NEUMATT – Milano

Nuclear matter and compact stellar objects

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Istituto Nazionale di Fisica Nucleare

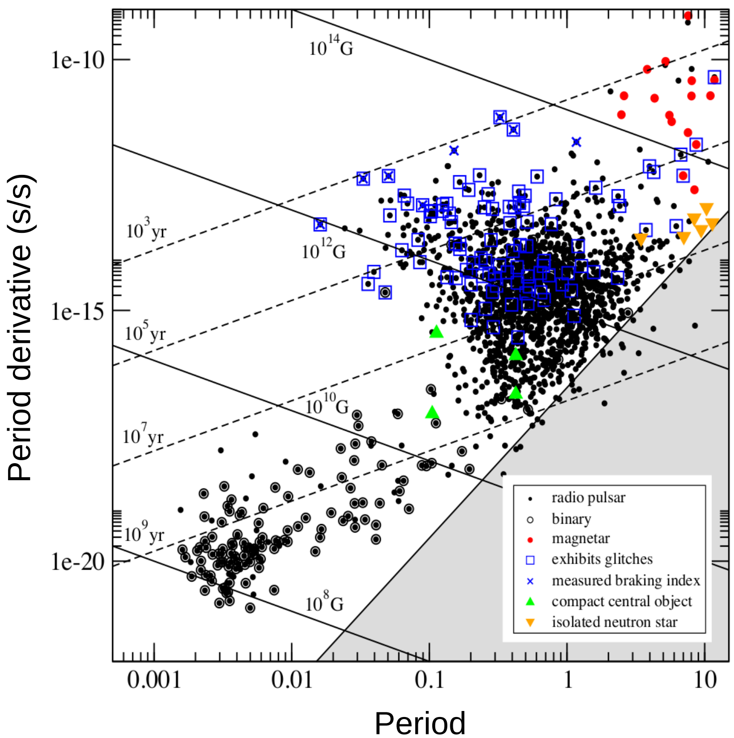
CAMK PAN

PHAROS THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

Neutron stars

What we observe, since:

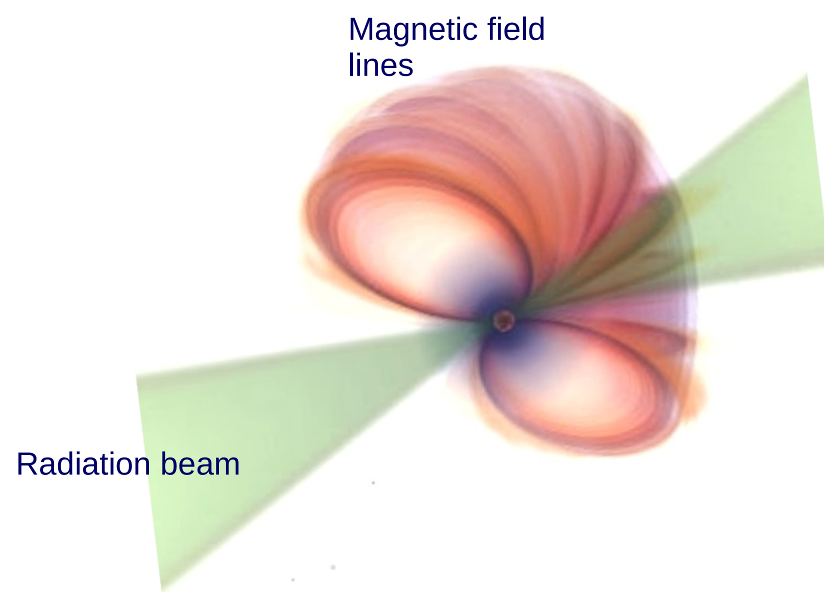
Hewish, Bell et al., *Observation of a rapidly pulsating radio source* (1968)



What we think it is, since:

Pacini, *Energy emission from a neutron star* (1967)

Gold, *Rotating neutron stars as the origin of the pulsating radio sources* (1968)



Pulsars are rotating magnetized neutron stars:

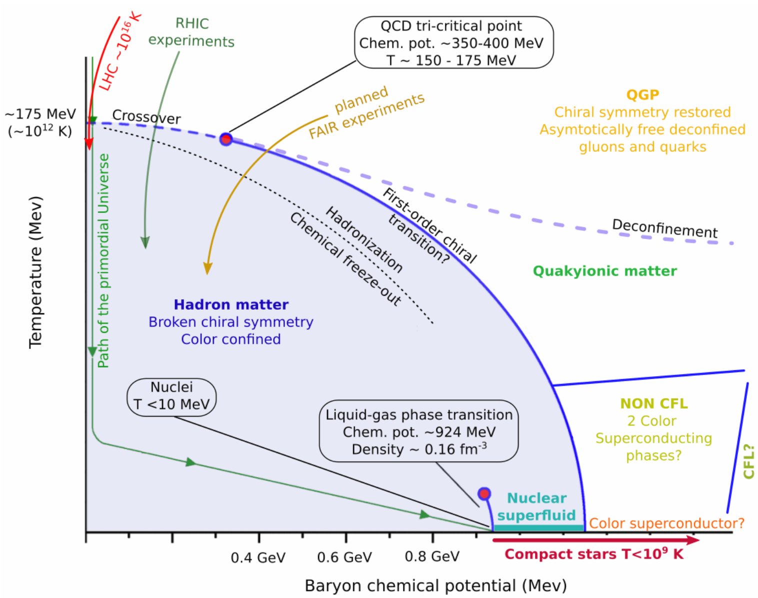
Coherent (non-thermal) emission + brightness + small preiod: only possible for very compact objects!

A vibrating WD or NS? excluded by pulsar-timing data: P increases with time.

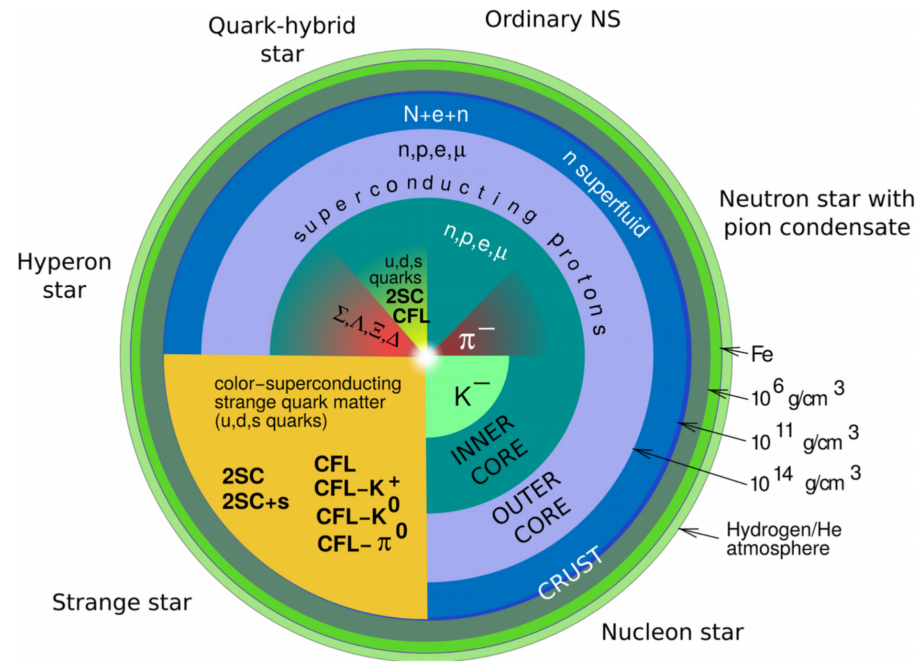
BH accretion? No regular pulses...

Neutron stars

Sketch of nuclear matter phase diagram



What we can not observe: internal structure



Compressing matter liberates degrees of freedom.

Open issue: composition of the inner crust and possible transitions in the core.

