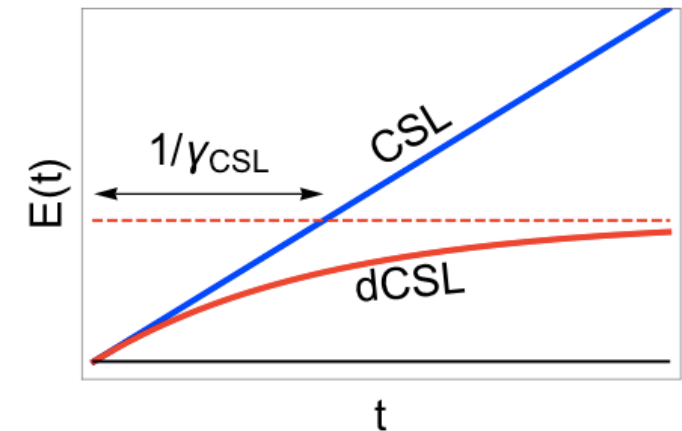
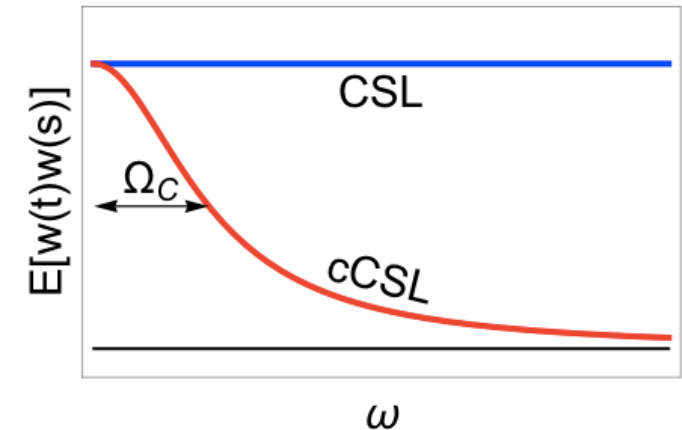
- The noise spectrum is flat (white noise)
 - Approximation of a realistic noise, which is instead characterized by a frequency cutoff
- Colored extension of the model



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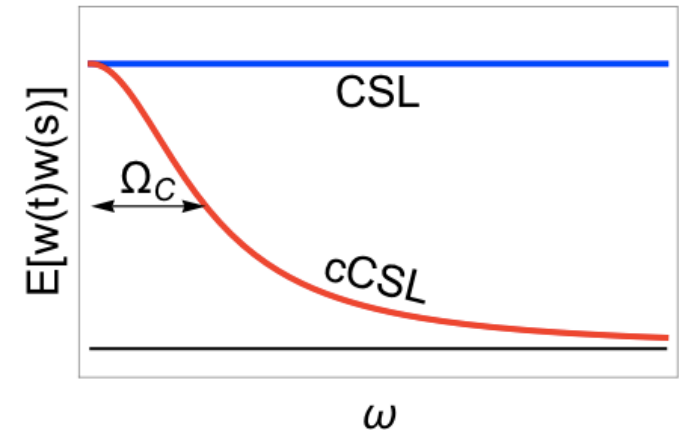
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 - Colored extension of the model
- The noise leads to infinite energy for the system
 - Approximation of a finite temperature noise
 - Dissipative extension of the model



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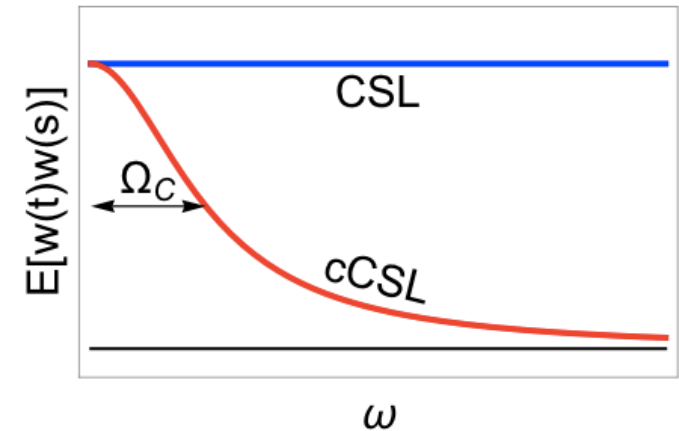
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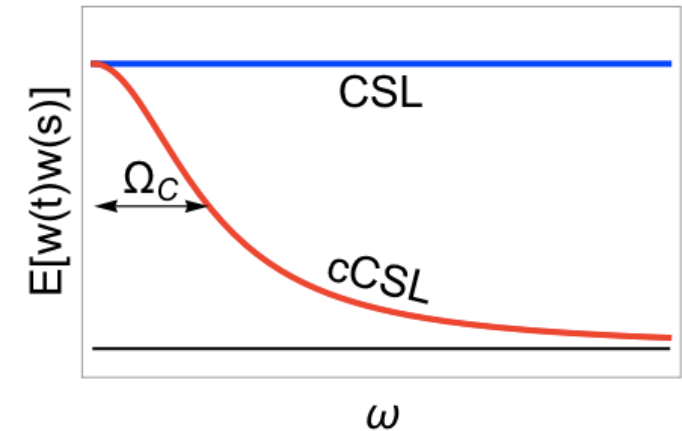
$$\frac{d|\psi_t\rangle}{dt} = \left[-\frac{i}{\hbar}\hat{H} + \frac{\sqrt{\lambda}}{m_0} \int d\mathbf{x}\hat{M}(\mathbf{x})w(\mathbf{x}, t) - \frac{2\lambda}{m_0^2} \int d\mathbf{x}\hat{M}(\mathbf{x}) \int ds f(t-s) \frac{\delta}{\delta w(\mathbf{x}, s)} \right] |\psi_t\rangle$$

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Colored CSL model

Langevin equations

$$\frac{d}{dt}\hat{x}(t) = \frac{\hat{p}(t)}{M}$$

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Density Noise Spectrum

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White noise case

$$\mathbb{E} [F_{\text{CSL}}(t)F_{\text{CSL}}(s)] = \hbar^2\eta\delta(t-s)$$

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Density Noise Spectrum

$$S_{xx}(\omega) = \frac{1}{4\pi} \int d\Omega \langle \{\tilde{x}(\omega), \tilde{x}(\Omega)\} \rangle$$

$$= \frac{1}{m^2} \frac{2m\gamma k_B T + S_{FF}(\omega)}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$$

$$S_{FF}(\omega) = \frac{1}{4\pi} \int d\Omega \langle \{\tilde{F}(\omega), \tilde{F}(\Omega)\} \rangle$$

