A dynamical description of the quantum measurement process

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Outline

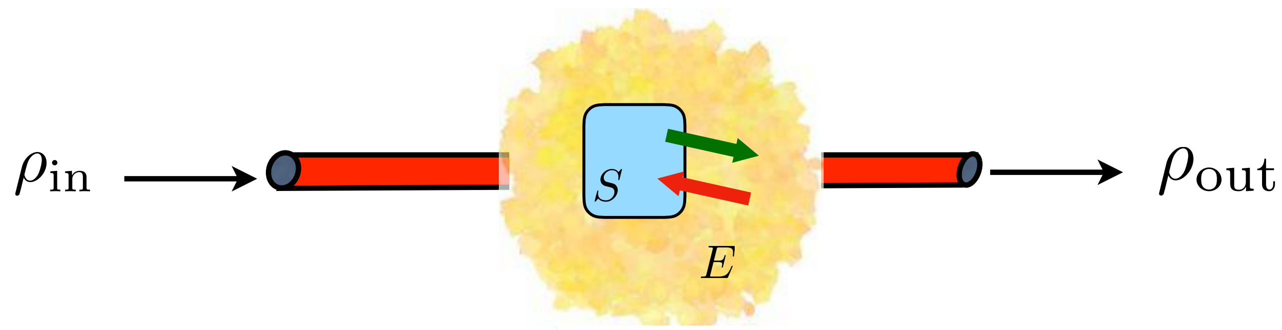
transmission of correlations

D. Gatto, A.D.P., V. Giovannetti, *arXiv:1806.07468*

- entanglement transmission as a criterium for quantifying the noise level of a q. evolution

- degradation of entanglement in Markovian noise

Evolution of quantum systems



$$\rho_{\mathrm{out}} = \Phi[\rho_{\mathrm{in}}]$$

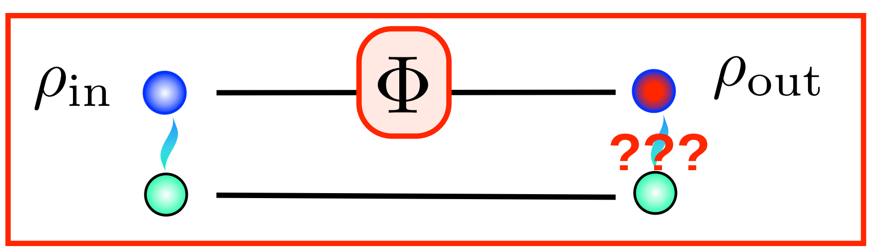
What is the "quality" of such evolution?

- **fidelity:** $\mathcal{F}\left(\rho_{\mathrm{in}}, \Phi[\rho_{\mathrm{in}}]\right) \xrightarrow[\text{convexity}]{} \mathcal{F}(\Phi) = \min\limits_{\rho_{\mathrm{in}}=|\psi\rangle\langle\psi|} \mathcal{F}\left(|\psi\rangle, \Phi[|\psi\rangle\langle\psi|]\right)$

- **capacity:** optimal communication rate in parallel on multiple copies of S

⋮

- **entaglement transmission:**



A. S. Holevo, Russian. Math. Surveys **53**, 1295 (1999); M. Horodecki, P.W. Shor, M. B. Ruskai, Rev. Math. Phys **15**, 629 (2003);
A. De Pasquale and V. Giovannetti, Phys. Rev. A **86**, 052302 (2012)

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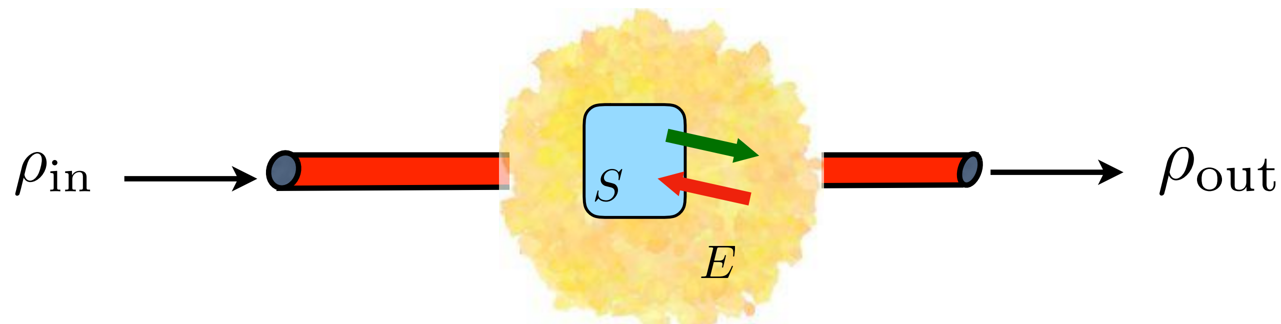

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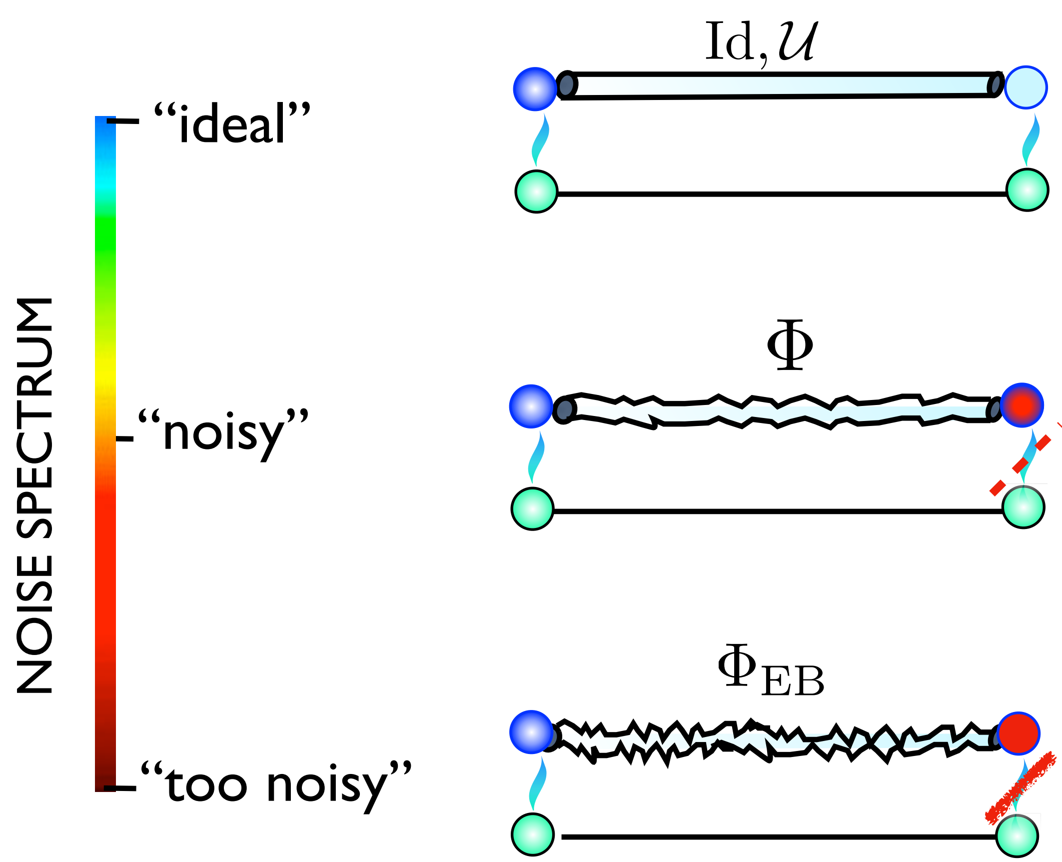

