

Loop Quantum Gravity

Planckian discreteness of spacetime geometry and possible implications

EPS Gravity Meeting

Rome

February 2019

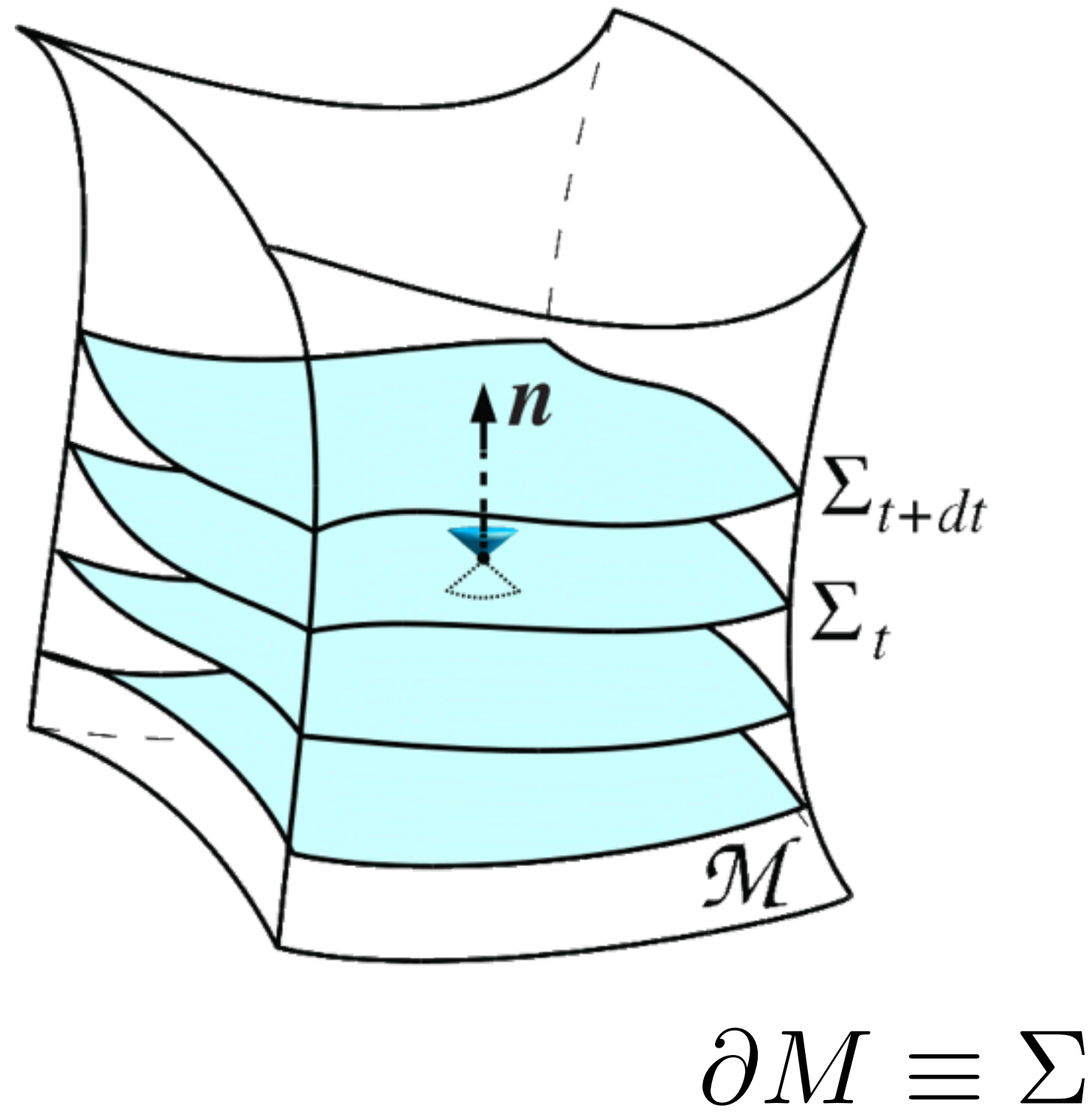
Alejandro Perez

**Centre de Physique Théorique,
Marseille, France.**

Loop Quantum Gravity

Planckian discreteness in a nut-shell

The phase space of GR in connection variable

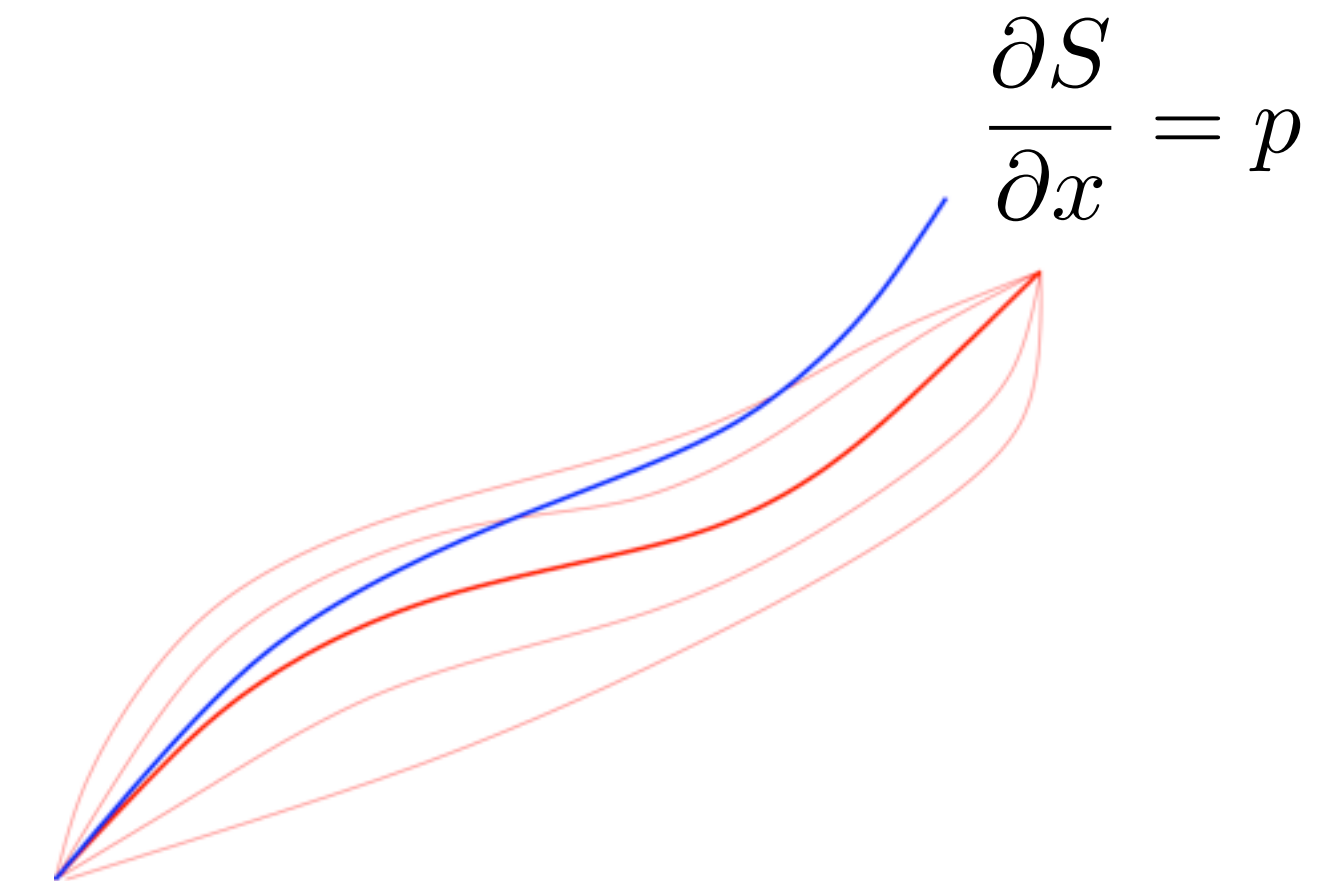


$$S[e, \omega] = \int_M \epsilon_{IJKL} e^I \wedge e^K \wedge F^{KL}(\omega)$$

$$\begin{aligned} \delta S &= \int_M 2\delta e^I (\epsilon_{IJKL} \wedge e^K \wedge F^{KL}(\omega)) + e^I \wedge e^K \wedge d_\omega(\delta\omega^{KL}) = \\ &= \int_M 2\delta e^I (\epsilon_{IJKL} \wedge e^K \wedge F^{KL}(\omega)) - (d_\omega(\epsilon_{IJKL} e^I \wedge e^K)) \wedge \delta\omega^{KL} \\ &+ \underbrace{\int_{\partial M} (\epsilon_{IJKL} e^I \wedge e^K) \wedge \delta\omega^{KL}}_{p\delta x} \end{aligned}$$

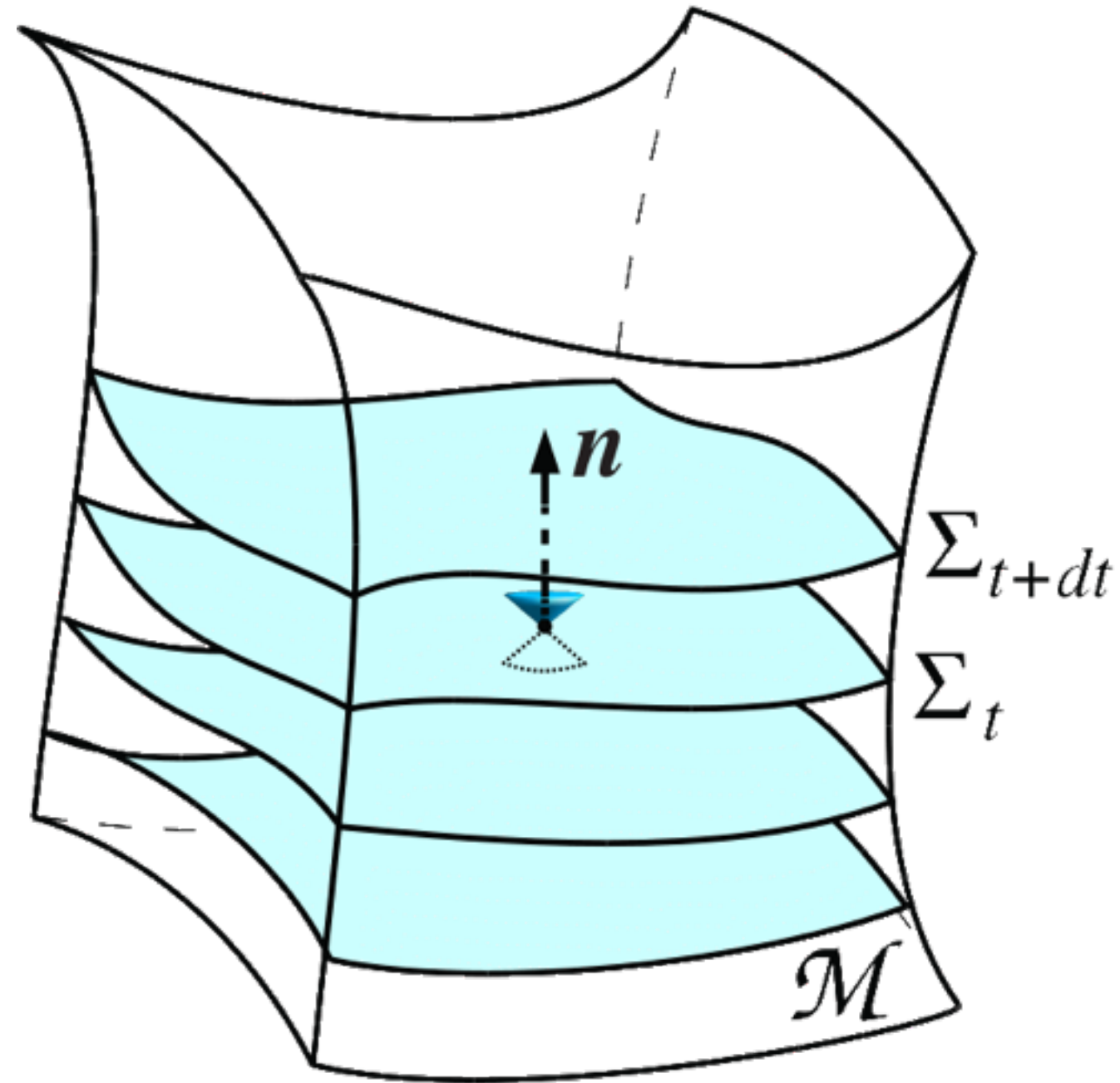
The symplectic potential

$$\phi(\delta) = \int_\Sigma (\epsilon_{IJKL} e^I \wedge e^K) \wedge \delta\omega^{KL}$$



