Corpuscular gravity
from cosmology to black holes

Roberto Casadio
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I EPS Conference on Gravitation
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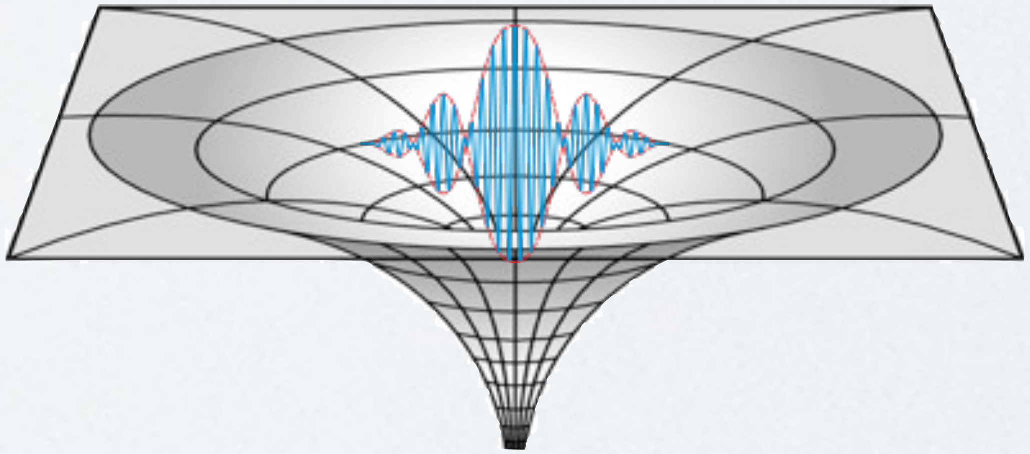
La Sapienza - Roma



Gravitational collapse = Singularities

Theorem: GR predicts (infinitely dense) singularity at the end of the collapse.

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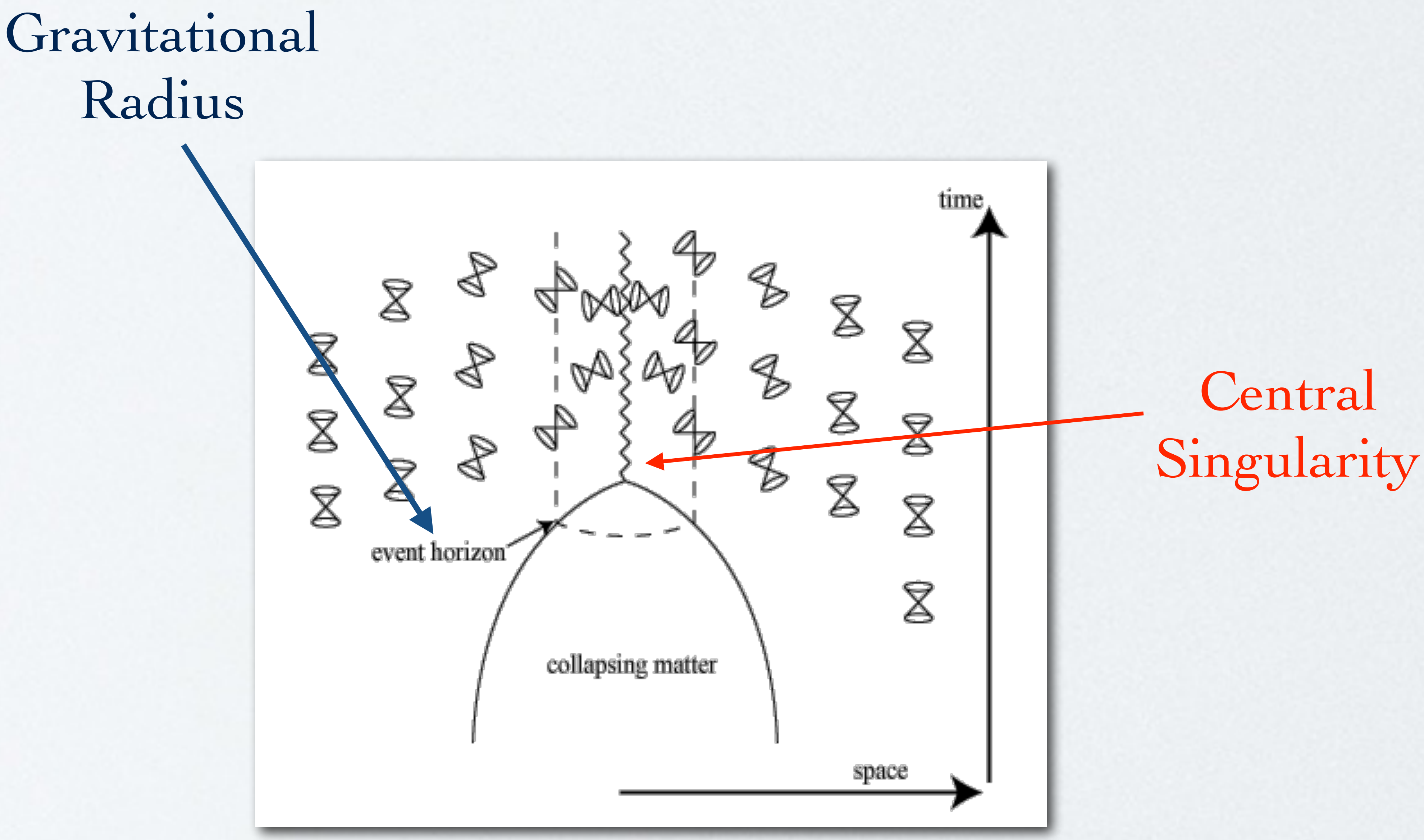

