Beyond GR

Claudia de Rham
Imperial College London

1st EPS Gravitation meeting
Rome
20 Feb 2019

Thanks to collaborators

Tate Deskins
(@ CWRU)

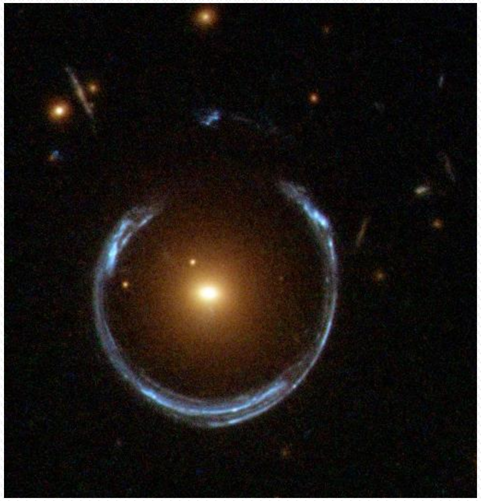
Shuang-Yong Zhou
(@ USTC)

Andrew Tolley
(@ Imperial)

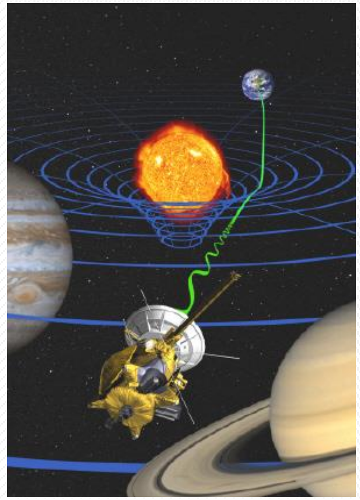
Scott Melville
(@ Cambridge)

CdR, Deskins, Tolley & Zhou, 1606.08462, RMP
CdR, Melville, Tolley & Zhou, 1702.06134 & 1702.08577
CdR, Melville, Tolley & Zhou, 1706.02712 & 1804.10624
CdR, Melville & Tolley, 1710.09611
Dar, CdR, Deskins, Giblin & Tolley, 1808.02165
CdR & Melville, 1806.09417

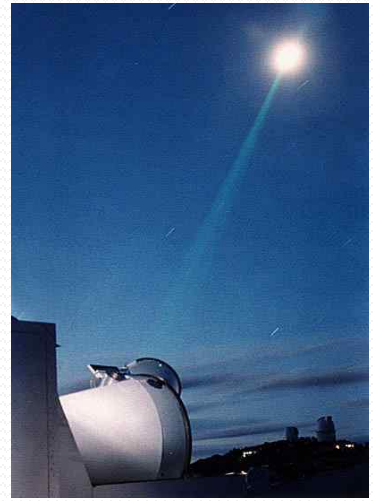
Strong Evidence for General Relativity



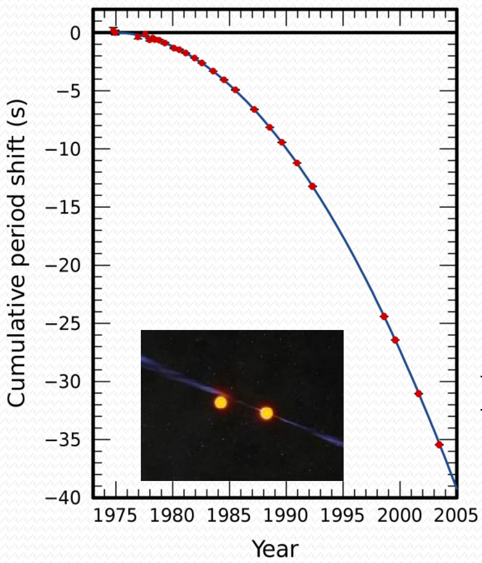
Gravitational Lensing



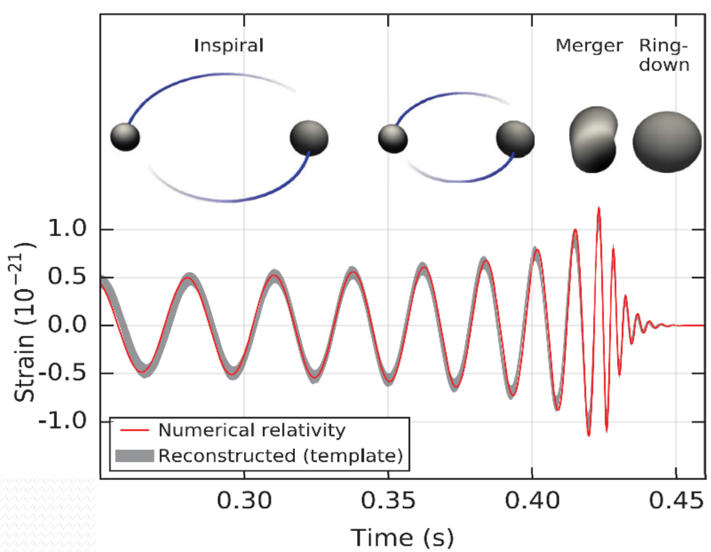
Frame Dragging
(from Earth Rotation)



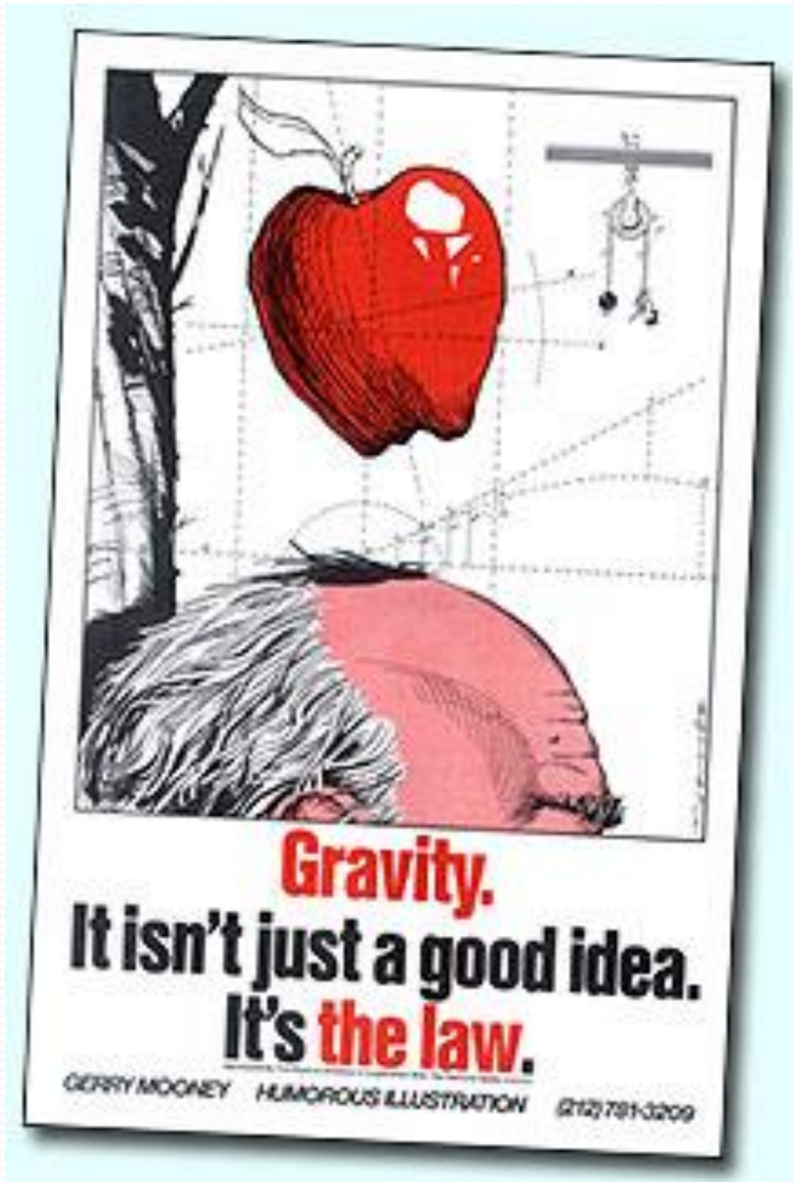
Measure of the advance of the Perihelion



Binary Pulsar
spin-down



GR isn’t just a good idea, it’s the law !



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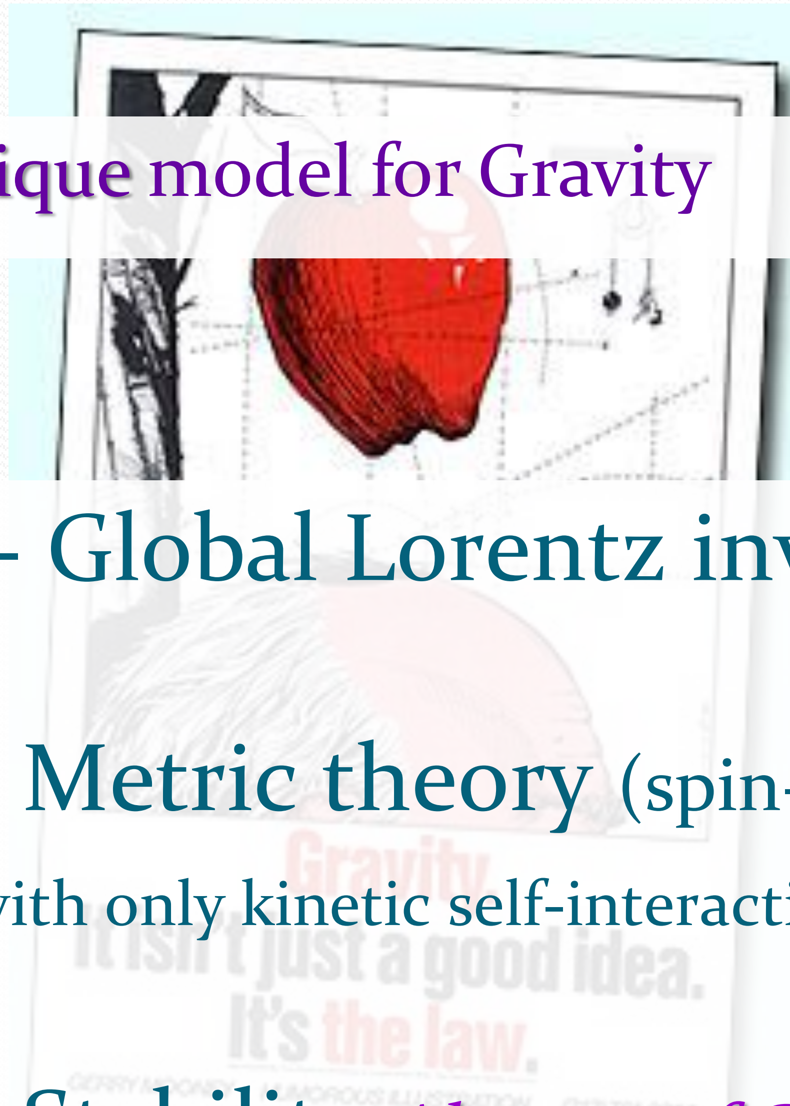