The phase space of GR in connection variables

A. Ashtekar, PRL 57 (1986)



$$\partial M \equiv \Sigma$$

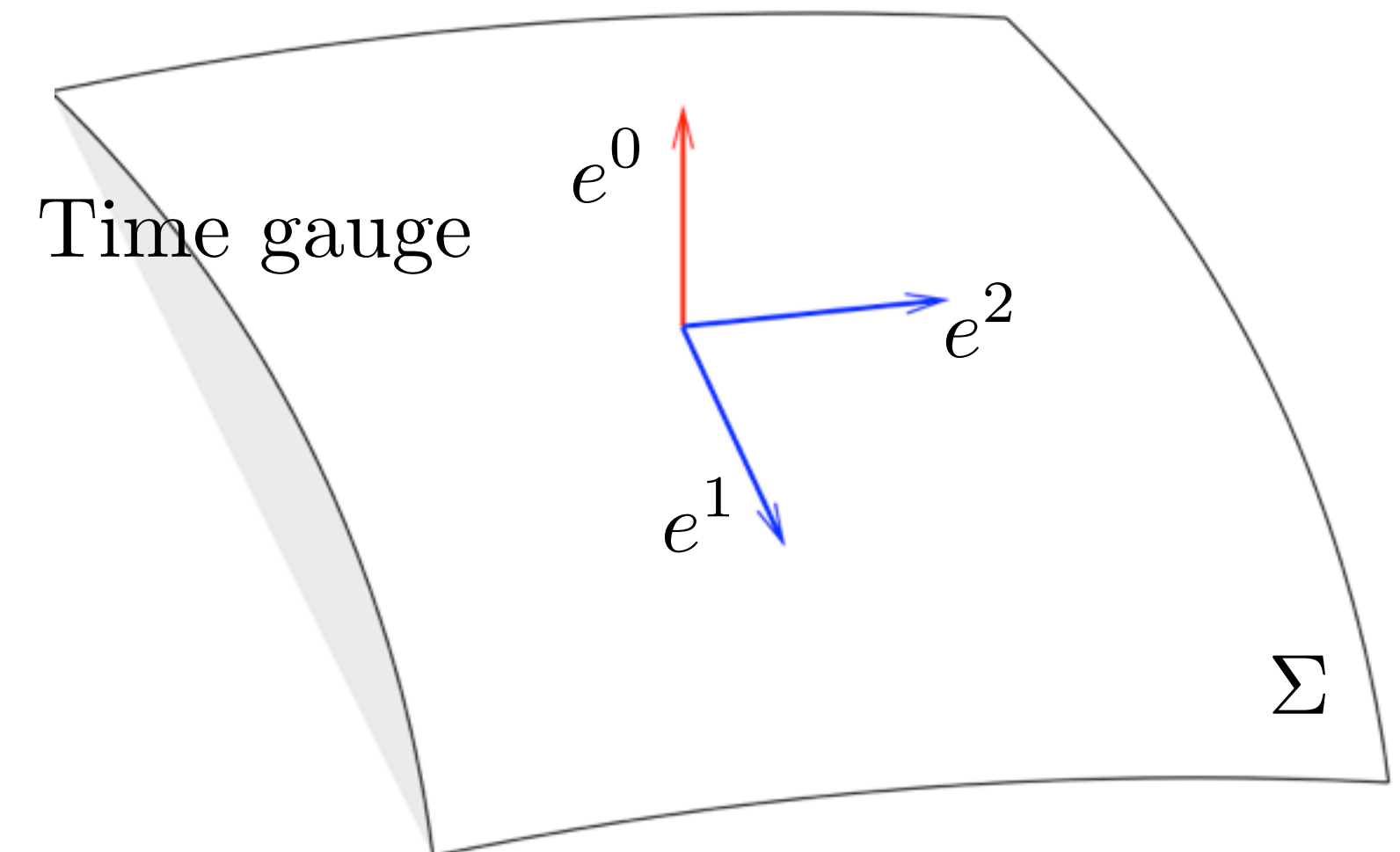
$$S[e, \omega] = \int_M \epsilon_{IJKL} e^I \wedge e^K \wedge F^{KL}(\omega)$$

$$\begin{aligned} \delta S &= \int_M 2\delta e^I (\epsilon_{IJKL} \wedge e^K \wedge F^{KL}(\omega)) + e^I \wedge e^K \wedge d_\omega(\delta\omega^{KL}) = \\ &= \int_M 2\delta e^I (\epsilon_{IJKL} \wedge e^K \wedge F^{KL}(\omega)) - (d_\omega(\epsilon_{IJKL} e^I \wedge e^K)) \wedge \delta\omega^{KL} \\ &\quad + \int_{\partial M} (\epsilon_{IJKL} e^I \wedge e^K) \wedge \delta\omega^{KL} \end{aligned}$$

The symplectic potential

$$\begin{aligned} \phi(\delta) &= \int_\Sigma (\epsilon_{IJKL} e^I \wedge e^K) \wedge \delta\omega^{KL} = \\ &= \int_\Sigma (\epsilon_{0jkl} e^0 \wedge e^j) \wedge \delta\omega^{kl} + (\epsilon_{0jkl} e^j \wedge e^k) \wedge \delta\omega^{l0} = \\ &= \int_\Sigma (\epsilon_{jkl} e^j \wedge e^k) \wedge \delta\omega^{l0} \end{aligned}$$

Number of components:
12 for e_a^I versus 18 for ω_a^{IJ}



A canonical transformation to get back the connection variables

The symplectic potential

$$\phi(\delta) = \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta\omega^{k0}$$

Une transformation canonique pour retrouver les variables de connexion

$\gamma \equiv$ Immirzi parameter

The symplectic potential

$$\begin{aligned}\phi(\delta) &= \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \omega^{k0} \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0})\end{aligned}$$

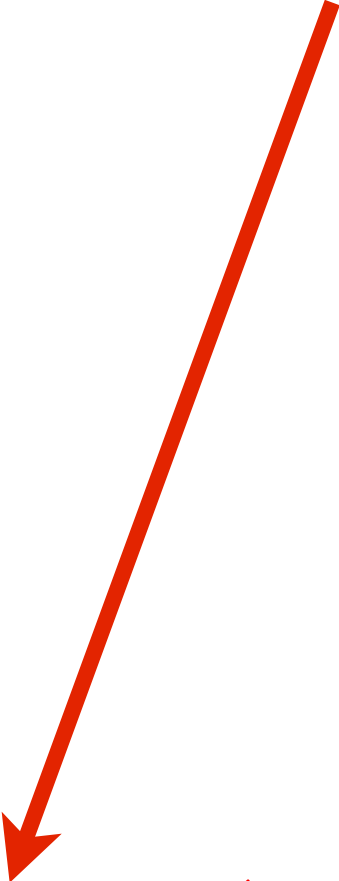
Une transformation canonique pour retrouver les variables de connexion

$\gamma \equiv$ Immirzi parameter

The spin connection

$$de^i + \epsilon^{ijk} \Gamma_j \wedge e_k = 0$$

The symplectic potential

$$\begin{aligned} \phi(\delta) &= \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \omega^{k0} \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0}) \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0} + \Gamma^k(e)) - \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \Gamma^k(e) \end{aligned}$$


Une transformation canonique pour retrouver les variables de connexion

$\gamma \equiv$ Immirzi parameter

The spin connection

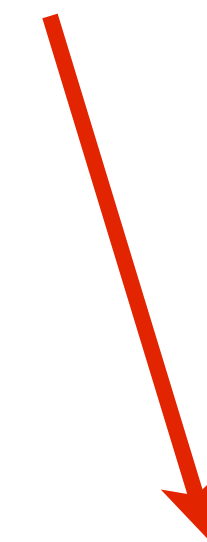
$$de^i + \epsilon^{ijk} \Gamma_j \wedge e_k = 0$$

$$d(\delta e^i) + \epsilon^{ijk} \delta \Gamma_j \wedge e_k + \epsilon^{ijk} \Gamma_j \wedge \delta e_k = 0$$

$$(\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \Gamma^k = d(e^i \wedge \delta e_i)$$

The symplectic potential

$$\begin{aligned} \phi(\delta) &= \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \omega^{k0} \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0}) \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0} + \Gamma^k(e)) - \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \Gamma^k(e) \end{aligned}$$



Une transformation canonique pour retrouver les variables de connexion

$\gamma \equiv$ le paramètre d'Immirzi

The spin connection

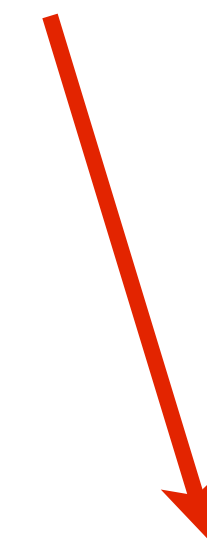
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The symplectic potential

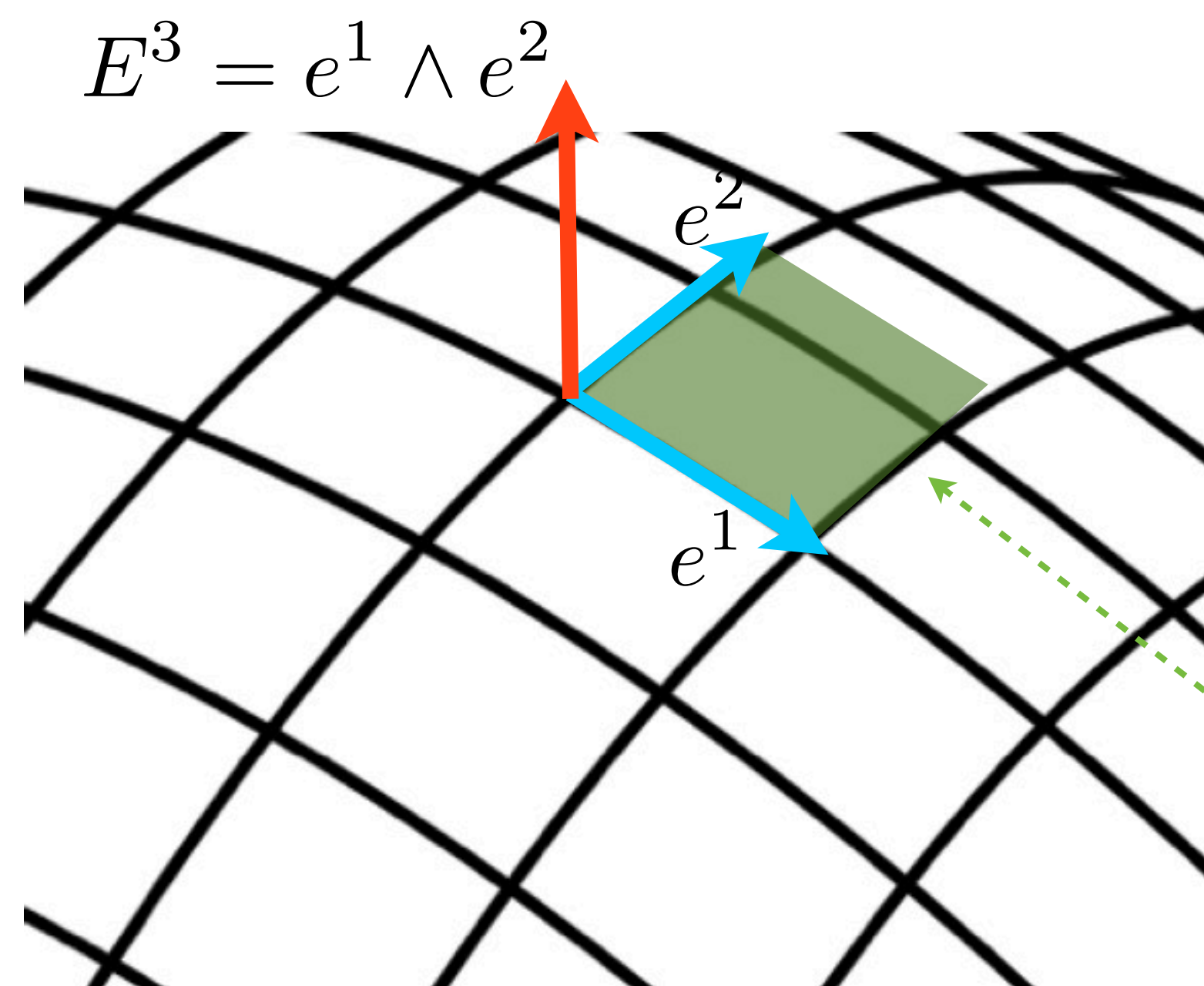
$$\begin{aligned} \phi(\delta) &= \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \omega^{k0} \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0}) \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0} + \Gamma^k(e)) - \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \Gamma^k(e) \\ &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta A^k - \frac{1}{\gamma} \int_{\partial \Sigma} e^i \wedge \delta e^i \end{aligned}$$



Quantization of area in a nut-shell

J. Engle, Noui, AP, D. Pranzetti
[Phys.Rev. D82 \(2010\) 044050](#)

$$\begin{aligned}
 \phi(\delta) &= \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta\omega^{k0} \\
 &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma\omega^{k0}) \\
 &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma\omega^{k0} + \Gamma^k(e)) - \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta\Gamma^k(e) \\
 &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta A^k - \frac{1}{\gamma} \int_{\partial\Sigma} e^i \wedge \delta e^i
 \end{aligned}$$



area element

$$dS^2[e]$$

In a 2-boundary

$$\{e_a^i(x), e_b^j(y)\} = \gamma \epsilon_{ab} \delta^{ij} \delta^{(2)}(x, y)$$

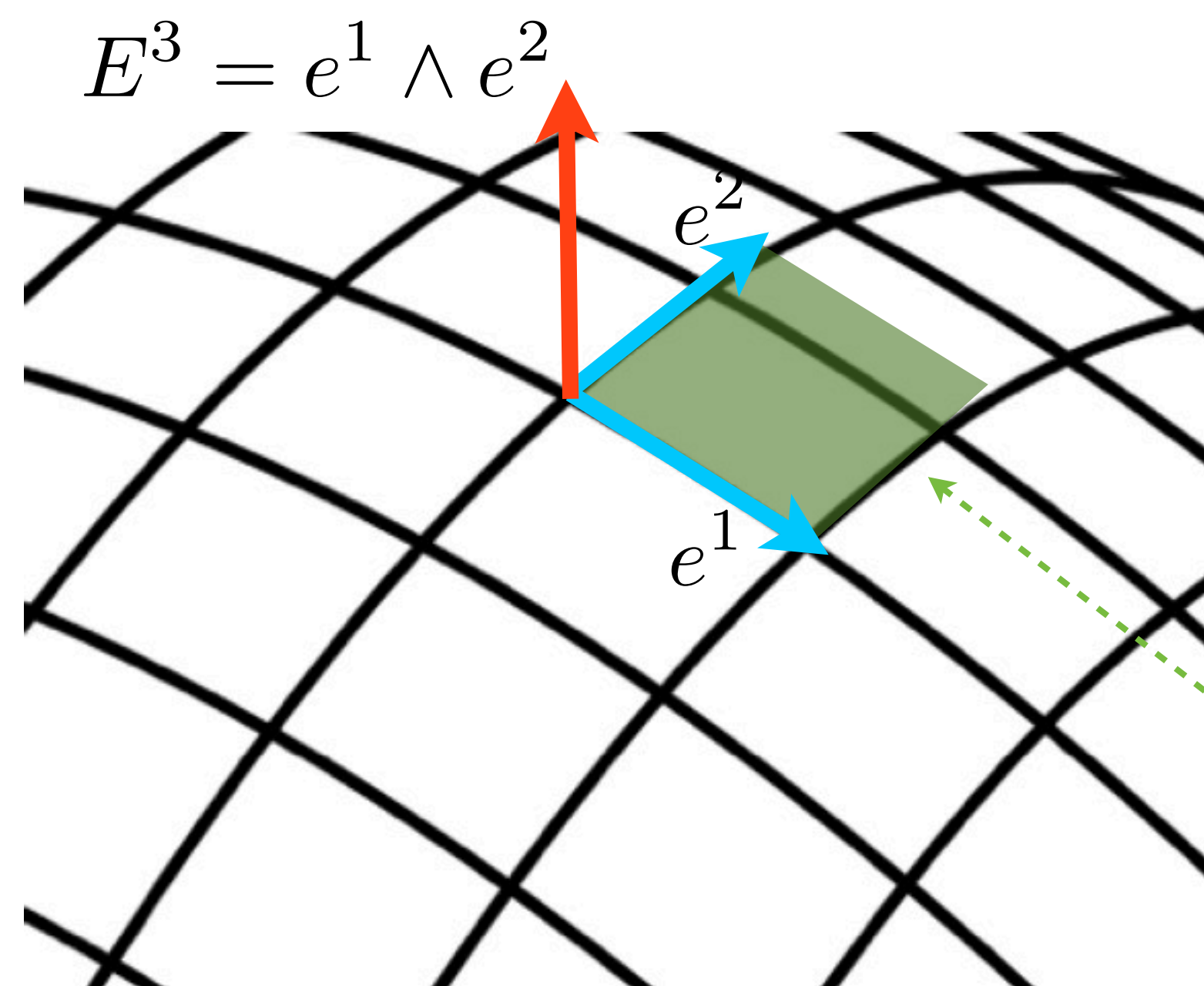
$$E^i \equiv \frac{1}{\gamma} \epsilon_{jkl} e^j \wedge e^k$$

$$\gamma \sqrt{E^i E_i} = \sqrt{h^{(2)}}$$

$$\{E^i(x), E^j(y)\} = \epsilon_{ijk} E^k \delta^{(2)}(x, y)$$

Quantization of area in a nut-shell

$$\begin{aligned}
 \phi(\delta) &= \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \omega^{k0} \\
 &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0}) \\
 &= \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta(\gamma \omega^{k0} + \Gamma^k(e)) - \frac{1}{\gamma} \int_{\Sigma} (\epsilon_{ijk} e^i \wedge e^j) \wedge \delta \Gamma^k(e) \\
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 \end{aligned}$$



