LTD 2019:
Nonlinear Properties of Supercurrent-Carrying Single and Multi-Layer Thin-Film Superconductors

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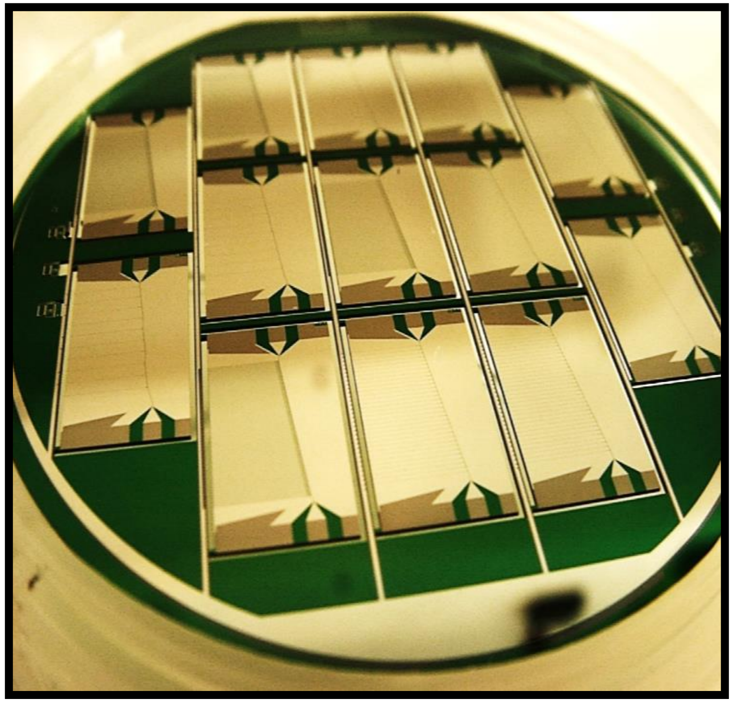
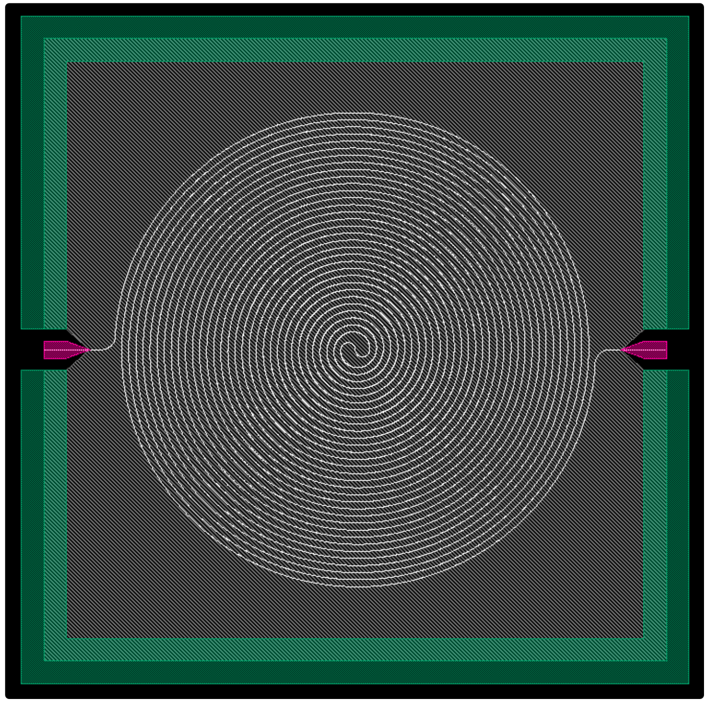
Thin-Film Devices

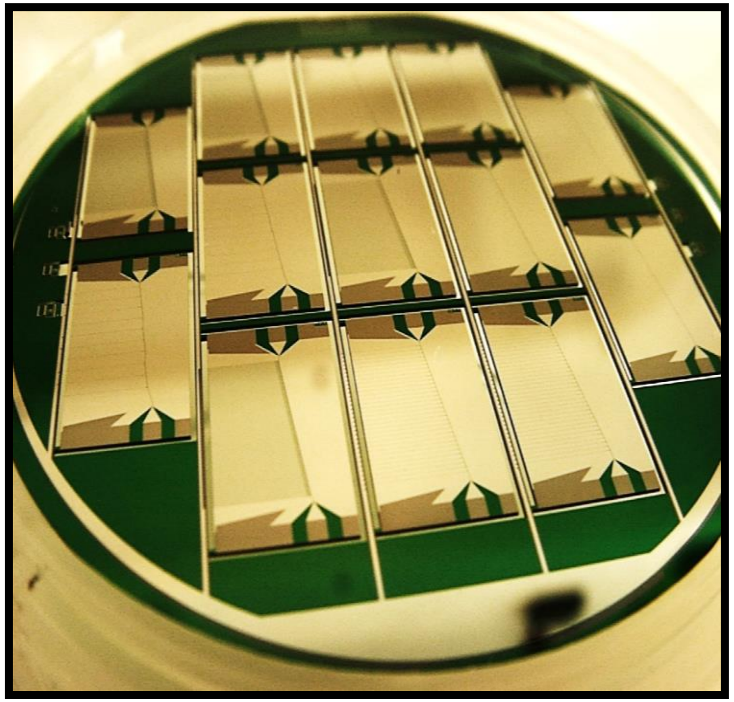


Resonator Devices



Travelling-Wave Devices

Thin-Film Devices

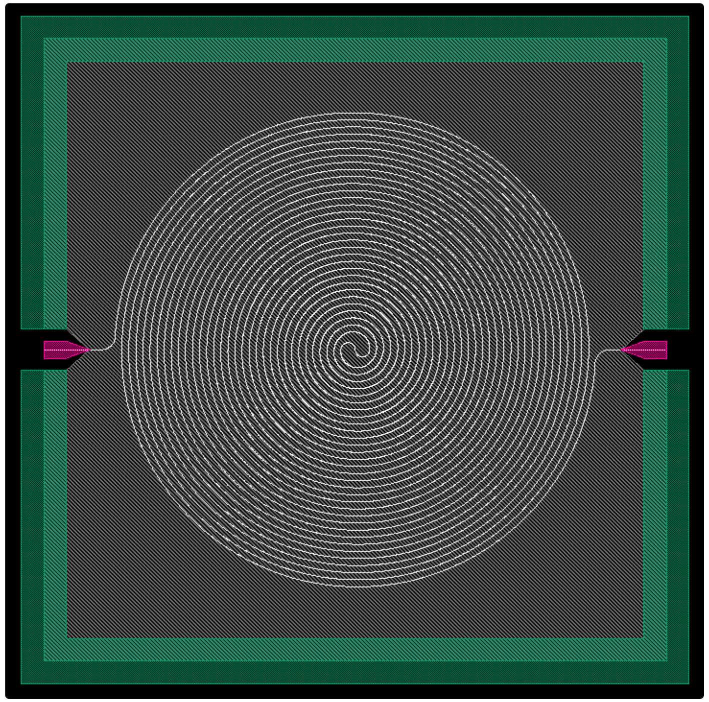


Resonator Devices

- Narrow band
- High quality factor
- Often easy to multiplex
- Kinetic Inductance Detectors (KIDs)
- Quantum bits (Qubits)

Thin-Film Devices

- Wide band
- Useful readout components
- Parametric Amplifiers
- Parametric Up Converters
- Low-loss delay lines



Travelling-Wave Devices

Kinetic Inductance Nonlinearity

$$L = L_0 \left[1 + \left(\frac{I}{I_*} \right)^2 + \left(\frac{I}{I_{*,4}} \right)^4 + \cdots \right]$$

L	Inductance
L_0	Inductance when $I = 0\mathrm{A}$
I	Supercurrent
I_*	Quadratic nonlinearity factor
$I_{*,4}$	Quartic nonlinearity factor

Zmuidzinas, 2012:

"MKIDs are usually operated in a regime in which the microwave currents are strong and nonlinearity is becoming important….."

