SPRITZ:
a new fully general relativistic magnetohydrodynamic code

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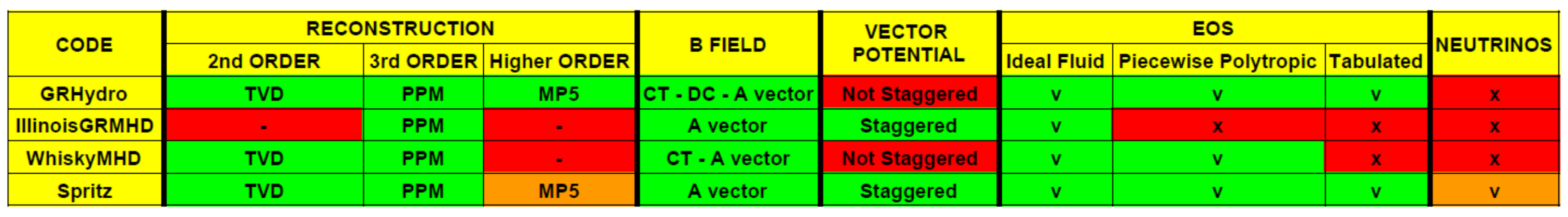
Introduction

Numerical modeling of Gravitational Wave (GW) sources, e.g. neutron star binaries (NS-NS), black hole binaries (BH-BH) and neutron star black hole binaries (NS-BH), has now become one of the most important fields of study in theoretical astrophysics, with the aim of comparing simulated data with observations. In particular, only a fully general relativistic magnetohydrodynamic (GRMHD) treatment may give an accurate picture of these scenarios. In addition, it is also important to consider a general treatment for the NS Equation Of State (EOS) that can take into account finite temperature effects.

Comparison with existing codes

In the literature, several fully general relativistic codes that perform GRMHD evolution of given sources already exist. The publicly available ones are the `IllinoisGRMHD` code [3] and the `GRHydro` code [5]. Our group in Trento has also its own GRMHD code: `WhiskyMHD` [4]. All these codes benefit from the publicly available `Einstein Toolkit` [1]. In the past years, they have been optimized and extensively used to simulate several GW sources. However, they all present limitations both from a technical (e.g. numerical methods to evolve the magnetic field, limited reconstruction order, etc...) and a physical (e.g. support for different EOSs, neutrinos, etc...) point of view. Starting from `WhiskyMHD`, we decided to improve this code in order to overstep the aforementioned limitations. See table 1 for a quick and comprehensive comparison of the mentioned codes.



CODE	RECONSTRUCTION			B FIELD	VECTOR POTENTIAL	EOS			NEUTRINOS
	2nd ORDER	3rd ORDER	Higher ORDER			Ideal Fluid	Piecewise Polytropic	Tabulated	
GRHydro	TVD	PPM	MP5	CT - DC - A vector	Not Staggered	v	v	v	x
IllinoisGRMHD	-	PPM	-	A vector	Staggered	v	x	x	x
WhiskyMHD	TVD	PPM	-	CT - A vector	Not Staggered	v	v	x	x
Spritz	TVD	PPM	MP5	A vector	Staggered	v	v	v	v

Table 1: Comparison Table for existing GRMHD codes' features. Color coding is as follows:
Green – implemented;
Red – not–implemented;
Orange – future implementation.

A staggered mesh grid

As in every numerical code, the space domain is divided in grid–cells of user specified dimensions. The fluid's state variables (e.g. ρ, P, $\vec{v}$) are stored in the grid–cells' centers. Usually, the electric and magnetic fields ($\vec{E}$ and $\vec{B}$) are instead stored respectively on cells' edges and faces. `Spritz` evolves the magnetic field as the curl of a given vector potential ($\vec{A}$), whose components are staggered just like $\vec{E}$ (see figure 1) and are evolved using the modified Lorenz gauge.

