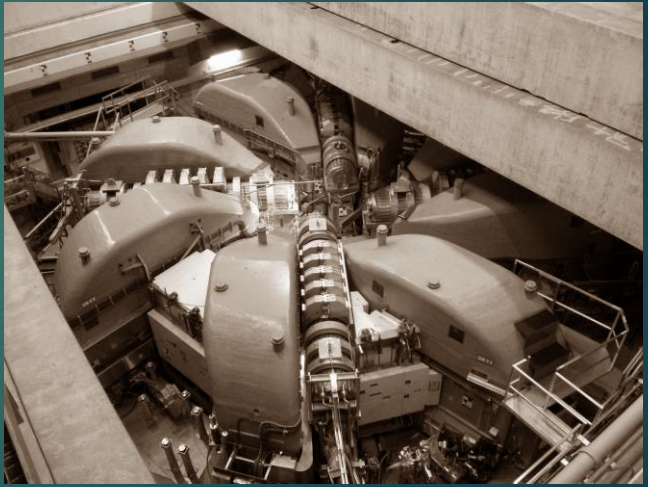
# Development of a Next-generation High-intensity Muon Beam at the Paul Scherrer Institut



FELIX ANTON BERG ON BEHALF OF THE HIMB GROUP

19.4.2018

MUON PRODUCTION AND BEAM INTERCEPTORS

ISTITUTO NAZIONALE DI FISICA NUCLEARE

IPA ETH zürich PAUL SCHERRER INSTITUT PSI

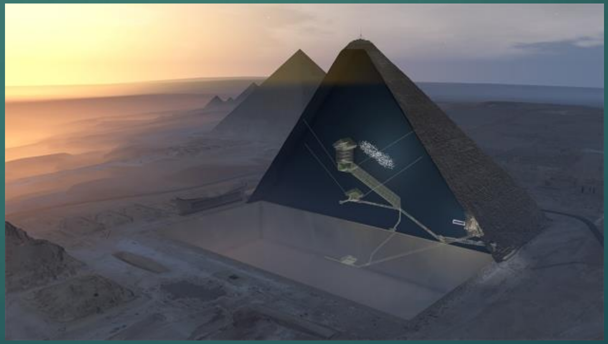
# Outline



- Overview on experimental demands on muon beam properties
- Conventional muon beam generation
- The High-intensity Muon Beam project at the Paul Scherrer institut
  - Initial Design
  - Modification of target station
  - High acceptance beam transport system

# Muons – versatile tools in different fields

- High energy muon collider
- Neutrino factories
- MuSR/ Solid State Physics / Surface Analysis
- Low energy particle physics:
    - Intensity / Precision frontier
- Tomography

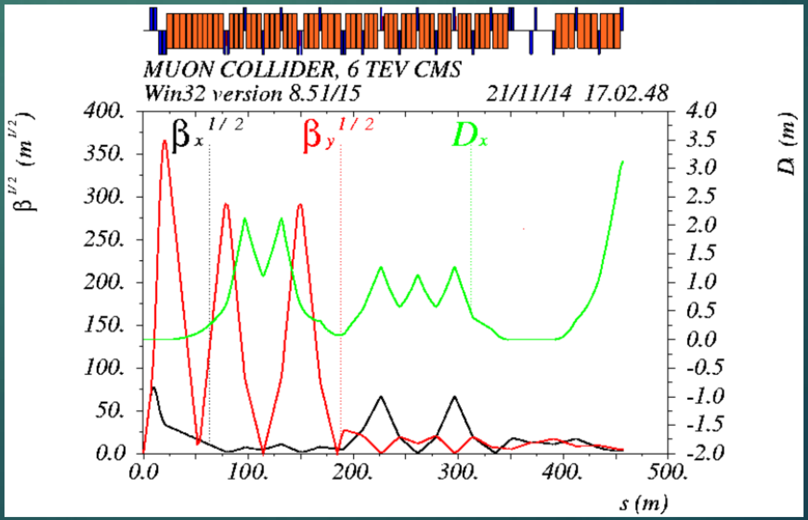


Discovery of a big void in Khufu's Pyramid by observation of cosmic-ray muons
DOI: 10.1038/nature24647

# Experimental demands - Muon Collider studies

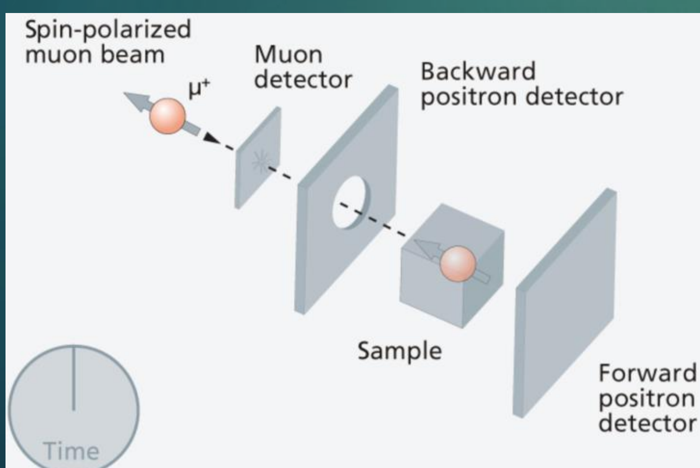
- $\mu^+$ & $\mu^-$
- Especially low emittance
- High muon flux
- High laboratory frame lifetime
- Pulsed structure

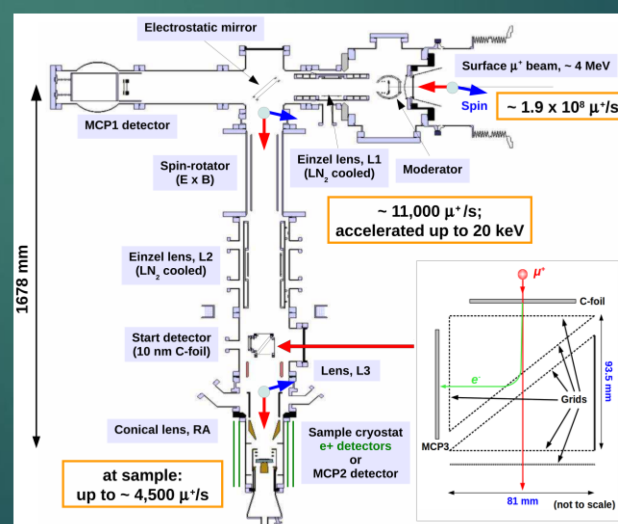
Design of a 6 TeV Muon Collider
DOI: 10.1088/1748-0221/11/09/P09003



# Experimental demands - Solid state Physiscs / µSR

- keV - MeV $\mu^+$
- Low emittance
- Low-High muon flux
- High Polarization with tunable orientation
- Pulsed and CW



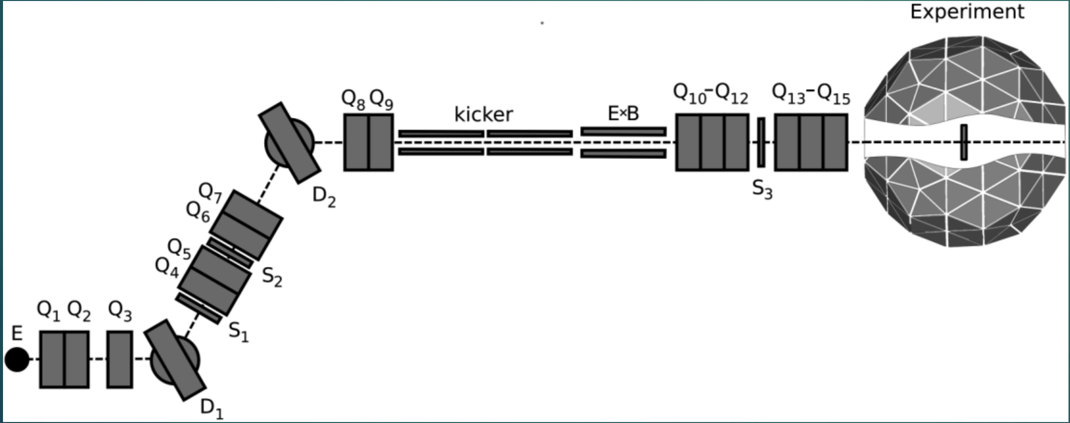

Geant4 simulation of the PSI LEM beam line: energy loss and muonium formation in thin foils and the impact of unmoderated muons on the µSR Spectrometer

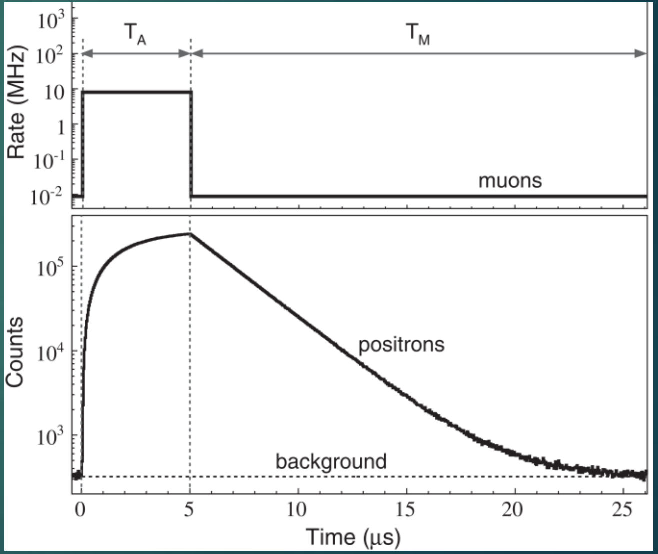
DOI: [10.1088/1748-0221/10/10/P10025](10.1088/1748-0221/10/10/P10025)

# Experimental demands - Muon Lifetime



- Intermediate surface $\mu^+$ rates
- Pulsed structure (μs)

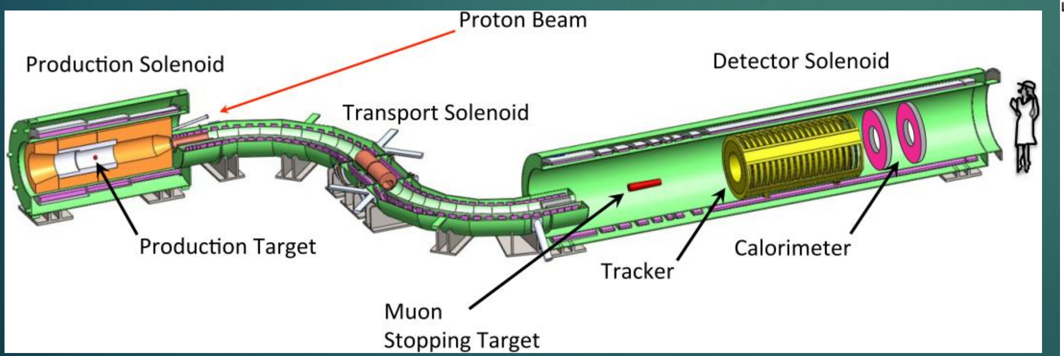


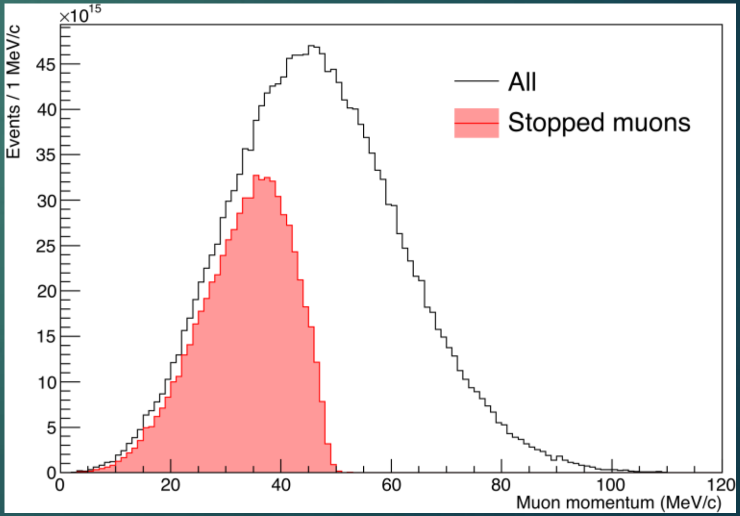




Detailed report of the MuLan measurement of the positive muon lifetime and determination of the Fermi constant
DOI: 10.1103/PhysRevD.87.052003

