Correlated photon emission by two excited atoms in a waveguide

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Information Geometry, Quantum Mechanics and Applications

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Outline

- Waveguide QED
  - Non-relativistic Hamiltonian $\&$ Dipolar interaction
  - Excitation sectors in the Friedrichs-Lee model
- One-excitation sector
  - Diagrams $\&$ Self-energy
  - Entanglement by relaxation
- Two-excitation sector
  - Perturbative expansion
  - Non-perturbative weak coupling limit

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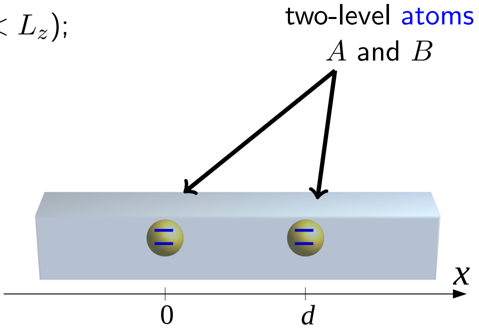
Waveguide QED

Multi-level quantum systems decay can be controlled by means of boundary conditions and dimensional reduction (e.g. embedding in a waveguide).

- Rectangular cross section $(L_y < L_z)$;
- Cut-off frequencies:
$$M_{n,m} = \pi\sqrt{\frac{n^2}{L_z^2} + \frac{m^2}{L_y^2}};$$
- Dispersion relation:
$$\omega(k) = \sqrt{k^2 + M_{n,m}^2}.$$

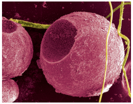


Electromagnetic field quantization:
$$[a(k), a^\dagger(k')] = \delta(k - k')$$

⇒ the model concerns photons in the lowest-cutoff energy $TE_{1,0}$ mode

Building block: the coupled qubit

Quantum Dot



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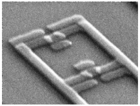

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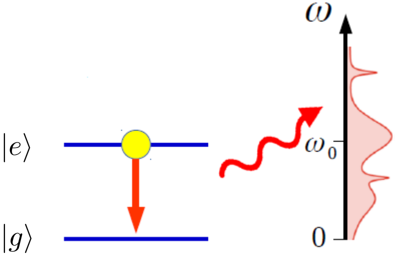
Artificial atoms engineering



Hilbert space: $\mathcal{H} \otimes \mathcal{H}_{\mathrm{B}}$

$$\begin{cases} \mathcal{H} = \mathrm{span}\{|e\rangle, |g\rangle\} \\ \mathcal{H}_{\mathrm{B}} = \mathrm{span}\{|\mathbf{k}\rangle\}, \text{ with } \mathbf{k} \in X \end{cases}$$

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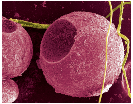

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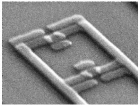

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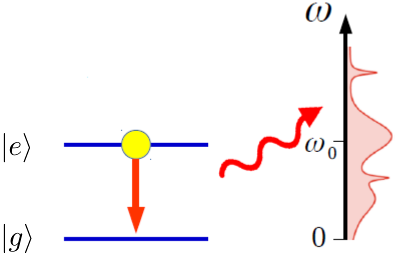

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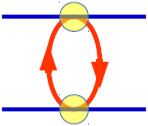
Structured bath

dispersion relation $\omega : X \to \mathbb{R}$

spectrum of a multiplication operator: $\Omega |\mathbf{k}\rangle = \omega(\mathbf{k}) |\mathbf{k}\rangle$

Examples:

- $\omega_0 \in \sigma_{ac}(\Omega)$: decay is energetically allowed (in most cases);
- $\omega_0 \notin \sigma_{ac}(\Omega)$: decay inhibition



Non-relativistic Hamiltonian & Dipolar interaction

Hamiltonian of an atom trapped in a potential $V(\mathbf{r})$:

$$H_{\rm at} = \frac{1}{2m_e}(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2 - V(\mathbf{r}) = H_0 - \frac{e}{m_e}\mathbf{p}\cdot\mathbf{A}(\mathbf{r}) + \mathcal{O}(e^2),$$

- nondynamical atoms center of mass position $\mathbf{r_0}$;
- dipolar approximation: atoms are assumed pointlike dipoles,

$$\begin{aligned}\langle e| H_{\rm int}^{(\rm dip)} |g\rangle &= -\frac{e}{m_e} A_z^{(1,0)}(\mathbf{r_0}) \langle e| p_z |g\rangle \\ &= -ie\omega_0 A_z^{(1,0)}(\mathbf{r_0}) z_{eg}.\end{aligned}$$