Entanglement-Breaking maps

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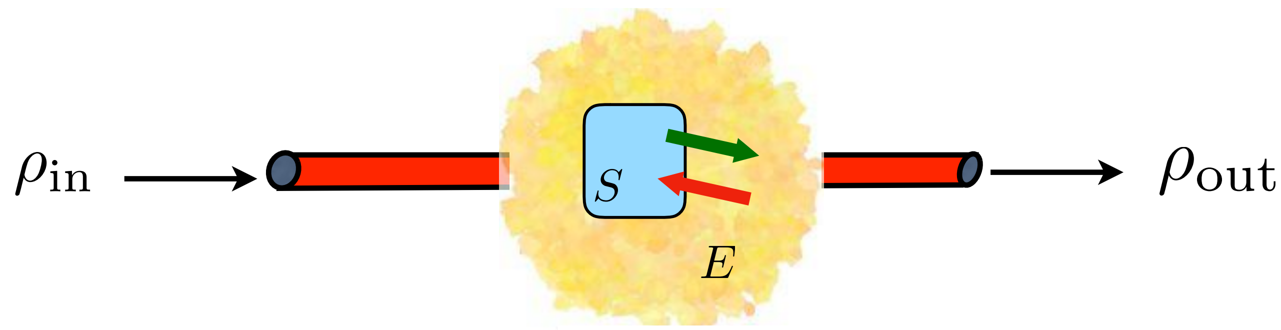

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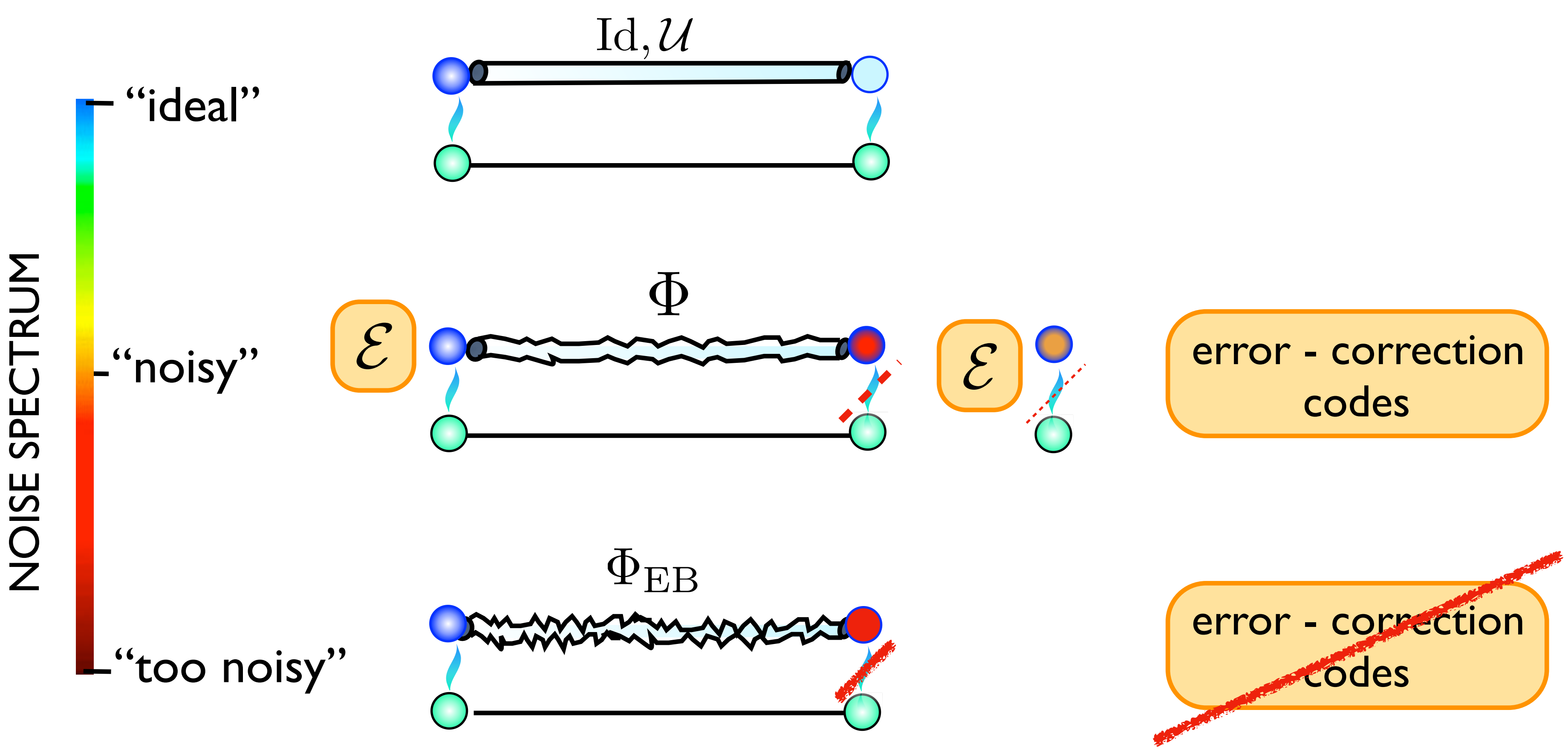

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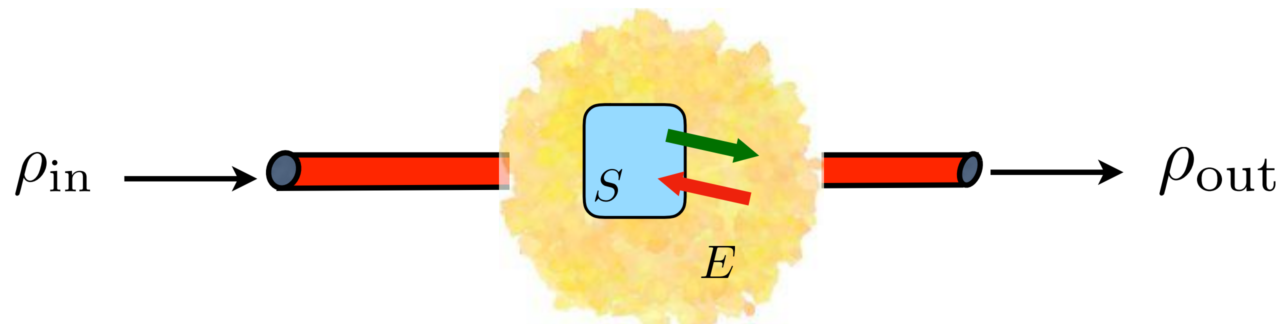

