Scattering of gravitational waves by compact bodies

Tom Stratton* and Sam Dolan†

School of Mathematics and Statistics, University of Sheffield

Introduction

- Beginning in 2015, multiple direct detections of gravitational waves (GWs) have been announced [1]. GWs are generated by highly energetic astrophysical sources, such as binary mergers or supernovae.
- In principle, GWs may be significantly scattered by sufficiently curved regions of spacetime [2].
- We examine a planar GW impinging on a spherically symmetric compact body, such as a neutron star.
- We find evidence for a **rainbow interference effect** [3], similar to that seen at the much smaller scales of heavy ion scattering.
- A **cusp caustic** forms downstream of the body. We compare with black hole scattering studies [2].

Model

- The compact body is modelled with a polytropic (PT) equation of state:

$$p = \kappa \rho^{1+1/n}. \tag{1}$$

- We consider two polytropic indices that model a neutron star $n \in \{0.5, 1\}$, and a constant density star $n = 0$.
- We fix the tenuity $\alpha := \frac{c^2 R}{GM} = 6$, where R is the objects' radius, and M its mass. For a neutron star $\alpha \approx 6$.

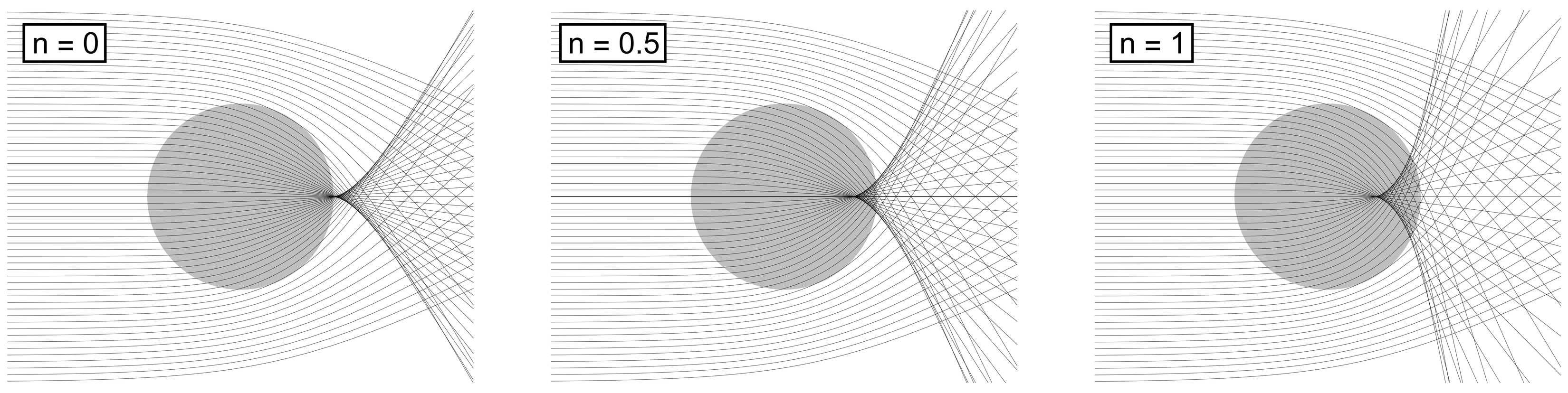

Figure: Geodesics for a massless particle, incident from the left on a variety of stellar models. Downstream, a caustic forms outlining a rainbow wedge, with a cusp near or inside the stellar surface. As the polytropic index n increases, the objects' mass is more concentrated to the centre, and the maximally deflected ray is scattered by a greater angle (defining the **rainbow angle**: θ_b).

Stars and black holes

- The metric perturbation for the scattered GW

$$h_{AB} \sim r \sum_{p=\pm 1} \sum_{l \geq 2}^{\infty} \sum_{m=-l}^{m=l} \Phi^p_{lm}(r,t) X^{lmp}_{AB}(\theta, \phi) \quad \text{as } r \to \infty, \tag{2}$$

where Φ^p_{lm} are **gauge invariant** master functions [4]. With this, the GW scattering cross section is calculated as the sum of two terms: **helicity preserving**, σ_P, and **reversing**, σ_R, scattered radiation.