Dipole moment $\quad d_{eg} = e|z_{eg}|$

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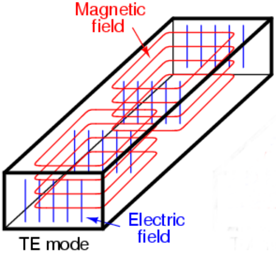

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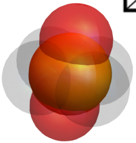

$$|g\rangle = |n=1, l=0, m=0\rangle$$
$$|e\rangle = |n=2, l=1, m=0\rangle$$

Friedrichs-Lee model

Vacuum instability description ⇨ Hamiltonian $H_g = H_0 + \lambda V_g$ in dipolar and rotating wave approximations:

$$H_0 = \omega_0 \left|e\right\rangle \left\langle e\right| \otimes \mathbb{1} + \mathbb{1} \otimes \int dk \; \omega(k) a^\dagger(k) a(k),$$

$$V_g = \int dk \left[\sigma^+ \otimes g(k) a(k) + \sigma^- \otimes g^*(k) a^\dagger(k)\right],$$

with form factor g, coupling constant λ, identity $\mathbb{1}$, ladder operators σ^+, σ^-.

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Excitation number: $\mathcal{N} = \left|e\right\rangle \left\langle e\right| + \int dk\; a^\dagger(k) a(k) \implies [H_g, \mathcal{N}] = 0$

One-excitation sector: $\mathcal{H} \otimes \mathcal{H}_{\mathrm{B}} = \mathbb{C} \oplus L^2(\mathbb{R})$ ➡ $\left|\psi\right\rangle = c \left|e\right\rangle \otimes \left|0\right\rangle + \left|g\right\rangle \otimes \left|\varphi\right\rangle$ with $c \in \mathbb{C},\; \varphi \in L^2(\mathbb{R})$

Vacuum state: $\left|vac\right\rangle = \left|e\right\rangle \otimes \left|0\right\rangle$

- Excited state probability $|c|^2$;
- One photon wavefunction φ.

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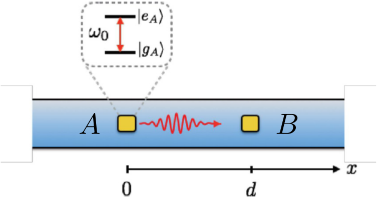
Infinite waveguide

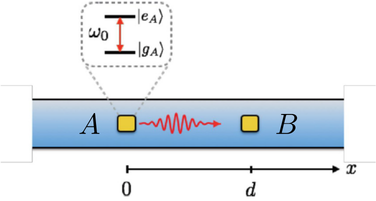


$$H_d = H_0 + \lambda V_d$$

$$H_0 = \omega_0 (|e_A\rangle \langle e_A| + |e_B\rangle \langle e_B|) + \int dk \; \omega(k) a^\dagger(k) a(k),$$

$$V_d = \int \frac{dk}{\sqrt{\omega(k)}} \left[\left(\sigma_A^+ + \sigma_B^+ e^{ikd} \right) a(k) + h.c. \right], \; g_x(k) = \frac{e^{ikx}}{\sqrt{\omega(k)}}$$

Infinite waveguide



$$H_d = H_0 + \lambda V_d$$

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Excitation number

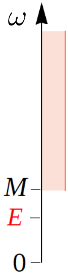
$$\mathcal{N} = |e_A\rangle\langle e_A| + |e_B\rangle\langle e_B| + \int dk\ a^\dagger(k) a(k)$$

still a constant of motion

$$\mathcal{N} = 1 :\ \ |\psi\rangle = (c_A |e_A, g_B\rangle + c_B |g_A, e_B\rangle) \otimes |0\rangle + |g_A, g_B\rangle \otimes |\varphi\rangle$$

with $\varphi \in L^2(\mathbb{R})$ s.t. $|\varphi\rangle = \int dk\ \varphi(k) a^\dagger(k) |0\rangle \in \mathcal{H}_\mathrm{B}$.

