UNIVERSITÀ DI PISA

Systematic errors in high precision gravity measurements by light-pulse atom interferometers on ground and in space

Anna M. Nobili

Dipartimento di Fisica "E. Fermi", Università di Pisa & INFN – Pisa, Italia

Alberto Anselmi

Thales Alenia Space Italy – Torino, Italia

1st EPS Conference on Gravitation

Roma, 19-21 February 2019

Fundamental Physics with Light-pulse Atom Interferometers (AIs)

Light-pulse AIs & fundamental physics

By measuring the gravitational accelerations of free-falling atoms, AIs can:

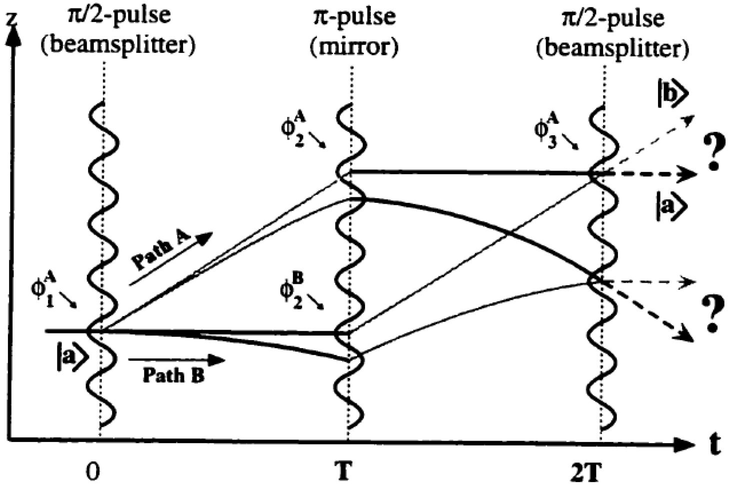
- Measure the absolute value of the local gravitational acceleration g (on ground)
- Test the Universality of Free Fall (UFF) or Weak Equivalence Principle (WEP), in the field of Earth (requires atoms of two different species, can be done on ground and in space inside s/c in low Earth orbit)

Arranged as gravity gradiometers AIs can:

- Measure the universal constant of gravitation G (on ground)
- Detect the effect of gravitational waves (on ground and in space)
- "Measure" the Earth from space (space geodesy)

Light-pulse AIs:
the principle, the theoretical prediction of the acceleration measured and the acceleration measurement error

Light-pulse AIs: the principle



- Light-pulse AIs are based on quantum mechanics

- As atoms fall, by means of **3 atom-light interactions** (Raman laser pulses) the atomic wave packet is split, redirected, and finally recombined.
- The phase that the atoms acquire during the interferometer sequence is proportional to the gravitational acceleration they are subjected to:

- **straight lines: without gravitational acceleration**
- **curved lines: with gravitational acceleration**

Analogy with falling corner-cube absolute gravimeter (I)

Metrologia **55** (2018) 1

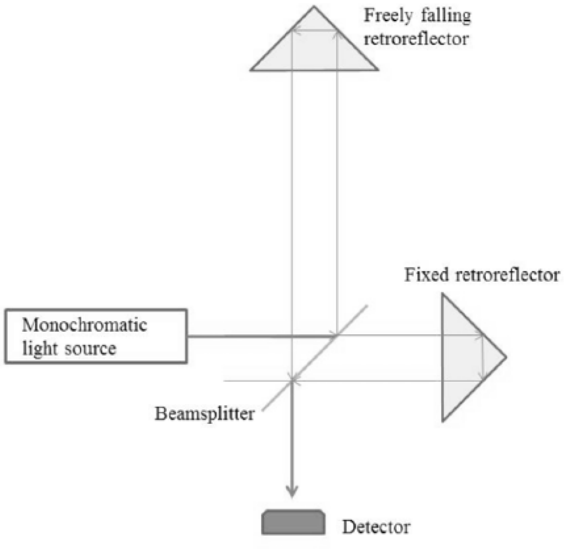


Figure 1. Simplified diagram of a freely falling cube gravimeter.