$$S_{FF}(\omega) = \hbar^2\eta\tilde{f}(\omega)$$



Frequency dependent
CSL contribution

Colored CSL model – Case study

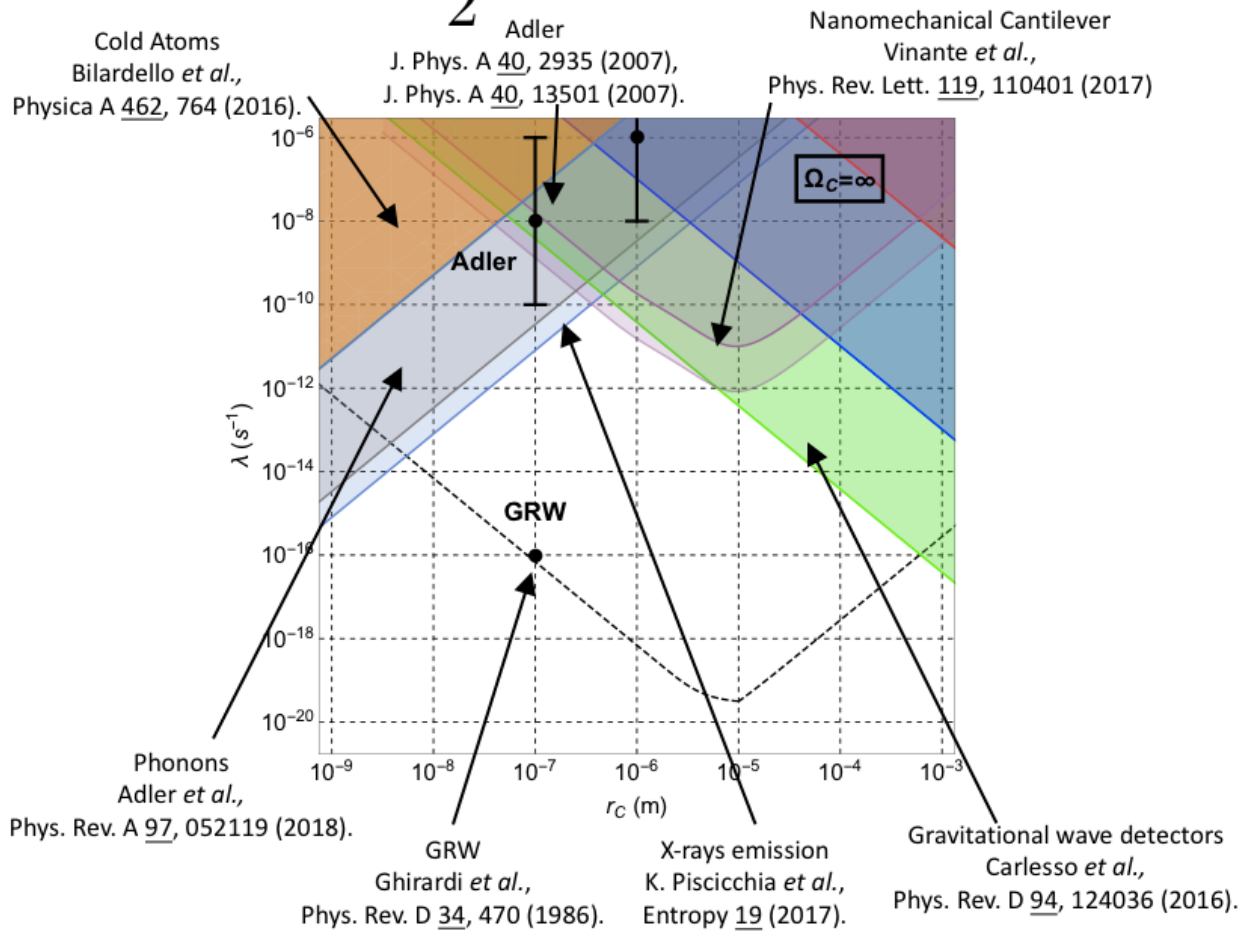
$$f(t - s) = \frac{\Omega_c}{2} e^{-\Omega_c |t-s|}$$

Colored CSL model – Case study

$$f(t - s) = \frac{\Omega_C}{2} e^{-\Omega_C |t-s|} \longrightarrow S_{FF}(\omega) = S_{FF}^{\text{CSL}} \times \frac{\Omega_C^2}{\Omega_C^2 + \omega^2}$$