Eom et al., 2012:

"A parametric amplifier that overcomes these limitations through the use of a travelling-wave geometry and the nonlinear kinetic inductance of a superconducting transmission line……"

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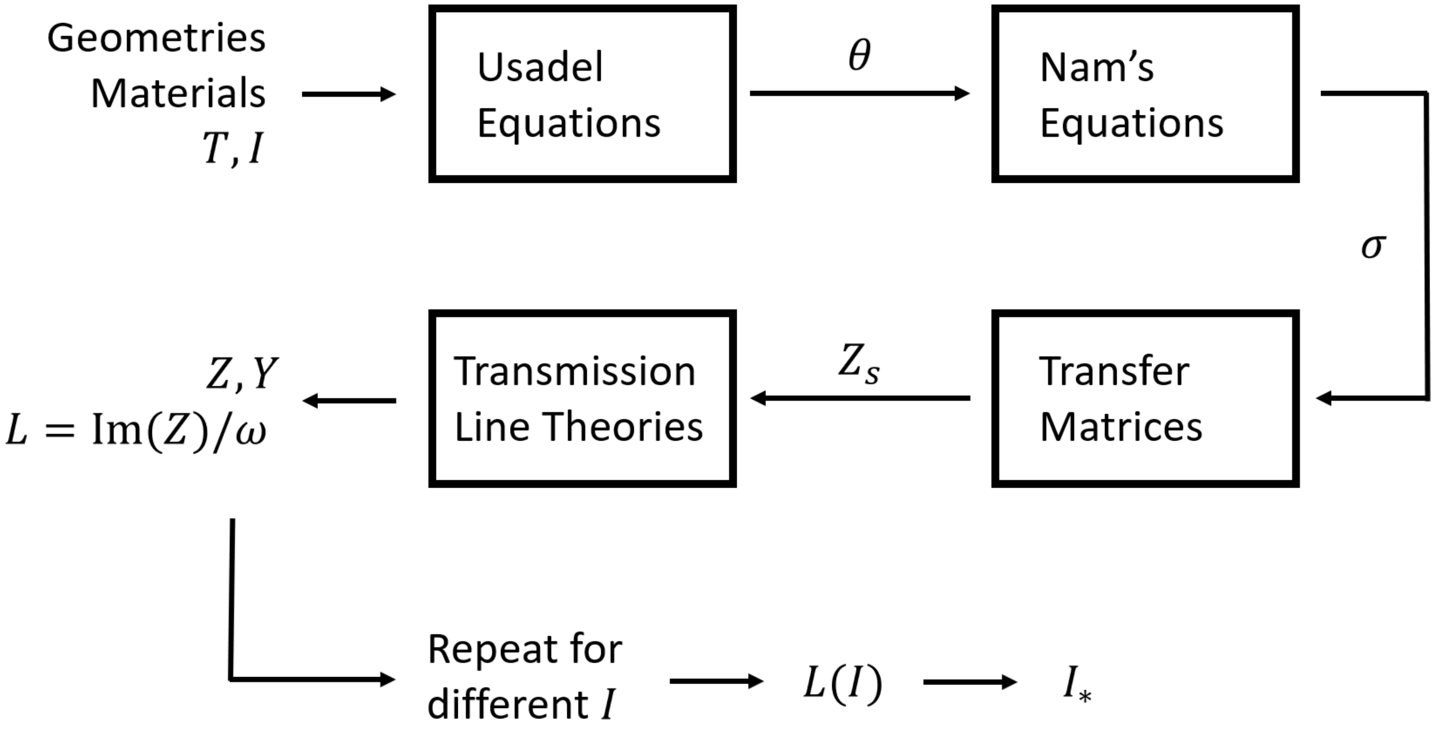
Analysis Routine



T	Temperature
θ	Green's functions
σ	Conductance
Z_s	Surface impedance
Z	Series impedance
Y	Shunt conductance

Numerically solving the Usadel equations

Superfluid Velocity:
$\vec{v}_s = D_s[\vec{\nabla}\phi - (2e/h)\vec{A}]$

Quasiparticle Density of States:
$N = N_0 \,\mathrm{Re}(\cos\theta)$

$$\frac{\hbar D_s}{2}\nabla^2\theta + iE\sin\theta + \Delta\cos\theta - \frac{\hbar}{2D_s}\vec{v}_s^{\,2}\cos\theta\sin\theta = 0$$

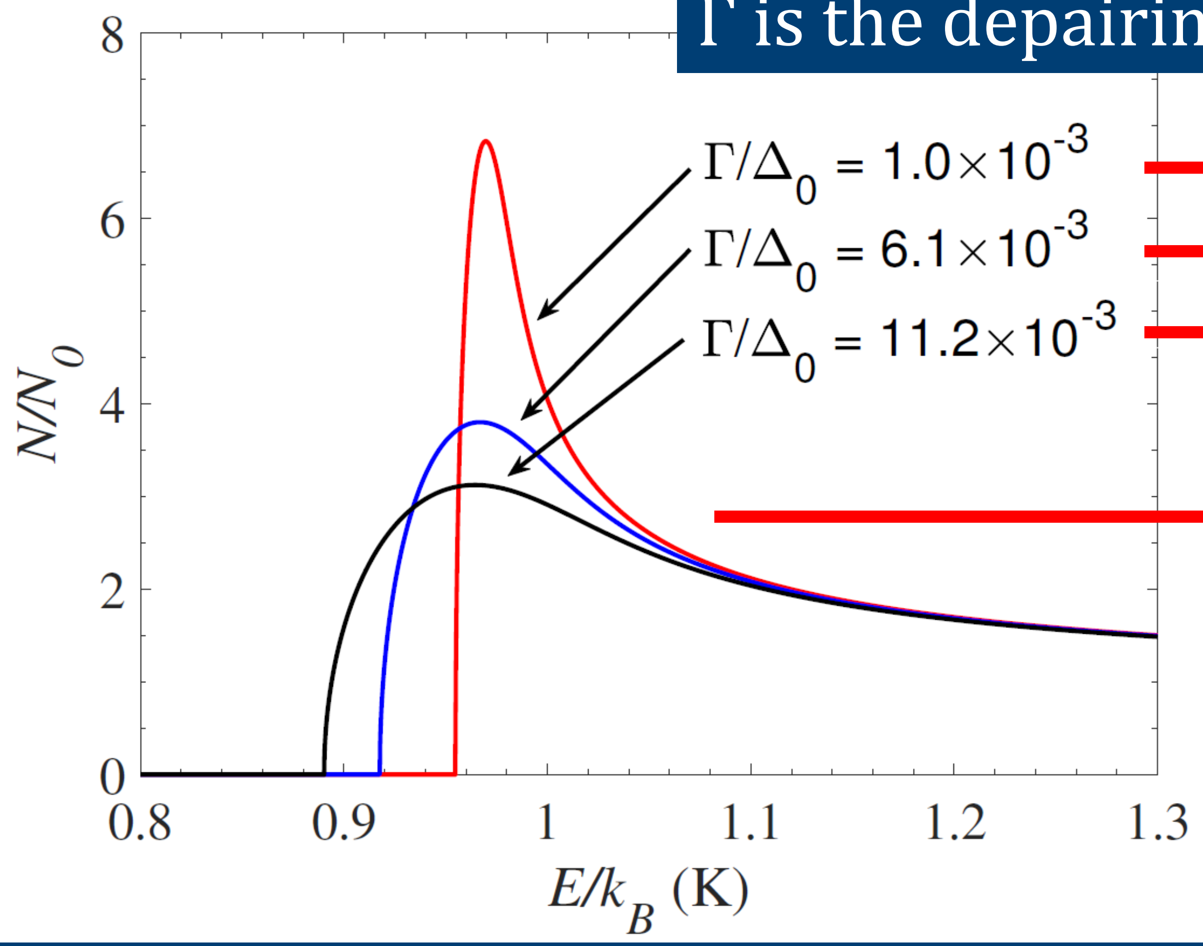
$$N_s V_{0,S}\int_0^{k_B\Theta_{D,S}} dE\,\tanh\left(\frac{E}{2k_BT}\right)\mathrm{Im}(\sin\theta) = \Delta$$