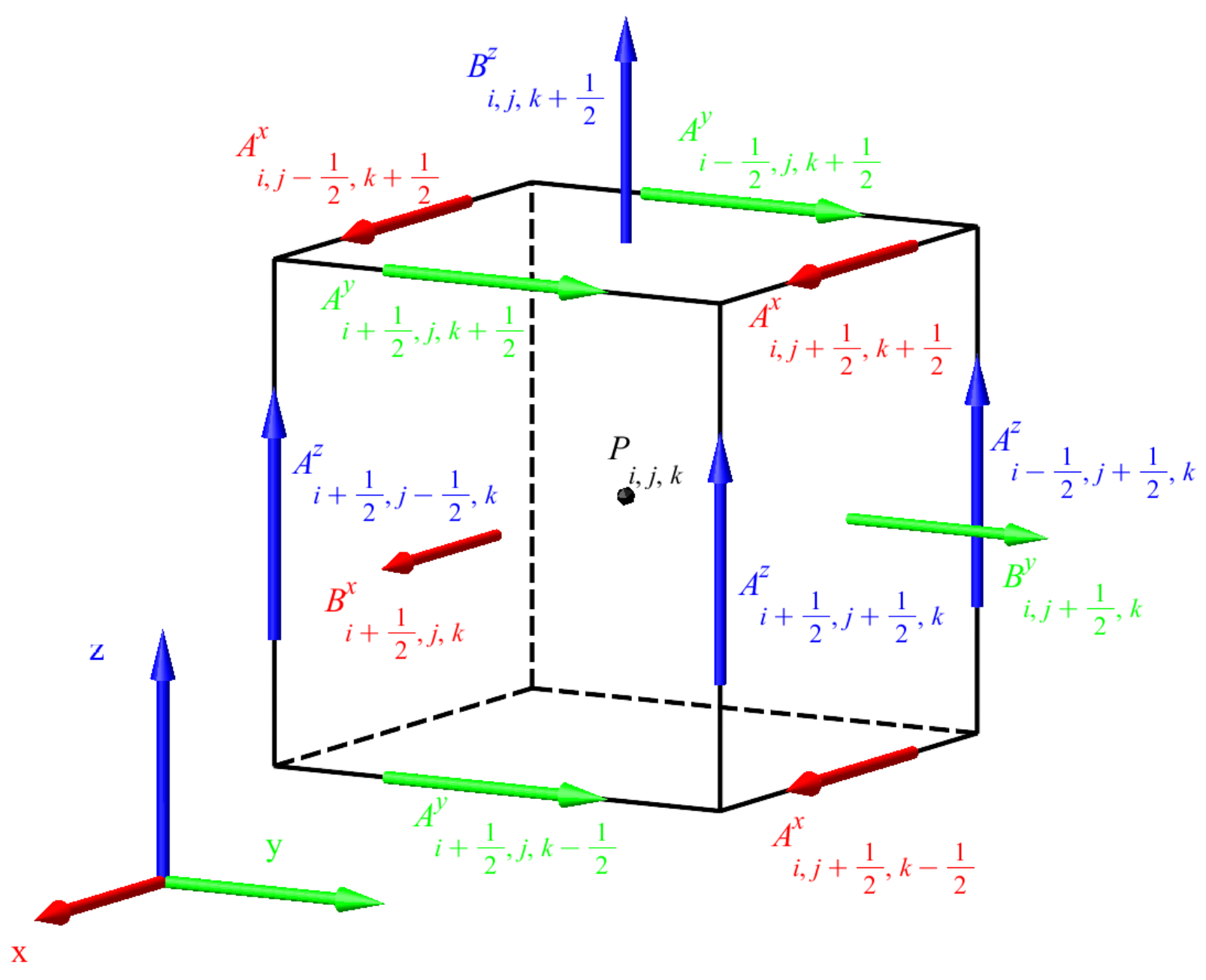


Figure 1: Example of grid–cell for the numerical algorithm, with clear representation of storage locations for magnetic field and vector potential components. Point $P_{i,j,k}$ represents the cell's center.

Since $\vec{B}$ is computed from the curl of $\vec{A}$, the divergence–free character of the magnetic field is automatically satisfied. An alternative scheme could store both $\vec{A}$ and $\vec{B}$ in the cells' centers (e.g. `WhiskyMHD`). An example of the different results for a shock–tube $1D$ test obtained via a staggered $\vec{A}$ and a not–staggered one is shown in figure 2.