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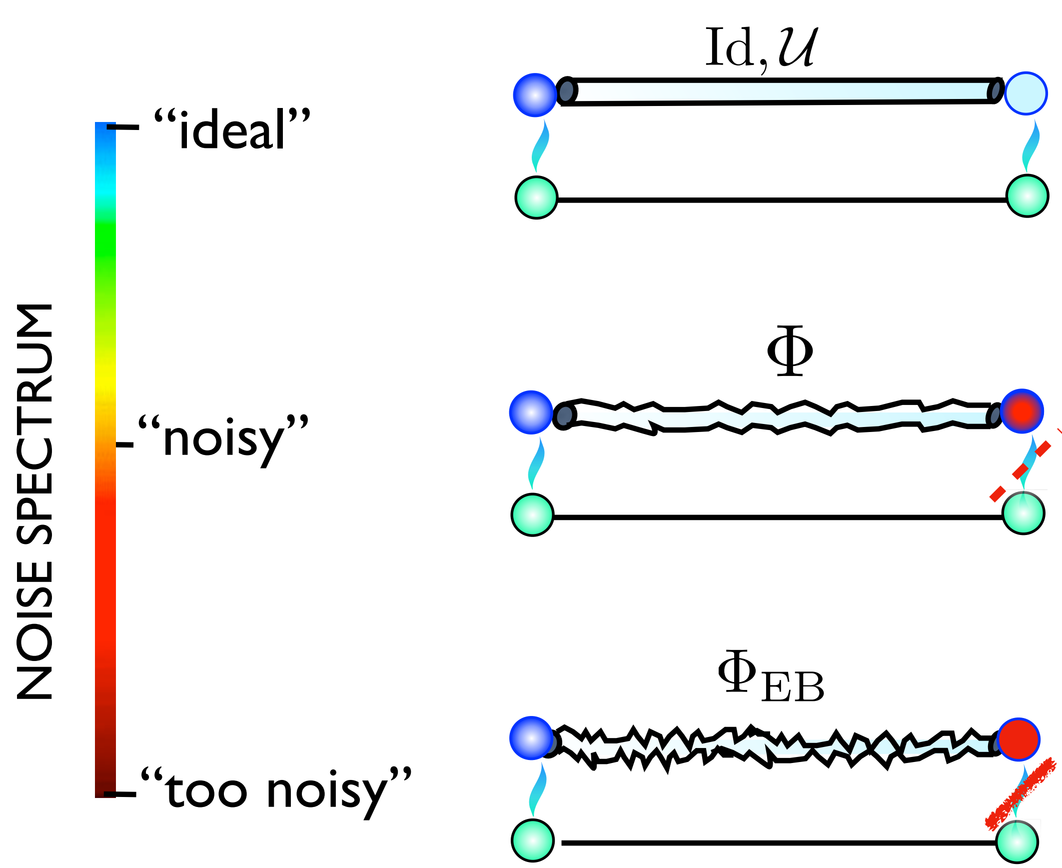

Entanglement-Breaking maps

How much entanglement survives?
Up to which time/length?

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Quantum dynamical Semigroups

coherent evolution:
(isolated systems)

Schrödinger equation

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}]$$

integrated in time

unitary CHANNEL

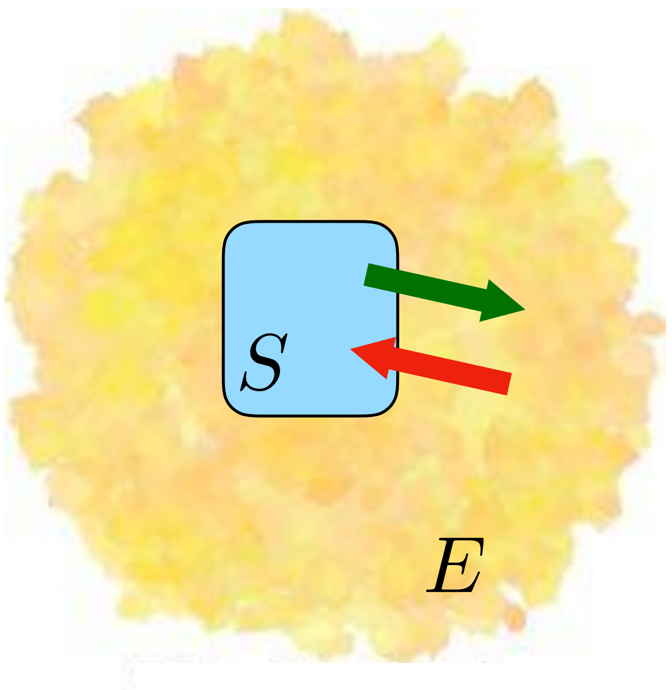
$$\mathcal{U}[\hat{\rho}_{\rm in}] = \hat{\rho}_{\rm out}$$

noisy evolution:
(open systems)

generic CHANNEL

$$\Phi[\hat{\rho}_{\rm in}] = \hat{\rho}_{\rm out} \cdots$$

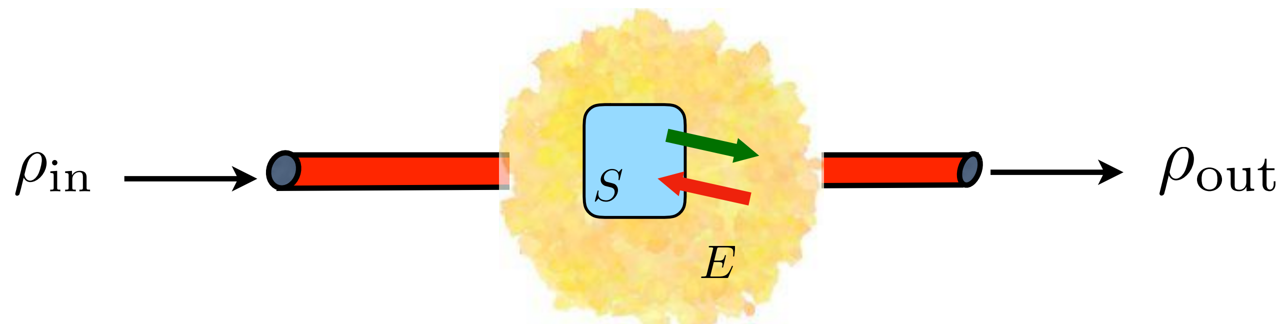
not necessarily **local** in time



$\hat{\rho}(t)$ depends on all the previous history,
and not only on $\hat{\rho}(t-dt)$

Lecture Notes for Physics Quantum Information and Computation, John Preskill, California Institute of Technology (1998)
H.-P. Breuer, and F. Petruccione, The Theory of Open Quantum Systems, Oxford University Press (2002)

Evolution of quantum systems



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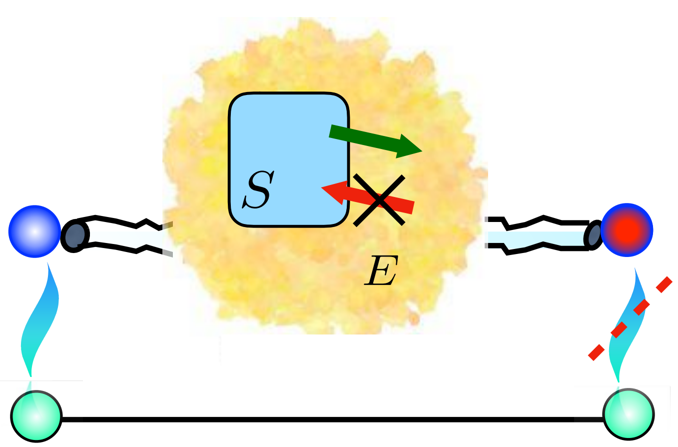

$$\Phi_t \otimes {\rm Id}$$

Weak coupling limit
Born - Markov approximation

$$\downarrow$$

$$(\Delta t)_{\rm dynamics} \gg (\Delta t)_{\rm coarse} \gg (\Delta t)_{\rm env}$$