“John’s interest in relativity was triggered in January 1951, when he studied the 1938-39 work of Robert Oppenheimer and …” (*J. A. Wheeler - 1911-2008*, by Kip S. Thorne)

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Uncertainty Principle: QM prevents singularities.

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↓

GR must fail (in some regimes)

Alternatives: aplenty…

Gravitational collapse = Singularities

Lesson from semiclassical theory: new effects already at the horizon scale

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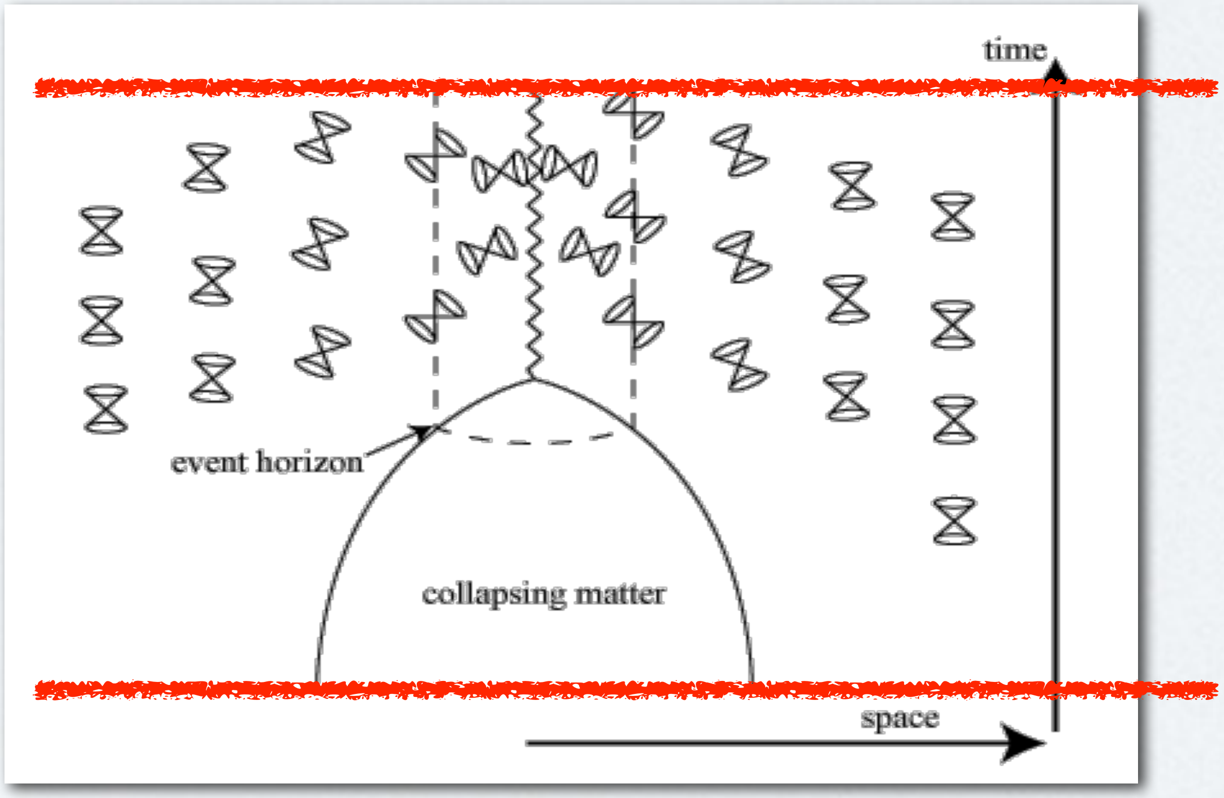