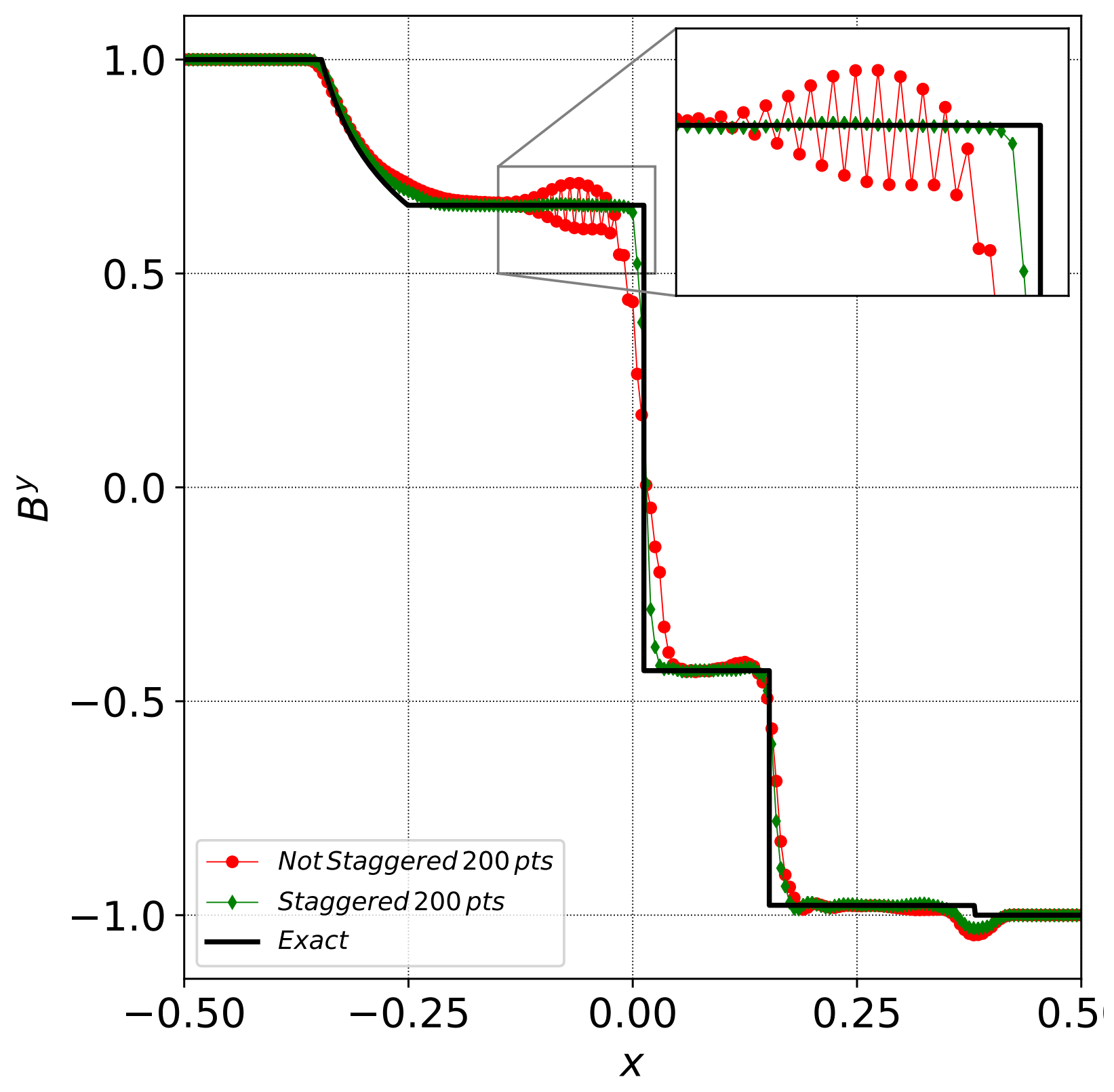


Figure 2: Comparison of results of `Balsara 1` test (from [2]), obtained via staggered and not–staggered vector potential.

Preliminary test results

We began an extensive code testing considering the most known $1D$ and $2D$ tests. In figure 3 we show the numerical results of the `Spritz` code for the $1D$ tests of Balsara [2].