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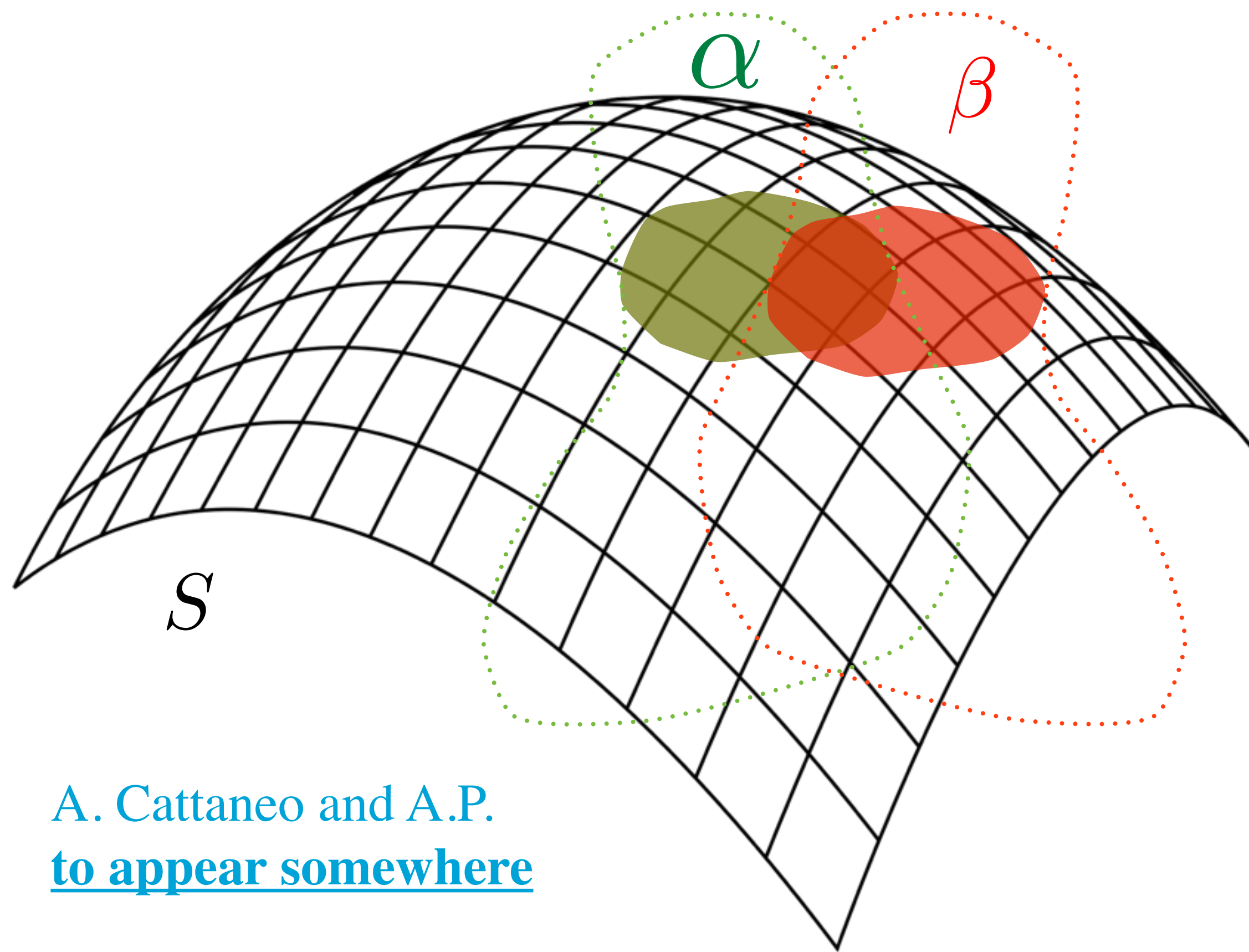
$$E^i \equiv \frac{1}{\gamma} \epsilon_{jkl} e^j \wedge e^k$$

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$$\{E^i(x), E^j(y)\} = \epsilon_{ijk} E^k \delta^{(2)}(x, y)$$

$$\text{area quantum} = \gamma \ell_p^2 \sqrt{j(j+1)}$$

Quantization of area in a nut-shell



A. Cattaneo and A.P.
to appear somewhere

$$E(\alpha, S) = \int_S \alpha^i e^j \wedge e^k \epsilon_{ijk}$$

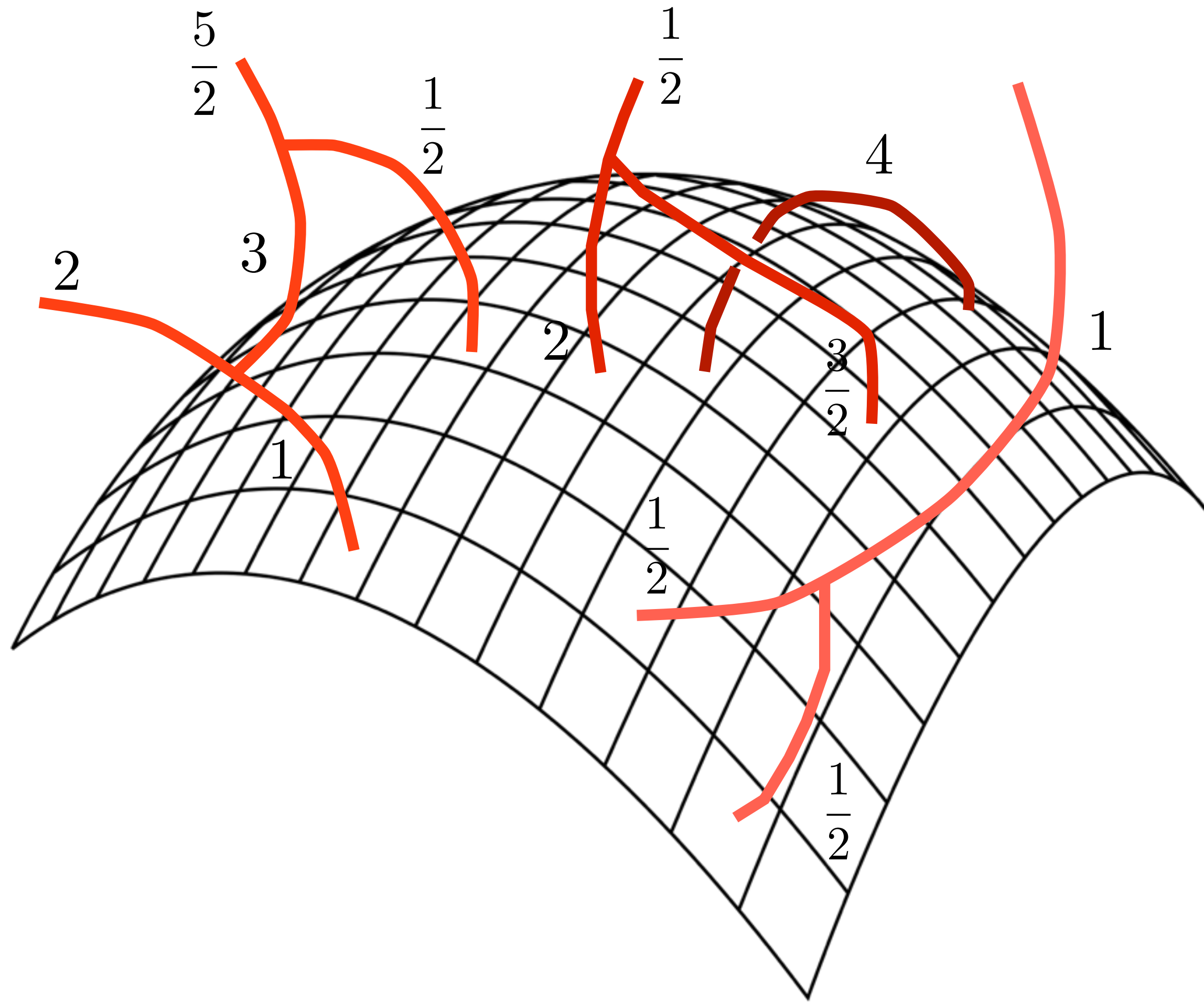
$$\begin{aligned} E(\alpha, S) &\equiv \int_S \text{Tr}[\alpha E] \\ &= \int_{\text{Int}(S)} d(\text{Tr}[\alpha E]) \\ &= \int_{\text{Int}(S)} (\text{Tr}[d_A(\alpha)E] + \text{Tr}[\alpha d_A(E)]) \\ &= \int_{\text{Int}(S)} \text{Tr}[(d\alpha + [A, \alpha])E] \end{aligned}$$

$$\begin{aligned} \{E(\alpha, S), E(\beta, S)\} &= \int_{\text{Int}(S)} \int_{\text{Int}(S)} \{\text{Tr}[(d\alpha + [A, \alpha])E], \text{Tr}[(d\beta + [A, \beta])E]\} \\ \{E(\alpha, S), E(\beta, S)\} &= E([\alpha, \beta], S) \end{aligned}$$

Quantization of area

C. Rovelli and L. Smolin. (1995)

A. Ashtekar and J. Lewandowski. (1997)



$\gamma \equiv$ Immirzi parameter

$$\text{area quantum} = \gamma \ell_p^2 \sqrt{j(j+1)}$$

Implications of discreteness

a brief status report

The black hole area spectrum

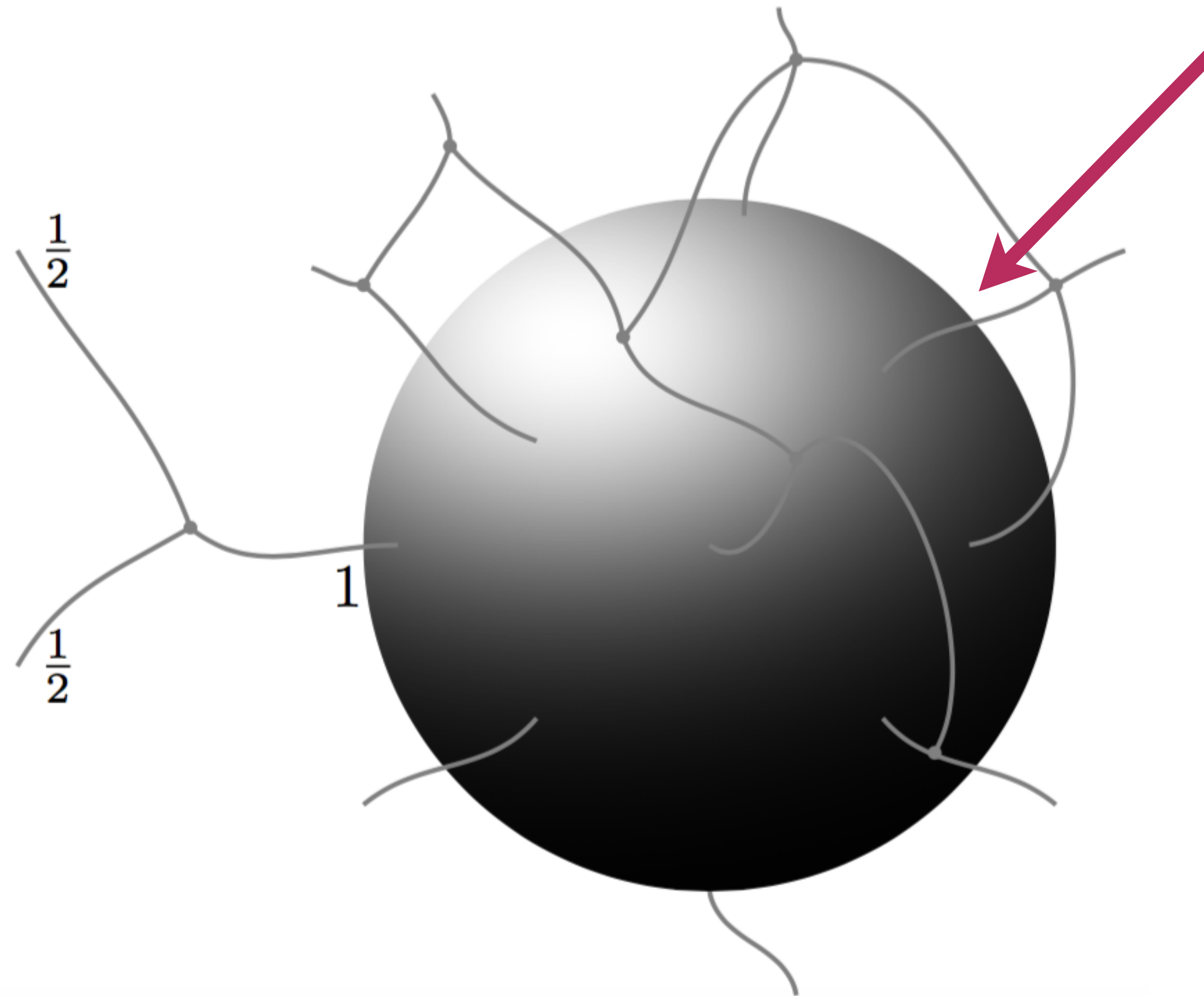
The area gap

$$\hat{A}_S |j_1, j_2 \dots\rangle = \left[8\pi\gamma\ell_p^2 \sum_p \sqrt{j_p(j_p + 1)} \right] |j_1, j_2 \dots\rangle$$