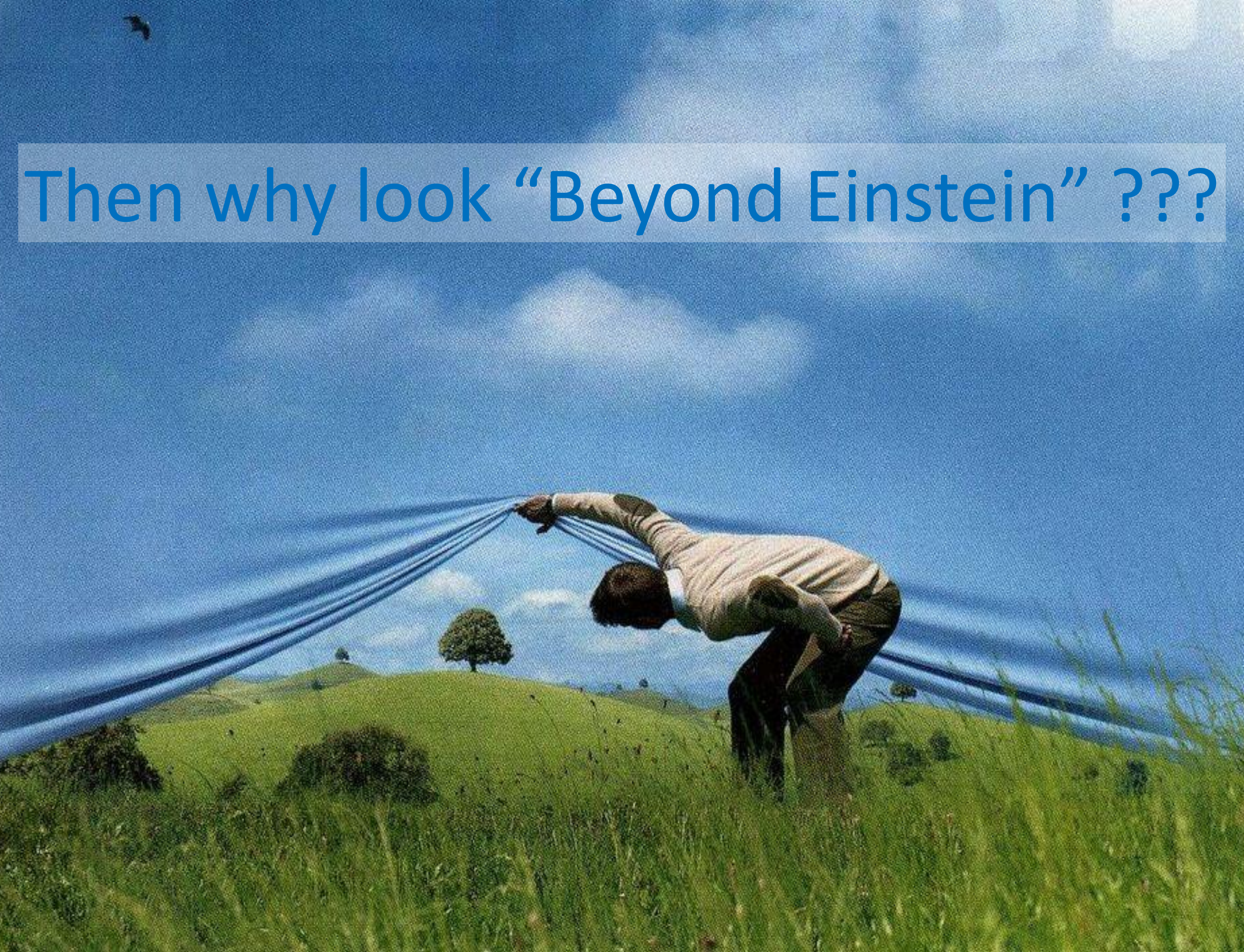
- GR is the **unique** model for Gravity

assuming: - Global Lorentz invariance

- Metric theory (spin-2)

with only kinetic self-interactions (massless)

- Stability *Absence of Ghosts*


Then why look “Beyond Einstein” ???

Why look "Beyond Einstein" ???

Open questions and puzzles of Cosmology...

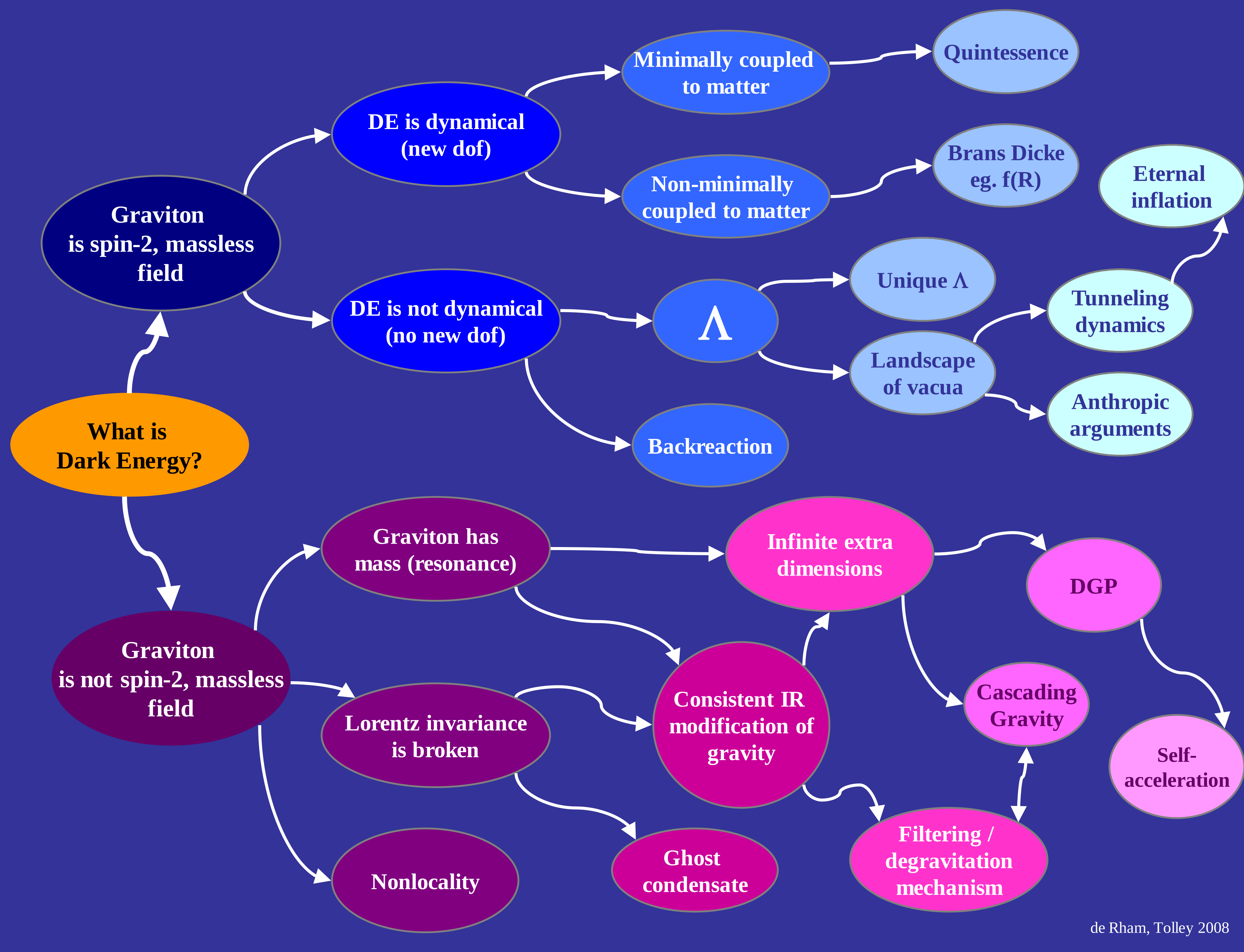
Inflaton or its alternative

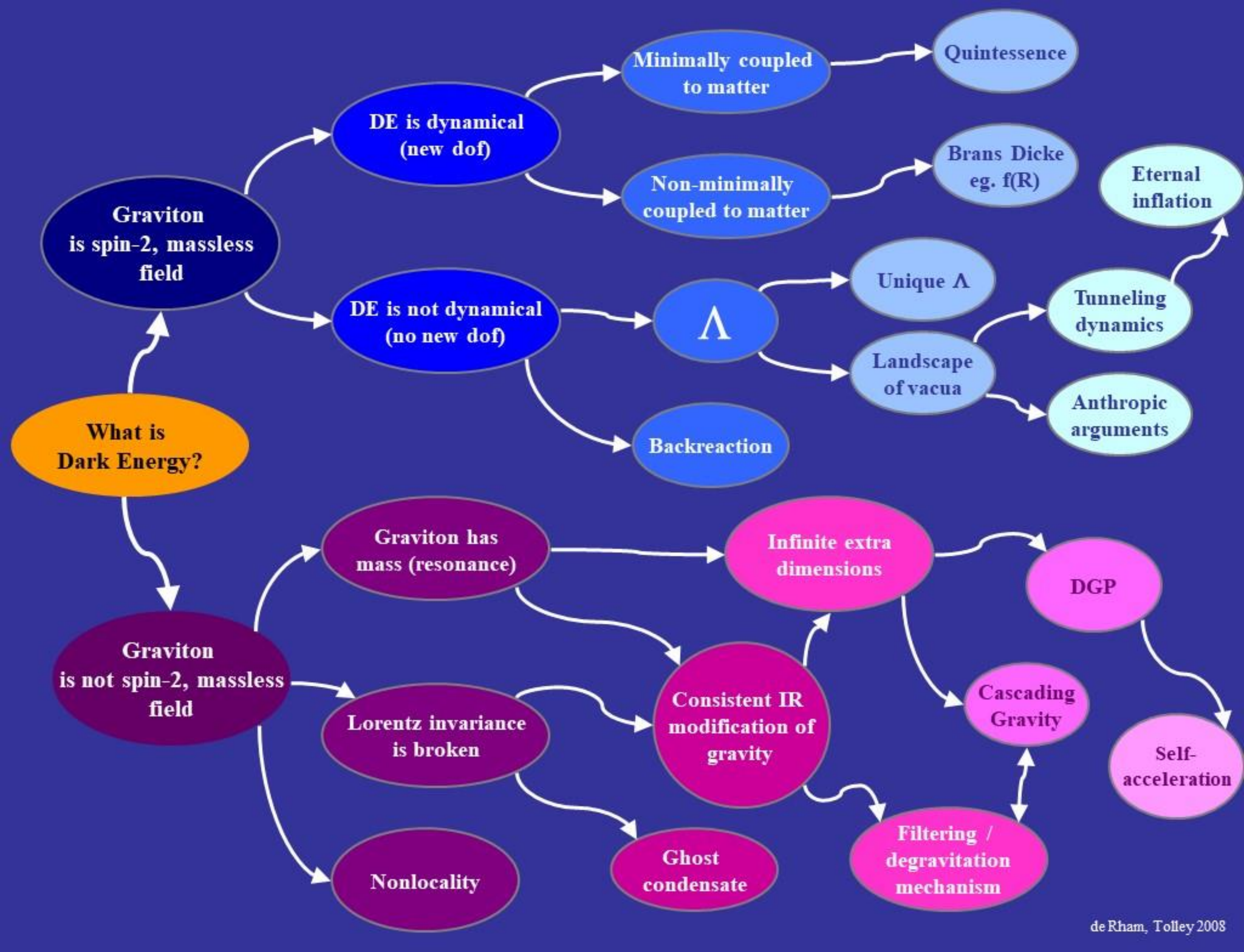
Dark Matter

Hierarchy Problem

Dark Energy

CC problem


What is Dark Energy?
Graviton is spin-2, massless field
DE is dynamical (new dof)
DE is not dynamical (no new dof)
Minimally coupled to matter
Non-minimally coupled to matter
Quintessence
Brans Dicke eg. f(R)
Λ
Unique Λ
Landscape of vacua
Tunneling dynamics
Eternal inflation
Anthropic arguments
Backreaction
Graviton is not spin-2, massless field
Graviton has mass (resonance)
Lorentz invariance is broken
Nonlocality
Infinite extra dimensions
Consistent IR modification of gravity
Ghost condensate
DGP
Cascading Gravity
Self-acceleration
Filtering / degravitation mechanism
de Rham, Tolley 2008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Beyond Horndeski
Horndeski
Galileons
Sym-metron
DHOST/ EST
Beyond Generalized Proca
Generalized Proca
biGravity/ Multi-gravity
Warped MG
Mass-Varying MG
Graviton has mass (local)
Quasi-dilaton
Partially Massless Gravity
Generalized Massive Gravity
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CdR, 2018