Bound states



$$H|\psi\rangle = E|\psi\rangle \text{ with } |\psi\rangle \propto \frac{|e_A, g_B\rangle \pm |g_A, e_B\rangle}{\sqrt{2}} \otimes |0\rangle + \dots$$

$E < M$: Evanescent mode interaction

- for ω_0 below the threshold there are both symmetric and antisymmetric eigenstates;
- the electromagnetic field contribution in these states is not negligible and $E = \omega_0 + \mathcal{O}(\lambda^2)$.

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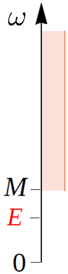

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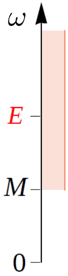

$E > M$: Propagating photon interaction

- a single atom decays if its excitation energy is immersed in the continuum;
- nontrivial bound states occur for particular values of inter-atomic distance.

Resonance condition

$$|\psi\rangle = \ (c_A\,|e_A, g_B\rangle + c_B\,|g_A, e_B\rangle) \otimes |0\rangle + |g_A, g_B\rangle \otimes |\varphi\rangle$$

This is a stationary state associated with the eigenvalue $E = \sqrt{\bar{k}^2 + M^2}$ if:

- atoms distance satisfy a resonance constraint

$$c_A + c_B e^{\pm i\bar{k}d} = 0 \quad \Longrightarrow \quad \bar{k}d = n\pi, \ n \in \mathbb{N};$$

- there are real solutions for

$$E = \omega_0 + \lambda^2 \int dk \frac{1 - (-1)^n e^{-ikd}}{\omega(k)(E - \omega(k))}.$$

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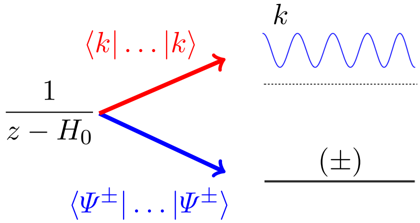
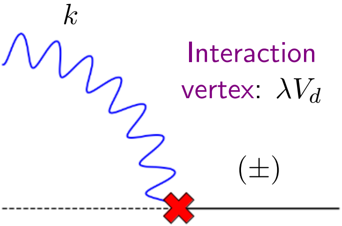
Triplet and singlet are entangled bound states:

$$|\psi_n\rangle = \sqrt{p_n}\frac{|e_A, g_B\rangle + (-1)^{n+1}\,|g_A, e_B\rangle}{\sqrt{2}} \otimes |0\rangle + \mathcal{O}(\lambda),$$

where $p_n \simeq \left(1 + n\frac{2\pi^2\lambda^2 M}{\bar{k}^3}\right)^{-1}$ is a descreasing function with distance.

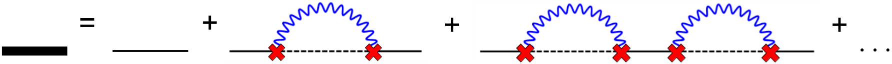
Diagrams & Self-energy



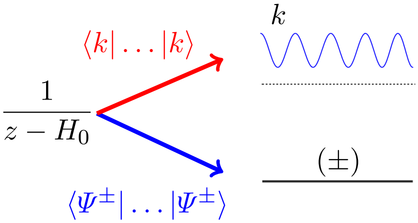


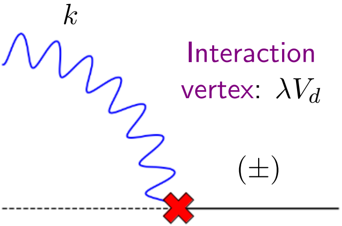
Interaction vertex: λV_d

Propagator rinormalization:



Diagrams & Self-energy





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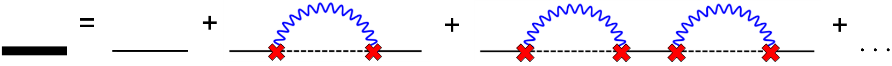

$$\langle \Psi^{\pm}| \frac{1}{z - H_d} |\Psi^{\pm}\rangle = \frac{1}{z - \omega_0} \sum_{n=0}^{\infty} \left(\lambda^2 \frac{\Sigma_{\pm}(z)}{z - \omega_0} \right)^n = \frac{1}{z - \omega_0 - \lambda^2 \Sigma_{\pm}(z)} = \mathcal{G}_{\pm}(z)$$

Self-energy: $\quad \Sigma_{\pm}(z) = \langle \Psi^{\pm}| V_d \frac{1}{z - H_0} V_d |\Psi^{\pm}\rangle$