- In a light-pulse AI a retroreflected laser beam is referenced to an atomic transition; this defines a ruler (whose graduations are spaced as λ of the laser) to which the trajectory of the free-falling atoms is compared so that its acceleration is measured.
- Analogy with classical absolute gravimeters in which the trajectory of a free-falling corner-cube is measured by a laser interferometer to obtain its gravitational acceleration g_{meas}.

Analogy with falling corner-cube absolute gravimeter (II)

- *Advantage*

- In AIs atoms provide at the same time the test mass and the read-out of its motion

- In the classical gravimeter a laser interferometer reads the motion of a falling body equipped as corner cube reflector

- *Disadvantage*

- The falling cube provides hundreds up to a thousand time-position measurements per drop

- AIs have only 3 position measurements per drop at their disposal, at pulse times $0, T, 2T$, from which to obtain the free-fall acceleration ... *and this is going to affect the measurement*

$\delta\phi$ *and* g_{meas} *can be predicted with a purely classical model*

A. Peters, PhD Thesis 1998 (Nature 1999, Metrologia 2001):

- The scale factor between the phase shift $\delta\phi$ and g_{meas} depends on Raman vector k and pulse time T, **which are both controlled experimentally**:

$$\delta\phi = kT^2 g_{meas}$$

- *"We can simply ignore the quantum nature of the atom and model it as a classical point particle that carries an internal clock and can measure the local phase of the light field"*
- **The purely classical description yields the same result as the exact path integral treatment (in closed form!).** The solution is then expanded in powers of the gravity gradient γ for convenience
- **The procedure is based on only 3 position measurements of the atom available at times $0, T, 2T$ of the 3 Raman pulses**
- Quantum mechanical details are needed only to account for smaller effects...

(e.g. finite length of pulse time)

The classical model revisited (I)

- *Include the possibility of WEP violation (hence UFF violation) in the model:*

$$m^g_{A,B} = m^i_{A,B}(1 + \eta_{A,B})$$

$\eta_A = \eta_B = 0 \quad$ if WEP holds

- *Eq. of motion and solution to first order in gradient* $\gamma = 2g_\circ/R_\oplus \simeq 3.1 \cdot 10^{-6}\,\mathrm{s}^{-2}$:

$$\ddot{z}_{A,B} \simeq -\Big[g_\circ(1 + \eta_{A,B}) + \gamma\Big(\frac{1}{2}g_\circ t^2 - v^\circ_{A,B}t - z^\circ_{A,B}\Big)\Big]$$

$$z_{A,B}(t) \simeq z^\circ_{A.B} + v^\circ_{A,B}t - \frac{1}{2}g_\circ(1 + \eta_{A,B})t^2 - \gamma t^2\Big(\frac{1}{24}g_\circ t^2 - \frac{1}{6}v^\circ_{A,B}t - \frac{1}{2}z^\circ_{A,B}\Big)$$

$z^\circ_{A,B}, v^\circ_{A,B}$ position and velocity errors of atoms at release, Initial Condition Errors - ICE

The classical model revisited (II)

- *Definition of phase shift measured by AI:*

$$\delta\phi_{A,B} = [\phi_{A,B}(2T) - 2\phi_{A,B}(T) - \phi_{A,B}(0)] = kT^2[z_{A,B}(2T) - 2z_{A,B}(T) - z_{A,B}(0)] =$$

$$= kT^2\Big[\frac{z_{A,B}(2T) - z_{A,B}(T)}{T^2} - \frac{z_{A,B}(T) - z_{A,B}(0)}{T^2}\Big]$$

- *With the solution $z_{A,B}(t)$ the predicted phase shift and $g_{A,B\,meas}$ are:*

$$\delta\phi_{A,B}(T) \simeq -kT^2\Big[g_\circ(1+\eta_{A,B}) + \gamma\Big(\frac{7}{12}g_\circ T^2 - v^\circ_{A,B}T - z^\circ_{A,B}\Big)\Big]$$

$$g_{A,B\,meas} \simeq g_\circ(1+\eta_{A,B}) + \gamma\Big(\frac{7}{12}g_\circ T^2 - v^\circ_{A,B}T - z^\circ_{A,B}\Big)$$

with $\eta_{A,B} = 0$ (no violation) this is the correct result, accepted by all scientists, for the free-fall acceleration of the atoms measured by AIs, no matter which approach is used to compute it, exact path integral or purely classical