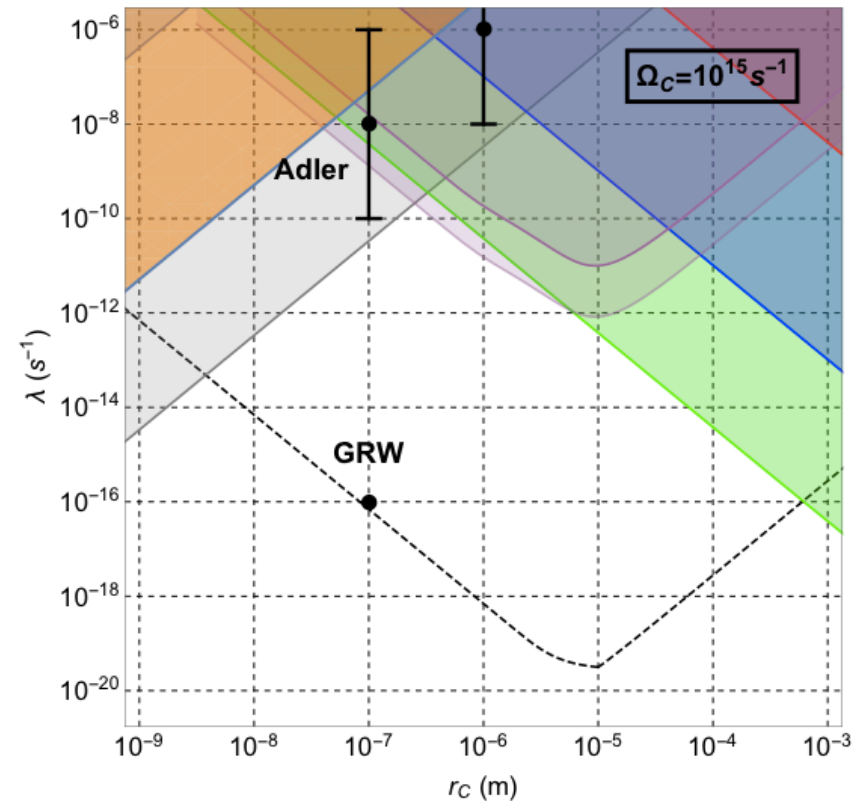
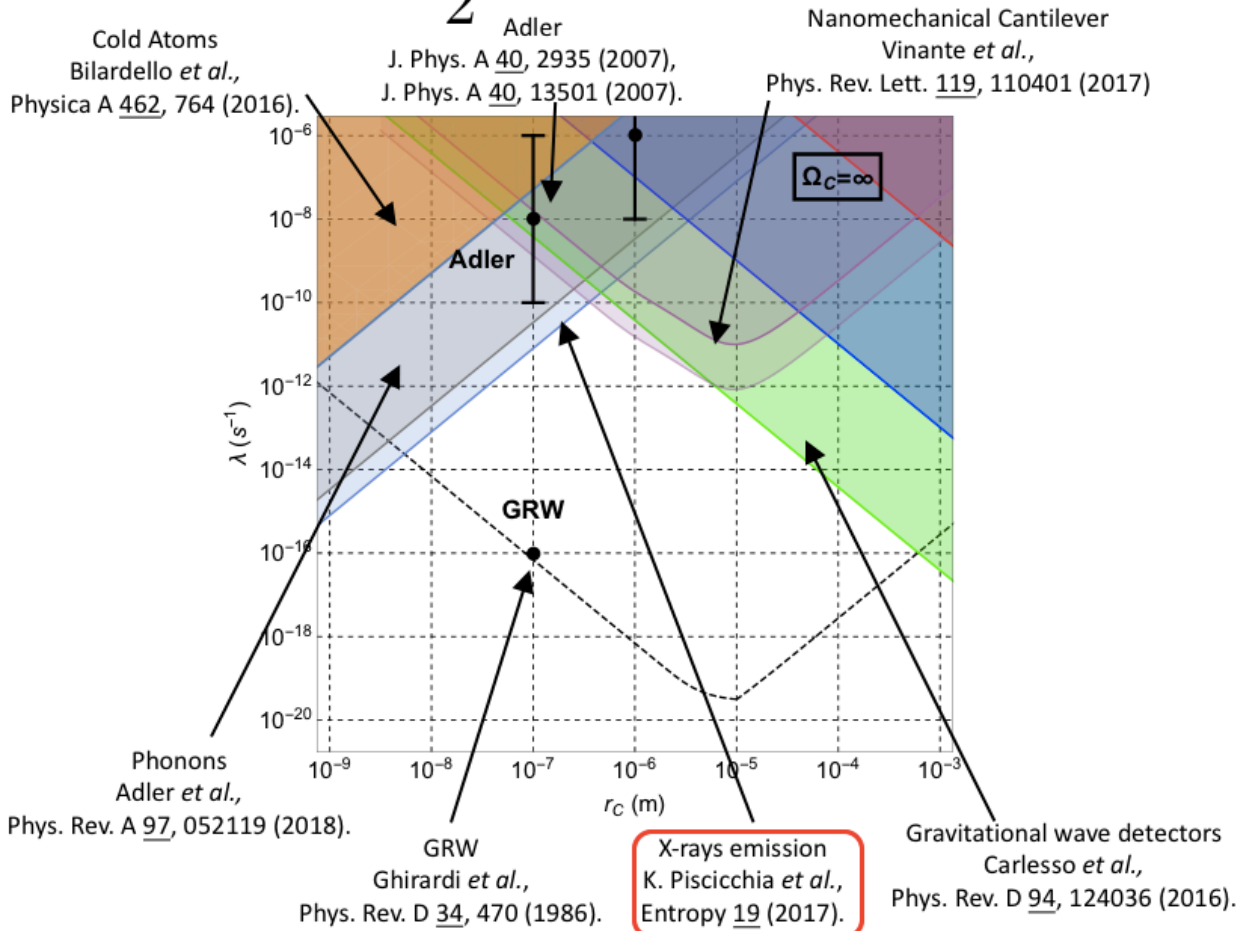
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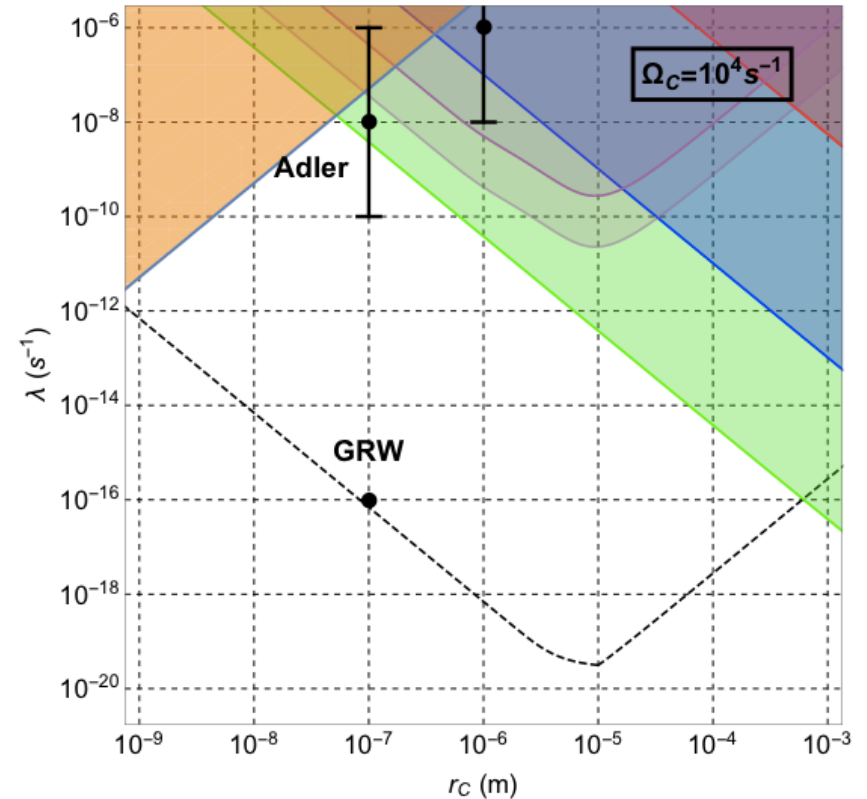
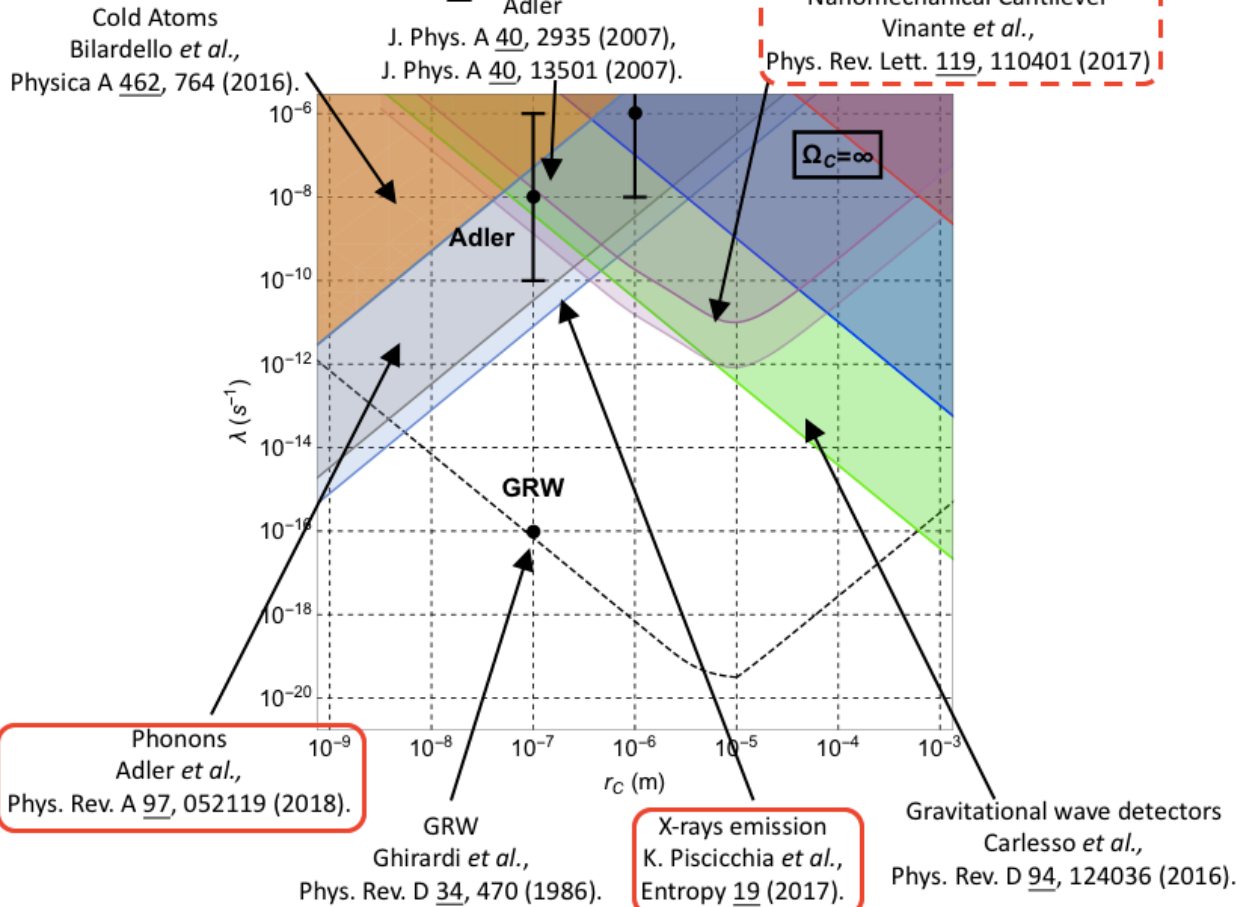
Carlesso, Ferialdi, Bassi, *ArXiv* 1805.10100 (2018)