$$|0; t = +\infty\rangle$$

$$e^{-\frac{i}{\hbar}\int \hat{H}\, dt}$$

$$|0; t = -\infty\rangle$$

$$|0; t = +\infty\rangle = \sum \text{excitations} = \text{Hawking radiation}$$

Hawking radiation ~ physical paradoxes ~ perhaps wrong background

Solvay Congress, June 1958: "... no escape is apparent except to assume that the nucleons at the center of a highly compressed mass must necessarily dissolve away into radiation at such a rate as to keep the total number of nucleons from exceeding a certain critical number." (*J. A. Wheeler - 1911-2008*, by Kip S. Thorne)

Corpuscular - Bootstrapped Newtonian Gravity

Newtonian gravity: linear interaction

$$\triangle V_{\mathrm{N}} = 4\,\pi\,\frac{\ell_{\mathrm{p}}}{m_{\mathrm{p}}}\,\rho$$

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Perturbative GR: gravitons self-interact

Einstein-Hilbert action: $$S_{\mathrm{EH}} = \int d^4x\,\sqrt{-g}\left(\frac{m_{\mathrm{p}}}{16\,\pi\,\ell_{\mathrm{p}}}\,\mathcal{R} + \mathcal{L}_{\mathrm{M}}\right) \qquad \mathcal{L}_{\mathrm{M}} = \rho$$

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1) Weak field
2) Static non-relativistic motion
3) De Donder gauge
4) Fierz-Pauli and some guessing …

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon\,h_{\mu\nu}$$

$$h_{\mu\nu} \simeq h_{00} = -2\,V$$

$$0 = 2\,\partial^\mu h_{\mu\nu} - \partial_\nu h \simeq \partial_t V$$

$$S[V] = 4\,\pi\int\epsilon\,dt\int_0^\infty r^2\,dr\left\{\frac{m_{\rm p}}{8\,\pi\,\ell_{\rm p}}\,V\,\triangle V - \rho\,V + \frac{\epsilon}{2}\left[\frac{m_{\rm p}}{4\,\pi\,\ell_{\rm p}}\,(V')^2 + V\,\rho\right]V\right\}$$

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Newtonian Lagrangian

R.C., A. Giugno, A. Giusti, M. Lenzi, PRD 96 (2017) 044010

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Self-interaction

R.C., A. Giugno, A. Giusti, M. Lenzi, PRD 96 (2017) 044010

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Bootstrapped Newtonian Gravity: add graviton self-interaction

