LTD 2019:
Nonlinear Properties of Supercurrent-Carrying Single and Multi-Layer Thin-Film Superconductors

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Thin-Film Devices

Resonator Devices

Travelling-Wave Devices
Thin-Film Devices

- Narrow band
- High quality factor
- Often easy to multiplex
- Kinetic Inductance Detectors (KIDs)
- Quantum bits (Qubits)
Thin-Film Devices

- Wide band
- Useful readout components
- Parametric Amplifiers
- Parametric Up Converters
- Low-loss delay lines
Kinetic Inductance Nonlinearity

\[ L = L_0 \left[ 1 + \left( \frac{I}{I_*} \right)^2 + \left( \frac{I}{I_{*,4}} \right)^4 + \cdots \right] \]

- \( L \): Inductance
- \( L_0 \): Inductance when \( I = 0 \)A
- \( I \): Supercurrent
- \( I_* \): Quadratic nonlinearity factor
- \( I_{*,4} \): Quartic nonlinearity factor
Zmuidzinas, 2012:

“MKIDs are usually operated in a regime in which the microwave currents are strong and nonlinearity is becoming important.....”

Eom et al., 2012:

“A parametric amplifier that overcomes these limitations through the use of a travelling-wave geometry and the nonlinear kinetic inductance of a superconducting transmission line......”
Analysis Routine

Geometries
Materials
$T, I$

Usadel Equations
$\theta$

Nam’s Equations
$\sigma$

Transmission Line Theories
$Z_s$

Transfer Matrices
$L(I)$

Repeat for different $I$

$L(I)$

$Y$

$T$ Temperature
$\theta$ Green’s functions
$\sigma$ Conductance
$Z_s$ Surface impedance
$Z$ Series impedance
$Y$ Shunt conductance
Numerically solving the Usadel equations

Superfluid Velocity:
\[ \vec{v}_s = D_s [\vec{\nabla} \phi - (2e/h)\vec{A}] \]

Quasiparticle Density of States:
\[ N = N_0 \Re(\cos \theta) \]

Solved iteratively to obtain \( \Delta, \theta \)

\[ \frac{\hbar D_s}{2} \nabla^2 \theta + iE \sin \theta + \Delta \cos \theta - \frac{\hbar}{2D_s} \vec{v}_s^2 \cos \theta \sin \theta = 0 \]

\[ N_s V_{0,S} \int_0^{k_B \Theta_{D,S}} dE \tanh \left( \frac{E}{2k_BT} \right) \Im(\sin \theta) = \Delta \]

\[ \frac{\sigma_N}{eD_s} \int_0^\infty dE \tanh \left( \frac{E}{2k_BT} \right) \Im(\sin^2 \theta) \vec{v}_s = \vec{j} \]
Usadel Equations → Densities of States

\[ \frac{\Gamma}{\Delta_0} = 1.0 \times 10^{-3} \]
\[ \frac{\Gamma}{\Delta_0} = 6.1 \times 10^{-3} \]
\[ \frac{\Gamma}{\Delta_0} = 11.2 \times 10^{-3} \]

DoS broadens!

\[ 1 - \frac{\Delta}{\Delta_0} = 0.6 \times 10^{-3} \]
\[ 1 - \frac{\Delta}{\Delta_0} = 4.5 \times 10^{-3} \]
\[ 1 - \frac{\Delta}{\Delta_0} = 8.4 \times 10^{-3} \]

$\Gamma$ is the depairing factor $\propto I^2$

$\Delta_g$ – DoS gap
$\Delta$ – order parameter

Move at different rate
Nam’s Equations → Complex Conductivity

\[ \frac{\sigma_2}{\sigma_N} \approx \frac{\pi \Delta}{\hbar \omega} \]

Full Nam’s conductivity integrals:

\[ \frac{\sigma_2}{\sigma_N} = \int dE \, fn(\sin \theta, \cos \theta, E) \]

\[ \frac{\sigma_2}{\sigma_N} \approx \frac{\pi \Delta_g}{\hbar \omega} \]

\( \Gamma \) is the depairing factor \( \propto I^2 \)
Transmission Line Theory → Inductance

- Find $L$ using transmission line model
- Vary input $I$ to obtain $L(I)$
- Polynomial fit to obtain $I_*$ (and $I_{*,4}$)

- Al layer decreases nonlinearity
- $d_{Al}$ Al
- 100 nm Ti

Multi-metal-layer

$\text{SiO}_2$
Romijin et al, 1982:

“...If the width is larger than the coherence length, vortex nucleation and vortex flow can be induced at high current densities....”

\[ \xi = \text{coherence length}, \xi_{Al} = 190 \text{ nm}, \xi_{Ti} = 60 \text{ nm} \]

**Previous Experiments:**
- \( w = 30 - 120 \text{ nm} \)
- \( t = 20 - 90 \text{ nm} \)

Near perfect agreement with theory

**Realistic Device Dimensions:**
- \( w \approx \text{order of a few \( \mu \text{m} \)} \)
- \( t \approx \text{order of } 10s - 100s \text{ nm} \)

How big can current be before result deviate from theory?
Experimental Method

1. Set fixed $I$
2. Increase $T$
3. Record $(I, T_c)$ at transition
Experimental Method

Nb magnetic shield: Mitigate environment influences
Experimental Method

Triple Al wirebond:
No current bottleneck!
Experimental Method

**Nb Magnetic Shield**

**Triple Al Wirebond:** No current bottleneck!

- **$V$** measurement pads
- **$I$** injection pads
- Thin, narrow test strip
Tc Measurements

$I_0$ - theoretical critical current at 0K
$I_{0,c}$ - experimental critical current at 0K

$w = 1 \, \mu m$ – good agreement for $I < \frac{2}{3} I_{0,c}$
$w = 3 \, \mu m$ – good agreement for $I < \frac{1}{2} I_{0,c}$

$I / I_0 \propto \left(1 - \frac{T_c}{T_{c,0}}\right)^{3/2}$ at small $I$
Tc Measurements

$I_0$ - theoretical critical current at 0K
$I_{0,c}$ - experimental critical current at 0K

$w = 3$ μm – good agreement for $I < \frac{1}{2} I_{0,c}$
$w = 4, 5$ μm – good agreement for $I < \frac{1}{3} I_{0,c}$

$I_c/I_0 \propto \left(1 - \frac{T_c}{T_{c,0}}\right)^{3/2}$

at small $I$

Vortex induced deviation?
Tc Measurements

- $I < \frac{1}{3} I_{0,c}$ covers most experimental applications

- Higher $I$ is avoided because:
  - Onset of dissipation
  - Bifurcation
  - Unaccounted higher order nonlinearity
Summary of Key Results

- Numerical routine for $I_*$
  - Full densities of states
  - Single or multi layer
  - Transmission line geometry

- Experiment comparing $T_c(I)$
  - Agreement with theory when $I < I_{c,0}/3$
  - Experimentally useful range

- Technique useful to understand, optimize, and design single / multi layer thin-film devices
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Questions?
No Data for Broken Lines
Critical Current Calculation