Exotics near thresholds and Hadron interactions in the heavy flavor sector

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in collaboration with

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Seminar@INFN Genova

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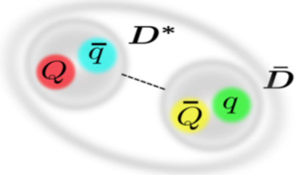
Outline

- Introduction
  - Exotic hadrons
  - $Z_c(3900)$
- Meson exchange model
  - Heavy quark symmetry and the meson exchange potential
  - Heavy meson exchange potential
- Quark exchange model
- Summary



Exotic hadron

Introduction

- Level structure of "matter"

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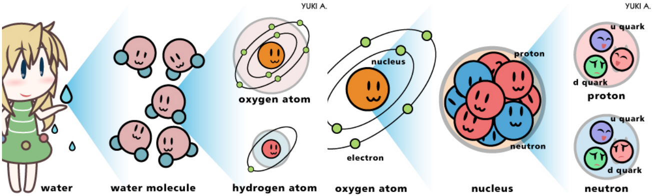

from HiggsTan.com

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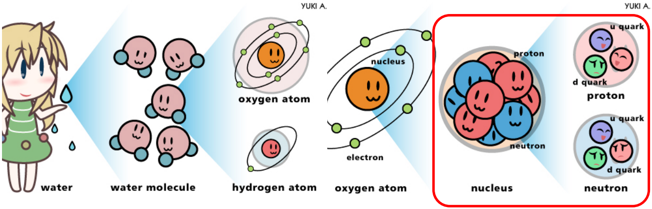

Nuclear and Hadron Phys

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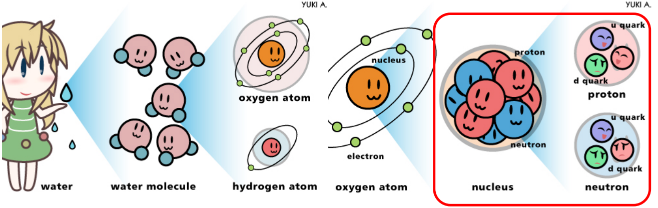

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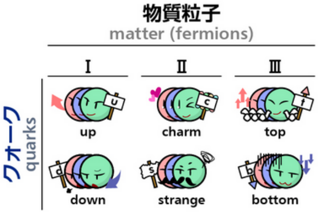
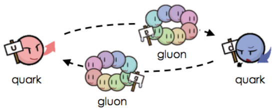
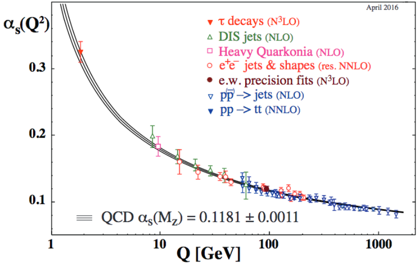


- Elementary particle “Quarks”
- $u, d \rightarrow$ Proton, Nucleon (Ordinary)
- **Systems with s, c or b**
  beyond ordinary matters

- Quantum ChromoDynamics (QCD)
  $\rightarrow$ describing the strong interaction of quark and gluon

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma^{\mu}(\partial_{\mu} + igA^{a}_{\mu}T^{a}) - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$





- Running coupling $\alpha_s = g^2/4\pi$

$\Rightarrow$ At low-energy, perturbative calculation breaks down
$\rightarrow$ **Quark confinement** inside "Hadron" (e.g. proton, nucleon,...)

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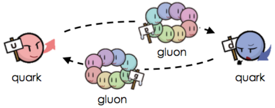
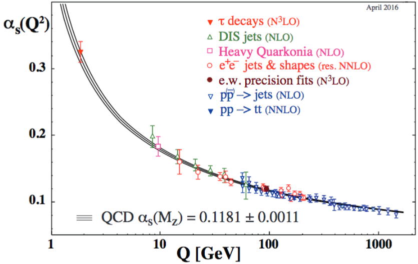


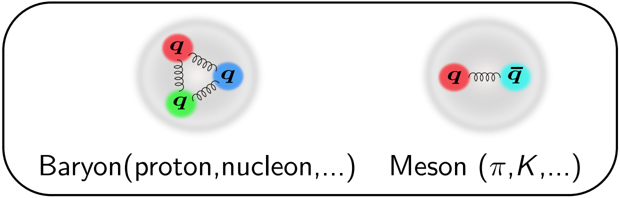


- Running coupling $\alpha_s = g^2/4\pi$

⇒ At low-energy, perturbative calculation breaks down
→ **Quark confinement** inside "Hadron" (e.g. proton, nucleon,...) **Difficulty of QCD** → **Model**, Lattice

Constituent quark model: simple but powerful
Introduction

- Simple description of Hadron structure → Constituent quark model
- Constituent quark model (Baryon(qqq) and Meson $q\bar{q}$) has been **successfully applied to the hadron spectra!**



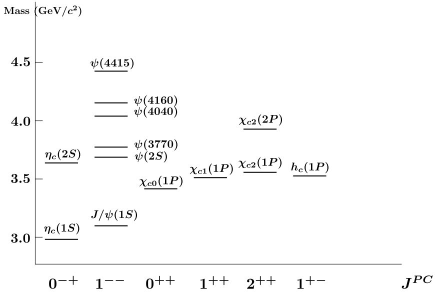
*q: "Constituent quark"

▷ Quark potential (One gluon exchange + Linear confinement)

$$V_q(r) = -\frac{a}{r} + br + c + d\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2}\delta^{(3)}(r) + \ldots$$

(Parameters are fixed to reproduce grand state hadrons)

▷ Charmonium ($c\bar{c}$)

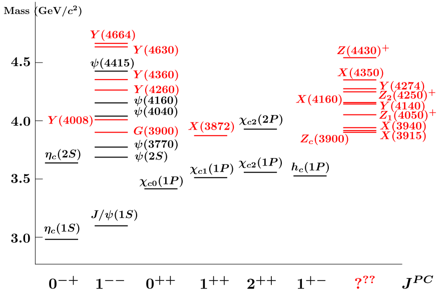


Charmonium $c\bar{c}$

N. Brambilla,*et al.* Eur.Phys.J.C **71**(2011)1534
S. Godfrey and N. Isgur, PRD**32**(1985)189

- Constituent quark model is powerful.