Systematic error in the acceleration measured by AIs

- *Free-fall acceleration at time T: theory*

$$g_{A,B\,theory} \simeq g_\circ(1+\eta_{A,B}) + \gamma\Big(\frac{1}{2}g_\circ T^2 - v^\circ_{A,B}T - z^\circ_{A.B}\Big)$$

- *Free-fall acceleration at time T: measured*

$$g_{A,B\,meas} \simeq g_\circ(1+\eta_{A,B}) + \gamma\Big(\frac{7}{12}g_\circ T^2 - v^\circ_{A,B}T - z^\circ_{A,B}\Big)$$

- *Systematic measurement error:*

$$\Delta a = \frac{1}{12}\gamma g_\circ T^2$$

Fractional error:

$$\frac{\Delta a}{g_\circ} = \frac{1}{12}\gamma T^2 \qquad (\gamma \simeq 3.1\cdot 10^{-6}\,s^{-2}\ ,\quad T \lesssim 1\,\mathrm{s}) \Rightarrow \text{not negligible}\ldots$$

UNIVERSITÀ DI PISA

Acceleration error in space

- *E.g. experiment inside ISS* ($r \simeq 6800\,\mathrm{km}$, $e \simeq 0$, $g_{orb} \simeq 8.7\,\mathrm{ms}^{-2}$), *Earth pointing, AI axis along radial direction, atoms dropped at nominal distance h from CM of s/c where*

$$\gamma_{orb} = 3\frac{g_{orb}}{r} \simeq 3.8 \cdot 10^{-6}\,\mathrm{s}^{-2} \qquad\qquad a_{tide} = \gamma_{orb} h$$

- *Free-fall acceleration at time T: theory* — *Nobili et al., GRG (2008)*

$$a_{A,B\,theory} \simeq a_{tide} + g_{orb}\eta_{A,B} + \gamma_{orb}\left(\frac{1}{2}a_{tide}T^2 - \Upsilon^{\circ}_{A,B}T - \zeta^{\circ}_{A.B}\right)$$

- *Free-fall acceleration at time T: measured*

$$a_{A,B\,meas} \simeq a_{tide} + g_{orb}\eta_{A,B} + \gamma_{orb}\left(\frac{7}{12}a_{tide}T^2 - \Upsilon^{\circ}_{A,B}T - \zeta^{\circ}_{A,B}\right)$$

- *Systematic measurement error:*

$$\Delta a_{orb} = \frac{1}{12}\gamma a_{tide}T^2 \quad , \quad \textit{Fractional error} \quad \frac{\Delta a_{orb}}{a_{tide}} = \frac{1}{12}\gamma_{orb}T^2 \quad \textbf{as on ground}$$

Position and velocity errors at atoms' release: effect, reduction & role of the acceleration measurement error

Reduction of the effect of ICE and gravity gradient (I)

