Asymptotic symmetries and their consequences

Marika Taylor

Mathematical Sciences and
STAG research centre, Southampton

February 20, 2019

Introduction

- This talk will be about analysing the asymptotics of AdS, dS and Ricci flat spacetimes.
- For **AdS**, the asymptotics play a crucial role in the holographic duality.
- For **asymptotically flat** spacetimes, the asymptotics are relevant to soft scattering theorems and gravitational memory effects.
- **dS** is relevant to our Universe!

STAG RESEARCH CENTRE

Introduction

- For asymptotically flat spacetimes, much of the analysis is tied specifically to four dimensions e.g. use of two dimensional celestial sphere.
- Long standing question: soft scattering theorems exist in all dimensions, but they have not been related to asymptotic symmetries for $d > 4$.
- What is the asymptotic symmetry structure in $d > 4$?

STAG RESEARCH CENTRE

Introduction

- The asymptotic analysis for AdS and flat spacetimes seems very different:
  - Timelike versus null conformal boundaries;
  - Fefferman-Graham versus Bondi-Sachs parameterization;
  - Sources/expectation values of operators versus Bondi mass, news etc.
- How are these analyses related? Can lessons from AdS holography be applied to flat spacetimes?

Introduction

- Anti-de Sitter and de Sitter are related by analytic continuations.
- Even if there is no dS/CFT, can we systematically reconstruct asymptotically locally dS spacetimes?
- Does this have implications for gravitational waves in an accelerating universe?

References

1. Federico Capone and Marika Taylor
"*Symmetries of asymptotically flat spacetimes*", 19xx.xxxxx
2. Aaron Poole, Kostas Skenderis and Marika Taylor
"*(A)dS*$_4$ in Bondi gauge", 1812.05369
and work in progress.

Complementary works include 1712.01204 by Pate, Raclariu and Stominger.

STAG RESEARCH CENTRE

Outline

- **Bondi-Sachs analysis**
- Asymptotically AdS spacetimes
- Asymptotic analysis for $d > 4$

Bondi gauge

- The 4d Minkowski metric can be written as

$$ds_M^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

where $u = (t - r)$ is the retarded time and $\gamma_{z\bar{z}}$ is the round metric on S^2.

- An asymptotically (locally) flat metric can be expressed in Bondi gauge as

$$ds^2 = -Xdu^2 - 2e^{2\beta}dudr + h_{AB}\left(d\theta^A + U^A du\right)\left(d\theta^B + U^B du\right)$$

where $ds^2 \to ds_M^2$ (locally) as $r \to \infty$.

STAG RESEARCH CENTRE

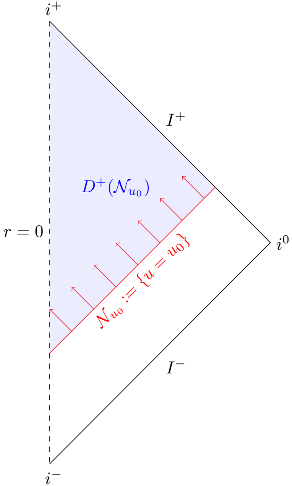
Bondi gauge for asymptotically flat 4d spacetimes

- Imposing one gauge condition on h_{AB},

$$\det\left(\frac{h_{AB}}{r^2}\right) = 1$$

  there are six unknown functions in (X, β, h_{AB}, U^A).
- The Einstein equations split into "main" equations and "supplementary" equations.
- If the latter are satisfied on a constant u hypersurface, they are automatically satisfied everywhere.

Bondi gauge for asymptotically flat spacetimes



- Given h_{AB} at constant $u = u_0$, main equations determine other metric functions.
- Final main equation determines the u evolution of h_{AB}.