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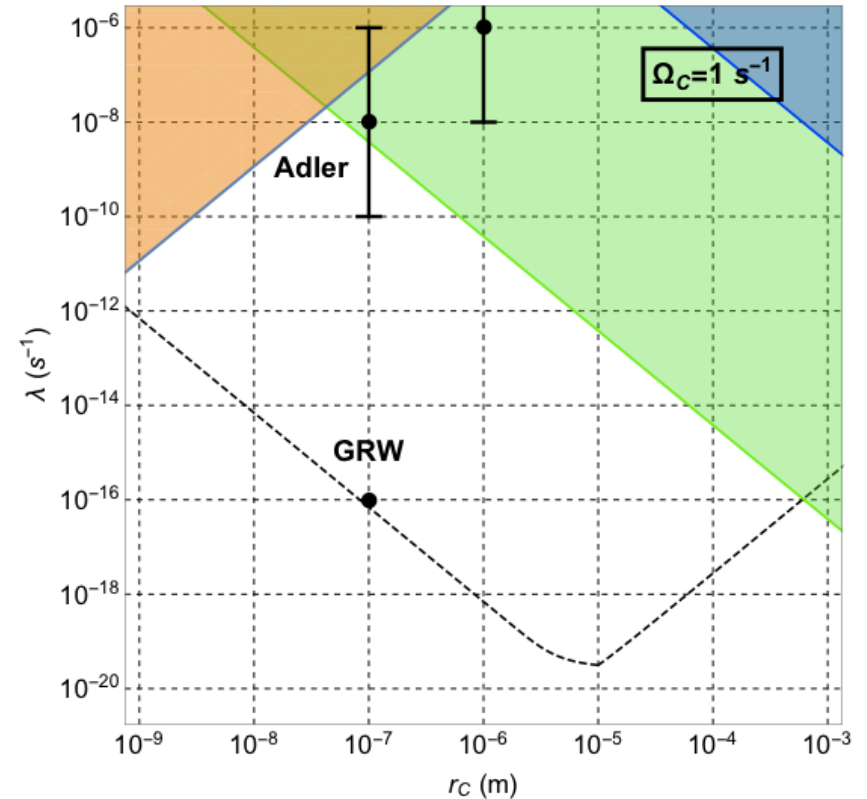
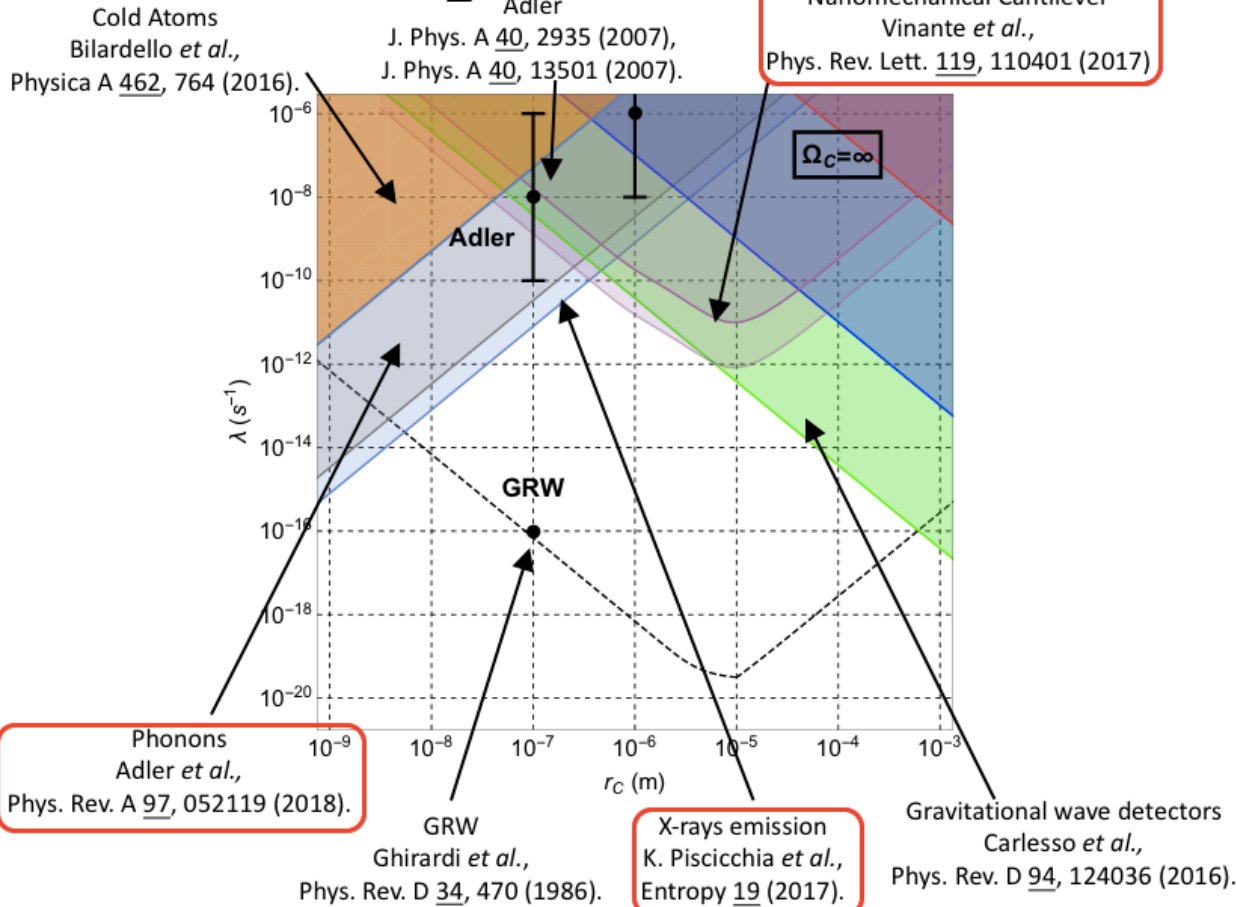
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Dissipative CSL model

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CSL model predicts an infinite energy increment!