- *Effect of gravity gradient:*

PRL **118,** 160401 (2017) PHYSICAL REVIEW LETTERS week ending 21 APRIL 2017

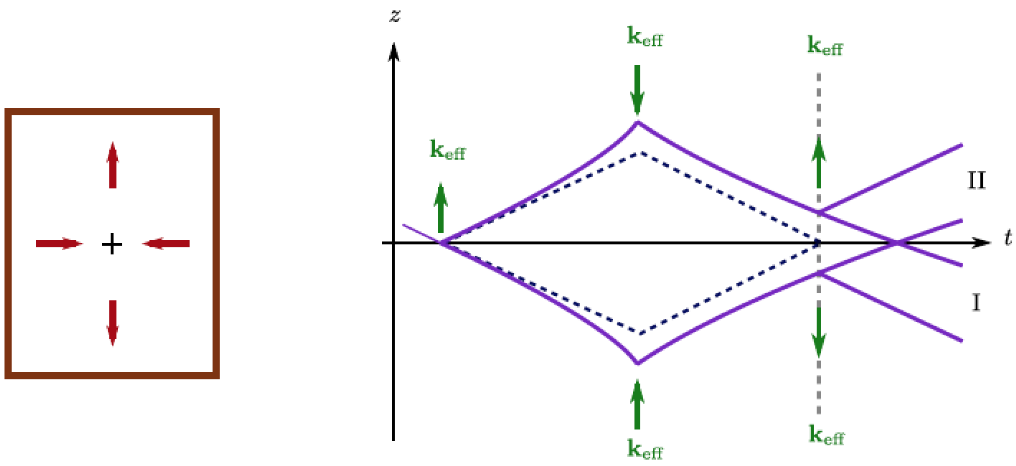


FIG. 2. Central trajectories for a Mach-Zehnder interferometer in the presence of gravity gradients as seen in a suitable freely falling frame. The tidal forces tend to open up the trajectories compared to the case without gravity gradients (dashed lines). The momentum transfer from the laser pulses is also indicated for the different branches. The picture on the left shows the tidal forces experienced by objects in a freely falling frame (the *Einstein elevator*) as a consequence of the gravity gradient.

Roura, PRL (2017)

Reduction of the effect of ICE and gravity gradient (II)

- An extra Δk at the second (middle) laser pulse compensates the gradient effect exactly if *(Roura, PRL (2017))*: $k_2 = k + \Delta k = k + \frac{1}{2}\gamma T^2 k$ exactly

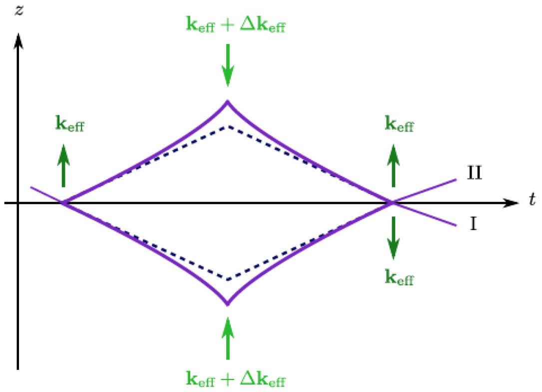


FIG. 3. Central trajectories for the same situation depicted in Fig. 2 but with a suitable adjustment of the momentum transfer from the second laser pulse so that a closed interferometer, with vanishing relative displacement between the interfering wave packets in each port, is recovered.

Gradient term that is not compensated: on ground

- *In reality compensation is not exact; in addition a residual un-compensated acceleration remains:*

$$\delta\phi^{\Delta k}_{A,B}(T) \simeq -kT^2\Big[g_\circ(1+\eta_{A,B}) - \gamma_{res}(z^\circ_{A,B} + v^\circ_{A,B}T) + \frac{1}{12}\gamma g_\circ T^2)\Big]$$

$\gamma_{res} \simeq 10^{-2}\gamma$ demonstrated experimentally by fine tuning of the laser frequency

(Kasevich's group, PRL (2018))

- *Term with un-compensated gradient remains:* $\Delta a = \frac{1}{12}\gamma g_\circ T^2$

Pointed out in a Comment by Dubetsky PRL (2018), accepted in Roura's Reply PRL (2018)

This work: The reason why it is not compensated is because the motion of the atoms is as predicted by theory, while the phase shift & acceleration used to compensate for the effect of gradient on their motion by fine tuning of laser frequency is wrong by this term $\Downarrow$
In fact, it SHOULD NOT be compensated!

Gradient term that is not compensated: in space

- *In space* $k_2 = k + \Delta k = k + \frac{1}{2}\gamma_{orb}T^2 k$ *for "exact" compensation:*

$$\delta\Phi^{\Delta k}_{A,B\,orb}(T) \simeq -kT^2\Big[a_{tide} + g_{orb}\eta_{A,B} - \gamma_{orb\,res}(\zeta^{\circ}_{A,B} + \Upsilon^{\circ}_{A,B}T) + \frac{1}{12}\gamma_{orb}a_{tide}T^2\Big]$$

$$\Delta a = \frac{1}{12}\gamma_{orb}a_{tide}T^2$$

fractional error $\qquad \dfrac{\Delta a}{a_{tide}} = \dfrac{1}{12}\gamma_{orb}T^2 \qquad$ as on ground