Isolated Horizon
boundary conditions

Ashtekar et al. (1999)

Boundary DOF described by
 $SU(2)$ Chern-Simons theory
Engle, AP, Noui, PRL 105 (2010)



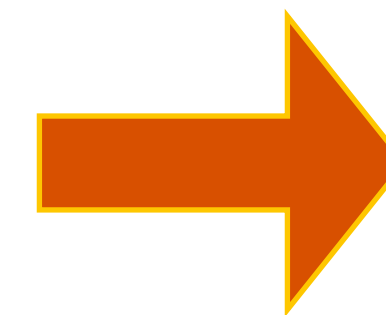
Entropy Calculation: the status in few lines

AP. Rept. Prog. Phys. **80** (2017)

Pure quantum geometry approach

$$S_{bh} = \frac{\gamma_0}{\gamma} \frac{A}{4\ell_p^2}$$

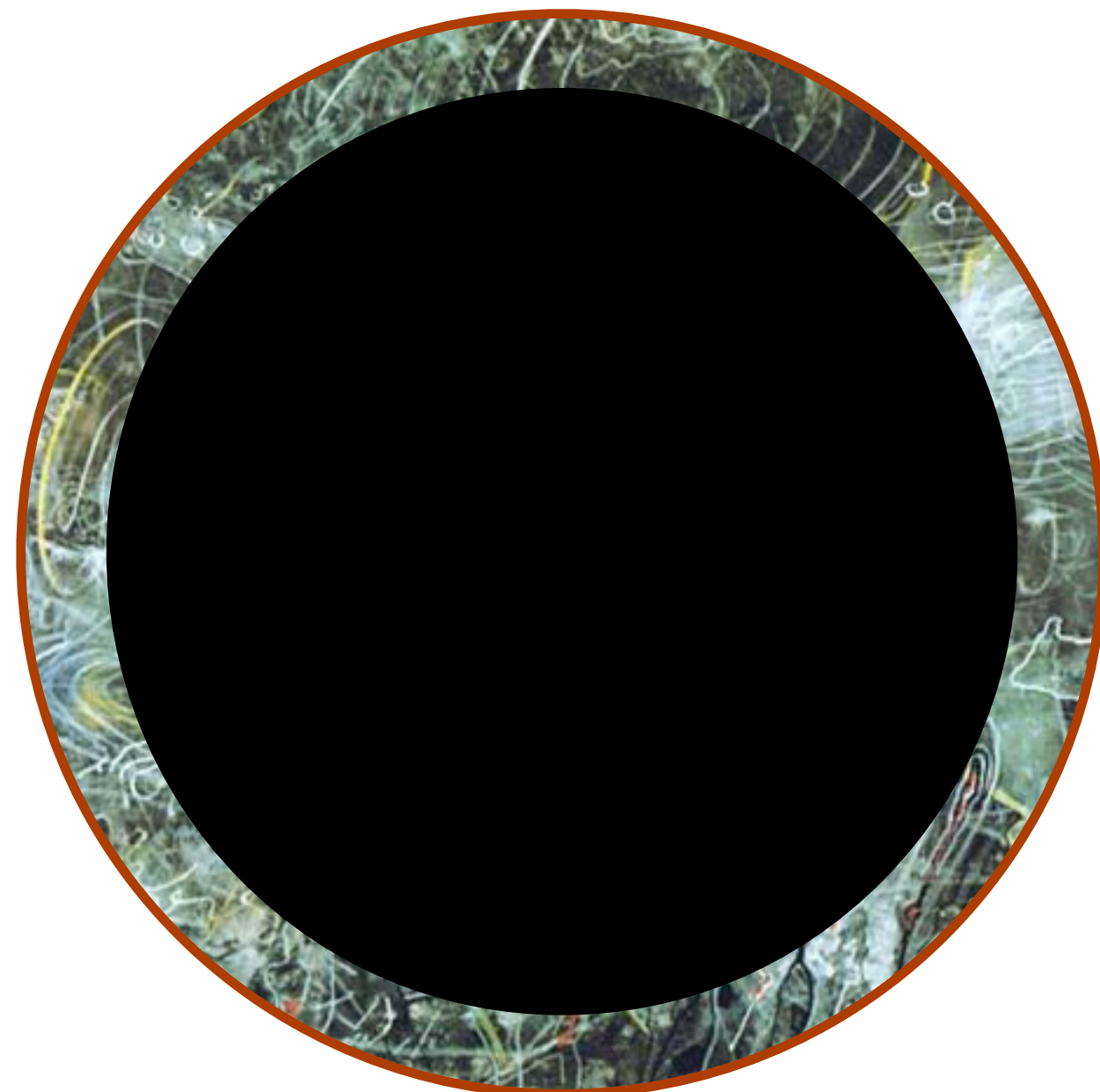
Rovelli (1996),
Ashtekar-Baez-Krasnov-Corichi (1999),
etc



Barbero-Villasenor (2008)

$$\gamma = \gamma_0$$

Taking into account matter excitations

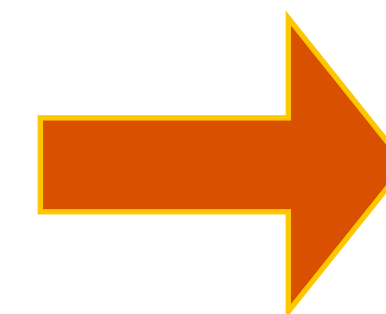


Analysis in standard QFT implies

$$S_{\text{matter}} = \lambda \frac{A}{\epsilon^2} + \text{corrections}$$

λ = undetermined constant due to:
UV regularisation dependence,
the species problem, etc.

ϵ = UV cut-off



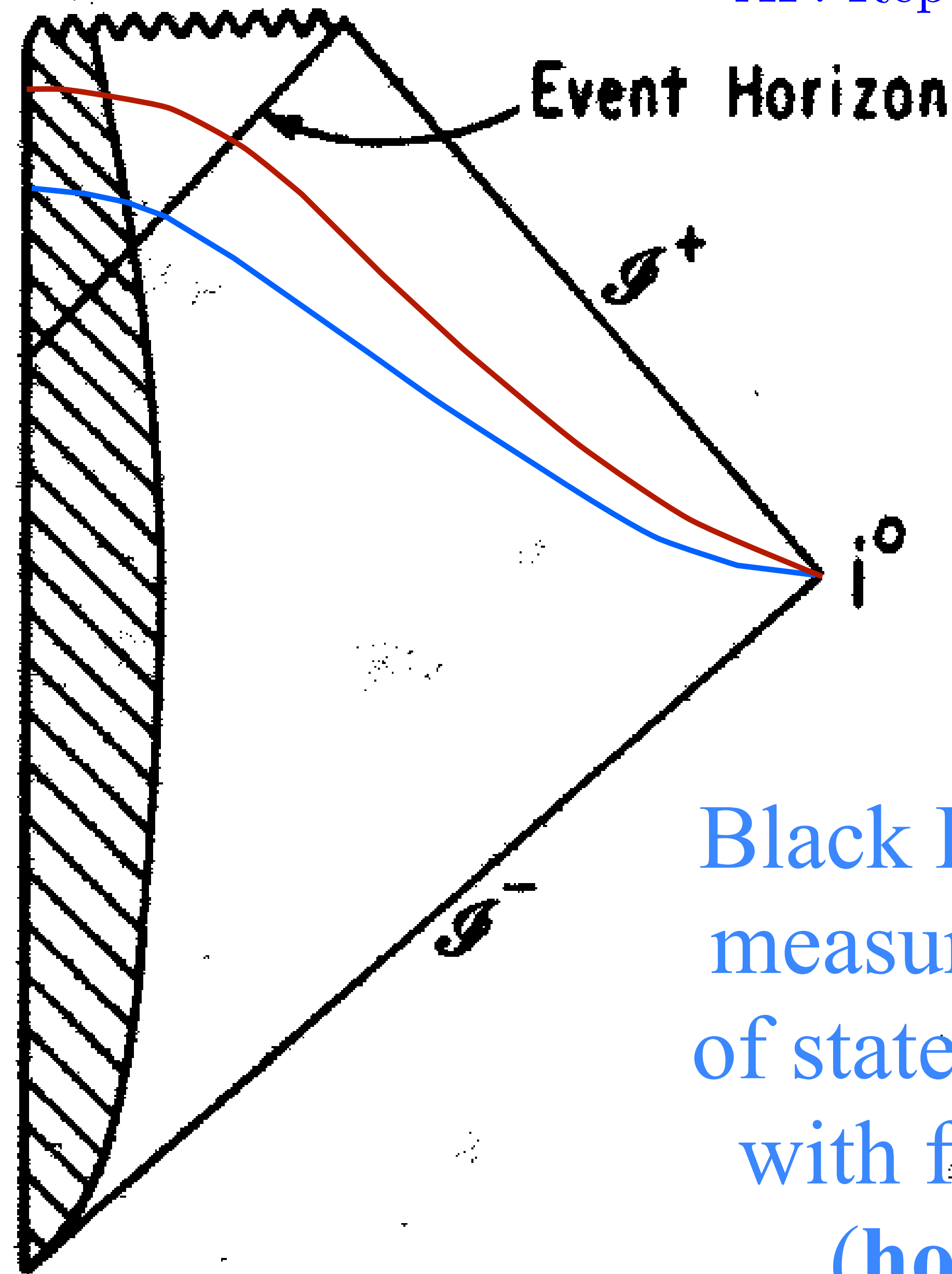
$$S_{bh} = \frac{A}{4G\hbar}$$

Ghosh-Noui-AP, PRD (2014)

Loop Quantum Gravity: a non-holographic approach

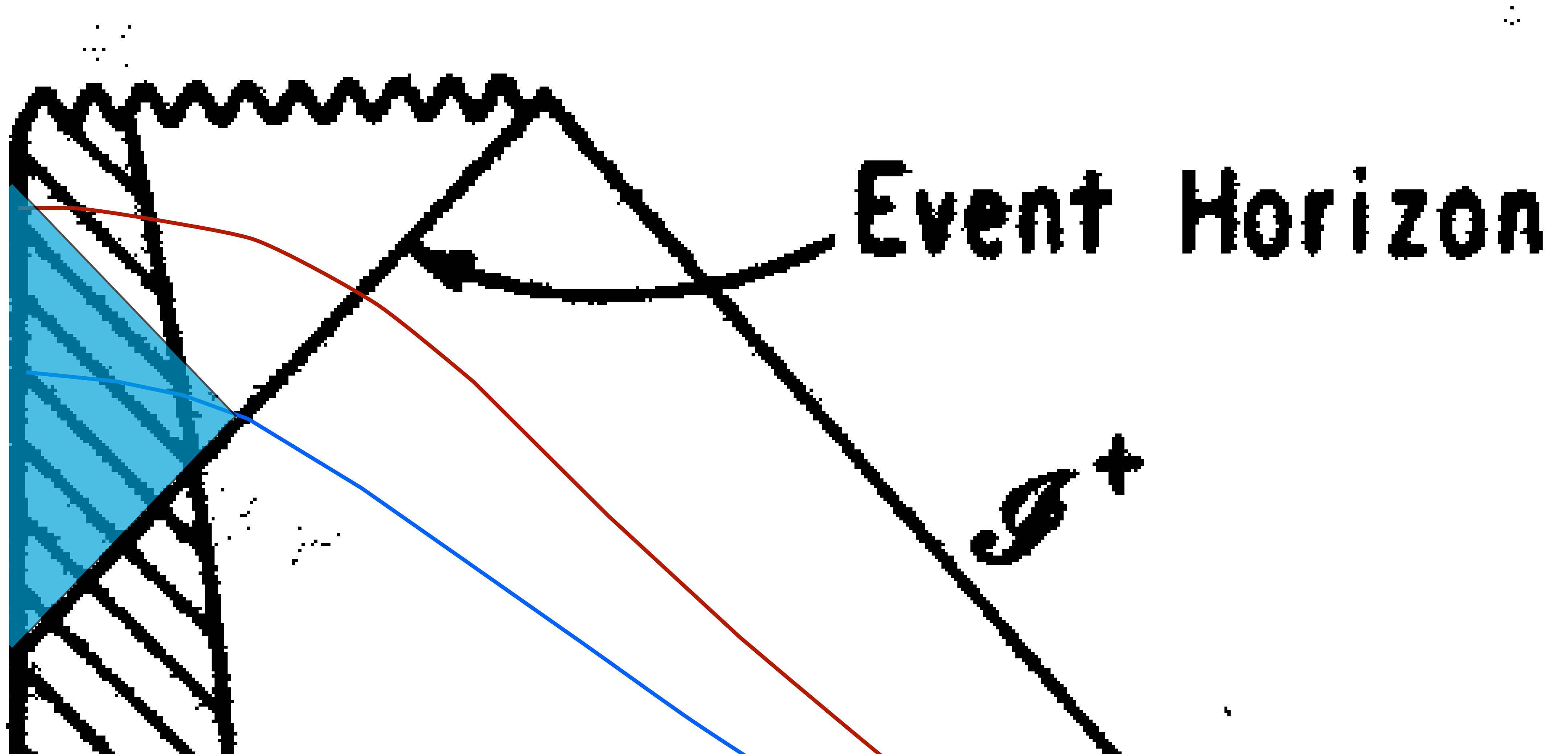
AP. Rept. Prog. Phys. 80 (2017)

Black Hole Entropy
is **not** a measure of
the number of
internal states of a
Black Hole