# Experimental demands - Mu2e conversion

- $\mu^-$ < 50 MeV/c
- High muon flux
- Pulsed structure → Backgroud reduction



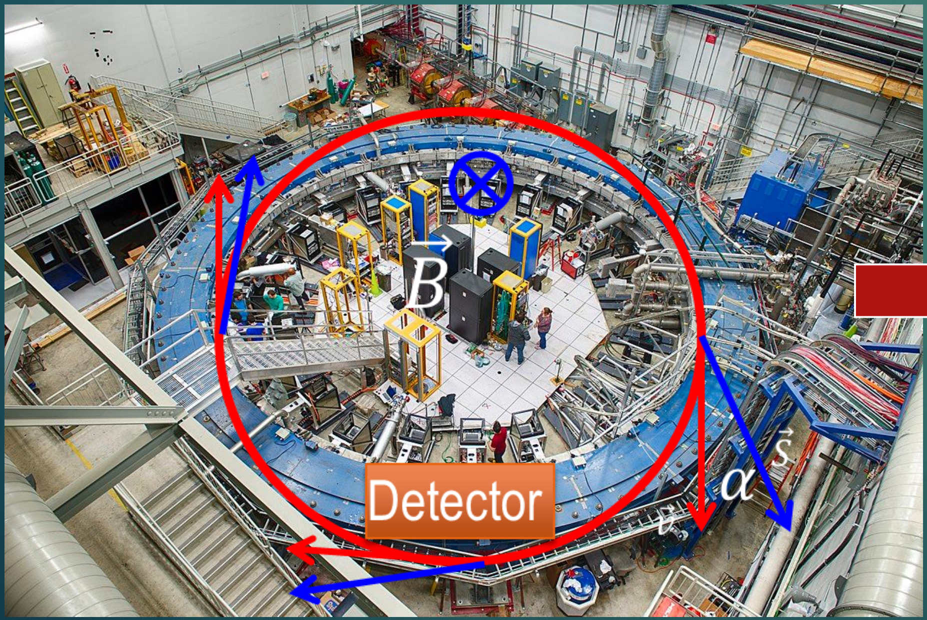

Mu2e Technical Design Report
arXiv:1501.05241

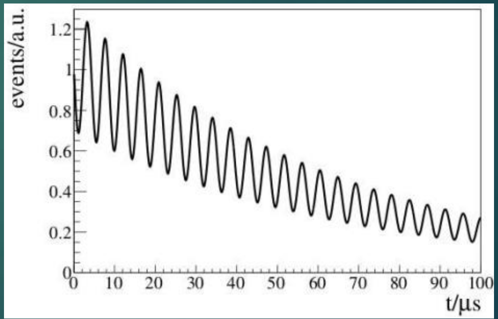
# Experimental demands - E989 g-2



- Magic momentum 3.09 GeV/c
- High polarization

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

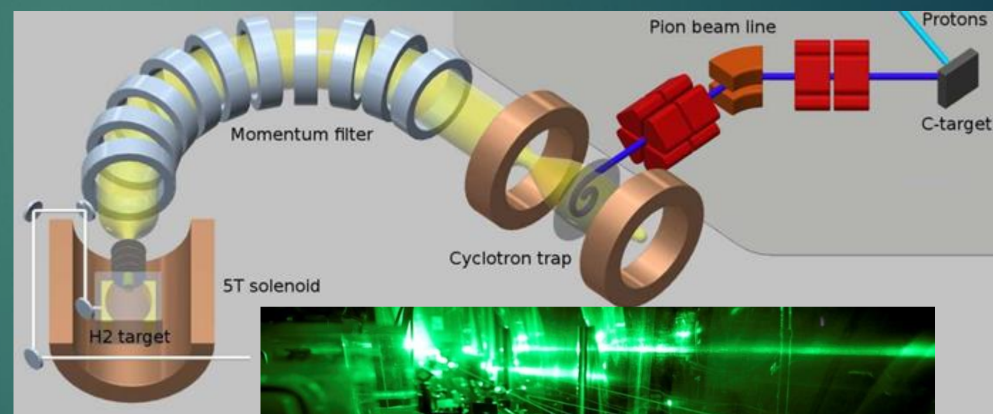


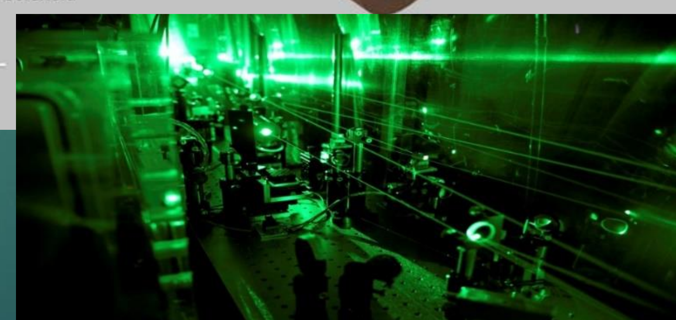




The Muon g – 2 experiment at Fermilab
DOI: 10.1016/j.nuclphysbps.2017.03.046

# Experimental demands - Muonic atoms

- keV – MeV
- $\mu^-$
- High muon flux

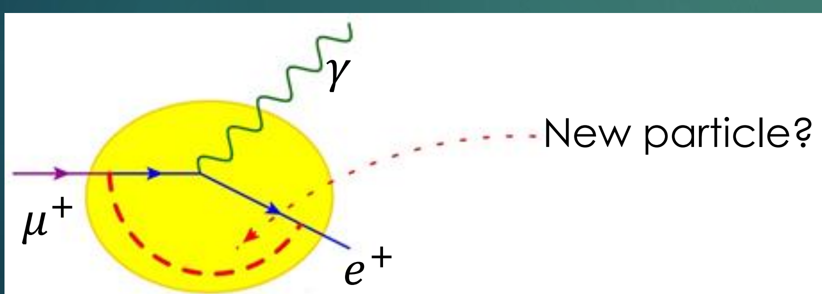



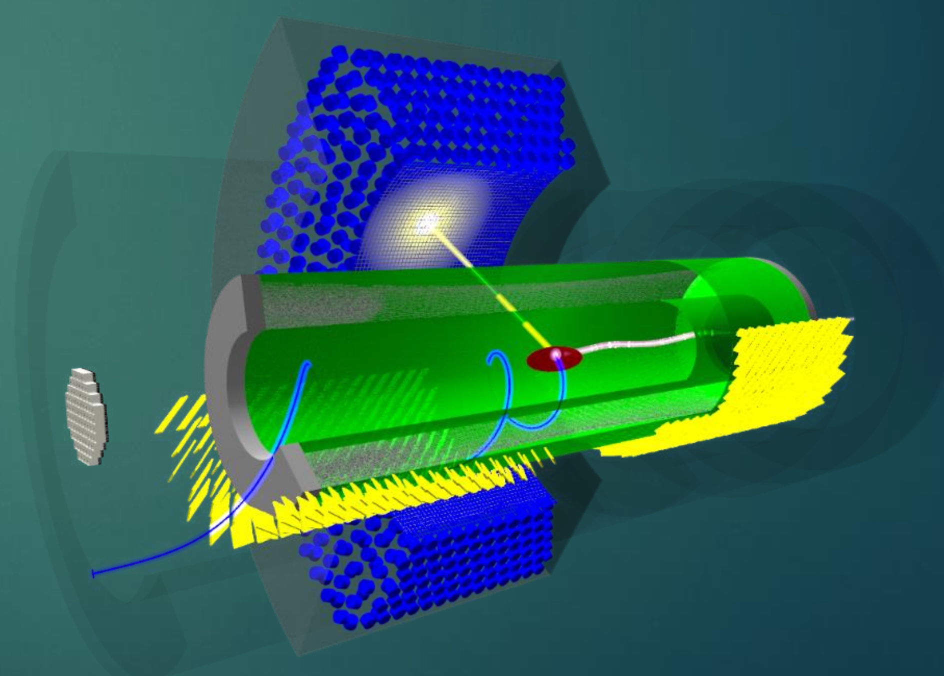



The size of the proton
DOI: 10.1038/nature09250

# Experimental demands - CLFV MEG II

- Surface muon 28 MeV/c $\mu^+$
- High muon flux
- CW beam



Goal: BR(μ+ → e+ γ) < 6·10-14 @ 90% CL

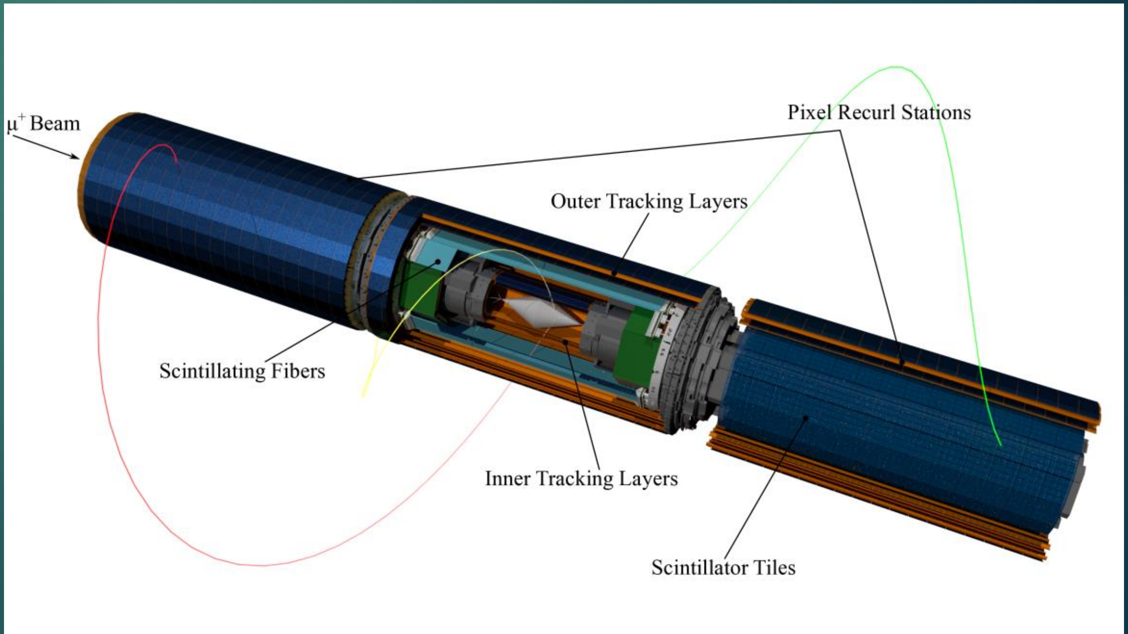
The design of the MEG II experiment
arXiv:1801.04688

# Experimental demands - CLFV Mu3e

- Surface muon 28 MeV/c $\mu^+$
- High muon flux
- CW beam

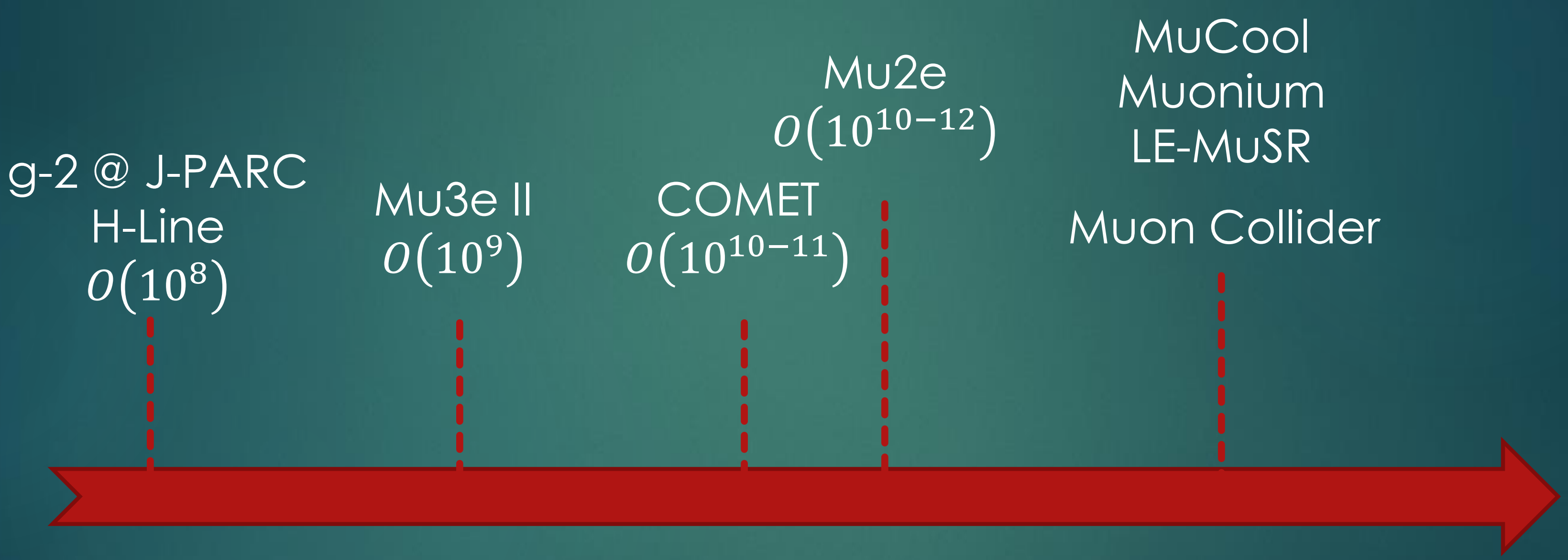
Final Sensitivity $\mathcal{O}(10^{-15} \rightarrow 10^{-16})$

Research Proposal for an Experiment to Search for the Decay
arXiv:1301.6113



# Future intensity demands





# Ideal Muon Beam



- Variety of experiments demand for variety of beam conditions
  - Pulsed vs. CW ←→ Background / pileup reduction in coincidence measurements
  - Momentum depends on application – stopped beams require low momenta

$$(\Delta R)_{tot} \cong a\left((0.09)^2 + \left(3.5\frac{\delta p}{p}\right)^2\right)^{\frac{1}{2}} \cdot p^{3.5}$$

A High Stopping Density $\mu^+$ Beam
DOI: 10.1016/0029-554X(76)90823-5

- Common demands :
  - High rate (with option of reduction)
  - Small phase space volume

# Muon beam facilities - Overview

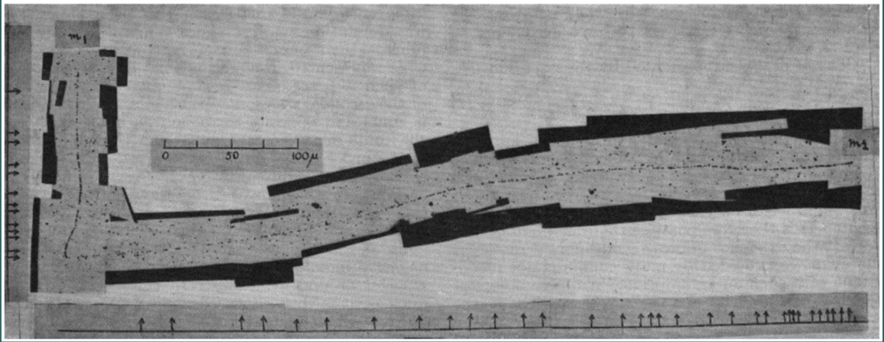


| Laboratory/Beamline | Energy/ Power | Present Muon $\mu^+/\mu^-$ Rates [Hz] | Future estimated $\mu^+/\mu^-$ Rate [Hz] |
|---|---|---|---|
| **PSI (CH)** | 590 MeV, 1.3 MW DC | | |
| LEM | | $4.2 \cdot 10^8$ $\mu^+$ | |
| πE5 | | $1.3 \cdot 10^8$ $\mu^+$ | |
| HiMB | | | $O(10^{10})$ $\mu^+$ / $O(10^8)$ $\mu^-$ |
| JPARC (JP) | 3 GeV, 1MW Pulsed<br>Reached 400kW | | $2 \cdot 10^8$ $\mu^+$ @ 1MW) |
| MUSE | | $8 \cdot 10^7$ / $4 \cdot 10^6$ | $10^7$ $\mu^-$ @ 1MW |
| COMET | 8 GeV, 56kW Pulsed | | $10^{11}$ $\mu^-$ 2019/2020 |
| FNAL (USA) | | | |
| Mu2e | 8GeV, 25kW Pulsed | | $5 \cdot 10^{10}$ $\mu^-$ 2019/2020 |
| RAON/RISP (KO) | 600 MeV, 400kW DC) | | $7 \cdot 10^8$ $\mu^+$ |
| CSNS (CN) | 1.6 GeV, 100kW Pulsed | | $10^{10}$ $\mu^+$ |
| TRIMUF (CA) | 500 MeV, 75kW, DC | | |
| M20/M9B | | $2 \cdot 10^6$ / $1.4 \cdot 10^6$ | |
| RAL ISIS (UK) | 800 MeV, 160kW, Pulsed | $1.5 \cdot 10^6$/ $7 \cdot 10^4$ | |
| RIKEN RAL | | | |
| RCNP Osaka Univ. (JP) | 400 MeV, 400W DC | | |
| MUSIC | | $10^6$/$1 \cdot 10^5$ | $4.2 \cdot 10^8$/$4.2 \cdot 10^7$ |

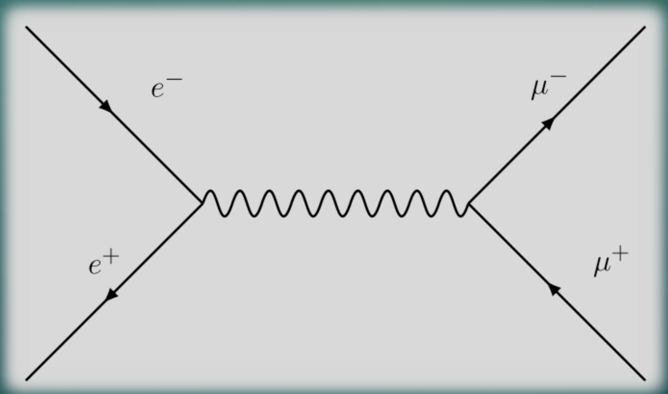
# Muon beam generation



- Proton nucleon interaction : Mainly from pion decays



- Direct production : $e^{+} + e^{-} \rightarrow \mu^{+} + \mu^{-}$



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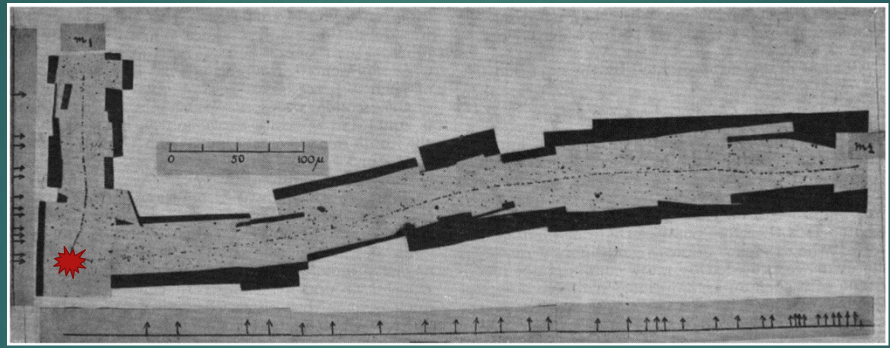


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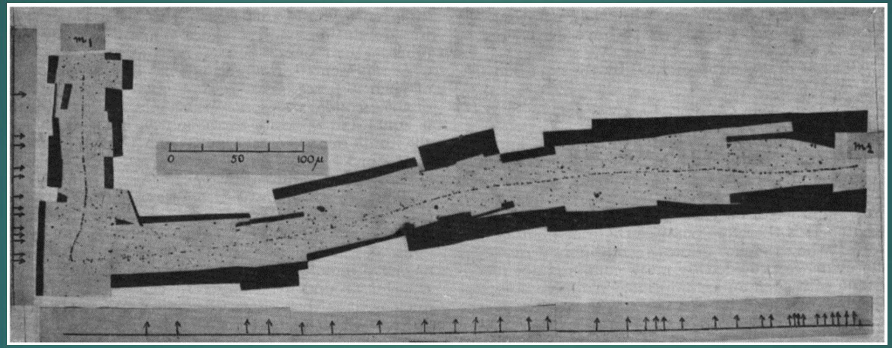
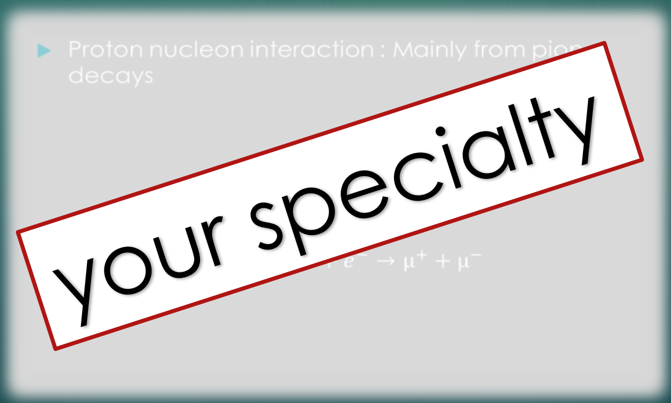
# Muon beam generation



- Proton nucleon interaction : Mainly from pion decays



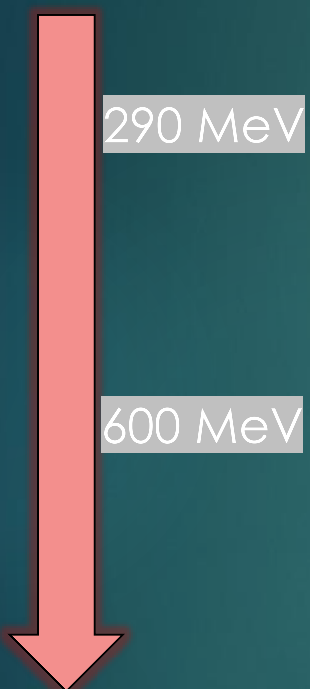

- Direct production : $e^+ + e^- \to \mu^+ + \mu^-$




Studies of a Scheme for Low Emittance Muon Beam Production From Positrons on Target
DOI: 10.18429/JACoW-IPAC2017-WEOBA3