▷ Charmonium ($c\bar{c}$) and **New Exotic hadrons *X*, *Y*, *Z***



Charmonium $c\bar{c}$

and

Exotic hadrons ($\neq c\bar{c}$)

X*, *Y*, *Z

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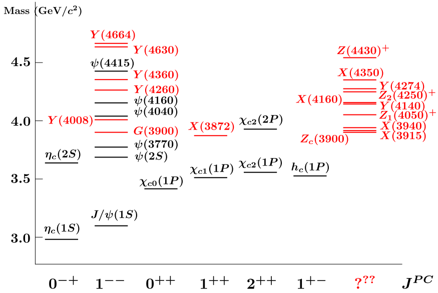

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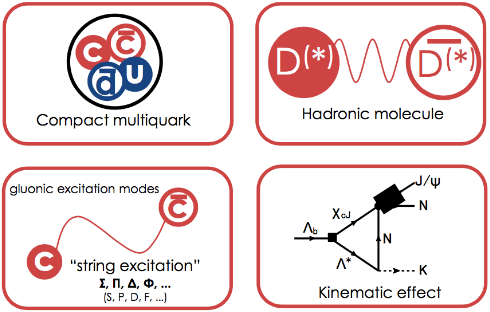
Charm $X(3872)$, Charged Z_c

Belle PRL **91** (2003) 262001. A. Hosaka *et al.* PTEP **2016** (2016) no.6, 062C01

Bottom $Z_b(10610)$ and $Z_b(10650)$ Belle PRL **108** (2012) 122001

- What is the structure of Exotic hadrons? Candidates:



- ▷ Multiquark states?:
  Compact mutiquark and/or Hadronic molecule
- ▷ Gluon excitation?: Hybrid state
- ▷ Kinematical effect?: It is not bound state!

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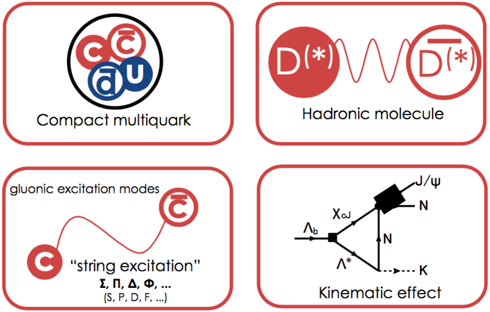

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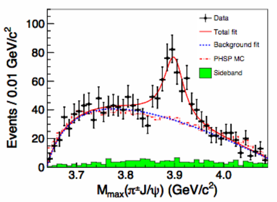
- Charged Charmonium??
- $Y(4260) \to Z_c(3900)\pi \to J/\psi\pi\pi$

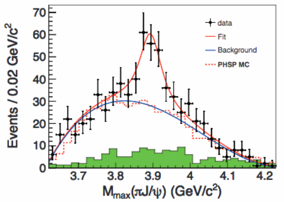
BESIII,PRL**110**(2013)252001



$M = 3899.0 \pm 3.6_{sta} \pm 4.9_{sys}$ MeV

$\Gamma = 46 \pm 10_{sta} \pm 20_{sys}$ MeV

Belle,PRL**110**(2013)252002



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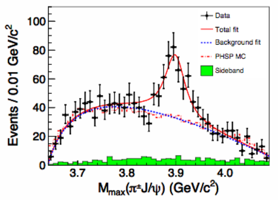

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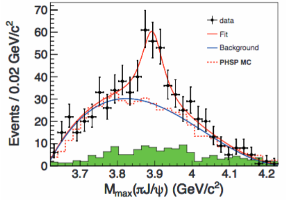

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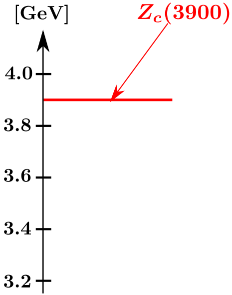
▷ Ordinal Charmonium $c\bar{c}$: no electric charge.
$\Rightarrow Z_c^+(3900)$: **Genuine Exotic State!?** $c\bar{c}u\bar{d}$

- $Z_c(3900)$ close to the hadron-hadron thresholds



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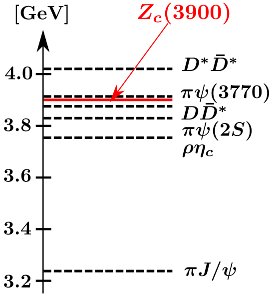

$Z_c(3900)$: Hadronic Molecules?

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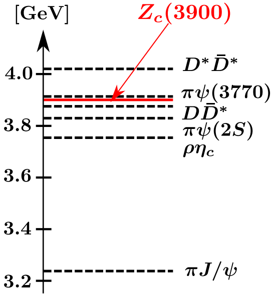
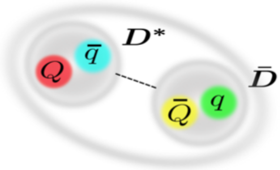


$\Rightarrow$



Hadronic molecule?

