The reference frame interpretation

Natalia Sánchez-Kuntz; Eduardo Nahmad-Achar

Institute for Nuclear Sciences

National Autonomous University of Mexico (UNAM)

Statement of the PBR theorem

Any model in which ψ represents mere information about an underlying physical state, must make predictions that contradict those of quantum theory.

Assumptions

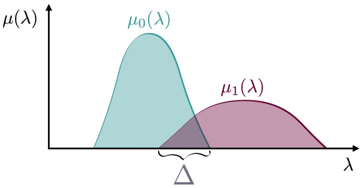
- There is an underlying physical state of the system;
- Systems that are prepared independently have independent physical states;

Characterisation of information

If λ is the phase space of physical states one can define the probability distribution of $|\psi_i\rangle$ over phase space,

$$\mu_i(\lambda)$$

If the distributions $\mu_0(\lambda)$ and $\mu_1(\lambda)$ of two non-orthogonal quantum states $|\psi_0\rangle$ and $|\psi_1\rangle$ overlap, then one can conclude that $|\psi_0\rangle$ and $|\psi_1\rangle$ represent mere information about the system they describe.



And vice versa.

Preparation devices and states

To elaborate their no-go theorem, they propose the following scenario:
Consider two identical and independent preparation devices; each device prepares a system in either the quantum state

$$|\psi_0\rangle = |0\rangle$$

or the quantum state

$$|\psi_1\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

so that when the two states are brought together, the complete system can be prepared in any of the four quantum states

$$|0\rangle \otimes |0\rangle, \; |0\rangle \otimes |+\rangle, \; |+\rangle \otimes |0\rangle, \; \text{and} \; |+\rangle \otimes |+\rangle \tag{1}$$

Measurement

The complete system can be measured, and for this they propose an entangled measurement with the four possible outcomes:

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle\Big]$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle\Big]$$

$$|\xi_3\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle\Big]$$

$$|\xi_4\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle\Big]$$

If $|\psi_0\rangle$ and $|\psi_1\rangle$ represent mere information, there is a probability $q^2 > 0$ that both systems result in physical states, λ_1 and λ_2, from the overlap region, Δ —from the previous characterisation.

Measurement and contradiction

But the probability that the quantum state

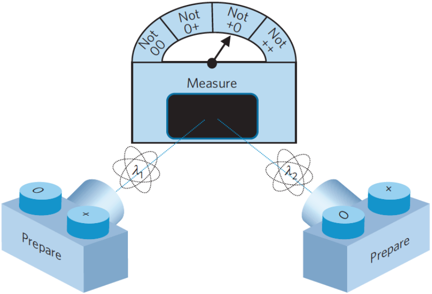
$$|0\rangle \otimes |0\rangle$$

results in

$$|\xi_1\rangle$$

is zero, same for $|0\rangle \otimes |+\rangle$ resulting in $|\xi_2\rangle$, for $|+\rangle \otimes |0\rangle$ resulting in $|\xi_3\rangle$, and for $|+\rangle \otimes |+\rangle$ resulting in $|\xi_4\rangle$.
This takes them to the conclusion that if the state $\lambda_1 \otimes \lambda_2$ that arrives to the detector is compatible with the four quantum states (1), then the measuring device could give a result that should, following simple QM, occur with zero probability.

Contradiction



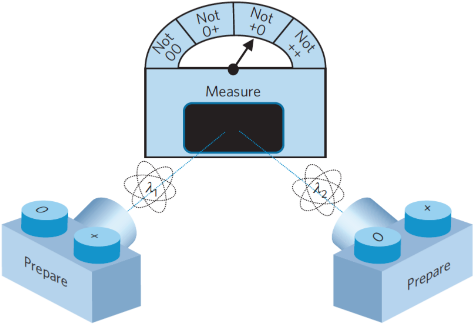
[1]

The previous contradiction arises only by assuming that the distributions of $|0\rangle$ and $|+\rangle$ overlap, so their distributions cannot overlap, and therefore $|\psi_0\rangle = |0\rangle$ and $|\psi_1\rangle = |+\rangle$ cannot represent mere information of an underlying physical system.

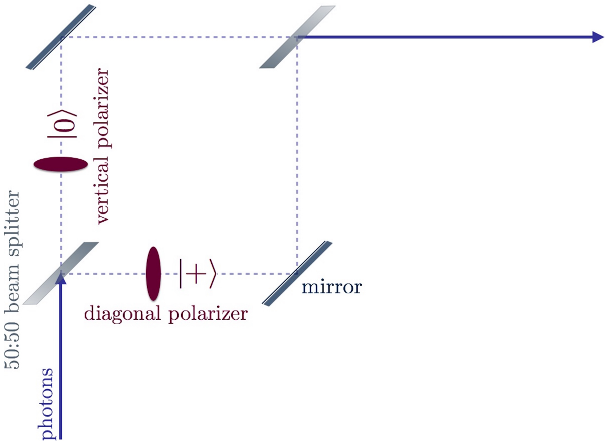
[1] Figure given by PBR in Nat. Phys. 8, 475 (2012)