STAG RESEARCH CENTRE

Asymptotically flat 4d metrics

- The asymptotic expansion of the metric as $r \to \infty$ is

$$ds^2 = ds_M^2 + \frac{2m_B}{r} du^2 + rC_{zz} dz^2 + rC_{\bar{z}\bar{z}} d\bar{z}^2$$

$$+ D^z C_{zz} du dz + D^{\bar{z}} C_{\bar{z}\bar{z}} du d\bar{z}$$

$$+ \frac{1}{r} \left(\frac{4}{3} (N_z + u \partial_z m_B) - \frac{1}{4} \partial_z (C_{zz} C^{zz}) \right) du dz + \cdots$$

where we use complex coordinates on the S^2.

- The highlighted terms indicate integration functions that arise in integrating the Einstein equations.

Asymptotic coefficients

- Bondi mass aspect $m_B(u, z, \bar{z})$; integrate over S^2 to get total Bondi mass M_B at time u.
- Traceless tensor $C_{AB}(u, z, \bar{z})$: captures gravitational memory effects, soft scattering theorems and gravitational waves.
- Angular momentum aspect $N^A(u, z, \bar{z})$; integrate over S^2 to get total angular momentum.

STAG RESEARCH CENTRE

Dynamical equations

- The Einstein equations give the evolution of the Bondi mass aspect

$$\partial_u m_B = \frac{1}{4}\left(D_A D_B(\partial_u C^{AB}) - \partial_u C^{AB}\partial_u C_{AB}\right)$$

  with a similar equation for the angular momentum aspect.
- Hence a non-zero news $N_{AB} = \partial_u C_{AB}$ leads to mass non-conservation (gravitational waves).

Superrotations and C_{AB}

- Superrotations act as meromorphic transformations on the S^2 coordinates i.e.

$$z \to \mathcal{Y}(z) \qquad \bar{z} \to \bar{\mathcal{Y}}(\bar{z})$$

- Such transformations change C_{AB}: ΔC_{zz} is expressed in terms of the Schwarzian derivative of $\mathcal{Y}$.
- Associated with the superrotations are (finite) superrotation charges. (Barnich and Troessart)

Conservation of superrotation charges $\leftrightarrow$ soft scattering theorems for scattering amplitudes. (Strominger et al)

STAG RESEARCH CENTRE

Outline

- Bondi-Sachs analysis for asymptotically flat 4d spacetimes
- **Asymptotically AdS and dS spacetimes**
- Asymptotically locally flat analysis for $d > 4$

Asymptotically AdS spacetimes

- A convenient parameterisation is Fefferman-Graham coordinates

$$ds^2 = \frac{d\rho^2}{\rho^2} + \frac{1}{\rho^2} g_{ij}(x,\rho) dx^i dx^j$$

in the neighbourhood of the conformal boundary $\rho \to 0$.

- Einstein equations expressed in terms of derivatives of g

$$g^{ij} \partial_\rho^2 g_{ij} = 0$$

and so on.

Asymptotically locally AdS$_4$ spacetimes

- In Fefferman-Graham coordinates:

$$ds^2 = \frac{d\rho^2}{\rho^2} + \frac{1}{\rho^2}\left(g_{(0)ij} + g_{(2)ij}\rho^2 + g_{(3)ij}\rho^3 + \cdots\right) dx^i dx^j$$

- Near boundary expansion reconstructed from $g_{(0)ij}$ (background metric for dual field theory) and $g_{(3)ij}$ (stress energy tensor for dual field theory).
- All other terms ($g_{(2)}$ etc) are expressed in terms of curvatures of this data (de Haro, Solodukhin, Skenderis, 2000).