- Exotic state may be **a loosely bound state (resonance)** of the meson-meson.

  $\Rightarrow$ Analogous to atomic nuclei (Deuteron: $B \sim$2.2 MeV)

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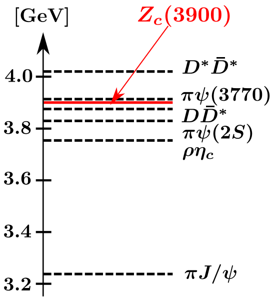

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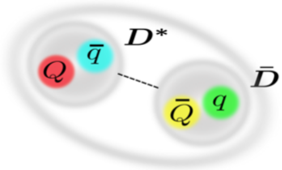

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- $D\bar{D}^*$ molecule? **π exchange**

ref. A. Hosaka *et al.* PTEP **2016** (2016)062C01, A.Esposito, *et al.*,Phys.Rept.**668**(2016)1,...

$Z_c(3900)$: Lattice QCD (Numerical Experiments)

- Lattice QCD simulation by HALQCD at $m_\pi = 410 - 700$ MeV

$\Rightarrow$ Coupled-channel $\pi J/\psi - \rho\eta_c - D\bar{D}^*$

PRL **117**, 242001 (2016) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2016

Fate of the Tetraquark Candidate $Z_c(3900)$ from Lattice QCD

Yoichi Ikeda,[1,2] Sinya Aoki,[3,4] Takumi Doi,[2] Shinya Gongyo,[3] Tetsuo Hatsuda,[2,5] Takashi Inoue,[6] Takumi Iritani,[7] Noriyoshi Ishii,[1] Keiko Murano,[1] and Kenji Sasaki[3,4]

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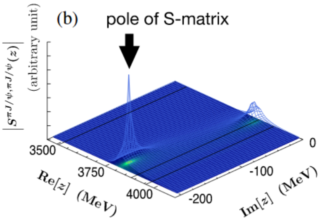

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Ikeda, *et al.*,PRL**117**(2016)242001

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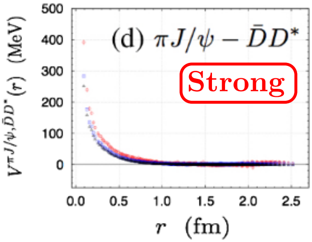


$\Rightarrow$ **Virtual state** is obtained.

- $Z_c(3900)$ is Threshold cusp induced by $\pi J/\psi - \bar{D}D^*$ **potential**

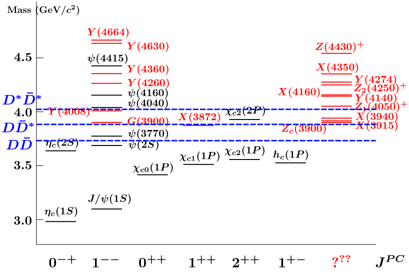
Charm quark exchange?

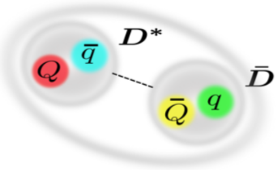
Ikeda, *et al.*,PRL**117**(2016)242001

Exotic structure:
Bound state? Cusp?

Exotic structure: Bound state? Cusp? $\Leftrightarrow$ **Hadron-hadron interaction**

- **Hadron-hadron interaction** is important to understand the nature of exotic states! not only Z_c but also others.





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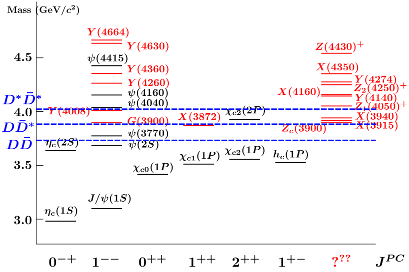

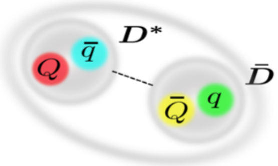

→ **Hadron int. is not established yet...**

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  Lightest meson π, Importance in the nuclear force,
  Heavy Quark Symmetry

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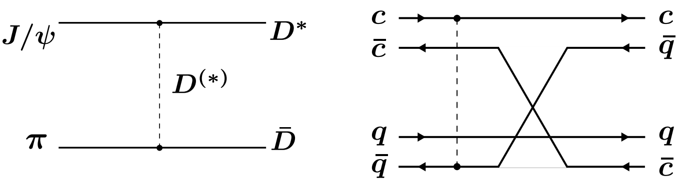
(a) $D^{(*)}$ meson exchange (b) Quark exchange



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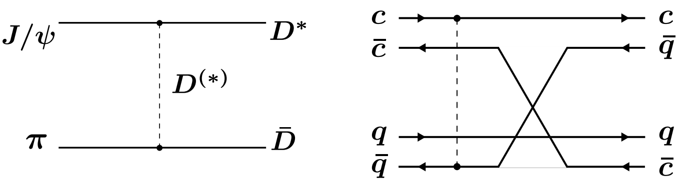
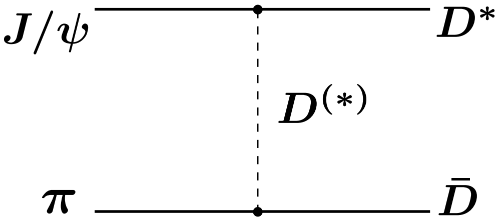


  $\rightarrow$ next sec. $\rightarrow$ next next sec.