Gravity, holds the star together

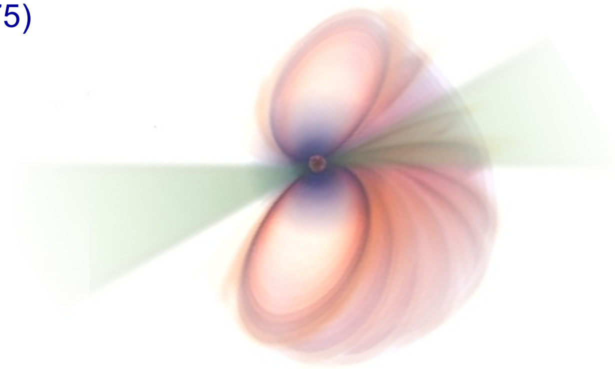
Electromagnetism, makes pulsars pulse and magnetars flare

Strong interaction, determines the internal composition and prevents gravitational collapse

Weak interaction, determines the internal composition and affects reaction rates (chemical equilibrium and neutrino cooling)

Glitch mechanism (vortex mediated)

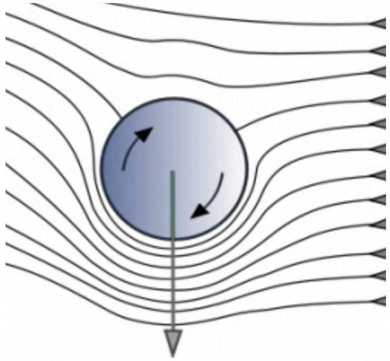
Anderson & Itoh *Pulsar glitches and restlessness as a hard superfluidity phenomenon* (1975)



- The charged component steadily looses angular momentum...

- If vortices are **pinned** the superfluid cannot spin-down:

  → the vortex lines corotate with the charged component
  → a velocity lag builds up
  → neutron current in the frame of the normal component



$$f_M = \kappa\, m_n n_n v_L$$

Magnus force

- Magnus force ≃ pinning force: the vortex line unpins:

$$|f_M| = f_P(r),$$

  → analogy between: *unpinning lag* in neutron stars and the *critical current* in superconductors

  → vortices can move: new dissipative channels open.

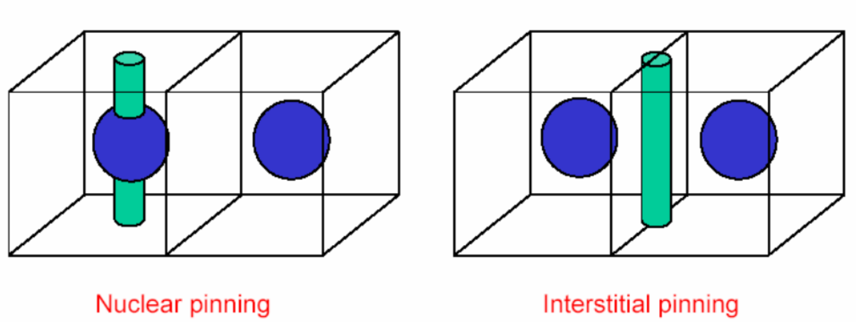
  → Only if vortices can move radially it is possible to exchange angular momentum between the two components.

Pinning forces

Seveso, Pizzochero, Grill, Haskell, *Mesoscopic pinning forces in neutron star crusts* (2016).

vortex-nucleus configurations in nuclear lattice

↓ ↓



↓ ↓

E_{NP} E_{IP}

Pinning energy: $\Delta E_{pin} = E_{NP} - E_{IP}$

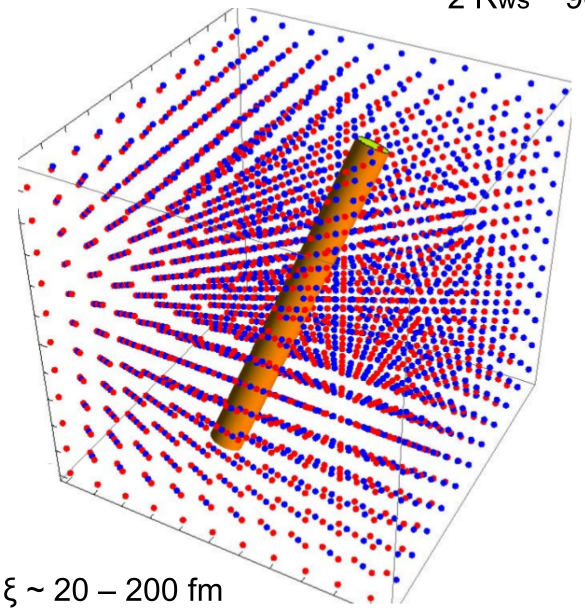
Semiclassical approach: static LDA calculation
(i.e. the local Fermi momentum is a function of n_n).

Energy contributions:

→ negative condensation energy ~ Δ^2 / E_F
→ kinetic energy of the irrotational vortex-induced flow
→ Fermi energy E_F of neutrons
→ nuclear cluster energy (Woods-Saxon potential)

Uncertain Δ: modifies the strength and location of the pinning energies.
Significant pinning occurs only in a restricted range: $0.07\, n_0 < n_B < 0.2\, n_0$
Donati & Pizzochero, Phys Lett B, 640 (2006)

2 R_{WS} ~ 90 – 20 fm



2 ξ ~ 20 – 200 fm

E_p ~ 3 – 0.02 Mev

Strong pinning if the coherence length ξ of a vortex is smaller than the lattice spacing.

IDEA: consider a segment of vortex line (the length L is given by the tension) and average over translations and rotations of the total pinning force divided by L

Entrainment coupling

- Andreev-Baskin (1975): Three-velocity Hydrodynamics of Superfluid solutions

Normal viscous mixture → different velocity fields cannot coexist inside the same fluid
Superfluid mixture → each superfluid can flow with its own velocity

It is a non-dissipative coupling: the particles are coupled but the presence of a gap hinders the formation of excitations. Therefore, the canonical momentum of one fluid is a linear combination of the kinematic velocities.

- Carter multi-fluid formalism (macroscopic variational approach):

$$\begin{aligned} \frac{\bar{p}_i^{\mathrm{n}}}{m_{\mathrm{n}}} &= v_i^{\mathrm{n}} + \varepsilon_{\mathrm{n}}(v_i^{\mathrm{p}} - v_i^{\mathrm{n}}) \\ \frac{\bar{p}_i^{\mathrm{p}}}{m_{\mathrm{p}}} &= v_i^{\mathrm{p}} + \varepsilon_{\mathrm{p}}(v_i^{\mathrm{n}} - v_i^{\mathrm{p}}) \end{aligned}$$

Thermodynamical quantities: always defined in the rest frame of the fluid.

If two different velocities can coexist, there is no common frame!