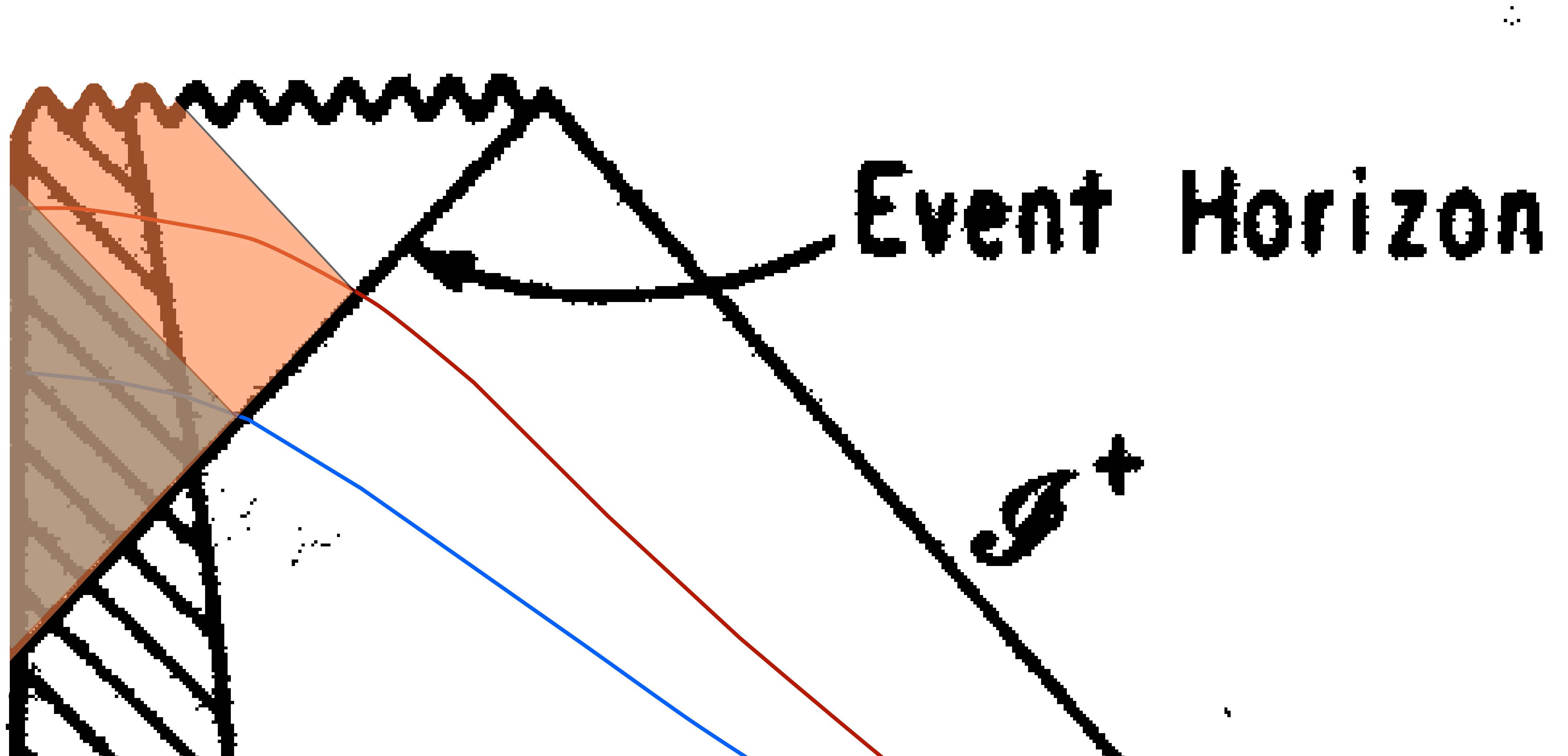


Black Hole entropy is a
measure of the number
of states we can interact
with from the outside
(**horizon states**)

Loop Quantum Gravity: a non-holographic approach



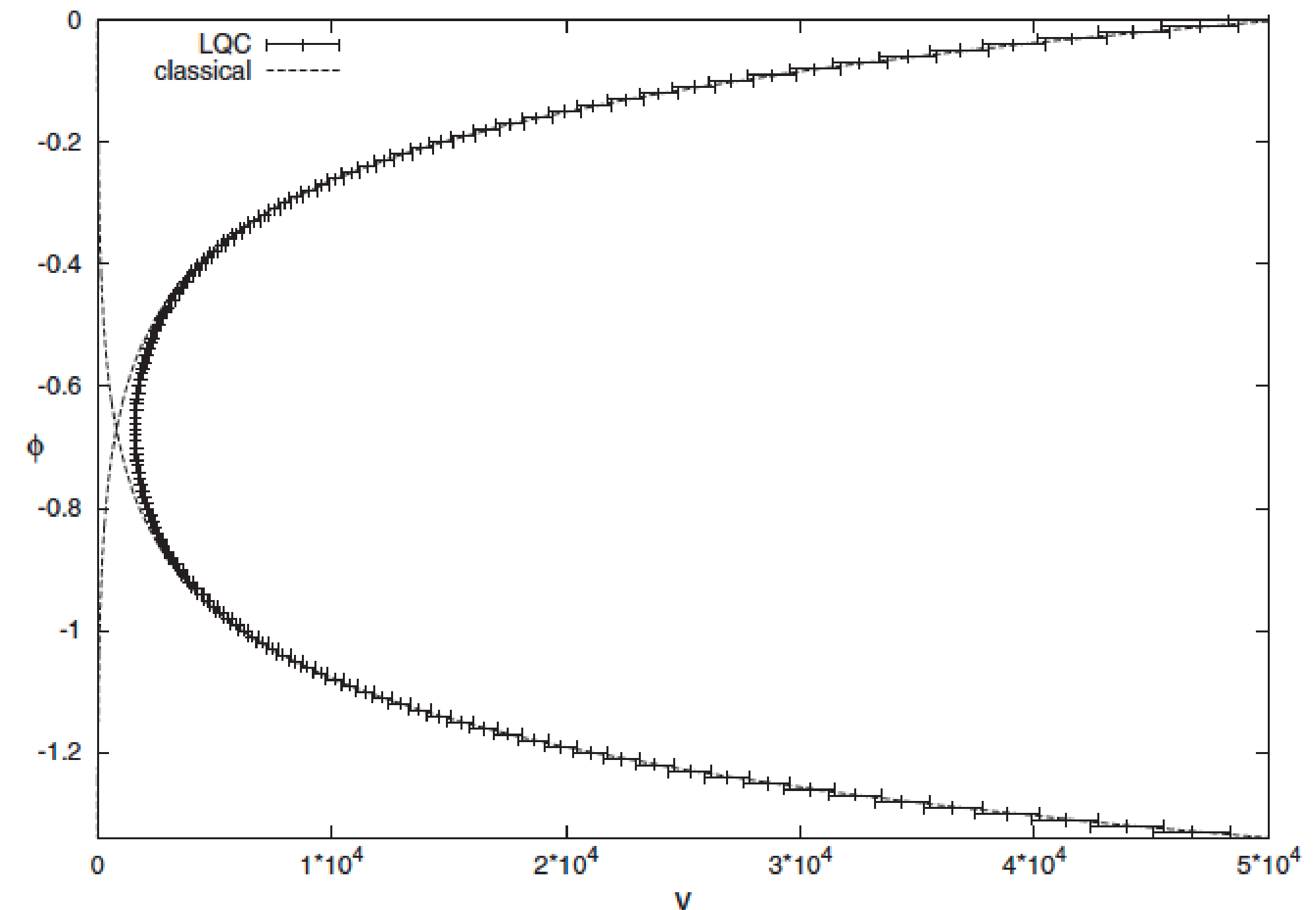
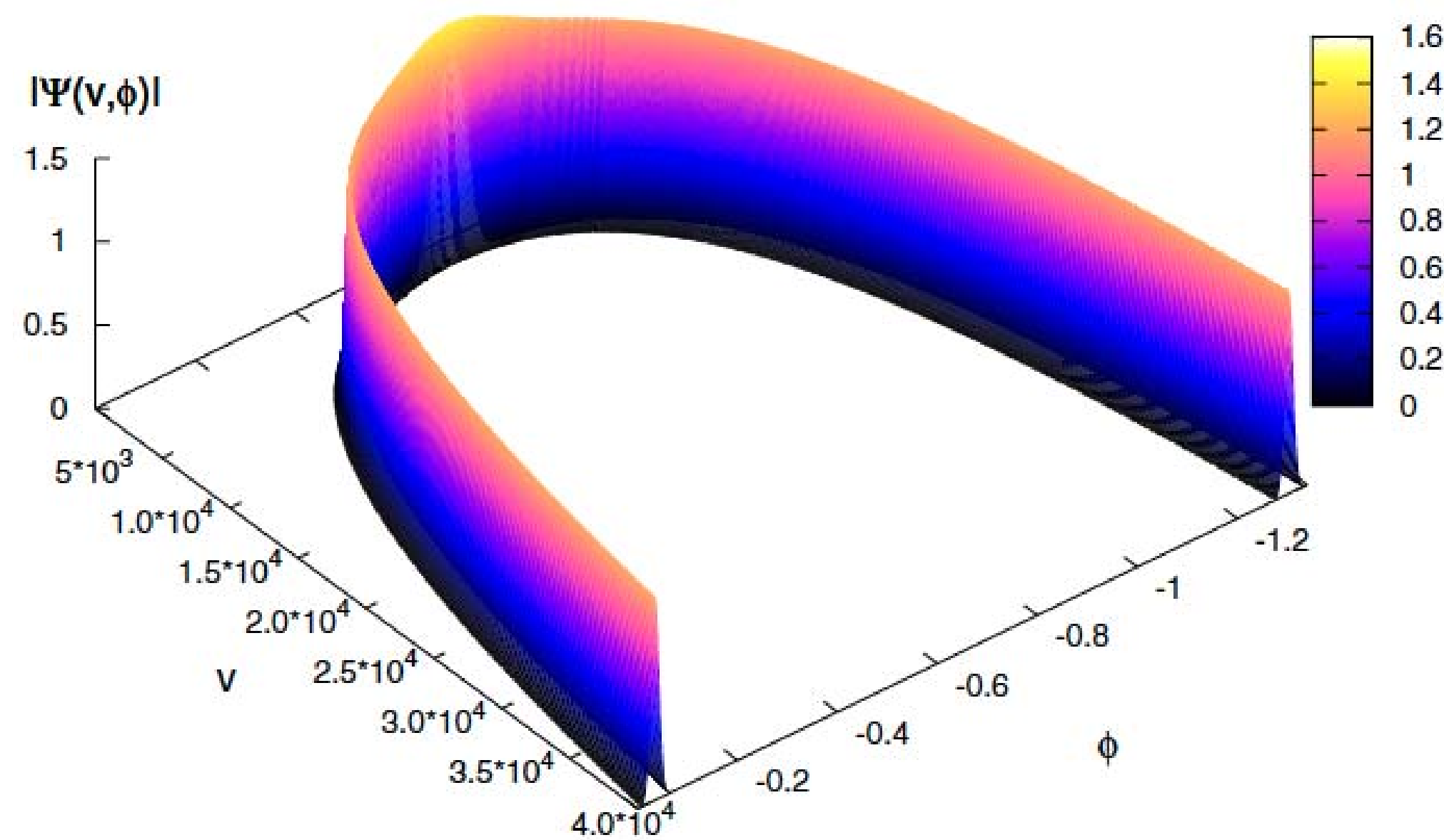
Loop Quantum Gravity: a non-holographic approach



Loop Quantum Gravity: in cosmology

Planckian discreteness resolves **big-bang** singularity.

Bojowald (2001), Ashtekar, Singh, etc.



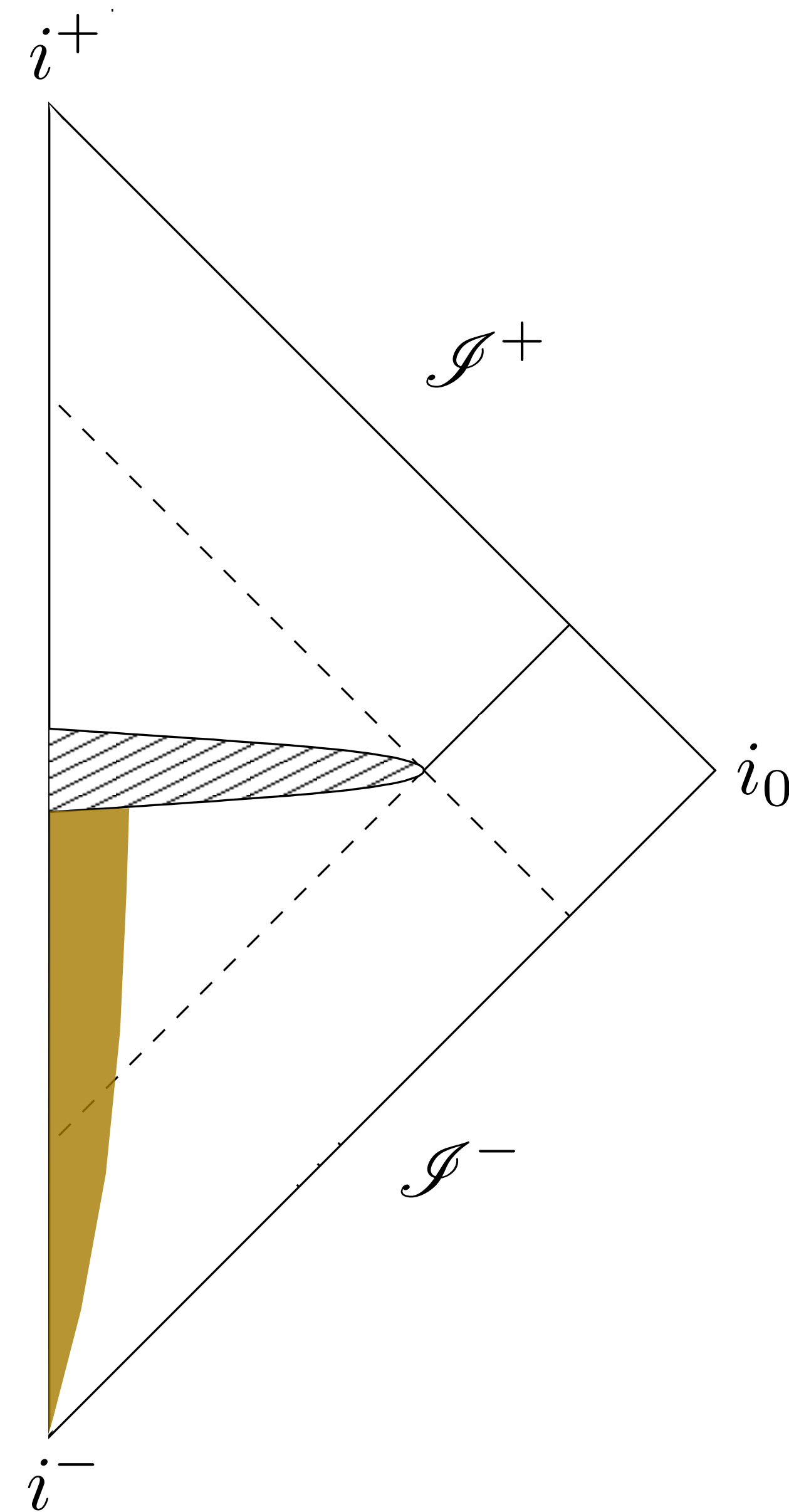
From Ashtekar, Pawłowski, Singh PRD
(2006)

Loop Quantum Gravity: collapse, singularities, and information.

