LTD 2019: Nonlinear Properties of Supercurrent-Carrying Single and Multi-Layer Thin-Film Superconductors

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Thin-Film Devices

Resonator Devices

Travelling-Wave Devices
Thin-Film Devices

- Narrow band
- High quality factor
- Often easy to multiplex
- Kinetic Inductance Detectors (KIDs)
- Quantum bits (Qubits)
Thin-Film Devices

- Wide band
- Useful readout components
- Parametric Amplifiers
- Parametric Up Converters
- Low-loss delay lines
Kinetic Inductance Nonlinearity

\[
L = L_0 \left[ 1 + \left( \frac{I}{I_*} \right)^2 + \left( \frac{I}{I_{*,4}} \right)^4 + \cdots \right]
\]

- \( L \) Inductance
- \( L_0 \) Inductance when \( I = 0 \)A
- \( I \) Supercurrent
- \( I_* \) Quadratic nonlinearity factor
- \( I_{*,4} \) Quartic nonlinearity factor
Zmuidzinas, 2012:

“MKIDs are usually operated in a regime in which the microwave currents are strong and nonlinearity is becoming important.....”

Eom et al., 2012:

“A parametric amplifier that overcomes these limitations through the use of a travelling-wave geometry and the nonlinear kinetic inductance of a superconducting transmission line......”
Analysis Routine

Geometries
Materials
$T, I$

$Z, Y$
$L = \text{Im}(Z)/\omega$

Repeat for different $I$

Usadel Equations

$\theta$

Nam's Equations

$\sigma$

Transfer Matrices

$L(I)$

$I_s$

$T$  Temperature

$\theta$  Green’s functions

$\sigma$  Conductance

$Z_s$  Surface impedance

$Z$  Series impedance

$Y$  Shunt conductance
Numerically solving the Usadel equations

Superfluid Velocity:
\[ \hat{\nu}_s = D_s [\nabla \phi - (2e/h) \hat{A}] \]

Quasiparticle Density of States:
\[ N = N_0 \text{Re}(\cos \theta) \]

\[ \frac{\hbar D_s}{2} \nabla^2 \theta + iE \sin \theta + \Delta \cos \theta - \frac{\hbar}{2D_s} \hat{\nu}_s^2 \cos \theta \sin \theta = 0 \]

\[ N_s V_{0,S} \int_0^{k_{B}\Theta_{D,S}} dE \tanh \left( \frac{E}{2k_B T} \right) \text{Im}(\sin \theta) = \Delta \]

\[ \frac{\sigma_N}{eD_s} \int_0^{\infty} dE \tanh \left( \frac{E}{2k_B T} \right) \text{Im}(\sin^2 \theta) \hat{\nu}_s = \hat{j} \]

Solved iteratively to obtain \( \Delta, \theta \)

Relates solution to \( \hat{j} \)
Usadel Equations → Densities of States

\[ \Gamma/\Delta_0 = 1.0 \times 10^{-3} \]
\[ \Gamma/\Delta_0 = 6.1 \times 10^{-3} \]
\[ \Gamma/\Delta_0 = 11.2 \times 10^{-3} \]

1 - \Delta/\Delta_0 = 0.6 \times 10^{-3}
1 - \Delta/\Delta_0 = 4.5 \times 10^{-3}
1 - \Delta/\Delta_0 = 8.4 \times 10^{-3}

**DoS broadens!**

\( \Delta_g \) – DoS gap
\( \Delta \) – order parameter

\( \Gamma \) is the depairing factor \( \propto I^2 \)

Move at different rate
Nam’s Equations → Complex Conductivity

\[ \sigma_2 \approx \pi \Delta \frac{\sigma_N}{\hbar \omega} \]

Full Nam’s conductivity integrals:

\[ \frac{\sigma_2}{\sigma_N} = \int dE \ fn(\sin \theta, \cos \theta, E) \]

\[ \frac{\sigma_2}{\sigma_N} \approx \pi \Delta_g \frac{\sigma_N}{\hbar \omega} \]

Γ is the depairing factor \( \propto I^2 \)
Transmission Line Theory → Inductance

- Find $L$ using transmission line model
- Vary input $I$ to obtain $L(I)$
- Polynomial fit to obtain $I_*$ (and $I_{*,4}$)

Al layer decreases nonlinearity
Romijin et al, 1982:

“*If the width is larger than the coherence length, vortex nucleation and vortex flow can be induced at high current densities...”*

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = \text{coherence length}$</td>
<td>$\xi_{Al} = 190 \text{ nm}$, $\xi_{Ti} = 60 \text{ nm}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Previous Experiments:</th>
<th>Realistic Device Dimensions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w = 30 - 120 \text{ nm}$</td>
<td>$w \approx \text{order of a few } \mu\text{m}$</td>
</tr>
<tr>
<td>$t = 20 - 90 \text{ nm}$</td>
<td>$t \approx \text{order of } 10s - 100s \text{ nm}$</td>
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</tbody>
</table>

Near perfect agreement with theory

How big can current be before result deviate from theory?
Experimental Method

1. Set fixed $I$
2. Increase $T$
3. Record $(I, T_c)$ at transition
Experimental Method

Nb magnetic shield:
Mitigate environment influences
Experimental Method

Triple Al wirebond:
No current bottleneck!
Experimental Method

- Magnetic shield
- ***Triple*** Al wirebond:
  - No current bottleneck!
- **$V$** measurement pads
- **$I$** injection pads
- Thin, narrow test strip
Tc Measurements

$I_0$ - theoretical critical current at 0K
$I_{0,c}$ - experimental critical current at 0K

$w = 1 \text{ \mu m} - \text{good agreement for } I < \frac{2}{3} I_{0,c}$

$w = 3 \text{ \mu m} - \text{good agreement for } I < \frac{1}{2} I_{0,c}$

Vortex induced deviation?
**Tc Measurements**

$I_0$ - theoretical critical current at 0K
$I_{0,c}$ - experimental critical current at 0K

$w = 3 \, \mu\text{m} - \text{good agreement for } I < \frac{1}{2} I_{0,c}$

$w = 4, 5 \, \mu\text{m} - \text{good agreement for } I < \frac{1}{3} I_{0,c}$

**Graphical Representation**

- $\frac{I}{I_0} \propto \left(1 - \frac{T_c}{T_{c,0}}\right)^{3/2}$ at small $I$

**Layer Information**

- **25 nm Al**
- **100 nm Ti**
Tc Measurements

- $I < \frac{1}{3} I_{0,c}$ covers most experimental applications
- Higher $I$ is avoided because:
  - Onset of dissipation
  - Bifurcation
  - Unaccounted higher order nonlinearity

$\text{W}$

$100 \text{ nm} \quad \text{Ti}$

$\text{W}$

$25 \text{ nm} \quad \text{Al}$

$100 \text{ nm} \quad \text{Ti}$
Summary of Key Results

- Numerical routine for $I_*$
  - Full densities of states
  - Single or multi layer
  - Transmission line geometry

- Experiment comparing $T_c(I)$
  - Agreement with theory when $I < I_{c,0}/3$
  - Experimentally useful range

- Technique useful to understand, optimize, and design single / multi layer thin-film devices
Acknowledgements

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Questions?
No Data for Broken Lines
Critical Current Calculation