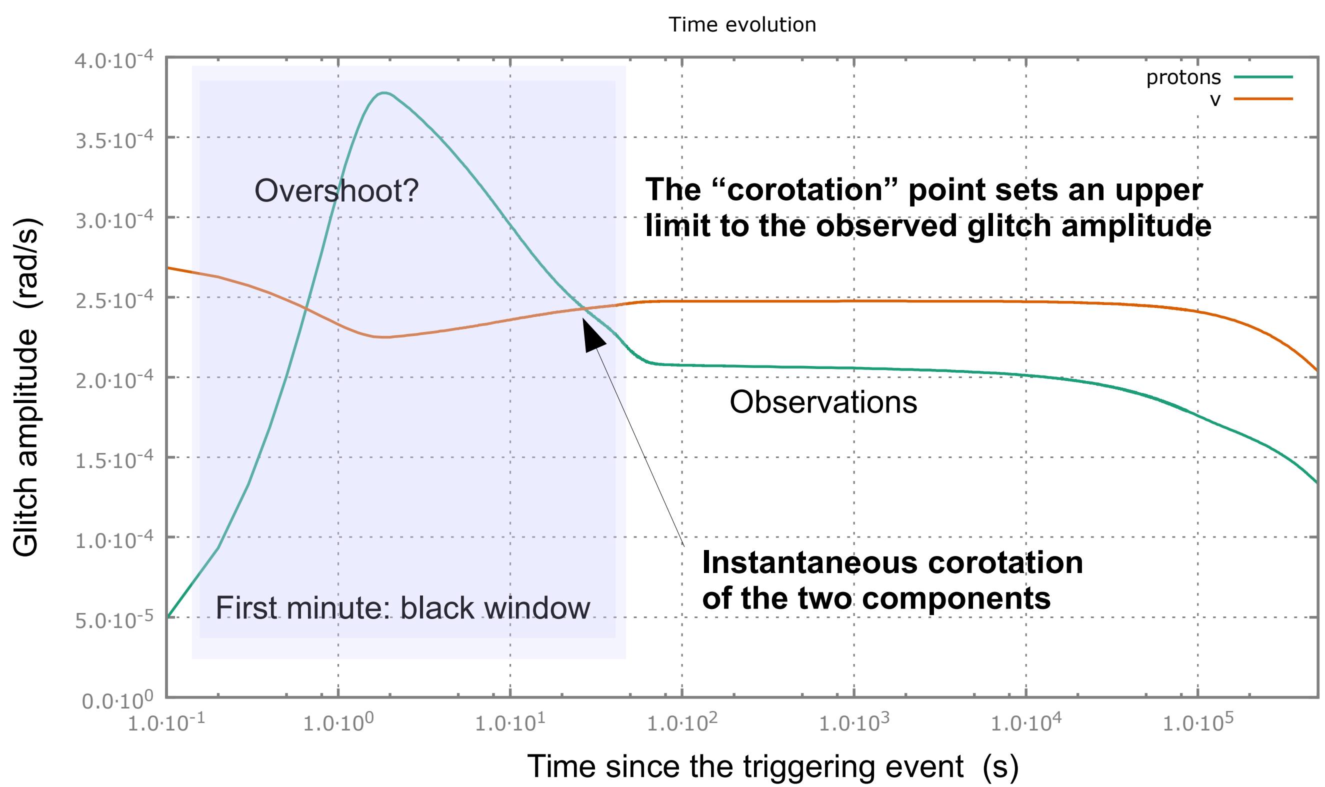
The energy density must depend on the velocity lag.

Variations of this “generalized” energy gives the usual linear relation between momenta and velocities.

Constraints on pulsar masses from the maximum observed glitch

P. M. Pizzochero[1,2*], M. Antonelli[1,2], B. Haskell[3] and S. Seveso[1]

$$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa I}\int dr\, r^3\, f_P(r)$$



Calculation of the maximum glitch amplitude

Antonelli M., Montoli A., Pizzochero P.M. *Effects of general relativity on glitch amplitudes and pulsar mass upper bounds* (2018)

$$\mathrm{d}s^2 = -e^{2\Phi(r)}\mathrm{d}t^2 + e^{2\Lambda(r)}\mathrm{d}r^2 + r^2\left[\mathrm{d}\vartheta^2 + \sin^2\vartheta(\mathrm{d}\varphi - \omega(r)\mathrm{d}t)^2\right]$$

$$L = -\int \left(T_{\alpha\beta} - \frac{1}{2}T^{\nu}_{\nu}g_{\alpha\beta}\right)(\partial_\varphi)^\alpha\, z_0^\beta\, dV$$

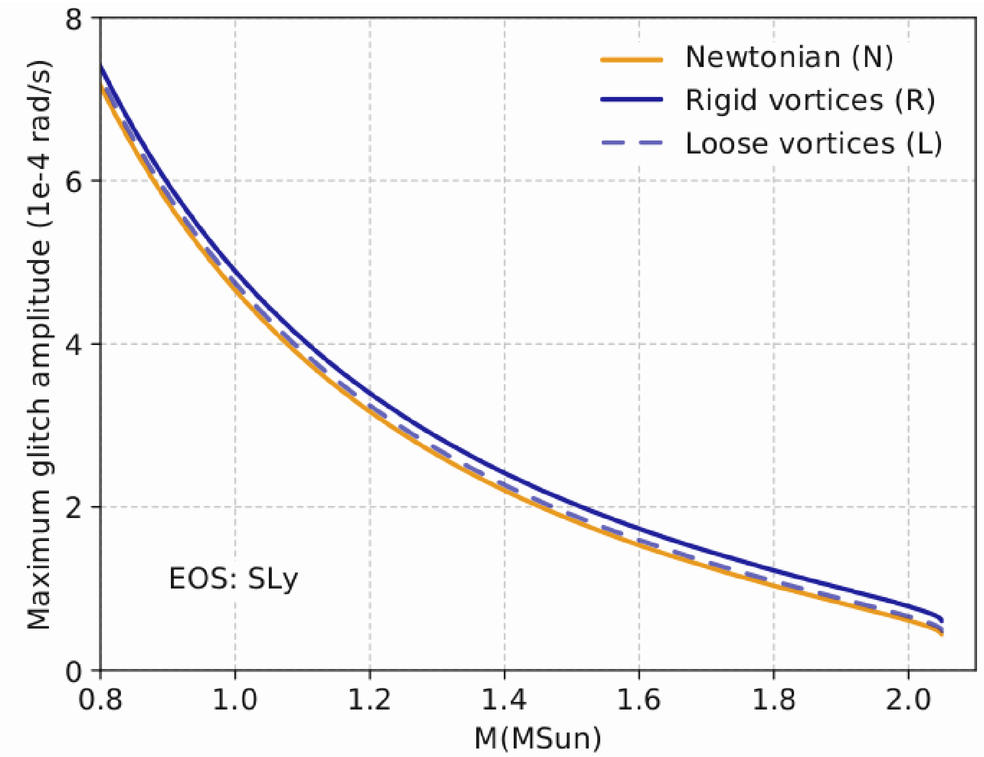
$$\begin{aligned} p_{n\alpha}/\mu_n &= (1-\epsilon_n)u_{n\alpha} + (\epsilon_n/\Gamma)u_{p\alpha} \\ p_{p\alpha}/\mu_p &= (1-\epsilon_p)u_{p\alpha} + (\epsilon_p/\Gamma)u_{n\alpha} \end{aligned}$$

The unpinning condition on the lag is:

$$\Omega^{cr}_{np}(r,\theta) = \frac{f_P(r)\, e^{\Phi(r)}}{\kappa\, r \sin\theta\, m_n n_n(r)}$$