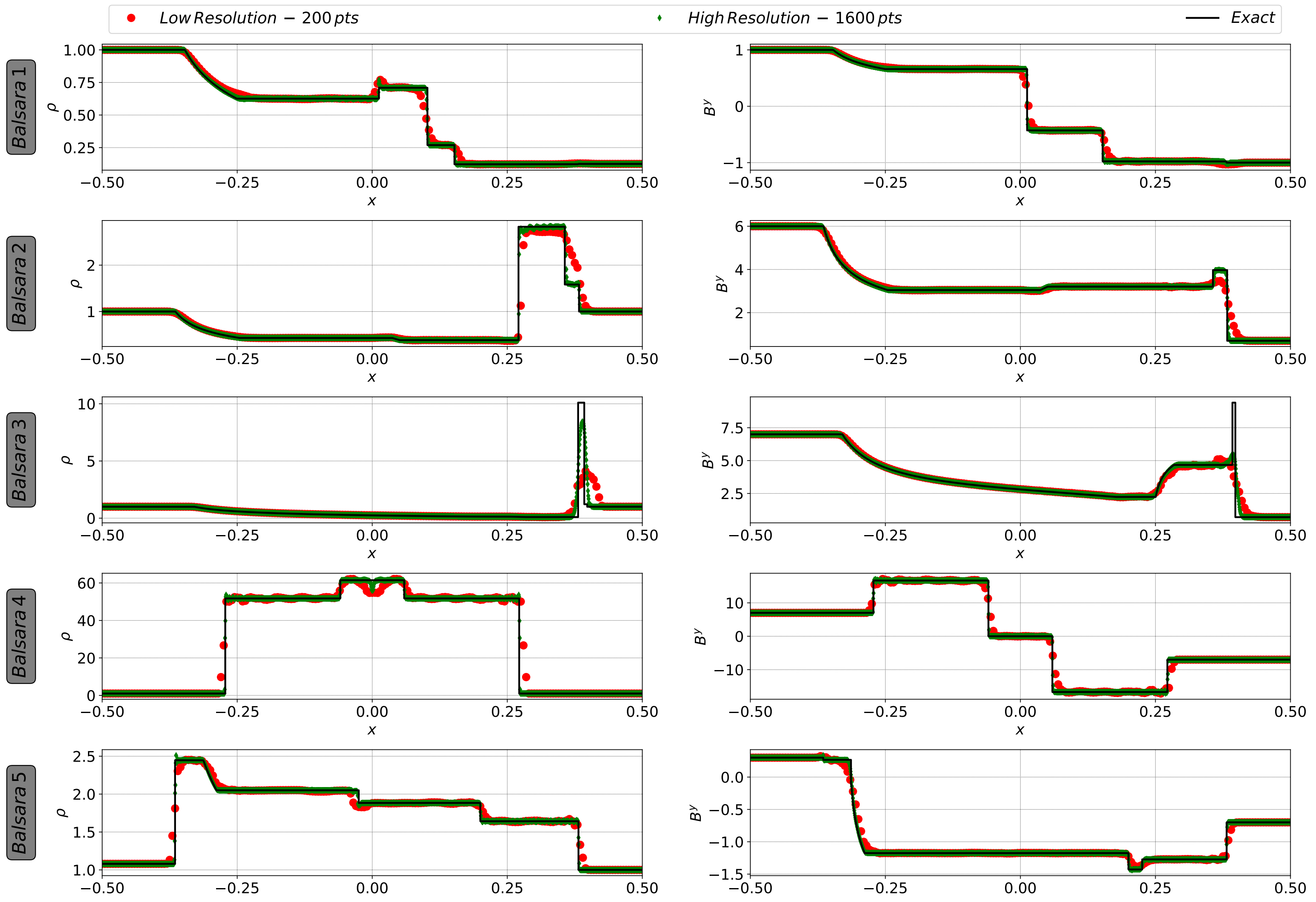


Figure 3: Results of $1D$ tests from [2] with low–resolution (200 grid–points, red dots) and high–resolution (1600 grid–points, green diamonds). The continuous black line represents the exact solution.

In figure 4 we show instead the evolution of the pressure on the equatorial plane for the 2D cylindrical explosion test [5].