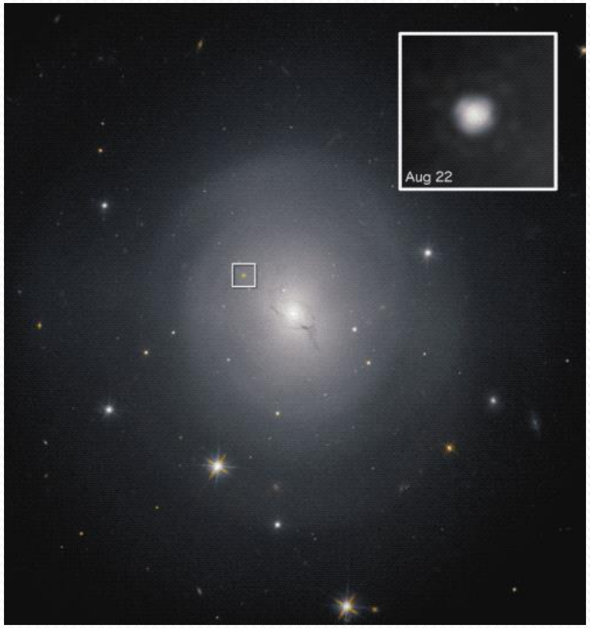
Setting different models apart

GW detections already made big impact

$$-3 \times 10^{-15} \leq \frac{c_T}{c_\gamma} - 1 \leq 7 \times 10^{-16}$$

GW&GBR 170817

Abbott *et. al.* 1710.05832, 1710.05833, 1710.05834



Setting different EFTs apart

GW detections already made big impact

$$-3\times10^{-15}\le\frac{c_T}{c_\gamma}-1\le7\times10^{-16}$$

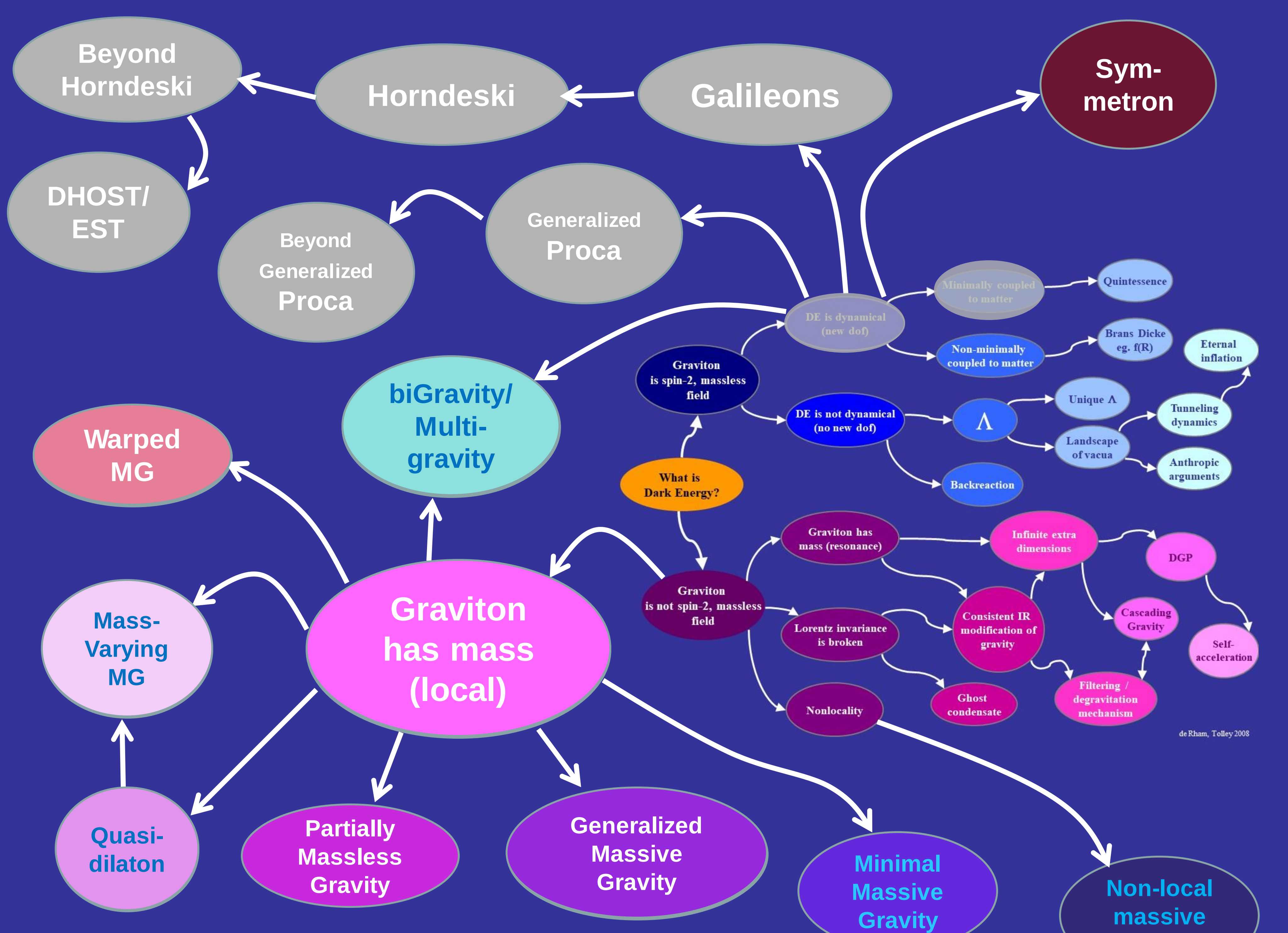
GW&GBR 170817

1509.08458, 1602.07670, 1710.05877, 1710.05893, 1710.05901, 1710.06394 (constraints from GWs speed)

\+ 1809.03484 (constraints from GWs decay into DE)

Horndeski is no longer valid as a dark energy EFT

Either it predicts $c_T \neq 1$ or GWs would decay in DE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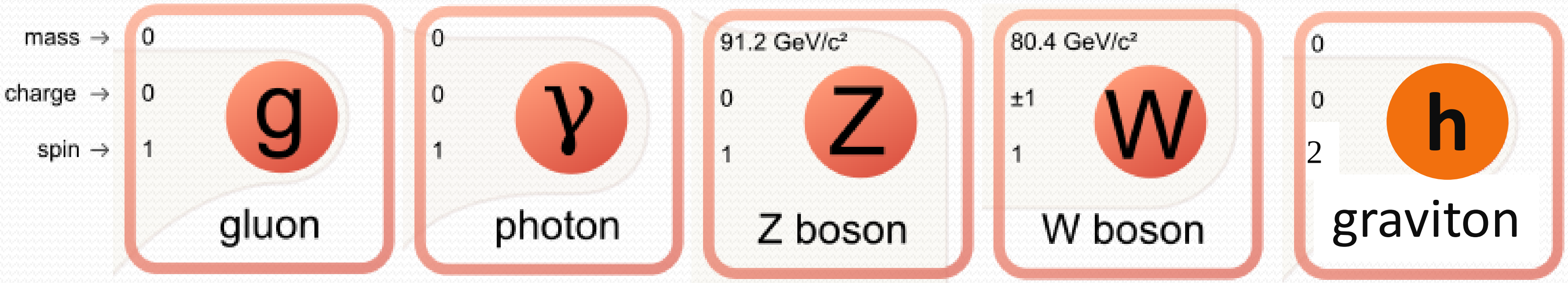
Could the graviton have mass ?



The force carriers don't need to be massless

It is not inconceivable that the photon has a mass

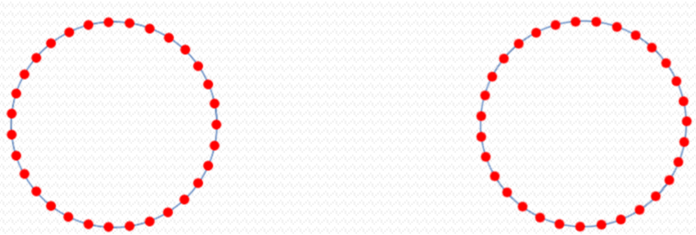
$$m_\gamma \lesssim 10^{-20}\mathrm{eV}$$

Could the graviton also have a mass ?
Higgs mechanism for gravity ?

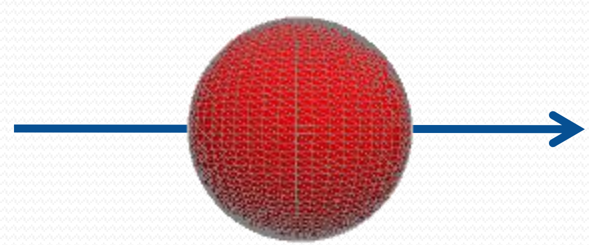
General Relativity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2} \;\; R$$

- **GR: 2** polarizations



Straight on view



Side view

Massive Gravity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \text{Mass Term}\right)$$

- The notion of mass requires a ***reference*** !

Massive Gravity

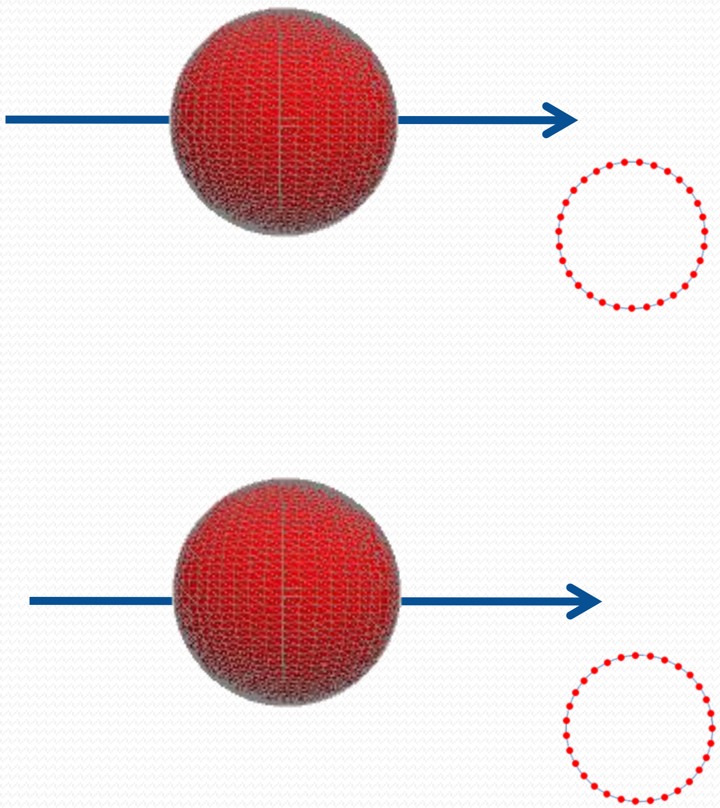
$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \text{Mass Term}\right)$$

- The notion of mass requires a ***reference*** !

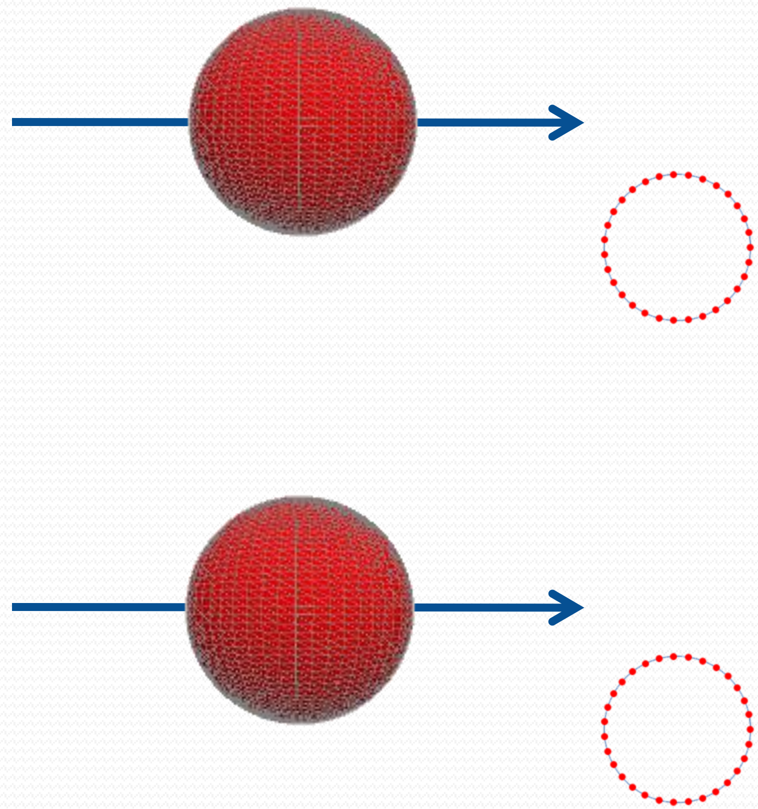
- Generates new dof

$$\underbrace{2}_{\text{GR}} + \underbrace{4}_{\text{Loss of 4 sym}} = 6$$

In principle GW could have 4 other polarizations



2 ‘vectors’



2 ‘scalars’

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\mathrm{FP}} = h_{\mu\nu}^2 - h^2$$

- Mass term for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Fierz & Pauli, Proc.Roy.Soc.Lond.A **173**, 211 (1939)

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\rm FP} = h_{\mu\nu}^2 - h^2$$

- Mass term for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Transforms under a change of coordinate

$$x^\mu \;\rightarrow\; x^\mu + \partial^\mu \xi$$

$$h_{\mu\nu} \;\rightarrow\; h_{\mu\nu} + 2\partial_\mu\partial_\nu\xi + \partial_\mu\partial_\alpha\xi\partial_\nu\partial^\alpha\xi$$

Typically involves some higher derivatives which leads to a ghost

Deffayet & Rombouts, 2005; Creminelli et. al. 2005

Massive Gravity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \text{Mass Term}\right)$$

- The notion of mass requires a ***reference*** !