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For a nucleon we have $\Delta E = 10^{-15}$ K in one year with $\begin{cases} \lambda = 10^{-17} \text{ s}^{-1} \\ r_C = 10^{-7} \text{ m} \end{cases}$

Dissipative CSL model

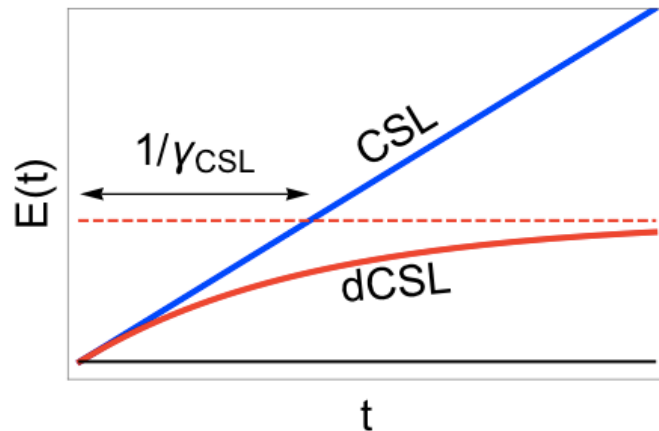
CSL model predicts an infinite energy increment!

For a nucleon we have $\Delta E = 10^{-15}$ K in one year with $\begin{cases} \lambda = 10^{-17} \text{ s}^{-1} \\ r_C = 10^{-7} \text{ m} \end{cases}$

A new parameter is introduced to solve the problem: the CSL temperature T_{CSL}

$$E(t) = e^{-\beta t} (E_0 - E_{\text{as}}) + E_{\text{as}},$$

$$E_{\text{as}} = \frac{3}{2} k_B T_{\text{CSL}}$$



$$\beta = 4\chi \frac{\lambda}{(1 + \chi)^5}$$

$$\chi = \frac{\hbar^2}{8m_0 k_B T_{\text{CSL}} r_C^2}$$

Dissipative CSL model

A. Smirne and A. Bassi, *Sci. Rep.* 5, 12518 (2015).

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} \hat{H} dt + \frac{\sqrt{\gamma}}{m_0} \int d\mathbf{y} \left(\hat{L}(\mathbf{y}) - r_t(\mathbf{y}) \right) dW_t(\mathbf{y}) + \right. \\ \left. - \frac{\gamma}{2m_0^2} \int d\mathbf{y} \left(\hat{L}^\dagger(\mathbf{y}) \hat{L}(\mathbf{y}) + r_t^2(\mathbf{y}) - 2r_t(\mathbf{y}) \hat{L}(\mathbf{y}) \right) dt \right] |\psi_t\rangle$$

$$\hat{L}(\mathbf{y}) = \frac{m}{(2\pi\hbar)^3} \sum_{\alpha} \int d\mathbf{Q} e^{\frac{i}{\hbar} \mathbf{Q} \cdot (\hat{\mathbf{x}}_{\alpha} - \mathbf{y})} \exp \left(-\frac{r_C^2}{2\hbar^2} |(1 + \chi)\mathbf{Q} + 2\chi\hat{\mathbf{p}}_{\alpha}|^2 \right)$$

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CSL Temperature dependence

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CSL model:

~~$$d|\psi_t\rangle = -\frac{i}{\hbar} \left(\hat{H} + \hat{V}_{\text{CSL}} \right) dt |\psi_t\rangle$$~~

~~$$\hat{V}_{\text{CSL}} = -\frac{\hbar\sqrt{\lambda}}{\pi^{3/4}r_c^{3/2}m_0} \int d\mathbf{y} \hat{M}(\mathbf{y}) w(\mathbf{y}, t)$$~~

~~$$\mathbb{E}[w(\mathbf{x}, t) w(\mathbf{y}, s)] = \delta(\mathbf{x} - \mathbf{y}) \delta(t - s)$$~~

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CSL model:

$$d|\psi_t\rangle = \left\{ -\frac{i}{\hbar} \hat{H} dt + d\hat{C} - \frac{1}{2} \mathbb{E} \left[d\hat{C}^\dagger d\hat{C} \right] \right\} |\psi_t\rangle$$

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$$\mathbb{E}[w(\mathbf{x}, t) w(\mathbf{y}, s)] = \delta(\mathbf{x} - \mathbf{y}) \delta(t - s)$$

$$d\hat{C} = \frac{\sqrt{\lambda r_C^3 (4\pi)^{3/2}}}{m_0} \int d\mathbf{y} \left(\hat{L}(\mathbf{y}) d\hat{B}^\dagger(\mathbf{y}) - \hat{L}^\dagger(\mathbf{y}) d\hat{B}(\mathbf{y}) \right),$$

Linear stochastic unravelling
with a quantum noise

$$\mathbb{E}[d\hat{B}_t(\mathbf{x})] = \mathbb{E}[d\hat{B}_t^\dagger(\mathbf{x})] = \mathbb{E}[d\hat{B}_t^\dagger(\mathbf{y}) d\hat{B}_t(\mathbf{x})] = 0 \\ \mathbb{E}[d\hat{B}_t(\mathbf{y}) d\hat{B}_t^\dagger(\mathbf{x})] = \delta(\mathbf{y} - \mathbf{x}) dt$$

Dissipative CSL model

$$d|\psi_t\rangle = \left\{ -\frac{i}{\hbar}\hat{H}dt + d\hat{C} - \frac{1}{2}\mathbb{E}\left[d\hat{C}^\dagger d\hat{C}\right] \right\} |\psi_t\rangle = d\hat{U}_t |\psi\rangle$$

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$$d\hat{O}(t) = d\hat{U}_t^\dagger \hat{O} \hat{U}_t + \hat{U}_t^\dagger \hat{O} d\hat{U}_t + \mathbb{E}[d\hat{U}_t^\dagger \hat{O} d\hat{U}_t],$$



$$\frac{d}{dt}\hat{x}(t) = \frac{\hat{p}(t)}{M} + \frac{\Delta\hat{p}_{\text{CSL}}(t)}{M}$$

$$\frac{d}{dt}\hat{p}(t) = -M\omega_0^2\hat{x}(t) - \gamma\hat{p}(t) + \xi(t) + F_{\text{CSL}}(t)$$

Dissipative CSL model

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$$S(\omega) = \frac{1}{m^2} \frac{2m\gamma_m k_B T + \hbar^2 \eta [1 + \varkappa^2 m^2 (\gamma^2 + \omega^2)]}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

Nobakht, M.C., Donadi, Paternostro and Bassi, *To appear*.