Eg. ion-traps, cavity QED, thermalization, optical fibers

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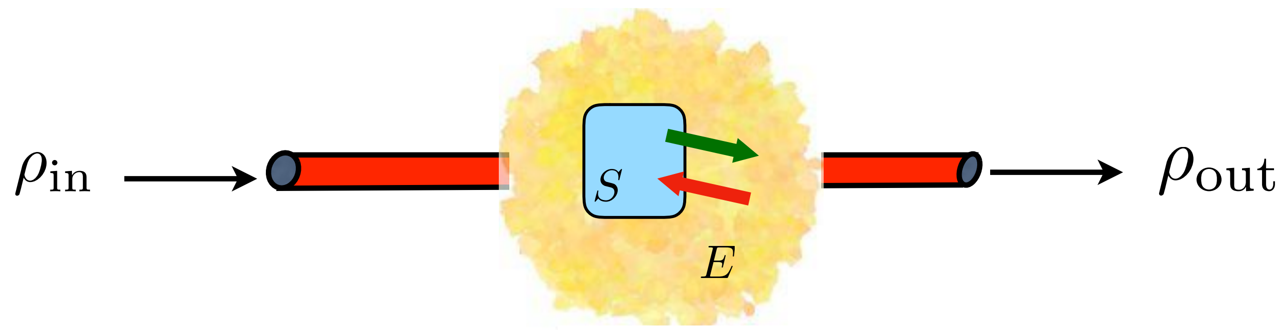

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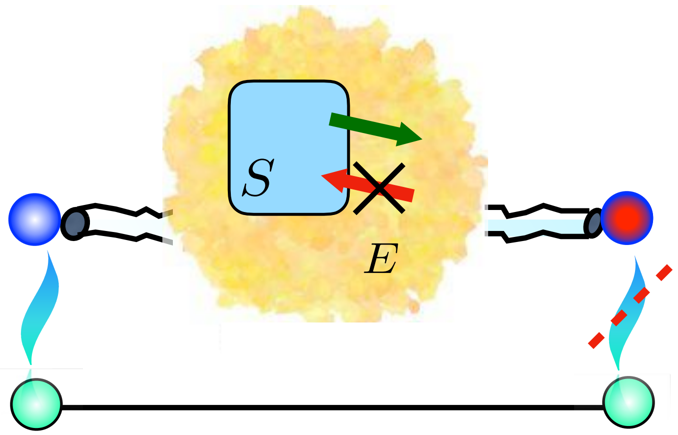

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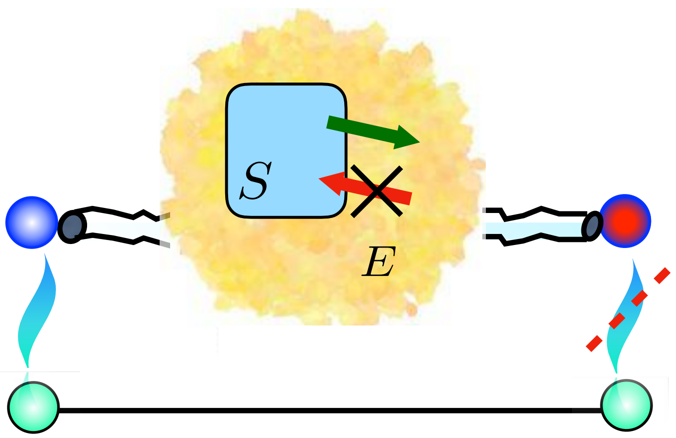
entanglement survival time τ_{ent} $\qquad t \geq \tau_{ent} : \Phi_t \in \mathrm{EB}$

D. Gatto, A. D.P., V. Giovannetti, *arXiv 1806.07468*

Degradation of entanglement in Markovian noise



$$\Phi_t := \Phi_{t,0}$$

entanglement survival time τ_{ent} $\quad t \geq \tau_{ent} : \Phi_t \in \mathrm{EB}$

GKSL generator

$\tau_{ent}(\mathcal{L})$

Markov evol.: divisibility $\quad \Phi_{t,0} = \Lambda_{t,t'} \circ \Phi_{t',0} \quad \forall t \geq t' \geq 0$

invariance under translations in time $\quad \Lambda_{t,t'} = \Lambda_{t-t',0} := \Phi_{t-t',0} \longrightarrow$ **dynamical semigroup** $\quad \dot{\Phi}_t = \mathcal{L} \circ \Phi_t$

- V. Gorini, A. Kossakowski and E. C. G. Sudarshan, J. Math. Phys. **17**, 821 (1976);
- G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).

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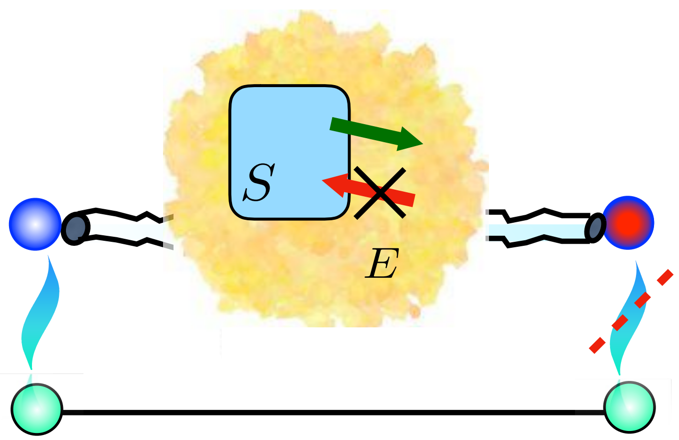

entanglement survival time τ_{ent} $\qquad t \geq \tau_{ent} : \Phi_t \in \mathrm{EB}$

$$\Phi_t = e^{t\mathcal{L}} \longrightarrow \tau_{ent}(\mathcal{L})$$

$$\mathcal{L}[\ldots] = \gamma \mathcal{D}[\ldots] - i\omega[H, \ldots]$$

dissipation **driving**

- $\tau_{ent}(q\mathcal{L}) = \tau_{ent}(\mathcal{L})/q \implies \tau_{ent}(\mathcal{L}) = \mathcal{T}_{ent}(k)/\gamma, \quad k := \omega/\gamma$
- $\mathcal{D}[\ldots] = \sum_j \left(L_j[\ldots]L_j^\dagger - \frac{1}{2}[L_j^\dagger L_j, \ldots] \right) \longrightarrow L_j$ **traceless**
- $\tau_{ent}[\Phi_t] \leftrightarrow \tau_{ent}[\mathcal{V}_t \otimes \Phi_t \otimes \mathcal{U}_t], \forall t$

  $\tau_{ent}(\mathcal{L}) = \tau_{ent}(\mathcal{U}^{-1} \circ \mathcal{L} \circ \mathcal{U}) \longrightarrow$ **class of drivings**

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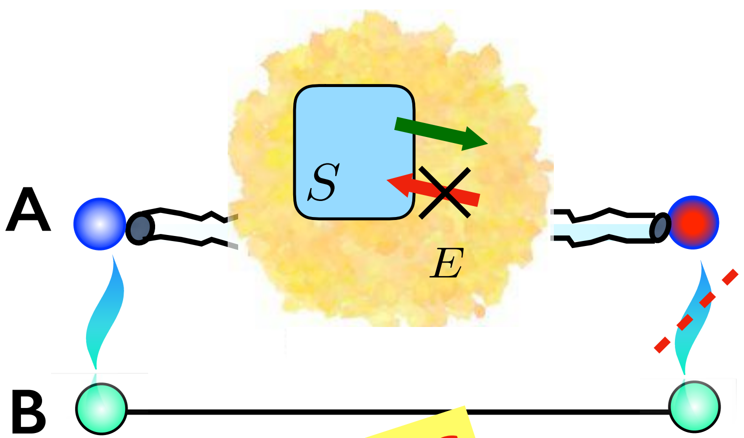

QUBIT SYSTEMS

entanglement survival time τ_{ent} $\quad t \geq \tau_{ent} : \Phi_t \in \mathrm{EB}$

Choi–Jamiolkowski isom. $\quad \Phi_t \to \rho_{AB}^{(\Phi_t)} = \Phi_t \otimes \mathrm{Id}\,(|\Omega\rangle_{AB}\langle\Omega|)\,, \; |\Omega\rangle = \frac{1}{\sqrt{d}} \sum_i |i\rangle_A \otimes |i\rangle_B$