- Generates new dof

$$2 + 4 = 6 = 5 + 1$$

Boulware & Deser, PRD6, 3368 (1972)

Massive Gravity

While it is true that most model of massive gravity suffer from ghost pathologies, there is a special class of theory for which the mode is fully absent

CdR & Gabadadze, 2010
CdR, Gabadadze & Tolley, 2011

Massive Gravity

While it is true that most model of massive gravity suffer from ghost pathologies, there is a special class of theory for which the mode is fully absent

$+m^2 M_{\mu\nu}$

Kinetic term has to be identical as in GR

With Andrew Matas & Tolley, 2013, 2015, 2015, 2015

Massive Gravity

While it is true that most model of massive gravity suffer from ghost pathologies, there is a special class of theory for which the mode is fully absent

Matter coupling has to be identical as in GR

Massive Gravity

While it is true that most model of massive gravity suffer from ghost pathologies, there is a special class of theory for which the mode is fully absent

Can we test such a theory ???

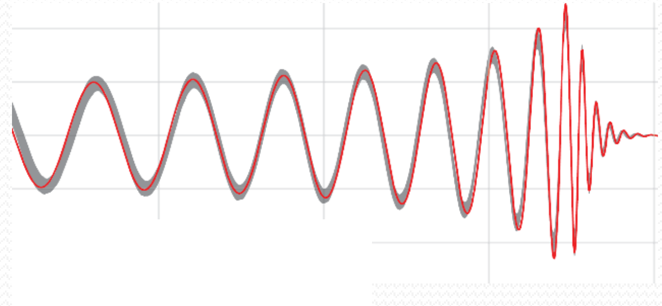
$+m^2 M_{\mu\nu}$

Only 2-parameters + mass scale

How light is gravity ???

Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

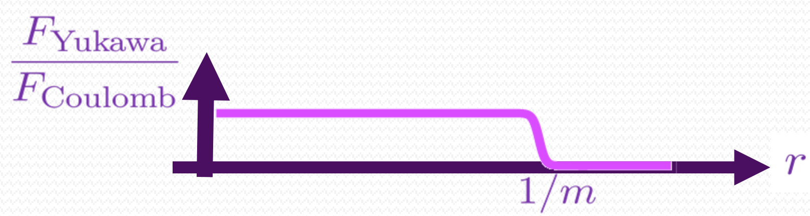


Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-29}	10^{19}	Bound clusters



CdR, Deskins, Tolley, Zhou, 1606.08462, RMP

How light is gravity ???

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Cleanest
(least model dependent)

Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Only for models
that carry a helicity-0 mode
(ie. For Local and Lorentz-invariant models)

Yukawa

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CdR, Deskins, Tolley, Zhou, 1606.08462, RMP

Direct detection of GWs

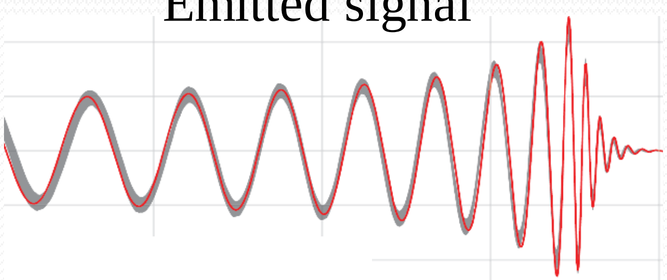
Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

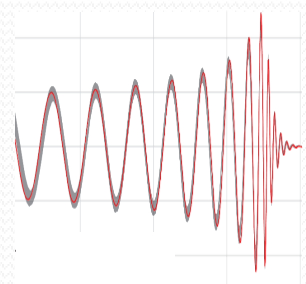
Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity

(including resonances at the level of spectral representation)

Emitted signal



Signal received for $m{\sim}0$



Signal received for $m{\sim}0.1\ d^{-1}$

Direct detection of GWs

modifications of the dispersion relation put a bound on the graviton mass

For GW150914,

$$D \sim 400\text{Mpc}, \;\; f \sim 100\text{Hz}, \;\; \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\text{eV}$$

For GW151226, ρ is smaller and the BHs are lighter so f is larger $\longrightarrow$ not as competitive

Will 1998
Abbott et al., 2016

Direct detection of GWs

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For GW170817 & GRB170817A

$$\Delta c = |c_\gamma - c_{\mathrm{GW}}| < 10^{-15} \quad \Rightarrow \quad m_g \lesssim 10^{-21}\mathrm{eV}$$

Direct detection of GWs

modifications of the dispersion relation put a bound on the graviton mass

For GW150914,

$$D \sim 400\mathrm{Mpc}, \;\; f \sim 100\mathrm{Hz}, \;\; \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\mathrm{eV}$$

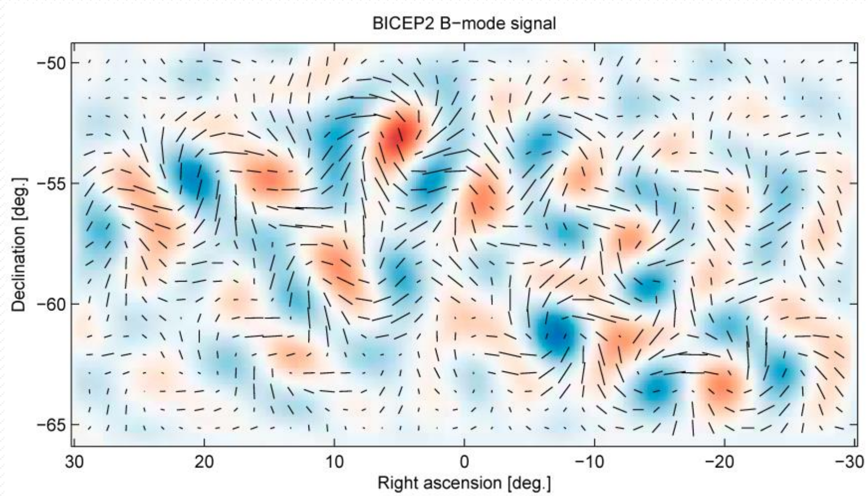
For LISA, could have

$$\rho \sim 10^3$$

$$D \sim 3\mathrm{Gpc} \quad \Longrightarrow \quad m_g \lesssim 10^{-26}\mathrm{eV}$$

$$f \sim 10^{-3}\mathrm{Hz}$$

Bounds from Primordial Gravitational Waves



if ever detected…

would imply the graviton is effectively massless at the time of recombination

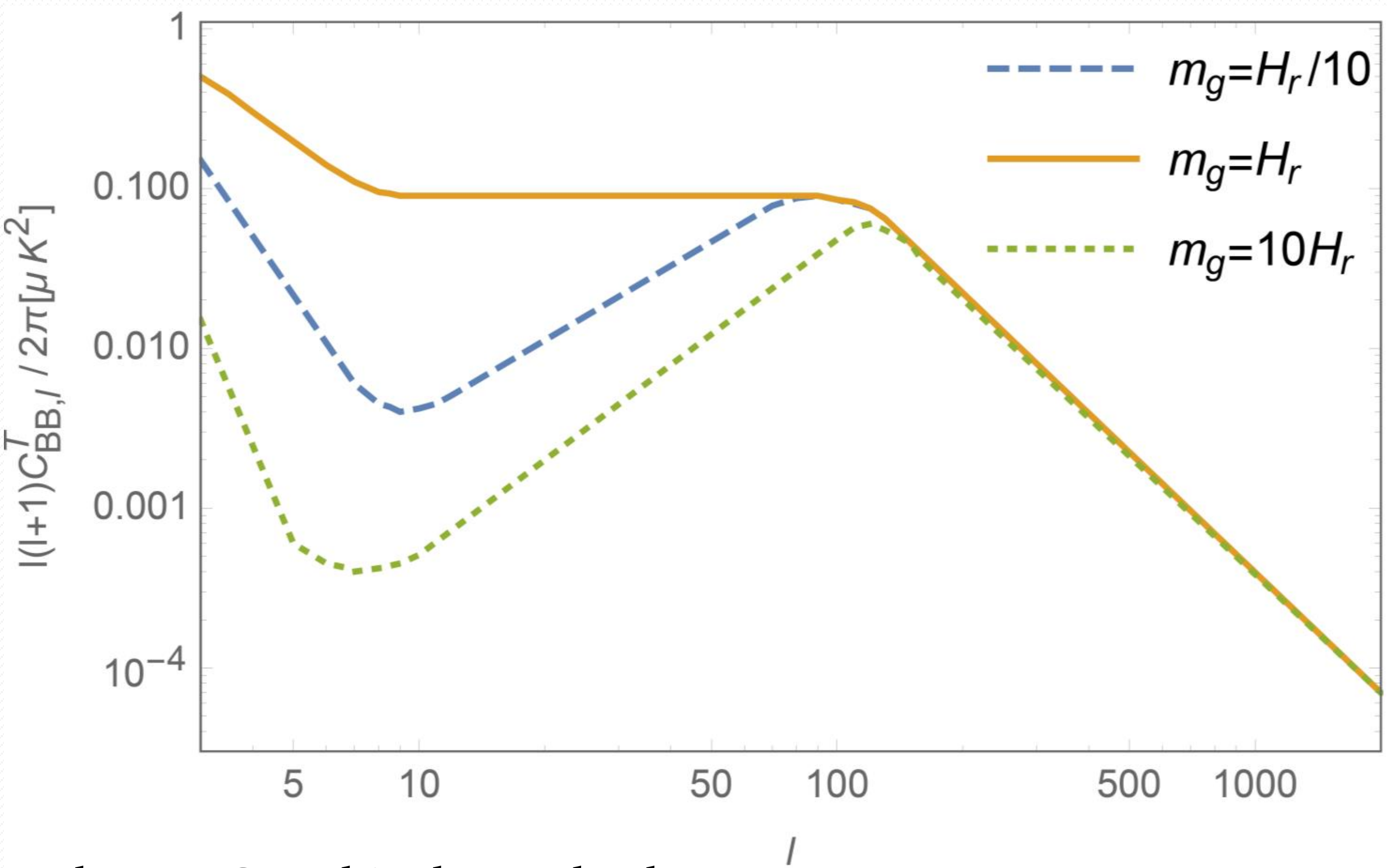
$$m_{\text{eff}} \ll 10^{-29}\text{eV}$$

Dubovsky, Flauger, Starobinsky & Tkachev, 2010
Fasiello & Ribeiro, 2015, (for bi-gravity)
Lin&Ishak, 2016 (Testing gravity using tensor perturbations)

Bounds from Primordial Gravitational Waves

Modification to the tensor mode evolution

$$\mathcal{D}_q''(\tau) + 2\frac{a'}{a}\mathcal{D}_q'(\tau) + \left(q^2 + a^2 m_g^2\right)\mathcal{D}_q(\tau) = J_q(\tau)$$