- How can we understand **strong $\pi J/\psi - D\bar{D}^*$ potential?**
  $\rightarrow$Very short range interaction due to large c mass

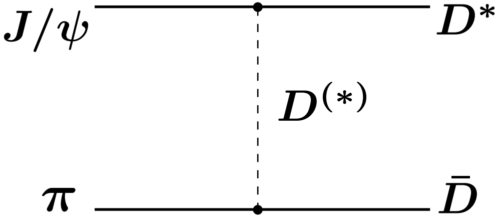
Today: Let us focus on **$Z_c(3900)$ as $\pi J/\psi - D\bar{D}^*$**

Meson exchange model



- $D^{(*)}\bar{D}^{(*)} - D^{(*)}\bar{D}^{(*)}$: π, ρ, ω
- $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$: D, D^*

Meson exchange model



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Heavy Quark Spin Symmetry

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Charm (c), Bottom (b), Top (t)

Heavy Quark Spin Symmetry

Charm (c), Bottom (b), Top (t)

$\Downarrow$

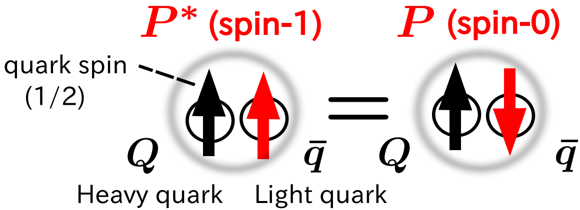
1. Coupled channels
2. Tensor force (OPEP)

Heavy Quark Spin Symmetry (HQS) N.Isgur,M.B.Wise,PLB**232**(1989)113

- **Suppression of Spin-spin force** in $m_Q \to \infty$.

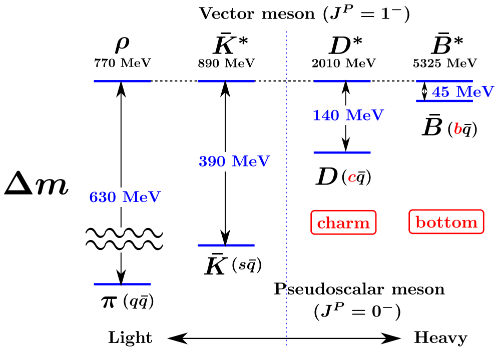
  $\Rightarrow$ **Mass degeneracy** of hadrons with the different J

- e.g. $Q\bar{q}$ meson



$\Rightarrow$ Mass degeneracy of spin-0 and spin-1 states!

- Mass difference between vector and pseudoscalar mesons. $(Q\bar{q},\ q = u, d)$



▷ Δm decreases when the quark mass increases.

Mass degeneracy of heavy hadrons
Meson exchange model

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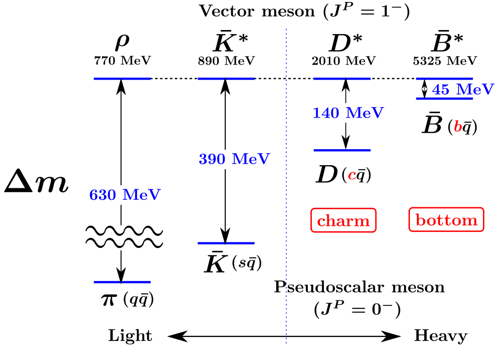


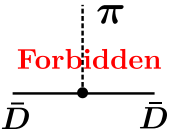
▷ Δm decreases when the quark mass increases.
$\Rightarrow$ **Degeneracy of Heavy hadrons!**

For $Z_c(3900)$, $D - D^*$ mixing

Heavy hadron-π coupling
Meson exchange model

- Effective Lagrangians: Heavy hadron and π

R. Casalbuoni *et al.*, Phys.Rept.**281** (1997)145, T. M. Yan, *et al.*, PRD**46**(1992)1148



▷ Heavy meson: $\bar{D}^{(*)}\bar{D}^{(*)}\pi$ **($DD\pi$: Parity violation)**

$$\mathcal{L}_{\pi HH} = -\frac{g_\pi}{2f_\pi}\mathrm{Tr}\left[H\gamma_\mu\gamma_5\partial^\mu\hat{\pi}\bar{H}\right], \quad H = \frac{1+\not{v}}{2}\left[D^*_\mu\gamma^\mu - D\gamma_5\right]$$

- Doublet D and D^* with one coupling const. $g_\pi = 0.59$ (from $D^* \to D\pi$ decay)

▷ Form factor (Hadron has **finite size**)

$$F(q^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2}, \quad \Lambda_{\bar{D}} \sim 1130 \text{ MeV}$$

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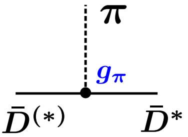
R. Casalbuoni *et al.*, Phys.Rept.**281** (1997)145, T. M. Yan, *et al.*, PRD**46**(1992)1148



▷ Heavy meson: $\bar{D}^{(*)}\bar{D}^{(*)}\pi$ **($DD\pi$: Parity violation)**

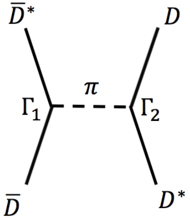
$$\mathcal{L}_{\pi HH} = -\frac{g_\pi}{2f_\pi}\mathrm{Tr}\left[H\gamma_\mu\gamma_5\partial^\mu\hat{\pi}\bar{H}\right], \quad \mathbf{H} = \frac{1+\not{v}}{2}\left[\mathbf{D}^*_\mu\gamma^\mu - \mathbf{D}\gamma_5\right]$$

- Doublet D and D^* with one coupling const. $g_\pi = 0.59$ (from $D^* \to D\pi$ decay)

▷ Form factor (Hadron has **finite size**)

$$F(q^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2}, \quad \Lambda_{\bar{D}} \sim 1130 \text{ MeV}$$

- One boson exchange potential (OBEP)



Born amplitude

$\rightarrow$ Non-relativistic Potential

OPEP

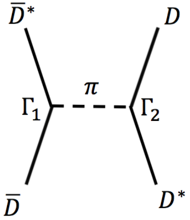
$$V^\pi = -\frac{1}{2}\left(\frac{g_\pi}{f_\pi}\right)^2 \left[\vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\hat{r}) T(r)\right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$C(r) = m_\pi^2 \left(\frac{e^{-m_\pi r}}{r} - \frac{e^{-\Lambda r}}{r} - \frac{\Lambda^2 - m_\pi^2}{2\Lambda} e^{-\Lambda r}\right)$$

Comments

- HQS induces ***$D\bar{D}^* - D^*\bar{D}^*$ couplings*** $\rightarrow$ OPEP works!