$t \geq \tau_{ent} \implies \rho_{AB}^{(\Phi_t)}$ separable

PPT criterion $\quad \rho_{AB}^{(\Phi_t)}$ separable iff $\det\left(\left[\rho_{AB}^{(\Phi_t)}\right]^{T_B}\right) \geq 0$

Ent. Sudden death criterion: If the relaxation state ($t \to \infty$) is not pure, τ_{ent} is finite

* $\lim_{t\to\infty} \Phi_t(\rho_A(0)) = \bar{\rho}_A$

t

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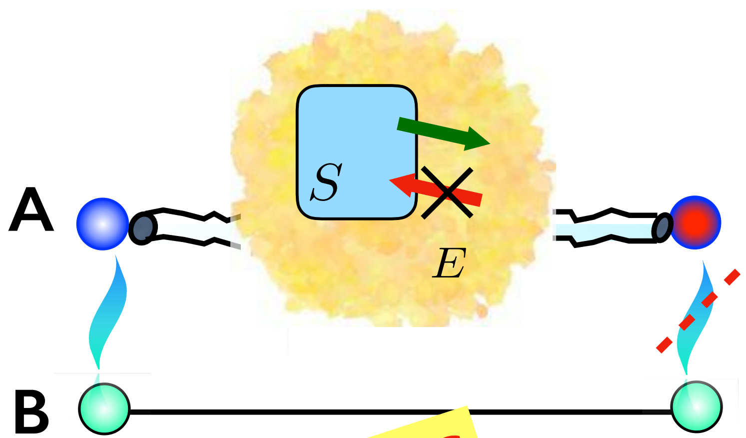

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$* \quad \lim_{t\rightarrow\infty} \rho_{AB}^{\Phi(t)} = \bar{\rho}_A \otimes \mathbb{I}_B$

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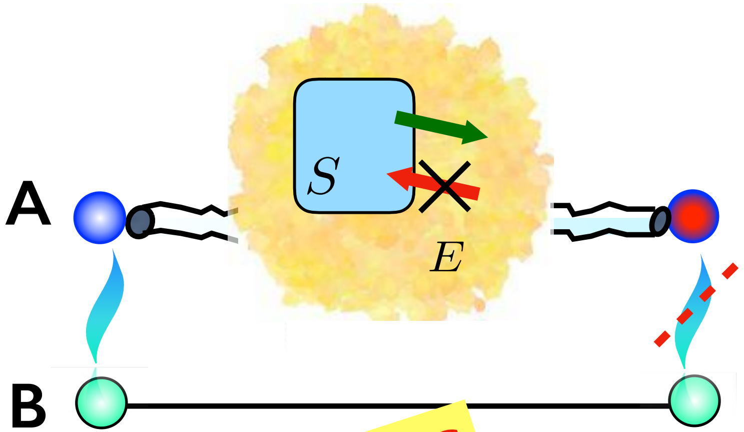

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$$\boxed{t \geq \tau_{ent} \implies \rho_{AB}^{(\Phi_t)} \text{ separable}}$$

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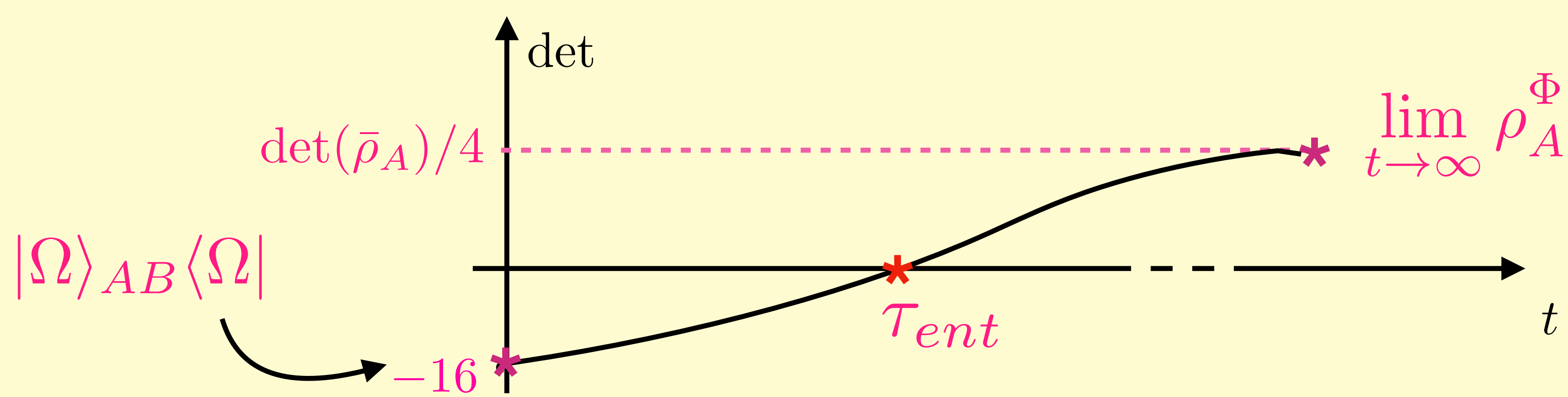

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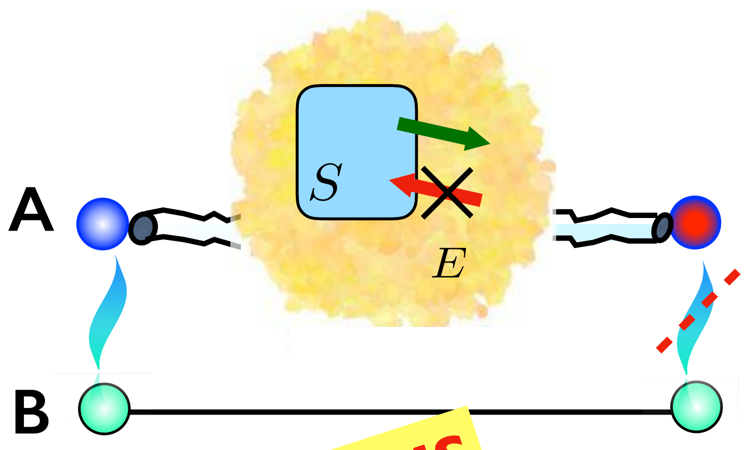

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Negativity $\mathcal{N}(\rho_{AB}^{(\Phi_t)}) = \frac{1}{2} \sum_\ell |\lambda_\ell^t| - \lambda_\ell^t$ entanglement monotone

$$\tau_{ent} = \min\left\{t \geq 0 \,\mathrm{s.t.}\, \mathcal{N}(\rho_{AB}^{\Phi_t}) = 0\right\}$$

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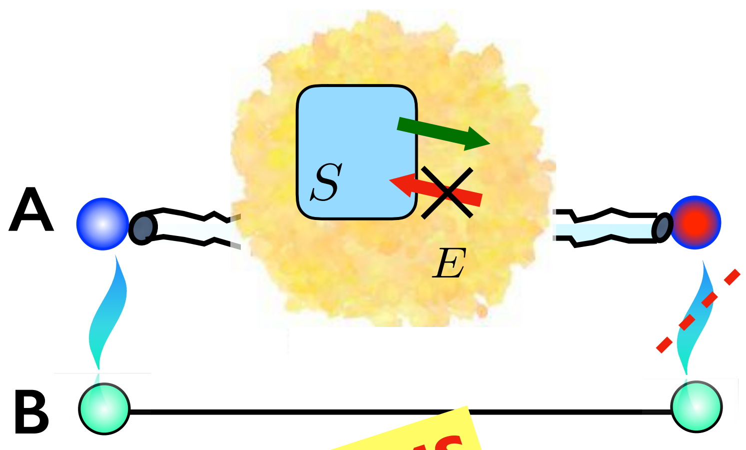