# Pion production $(p^+ + nucl.)$






- Positive Pions

$$p + p \rightarrow p + n + \pi^+$$
$$p + n \rightarrow n + n + \pi^+$$
$$p + p \rightarrow d + \pi^+$$

- Negative & neutral Pions

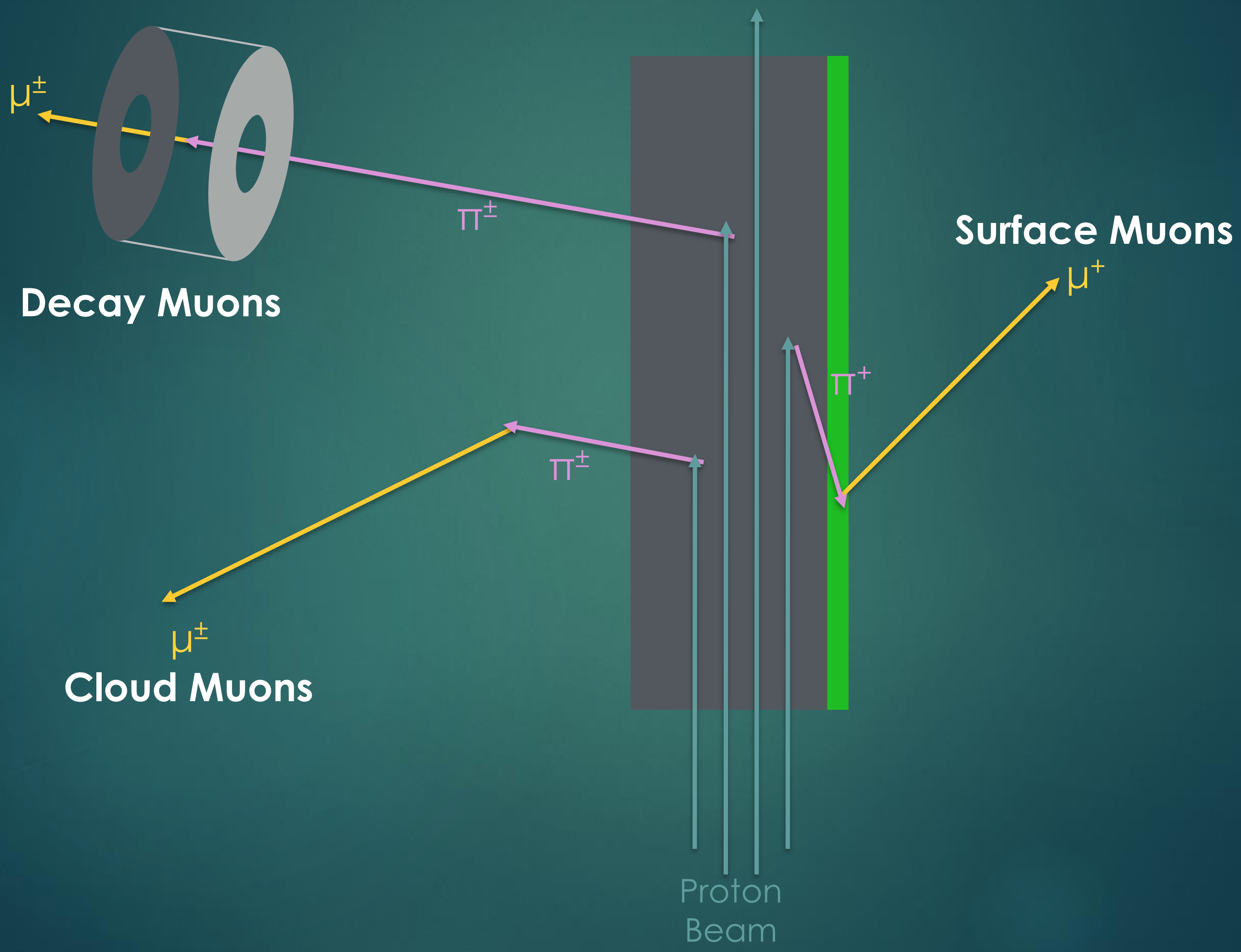
$$p + n \rightarrow p + p + \pi^-$$
$$p + n \rightarrow p + n + \pi^0$$
$$p + p \rightarrow p + p + \pi^0$$
$$p + n \rightarrow d + \pi^0$$

- Double Pion production

$$p + p \rightarrow p + p + \pi^+ + \pi^-$$
$$p + p \rightarrow p + p + \pi^0 + \pi^0$$
$$p + p \rightarrow n + n + \pi^+ + \pi^+$$
$$p + p \rightarrow n + p + \pi^+ + \pi^0$$
$$p + p \rightarrow d + \pi^+ + \pi^0$$
$$p + n \rightarrow p + p + \pi^- + \pi^0$$
$$p + n \rightarrow p + n + \pi^+ + \pi^-$$
$$p + n \rightarrow p + n + \pi^0 + \pi^0$$
$$p + n \rightarrow n + n + \pi^+ + \pi^0$$
$$p + n \rightarrow d + \pi^- + \pi^+$$
$$p + n \rightarrow d + \pi^0 + \pi^0$$

# Muon production

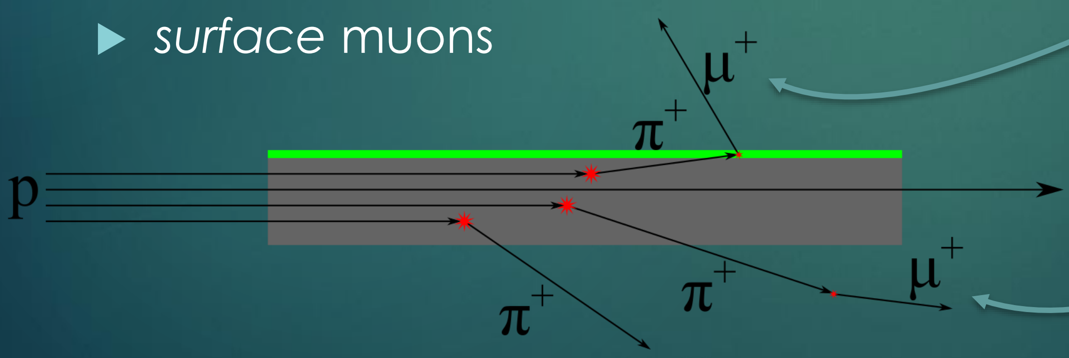


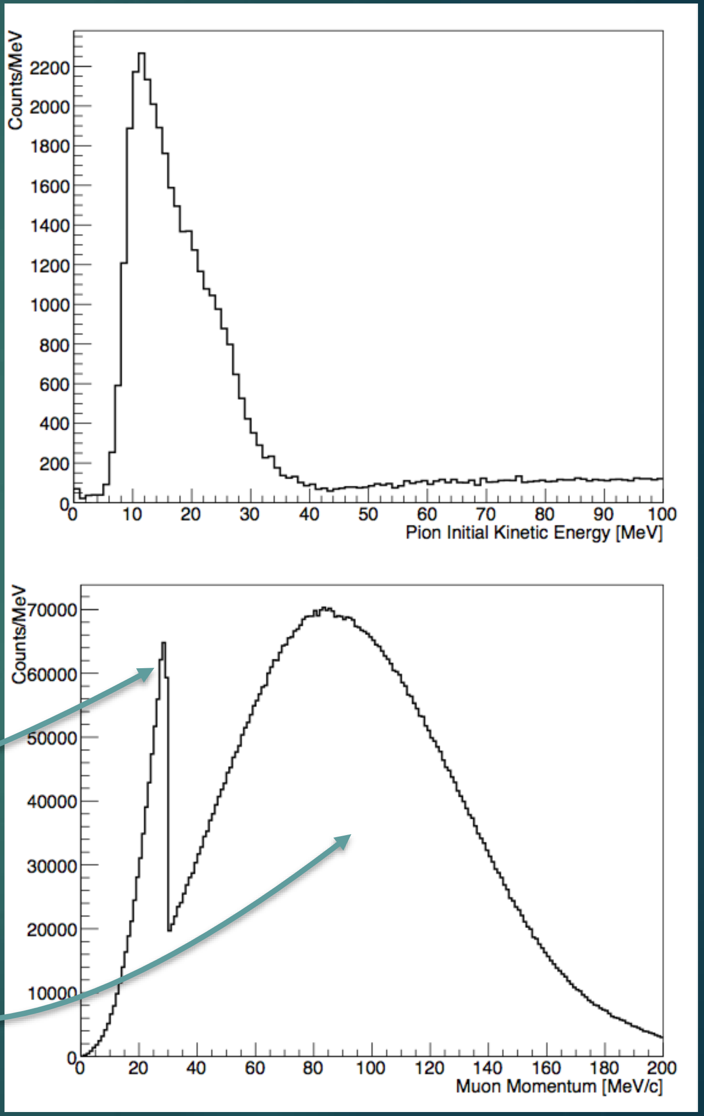


# Pion production ($p^+ + nucl.$)



- Depending on $T_\pi$ either
  - **stopped inside** the target or
  - **leave** the **target** volume completely
- $\pi^+$ **decay** in region **around target**
  - *cloud* muons
- $\pi^+$ **captured** in secondary **beamline**
  - *decay* muons
- $\pi^+$ **stop inside** the **target** and decay
  - *surface* muons

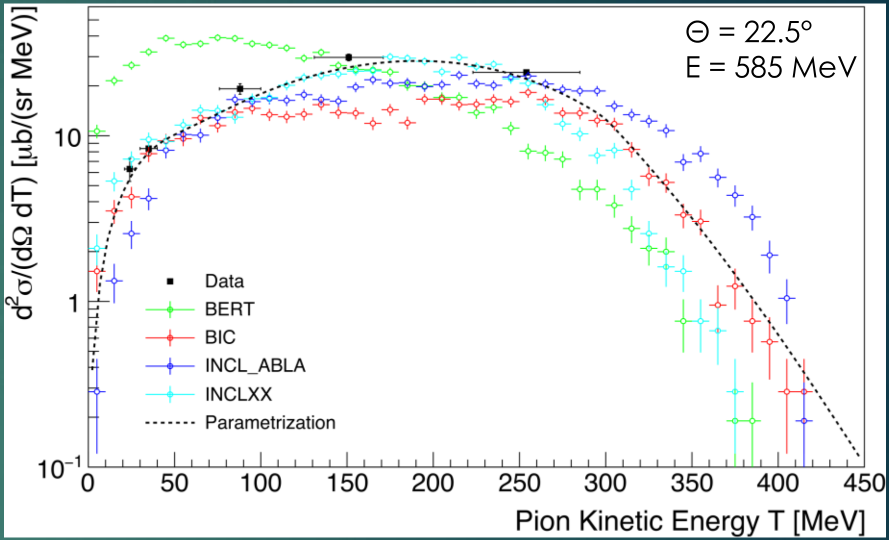


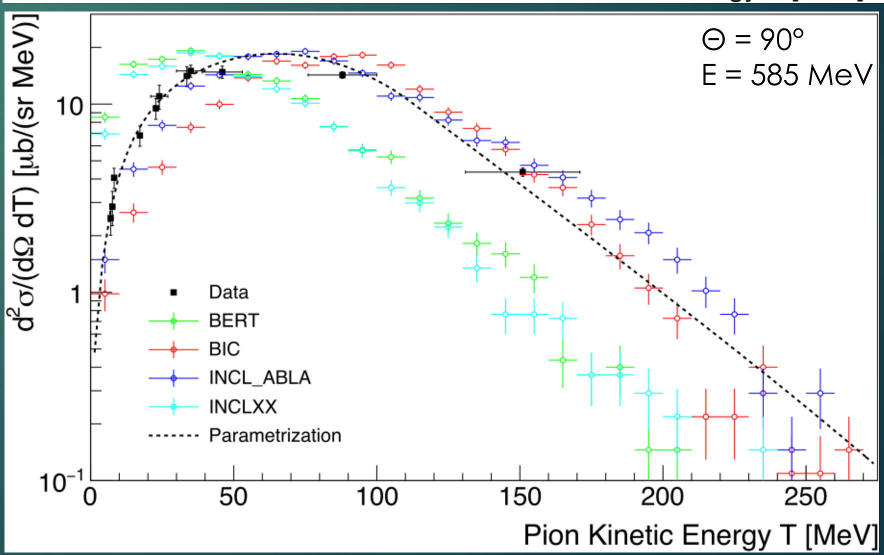



# Pion production (Geant4)



- Geant4 pion production cross sections not optimized for low energies
- HiMB-Modell :
  - Parametrization of cross sections to existing data
  - Accuracy $\mathcal{O}(10\%)$

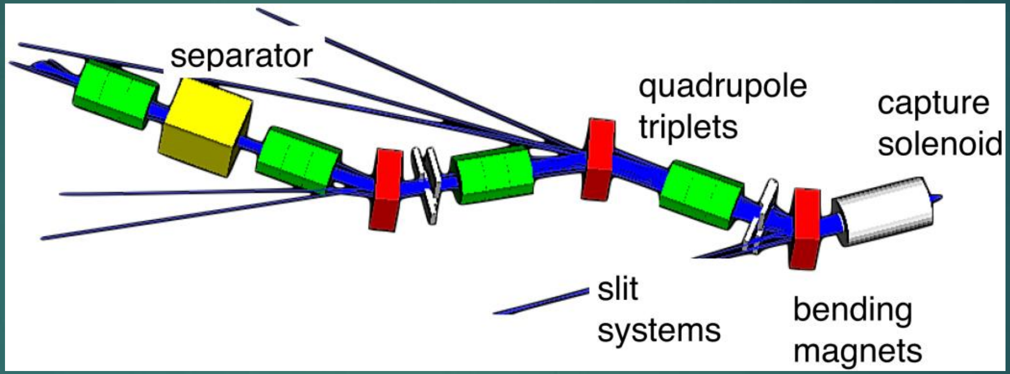


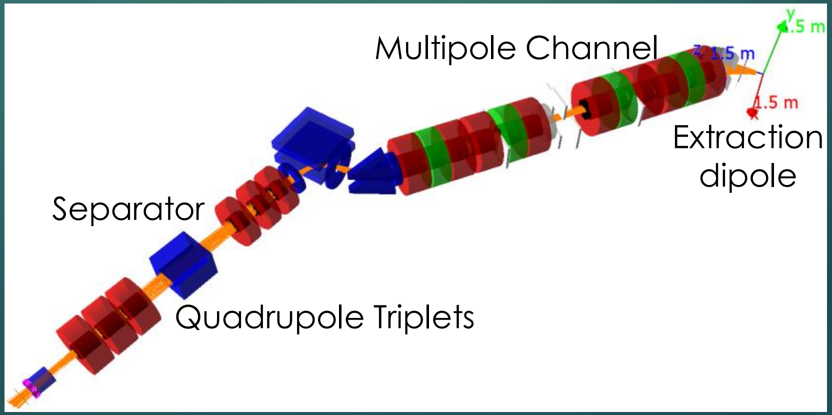

Target Studies for Surface Muon Production
DOI: 10.1103/PhysRevAccelBeams.19.024701