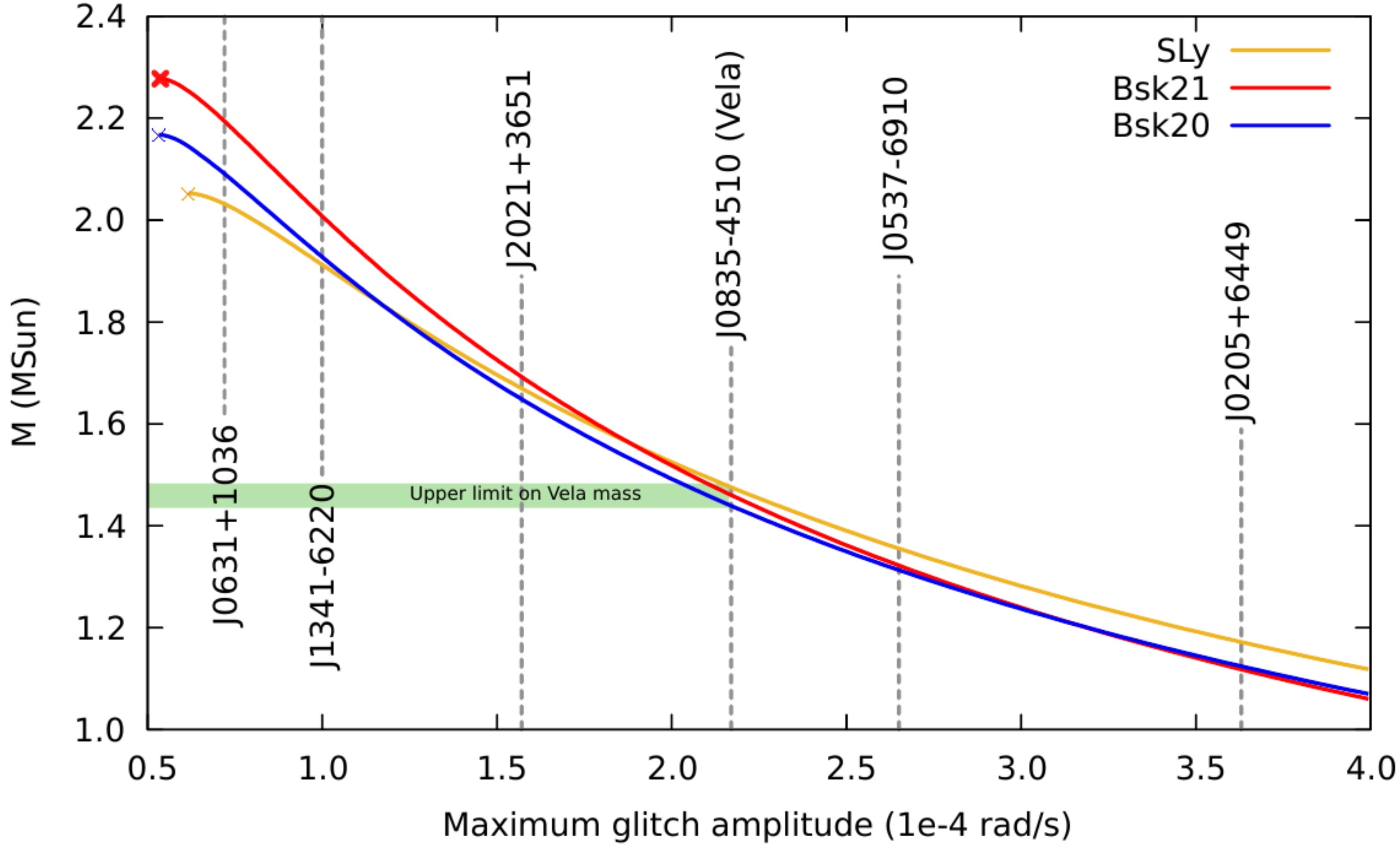
The corresponding maximum glitch amplitude is:

$$\boxed{\Delta\Omega_{\rm abs} = \frac{\pi^2}{I\kappa}\int_0^{R_d} dr\, r^3\, e^{\Lambda(r)}\,\frac{\mathcal{E}(r)+P(r)}{m_n\, n_B(r)\, c^2}\, f_P(r)}$$



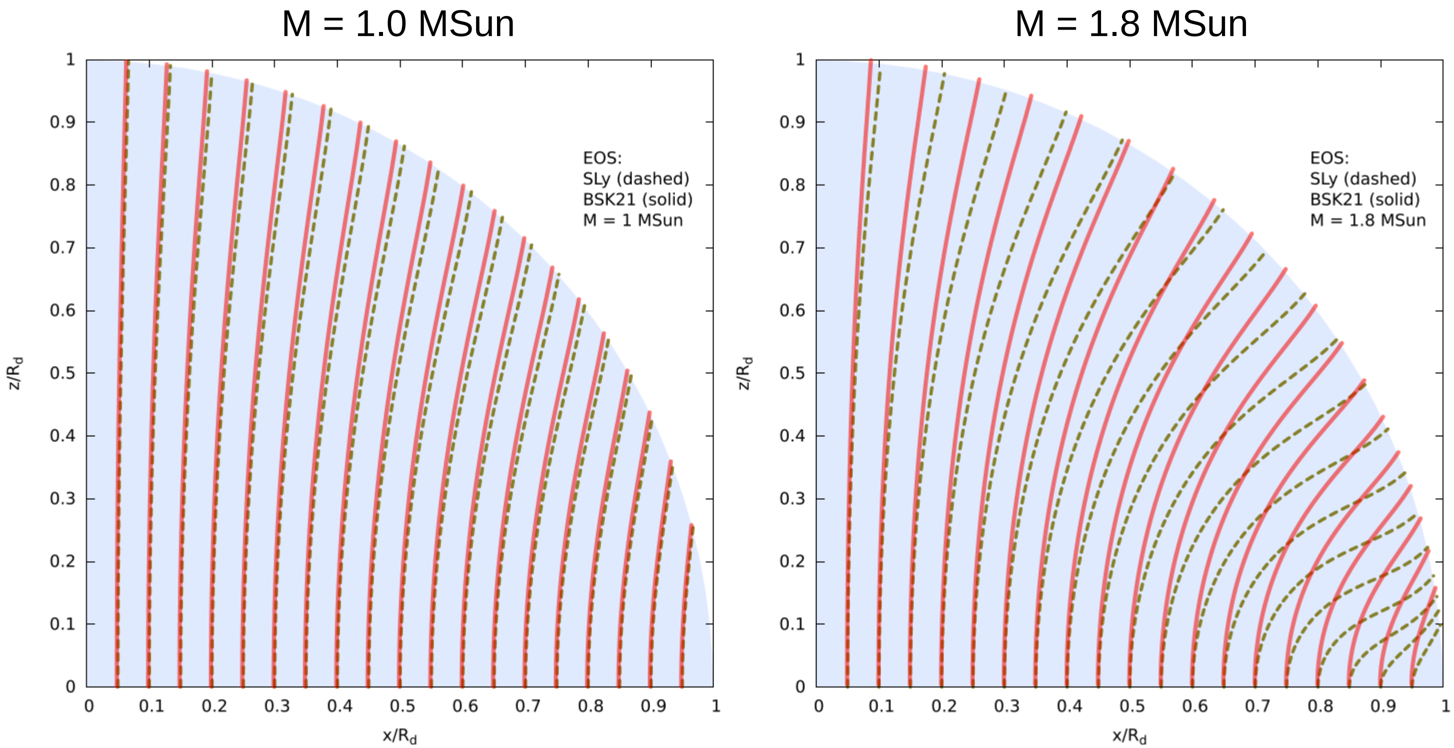
Upper bounds on pulsar masses

Antonelli M., Montoli A., Pizzochero P.M. *Effects of general relativity on glitch amplitudes and pulsar mass upper bounds* (2018)



$$\Delta\Omega < \frac{I_v}{I}\langle\Omega_{vp}^{cr}\rangle = \frac{I_n}{I}\langle\Omega_{np}^{cr}\rangle$$

Relativistic corrections: bent vortices



$$\omega_\mu := *dp_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} dp^{\alpha\beta} u_\nu$$

$$\kappa n_v = \sqrt{\omega_\mu \omega^\mu} = \sqrt{\frac{dp^{\alpha\beta} dp_{\alpha\beta}}{2}}$$

Relativistic vorticity lines for two stars of 1 Msun and 1.8 Msun (EOSs: SLy dashed, BSk21 solid): effect of stratification (enthalpy) and curved spacetime. Slow and uniform rotation has been assumed (the shear tensor vanishes).