Solved iteratively to obtain Δ, θ

$$\frac{\sigma_N}{eD_s}\int_0^{\infty} dE\,\tanh\left(\frac{E}{2k_BT}\right)\mathrm{Im}(\sin^2\theta)\vec{v}_s = \vec{j}$$

Relates solution to $\vec{j}$

Usadel Equations → Densities of States



Γ is the depairing factor $\propto I^2$

$1 - \Delta/\Delta_0 = 0.6 \times 10^{-3}$

$1 - \Delta/\Delta_0 = 4.5 \times 10^{-3}$

$1 - \Delta/\Delta_0 = 8.4 \times 10^{-3}$

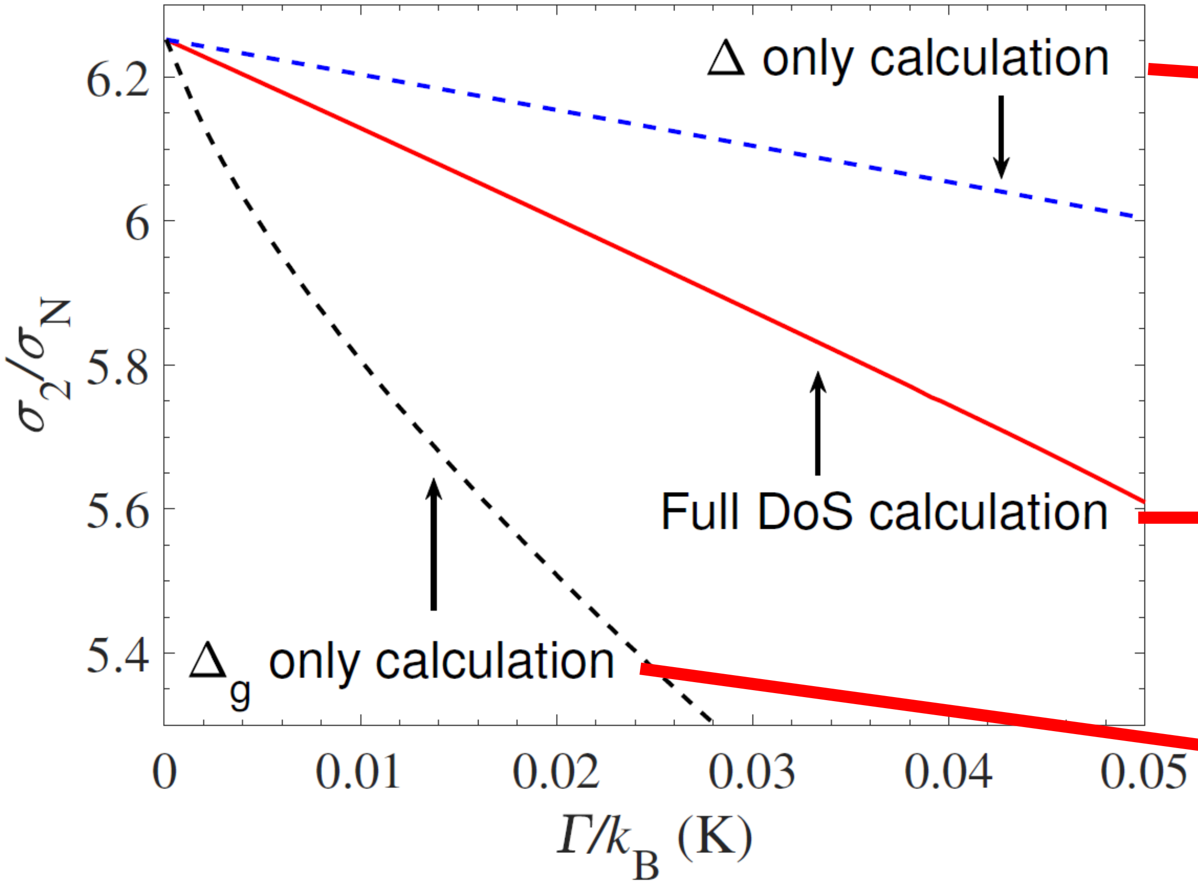
DoS broadens!

Δ_g – DoS gap

Δ – order parameter

Move at different rate

Nam's Equations → Complex Conductivity



Γ is the depairing factor $\propto I^2$

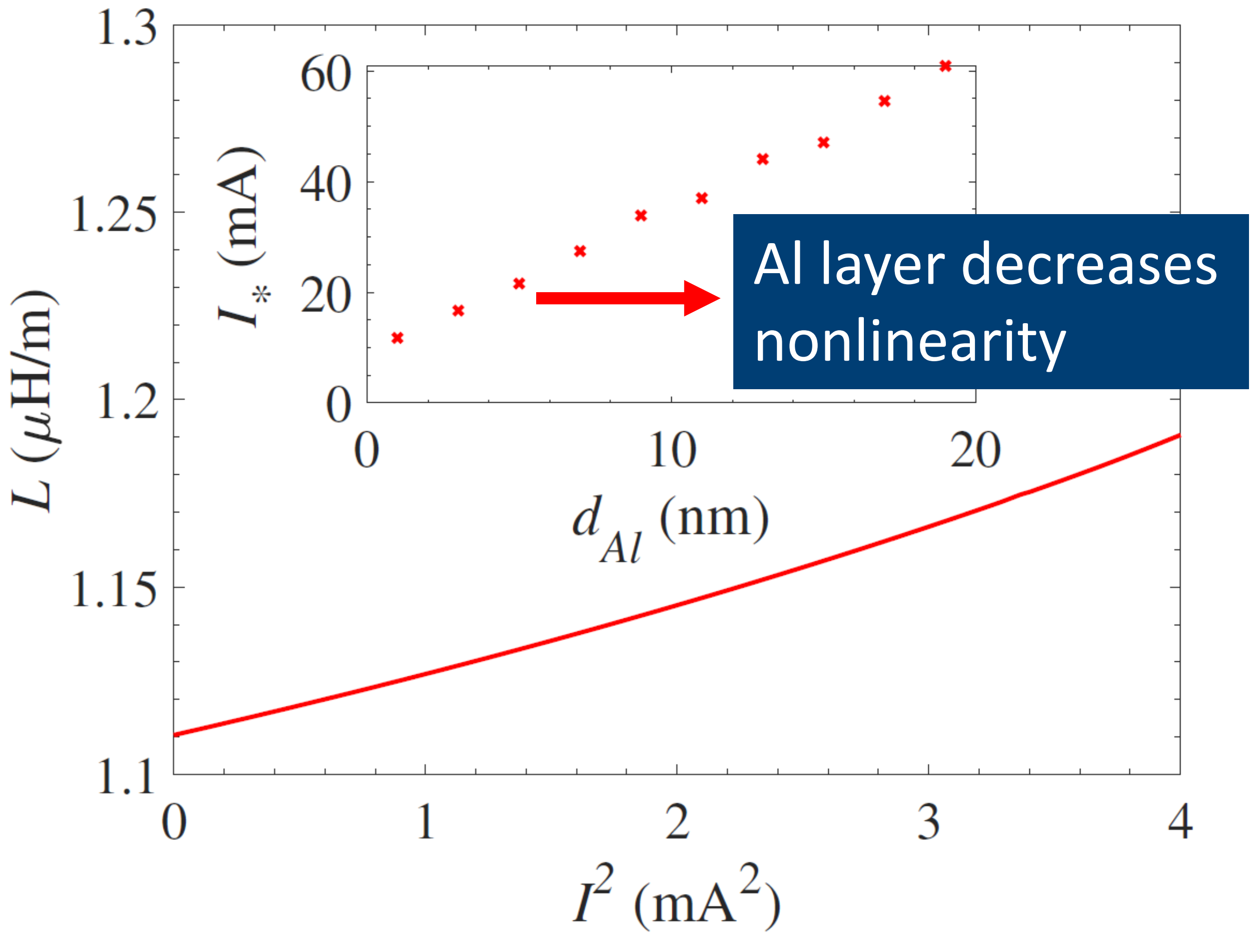
$$\frac{\sigma_2}{\sigma_N} \approx \frac{\pi\Delta}{\hbar\omega}$$