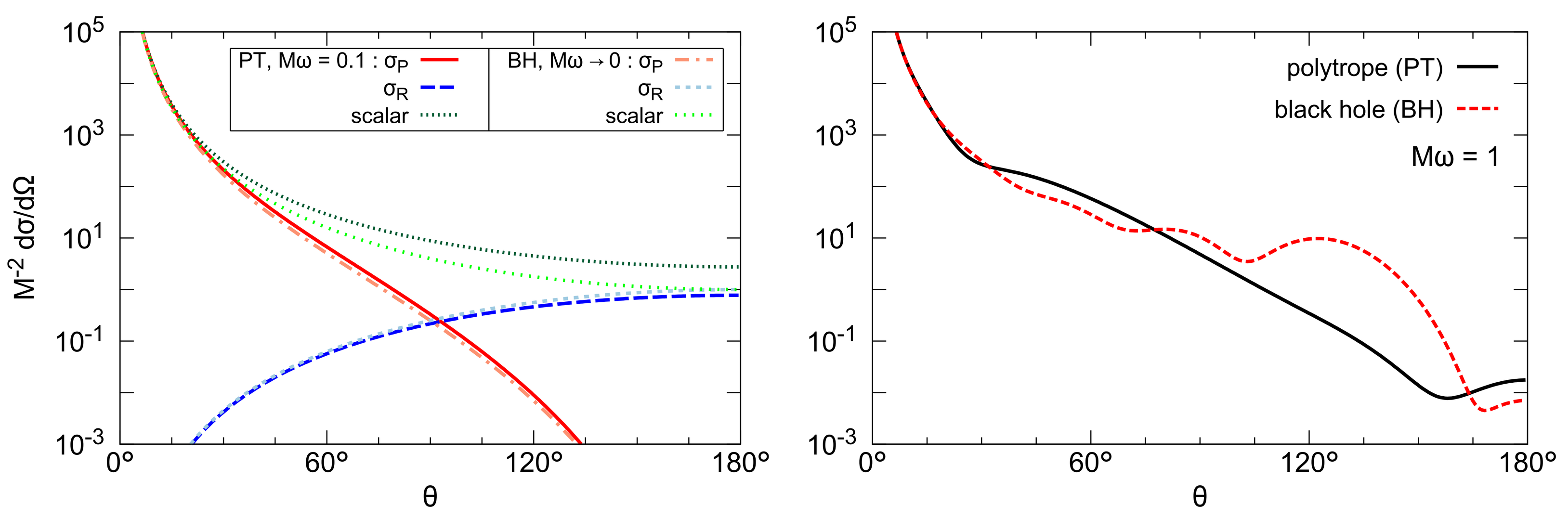

Figure: (a) Cross sections for a wave with low frequency, $M\omega = 0.1$, incident on a star with $\alpha = 6$ and $n = 1$. Low frequency approximations for Schwarzschild black hole (BH) scattering cross sections are also shown. It appears the stars' cross section approaches the same limit as $M\omega \to 0$. (b) For couplings $M\omega \gtrsim 1$, the cross sections become qualitatively different for stars and BHs.