Vorticity in GR: exterior derivative of the momentum per particle: $(dp)_{\mu\nu} = \tilde{h}_{,\mu} u_\nu - \tilde{h}_{,\nu} u_\mu + \tilde{h}\,(u_{\nu,\mu} - u_{\mu,\nu})$

Feynman-Onsager relation $$\Omega - \omega(r) = \frac{\kappa\, e^{\Phi(r)}\, \mathcal{N}(r,\theta)}{2\pi\,(r\sin\theta)^2(\mu(r)/m_n)} + O(\Omega^2)$$

People and COST Action PHAROS

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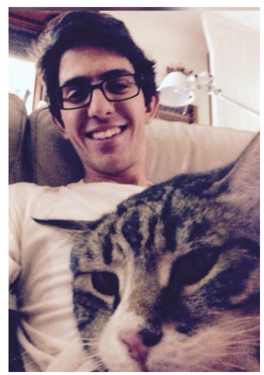

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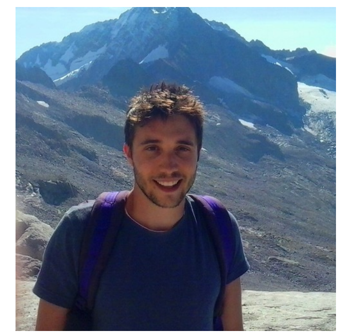

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The COST Action PHAROS (CA16214) has the goal of attacking key challenges in the physics involved in neutron stars by facing them via a problem based approach:

WG1: Equation of State of dense matter

WG2: Superconductivity/Superfluidity in dense matter and transport coefficients

PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

http://www.pharos.ice.csic.es

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Seveso S., Pizzochero P.M., Haskell B., Antonelli M., *Mesoscopic pinning forces in magnetized neutron star core.* (IN PREPARATION!)

Pizzochero P.M., **Antonelli M.**, Haskell B., Seveso S. (2017). *Constraints on pulsar masses from the maximum observed glitch.*

Superfluidity in neutron stars

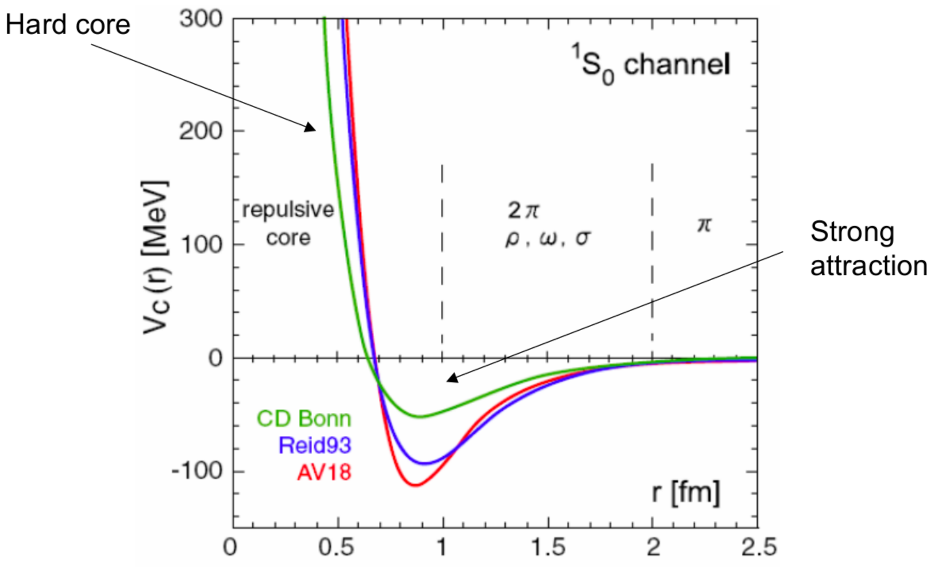
Superfluidity and superconductivity are associated with Bose-Einstein condensation.

In the case of fermions, the condensation proceeds through the formation of Cooper pairs.

Fermi surface is unstable against pairing:

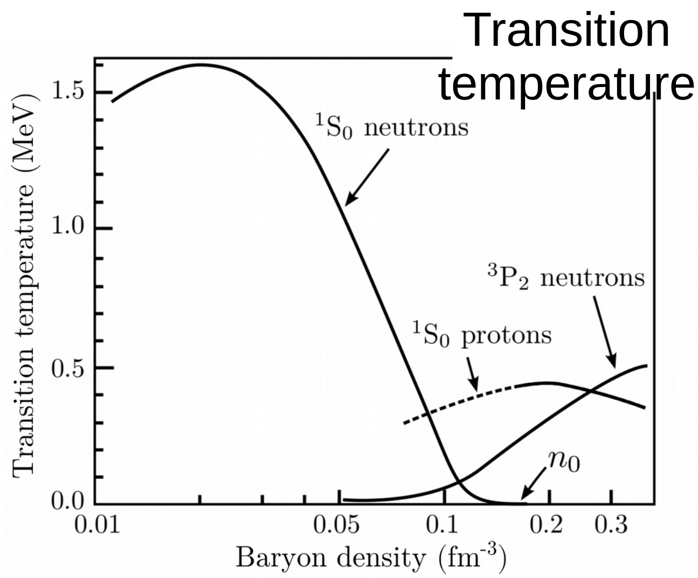
- metals: paired electrons (via phonon exchange)

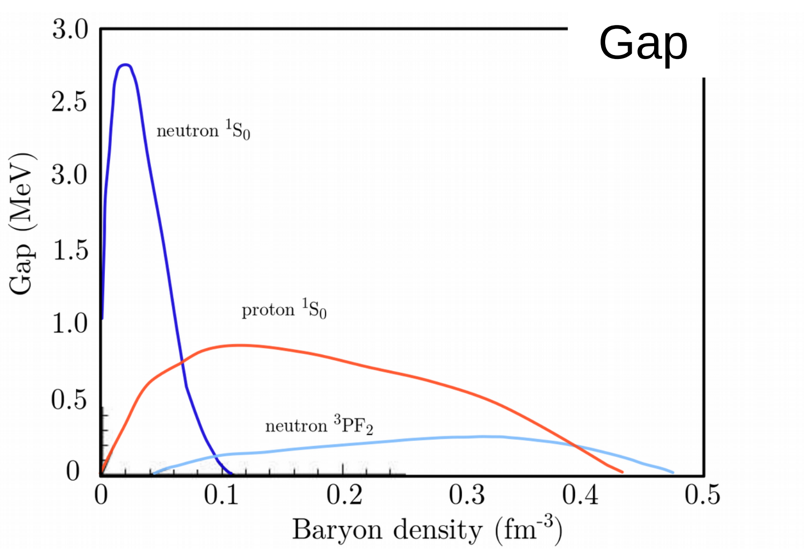
- neutrons in neutron stars: attractive components of the nuclear force



N. Ishii, S. Aoki, T. Hatsuda, PRL 99, 022001 (2007)

Energy gap: difficult to excite quasiparticles!





Superfluid hydrodynamics in neutron stars

$$\frac{\bar{p}_i^{\rm n}}{m_{\rm n}} = v_i^{\rm n} + \varepsilon_{\rm n}(v_i^{\rm p} - v_i^{\rm n})$$
$$\frac{\bar{p}_i^{\rm p}}{m_{\rm p}} = v_i^{\rm p} + \varepsilon_{\rm p}(v_i^{\rm n} - v_i^{\rm p})$$

The dynamics of vortices is hidden inside this term. Pinning modifies the mutual friction (when all vortices are perfectly pinned the mutual friction is zero)

$$\partial_t \rho_{\rm x} + \nabla_i(\rho_{\rm x} v_{\rm x}^i) = 0$$

$$(\partial_t + v_{\rm x}^j \nabla_j)(v_i^{\rm x} + \varepsilon_{\rm x} w_i^{\rm yx}) + \nabla_i(\tilde{\mu}_{\rm x} + \Phi) + \varepsilon_{\rm x} w_{\rm yx}^j \nabla_i v_j^{\rm x} = f_i^{\rm x}/\rho_{\rm x}$$

$$f_i^{\rm x} = 2\rho_{\rm n}\mathcal{B}'\epsilon_{ijk}\Omega^j w_{\rm xy}^k + 2\rho_{\rm n}\mathcal{B}\epsilon_{ijk}\hat{\Omega}^j\epsilon^{klm}\Omega_l w_m^{\rm xy}$$

Example: vortex mediated mutual friction for an unpinned and straight vortex bundle

Assumption: the charged component (and possibly the superfluid core) rotates as a rigid body

On the other hand, the superfluid in the crust can rotate non-uniformly.