STAG RESEARCH CENTRE

Bondi gauge

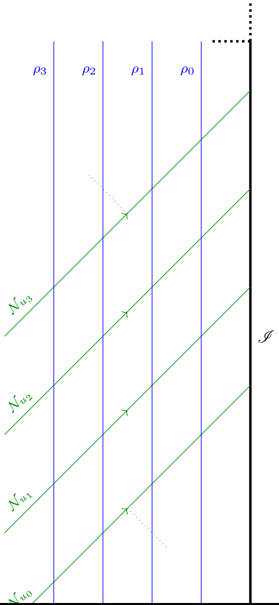
- We can also parameterise an asymptotically locally AdS_4 spacetime in Bondi gauge as

$$ds^2 = -Xdu^2 - 2e^{2\beta} du dr + h_{AB} \left(d\theta^A + U^A du \right) \left(d\theta^B + U^B du \right)$$

as $r \to \infty$.

- Now the spacetime is foliated by hypersurfaces of constant u in the vicinity of the conformal boundary.

Bondi gauge



- Nested structure of Einstein equations persists for $\Lambda \neq 0$ (although is usually broken by additional matter).
- Bondi gauge is natural choice for numerical AdS simulations involving horizons. (See e.g. Chesler and Yaffe)

STAG RESEARCH CENTRE

Results of asymptotic analysis

- The metric functions admit analytic expansions in $1/r$

$$X = r^2 \sum_{n=0}^{\infty} \frac{X_{(n)}(u, \theta^A)}{r^n} \qquad \beta = \sum_{n=0}^{\infty} \frac{\beta_{(n)}(u, \theta^A)}{r^n}$$

$$h_{AB} = \sum_{n=0}^{\infty} \frac{h_{AB(n)}(u, \theta^A)}{r^n} \qquad U_A = \sum_{n=0}^{\infty} \frac{U_{A(n)}(u, \theta^A)}{r^n}$$

- The entire expansion can be determined algebraically from knowledge of coefficients at order zero and three.

Interpretation

1. The cosmological constant (as expected) changes the structure of the asymptotic expansions.
2. The integration functions at order zero correspond to the (constrained) background metric for the 3d QFT.
3. The other integration functions at order three would be termed "Bondi mass aspect" and "Bondi angular momentum aspect" by relativists but are related algebraically to the dual QFT stress tensor.

STAG RESEARCH CENTRE

Boundary metric: asymptotically locally AdS

- Non-trivial $(X_{(0)}(u, \theta^A), \beta_{(0)}(u, \theta^A), \cdots)$ corresponds to a curved, time dependent background metric for the CFT.
- From an AdS/CFT perspective, the determinant restriction on the S^2 is very unnatural: excludes ' breathing" modes for the sphere, that are e.g. relevant for quark gluon plasma simulations.

STAG RESEARCH CENTRE

Bondi aspects v QFT data

- Relativists define a Bondi mass aspect $m_B(u, z, \bar{z}) \sim X_{(3)}(u, z, \bar{z})$, in analogy to asymptotically flat spacetimes.
- For asymptotically locally AdS_4 the relation between m_B and the dual stress energy tensor is in general (very) complicated:

$$T_{tt} \sim m_B + \mathcal{T}(X_{(0)}, \beta_{(0)}, U_{A(0)}, h_{AB(0)})$$

- Lesson: Bondi mass aspect is not natural from holographic perspective!

STAG RESEARCH CENTRE

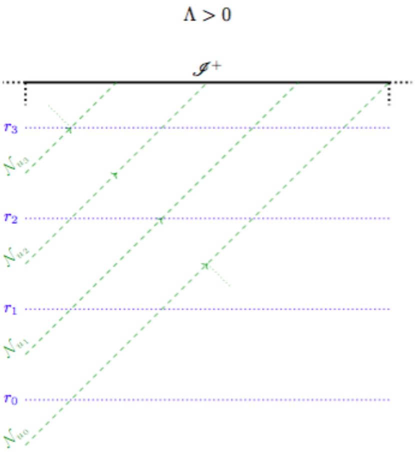
Applications of Bondi → QFT map

- For simulations carried out in Bondi gauge, one can now read off directly the QFT data.
- Analysis is relevant for AdS Robinson-Trautmann metrics

$$ds^2 = F(r, u, z, \bar{z})du^2 - 2dudr + 2r^2 e^{\phi(u,z,\bar{z})} dz d\bar{z}$$

Admit geodesic congruence with zero shear, twist and non-vanishing divergence; relax to black hole at late times; describe QFT thermalization.

dS analysis



- Analytic continuation: general notion of asymptotically locally de Sitter (generalizes Ashtekar et al).
- Associated conserved charges: interpretation for gravitational waves?