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QUBIT SYSTEMS

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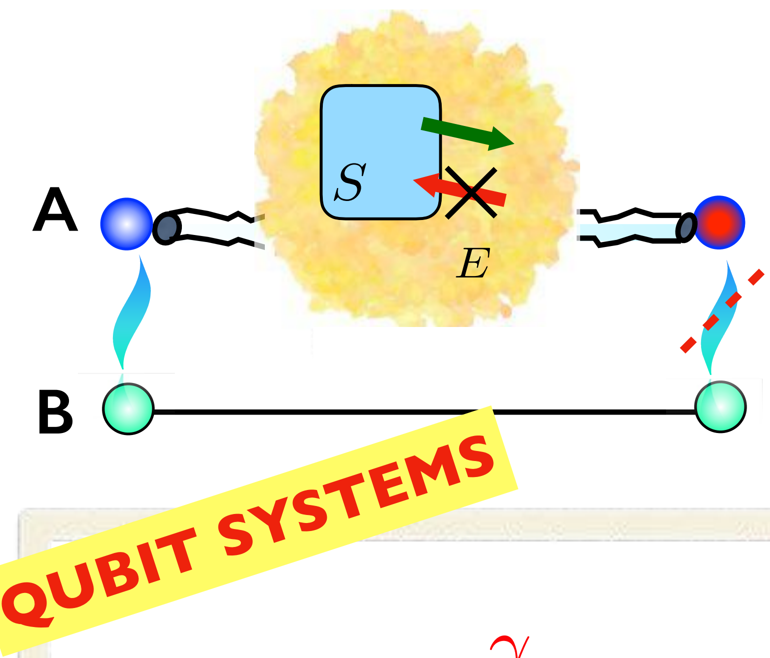

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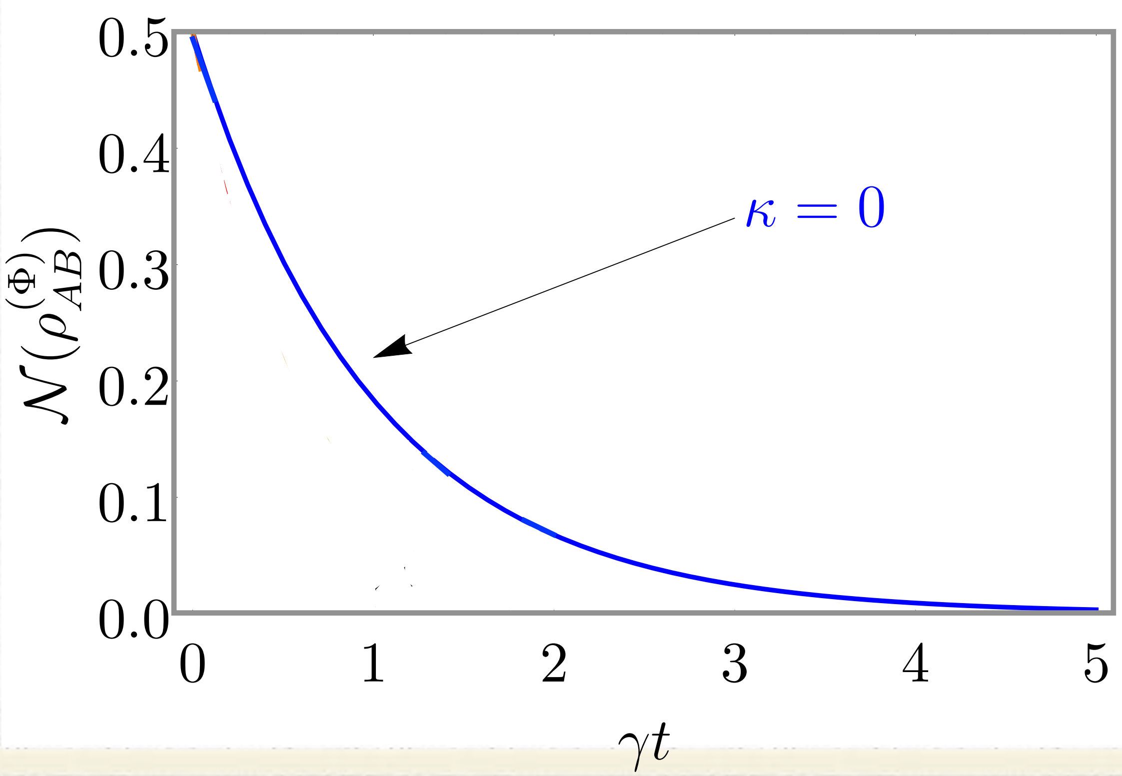

$$\mathcal{N} = e^{-\gamma t}/2 \implies \tau_{ent} \to \infty$$

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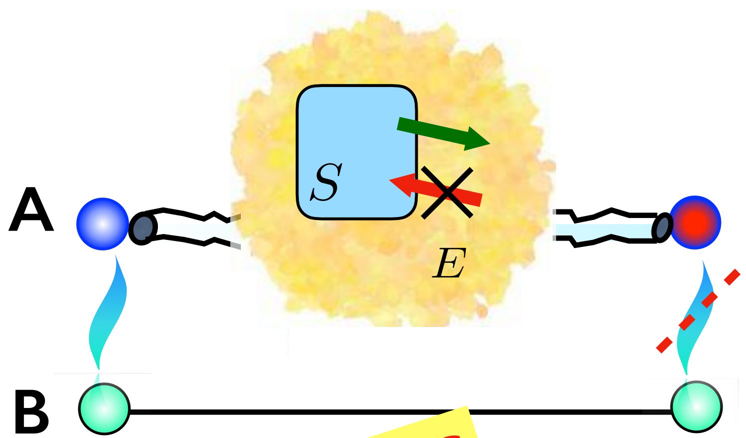

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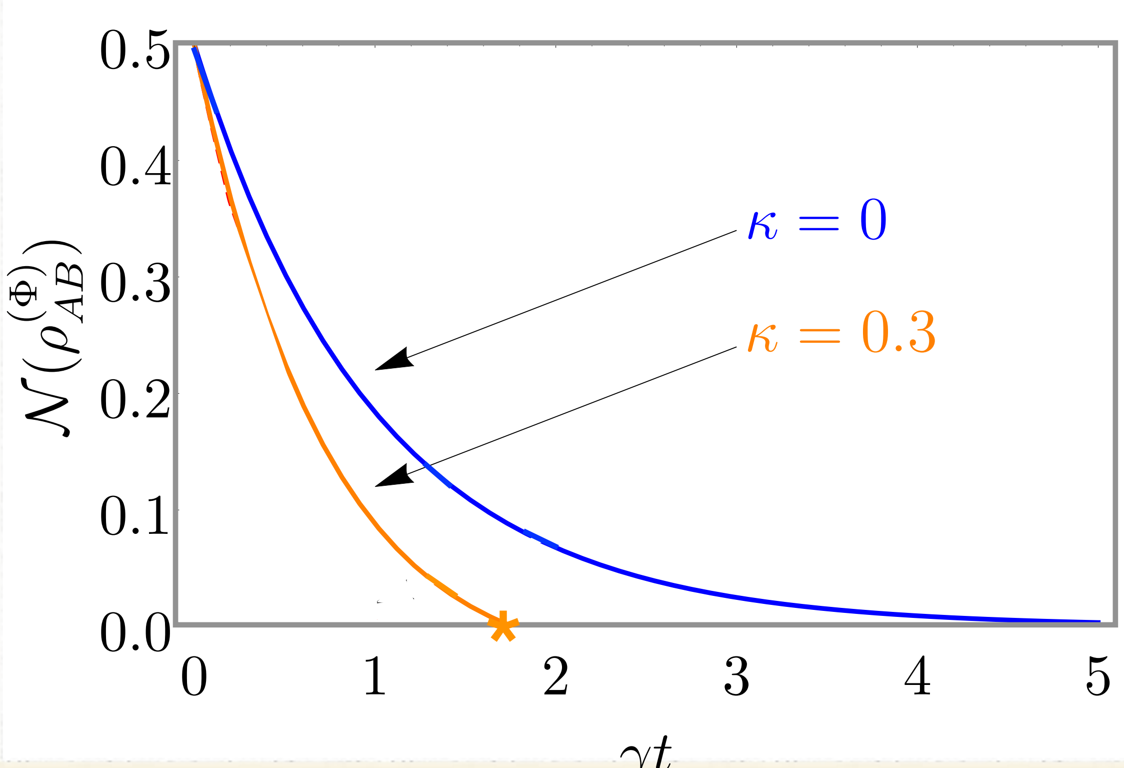