Widely used approximation: circular flow (no meridional circulation)

$$p_{n\varphi} = m_n\, x\, (\Omega_p + (1-\epsilon_n)\Omega_{np})\,,$$
$$p_{p\varphi} = m_p\, x\, (\Omega_p + \epsilon_p \Omega_{np})\,,$$

$$L = \int x\, \pi_\varphi\, d^3x = I\,\Omega_p + \Delta L[\Omega_{np}]$$

$$\pi_\varphi = n_n p_{n\varphi} + n_p\, p_{p\varphi} = m_n x(n_B \Omega_p + n_n \Omega_{np})$$

Entrainment coupling: crust and core

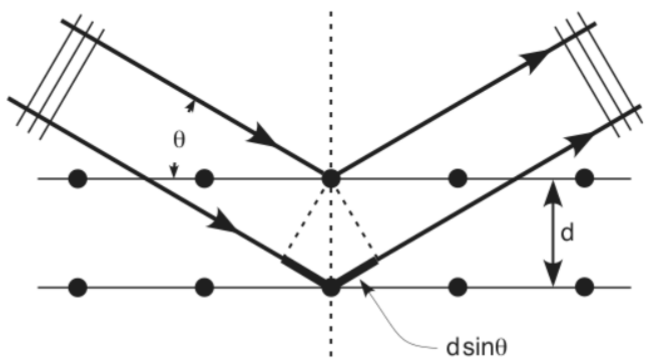
- In the crust:

Chamel N. *Neutron conduction in the inner crust of a neutron star in the framework of the band theory of solids,* Phys Rev C 85 (2012)

- Bragg scattering by crustal lattice, non-local m* > 1

→ Consequence: the crustal superfluid is entrained by the normal component: reduced mobility of free neutrons is a potential problem for pulsar glitch theory.

$$d\sin\theta = N\pi/k$$



- In the core:

Chamel N., Haensel P. *Entrainment parameters in a cold superfluid neutron star core,* Phys. Rev. C 73 (2006).

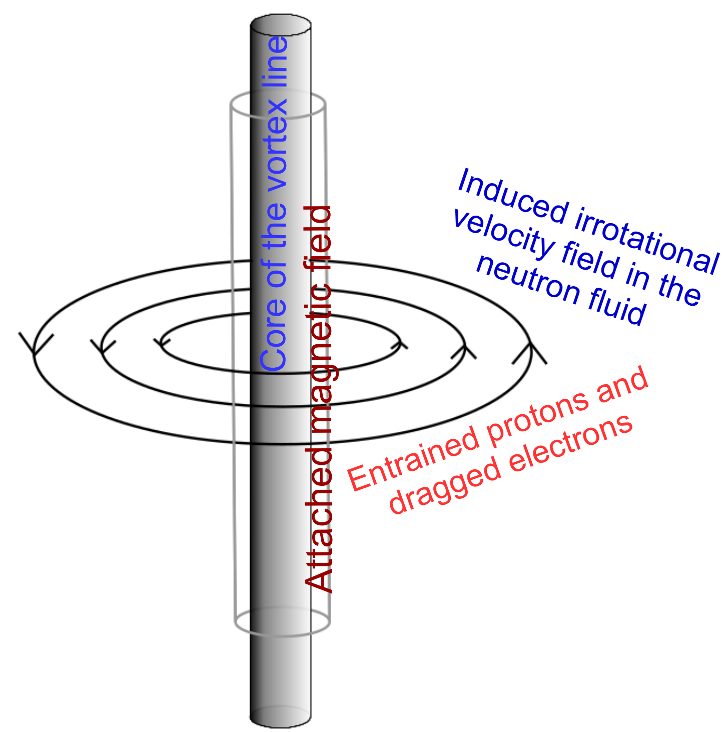
- Entrainment is due to the strong interaction between protons and neutrons

- Very different mechanism: actually more similar to the original A&B idea

- Local effect, m*<1

→ Consequence #1: Scattering of electrons off vortex cores: the core is coupled to the crust on the timescale of a second.
Alpar et al. *Rapid postglitch spin-up of the superfluid core in pulsars(1984)*

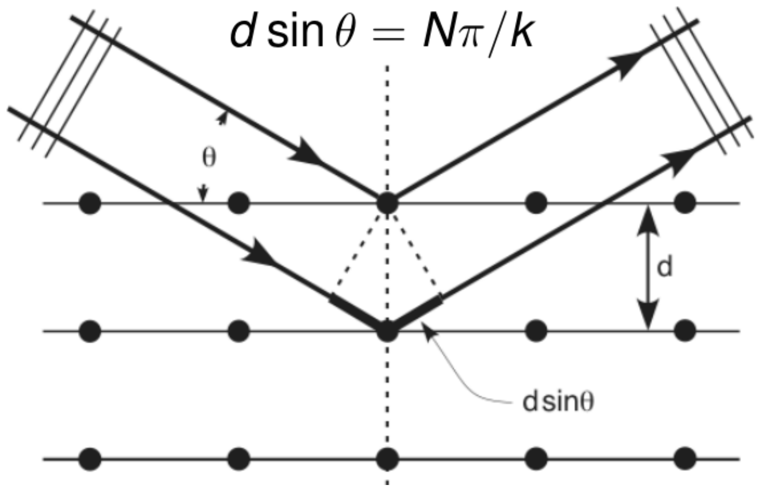
→ Consequence #2: Dipole-dipole interaction with flux-tubes (core pinning?)



Entrainment in neutron-star crusts

Despite the absence of viscous drag, the crust can still resist the flow of the superfluid due to non-local and non-dissipative entrainment effects.

Carter, Chamel, Haensel – Nucl.Phys.A (2005)
Entrainment coefficient and effective mass for conduction neutrons in NS crust: simple microscopic models



A neutron wave can be scattered by the lattice:

- A neutron can be coherently scattered if $k > \pi/d$.

- In a NS, neutrons have momenta up to k_F. Typically $k_F > \pi/d$ in all regions of the inner crust but the shallowest. Therefore, Bragg scattering should be taken into account.

Neutrons in neutron-star crusts are analogous to electrons in ordinary solids. Their properties can thus be determined by the band theory of solids. The average density of “conduction” neutrons is proportional to the average group velocity over the Fermi surface

$$n_n^{\mathrm{c}} = \frac{m_n}{24\pi^3\hbar^2} \sum_\alpha \int_{\mathrm{F}} |\boldsymbol{\nabla}_{\boldsymbol{k}} \varepsilon_{\alpha\boldsymbol{k}}| \mathrm{d}\mathcal{S}^{(\alpha)}$$