# HiMB Model Validation

- MuE4 surface muon beamline



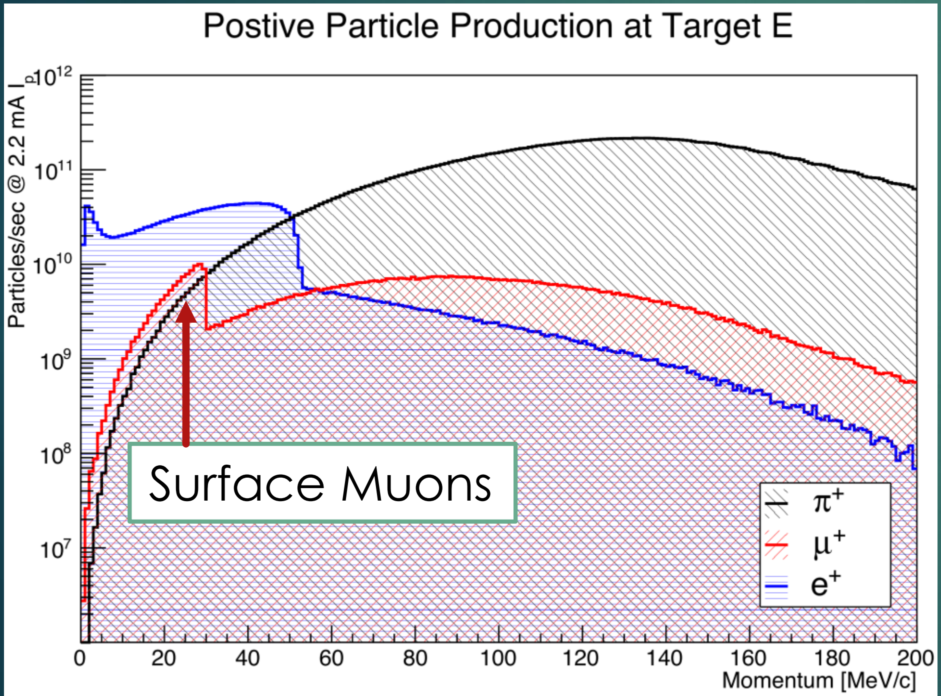

- PiE5 surface muon beamline

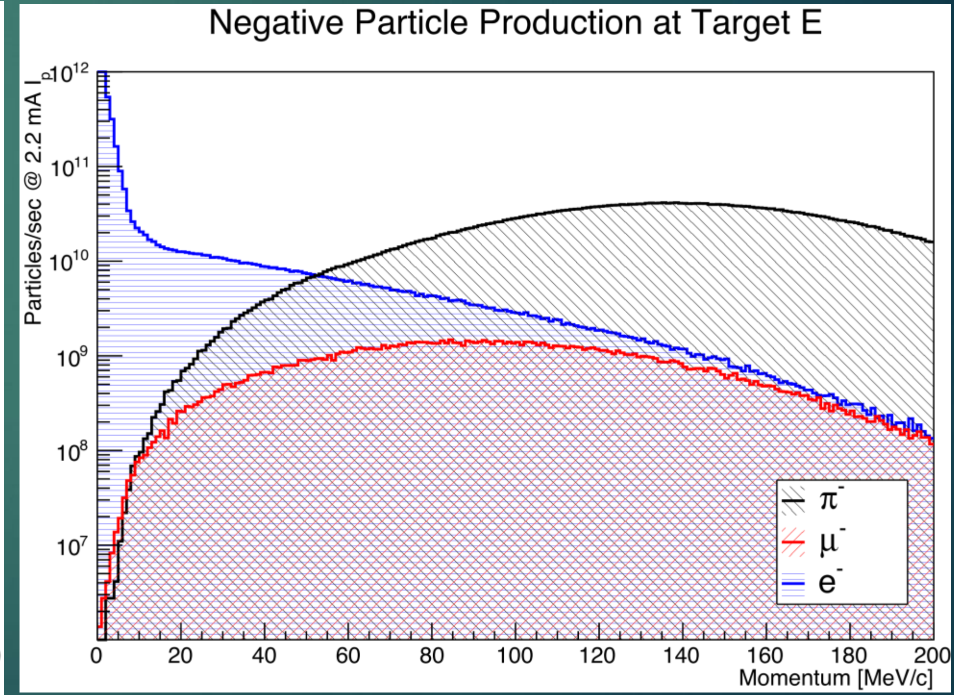


# Particle production from standard graphite target



- Beam composition seen by the PiE5 channel acceptance

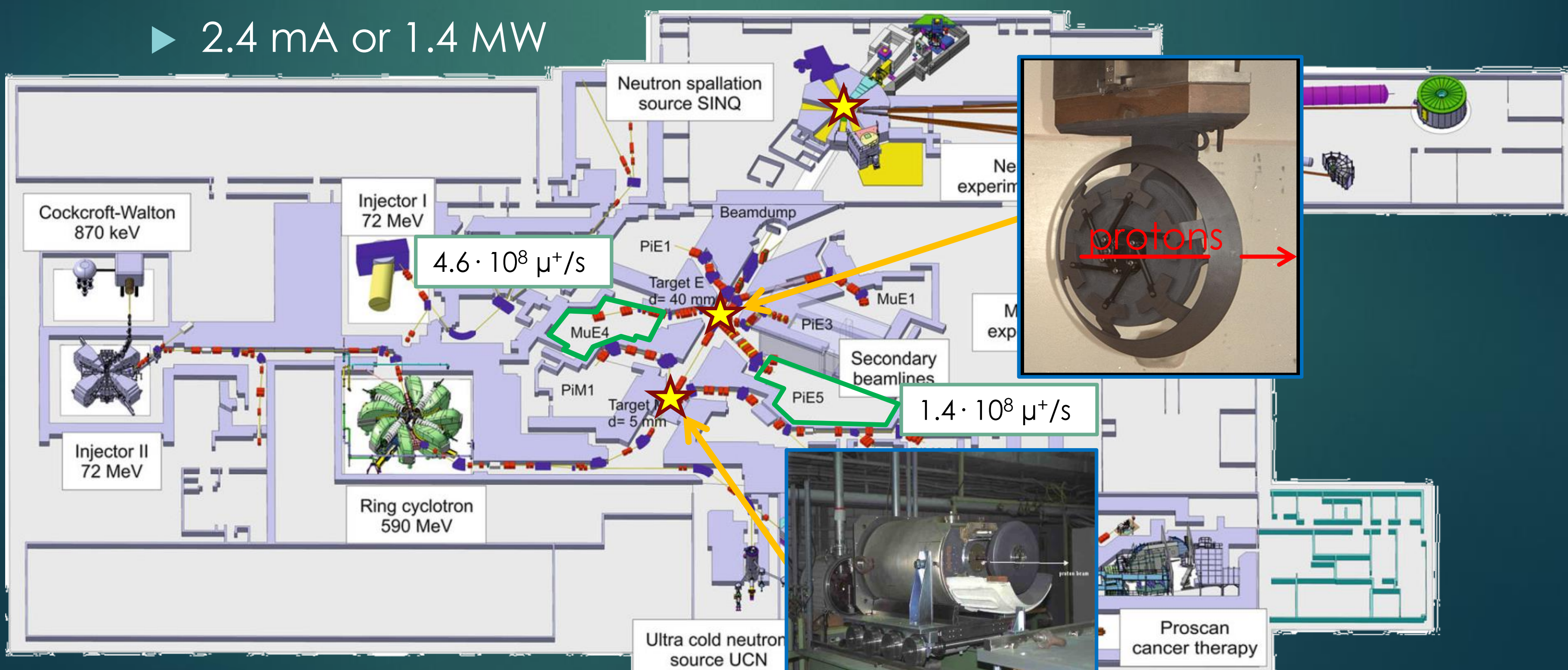



PRODUCTION, CHARACTERIZATION, AND MONITORING OF SURFACE MUON BEAMS FOR CHARGED LEPTON FLAVOR VIOLATION EXPERIMENTS

# PSI - the world's highest intensity surface $\mu^+$ beams

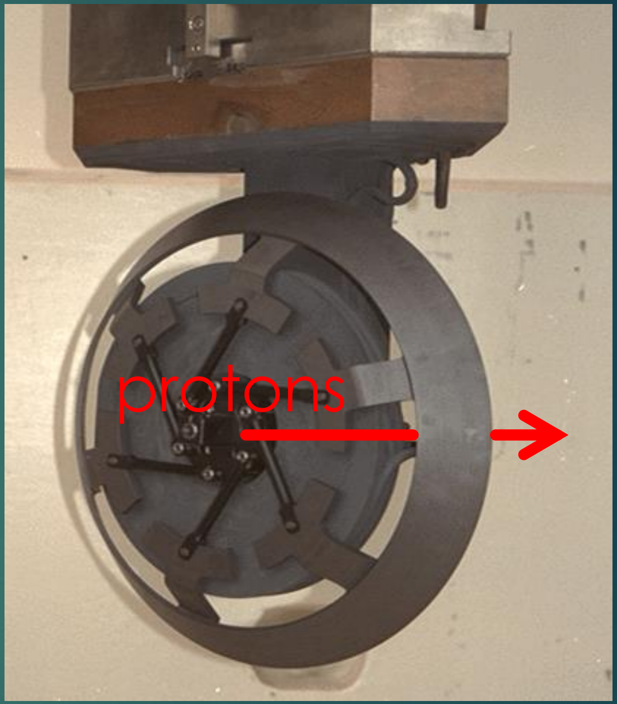
- Proton beam :
  - 590 MeV
  - 50 MHz / 20 ns → CW surface muons
  - 2.4 mA or 1.4 MW
- 2 Production Targets
- SINQ neutron source



# The Muon production target E



- Rotating target (1 Hz)
- Polycrystalline graphite
- 40 mm length in beam direction
- 50 kW proton beam energy deposit
- 1700 K radiation cooled
- 30 % loss of protons
- Delivers world most intense surface muon beams



# The High-intensity Muon Beam (HiMB) project

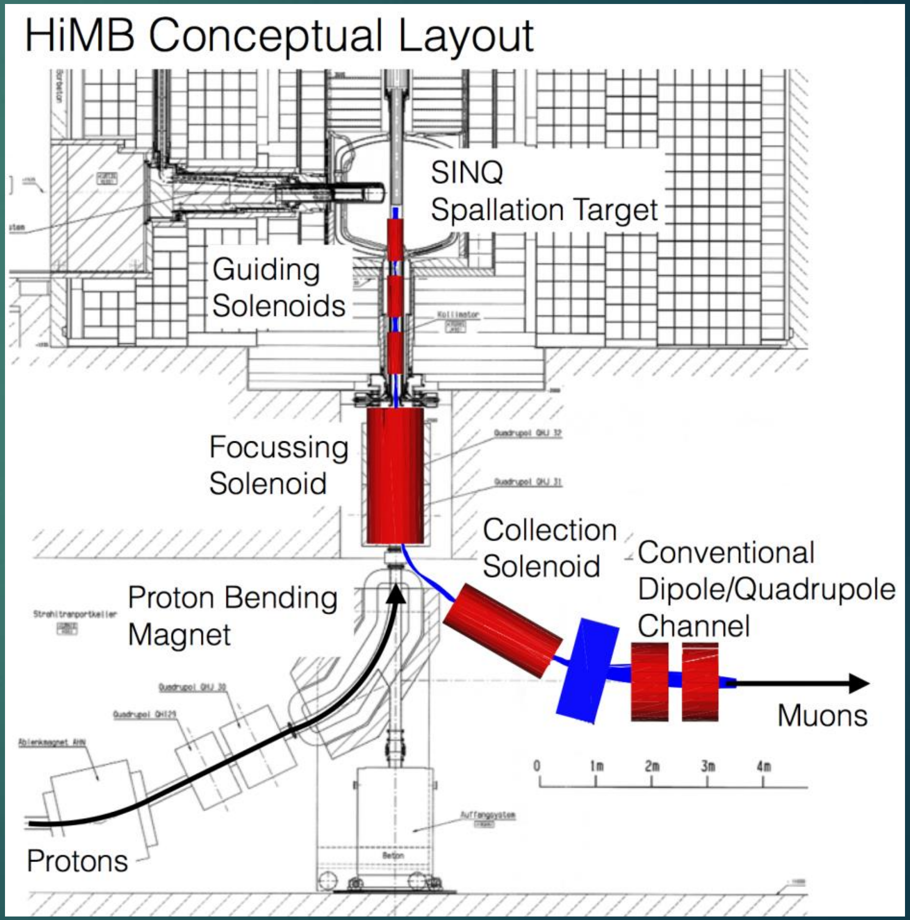


- Next-generation experiments demand for several orders of magnitude higher surface $\mu^+$ flux
- Two-pronged approach :
  - High-yield surface muon production target
  - High capture efficiency & large-phasespace acceptance transport channel

# HiMB @ SINQ



- Residual proton beam (~1 MW) dumped on SINQ
- Replace existing quadrupoles with solenoids:
  - Preserve proton beam footprint
  - Capture backward travelling surface muons
- Extract muons in Dipole fringe field
- $9 \cdot 10^{10}$ surface $\mu^+$/s @1.7 mA proton beam current

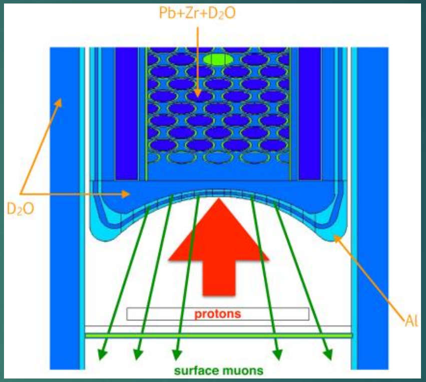


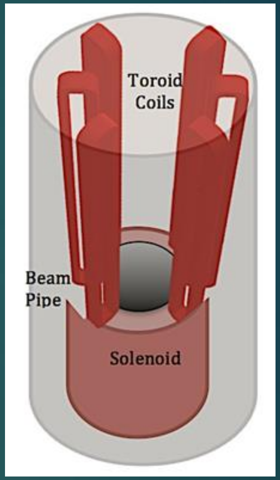
# HiMB @ SINQ

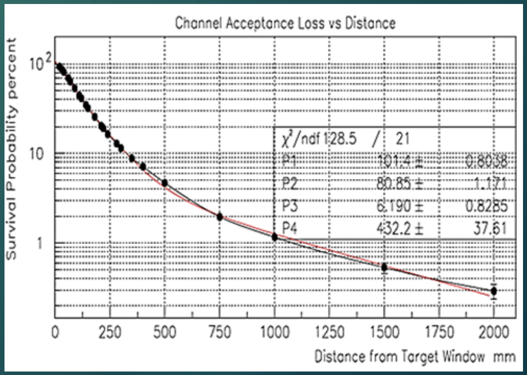


- Backward travelling pions stopped in beam window
- Capturing turned out to be difficult :
  - Large phasespace (divergence & 'source' extent)
  - Capture solenoid aperture needed to be increased, but constrained by moderator tank
- High radiation level close to target
- Several iterations with different capturing elements

  → Sought alternative solution
  →HiMB @ EH



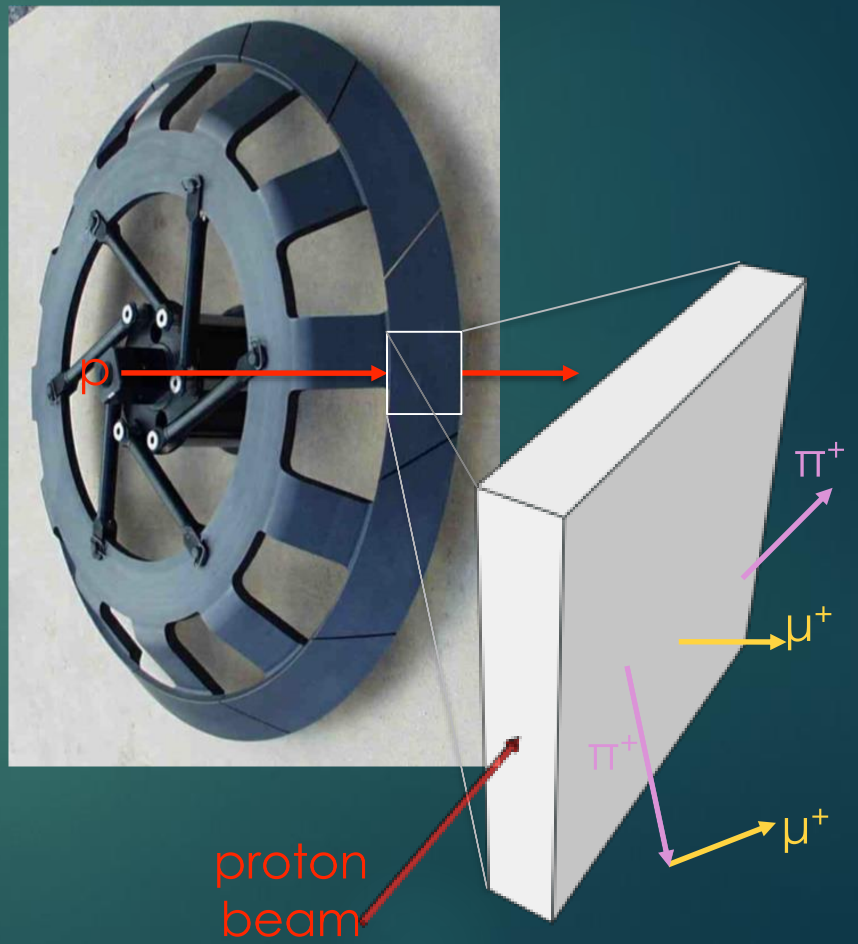



# Reconsider production target E



- Simulation with new model showed $\mathcal{O}(10^{11})$ surface $\mu^+/s$
- HiMB with ‘conventional‘ production target and high-acceptance solenoidal channel upstream SINQ