There is the expectation that the same would hold for **Black Hole singularities**

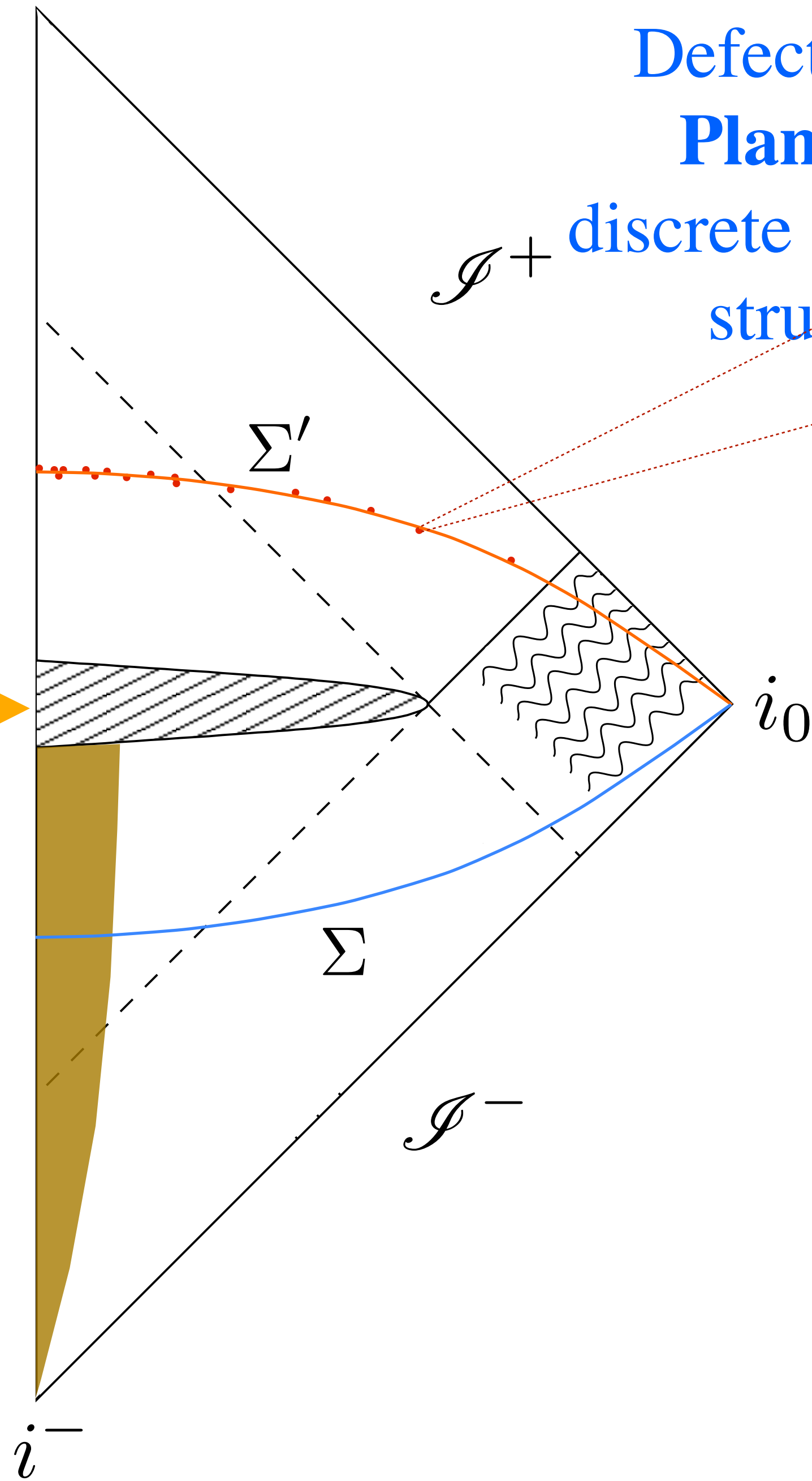
Simplified models support this expectation
(Modesto, Ashtekar-Singh, Rovelli-Haggard, Rovelli, Pullin, Corichi-Singh, etc.)

Unitarity: Information should be recovered after BH evaporation

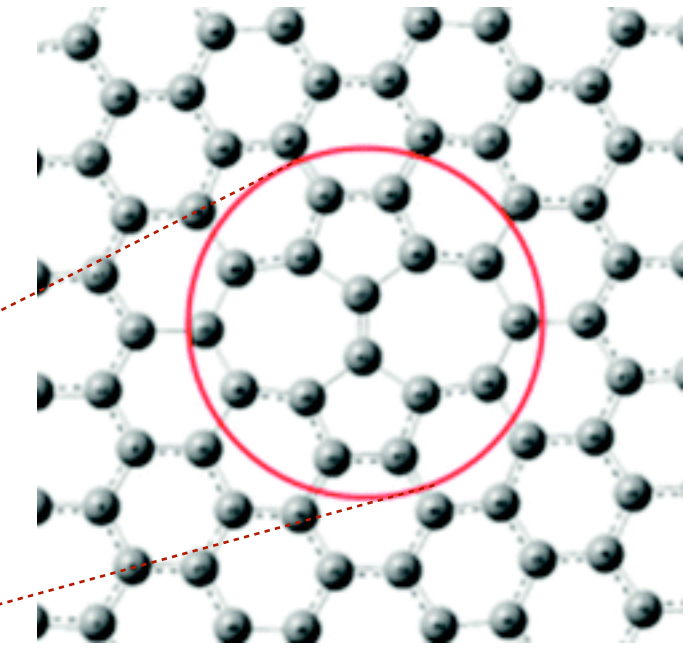


New perspective on the information paradox

AP, *Class. Quant. Grav.*
32, 2015.



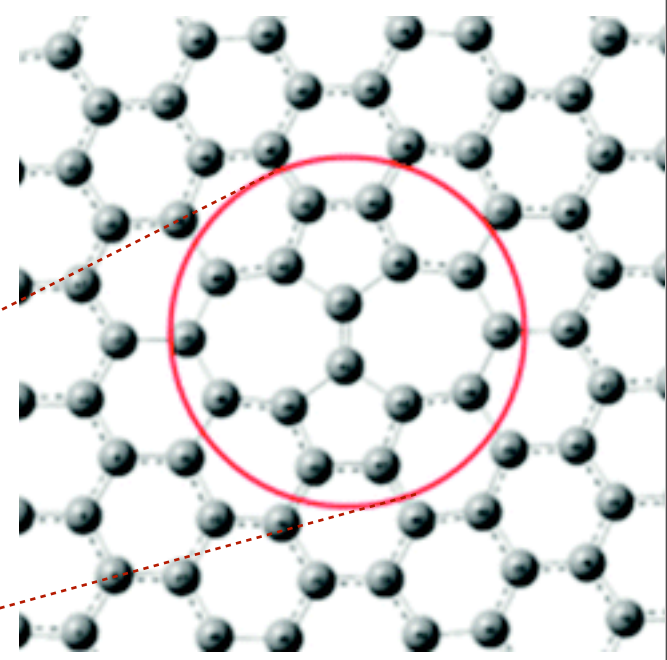
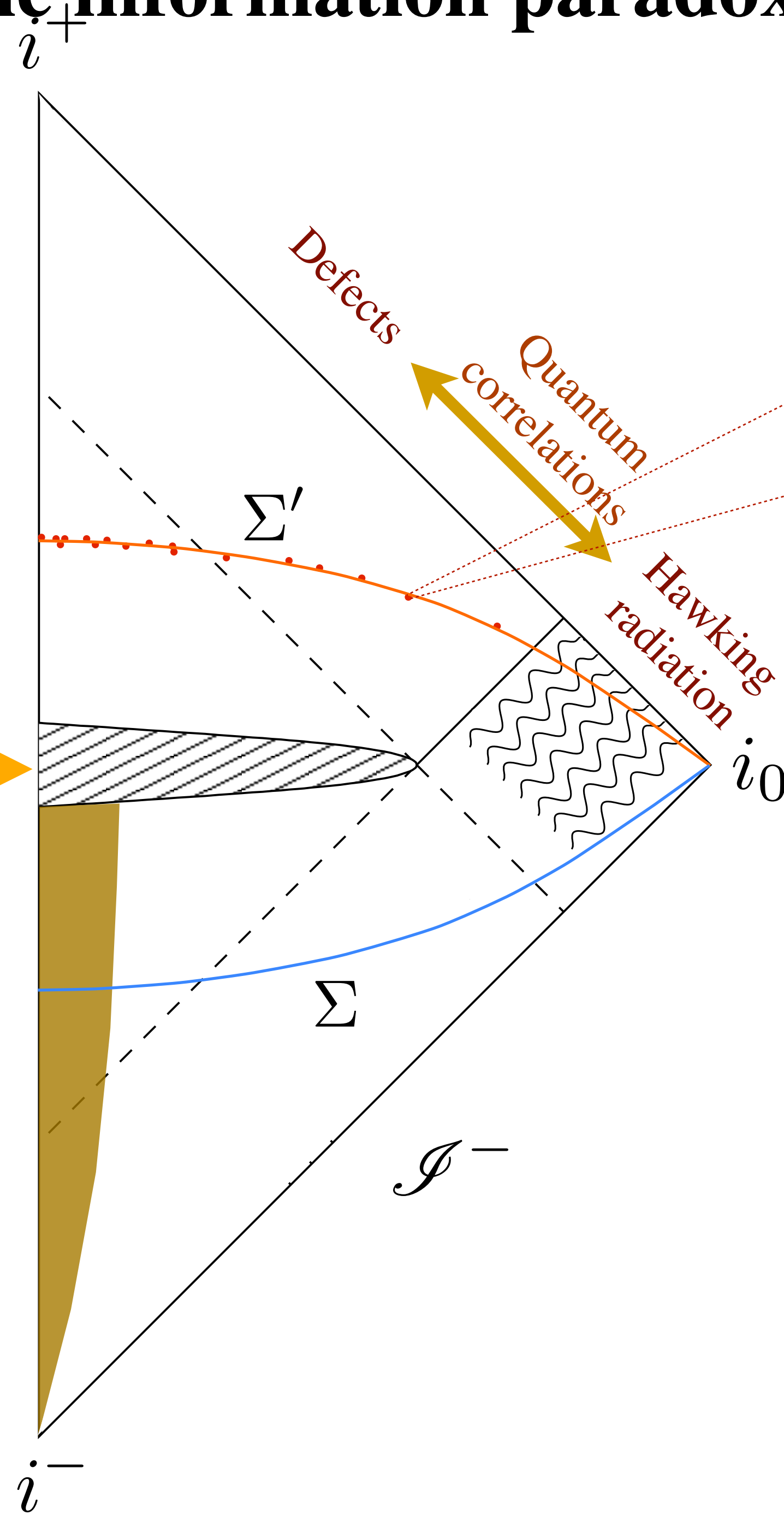
Defects in the
Planckian
discrete spacetime
structure



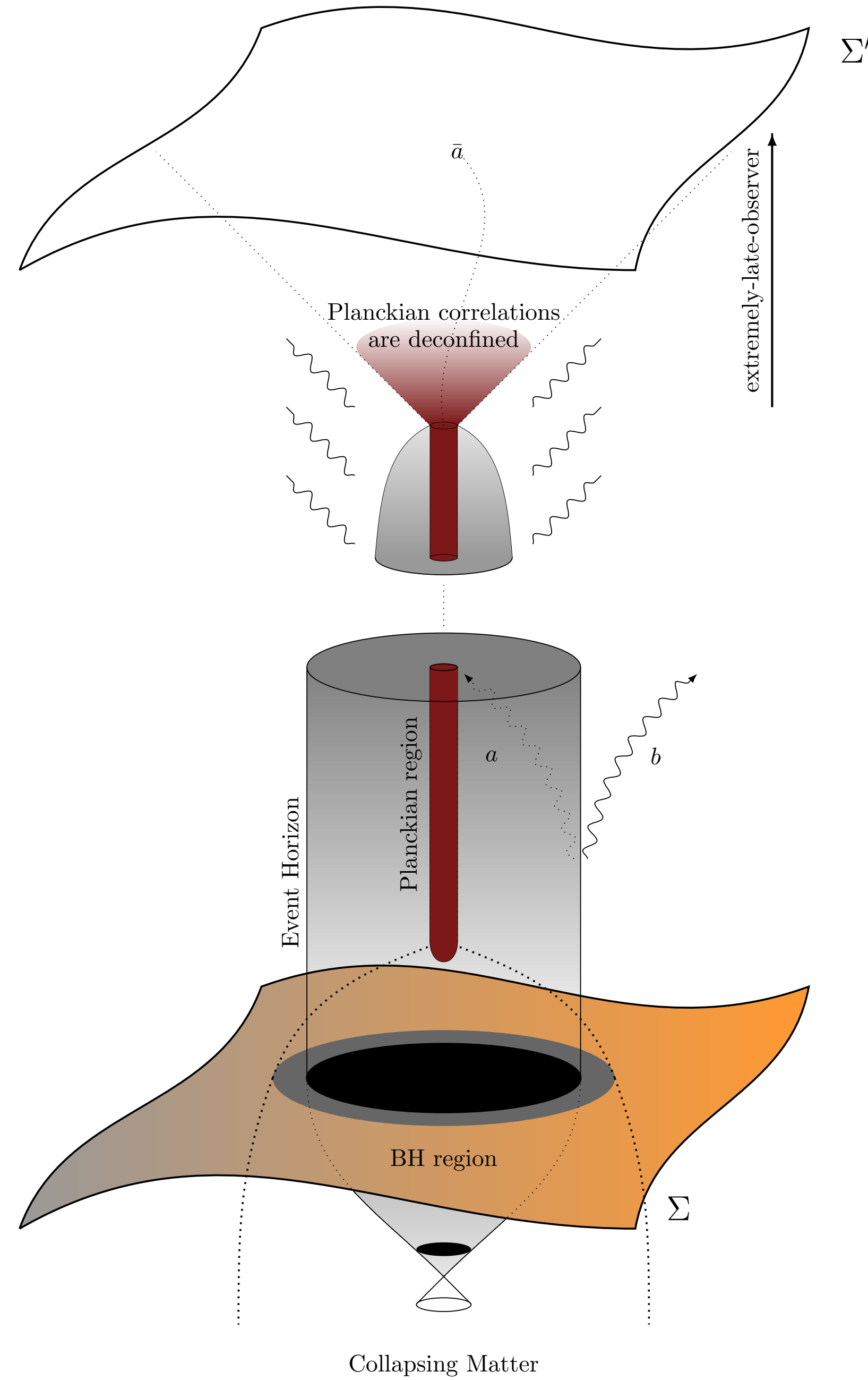
Defects are **hidden**
for the probes of low energy
observers. No nontinuum
field theory description. They
can be essentially “**zero
energy**”

New perspective on the information paradox

AP, *Class. Quant. Grav.*
32, 2015.