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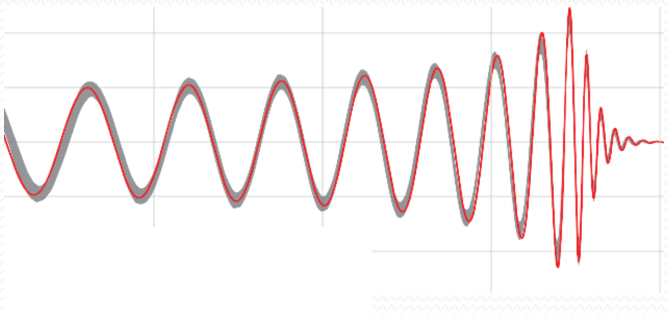
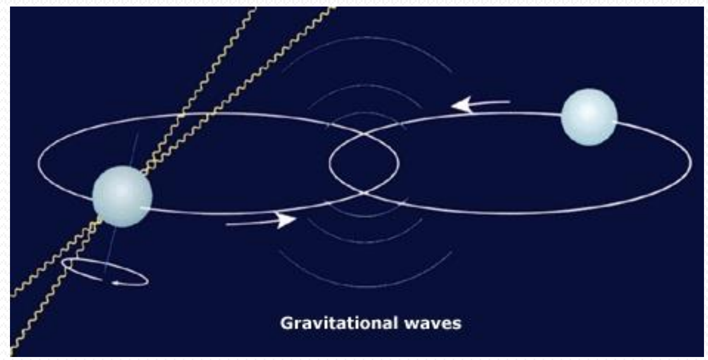
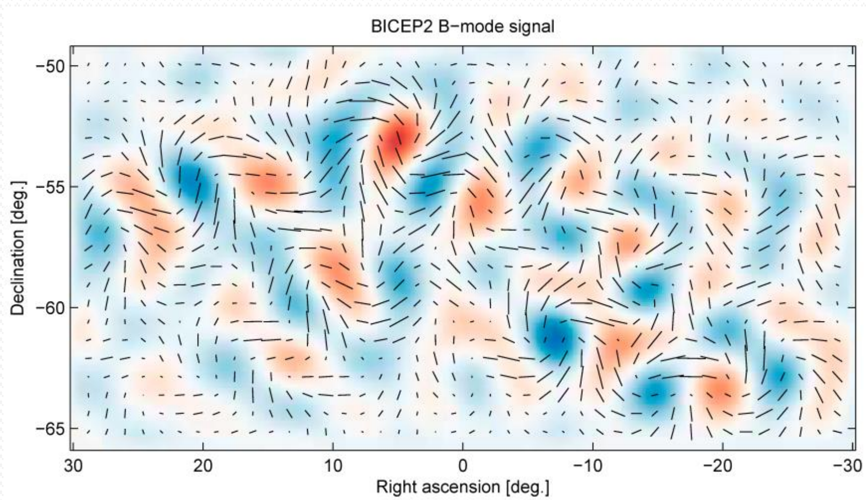
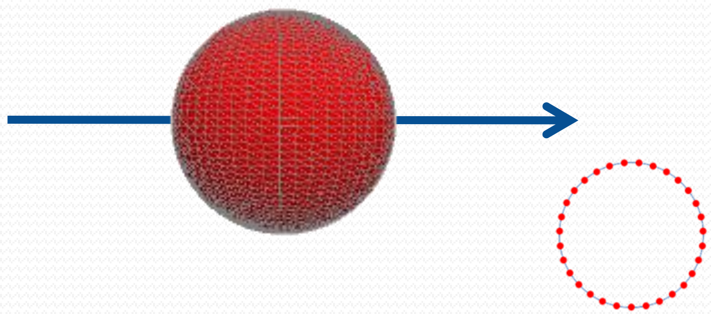


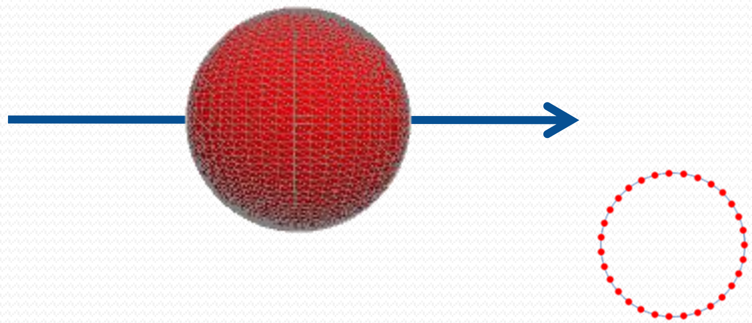




Scalar and Vector modes of the graviton

In a Lorentz invariant theory, a massive graviton also carries a helicity-0 and 2 helicity-1 modes.





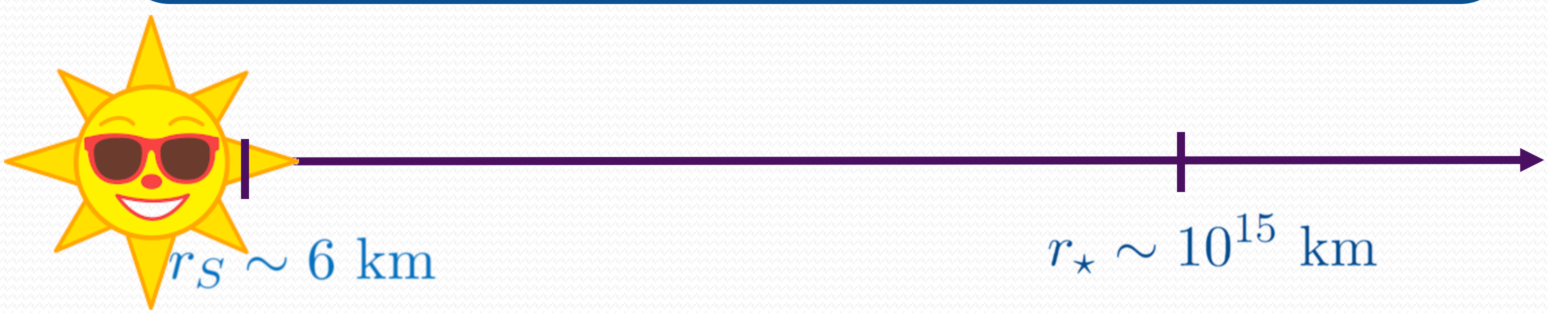
Helicity-0 mode propagates an additional gravitational force that can be very well tested (particularly in the Solar System)

Screened via a Vainshtein mechanism

Vainshtein mechanism

- Well understood for Static & Spherically Symmetric configurations $\quad e.g. \quad T = -M_\oplus\, \delta^{(3)}(r)$
- Force mediated by the helicity-0 mode $\phi'(r)$

$$\frac{\phi'(r)}{r} + \frac{1}{M_{\rm Pl} m^2}\left(\frac{\phi'(r)}{r}\right)^2 = \frac{M_\oplus}{4\pi M_{\rm Pl} r^3}$$



Vainshtein radius: $r_*^3 = \frac{1}{M_{\rm Pl} m^2}\frac{M_\oplus}{M_{\rm Pl}}$

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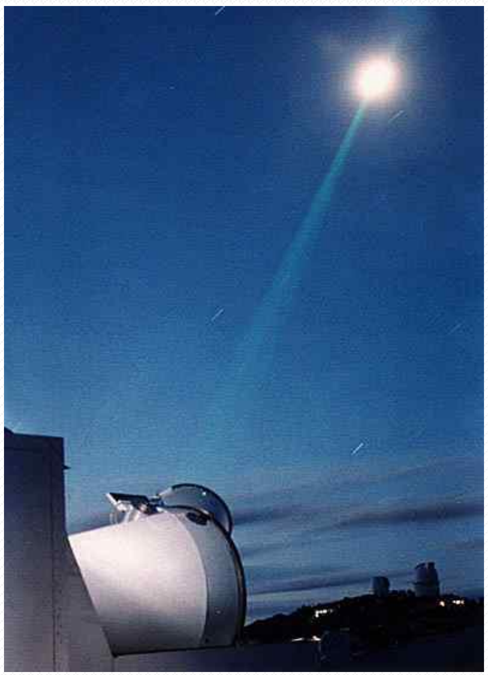
Vainshtein radius:

$$r_*^3 = \frac{1}{M_{\rm Pl} m^2}\frac{M_\oplus}{M_{\rm Pl}}$$

$$\text{for} \quad r \gg r_* \,, \quad \phi'(r) \sim \frac{M_\oplus}{M_{\rm Pl}}\frac{1}{r^2}$$

$$\text{for} \quad r \ll r_* \,, \quad \phi'(r) \sim \frac{M_\oplus}{M_{\rm Pl}}\frac{1}{r_*^{3/2}\sqrt{r}}$$

Lunar Laser Ranging bounds



For DGP, (cubic Galileon)

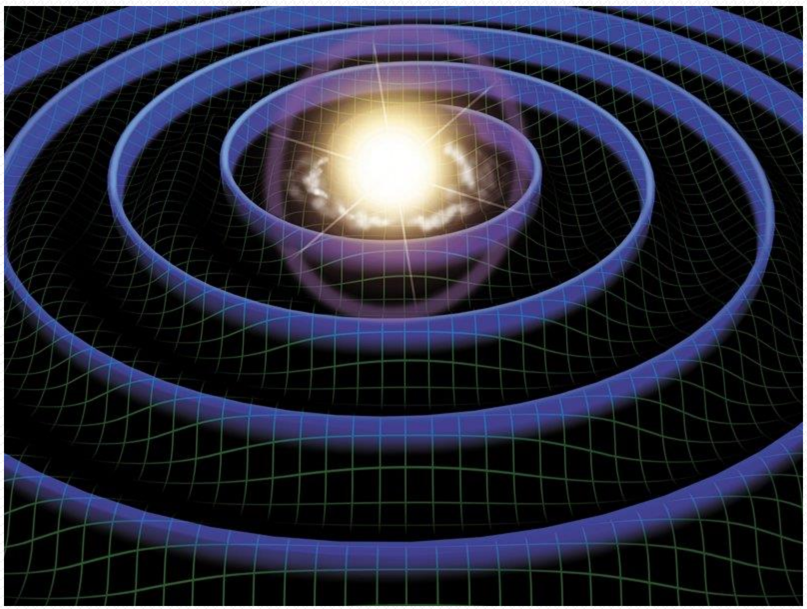
$$m_g < \delta\phi \left(\frac{r_{S,\oplus}}{a^3}\right)^{1/2} \qquad m_g \lesssim 10^{-32}\mathrm{eV}$$

For hard mass graviton, (~ quartic Galileon)

$$m_g < \delta\phi^{3/4} \left(\frac{r_{S,\oplus}}{a^3}\right)^{1/2} \qquad m_g \lesssim 10^{-30}\mathrm{eV}$$

Radiation into the scalar mode of the graviton

The existence of a scalar mode means new channels of radiation



Monopole & dipole exist but are suppressed by conservation of energy & momentum.

Quadrupole emitted by helicity-0 mode is suppressed by Vainshtein mechanism
(best understood in a Galileon approximation)

Work with Furqan Dar, Tate Deskins, John Tom Giblin & Andrew Tolley

Contours of $\dot{\phi}^2$

For the cubic Galileon:
Power still in the quadrupole as in GR
Corrections to GR are very suppressed

Galileon Quadrupole emission

$$P_{\text{Quadrupole}} \sim \frac{(\Omega_P \bar{r})^3}{(\Omega_P r_\star)^{3/2}} \frac{\mathcal{M}^2}{M_{\text{Pl}}^2} \Omega_P^2 \qquad r_*^3 = \frac{1}{M_{\text{Pl}} m^2} \frac{M_{\text{Binary}}}{M_{\text{Pl}}}$$

For the Hulse-Taylor Pulsar $\quad m_g \lesssim 10^{-27} \text{eV}$

- For the Cubic Galileon, higher multipoles are suppressed by additional powers of velocity

Galileon Quadrupole emission

$$P_{\text{Quadrupole}} \sim \frac{(\Omega_P \bar{r})^3}{(\Omega_P r_\star)^{3/2}} \frac{\mathcal{M}^2}{M_{\text{Pl}}^2} \Omega_P^2 \qquad r_*^3 = \frac{1}{M_{\text{Pl}} m^2} \frac{M_{\text{Binary}}}{M_{\text{Pl}}}$$

For the Hulse-Taylor Pulsar $m_g \lesssim 10^{-27}\text{eV}$

- For the Cubic Galileon, higher multipoles are suppressed by additional powers of velocity
- Massive gravity and stable self-accelerating models always include *at least* a *quartic Galileon*
- In the Quartic Galileon, the angular direction is *not screened as much* as the others $\longrightarrow$ many multipoles contribute to the power with the same magnitude…

$\longrightarrow$ Multipole expansion breaks down

How light is gravity ???

Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

Cleanest
(least model dependent)

Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Only for models
that carry a helicity-0 mode
(ie. For Local and Lorentz-
invariant models)

Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-29}	10^{19}	Bound clusters

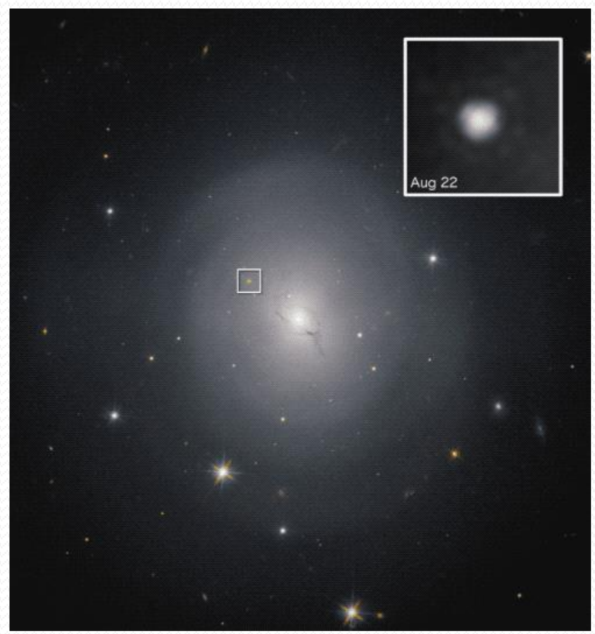
CdR, Deskins, Tolley, Zhou, 1606.08462, RMP

Setting different EFTs apart

- We could simply wait for observations to tell them apart

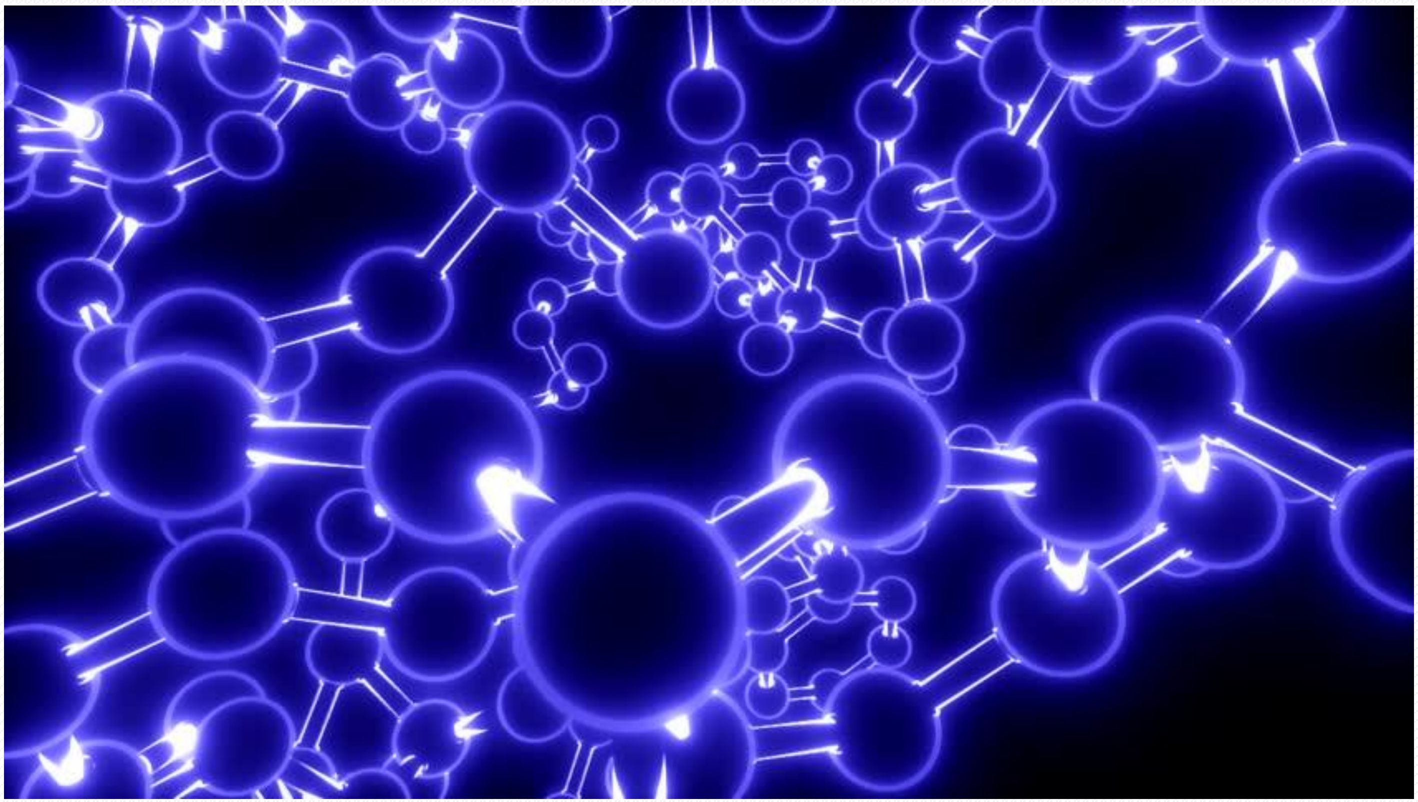
GW&GBR 170817

(eg. DBI, K-inflation, G-inflation, gauge inflation, ghost inflation, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, ~~generalized galileon~~, ~~Horndeski~~, beyond Horndeski, beyond beyond Horndeski, ~~Fab4~~, beyond Fab4, EST, DHOST, K-essence, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, mimetic gravity, unimodular gravity, dipolar dark matter, ..., ...)

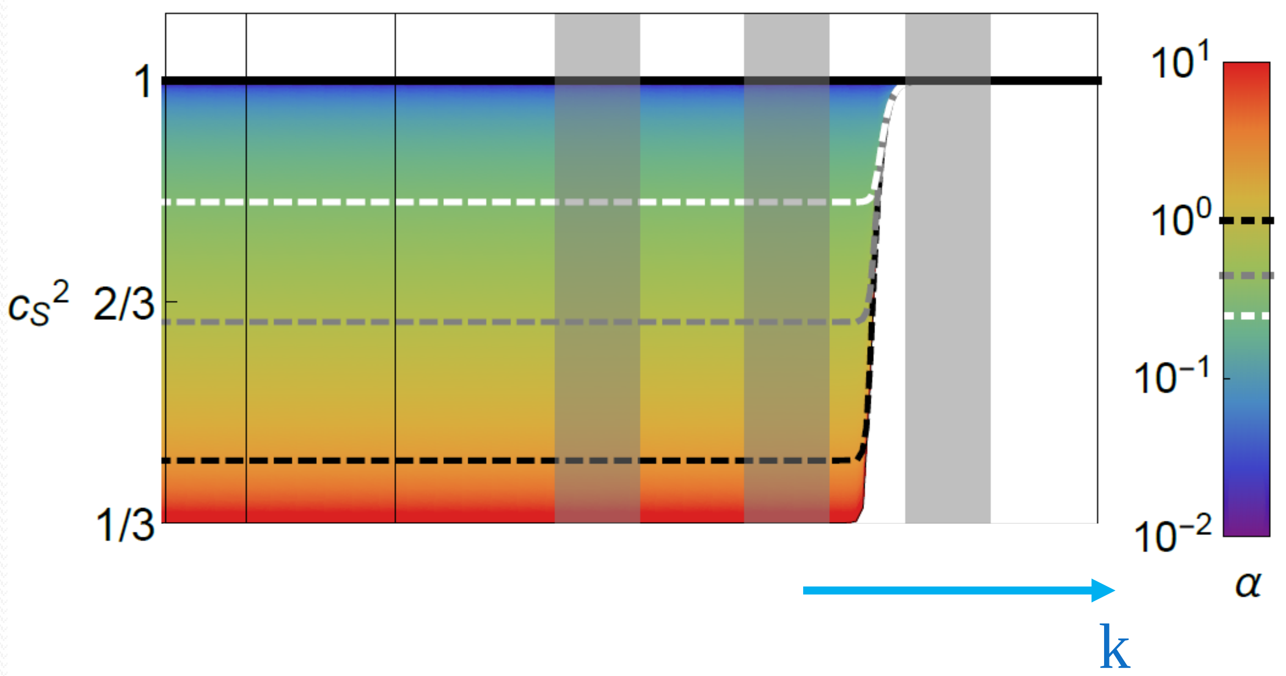


- Horndeski predicts $c_T \neq 1$

- At sufficiently high energy we would expect the spontaneously Lorentz breaking cosmology to be irrelevant



- Horndeski predicts $c_T \neq 1$ – frequency dependent statement !
- At sufficiently high energy we would expect the spontaneously Lorentz breaking cosmology to be irrelevant



Sound speed for a scalar field analogue and known (partial) Lorentz-invariant completion

with Scott Melville, 1806.09417

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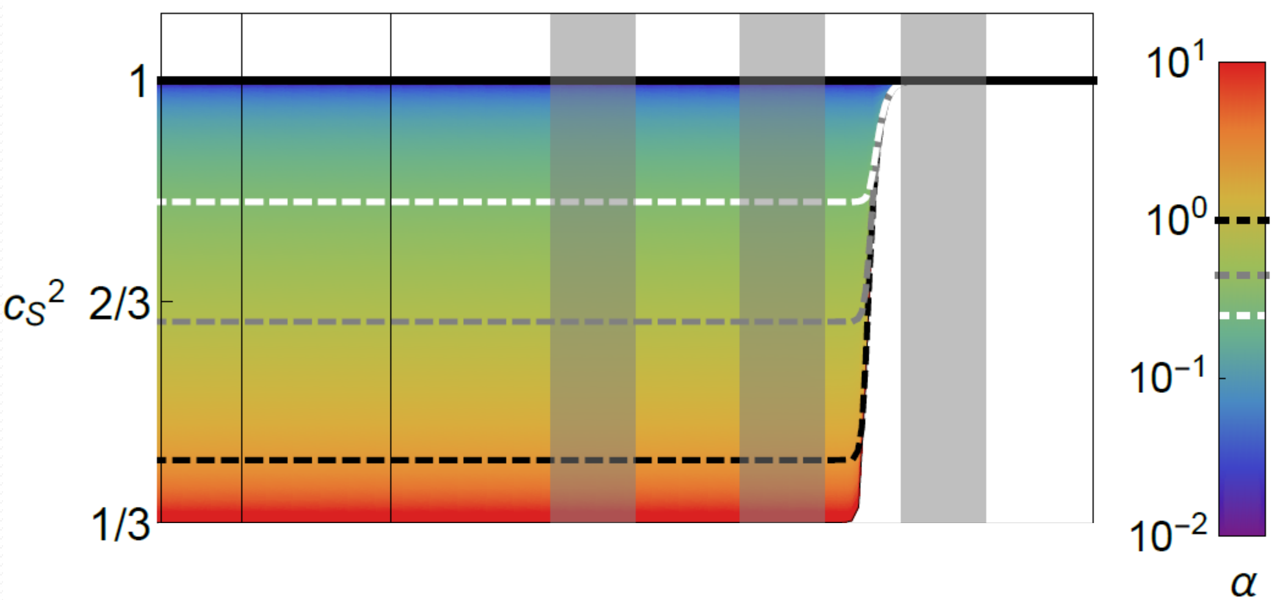
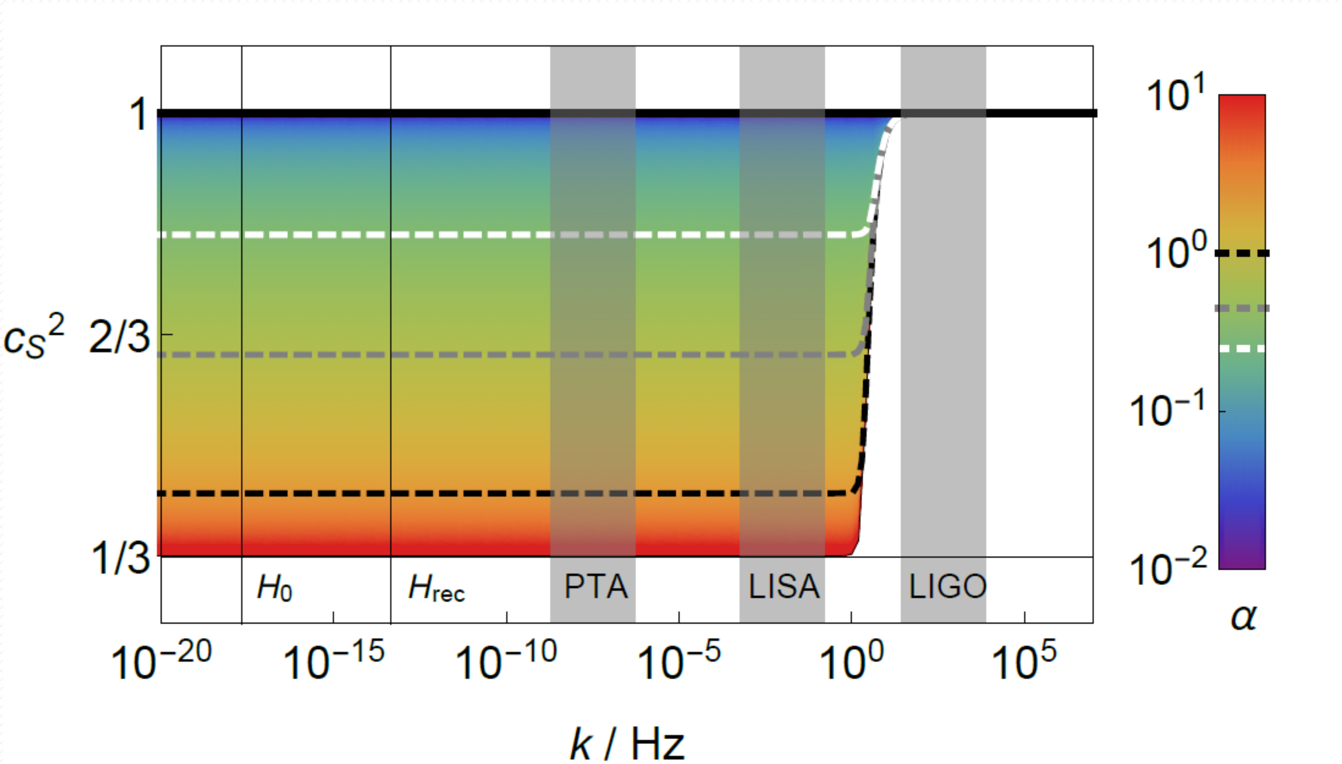