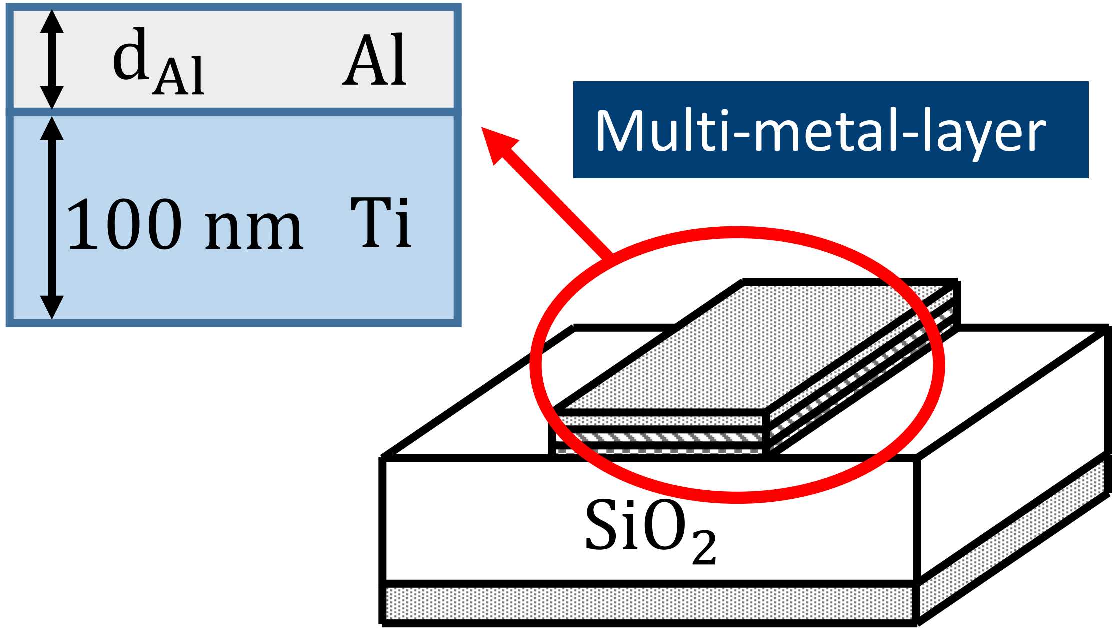
Full Nam's conductivity integrals:

$$\frac{\sigma_2}{\sigma_N} = \int dE \,\mathrm{fn}(\sin\theta\,, \cos\theta\,, E)$$

$$\frac{\sigma_2}{\sigma_N} \approx \frac{\pi\Delta_g}{\hbar\omega}$$

Transmission Line Theory → Inductance





- Find L using transmission line model
- Vary input I to obtain $L(I)$
- Polynomial fit to obtain I_* (and $I_{*,4}$)

Romijin et al, 1982:

" If the width is larger than the coherence length, vortex nucleation and vortex flow can be induced at high current densities...."

ξ = coherehnce length, $\xi_{Al} = 190$ nm , $\xi_{Ti} = 60$ nm

Previous Experiments:
$w = 30 - 120$ nm
$t = 20 - 90$ nm

Near perfect agreement with theory

Realistic Device Dimensions:
$w \approx$ order of a few μm
$t \approx$ order of 10s – 100s nm

How big can current be before result deviate from theory?