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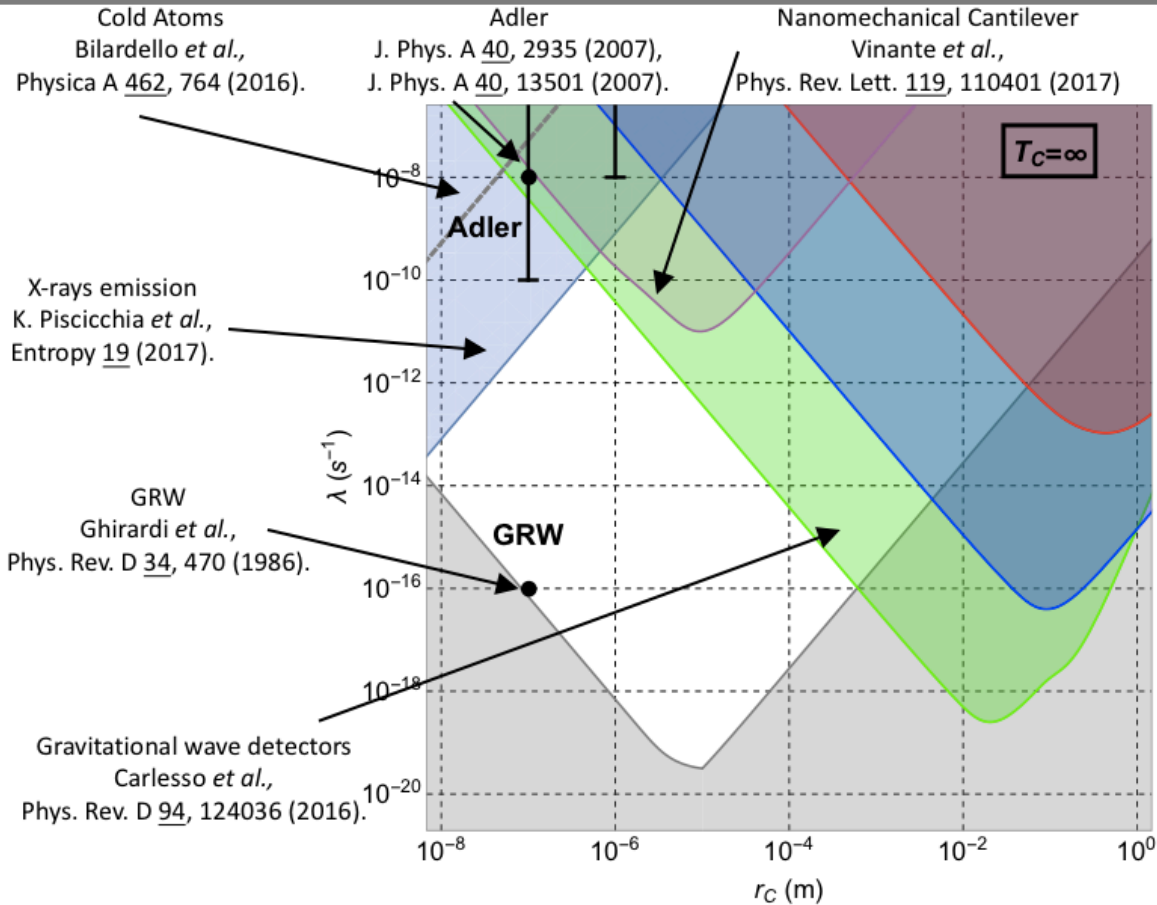
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$$\gamma = \gamma_m + \gamma_{\text{CSL}}(\lambda, r_C, T_{\text{CSL}})$$

$$\Delta\hat{p}_{\text{CSL}}(t) = -M\hbar \varkappa(\lambda, r_C, T_{\text{CSL}}) w(t)$$

$$\mathbb{E}[F_{\text{CSL}}(t)F_{\text{CSL}}(s)] = \hbar^2 \eta(\lambda, r_C, T_{\text{CSL}}) \delta(t - s)$$

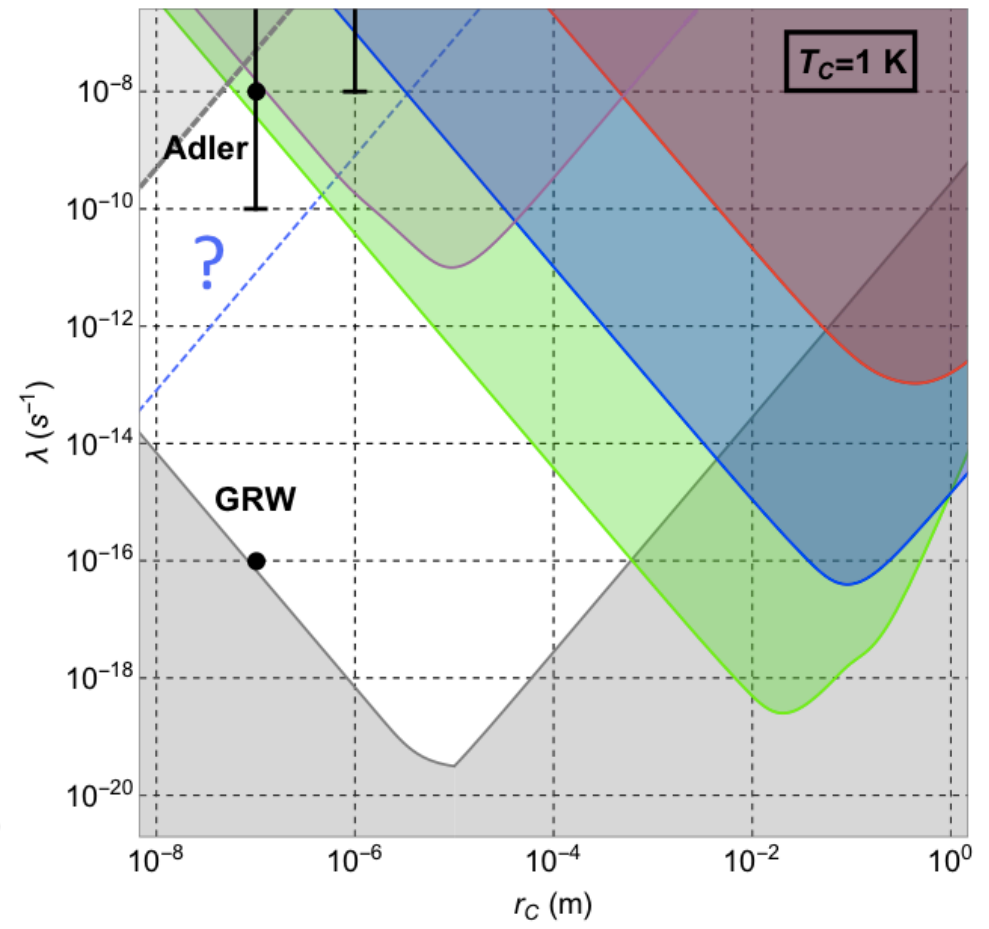
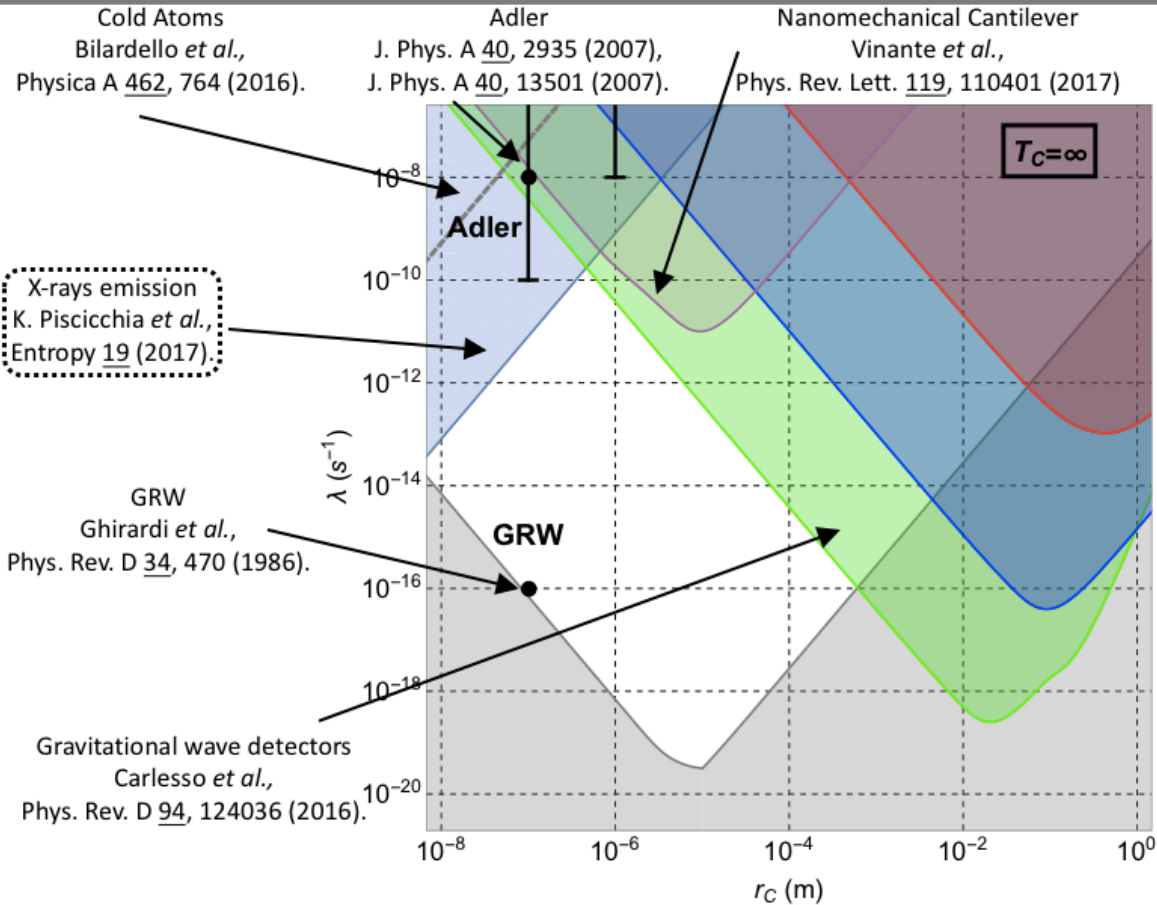
Dissipative CSL model