New perspective on the information paradox



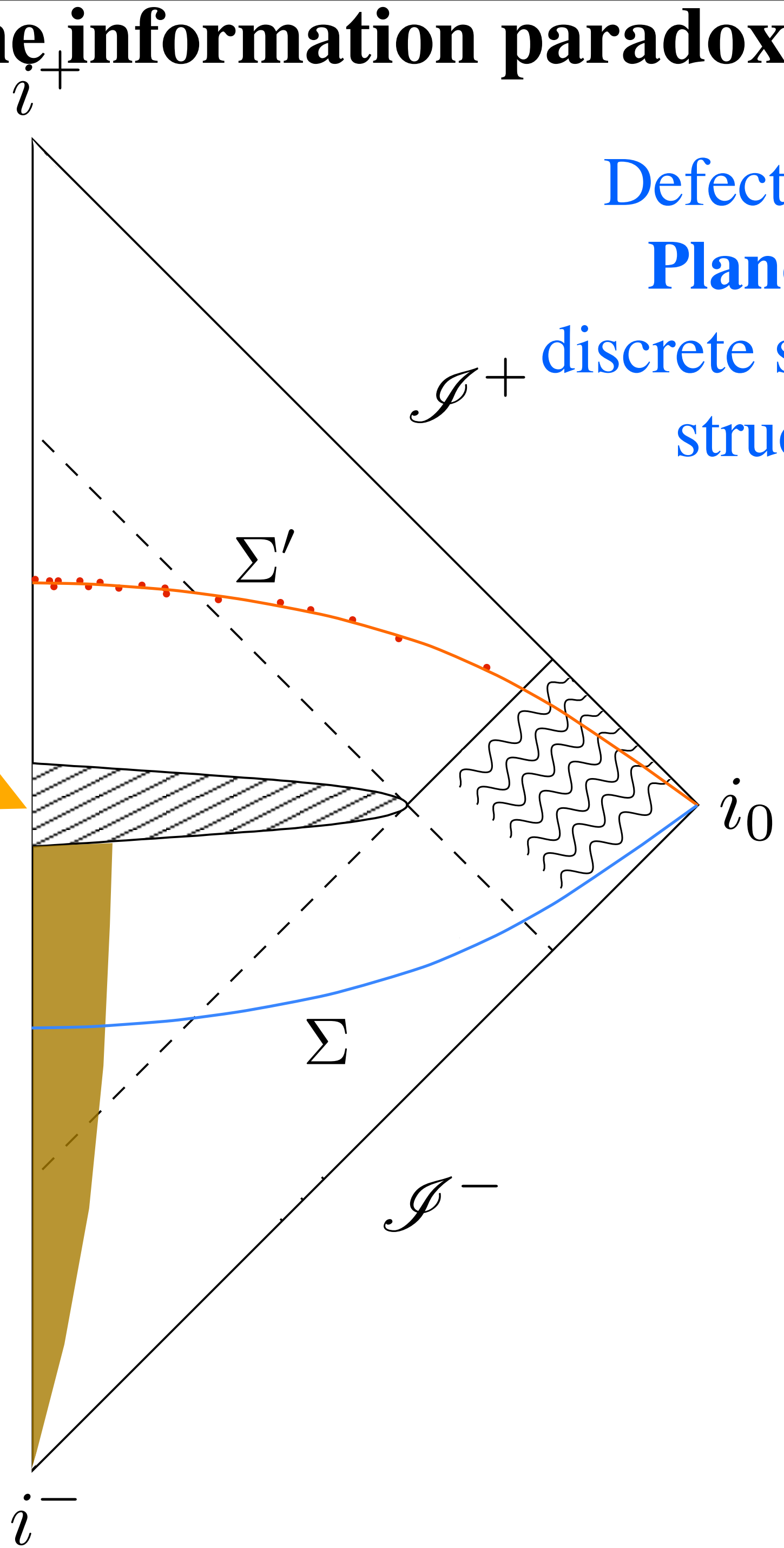
New perspective on the information paradox

AP, *Class. Quant. Grav.*
32, 2015.

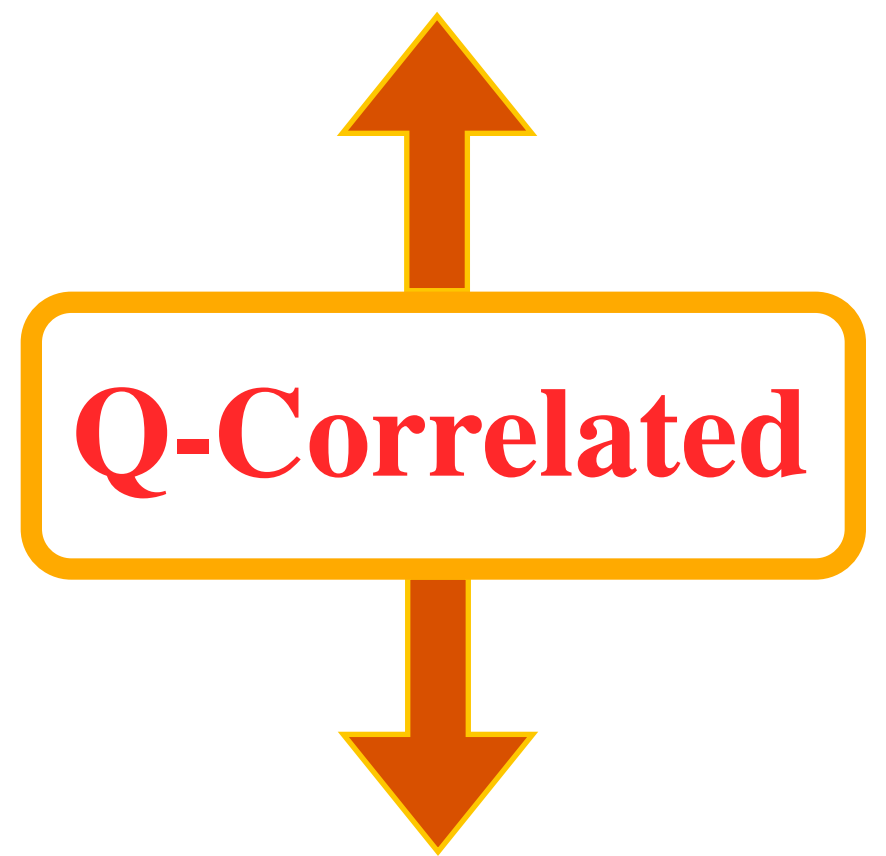
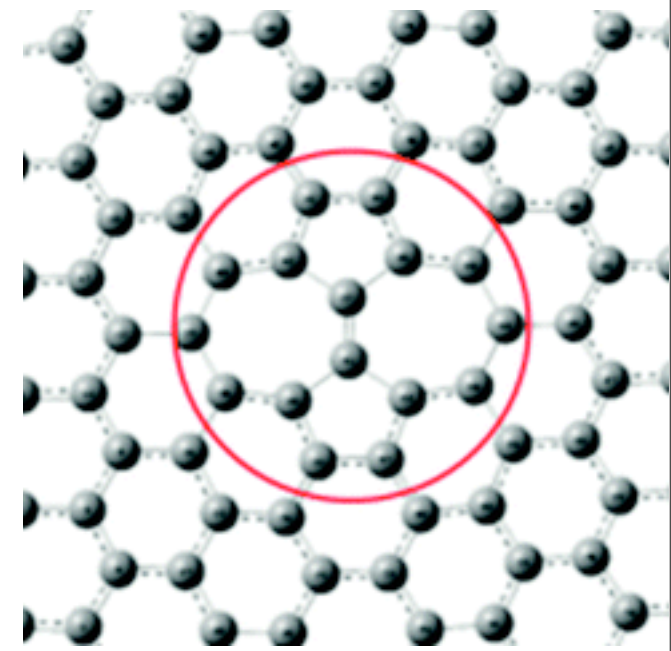


Decoherence with discrete micro-structure: **violations of energy conservation in the smooth effective description!**

Banks, Peskin, Susskind (1984) - Unruh, Wald (1995)



Defects in the
Planckian
discrete spacetime
structure



Hawking
thermal
radiation

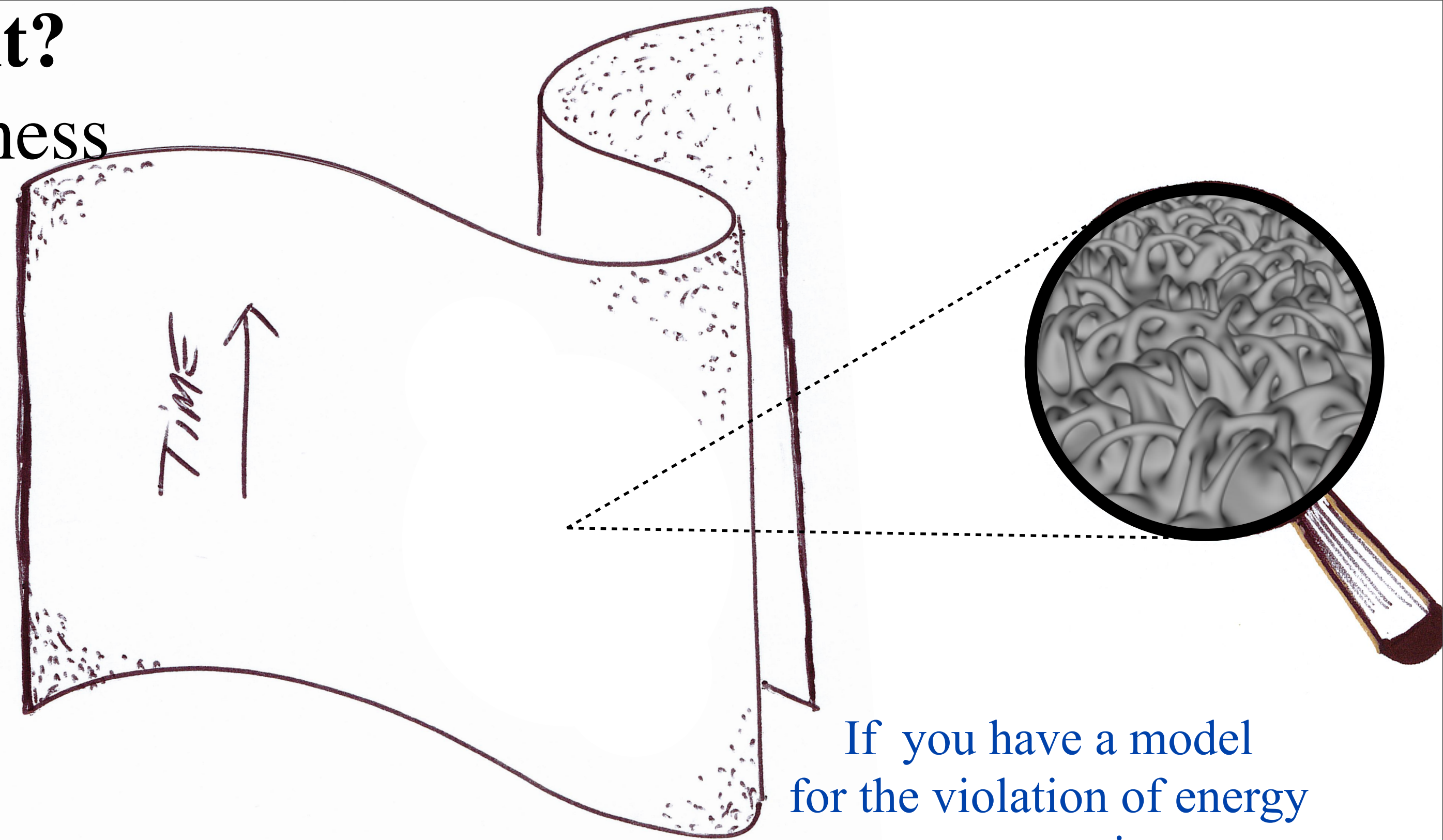
An emergent cosmological constant?

phenomenology arising from discreteness

T. Josset, AP, D. Sudarsky.
PRL 118, (2017)

AP, D. Sudarsky.
arXiv:1711.05183, (2017)

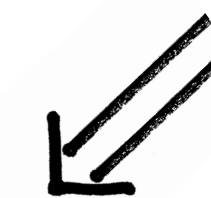
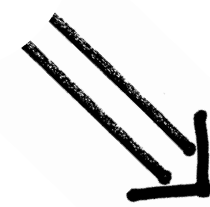
AP, D. Sudarsky, J.D. Bjorken.
Int.J.Mod.Phys. D27 (2018)