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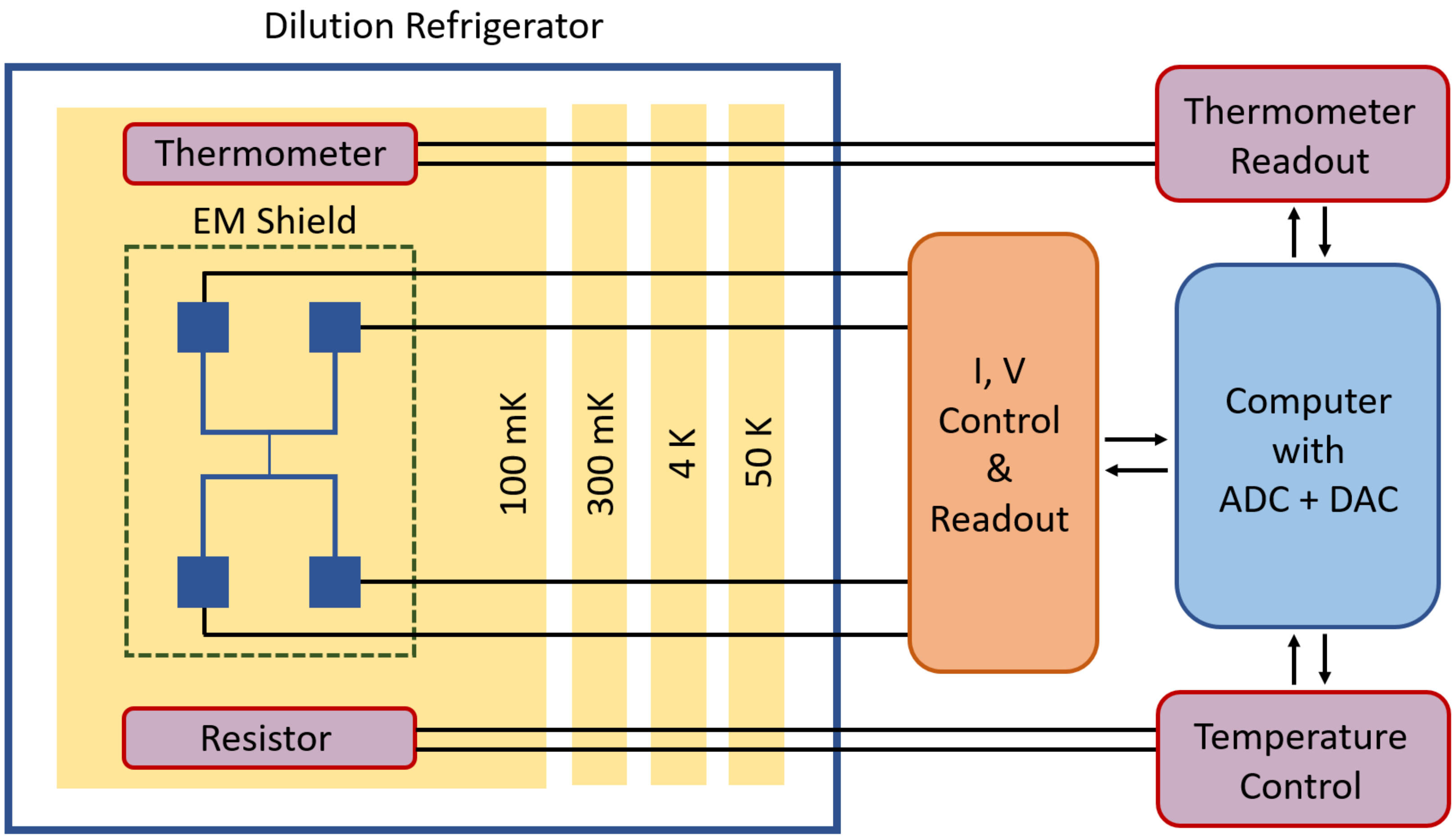
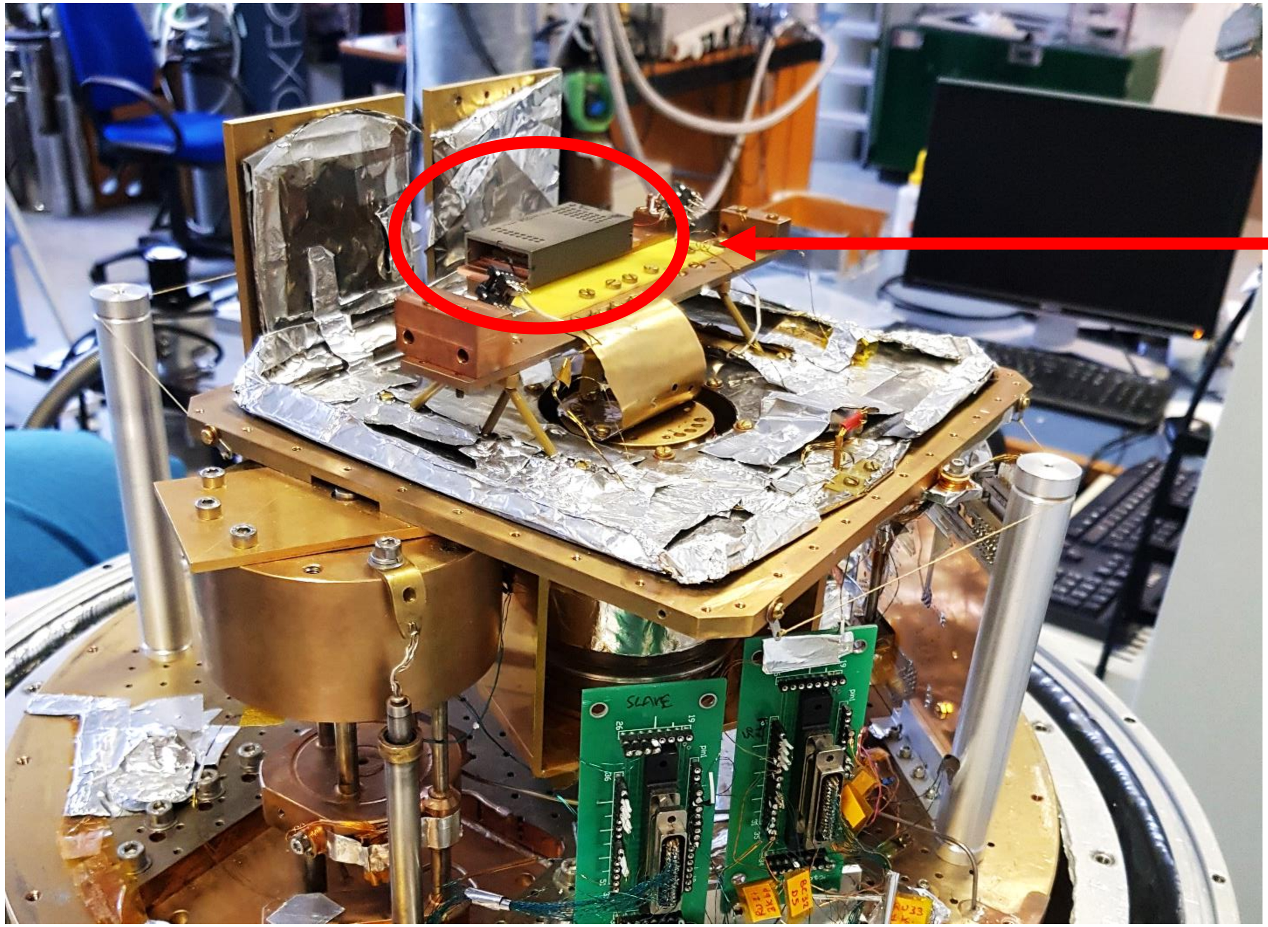
Experimental Method



1. Set fixed I
2. Increase T
3. Record $(\mathrm{I}, \mathrm{T_c})$ at transition

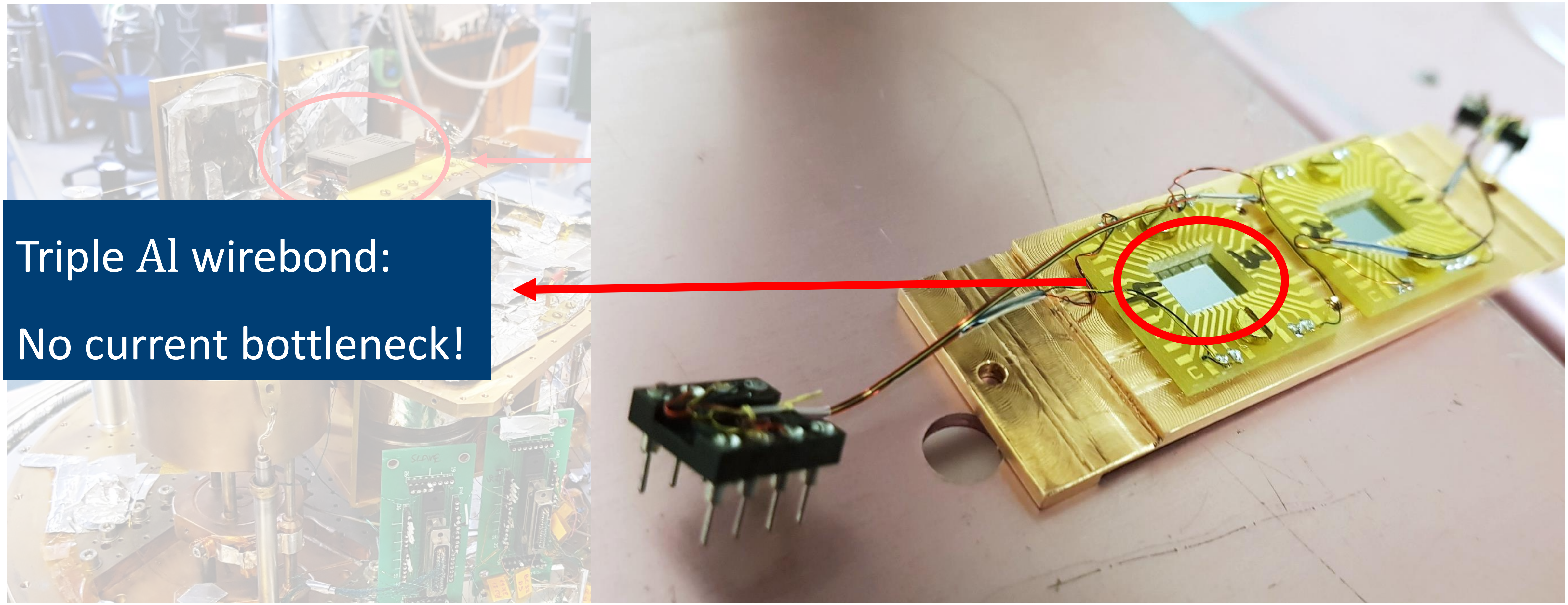
Experimental Method



Nb magnetic shield:

Mitigate environment influences

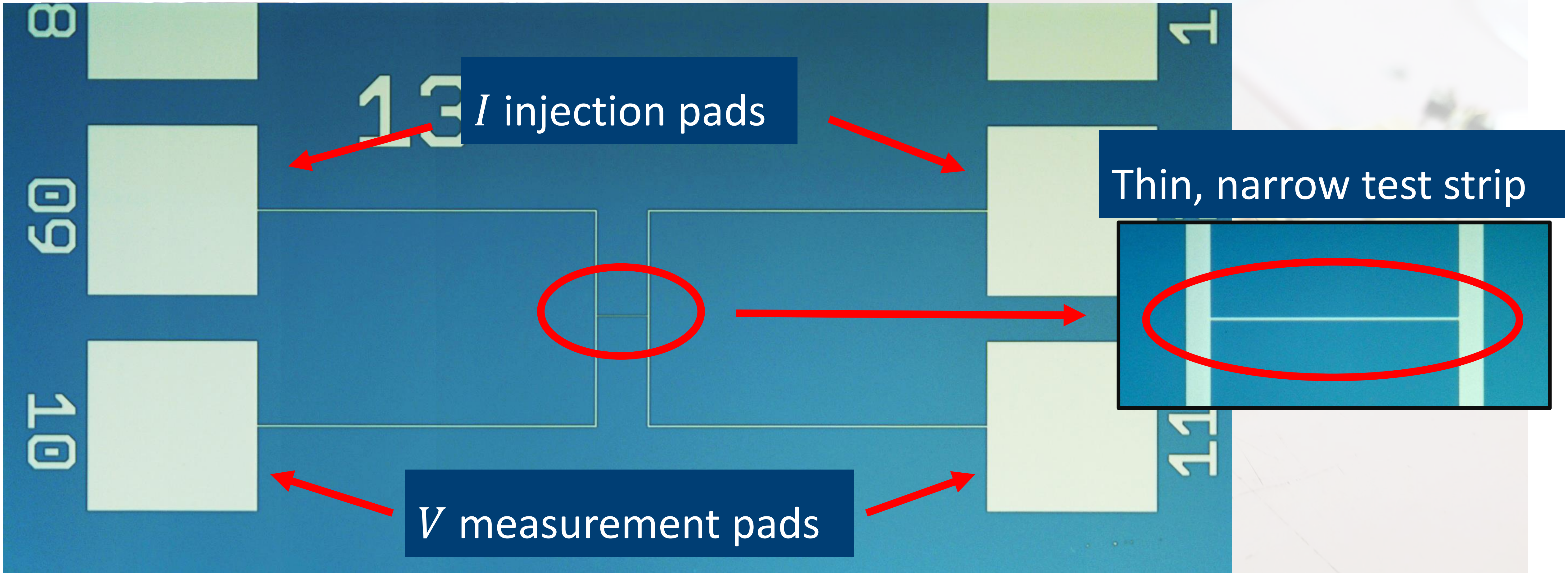
Experimental Method



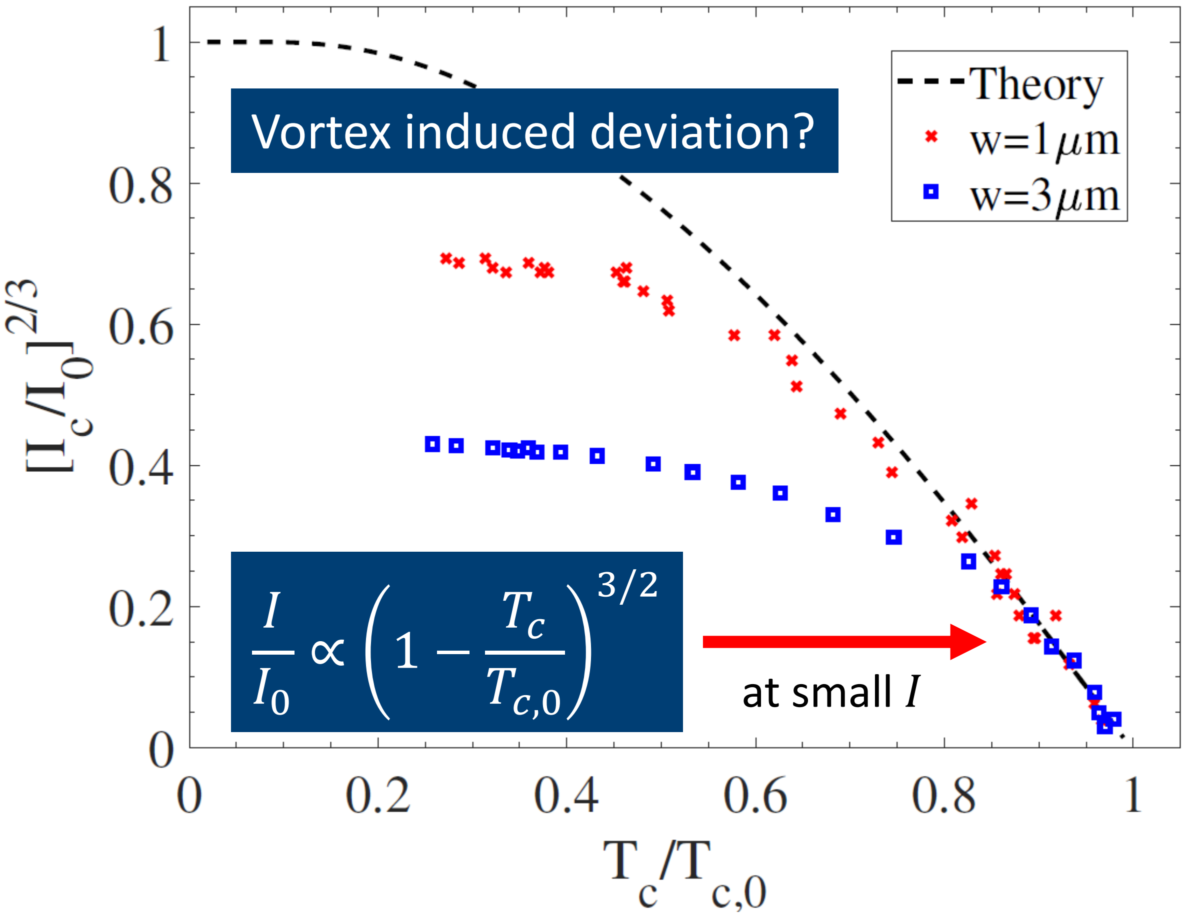
Triple Al wirebond:

No current bottleneck!