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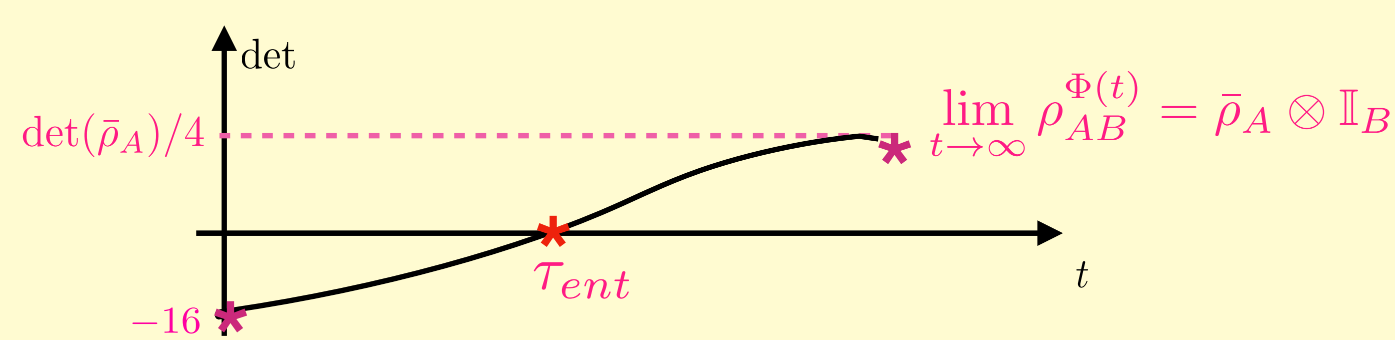
Ent. Sudden death criterion



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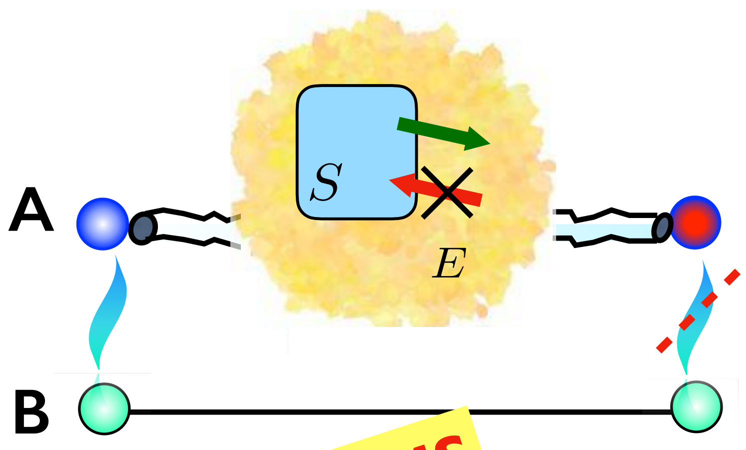

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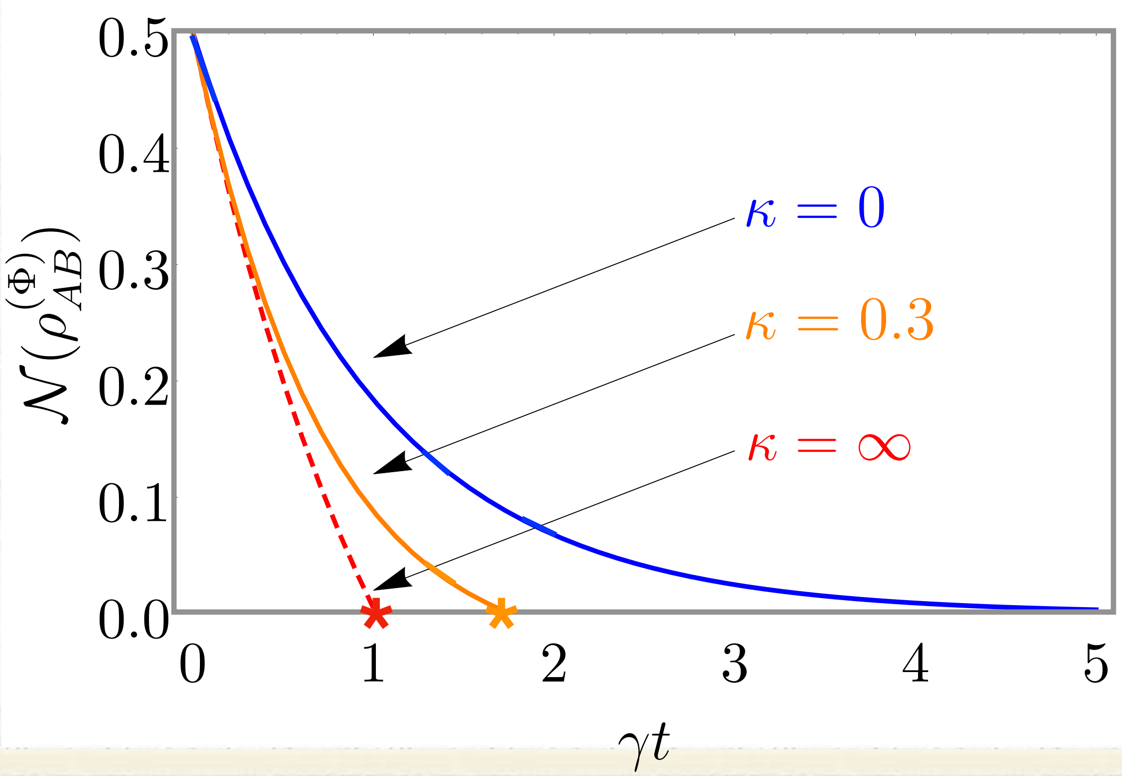

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$$k \to \infty \implies \mathcal{T}_{ent}(k) = \mathrm{arcosh}(3)$$

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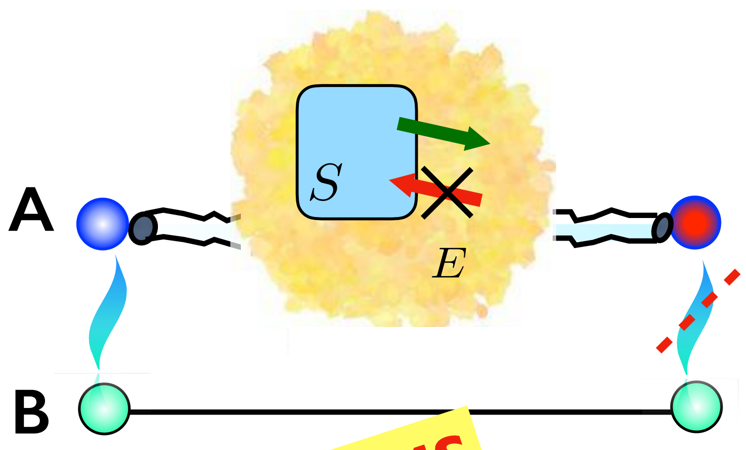