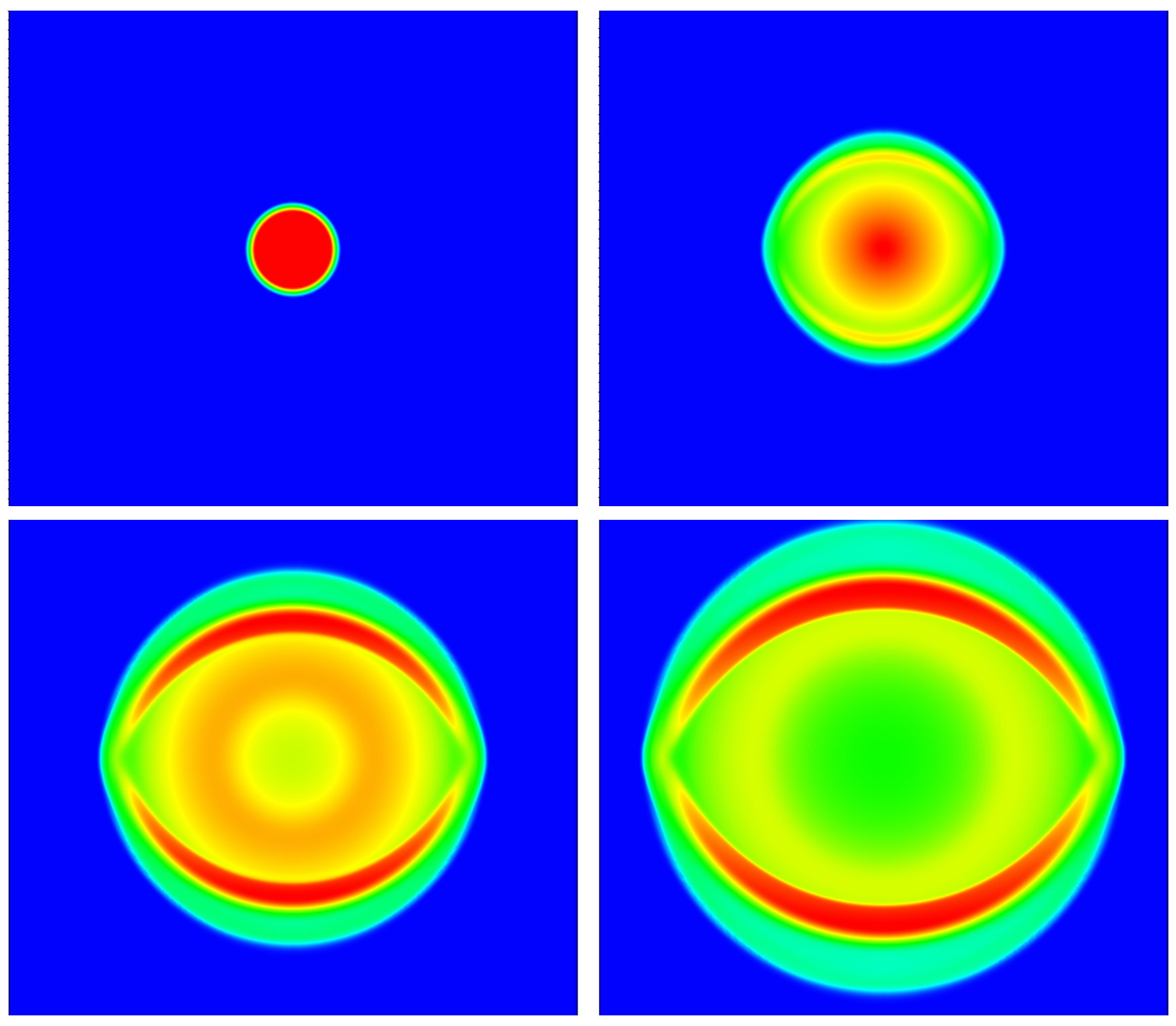


Figure 4: Evolution of the cylindrical explosion $2D$ test. From up left to bottom right the images show configuration at time $0.0, 1.5, 3.0, 4.0$.

Forthcoming Research

We will continue the exensive testing further with other already known $2D$ tests. After that we will implement also a fifth order reconstruction (MP5) and proceed with fully general relativistic $3D$ tests (e.g. magnetized TOV, BH accretion, and NS binary). At the same time, we are also implementing a leackage scheme to take into account neutrino emission. The final aim of this work is to obtain a GRMHD code that can evolve magnetized binary NS mergers taking into account neutrino emission and finite temperature EOS. Up to now, there been only one such simulation (see [6]). The `Spritz` code will allow us to considerably extend the range of parameters we can explore and compute more accurate GWs and electromagnetic signals (from short gamma-ray bursts to kilonovae).

References

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