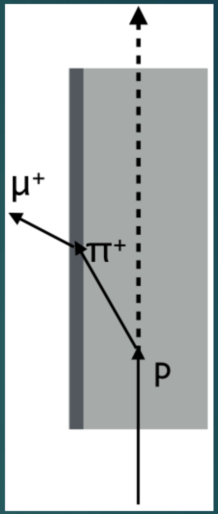


# Change of target material

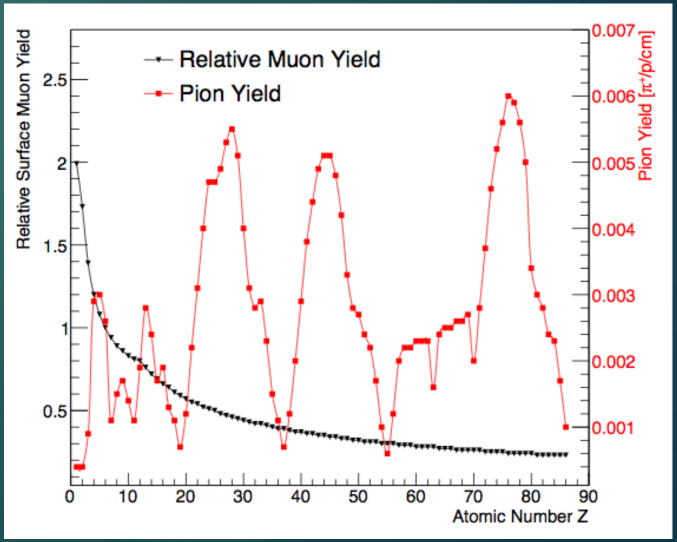


- Optimize material choice to increase surface muon rate

$$Relative\ \mu^{+}\ Yield:$$
$$\propto\ \pi^{+}\ Stop\ Density \times \mu^{+}\ Range \times Relative\ Target\ Length$$
$$\propto n \cdot \sigma_{\pi^{+}} \cdot \left(\frac{dE}{dx}\right)_{\pi^{+}} \times \left(\frac{dE}{dx}\right)_{\mu^{+}}^{-1} \times \frac{\rho_{C}(Z/A)_{C}}{\rho_{target}(Z/A)_{target}}$$




- Light elements favoured
- Material must withstand harsh environment
- Choice of $B_4C$ or $Be_2C$ shows 10 / 14 % gain while preserving density for proton beam

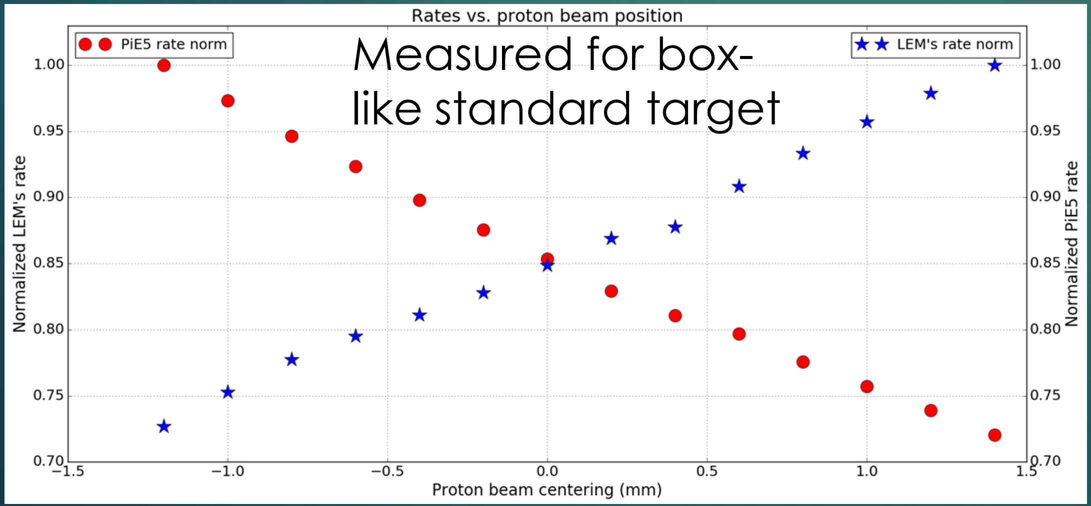


# Production target geometry

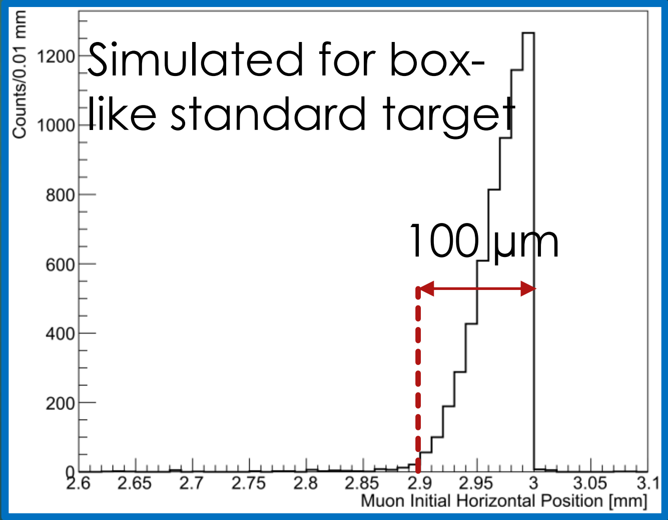


- Thinning down target width effectively increases the overlap of proton beam and surface muon production layer
- Maintain material budget seen by proton beam

Proton beam centring vs. rates in opposing beamlines



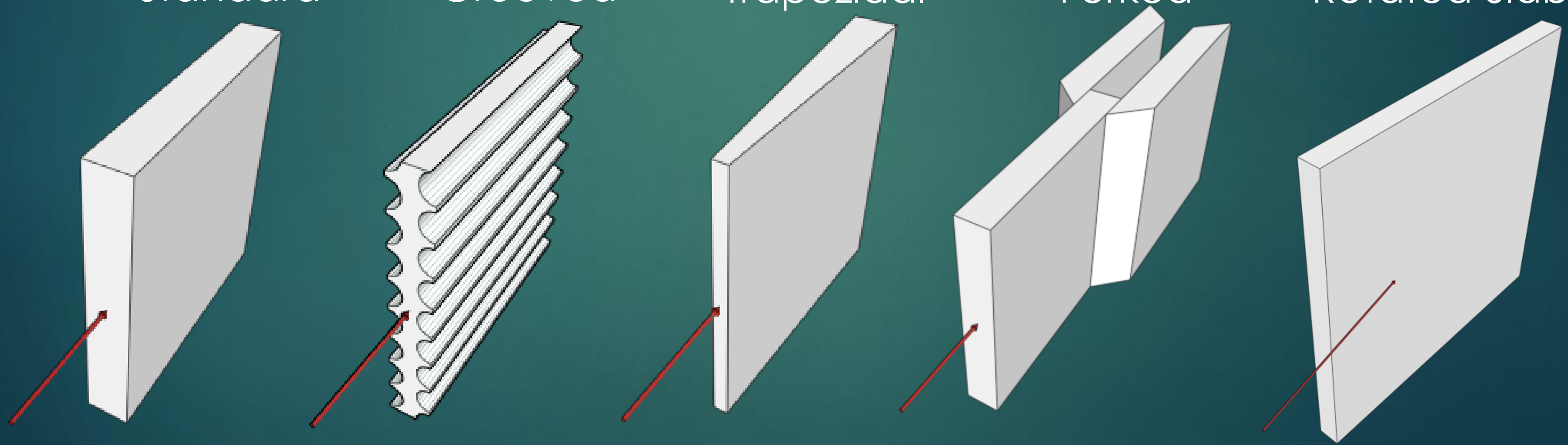

Transverse surface muon production depth



# Production target geometry

- Comparison studies of different target geometries
- Enhancements normalized to standard target

Standard | Grooved | Trapezidal | Forked | Rotated Slab

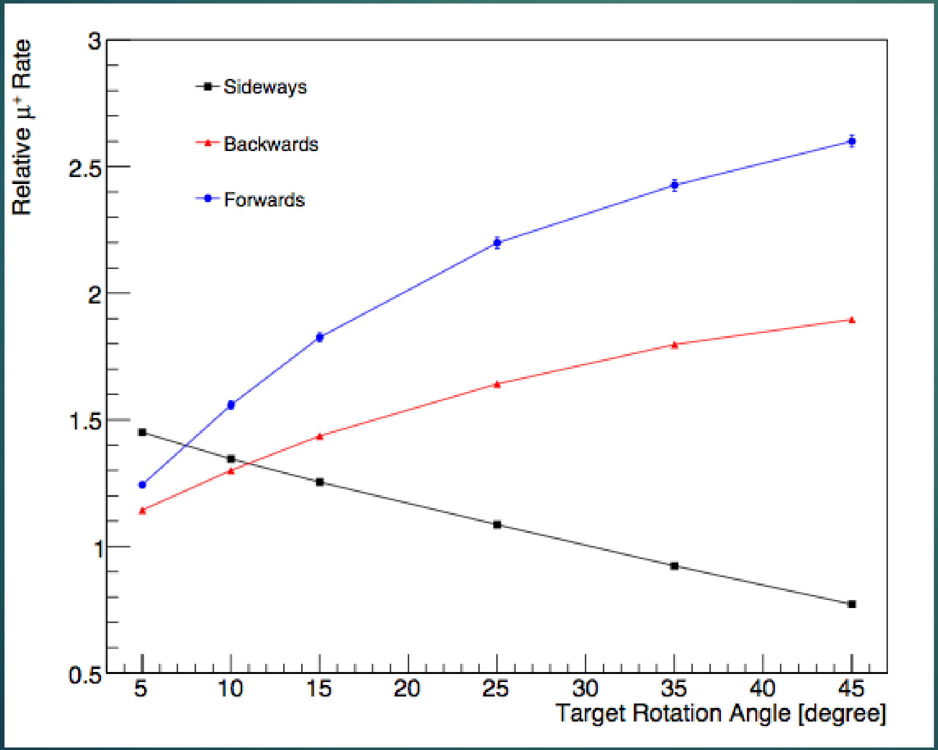


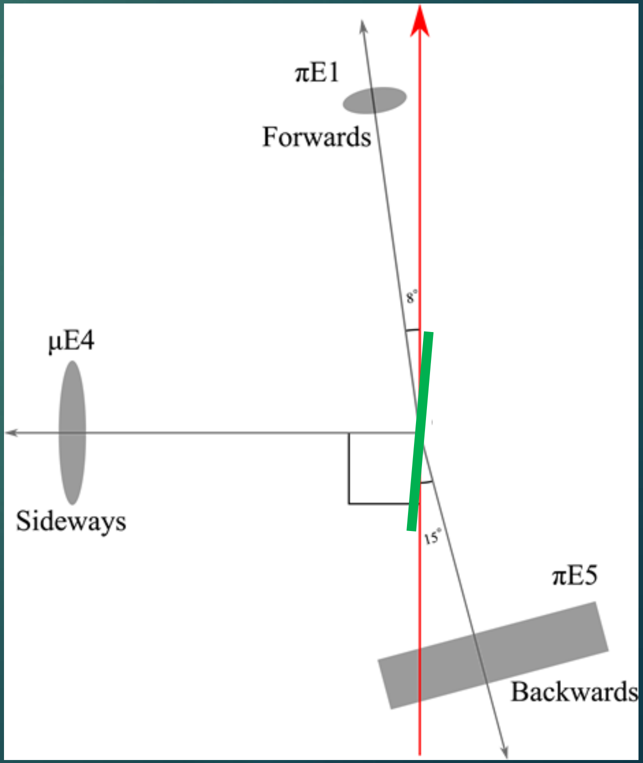
x 1.0 | x 1.1 | x 1.4 | x 1.5

# Rotated slab target



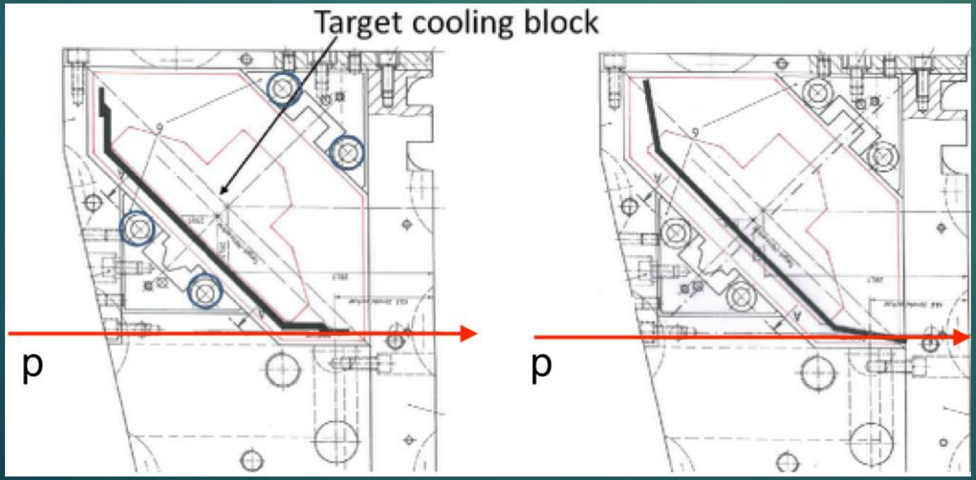
- Thin down target width without comprimising proton beam footprint →
- Muon yield depends on rotation angle

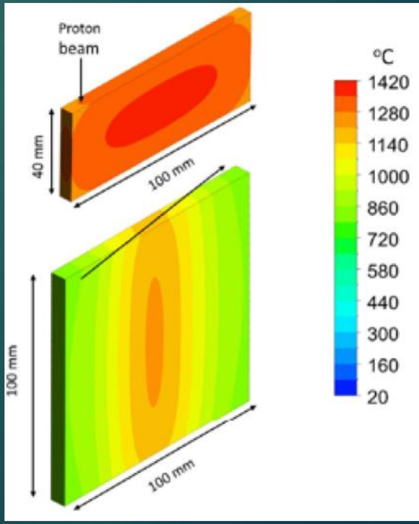


# Rotated slab target - Prototype test

- Upgrade existing graphite production target E 40 mm
  - 8° slanting angle → Measurement in forward / **backward** / sideways direction
  - Production and implementation feasible
  - Thermal simulation ongoing