Sound speed for a scalar field analogue and known (partial) Lorentz-invariant completion

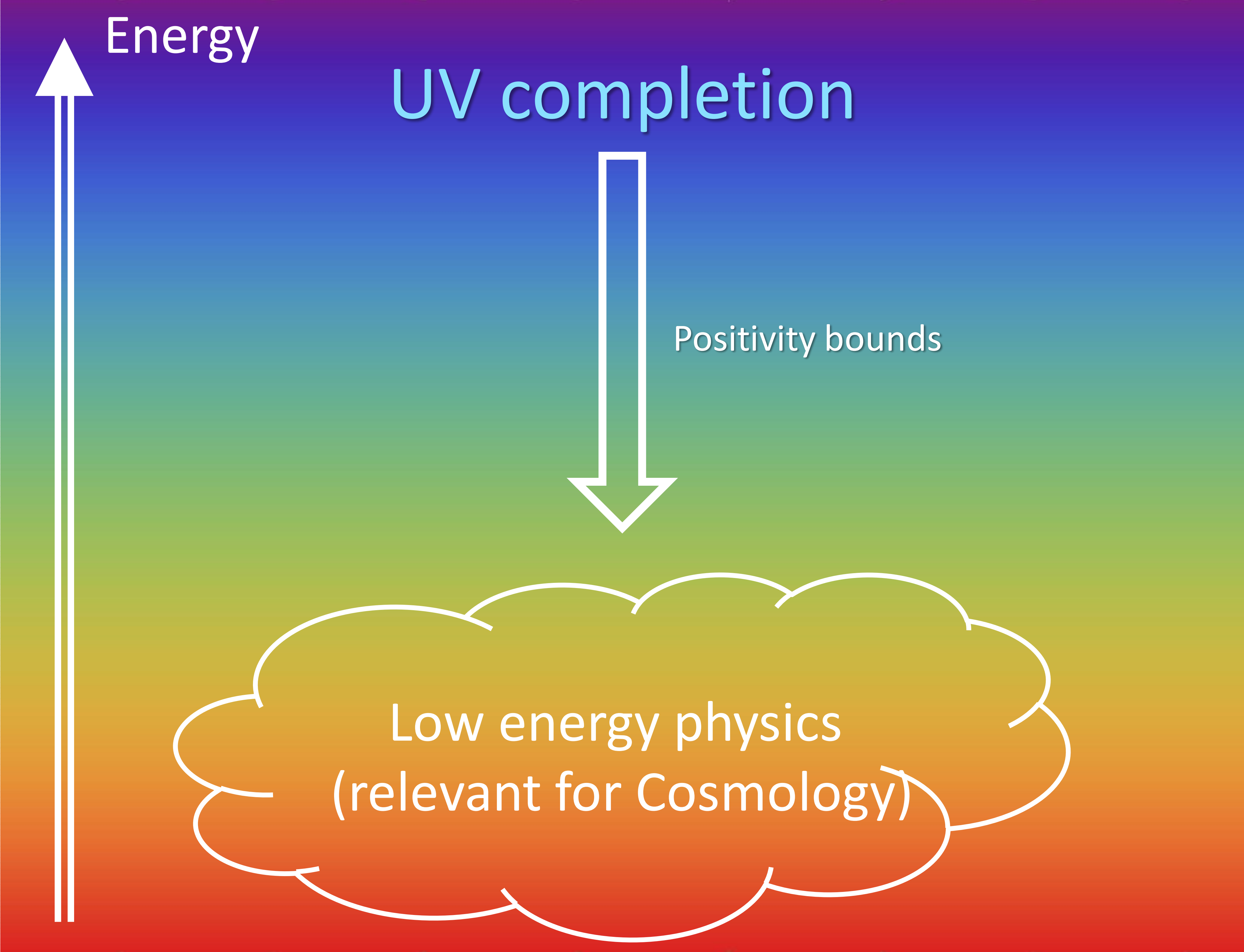
For Horndeski models of DE the cutoff has to be $\ll (M_{\rm Pl} H_0^2)^{1/3} \sim 260\text{Hz}$

- Horndeski predicts $c_T \neq 1$ – frequency dependent statement !
- At sufficiently high energy we would expect the spontaneously Lorentz breaking cosmology to be irrelevant



Sound speed for a scalar field analogue and known (partial) Lorentz-invariant completion Λ~Hz

with Scott Melville, 1806.09417


Energy
UV completion
Positivity bounds
Low energy physics
(relevant for Cosmology)

Summary

- Cosmology has motivated the (re)development of entire new classes of scalar EFTs
- Observations already put strong constraints on some of these models, and particularly on the (effective) graviton mass
- (perturbative) unitarity & analyticity can allow for a better segregation
- Within the context of massive gravity, current observations already put an interesting bound on the graviton mass.
- Future observations could constrain the graviton mass on close to cosmological scales.

How light is gravity ???

Yukawa

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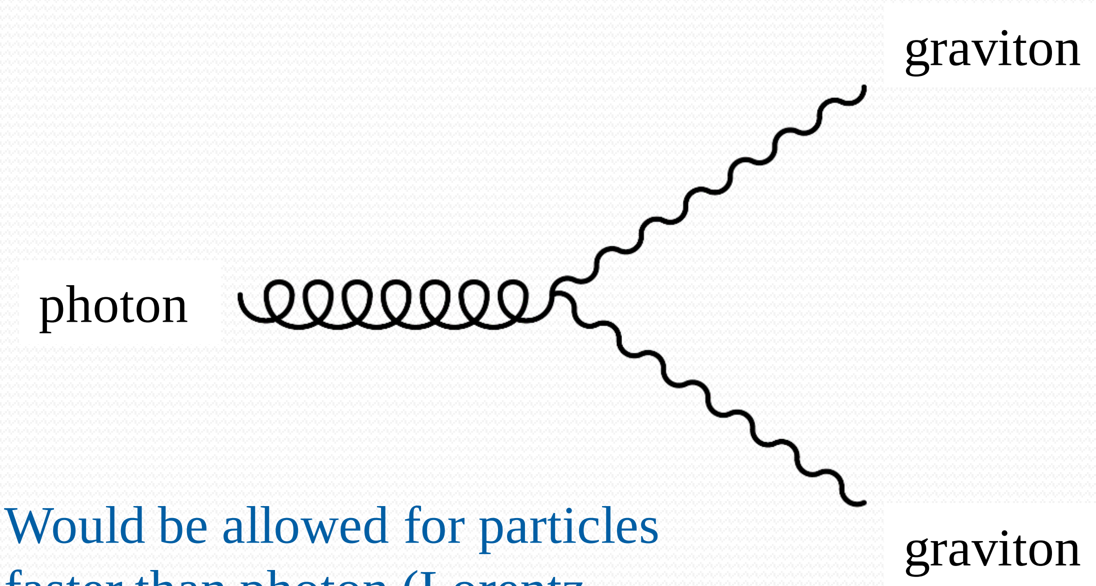
Cherenkov Radiation

Particles traveling faster than GWs could decay into GWs



Forbidden process in
Lorentz invariant models
(if the photon is massless)

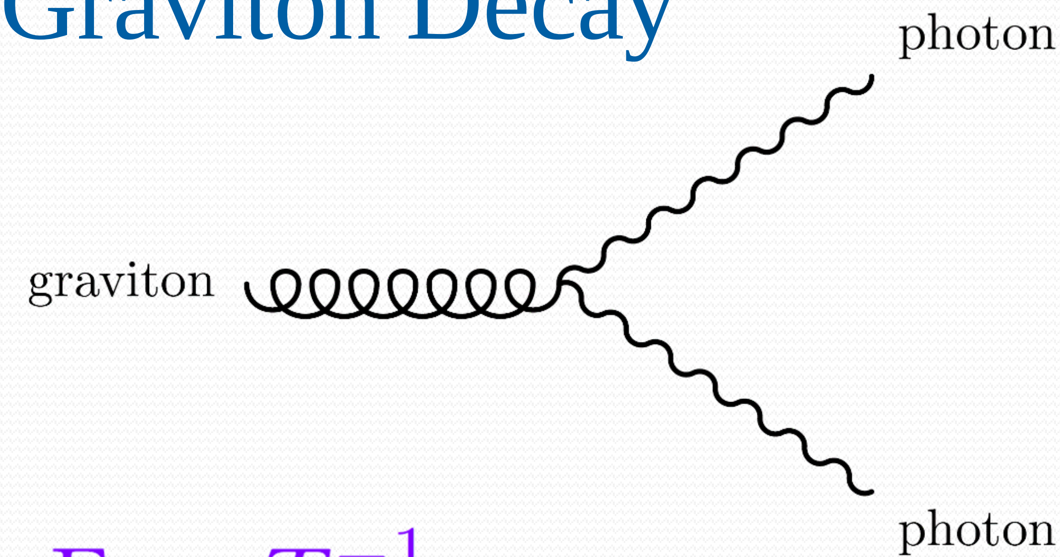
Would be allowed for particles
faster than photon (Lorentz
violating models)

eg. Blas, Ivanov, Sawicki, Sibiryakov1602.04188

Can be used to put bounds on the difference of speeds
but those translate into very weak bounds on the graviton mass

Graviton Decay

If the graviton has a mass:



aLIGO direct detection: $\Gamma \ll T^{-1}_{\mathrm{GW\ travel\ time}}$

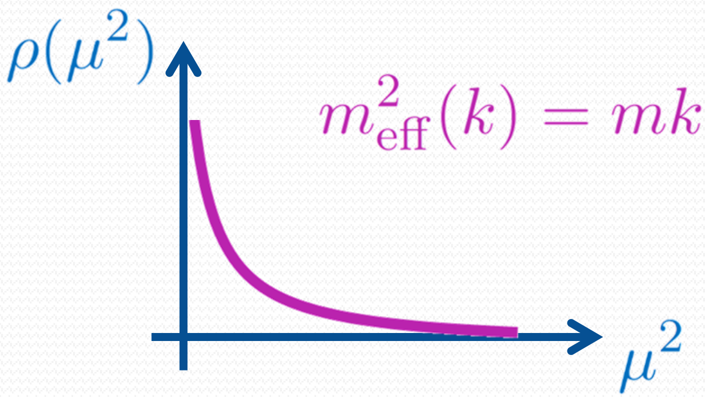
Very weak bound…

Constraints from cosmology: $\Gamma \ll H_{\mathrm{today}}$

$$\mathrm{Im}[m_g^2] \ll H_{\mathrm{today}} \sqrt{\mathrm{Re}[m_g^2]}$$

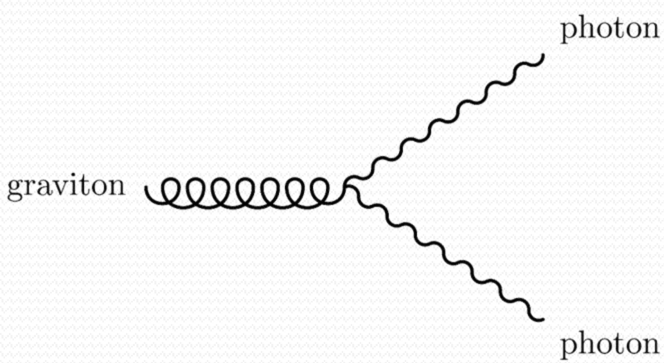
Graviton Decay

If the graviton is a resonance (eg. in DGP, Cascading Gravity,…)