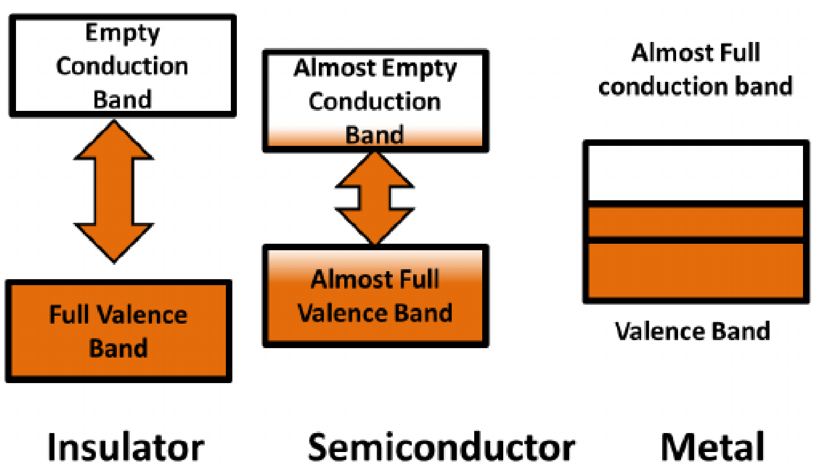
Band theory of solids

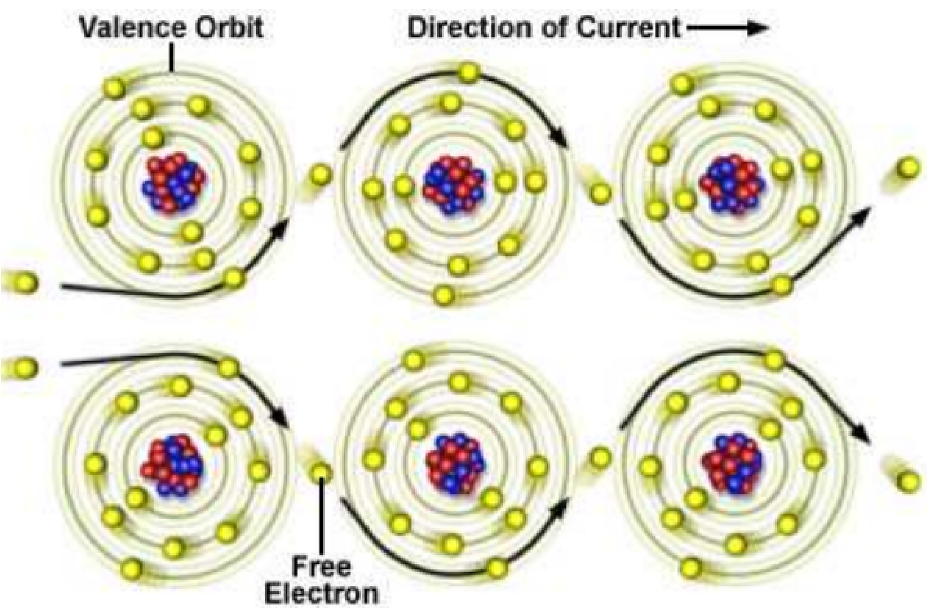
Due to the interactions with the periodic lattice, neutrons move in the inner crust as if they had an effective mass m*.

At the highest energies of the valence band (or at the lowest energies of the conduction band), the band structure E(K) of an electron in a solid can be approximated as

"Band structure" → $$E(\mathbf{k}) = E_{edge} + \frac{\hbar^2 k^2}{2m^*}$$ ← Electrons behave as nearly free particles but with a different mass

Energy of the edge of the band





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“Band structure”

$$E(\mathbf{k}) = E_{edge} + \frac{\hbar^2 k^2}{2m^*}$$

Energy of the edge of the band

Electrons behave as nearly free particles but with a different mass

Group velocity (velocity of a “modulation” in the electron wave): $\mathbf{v}_G = \partial\omega(\mathbf{k})/\partial\mathbf{k} = \hbar^{-1}\partial E(\mathbf{k})/\partial\mathbf{k}$

Semiclassical (single particle) approach: $\frac{d}{dt}\mathbf{v}_G = \mathbf{F}_{ext}$ where $\frac{d}{dt}\mathbf{v}_G = \hbar^{-1}\frac{d}{dt}\frac{\partial E(\mathbf{k})}{\partial\mathbf{k}}$

Using $\frac{d}{dt}\mathbf{k} = \hbar^{-1}\mathbf{F}_{ext}$ it is easy to obtain the semiclassical analogue of the relation “a = F / m”

$$\frac{d}{dt}v_G^i = \hbar^{-2}\frac{\partial^2 E(k)}{\partial k_i \partial k_j}F_j$$

Maximum glitch method

Consider the total angular momentum equation: $$\partial_t \Delta L[\Omega_{vp}] + I \partial_t \Omega_p = -I|\dot{\Omega}_\infty|$$

Conserve angular momentum during a glitch: $$\Delta\Omega_p = \frac{1}{I}(\Delta L[\Omega_{vp}^{pre}] - \Delta L[\Omega_{vp}^{post}])$$

Maximize it! $$\Delta L[\Omega_{vp}^{pre}] = \Delta L[\Omega_{vp}^{cr}] \ , \quad \Delta L[\Omega_{vp}^{post}] = 0 \quad \Rightarrow \quad \boxed{\Delta\Omega_{max} = \frac{1}{I}\Delta L[\Omega_{vp}^{cr}]}$$

Slack vortices:

Vortices are tensionless at the hydrodynamical scale.

Local unpinning condition:

$$|\mathbf{f}_M| = f_P(r) \quad \Rightarrow \quad \Omega_{vp}^{cr} = \frac{f_P(r)\, m^*(r)}{\kappa \, \sin\theta \, r \, \rho_n(r)}$$

Rigid (straight) vortices:

Vortices are rigid at the hydrodynamical scale.

Non-local unpinning condition:

$$\int_{\gamma_x} |\mathbf{f}_M| = \int_{\gamma_x} f_P \quad \Rightarrow \quad \Omega_{vp}^{cr}(x) = \frac{\int_{\gamma_x} f_P}{\kappa \, x \int_{\gamma_x} \rho_n / m^*}$$

In both cases it is simple to show that $$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa \, I} \int dr \, r^3 \, f_P(r)$$

Maximum glitch method

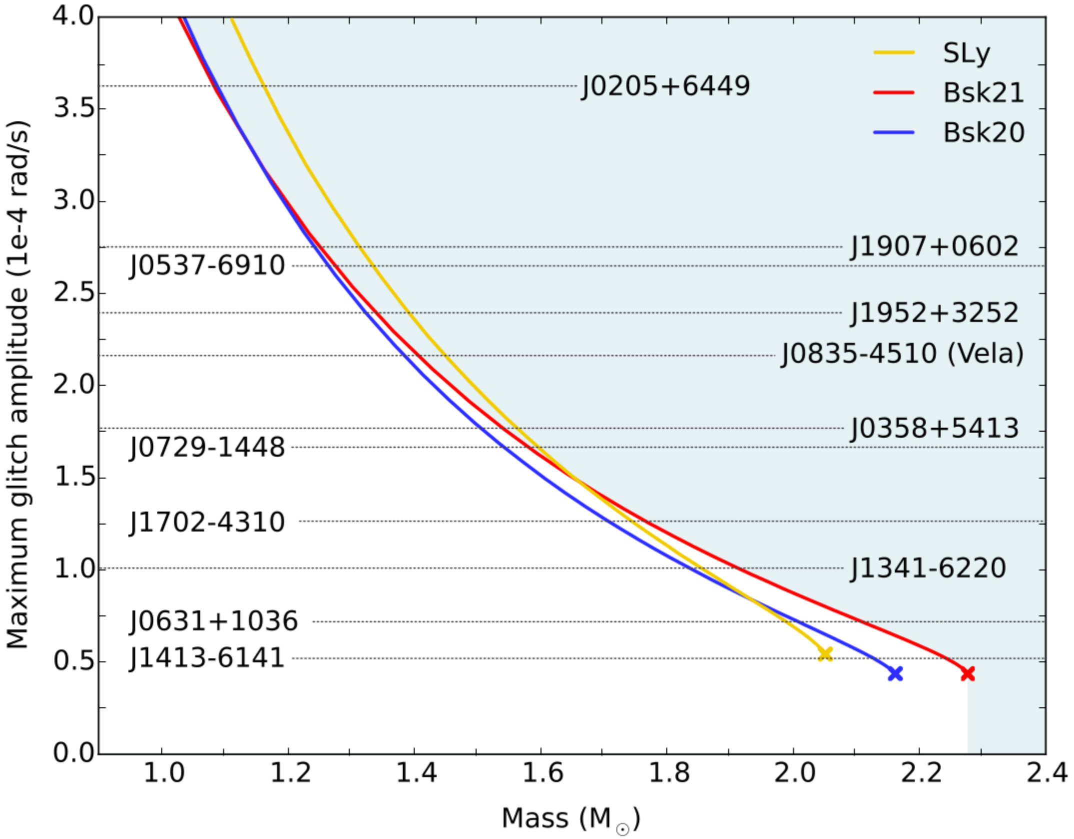
“Maximum glitch amplitude” at corotation:

→ Only dependent on pinning forces
and on the mass of the star

→ Entrainment independent!
Analitically for slack or parallel lines,
numerically also in intermediate cases.