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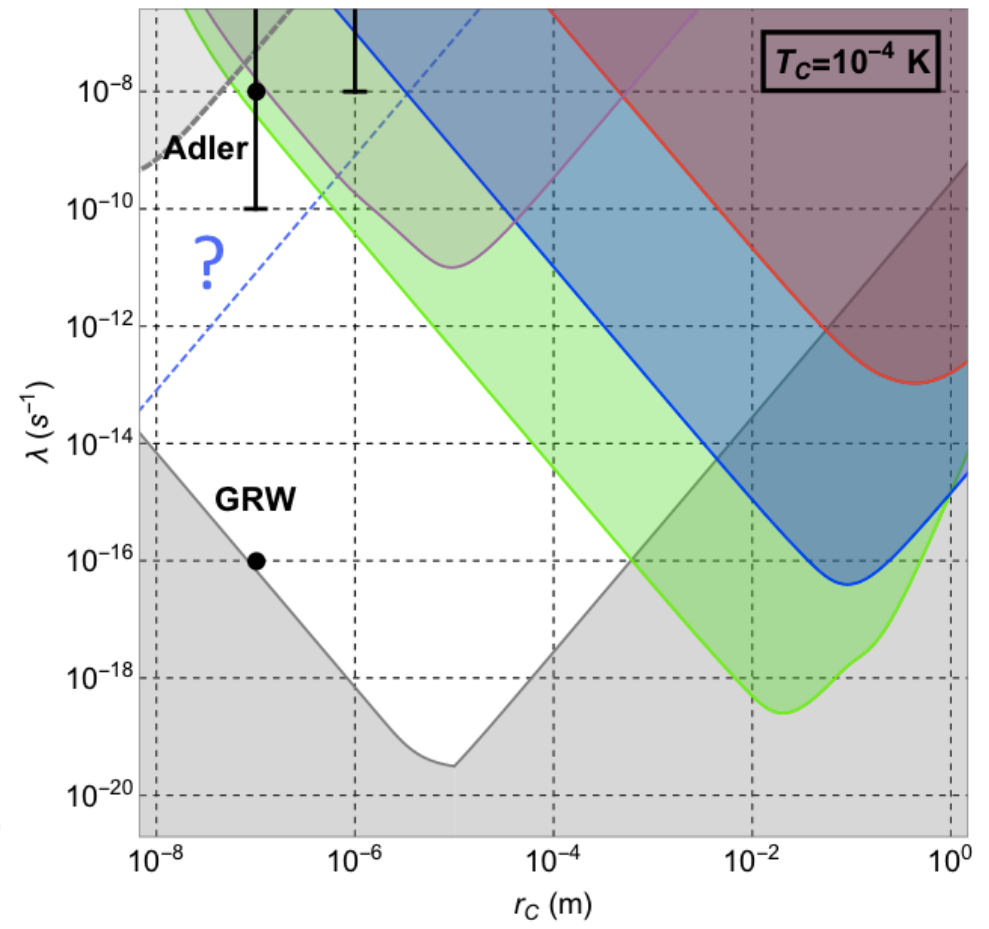
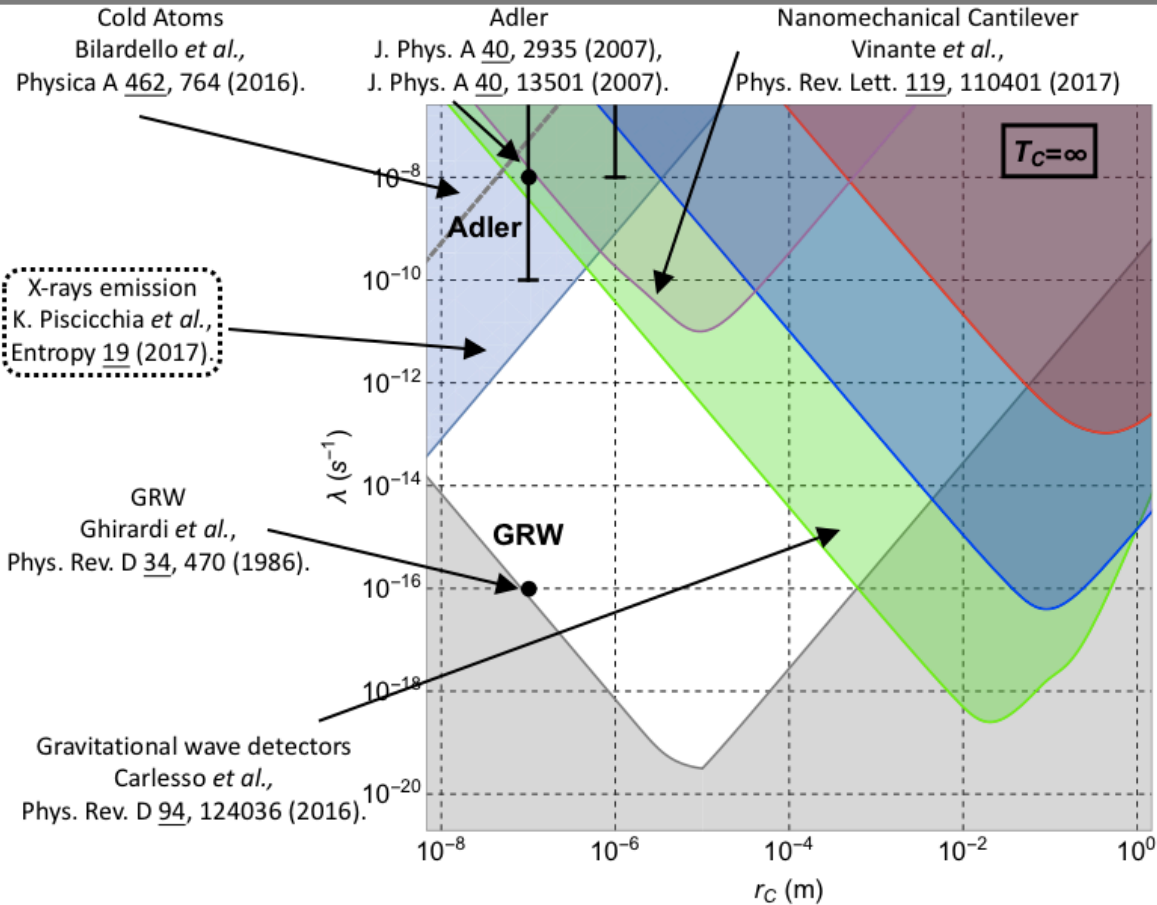
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Dissipative CSL model