The graviton already has a finite lifetime even without taking into account its possible decay into photons

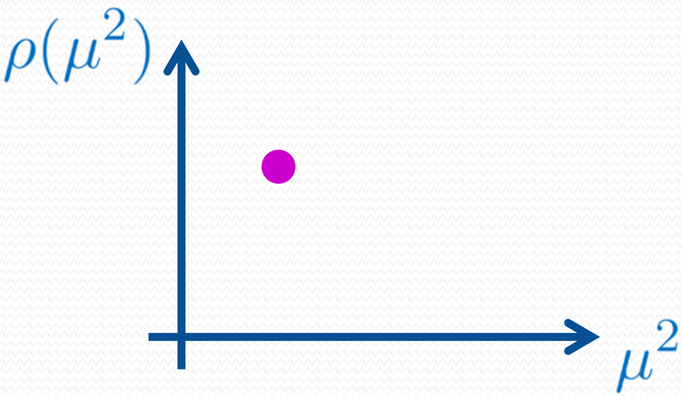
$$m \lesssim H_{\rm today}$$



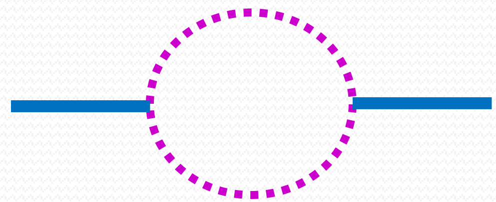
Graviton Decay

For a hard mass graviton

At tree-level, $\mathrm{Im}[m_g^2] = \Gamma = 0$



loop-effect on graviton self-energy

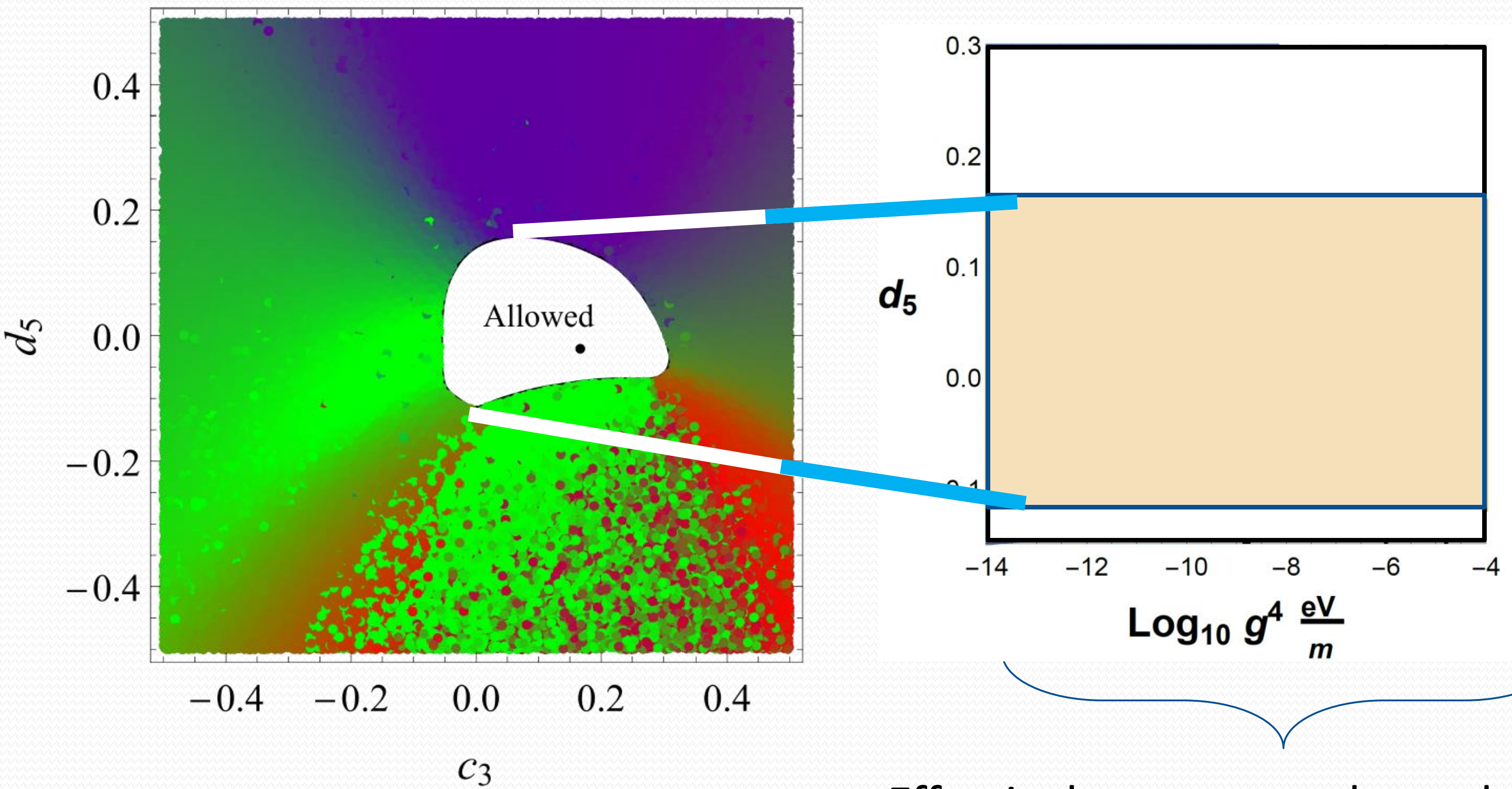


$$\Gamma \sim N \frac{m_g^3}{M_{\mathrm{Pl}}^2}$$

N: total number of light particles that may exist
(photon + axion, hidden sector not subject to SM constraints,…)

$$m_g \lesssim 10^7 \mathrm{eV} \times N^{-1/3}$$

“Standard” Positivity bounds

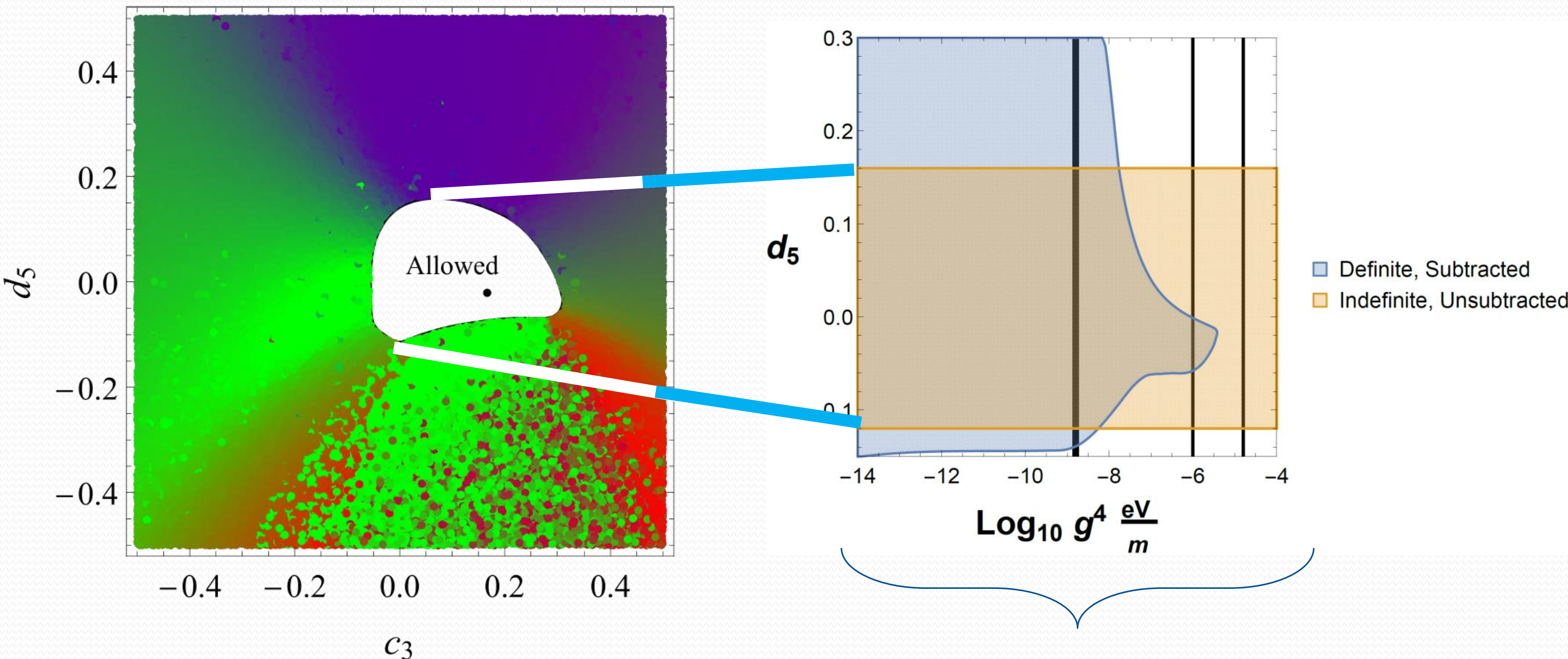


Effectively measures the scale of the cutoff

Cheung & Remmen, JHEP 1604 (2016)

Improved positivity bounds

$$B^{(2,0)}(0) > \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\,A(\mu,t)}{(\mu-2m^2)^3} = \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \sqrt{1-\frac{4m^2}{\mu^2}}\frac{\mu\sigma_{\mathrm{total}}(\mu)}{(\mu-2m^2)^3}\,.$$



Effectively measures the scale of the cutoff

CdR, Melville, Tolley, 1710.09611

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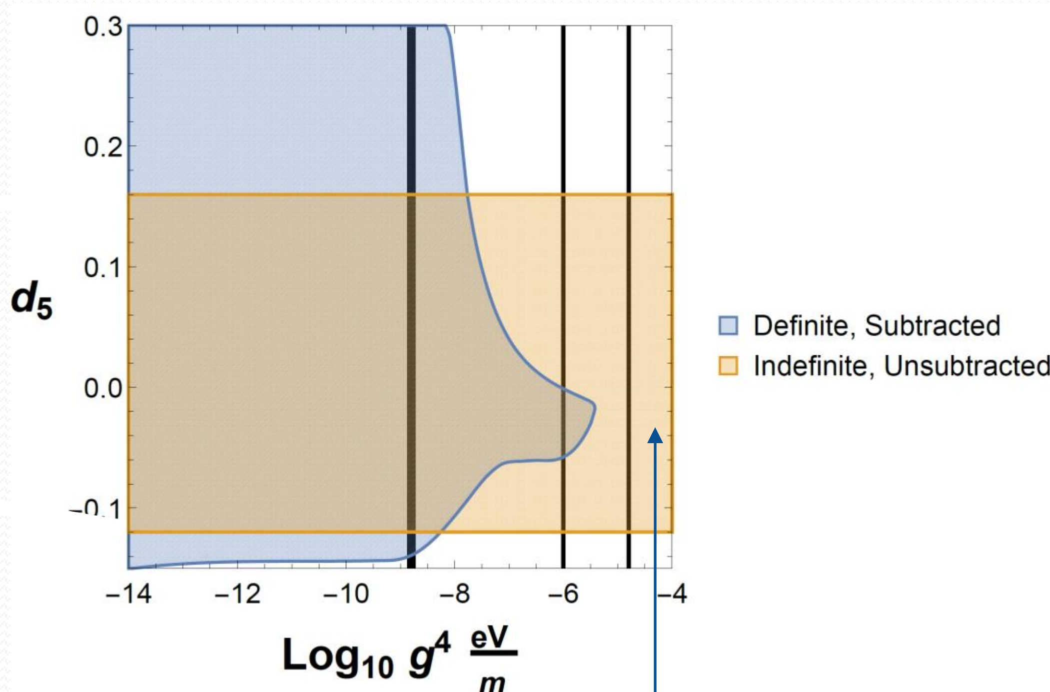

Bellazzini, Riva, Serra, Sgarlata 1710.0253
Assuming a large enough g, ***the improved positivity*** bounds can rule out the allowed parameter space

CdR, Melville, Tolley, 1710.09611: the **improved positivity bounds** should be seen as a **constrain on the value of the cutoff !**