GENERALIZED ROBERTSON -WALKER AND TWISTED SPACETIMES

Luca G. Molinari & Carlo A. Mantica
Dipartimento di Fisica UniMi e I.N.F.N. Milano

July 9, 2018

A hierarchy of space-times

Warped and twisted metrics have the form

$$\boxed{ds^2 = -dt^2 + a^2 g^*_{\mu\nu} dx^\mu dx^\nu}$$

where **a** is the scale factor.

1) Robertson-Walker: $a(t)$, (M^*, g^*) maximally symmetric.

2) Generalized RW: $a(t)$, (M^*, g^*) a Riemannian manifold.

3) Twisted: $a(\mathbf{x}, t)$, (M^*, g^*) a Riemannian manifold.

Bang-Yen Chen gave covariant characterizations of GRW and for twisted.

An alternative unifying scheme:

Theorem (Mantica & Molinari, 2017)

A Lorentzian manifold is **Twisted** *iff* $\exists$ *a velocity field* ($u^j u_j = -1$) *such that* $\nabla_i u_j = \varphi(g_{ij} + u_i u_j)$. φ *is Hubble's parameter* H.

- the manifold is **GRW** *if* $R_i{}^j u_j = \xi u_i$.

- the manifold is **RW** *if also* $C_{ijkl} = 0$.

RICCI and WEYL TENSORS

General form of the Ricci tensor in a twisted space-time:

$$R_{ij} = Ag_{ij} + Bu_i u_j - (n-2)(u_i v_j + v_i u_j - C_{kijm} u^k u^m)$$

GRW: $v_i = 0$ i.e. u_i is eigenvector;
RW: $v_i = 0$ and $C_{jklm} = 0$ (perfect fluid).
A simple statement (with long proof):

Theorem (Molinari & Mantica, 2017)

In a twisted space-time

$$\nabla^m C_{jklm} = 0 \iff u^m C_{jklm} = 0$$

f(R) gravity

The Einstein equations $R_{ij} - \frac{R}{2} g_{ij} = \kappa T_{ij}$ descend from minimal action:

$$S = \int d^4x \sqrt{g}\, R(x) + S_{\text{matt}}$$

$\Rightarrow$ a Ricci tensor of the form $R_{ij} = A u_i u_j + B g_{ij}$ corresponds to a matter tensor $T_{ij} = (p + \mu) u_i u_j + p g_{ij}$ (perfect fluid).
In $f(R)$ gravity the scalar curvature R is replaced by a function $f(R)$ that modifies the Einstein eqs. It is a viable route to describe DM effects without new matter fields (Capozziello).

Theorem (Mantica & Molinari, to appear)

A perfect fluid Ricci tensor implies a perfect fluid matter tensor iff the space-time is a GRW.

Publications 2017

- C.A.Mantica and L.G.Molinari, *Simple conformally recurrent space-times are conformally recurrent pp-waves*, Coll. Math. **150** (1) (2017) 9–20.
- C.A. Mantica and L.G.Molinari, *Generalized Robertson Walker spacetimes: a survey*, Int. J. Geom. Meth. Mod. Phys. **14** (2017) 1730001 (27 pp.)
- C.A.Mantica and L.G.Molinari, *Twisted Lorentzian manifolds: a characterisation with torse-forming time-like unit vectors*, Gen. Relativ. Gravit. **49** (2017) 51 (7 pp).
- L.G.Molinari and C.A.Mantica, *A simple property of the Weyl tensor for a shear, vorticity and acceleration-free velocity field*, Gen. Relativ. Gravit. **50** (2018) 81 (7 pp.)