Experimental Method

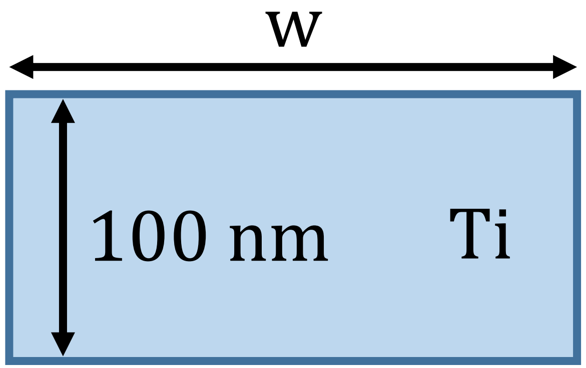


Tc Measurements

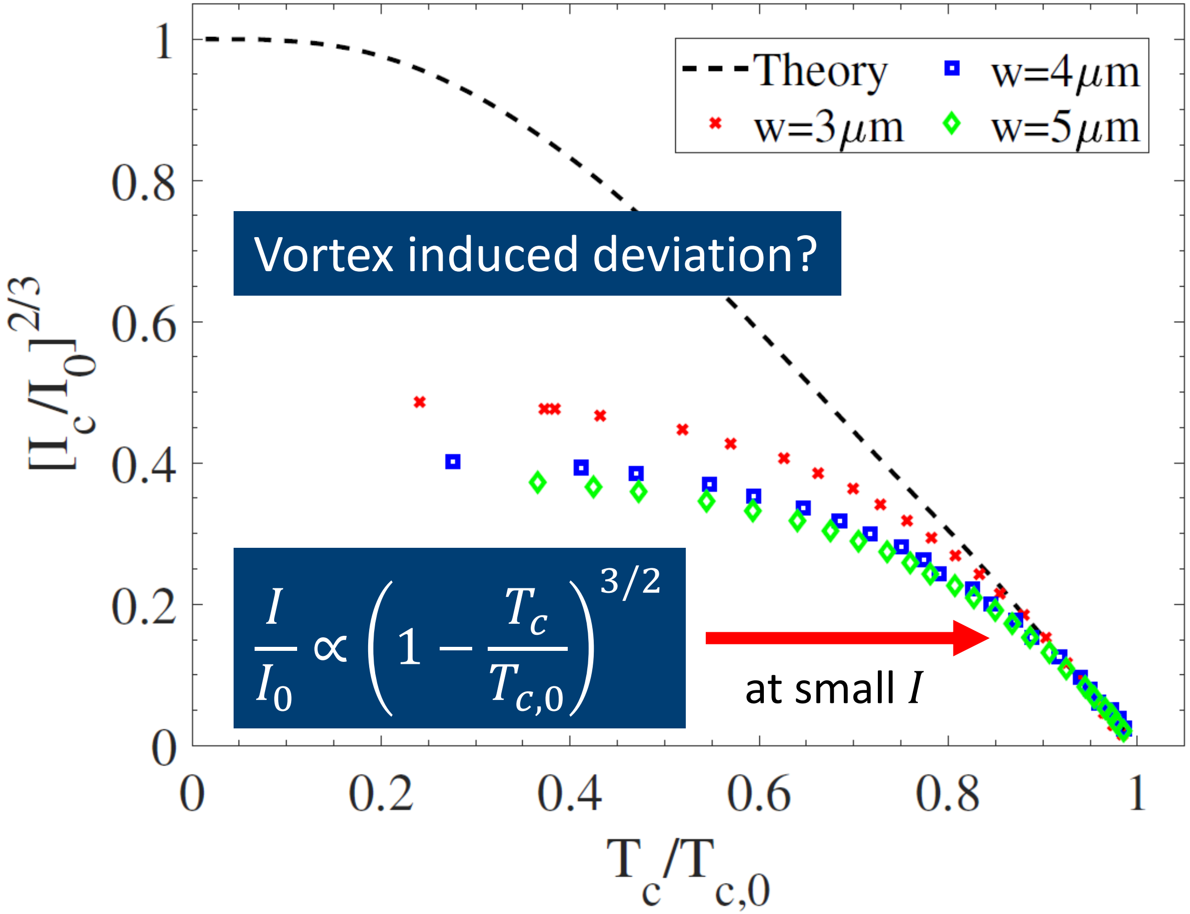


I_0 - theoretical critical current at 0K
$I_{0,c}$ - experimental critical current at 0K

$w = 1$ µm – good agreement for $I < \frac{2}{3} I_{0,c}$
$w = 3$ µm – good agreement for $I < \frac{1}{2} I_{0,c}$



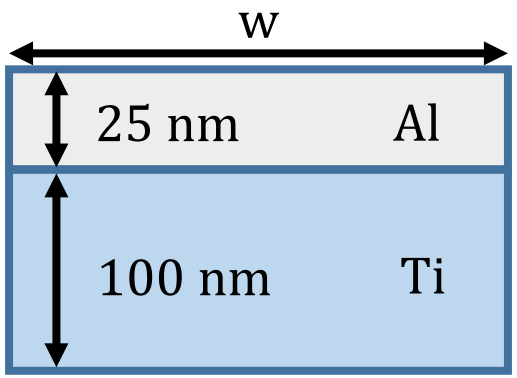
Tc Measurements



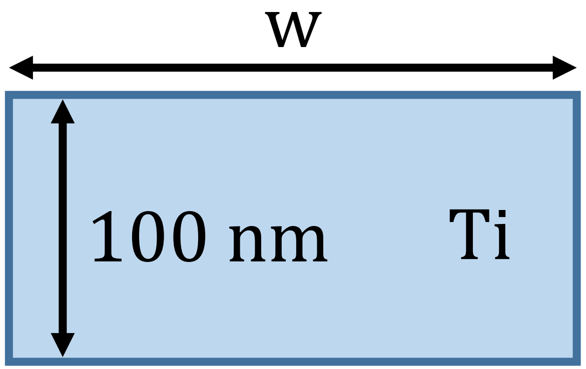
$$\frac{I}{I_0} \propto \left(1 - \frac{T_c}{T_{c,0}}\right)^{3/2}$$

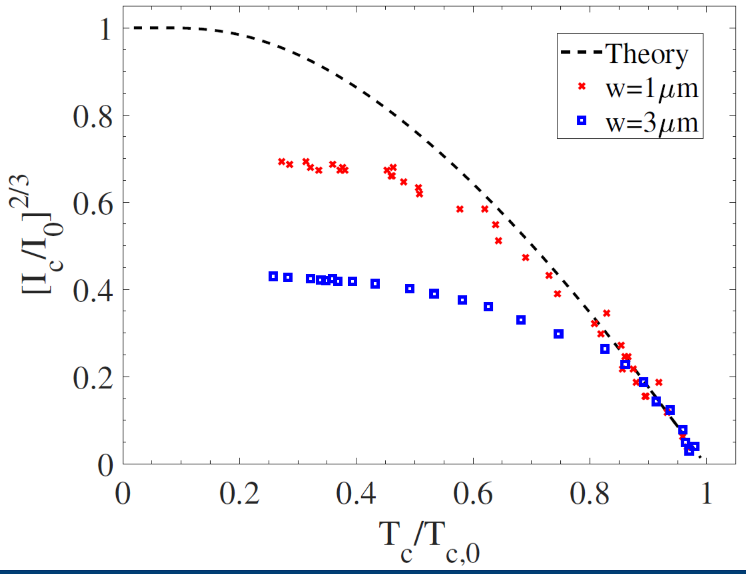
I_0 - theoretical critical current at 0K
$I_{0,c}$ - experimental critical current at 0K

$w = 3\ \mu\text{m}$ – good agreement for $I < \frac{1}{2} I_{0,c}$
$w = 4, 5\ \mu\text{m}$ – good agreement for $I < \frac{1}{3} I_{0,c}$