Consequences of the acceleration measurement error

Effect of acceleration error on absolute measurement of g

- Freely falling corner cube retroreflector monitored by laser interferometry (FG5 absolute gravimeter):
  $\frac{\Delta g}{g} = 1.1 \times 10^{-9}$ *Niebauer et al., Metrologia (1995)*

- Dropped Cs cold atom cloud monitored by light-pulse atom interferometry: $\frac{\Delta g}{g} \simeq 3 \times 10^{-9}$
  Peters, Chung & Chu, Nature (1999); Metrologia (2001)

$$g_{meas} \simeq g_\circ + \gamma\left(\frac{7}{12} g_\circ T^2 - v^\circ T - z^\circ\right)$$

At this level (and with $T = 160\,\mathrm{ms}$) the systematic effects to order γ have required careful checks by means of many measurements in different experimental conditions (e.g. with different free fall times) to model and reduce them below the target.... then, acceleration error $\frac{1}{12}\gamma g_\circ T^2$ also below the target

- If T can be increased, for better sensitivity, and Roura's scheme is applied to reduce the gradient and relax requirement on ICE (as $\gamma/\gamma_{res} \simeq 10^2$), then the acceleration error $\frac{1}{12}\gamma g_\circ T^2$ becomes dominant:

$$g_{meas}^{\Delta k} \simeq g_\circ - \gamma_{res}\left(z^\circ + v^\circ T\right) + \frac{1}{12} g_\circ \gamma T^2$$

Dominant errors in tests of UFF by dual AI on ground (I)

- **Different atoms A, B are dropped in a Dual Atom Interferometer (DAI) individual phase shifts are measured, their difference is calculated and checked to detect a violation. The gradient term with $7/12$ or $1/12$ coefficient cancels out in the difference.**
- **Unless the same laser is used for both species the time pulse interval may differ by ΔT, and there is a requirement on it:**

$$\delta\phi_B^{\Delta T} - \delta\phi_A \simeq -kT^2\left[g_\circ\left(\eta + 2\frac{\Delta T}{T}\right) - \gamma\left(z_B^\circ - z_A^\circ + (v_B^\circ - v_A^\circ)T\right)\right]$$

$\frac{\Delta T}{T}$ competes directly with $\eta = \eta_B - \eta_A$ because the phase shift grows as T^2:

$$\frac{\Delta T}{T} < \frac{\eta}{2} = 5 \cdot 10^{-14} \quad \text{for} \quad \eta_{current} \simeq 10^{-13}$$

…only to match current best ground tests by torsion balances. **Extremely hard to meet** $\Rightarrow$

extremely hard to test very different atom species!!

Dominant errors in tests of UFF by dual AI on ground (II)

- Most DAIs test ^{87}Rb and ^{85}Rb, same laser can be used, no synchronization problem (but atoms differ by 2 neutrons only...)
- The dominant error is due to position and velocity offsets at release coupled with γ.
- Roura's scheme can be applied to reduce it:

$$\delta\phi_B^{\Delta k} - \delta\phi_A^{\Delta k} \simeq -kT^2\Big[g_\circ\eta - \gamma_{res}\Big(z_B^\circ - z_A^\circ + (v_B^\circ - v_A^\circ)T\Big)\Big]$$

$$z_B^\circ - z_A^\circ + (v_B^\circ - v_A^\circ)T < \eta\frac{g_\circ}{\gamma_{res}}$$

- **The requirement is relaxed as $\gamma/\gamma_{res} \simeq 10^2$ but initial offsets must meet it in all drops, and must be measured for demonstration, otherwise no violation can ever be claimed**

Nobili, PRA (2016), Roura PRL (2017)