Resolvent formalism

The propagator elements for the atomic excitation is:

$$\mathcal{G}_{ij}(z) = \langle i| \frac{1}{z - H_d} |j\rangle \text{ with } i,j = 1,2 = (e_A, g_B), (g_A, e_B).$$

Free case $(\lambda = 0)$:

propagator in the basis $\{|e_A, g_B\rangle, |g_A, e_B\rangle\}$

$$\mathcal{G}_0(z) = \frac{1}{z - \omega_0} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \text{unique pole } z = \omega_0$$

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unique pole $z = \omega_0$

Interacting case ($\lambda > 0$):

$$\mathcal{G}(z) = [\mathcal{G}_0(z)^{-1} - \lambda^2 \Sigma(z)]^{-1} \Rightarrow$$

the self-energy $\Sigma(z)$ lifts the degeneracy

the propagator is diagonalized in the Bell basis $|\Psi^\pm\rangle = \frac{|e_A, g_B\rangle \pm |g_A, e_B\rangle}{\sqrt{2}}$:

$$\mathcal{G}(z) = \frac{|\Psi^+\rangle\langle\Psi^+|}{z - \omega_0 - \lambda^2 \Sigma_+(z)} + \frac{|\Psi^-\rangle\langle\Psi^-|}{z - \omega_0 - \lambda^2 \Sigma_-(z)}.$$

Entanglement by relaxation

$$|e_A, g_B\rangle = \frac{|e_A, g_B\rangle + |g_A, e_B\rangle}{2} + \frac{|e_A, g_B\rangle - |g_A, e_B\rangle}{2}$$

n even — decays to ground states + photon

n odd — relaxes to the entangled bound state

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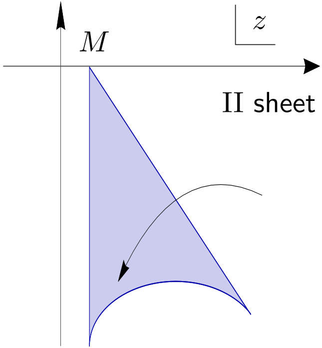
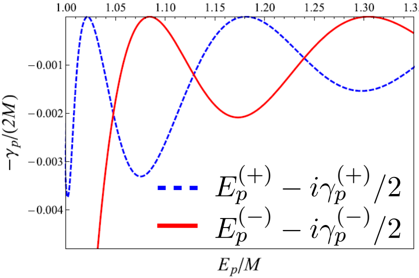
$$\Sigma_{\pm}^{\mathrm{II}}(z - i0^+) = \Sigma_{\pm}(z + i0^+)$$



$$U(t)\theta(t) = \frac{i}{2\pi}\int_{-\infty+i0^+}^{+\infty+i0^+} dz\; e^{-izt}\mathcal{G}(z)$$



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Two-excitation sector

For $\mathcal{N}=2$ the state reads

$$|\psi\rangle = c_{AB}\,|e_A,e_B\rangle + \sum_{s=\pm}\int dk\ B_s(k)\,|\Psi^s;k\rangle$$
$$+\frac{1}{\sqrt{2}}\int dkdk'\ A(k,k')\,|k,k'\rangle \text{ with } B_s\in L^2(\mathbb{R}), A\in L^2(\mathbb{R})^{\odot 2}.$$

This expression implies a dipolar interaction with rotated couplings:

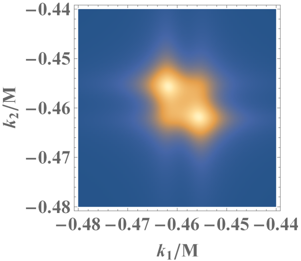
$$V_d=\int dk\ \sum_{s=\pm}\frac{1+se^{ikd}}{\sqrt{2\omega(k)}}b_s\ a^\dagger(k)+\ \text{H.c.}\quad \text{with}\quad g_s(k)=\frac{g_0(k)+sg_d(k)}{\sqrt{2}}$$

where $b_s^\dagger\,|g_A,g_B\rangle = \dfrac{\sigma_A^+ + s\sigma_B^+}{\sqrt{2}}\,|g_A,g_B\rangle = |\Psi^s\rangle$.