- One boson exchange potential (OBEP) **with Tensor force!**



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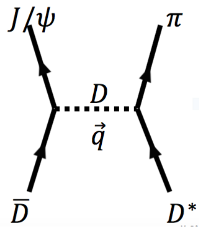
- HQS induces $\boldsymbol{D\bar{D}^* - D^*\bar{D}^*}$ **couplings** $\rightarrow$ OPEP works!
- Tensor force $T(r) \Rightarrow$ **the driving force** in atomic nuclei
  $S_{12}(\hat{r}) = 3(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r}) - \vec{S}_1\cdot\vec{S}_2 \rightarrow S$-$D$ mixing

Heavy meson exchange potential
Meson exchange model

- $D^{(*)}$ meson exchange potential in $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$



D exchange

$$V^D = \frac{2}{3}\frac{g_\psi g_\pi}{f_\pi\sqrt{E_\pi}}\left[\vec{S}_1\cdot\vec{S}_2 C(r) + S_{12}(\hat{r})T(r)\right]$$

D^* exchange

$$V^{D^*} = \frac{2}{3}\frac{g_\psi g_\pi}{f_\pi\sqrt{E_\pi}}\left[2\vec{S}_1\cdot\vec{S}_2 C(r) - S_{12}(\hat{r})T(r)\right]$$

$g_\psi = 8$

A. Deandrea, G. Nardulli and A. D. Polosa, PRD**68**(2003)034002

Comments

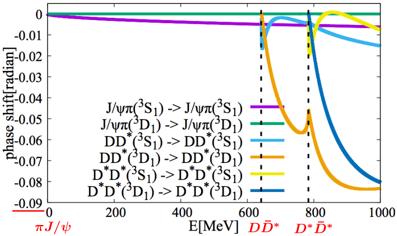
- $D^{(*)}$ meson exchange gives the $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$ potential. Hidden $\leftrightarrow$ Open-Open
- $D^{(*)}$ mass $\sim$ 2 GeV $\Leftrightarrow 1/m_{D^{(*)}} \sim 0.1$ fm
  Does it work?

Numerical results: Phase shift

Meson exchange model

- We found...

- We found... **No Bound state, No Resonance**
  Very Small phse shift $|\delta| < 0.09$ [rad]



- $D^{(*)}\bar{D}^{(*)}$ channel: **Small** contribution from OPEP

- $\pi J/\psi$ channel: $D^{(*)}$ exchange is **Negligible**

Numerical results: Phase shift

Meson exchange model

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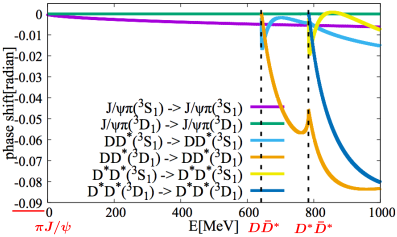


- $D^{(*)}\bar{D}^{(*)}$ channel: **Small** contribution from OPEP
  Why?: Isospin factor $\vec{\tau}_1 \cdot \vec{\tau}_2$, $I = 0$: -3, but $I = 1$: $+1$
- $\pi J/\psi$ channel: $D^{(*)}$ exchange is **Negligible**
  Why?: Suppression by the form factor (finite hadron size)

- No resonance $\leftarrow$ agreeing with Lattice QCD result

$\Leftrightarrow$ We cannot explain the strong $\pi J/\psi - D\bar{D}^*$ potential.

- No resonance $\leftarrow$ agreeing with Lattice QCD result

$\Leftrightarrow$ We cannot explain the strong $\pi J/\psi - D\bar{D}^*$ potential.

Problems in $D^{(*)}$ exchange potential

- Too large mass, $1/m_{D^{(*)}} \sim 0.1$ fm
- In such short range region, "Hadron" is not good effective d.o.f. ?

$\Downarrow$

Quark exchange interaction!

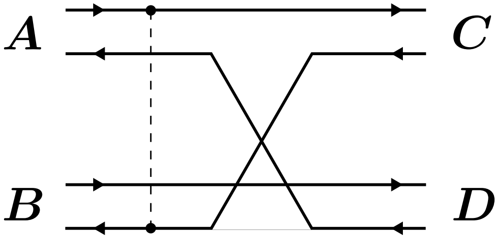
$\rightarrow$next section

Quark exchange model (on progress...)

- **Born-order quark-exchange diagram**

  T. Barnes and E. S. Swanson, PRD**46**(1992)131. Swanson, Ann. Phys. **220**(1992)73.

- $AB \to CD$ scattering $\quad \mathcal{M}_{fi} \propto \langle C, D | H_I | A, B \rangle$



- Ingredients: Meson Wavefunctions(A, B, C, D)
  Quark interaction (Quark Model)
- Born amplitude $\Rightarrow$ Meson-meson Potential can be obtained

- Quark Hamiltonian (One gluon exchange + Linear potentials)

Barnes and Swanson, PRD**46**(1992)131.; Swanson, Ann. Phys. **220**(1992)73.