Dominant errors in tests of UFF by dual AI in space

- **If 2 lasers must be used for 2 atoms species, the requirement on ΔT in space in not a show stopper as it is as on ground:**

$$\delta\phi_{B\,orb} - \delta\phi_{A\,orb} \simeq kT^2\Big[g_{orb}\eta + 2\frac{\Delta T}{T}a_{tide} - \gamma_{orb}\Big(\zeta_B^\circ - \zeta_A^\circ + (\Upsilon_B^\circ - \Upsilon_A^\circ)T\Big)\Big]$$

$$\frac{\Delta T}{T} < \frac{\eta}{2}\frac{g_{orb}}{a_{tide}} \simeq 1.1 \cdot 10^{-9}$$

for $\eta = 2 \times 10^{-15}$, $h \simeq 2\,\mathrm{m}$ $(a_{tide} = \gamma_{orb}h)$

- **Applying Roura's scheme to reduce the gradient, it must also be**

$$\zeta_B^\circ - \zeta_A^\circ + (\Upsilon_B^\circ - \Upsilon_A^\circ)T < \eta\frac{g_{orb}}{\gamma_{orb\,res}}$$

Effect of acceleration error in gravity gradiometry with AIs on ground

- A, B same species but vertical separation h; B higher than A where $g_\circ = \frac{GM_\oplus}{R_\oplus^2}$, $\gamma = \frac{2g_\circ}{R_\oplus}$; tidal acceleration in B is $a_{tide} = \gamma h$:

$$g_{\circ B} \simeq g_\circ - \gamma h \qquad \gamma_B \simeq \gamma - \frac{3}{2}\frac{a_{tide}}{R_\oplus}$$

- The gradiometer measures:

$$g_{B\,meas} - g_{A\,meas} \simeq -\Big[a_{tide} + \gamma\Big(\frac{49}{48}a_{tide}T^2 + (v_B^\circ - v_A^\circ)T + z_B^\circ - z_A^\circ\Big)\Big]$$

- While the theory predicts:

$$g_{B\,theory} - g_{A\,theory} \simeq -\Big[a_{tide} + \gamma\Big(\frac{7}{8}a_{tide}T^2 + (v_B^\circ - v_A^\circ)T + z_B^\circ - z_A^\circ\Big)\Big]$$

with a systematic fractional error that cannot be neglected because the gradiometer's signal is a_{tide}, not $g_\circ$:

$$\frac{\Delta a}{a_{tide}} \simeq \frac{7}{48}\gamma T^2$$

Effect of acceleration error in gravity gradiometry with AIs in space

- A, B **same species but radial separation** Δh **:**

$$\delta\phi_B - \delta\phi_A \simeq kT^2\Big[\gamma_{orb}\Delta h + \gamma_{orb}\Big(\frac{7}{12}\gamma_{orb}\Delta h T^2 + (\Upsilon^{\circ}_B - \Upsilon^{\circ}_A)T + \zeta^{\circ}_B - \zeta^{\circ}_A\Big)\Big]$$

- **If Roura's scheme is applied to reduce the gradient, still one un-compensated term remains:**

$$\delta\phi_B^{\Delta k} - \delta\phi_A^{\Delta k} \simeq kT^2\Big[\gamma_{orb}\Delta h + \gamma_{orb\,res}\Big(\Upsilon^{\circ}_B - \Upsilon^{\circ}_A)T + \zeta^{\circ}_B - \zeta^{\circ}_A\Big) + \frac{1}{12}\gamma_{orb}(\gamma_{orb}\Delta h)T^2\Big]$$

with a fractional systematic error to first order in γ_{orb}**, which is the target of the measurement:**

$$\frac{\Delta a}{\gamma_A \Delta h} = \frac{1}{12}\gamma_{orb}T^2$$

Conclusions

1st message

AIs are based on quantum mechanics but their measurement can be predicted with a purely classical model.

2nd message

Within the classical model it becomes apparent, beyond question, that the free-fall acceleration of the atoms is measured only approximately.
This is because –by its very nature– the AI relies on only 3 time-position measurements per drop. Unlike the falling corner-cube reflector of a classical gravimeter which can count on hundreds to a thousand data points per drop to which the theoretical time law is compared to obtain the free-fall measured acceleration.

3rd message

The result is a systematic acceleration error, both on ground and in space, which is important and deserves attention by all scientists using AIs for fundamental physics (on ground and in space) and for space geodesy.