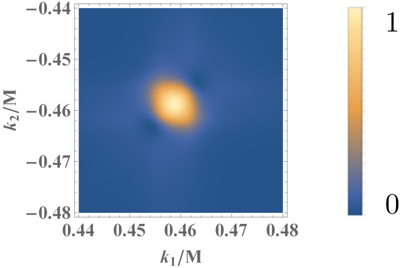
Properties: $(\sigma_{A,B})^2=0 \implies b_+b_-=0$ and $|e_A,e_B\rangle = \pm(b_\pm^\dagger)^2\,|g_A,g_B\rangle$

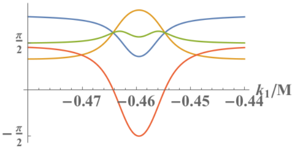
Non-perturbative weak coupling limit

Selected parameters: $\omega_0 = 1.1M$, $\lambda = 10^{-2}M$, $k_0 d = \frac{\pi}{2}$

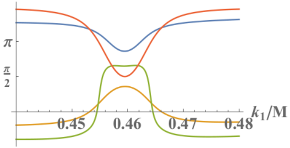


$$\frac{k_0}{M} \simeq 0.458$$



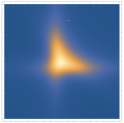


- $\phi_- = \mathrm{Arg}\{\mathcal{A}_-(k_1, k_2)\}$
- $\phi_+ = \mathrm{Arg}\{\mathcal{A}_+(k_1, k_2)\}$
- $\mathrm{Arg}\{\mathcal{A}(k_1, k_2)\}$
- $\phi_+ - \phi_-$



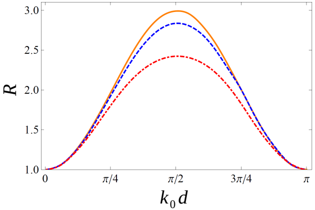
Self-energy non-perturbative effect

splitting: $2\,\Im m\{\Sigma_\pm(\omega_0 + i0^+)\}$

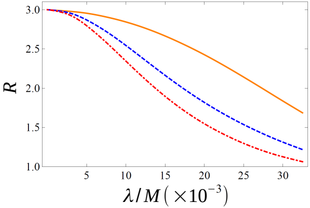
$s = +$: 

$s = -$: 

Non-perturbative weak coupling limit



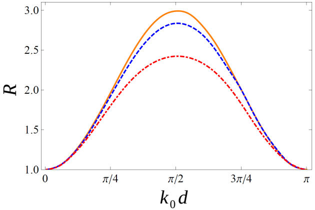
- $\lambda = 10^{-3}M$
- $\lambda = 10^{-2}M$
- $\lambda = 2\times 10^{-2}M$



- $k_0 d = \pi/2$
- $k_0 d = 3\pi/2$
- $k_0 d = 5\pi/2$

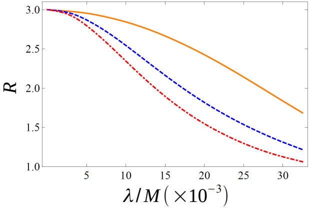
- $\omega_0 = 1.1M$;
- peaks in the variation of $k_0 d$ are points of the orange curve in the variation of λ;
- increased distances implement stronger coupling regimes.

Non-perturbative weak coupling limit



- $\lambda = 10^{-3} M$
- $\lambda = 10^{-2} M$
- $\lambda = 2 \times 10^{-2} M$

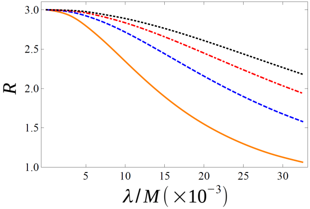
- $\omega_0 = 1.1M$;
- peaks in the variation of $k_0 d$ are points of the orange curve in the variation of λ;
- increased distances implement stronger coupling regimes.



- $k_0 d = \pi/2$
- $k_0 d = 3\pi/2$
- $k_0 d = 5\pi/2$

- $k_0 d = 5\pi/2$;
- increasing values for ω_0 push forward the plasmonic bound state.

- $\omega_0 = 1.1M$
- $\omega_0 = 1.2M$
- $\omega_0 = 1.3M$
- $\omega_0 = 1.4M$



Conclusions & Outlook

- An entangled bound state exists for discrete values of the inter-atomic distance, corresponding to the resonance condition in the one-excitation sector;

- in the two-excitation sector the two photon state is the asymptotic one in the weak coupling limit, showing direction correlation maximized in anti-resonance;

Thank you
for your attention