Rainbow scattering

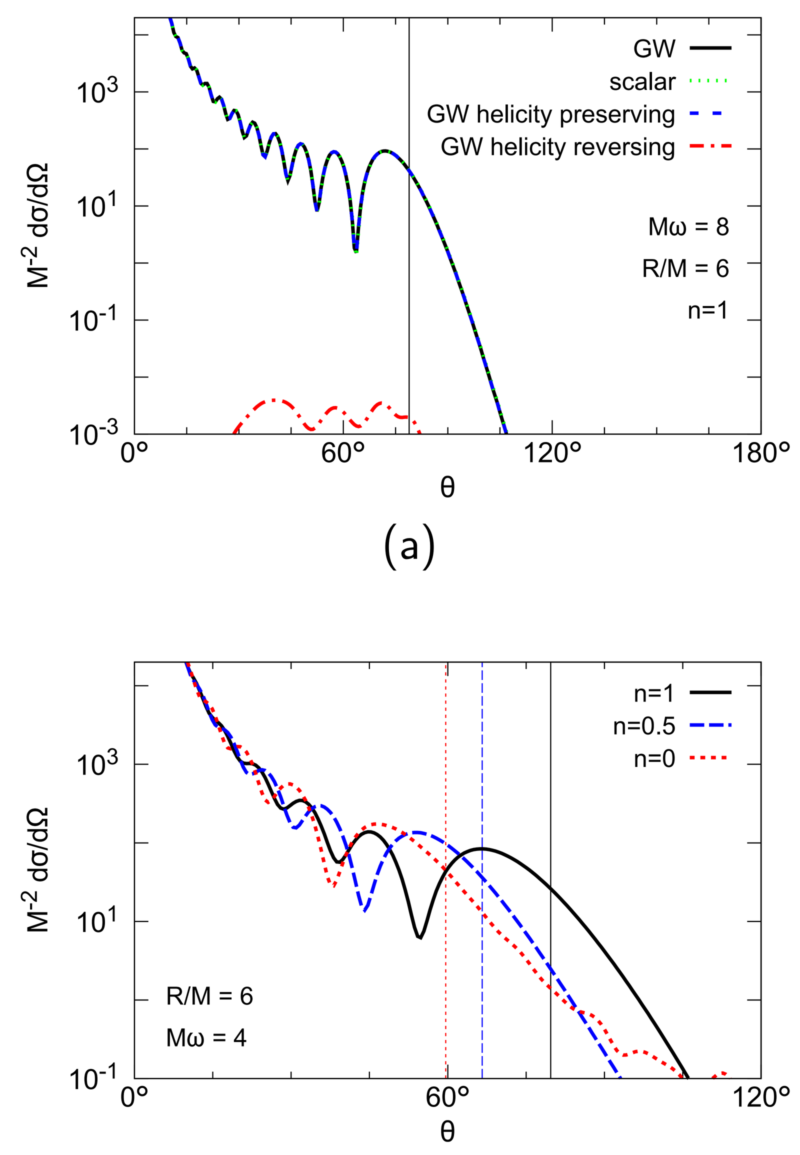

Figure: The scattering process is encapsulated by three dimensionless parameters: α, n, and $M\omega$.
(a) For $M\omega \gtrsim 1$ the cross section shows a rainbow pattern, both for GWs and scalar waves. Vertical lines mark the rainbow angles. Below this a primary maximum is followed by supernumerary troughs and peaks. Beyond the rainbow angle, the cross section falls off into the shadow zone. There is a forward divergence and negligible helicity reversal.
(b) The GW cross sections for the three different stellar models is qualitatively the same. However, the interference pattern (and θ_b) is shifted to greater angles for larger n.