Preparation devices

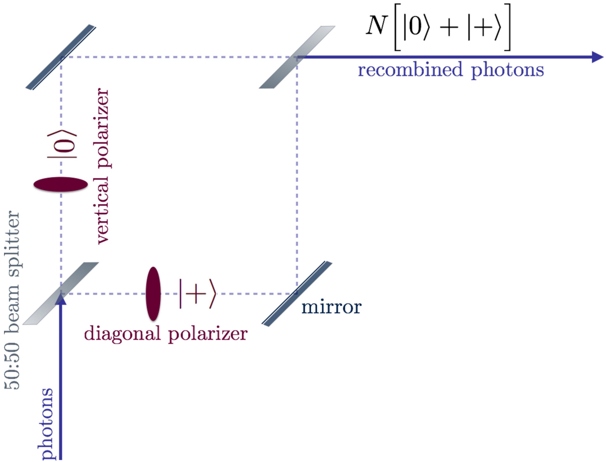
...but there is something peculiar about the PBR preparation devices.



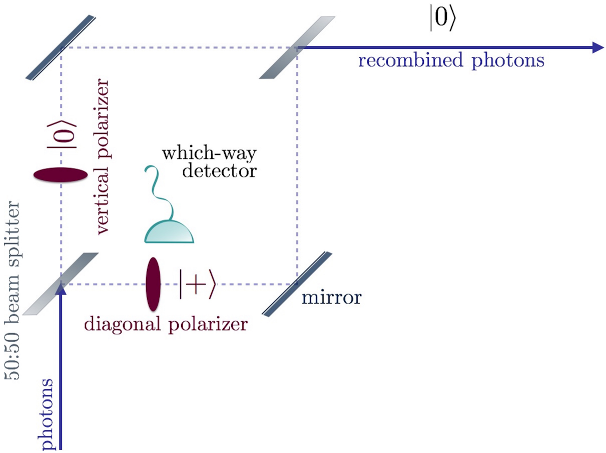
Preparation devices and states



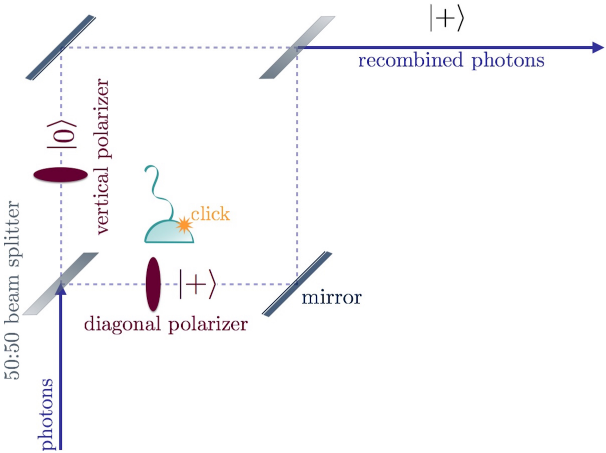
Preparation devices and states



Preparation devices and states



Preparation devices and states



No contradiction

In the case where there is no distinguishability between the preparation of $|0\rangle$ and the preparation of $|+\rangle$, the state that would arrive at the detector would be

$$|\Psi\rangle = |\psi\rangle \otimes |\psi\rangle = N^2\Big[|0\rangle + |+\rangle\Big] \otimes \Big[|0\rangle + |+\rangle\Big],$$

and not one of the states (1) assumed by PBR.
This state $|\Psi\rangle$ that arrives at the detector is compatible with the measurement basis used in the PBR theorem, in the sense that it may result in any of its elements $(|\xi_i\rangle)$ with non-zero probability.
Following the logic of PBR, no contradiction arises when regarding $|0\rangle$ and $|+\rangle$ as mere information.

Then, what does ψ represent?

We can conclude from the previous revision that the degree of "physicality" of a property depends directly on the measurement such property has undergone. A property that has not been measured can be regarded as mere information, while a property that has been measured has to be regarded as a physical property of a certain system, having a counterpart in reality.

The complementarity principle

> *In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science.*

Complementarity meant for Bohr an understanding of physical reality in regards to reference frames, the defining objects of reference frames being the measuring apparatuses and the quantities coming into being within these reference frames as complementary, meaning that two or more complementary quantities cannot manifest in one and the same reference frame, and that each quantity must manifest in its corresponding reference frame.

The reference frame interpretation

- A reference frame is determined by a complete set of commuting operators;
- Eigenstates of these complete set are **ontological states**;
- The states generated from linear combinations of different eigenstates of an observable are **quantum states**;

The reference frame interpretation

- A reference frame is determined by a complete set of commuting operators;
- Eigenstates of these complete set are **ontological states**;
- The states generated from linear combinations of different eigenstates of an observable are **quantum states**;
- The reference frame determines the ontological or informational character of ψ;

Complementary reference frames

Any non-commuting quantities define complementary reference frames.
So, the emblematic complementary quantities of position and momentum define complementary reference frames, since

$$[\hat{x}, \hat{p}] = -i\hbar$$

holds.