If you have a model
for the violation of energy
conservation

$$\mathbf{J}_b \equiv 8\pi \nabla^a \mathbf{T}_{ab}$$

$$\mathbf{R}_{ab} - \frac{1}{4} g_{ab} \mathbf{R} = 8\pi \left(\mathbf{T}_{ab} - \frac{1}{4} g_{ab} \mathbf{T} \right)$$

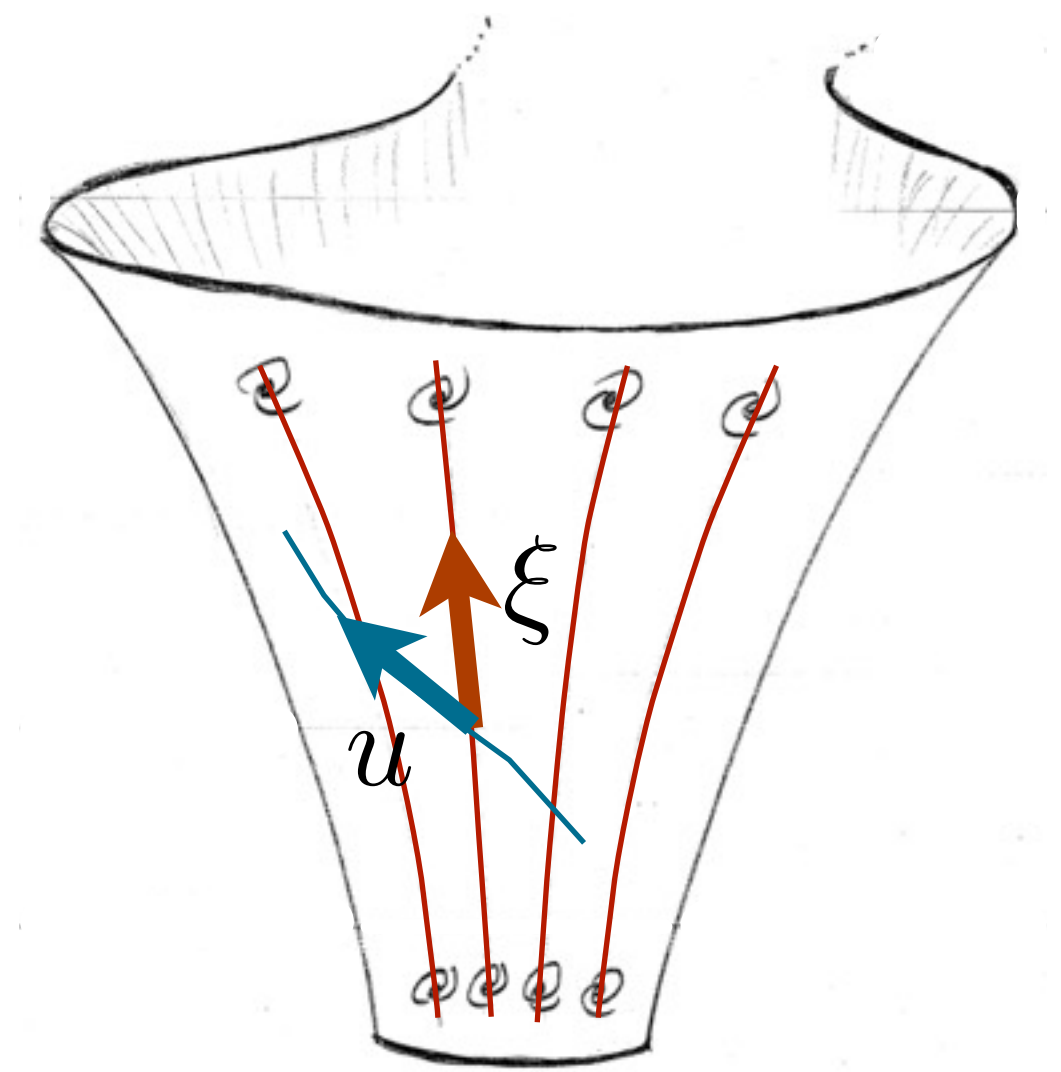


$$\mathbf{R}_{ab} - \frac{1}{2} \mathbf{R} g_{ab} = 8\pi \mathbf{T}_{ab} - \underbrace{\left[\Lambda_0 + \int_{\ell} \mathbf{J} \right]}_{\text{Dark Energy } \Lambda} g_{ab}$$

Diffusion from Planckian discretenss

Langevin-Papapetrou like equation; noisy diffusion due to discreteness.

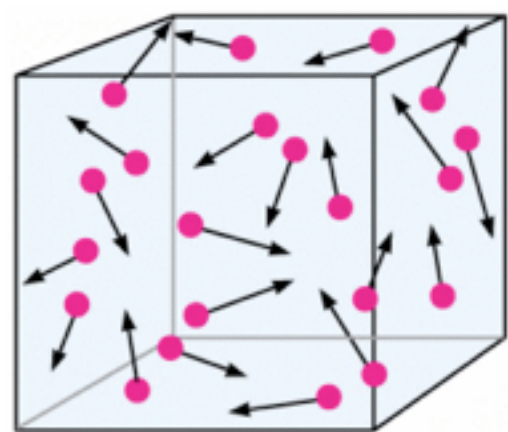
$$a^b = u^a \nabla_a u^b = \alpha \text{sign}(s \cdot \xi) \frac{m\mathbf{R}}{m_p^2} s^b$$



Single particle energy diffusion.

$$\dot{E} \equiv -m u^\mu \nabla_\mu (u^\nu \xi_\nu) = -\alpha \frac{m^2}{m_p^2} |(s \cdot \xi)| \mathbf{R} - m u^\mu u^\nu \nabla_{(\mu} \xi_{\nu)}$$

Multiparticle Energy-Momentum diffusion in cosmology.



$$\mathbf{J}_b \equiv 8\pi \nabla^a \mathbf{T}_{ab} = -4\pi \alpha \hbar \frac{T\mathbf{R}}{m_p^2} \left[8\pi G \sum_i |s^i| \mathbf{T}_i \right] \xi_b$$

$$\approx 2\pi \alpha \hbar \frac{T\mathbf{R}^2}{m_p^2} \xi_b$$

Top quark approximation

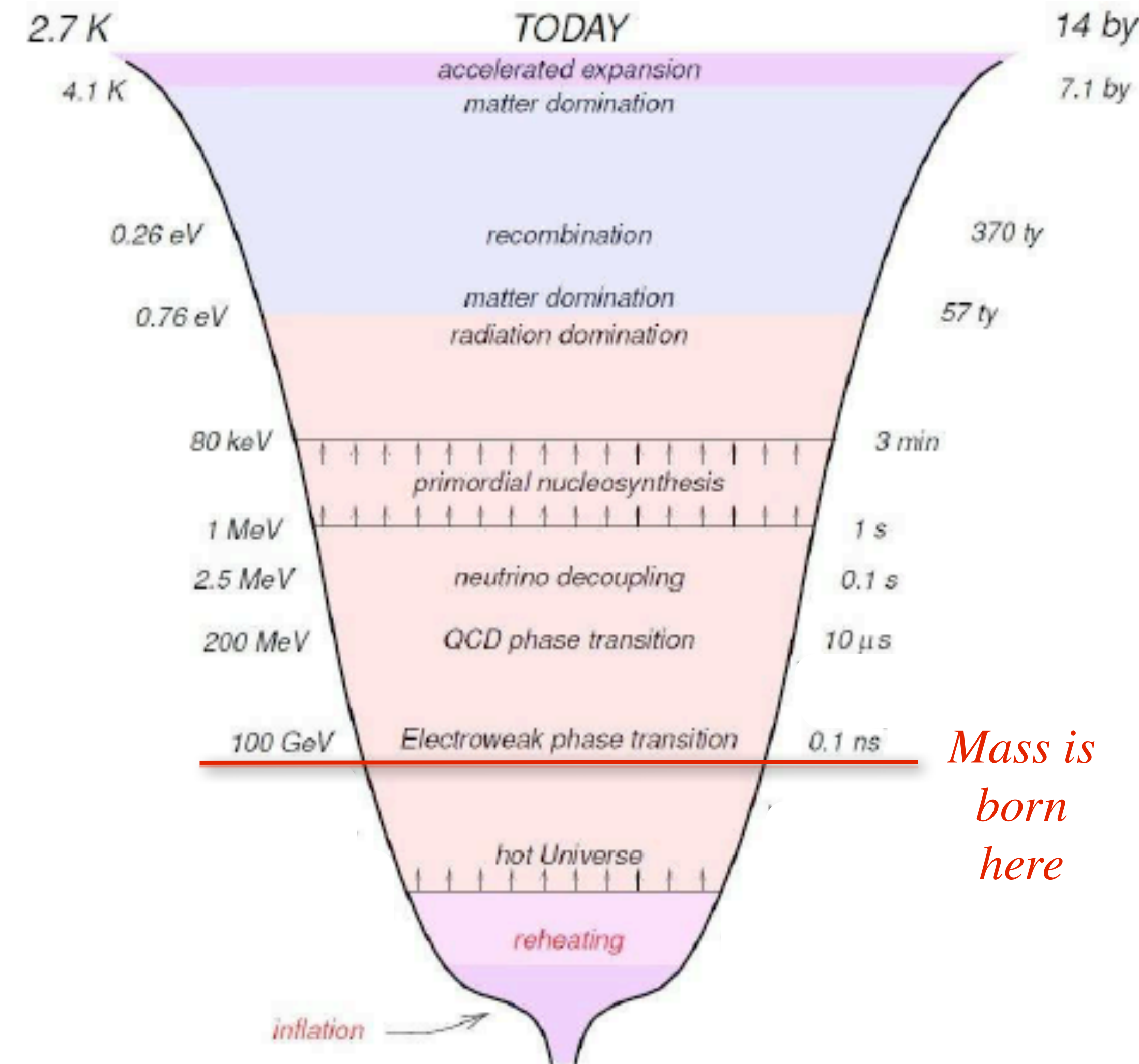
Sum over species of the standard model

Results are in suggesting agreement with observations

$$\mathbf{J}_b = \frac{2\pi\alpha\hbar}{m_p^2} T\mathbf{R}^2 \xi_b$$

$$\Lambda = \int \mathbf{J}_b dx^b = \frac{2\pi\alpha\hbar}{m_p^2} \int_{t_0}^t T\mathbf{R}^2 dt$$

$$\Lambda \approx \frac{\bar{m}_t^4 T_{ew}^3}{m_p^7} m_p^2 \approx \underbrace{\left(\frac{T_{ew}}{m_p}\right)^7}_{10^{-120}} m_p^2$$



Results are in suggesting agreement with observations

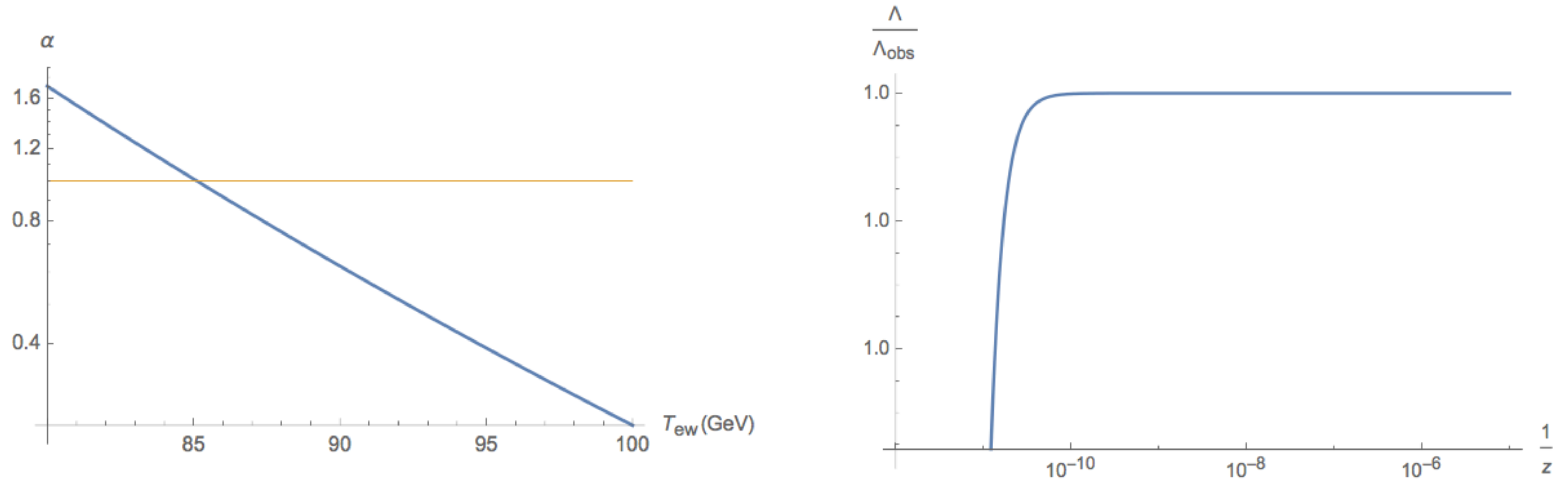


Figure 1: **Left:** The value of the phenomenological parameter α that fits the observed value of Λ_{obs} as a function of the EW transition scale T_{ew} in GeV. We see that for $T_{ew} \approx 100\text{GeV}$ $\alpha \approx 1$.

Right: The time dependence of Λ expressed in terms of the inverse redshift factor $1/z$.

Discussion

1. Loop Quantum Gravity predicts discreteness of spacetime geometry at the Planck scale ([Rovelli-Smolin](#)).
2. Discreteness opens the way for a fundamental account of black hole entropy. The approach of LQG is fundamentally **not** holographic. This avoids contradictions with standard QFT and GR in the regimes where we expect both to be valid approximations (e.g. [the firewall problem](#)).
3. The resolution of information problem requires dynamics across the singularity. Discreteness of LQG regularises singularities (in models of cosmology ([Ashtekar, Bojowald, Singh,...](#)) and BH collapse ([Ashtekar, Rovelli, Pullin, ...](#))).
4. Decoherence with discrete microstructure is a natural and provides a resolution of the information problem. But decoherence implies diffusion; this leads to a simple phenomenological model for an emergent cosmological constant which agrees with observations.
5. Open hard problems in LQG: The continuum limit (E. Bianchi, B. Dittrich, ...), dynamics (spin-foams, [S. Speziale, M. Han,...](#)), fundamental observables ([L. Freidel, J. Lewandowski, T. Thiemann, K. Giesel,...](#)) description of matter...

Thank you!