→ No need to consider straight
vortex lines

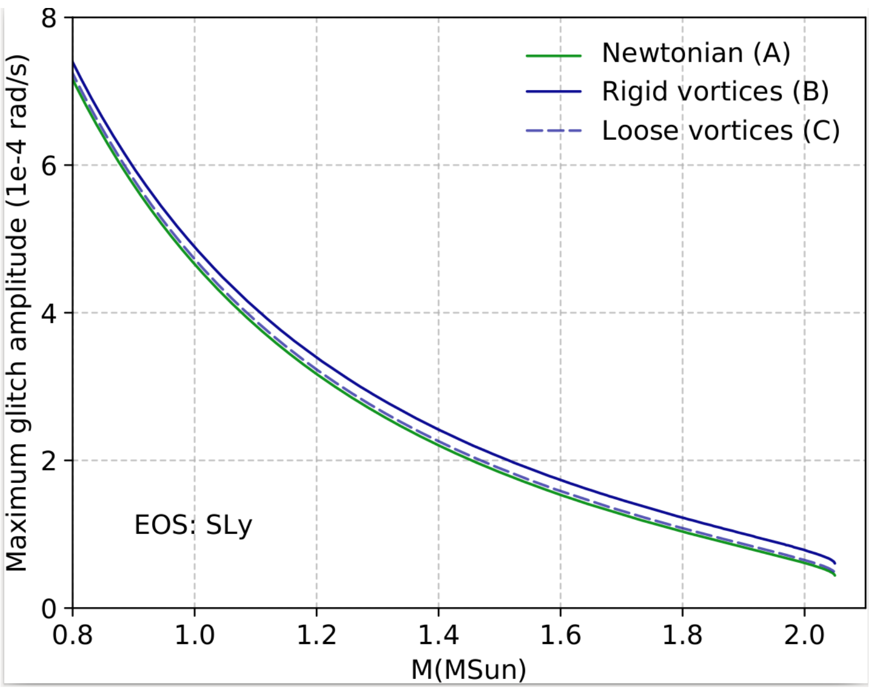
→ As long as pinning is crust-confined
you (analytically) obtain the same
maximum glitch amplitude also for vortices
that extend also into the core.



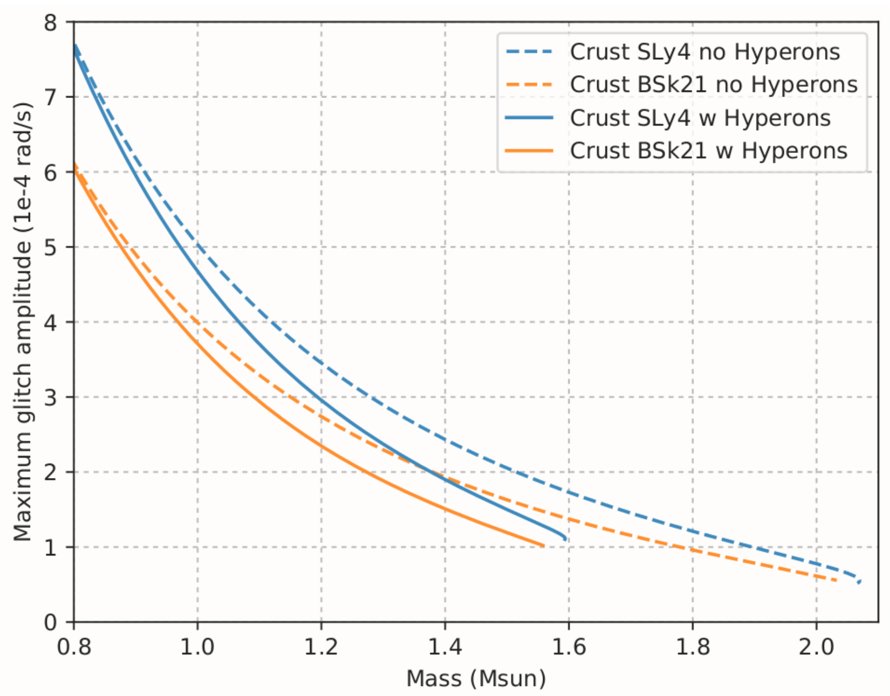
$$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa I} \int dr\, r^3\, f_P(r)$$

Pizzochero P.M., Antonelli M., Haskell B., Seveso S. (2017). *Constraints on pulsar masses from the maximum observed glitch.*

Effect of hyperons



Unified EOS (SLy)



Relativistic mean field with hyperons

Test of the SFHo model with and without hyperons, Steiner, Hempel, Fischer, Astrophys. J. 774, 17 (2013).

Slow rotation approximation

$$\frac{R^3\Omega^2}{GM} \ll 1$$

Slow rotation condition, Hartle (1967)

Extreme cases (millisecond pulsars, $M = 1.4M_{\odot}$, $R = 10km$):

- J1748-2446ad, $\Omega = 4501$ rad s^{-1} (not seen glitching):

$$R^3\Omega^2/(GM) \approx 0.11$$

- J1824-2452A, $\Omega = 2057$ rad s^{-1} (seen glitching, Cognard & Backer 2004):

$$R^3\Omega^2/(GM) \approx 0.023$$

Kepler frequency (rotation rate at which mass-shedding sets in at the equator) deduced from calculations using realistic EOS

$$\Omega_K \approx 7600 - 7700 \left(\frac{M}{M_{\odot}}\right)^{1/2} \left(\frac{10\ \mathrm{km}}{R}\right)^{3/2} \quad \mathrm{s}^{-1}$$

Original framework of Hartle & Sharp, 1967: one perfect fluid that is rigidly rotating

Two-components generalization: Andersson & Comer, *Slowly Rotating General Relativistic Superfluid Nss* (2000)

$$\Omega_n^2 \text{ or } \Omega_p^2 \text{ or } \Omega_n\Omega_p \quad << \quad \left(\frac{c}{R}\right)^2 \frac{GM}{Rc^2}$$

$$10^{-2}\ \mathrm{rad/s} \le |\Omega_n - \Omega_p| \le 10\ \mathrm{rad/s}.$$

Relativistic corrections in a nutshell

Each component has 4-velocity (p is rigid, v or n can rotate non-uniformly)

$$u_x = W_x \, e^{-\Phi} \, (\partial_t + \Omega_x \partial_\varphi)$$

The idea is to select the contribution of the superfluid to the total angular momentum

$$L = -\int \left(T_{\alpha\beta} - \frac{1}{2} T^{\nu}_{\nu} g_{\alpha\beta} \right) (\partial_\varphi)^\alpha \, z_0^\beta \, dV$$

Namely

$$L \;=\; I \, \Omega_p \;+\; \Delta L[\Omega_{np}]$$

… but the momenta contain the entrainment coupling and a Lorentz factor $\Gamma = -u_n^\alpha u_{p\alpha}$

$$\begin{aligned} p_{n\alpha}/\mu_n &= (1-\epsilon_n) u_{n\alpha} + (\epsilon_n/\Gamma) u_{p\alpha} \\ p_{p\alpha}/\mu_p &= (1-\epsilon_p) u_{p\alpha} + (\epsilon_p/\Gamma) u_{n\alpha} \end{aligned}$$

$$\epsilon_x = \frac{2\,\alpha}{\Gamma^2 \, n_x \, \mu_x}$$

In principle the angular momentum is not a linear functional of the lag → Hartle slow rotation:

$$\Delta L[\Omega_{np}] \;=\; \int d^3x \; e^{\Lambda - \Phi} \, y_n \, h \, x^2 \, \Omega_{np}$$

$$\boxed{\begin{gathered} \Delta^{cr} \;=\; \frac{f_P}{\kappa \, m_n \, y_n \, n_B} \;\lesssim\; 10^{-5}\, c \\ \Delta^{cr} = x e^{-\Phi} \Omega^{cr}_{np} \end{gathered}}$$