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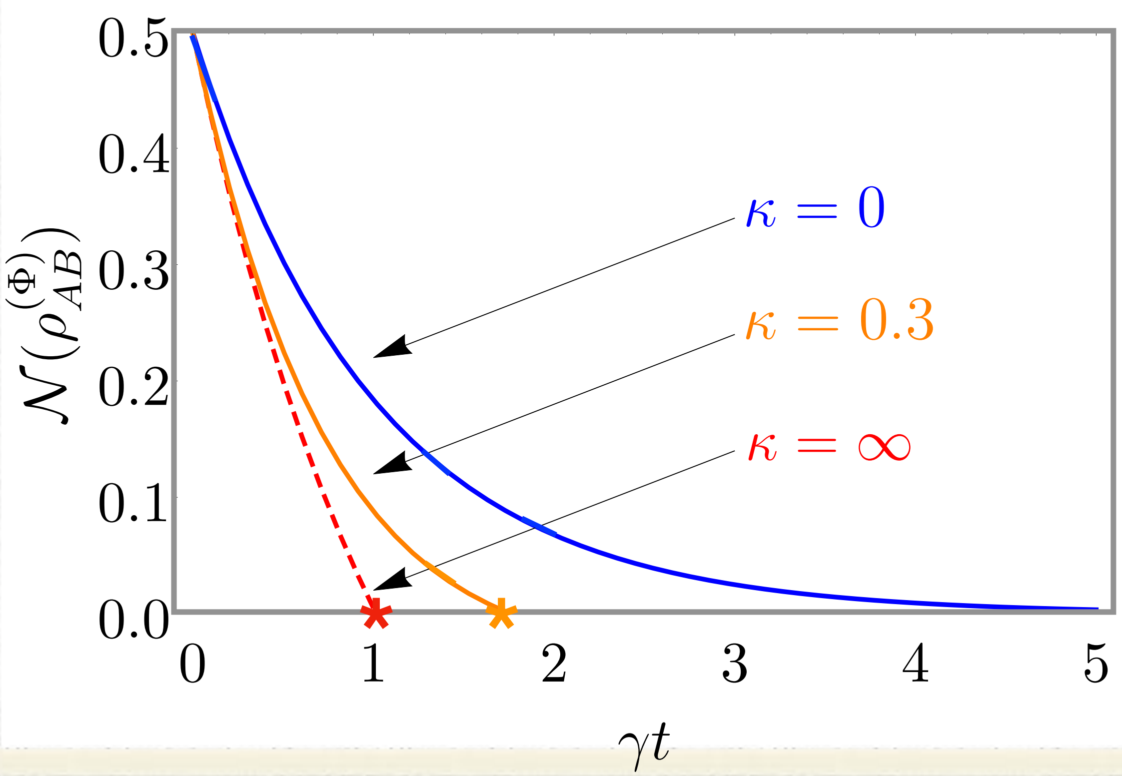

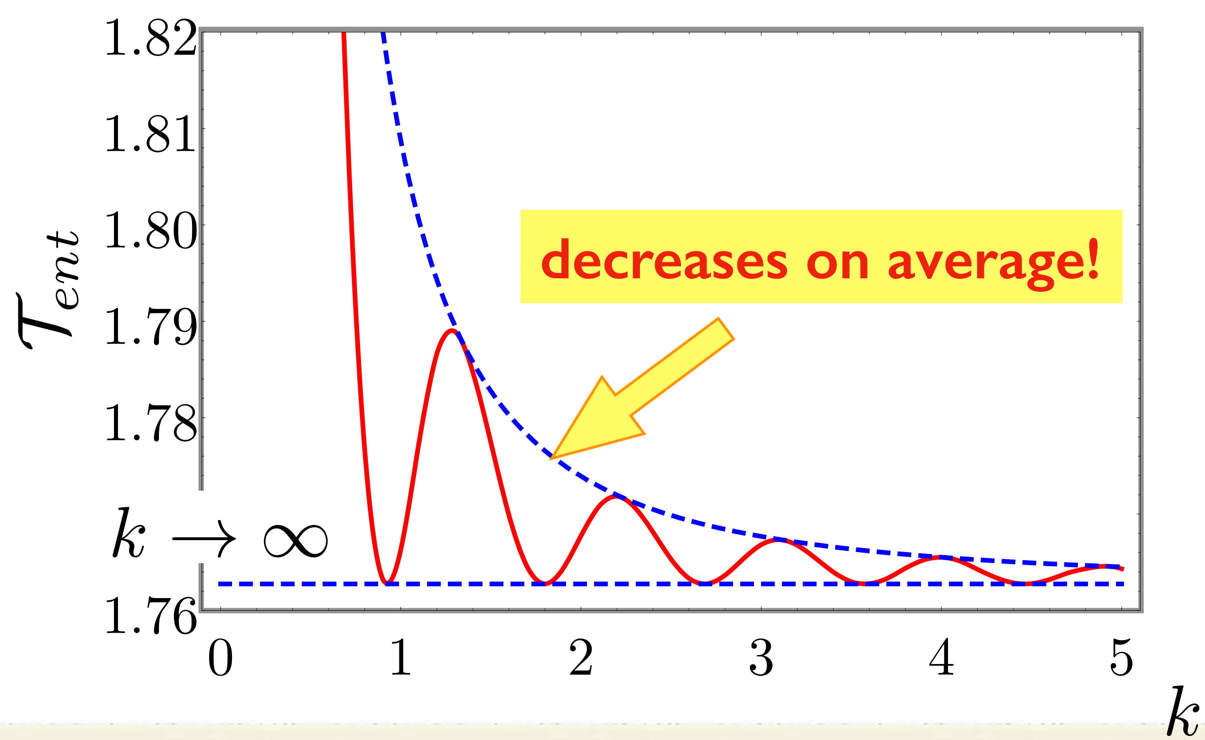

Conclusions

transmission of correlations

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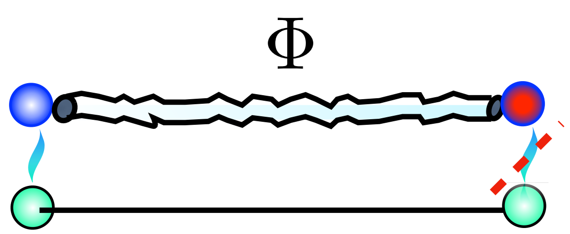
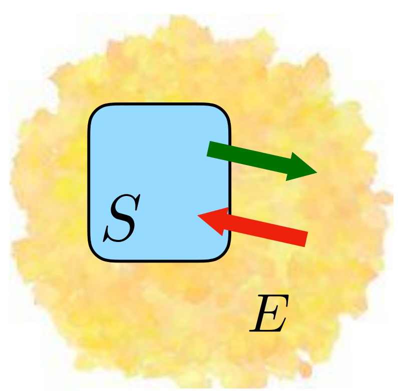
- entanglement transmission: a criterium for quantifying the noise level of a q. evolution



- degradation of entanglement in Markovian noise:



- τ_{ent} entanglment survival time: def + properties
- qubit map: adding a unitary contribution can reduce τ_{ent}