Complementary reference frames

Any non-commuting quantities define complementary reference frames.

So, the emblematic complementary quantities of position and momentum define complementary reference frames, since

$$[\hat{x}, \hat{p}] = -i\hbar$$

holds.

Time and energy also define complementary reference frames.

Complementary reference frames

Any non-commuting quantities define complementary reference frames.
So, the emblematic complementary quantities of position and momentum define complementary reference frames, since

$$[\hat{x}, \hat{p}] = -i\hbar$$

holds.
Time and energy also define complementary reference frames.
Eigenstates of position are defined in space-time, while eigenstates of momentum are defined in, what we call, momentum-energy.

The momentum-energy manifold

A momentum measurement would force the system being measured to stand in the momentum reference frame. In the position representation,

$$\psi_{\vec{p}}(\vec{x}, t) = e^{\frac{i}{\hbar}(\vec{p}_0 \cdot \vec{x} - E_0 t)}$$

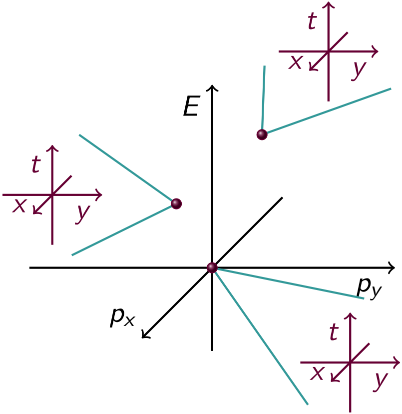
and, as we have seen, this state only depicts information of the particle's whereabouts in the position reference frame, while it is an ontological state in the momentum reference frame. Indeed,

$$\psi_{\vec{p}}(\vec{p}, E) = \delta^{(3)}(\vec{p} - \vec{p}_0)\delta(E - E_0)$$

For a free particle in a momentum eigenstate all of space-time is an equivalence class projected onto the location of the particle in momentum-energy.

The momentum-energy manifold

Four-dimensional space-time projected as an equivalence class to four-dimensional momentum-energy. As we know from SR, space and time are geometrically intertwined. We propose the same for energy and momentum.



The momentum-energy manifold

The tangent space to an event e in $(\mathcal{M}_x, \mathcal{G}_x)$, $T_e(\mathcal{M}_x)$, has the four basis vectors

$$\left\{ \frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right\}$$

which in QM are, precisely,

$$-\frac{i}{\hbar} \{p_0, p_1, p_2, p_3\}$$

So momentum-energy is simply the tangent space to space-time, and vice versa.

Conclusions

- PBR theorem needs distinguishability between states. So, in order to follow PBR's theorem to its consequence, the precise system described by the quantum state must have undergone a measurement;
- Built an interpretation with the previous result. The character of ψ is more subtle than just the division between ontological/epistemological;
- This interpretation can be applied to explain in a local way the violation of Bell's inequality, and to explain *in a less paradoxical manner* the double slit experiment and the measurement problem;
- One can we give a geometrical structure to momentum-energy, a manifold isomorphic to space-time;

All this is work in progress.

References

[1] Pusey, M. F., Barrett, J., Rudolph, T.: On the reality of the quantum state. Nat. Phys. 8, 475 (2012)

[2] Harrigan, N., Spekkens, R. W.: Einstein, incompleteness, and the epistemic view of quantum states. Found. Phys. 40, 125 (2010)

[3] Bohr, N.: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 48, 696 (1935)

[4] Faye, J.: Copenhagen interpretation of quantum mechanics. In *The Stanford Encyclopedia of Philosophy*, edited by E.N. Zalta, (2014)

[5] Bohr, N.: *Atomic Theory and the Description of Nature*, reprinted as *The Philosophical Writings of Niels Bohr, Vol. I*, (Ox Bow Press 1987)

[6] Dickson, M.: The EPR experiment: A prelude to Bohr's reply to EPR. In *History of Philosophy of Science, New Trends and Perspectives*, edited by M. Heidelberger and F. Stadler, 263 (Kluwer Academic Publishers 2002)

[7] Sánchez-Kuntz N., Nahmad-Achar E.: Quantum locality, rings a bell?: Bell's inequality meets local reality and true determinism. Found. Phys. 48, 27 (2018)

[8] Aharonov, Y.: Can we make sense out of the measurement process in relativistic quantum mechanics? Phys. Rev. D. 24, 359 (1981)

[9] Leifer, M.S.: Is the quantum state real? An extended review of ψ-ontology theorems. Quanta 3, 67 (2014)

The momentum-energy manifold

We propose that the geometrical structure of the four-dimensional manifold momentum-energy $(\mathcal{M}_p, \mathcal{G}_p)$ is identical to that of space-time $(\mathcal{M}_x, \mathcal{G}_x)$. That is, there exists a non-natural isomorphism ϕ which maps one space to the other.
All the properties of $(\mathcal{M}_p, \mathcal{G}_p)$ are inherited from Special Relativity. Just as events are defined by the position four-vector, swifts can be defined by the momentum four vector,

$$(p^0, \vec{p}) = (E, p_x, p_y, p_z).$$