Cold Atoms
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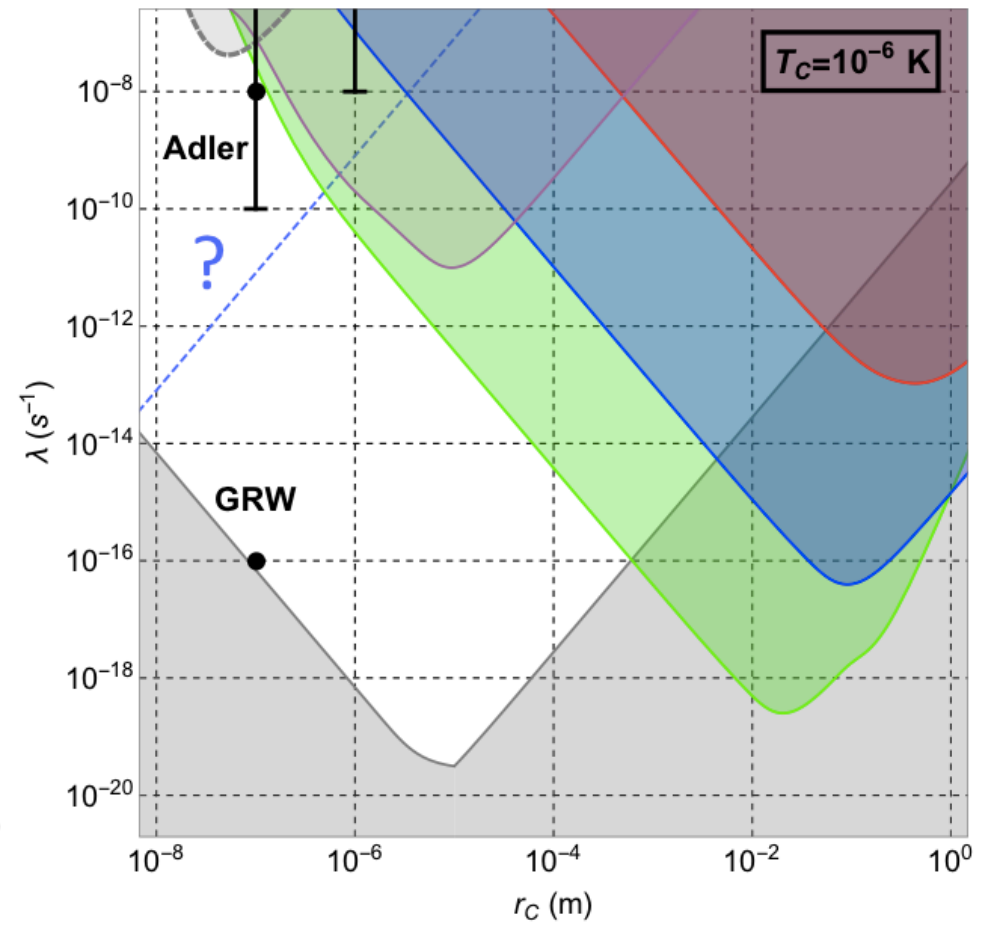
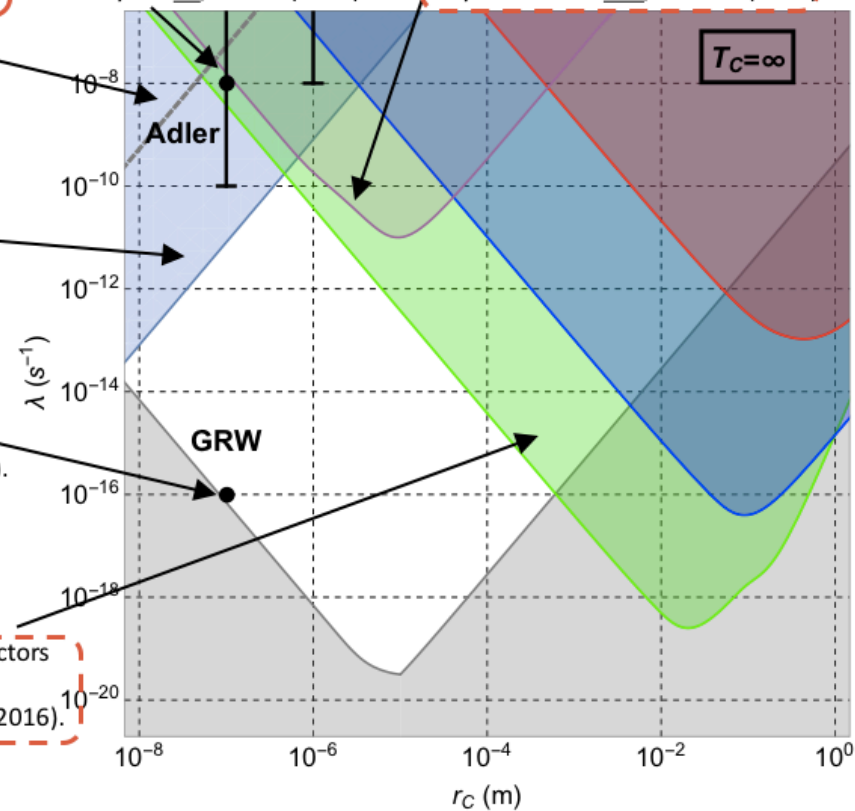
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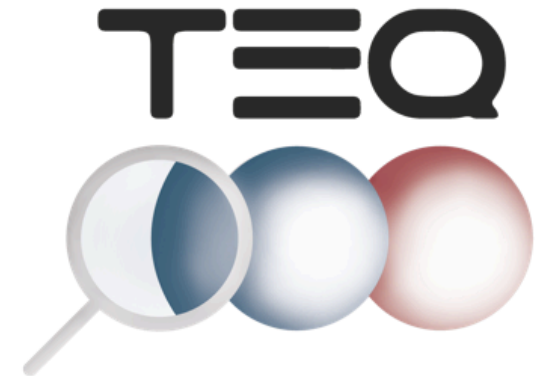
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