Outline

- Bondi-Sachs analysis of asymptotically flat spacetimes
- Asymptotically Anti-de Sitter spacetimes
- **Asymptotically flat analysis for $d > 4$**

STAG RESEARCH CENTRE

Bondi gauge analysis in higher dimensions

- We can again parameterise the metric as

$$ds^2 = -Xdu^2 - 2e^{2\beta}dudr + h_{AB}\left(d\theta^A + U^A du\right)\left(d\theta^B + U^B du\right)$$

  in terms of metric functions (X, β, h_{AB}, U^A).
- The nested structure of the Einstein equations persists.
- The key question is again the boundary conditions for $r \to \infty$ for $d > 4$.

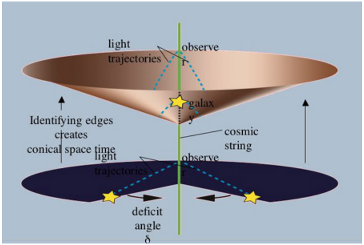
Asymptotic local flatness

Lessons from AdS:

- Imposing spacetime is asymptotic to Minkowski is over-restrictive.
- The determinant condition on the three sphere is unnatural.

Appropriate boundary conditions are asymptotically locally flat, allowing defects on the celestial sphere.

Superrotations and cosmic strings



- Strominger and Zhiboedov proposed that $d = 4$ superrotations should be interpreted as cosmic strings piercing the celestial sphere.
- Follows earlier work by Griffiths and Podolsky.

Superrotations and cosmic strings

- In $d = 4$ cosmic string metric

$$ds^2 = -du^2 - 2dudr + r^2(d\theta^2 + K^2 \sin^2\theta d\phi^2)$$

  where $K^2 = 1 - 2\delta$ can be expressed as $r \to \infty$ as

$$ds^2 = ds_M^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + \cdots$$

  with C_{AB} determined by a meromorphic transformation dependent on δ.
- In $d = 4$ superrotations arise at the same asymptotic order as gravitational wave effects.

Cosmic $(d-3)$-branes

- In general dimensions, cosmic $(d-3)$ branes are Ricci flat, except at location of brane, generalizing Vilenkin analysis.
- Relevant defects are hence cosmic branes

$$ds^2 = -du^2 - 2dudr + r^2(d\theta^2 + K^2 \sin^2\theta d\phi^2 + \cos^2\theta d\Omega^2_{d-4})$$

where again $K^2 = 1 - 2\delta$ characterizes the deficit.

STAG RESEARCH CENTRE

Cosmic $(d-3)$-branes

- Cosmic brane effects do not arise as the same asymptotic order as gravitational waves e.g. d=5

$$ds^2 = -du^2 - 2dudr$$

$$+r^2\left(h_{AB} + \frac{1}{r^{\frac{3}{2}}}C_{AB} + \frac{1}{r^2}\tilde{C}_{AB}\right)d\theta^A d\theta^B + \cdots$$

- C_{AB} gravitational waves; h_{AB} and $\tilde{C}_{AB}$ cosmic branes.
- Can the latter be related to soft scattering theorems?

STAG RESEARCH CENTRE

Summary and conclusions

- Asymptotic analysis for AdS in Bondi gauge useful for holography in dynamical situations.
- Asymptotically locally flat: integration functions associated with analogues of superrotations exist in all dimensions.
- For $d \neq 4$ these occur at a different order in the asymptotic expansion to gravitational waves.
- Do associated symmetry charges lead to soft scattering theorems?

STAG RESEARCH CENTRE