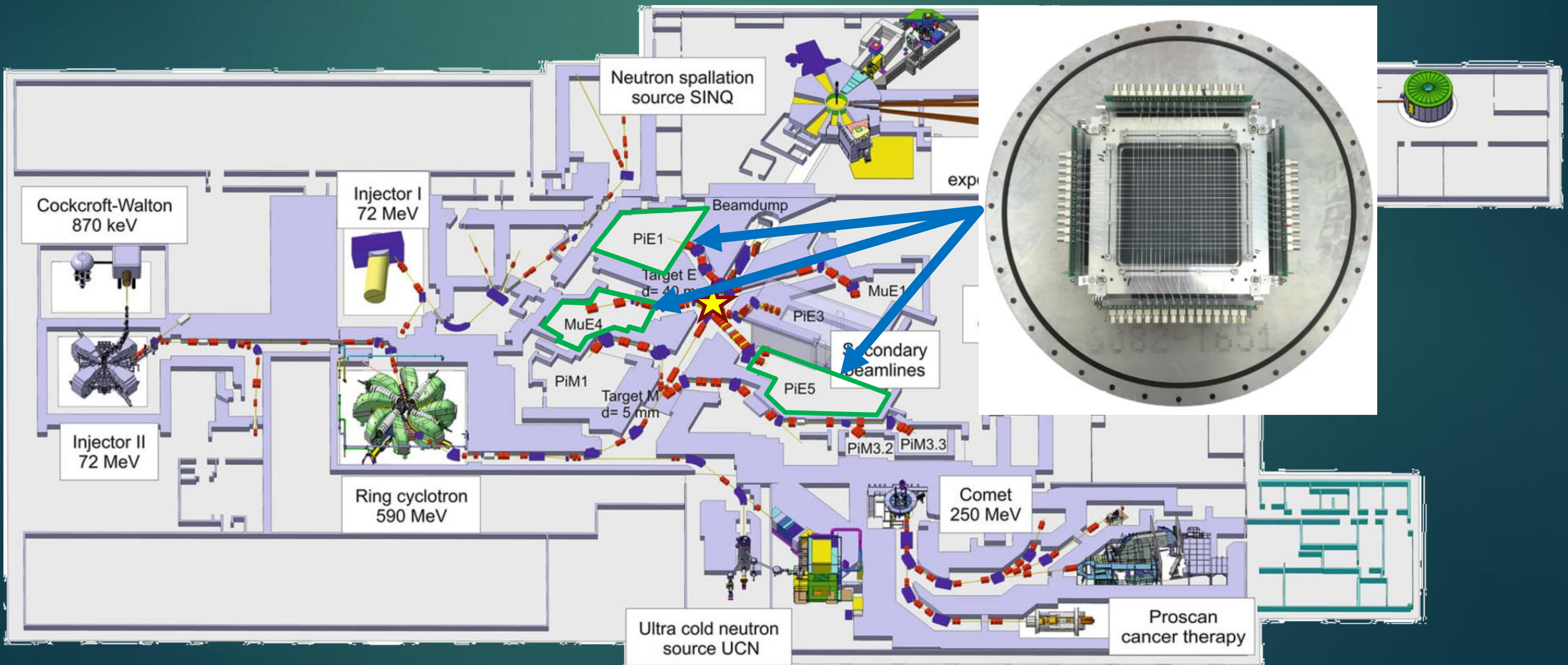


# Rotated slab target - Prototype test



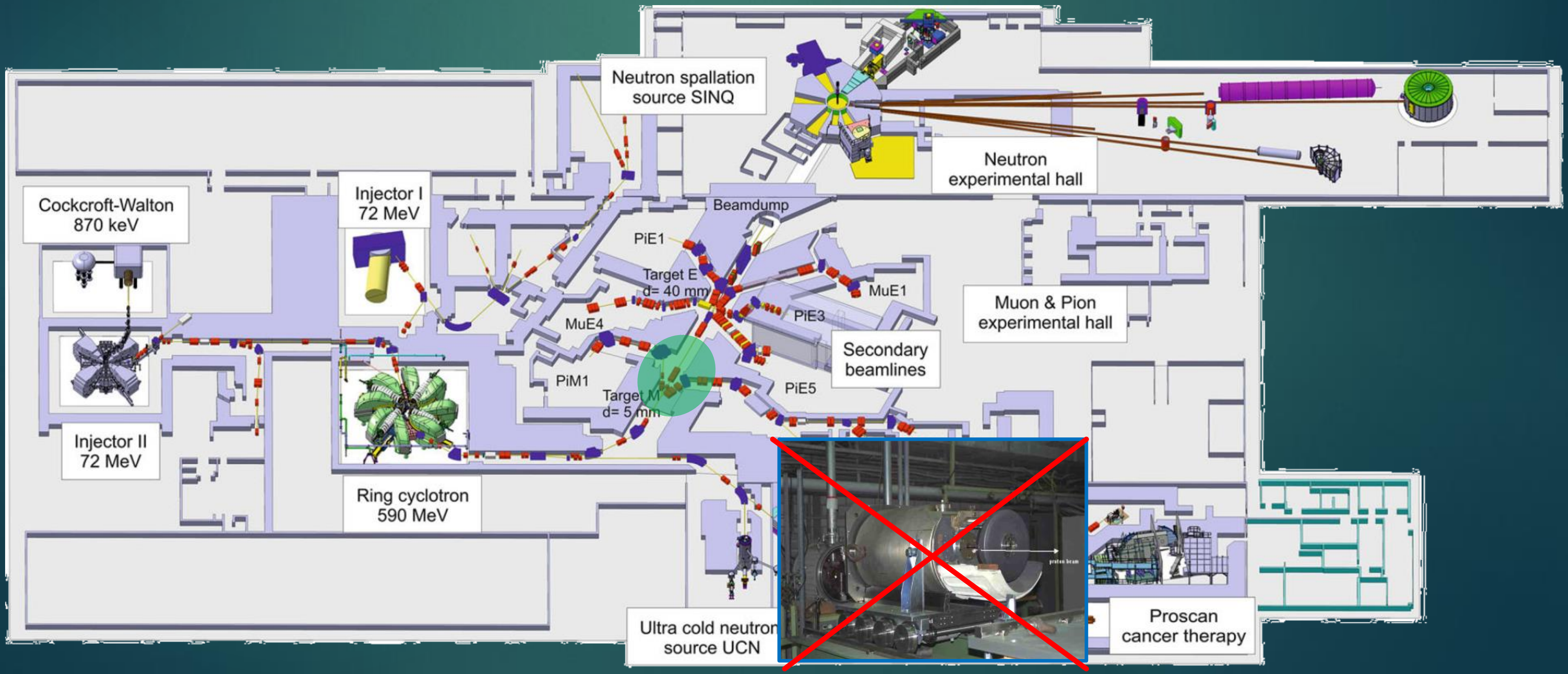
- Expect 30-50 % enhancement
- Measurement foreseen in three directions in 2019



# HiMB Project @ PSI



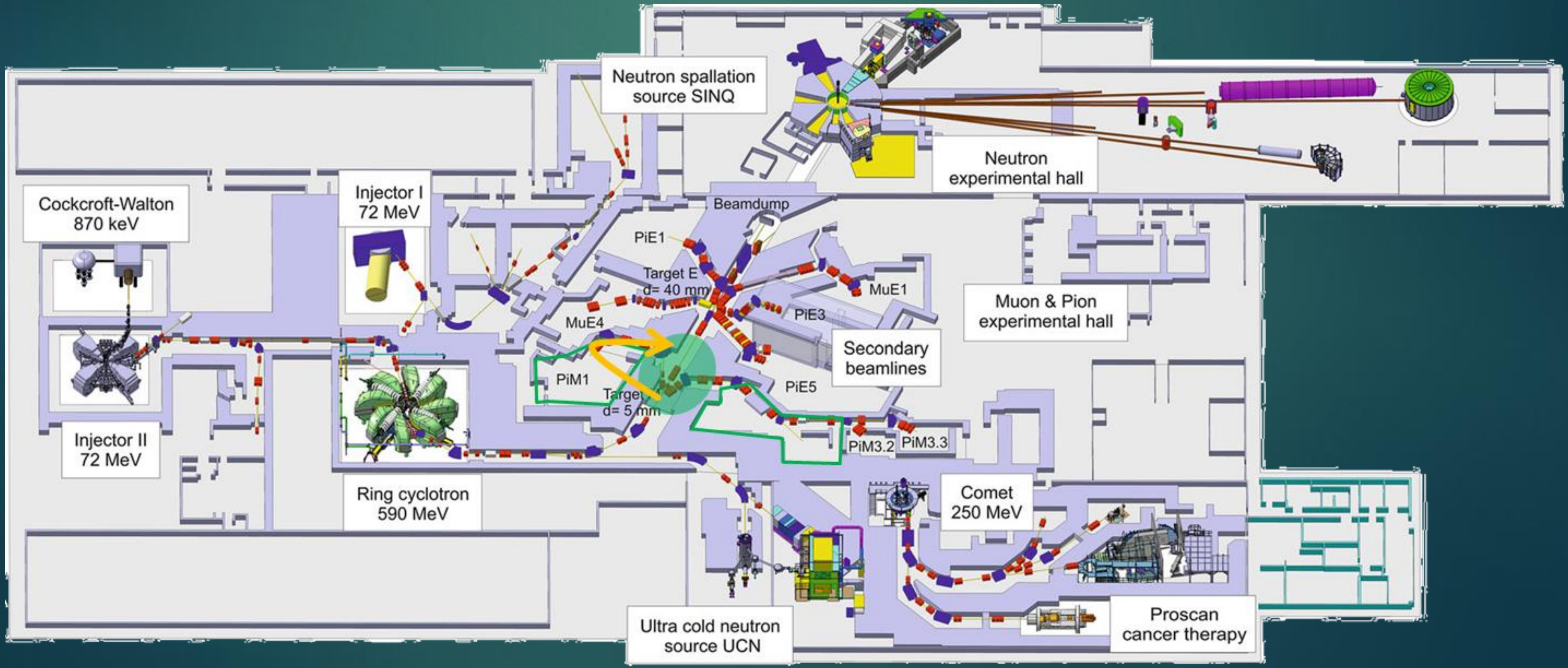
- Take out existing thin target M



# HiMB Project @ PSI



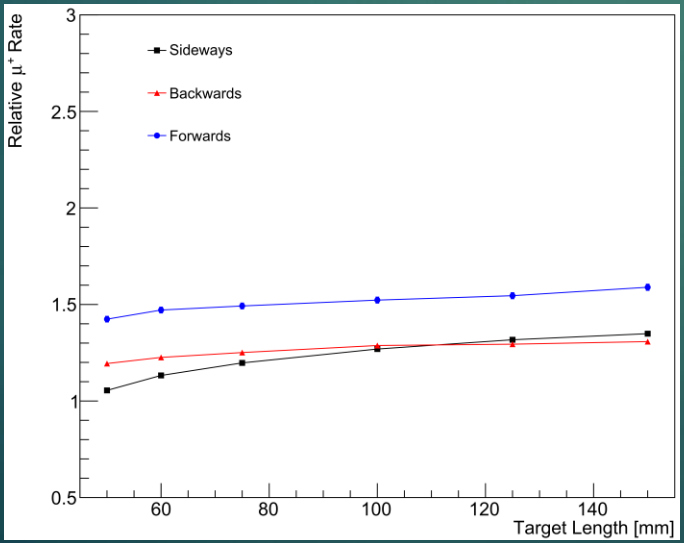
- New target station downstream present TgM location
- ~90° extraction to existing experimental areas
- Large phase space acceptance solenoidal channel

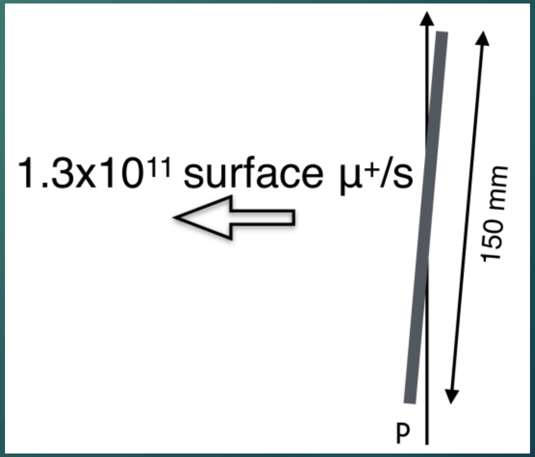


# HiMB project @ PSI - New target design



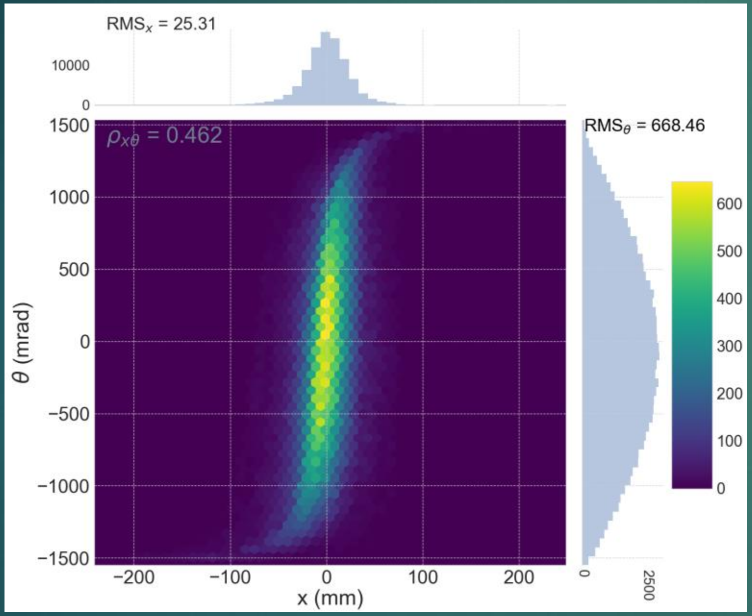
- Rotated slab target under 5°
- 20 mm effective length for proton beam
- 150 mm long
- ~90° extraction (~ +50 %)
- Probably alternate material (~ +14 %)

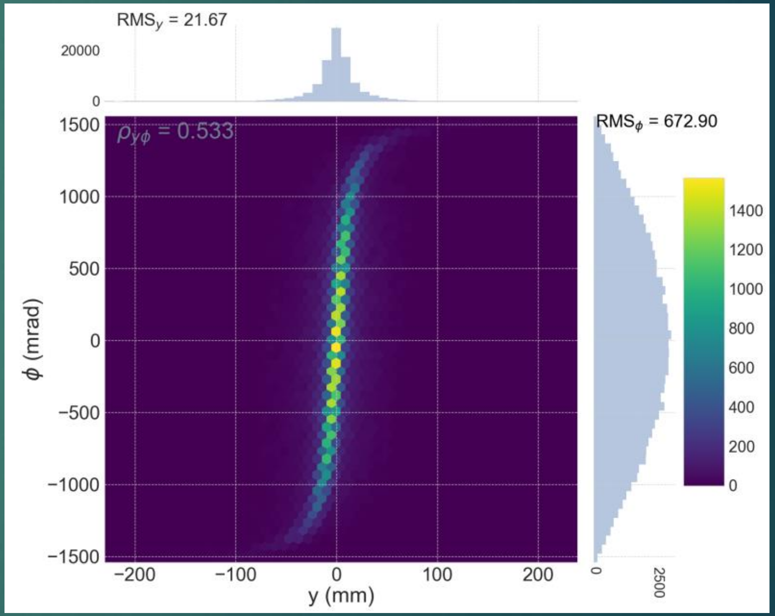


# HiMB rotated slab target - transverse phase space

- Phase space @ generation (10 mm ⊥ protons) :

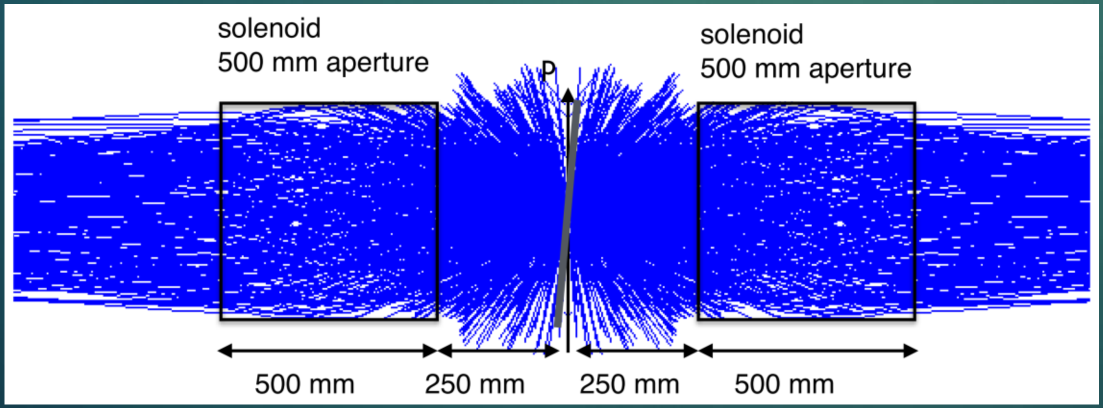


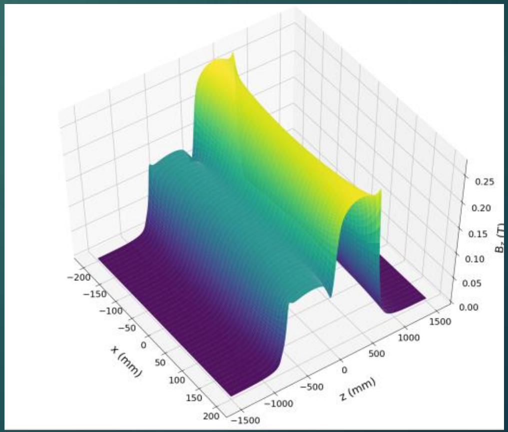




- $\varepsilon_x = 15000\,\pi\, mm\, mrad$
- $\varepsilon_y = 12000\,\pi\, mm\, mrad$
- $x' = 80\, mrad\ /\ 4.6°$

# Baseline Design for the Capture Solenoid

- Large aperture capture solenoid
- Close to target
- Radiation hardness
- Symmetric layout

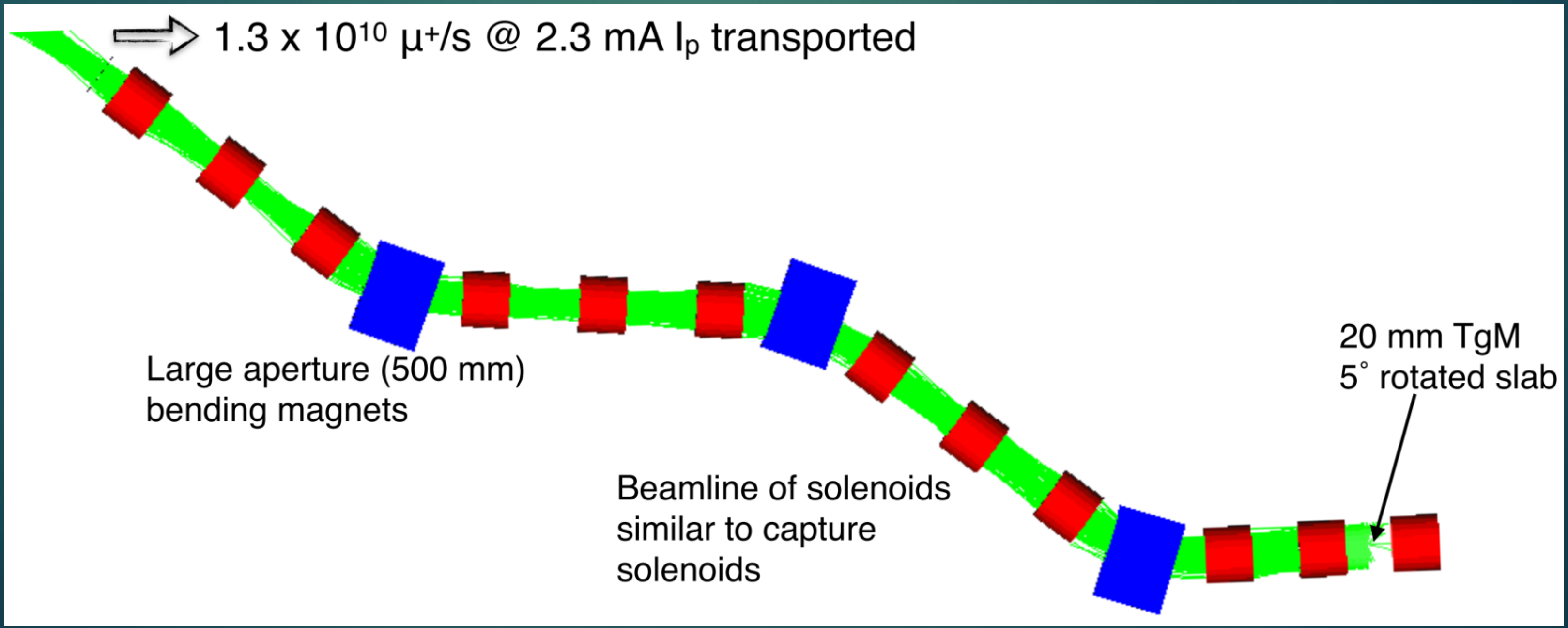


# The HiMB beam line layout - First G4Beamline simulation



- Large aperture solenoids and dipoles
- Large number of muons can be transported
- Almost parallel beam, no focus, no beam diagnostics, no separator