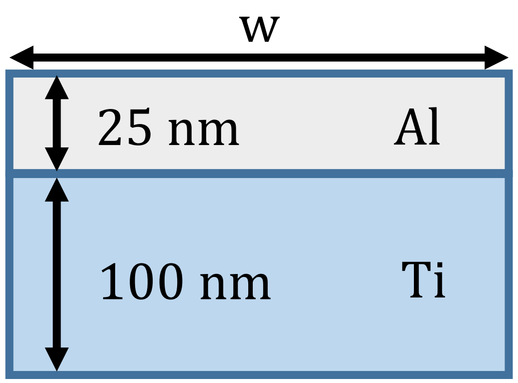


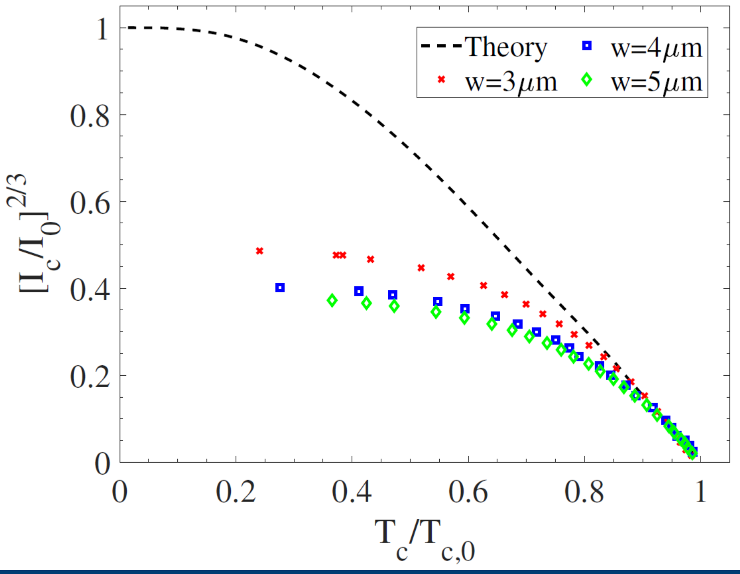
Tc Measurements





- $I < \frac{1}{3} I_{0,c}$ covers most experimental applications
- Higher I is avoided because:
  - Onset of dissipation
  - Bifurcation
  - Unaccounted higher order nonlinearity



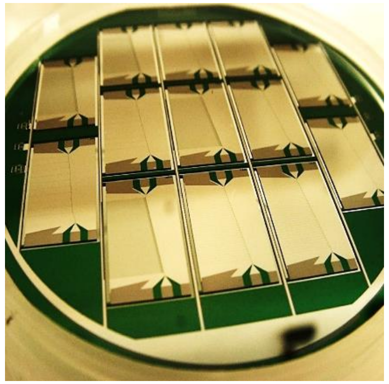
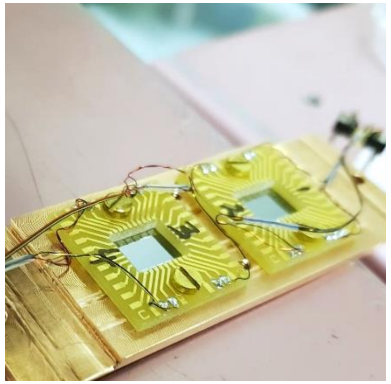


Summary of Key Results

- Numerical routine for I_*
  - Full densities of states
  - Single or multi layer
  - Transmission line geometry

- Experiment comparing $T_c(I)$
  - Agreement with theory when $I < I_{c,0}/3$
  - Experimentally useful range

- Technique useful to understand, optimize, and design single / multi layer thin-film devices

Acknowledgements

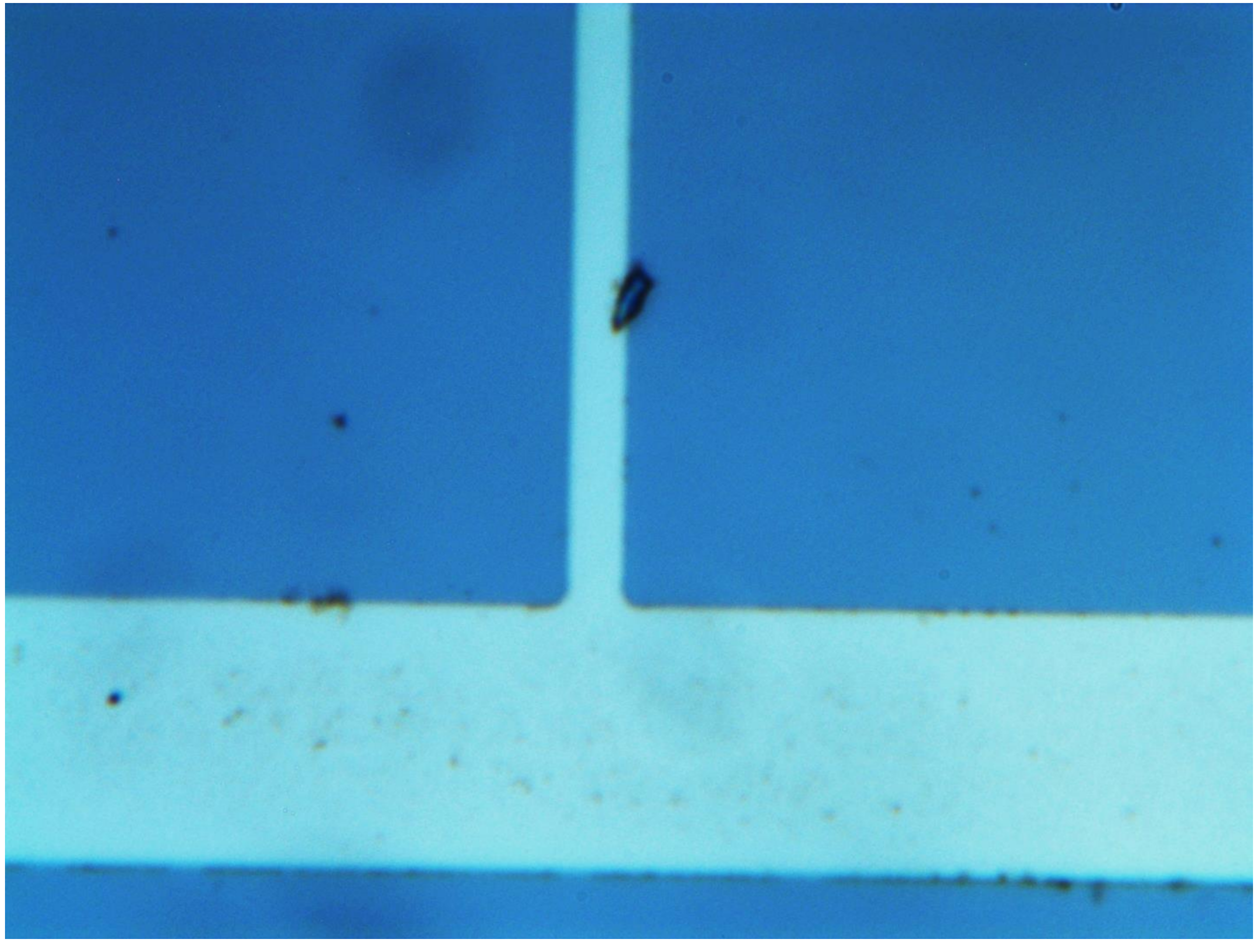
Co-Authors:
Stafford Withington, David Goldie, Chris Thomas

Test System:
David Sawford

Fabrication:
Chris Thomas, Michael Crane

Questions?

No Data for Broken Lines



Critical Current Calculation