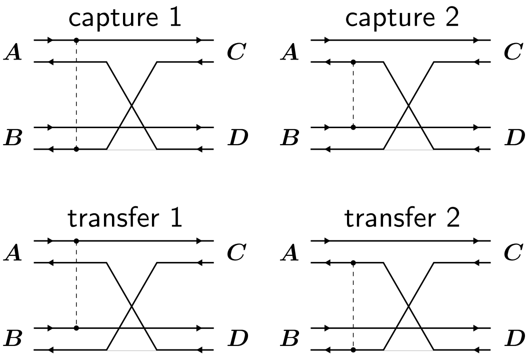
$$H^q_{ij} = K_q + \left(-\frac{3}{4}br + \frac{\alpha_s}{r} - C\right)\vec{F}_i \cdot \vec{F}_j - \frac{8\pi\alpha_h}{3m_i m_j}\left(\frac{\sigma^3}{\pi^{3/2}}e^{-\sigma^2 r_{ij}^2}\right)\vec{S}_i \cdot \vec{S}_j \vec{F}_i \cdot \vec{F}_j$$

▷ Parameters are fixed to reproduce the mass of mesons

Table: Quark Model Parameters from Ann.Phys.**220**(1992)73.

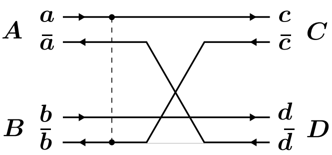
$m_q = 0.375$ GeV	$m_c = 1.9$ GeV
$\alpha_s = 0.857$	$\alpha_h = 0.840$
$b = 0.154$ GeV^{-2}	$C = -0.4358$ GeV
$\sigma = 0.70$ GeV	

- Born quark exchange diagrams T. Barnes and E. S. Swanson, PRD**46**, 131 (1992).
  Quark interaction between Mesons ⇒ Four diagrams



- Scattering Amplitude $\mathcal{M}_{fi} \propto \langle C, D | H^q | A, B \rangle$

$$\mathcal{M}_{fi}^{tot} = \mathcal{M}_{fi}^{capture1} + \mathcal{M}_{fi}^{capture2} + \mathcal{M}_{fi}^{transfer1} + \mathcal{M}_{fi}^{transfer2}$$

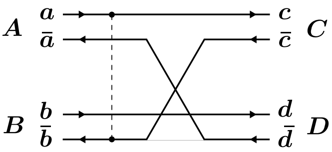


- Meson momenta: A, B, C, D
- Quark momenta: $a, \bar{a}, b, \bar{b}, c, \bar{c}, d, \bar{d}$
- Conservation: $A + B = C + D$, $\bar{a} = \bar{d}$, $b = d$

- Amplitude

$$\to \int\int d^3a d^3c \phi_C^*(2\vec{c} - \vec{C})\phi_D^*(2\vec{a} - 2\vec{A} - \vec{C})V(\vec{a} - \vec{c})\phi_A(2\vec{a} - \vec{A})\phi_B(2\vec{a} - \vec{A} - 2\vec{C})$$

capture 1



- Meson momenta: A, B, C, D
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- Conservation: $A + B = C + D$, $\bar{a} = \bar{d}$, $b = d$

- Amplitude

$$\to \int\int d^3a d^3c \phi^*_C(2\vec{c} - \vec{C})\phi^*_D(2\vec{a} - 2\vec{A} - \vec{C})V(\vec{a} - \vec{c})\phi_A(2\vec{a} - \vec{A})\phi_B(2\vec{a} - \vec{A} - 2\vec{C})$$

- Potentials (momentum space)

Coulomb: $V^{Coul}(q) = -\frac{\alpha_s}{2\pi^2}\frac{1}{\vec{q}^{\,2}}$, **Hyperfine:** $V^{Hyp}(q) = -\frac{8\pi\alpha_h}{3m_i m_j}e^{-\vec{q}^{\,2}/4\sigma^2}$

Linear (Regularized):

$$V^{Lin}(r) = br \times e^{-\varepsilon r} \to V^{Lin}(q) = b\left[\frac{-8\pi}{(\vec{q}^{\,2}+\varepsilon^2)^2} + \frac{32\pi\varepsilon^2}{(\vec{q}^{\,2}+\varepsilon^2)^3}\right]$$

Numerical Results

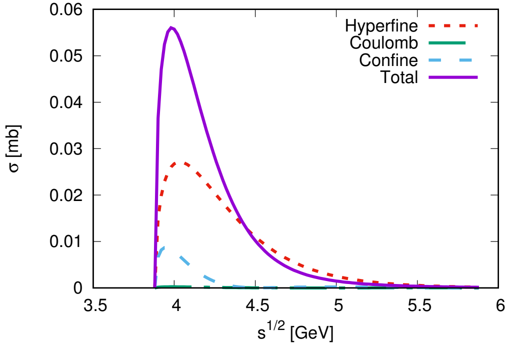
(on progress...)

- $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$ Cross Section (Born-term)

Cross Section (Born term): $\pi J/\psi - D\bar{D}^*$

Numerical Result

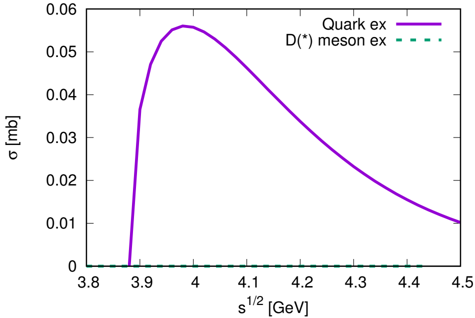
- $\pi J/\psi - D\bar{D}^*$: Amplitude $\rightarrow$ Cross section



- Cross section $\Rightarrow |(\text{Coulomb}) + (\text{Confine}) + (\text{Hyperfine})|^2$
- Dominant role of the Hyperfine (Spin-spin) term
  $\Leftrightarrow$ Minor role of the Coulomb one.