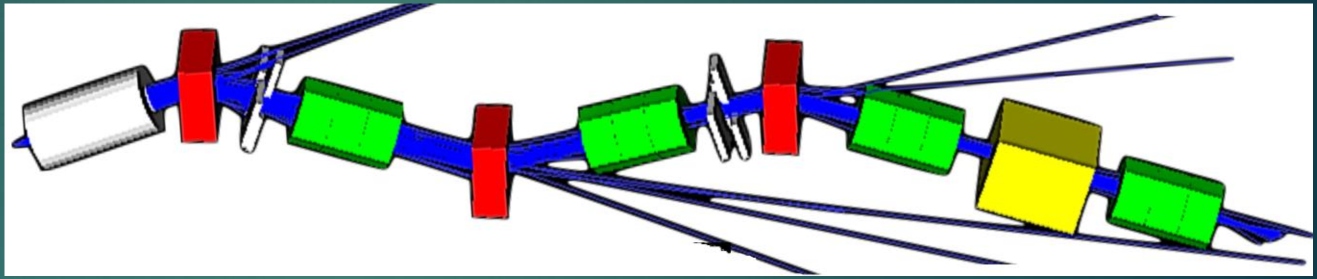
# The HiMB beam line layout - Comparison



$1.2 \cdot 10^{11} \rightarrow 7.2 \cdot 10^{9} \mu^{+}/s$

$5 \cdot 10^{8} \mu^{+}/s$

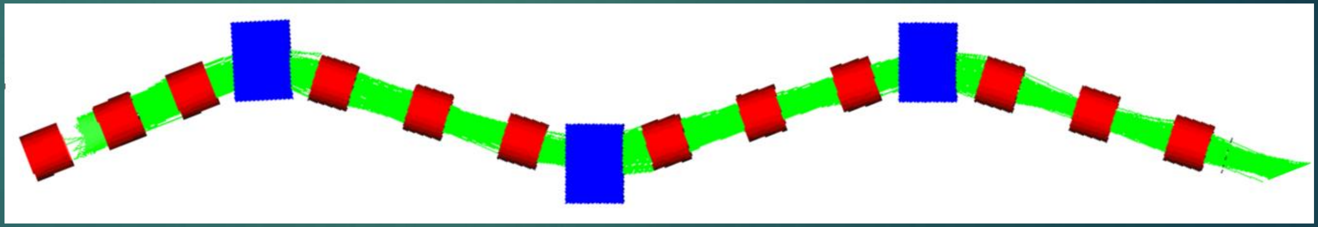
MuE4 Hybrid Channel
@ Target E 40 mm



$1.3 \cdot 10^{11} \rightarrow 3.4 \cdot 10^{10} \mu^{+}/s$

$1.3 \cdot 10^{10} \mu^{+}/s$

HiMB Initial Design
Solenoid Channel
@ 5° 20 mm slab

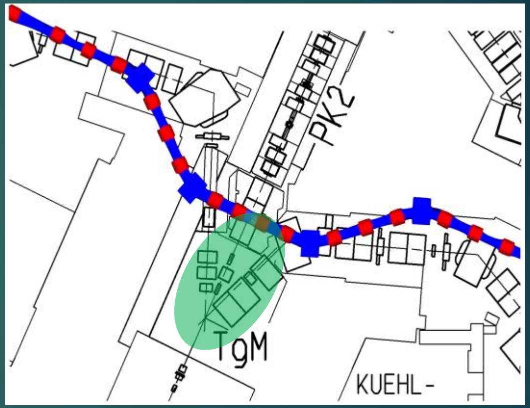


| | Capture efficiency | Transmission | Overall efficiency |
|---|---|---|---|
| MuE4 | ~ 6 % | ~ 7% | ~ 0.4 % |
| HiMB | ~ 26 % | ~ 40 % | ~10 % |

# ToDo



- Optimization of capturing
- Optimize final focussing
- Iterative Beam line optimization and implementation of beam monitoring and particle separator locations with max. transmission
- Minimize shielding modifications
- Particle separation
- Investigate impact on proton beam properties
- Study extraction angle
- Determine new target location
- Disposal of highly radioactive waste
- Study Mu3e setup phase space acceptance and optimize final focus properties
- Find solution with current users of Target M



# HiMB - Outlook



- HiMB aims at surface muon beam intensities $\mathcal{O}(10^{10})\ \mu^+/s$
- Initial simulations show that such rates are feasible
- $\mu^+$ - Beam optics and investigations on proton beam modifications underway
- HiMB opens the door to interesting physics opportunities for particle physics and materials science using high-intensity and high-brightness muon beams (Mu3e Phase II, Low energy MuSR, Muonium spectroscopy, …)

IPA ETH zürich PAUL SCHERRER INSTITUT PSI

# HiMB Project at PSI continues in pursuit of the Intensity Frontier!

Grazie mille per la vostra attenzione!

# Backup slides

# Gamma Factory

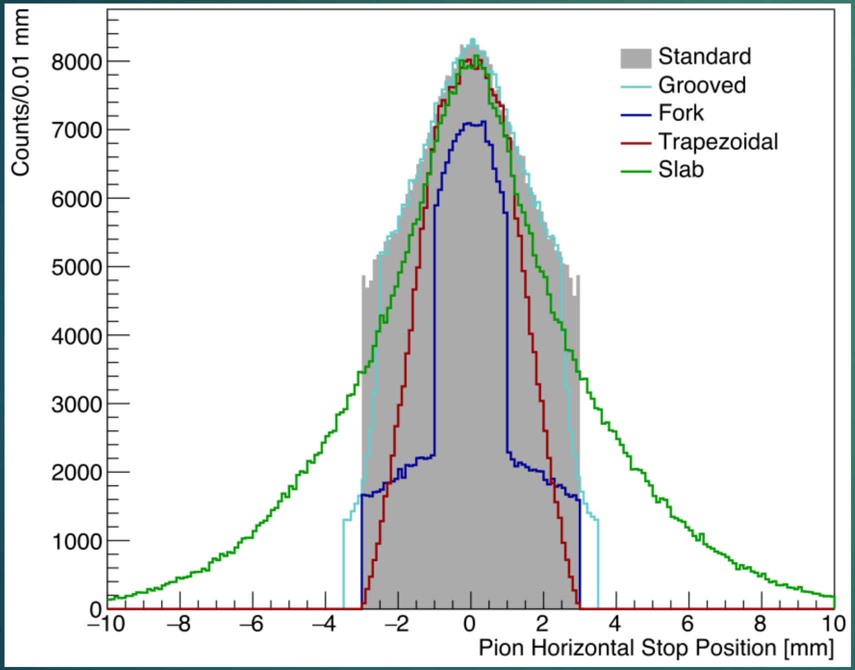


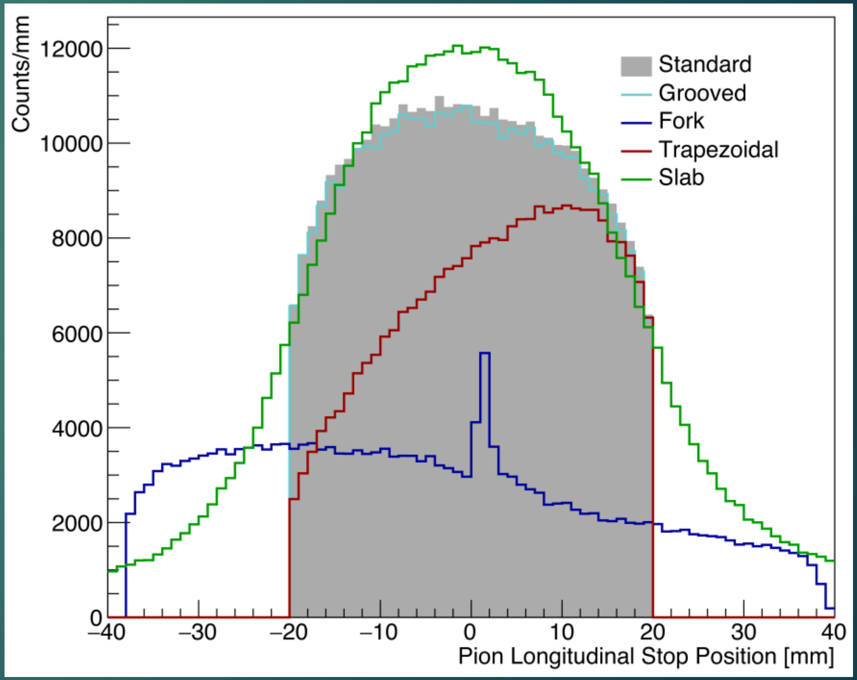
- Partially Stripped Ion (PSI) beams
- Resonant absorption of the laser photons by the PSI beam, followed by a spontaneous atomic-transition emissions of secondary photons
- The initial laser-photon frequency is boosted by a factor of up to $4 \times \gamma^2$
- Direct production of muons ($10^{12}$ muons/s) or via positrons ($10^{12}$ positrons/s)

The Gamma Factory proposal for CERN
arxiv.org/abs/1511.07794

# Pion stopping distribution for different target types





# Comparison of transverse emittance



| | $\varepsilon_x$ (mm mrad) | $\beta\gamma\varepsilon_x$ (mm mrad) | P (MeV/c) |
|---|---|---|---|
| $e^+ + e^- \to \mu^+ + \mu^-$ (1 cm Be @ 44 GeV $e^+$) | $0.17 \cdot 10^{-3}$ | $0.04$ | 22000 |
| PSI πE5 area (4 cm C @ 590 MeV $p^+$) | 3000 | 800 | 28 |
| PSI HiMB ($2cm \cdot \sin(5°)$ C @ 590 MeV $p^+$) | 45000 | 12000 | 28 |

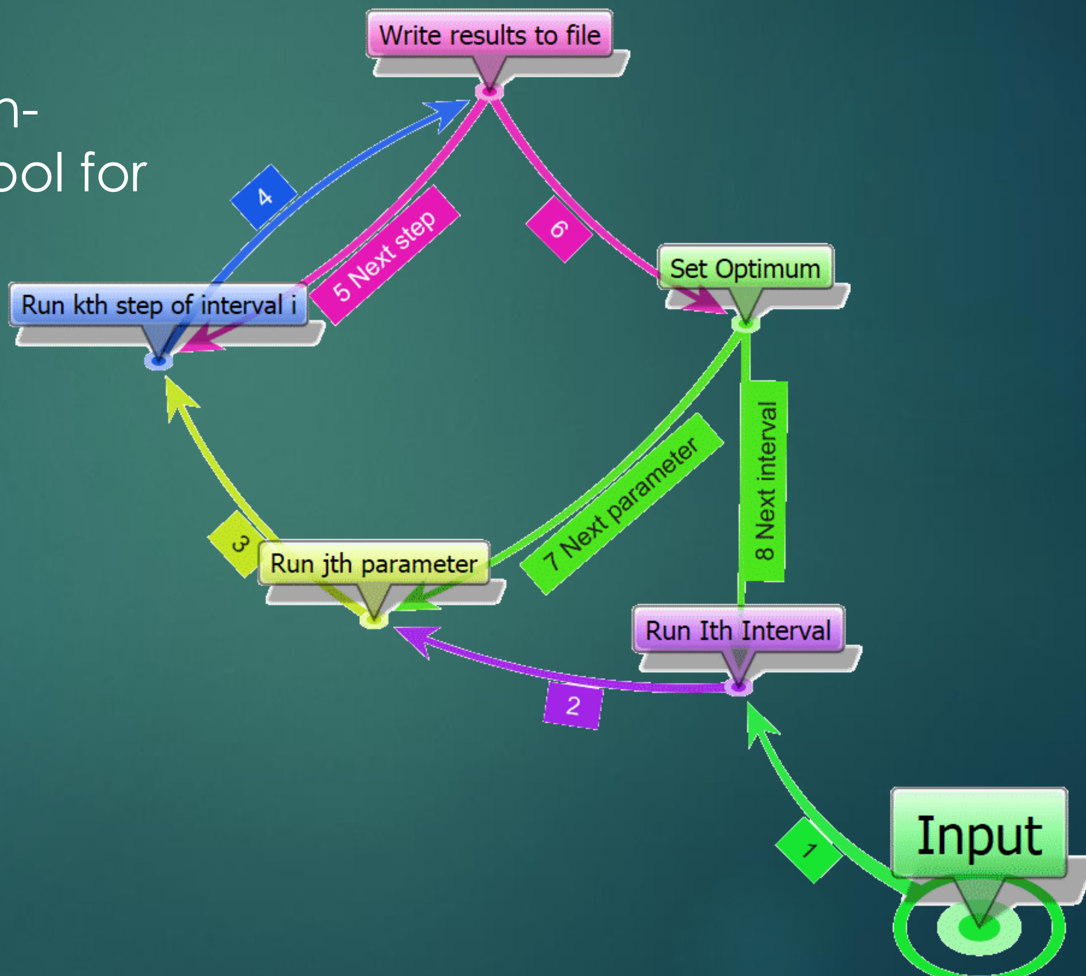
# Simulation tools for HiMB



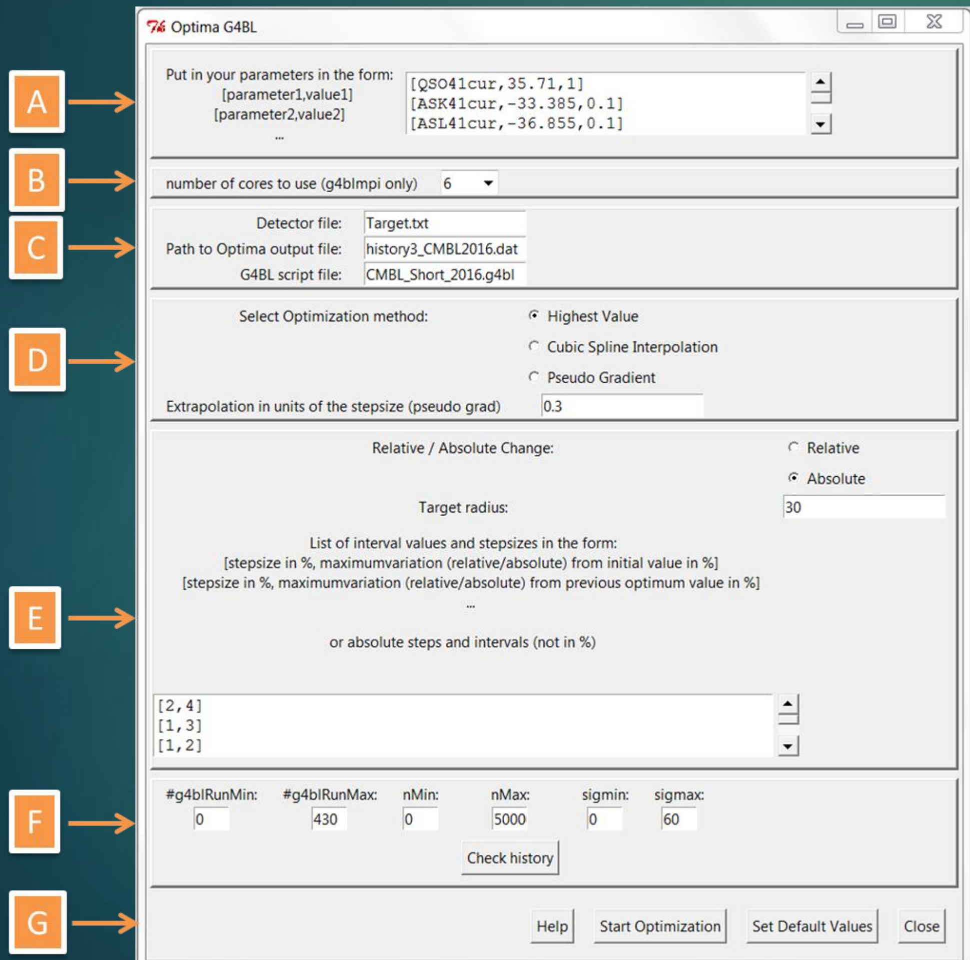
- TRANSPORT : matrix based phase space propagation
- TURTLE : matrix based single particle tracking
- G4Beamline : For final optics (highly non-linear beam line)
- Beamline optimization with G4BLOPTIMA