Near field scattering profile

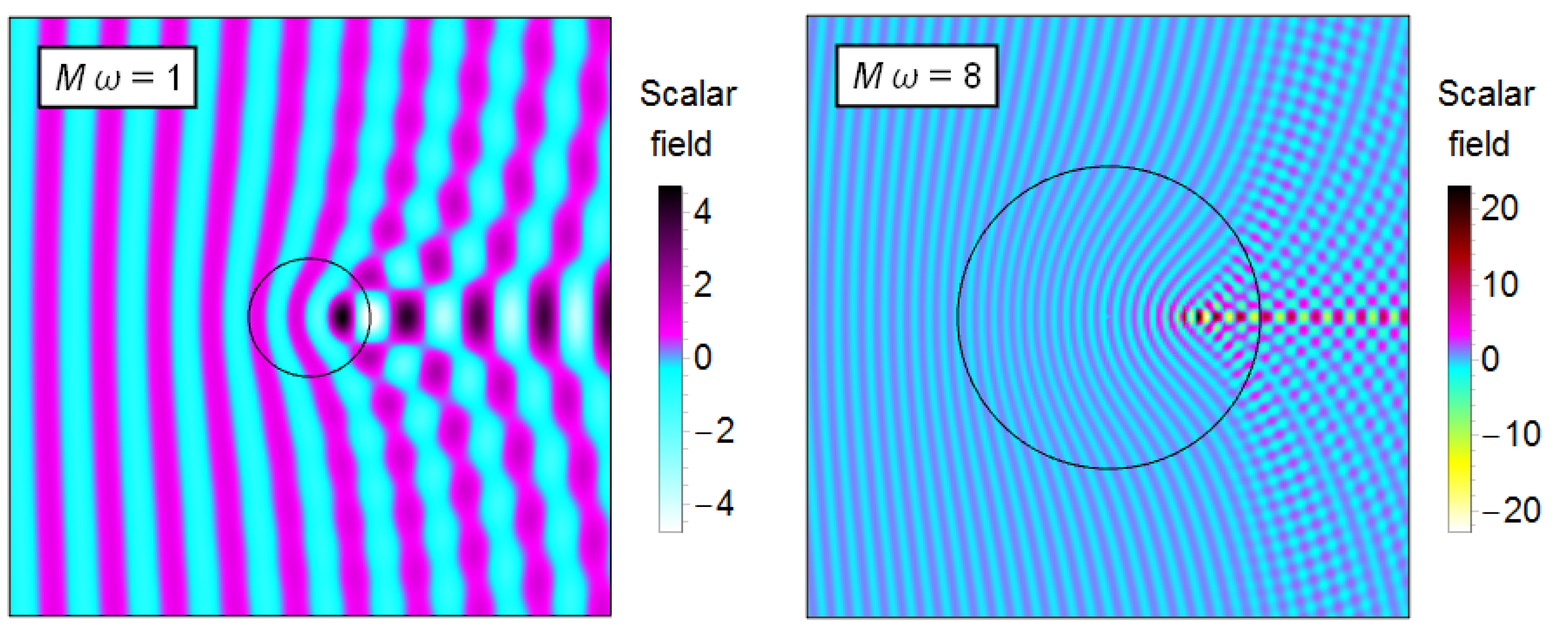

Figure: A unit amplitude scalar plane wave scattering off a polytropic star (outline in black), with $\alpha = 6$ and $n = 1$. The incident wave has coupling $M\omega = 1$ (a), and $M\omega = 8$ (b). At the cusp, the wave is amplified by a factor $A \approx 4$ (a), and $A \approx 20$ (b). In Fig. (b) one can just about see the caustic feature at $\theta_b \approx 79.7°$.

Discussion

- Time-independent scattering of gravitational plane waves by a compact body such as a neutron star exhibits a rainbow interference pattern when $M\omega \gtrsim 1$.
- The scattering cross section features depend on the structure of the central object, i.e. the equation of state.
- Scattering patterns for compact bodies and black holes (without rotation) are qualitatively different at high frequency, but similar at low frequency.
- In this model, a scalar wave is a good proxy for a GW when $M\omega \gtrsim 1$.
- Coupling to fluid perturbations seems to have a negligible effect on the cross section. The GW is scattered almost entirely by the spacetime curvature alone. Further work using the Inverse Cowling Approximation could confirm this.

References

[1] B. P. Abbott *et al.* (Virgo, LIGO Scientific), arXiv:1811.12907, (2018).

[2] J. A. H. Futterman *et al.*, *Scattering from black holes*, Cambridge University Press (1988).

[3] S. R. Dolan and T. Stratton, Phys. Rev. **D95**, 124055 (2017).

[4] K. Martel and E. Poisson, Phys. Rev. **D71**, 104003 (2005).

* tstratton1@sheffield.ac.uk , † s.dolan@sheffield.ac.uk