- $\pi J/\psi - D\bar{D}^*$: Amplitude $\rightarrow$ Cross section

(i) Quark ex vs $D^{(*)}$ ex

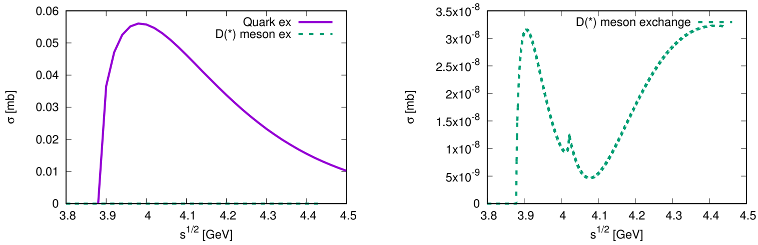


- Comparing results of Quark exchange and $D^{(*)}$ exchange
- Can you see a dashed line ($D^{(*)}$ exchange)?

Cross Section: Quark exchange vs $D^{(*)}$ exchange
Numerical Result

- $\pi J/\psi - D\bar{D}^*$: Amplitude $\rightarrow$ Cross section

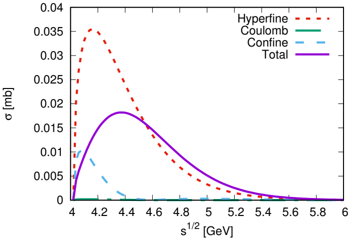
(i) Quark ex. vs $D^{(*)}$ ex. (ii) $D^{(*)}$ ex. (Zoom)



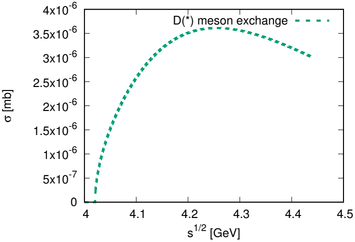
- Comparing results of Quark exchange and $D^{(*)}$ exchange
- Can you see a dashed line ($D^{(*)}$ exchange)? $< \mathbf{3.5 \times 10^{-8}}$ **mb**
- Large difference between Quark exchange and $D^{(*)}$ exchange!

- $\pi J/\psi - D^* \bar{D}^*$: Amplitude $\rightarrow$ Cross section

(i) Quark ex vs $D^{(*)}$ ex

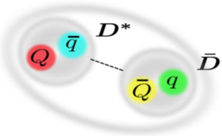


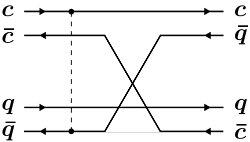
(ii) $D^{(*)}$ exchange



- Cross section $\Rightarrow$ $|(\text{Coulomb}) + (\text{Confine}) + (\text{Hyperfine})|^2$
- **Dominant role of the Spin-spin term**.
  Coulomb term $\Rightarrow$ Very small contribution.
- Large difference between Quark exchange and $D^{(*)}$ exchange!

Summary





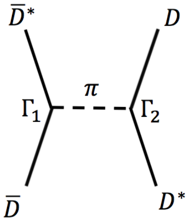
- Many exotic states near the threshold.
  $\rightarrow$ Understanding **the hadron-hadron interaction** is needed.
- Charged charmonium $Z_c(3900)$ has been discussed as the Hadronic molecules or the threshold cusp.
- OPEP contribution is not strong. $D^{(*)}$ meson exchange is **negligible**.
- Quark exchange interaction is introduced as Short range $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$ potential.
  We find **Large difference** between results from Quark exchange and $D^{(*)}$ meson exchange.

Back Up

One pion exchange potential in $D^{(*)}\bar{D}^{(*)}$

Meson exchange model

- One boson exchange potential (OBEP)



OPEP

$$V^\pi = -\frac{1}{2}\left(\frac{g_\pi}{f_\pi}\right)^2 \left[\vec{S}_1\cdot\vec{S}_2 C(r) + S_{12}(\hat{r})T(r)\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

Vector (ρ,ω) exchange

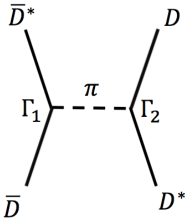
$$V^v = -\left(\frac{\lambda g_V}{\sqrt{3}}\right)^2 \left[2\vec{S}_1\cdot\vec{S}_2 C(r) - S_{12}(\hat{r})T(r)\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

Comments

- Tensor force $T(r) \Rightarrow$ **the driving force** in atomic nuclei
  $S_{12}(\hat{r}) = 3(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r}) - \vec{S}_1\cdot\vec{S}_2 \to S$-$D$ mixing

One pion exchange potential in $D^{(*)}\bar{D}^{(*)}$
Meson exchange model

- One boson exchange potential (OBEP) **with Tensor force!**



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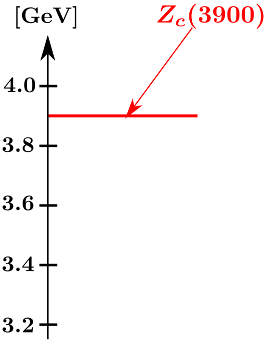
Comments

- Tensor force $T(r) \Rightarrow$ **the driving force** in atomic nuclei
  $S_{12}(\hat{r}) = 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2 \rightarrow S$-$D$ mixing
- $G-$parity of vector mesons: ρ $(G = -1)$, ω $(G = +1)$
  $\Rightarrow$ **Working against each other**, $\rho + \omega$ has a minor role...