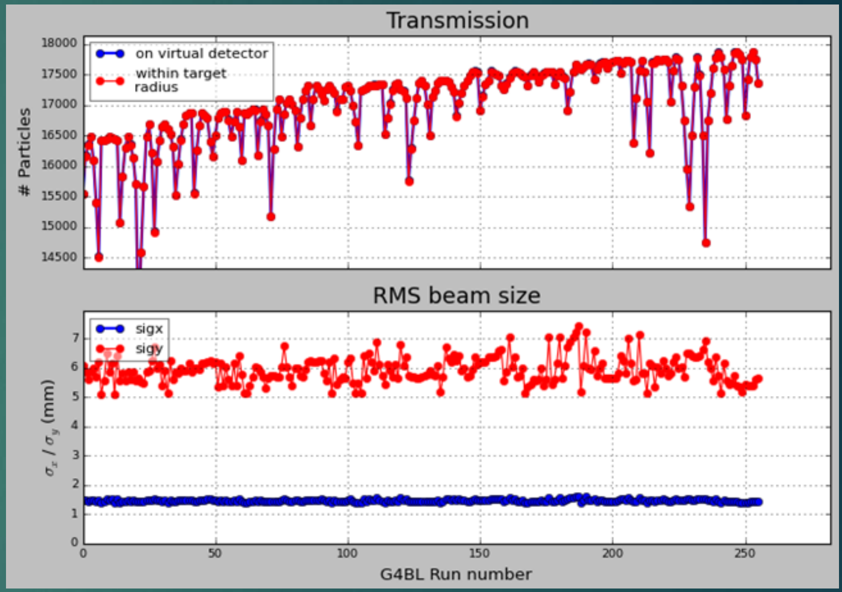
# G4BLOPTIMA G4Beamline Optimization

Cross-platform Python-based optimization tool for G4Beamline



# G4BLOPTIMA
# GUI & Online monitoring

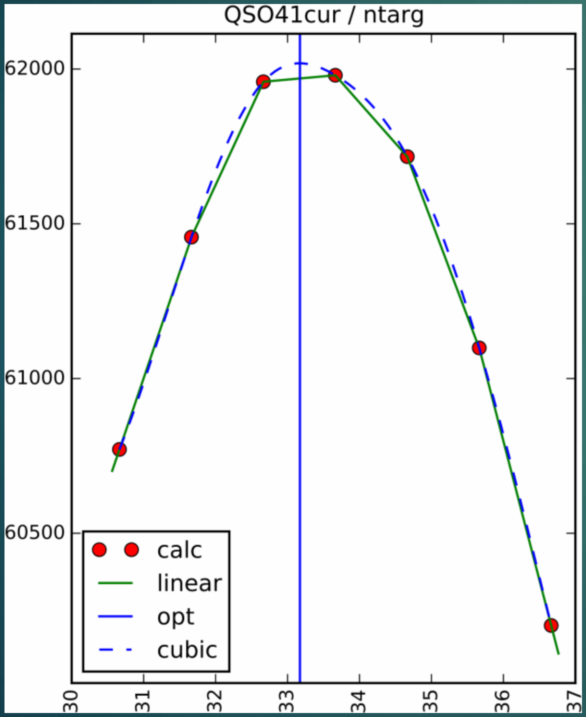



```
Parameters: [0.0801, -0.022, 0.58]  -  Number of particles on target: 167
Parameters: [0.08455, -0.022, 0.58]  -  Number of particles on target: 185
Parameters: [0.089, -0.022, 0.58]  -  Number of particles on target: 184
Parameters: [0.09345, -0.022, 0.58]  -  Number of particles on target: 177
Parameters: [0.0979, -0.022, 0.58]  -  Number of particles on target: 167
Optimum of parameter QSN1cur is 0.08455 with 185 particles on 15 mm radius target - (Opt=Max(calculated))
Parameters: [0.08455, -0.019799999999999998, 0.58]  -  Number of particles on target: 181
Parameters: [0.08455, -0.020900000000000002, 0.58]  -  Number of particles on target: 183
Parameters: [0.08455, -0.022, 0.58]  -  Number of particles on target: 185
Parameters: [0.08455, -0.0231, 0.58]  -  Number of particles on target: 186
Parameters: [0.08455, -0.0242, 0.58]  -  Number of particles on target: 180
Optimum of parameter QSN2cur is -0.0231 with 186 particles on 15 mm radius target - (Opt=Max(calculated))
Parameters: [0.08455, -0.0231, 0.522]  -  Number of particles on target: 172
Parameters: [0.08455, -0.0231, 0.551]  -  Number of particles on target: 184
Parameters: [0.08455, -0.0231, 0.58]  -  Number of particles on target: 186
Parameters: [0.08455, -0.0231, 0.609]  -  Number of particles on target: 174
Parameters: [0.08455, -0.0231, 0.638]  -  Number of particles on target: 158
Optimum of parameter QSMcur is 0.58 with 186 particles on 15 mm radius target - (Opt=Max(calculated))
Run 0 finished.
Parameters: [0.0820135, -0.0231, 0.58]  -  Number of particles on target: 173
```

# G4BLOPTIMA Output

PDF for all optimization parameter saved after each iteration




History File with relevant quantities for later analysis

history.dat

```
QSO42cur QSM41cur QSN41cur QSN42cur SML41cur ASL41cur QSO41cur ASK41cur sigx sigy meanx meany n ntarg20.0
-20.75 -122.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 35.2648793122 71.6748994059 -5.70331642886 0.148562888467 14746 765
-15.75 -122.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 37.3908254513 82.4012575124 -4.13063811736 1.10716991136 12214 525
-10.75 -122.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 38.2585913886 86.0408883658 -5.53304712982 0.691430803473 9513 398
-5.75 -122.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 37.9565209108 87.8777070621 -6.71763006769 1.02870064002 7633 286
-0.75 -122.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 37.780009386 88.952953528 -7.05060587236 1.27033337587 6311 229
-20.75 -134.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 46.9647957628 79.4400380126 -2.37677964852 0.473682529038 14162 475
-20.75 -128.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 41.1507814146 75.7238277813 -4.25068216831 0.184524970839 14478 598
-20.75 -122.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 35.2648793122 71.6748994059 -5.70331642886 0.148562888467 14746 765
-20.75 -116.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 29.3747449495 67.2209773143 -6.78081314554 0.140781668085 14946 1000
-20.75 -110.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 23.6901834435 62.1648726396 -7.52431736071 0.138827148693 15070 1441
-20.75 -110.5 -20.0 10.0 0.0 -37.88 50.46 -32.66 30.7589225292 130.423581385 -17.4198947762 -0.406389007913 15203 453
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-20.75 -110.5 -10.0 10.0 0.0 -37.88 50.46 -32.66 23.6901834435 62.1648726396 -7.52431736071 0.138827148693 15070 1441
-20.75 -110.5 -5.0 10.0 0.0 -37.88 50.46 -32.66 34.5448930958 136.270363435 0.445493062924 1.98812288741 12523 346
-20.75 -110.5 0.0 10.0 0.0 -37.88 50.46 -32.66 46.3992726288 148.833974534 7.14802428125 3.10269041166 7889 136
```
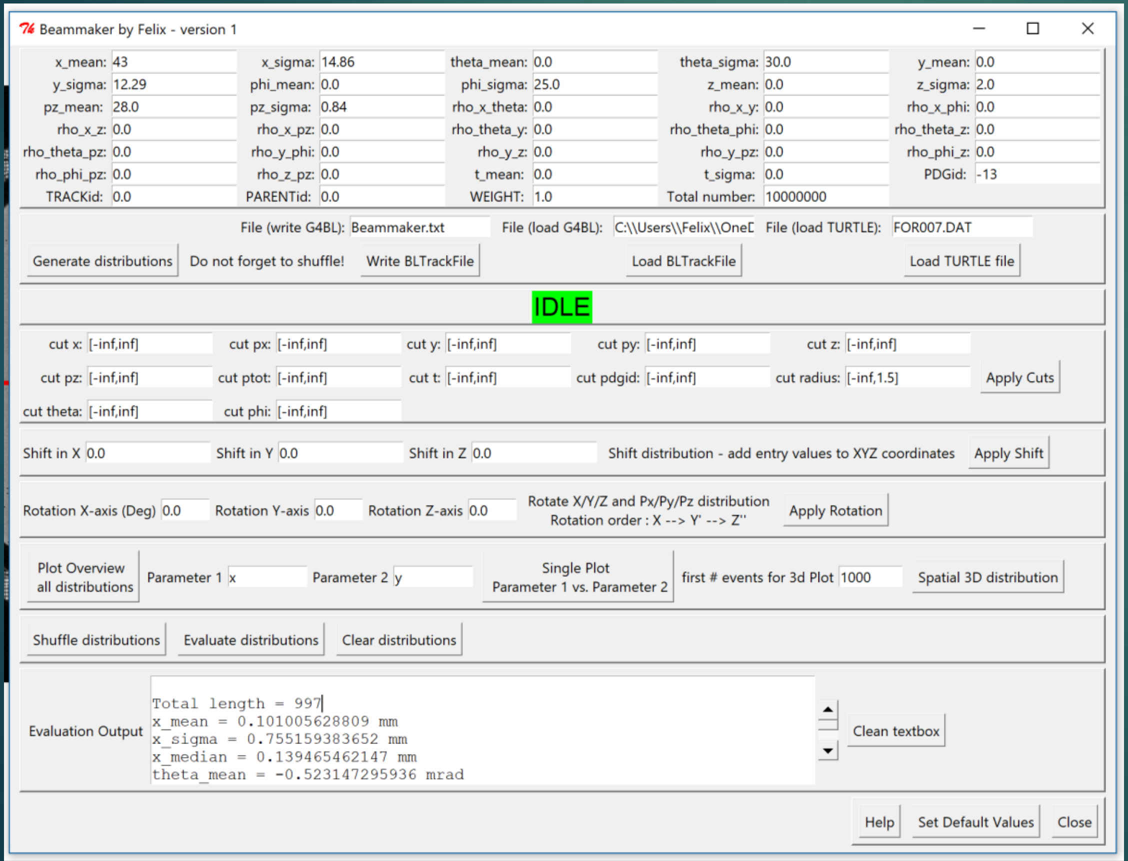
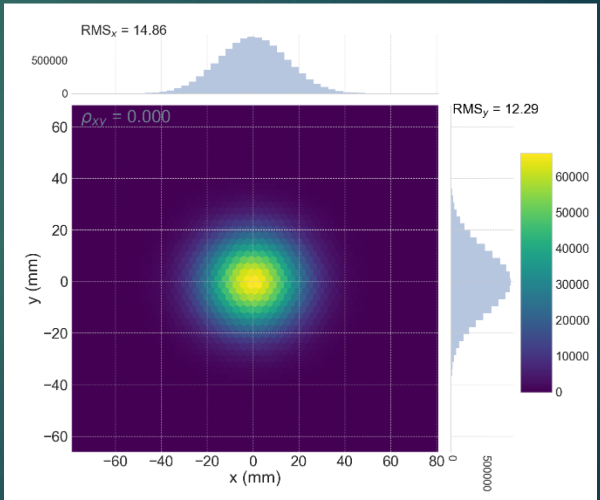

# G4BL Beammaker



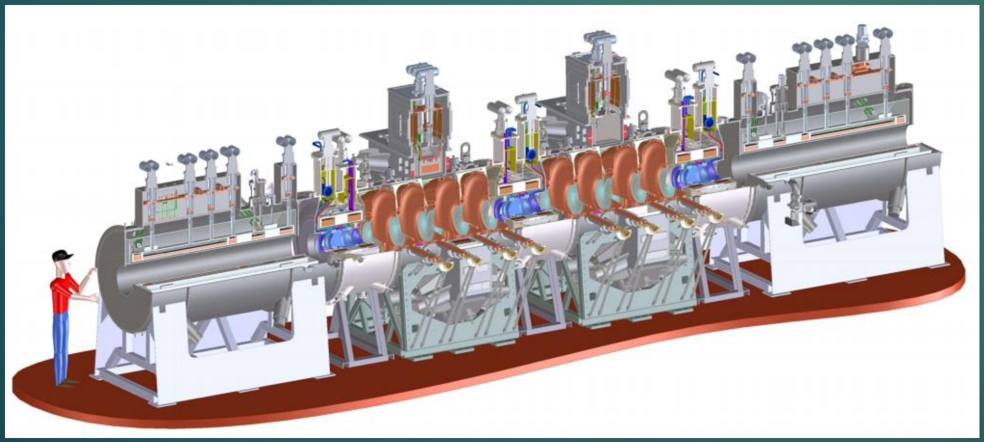
Python based beam analysis, manipulation of TURTLE / G4Beamline output and arbitrary phase space generation



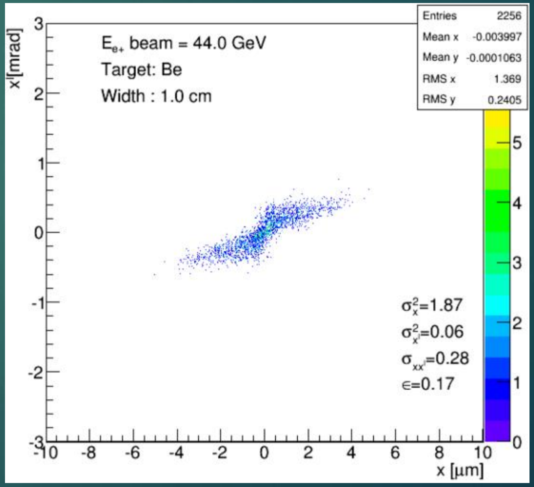



# Future Muon Beams for Colliders

- MICE (MUON IONIZATION COOLING EXPERIMENT) :
  - 6D Phase space cooling of 'Hot' muon beams from conventional pion decay beams
- Intrinsically cold muons from generation



Lattice design and expected performance of the Muon Ionization Cooling Experiment demonstration of ionization cooling
DOI: 10.1103/PhysRevAccelBeams.20.063501




Novel proposal for a low emittance muon beam using positron beam on target
DOI: 10.1016/j.nima.2015.10.097

# Change of target material



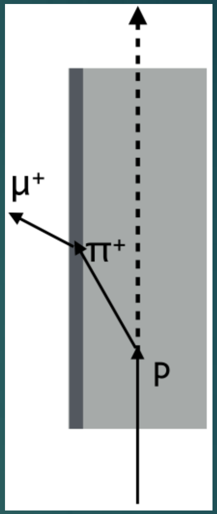
- Optimize material choice to increase surface muon rate

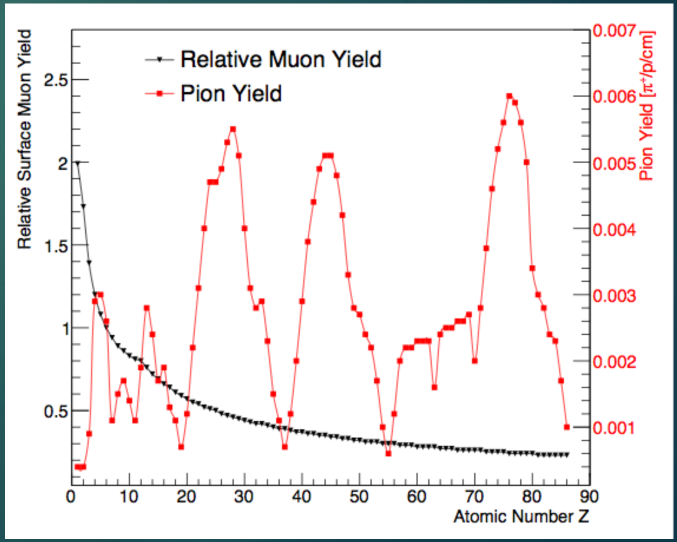
$$\textit{Relative } \mu^+ \textit{ Yield} \propto \pi^+ \textit{ Stop Density} \times \mu^+ \textit{ Range} \times \textit{Relative Target Length}$$

$$\propto n \cdot \sigma_{\pi^+} \cdot \left(\frac{dE}{dx}\right)_{\pi^+} \times \left(\frac{dE}{dx}\right)_{\mu^+}^{-1} \times \frac{\rho_C (Z/A)_C}{\rho_{target}(Z/A)_{target}}$$

$$\propto Z^{\frac{1}{3}} \cdot Z \cdot \frac{1}{Z} \cdot \frac{1}{Z} = \frac{1}{Z^{2/3}}$$

- Material must withstand harsh environment
- Choice of $B_4C$ or $Be_2C$ shows 10 / 14 % gain while preserving density for proton beam



# Surface muon momentum



$$m_{\pi^+}c^2 = \sqrt{p_{\mu^+}^2 c^2 + m_{\mu^+}^2 c^4} + \sqrt{p_{\nu_\mu}^2 c^2 + m_{\nu^+}^2 c^4}$$

$$p_{\nu_\mu} + p_{\mu^+} = 0$$

$$\rightarrow p_{\mu^+} = \frac{c}{2m_{\pi^+}}\left(m_{\pi^+}^2 - m_{\mu^+}^2\right) = 29.79\ MeV/c$$

# Survival probability



$$F(s, \beta, \gamma, \tau) = \mathrm{e}^{-\frac{s}{\beta\gamma c\tau}}$$