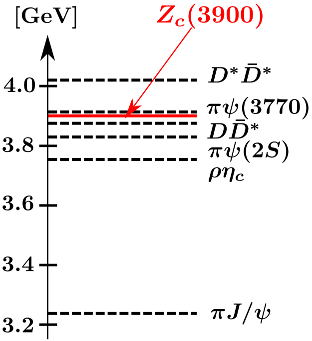
Meson-meson thresholds
Model Setup

- Meson-meson thresholds,



Meson-meson thresholds
Model Setup

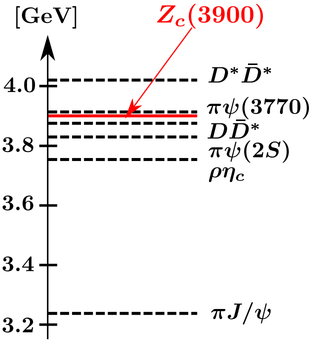
- Meson-meson thresholds, $\pi J/\psi, \rho\eta_c, \pi\psi(2S), D\bar{D}^*, D^*\bar{D}^*, \ldots$



Coupled-Channels

$$\left\{\begin{matrix} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{matrix}\right\} - \left\{\begin{matrix} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{matrix}\right\}$$

- Meson-meson thresholds, $\pi J/\psi, \rho\eta_c, \pi\psi(2S), D\bar{D}^*, D^*\bar{D}^*, \ldots$



Coupled-Channels

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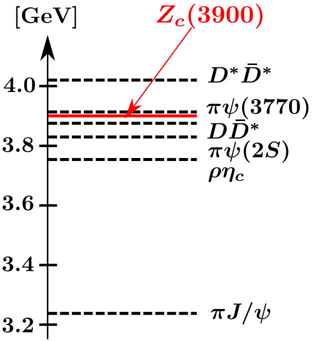
- Born-order quark-exchange
  $\Rightarrow$ Applicable to charm exchange (**Hidden**$\leftrightarrow$**Open-Open**)

- Meson-meson thresholds, $\pi J/\psi, \rho\eta_c, \pi\psi(2S), D\bar{D}^*, D^*\bar{D}^*, \ldots$



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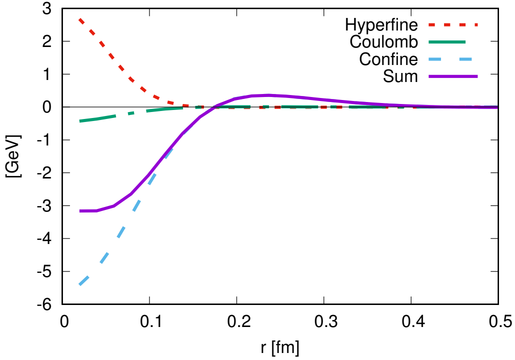
- Born-order quark-exchange
  $\Rightarrow$ Applicable to charm exchange (**Hidden**$\leftrightarrow$**Open-Open**)

Today: $\pi J/\psi - D\bar{D}^*$ and $\pi J/\psi - D^*\bar{D}^*$ (S-wave)

- Energy-dependent $\pi J/\psi - D\bar{D}^*$ potential

$\rightarrow$ Result@$\underline{E = m_D + m_{\bar{D}^*}}$ ($D\bar{D}^*$ threshold)



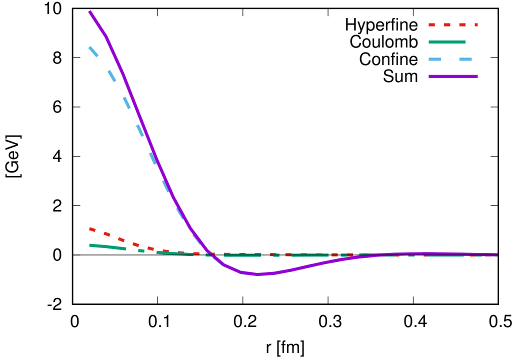
- Hyperfine (Spin-spin) and Confinement potentials
  $\Rightarrow$ **Working against each other**

$\pi J/\psi - D^*\bar{D}^*$ Potentials (on the threshold)

Numerical Result

- Energy-dependent $\pi J/\psi - D^*\bar{D}^*$ potential

$\rightarrow$ Result@$\underline{E = m_{D^*} + m_{\bar{D}^*}}$ ($D^*\bar{D}^*$ threshold)

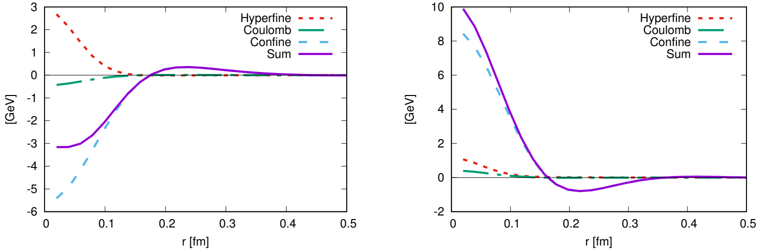


- Hyperfine (Spin-spin) and Confinement potentials
  $\Rightarrow$ **Cooperating with each other**

- Comparing $\pi J/\psi - D\bar{D}^*$ with $\pi J/\psi - D^*\bar{D}^*$

(i) $\pi J/\psi - D\bar{D}^*$ $(@E = m_D + m_{\bar{D}^*})$

(ii) $\pi J/\psi - D^*\bar{D}^*$ $(@E = m_{D^*} + m_{\bar{D}^*})$



- Similar?
- Degenerate in the heavy quark limit?