Gravitational potential energy:

$$U_{\rm N}(r) = 2\,\pi \int_0^r \bar{r}^2\,d\bar{r}\,\rho(\bar{r})\,V_{\rm N}(\bar{r})$$

Gravitational potential energy density :

$$\begin{aligned} J_V(r) &= \frac{1}{4\,\pi\,r^2}\,\frac{d}{dr} U_{\rm N}(r) \\ &= -\frac{m_{\rm p}}{8\,\pi\,\ell_{\rm p}}\,\left[V_{\rm N}'(r)\right]^2 \end{aligned}$$

Gravitational potential self-energy:

$$U_{\rm GG} \sim \int r^2\,dr\,J_V\,V$$

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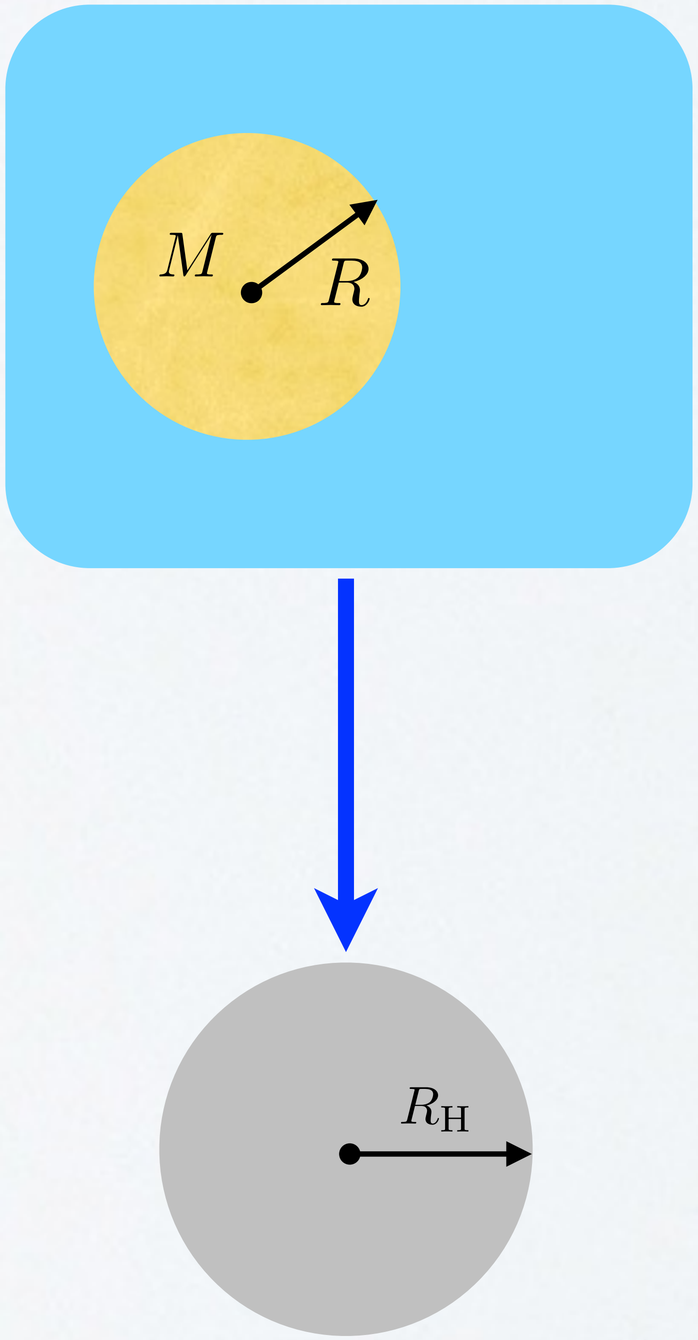
R.C., M. Lenzi, O. Micu, PRD 98 (2018) 104016

Corpuscular - Bootstrapped Newtonian Gravity

"Energy balance": $H \equiv H_{\mathrm{B}} + H_{\mathrm{G}} = M$

Star



Black Hole

R.C., A. Giugno, A. Giusti, PLB *763* (2016) *337*

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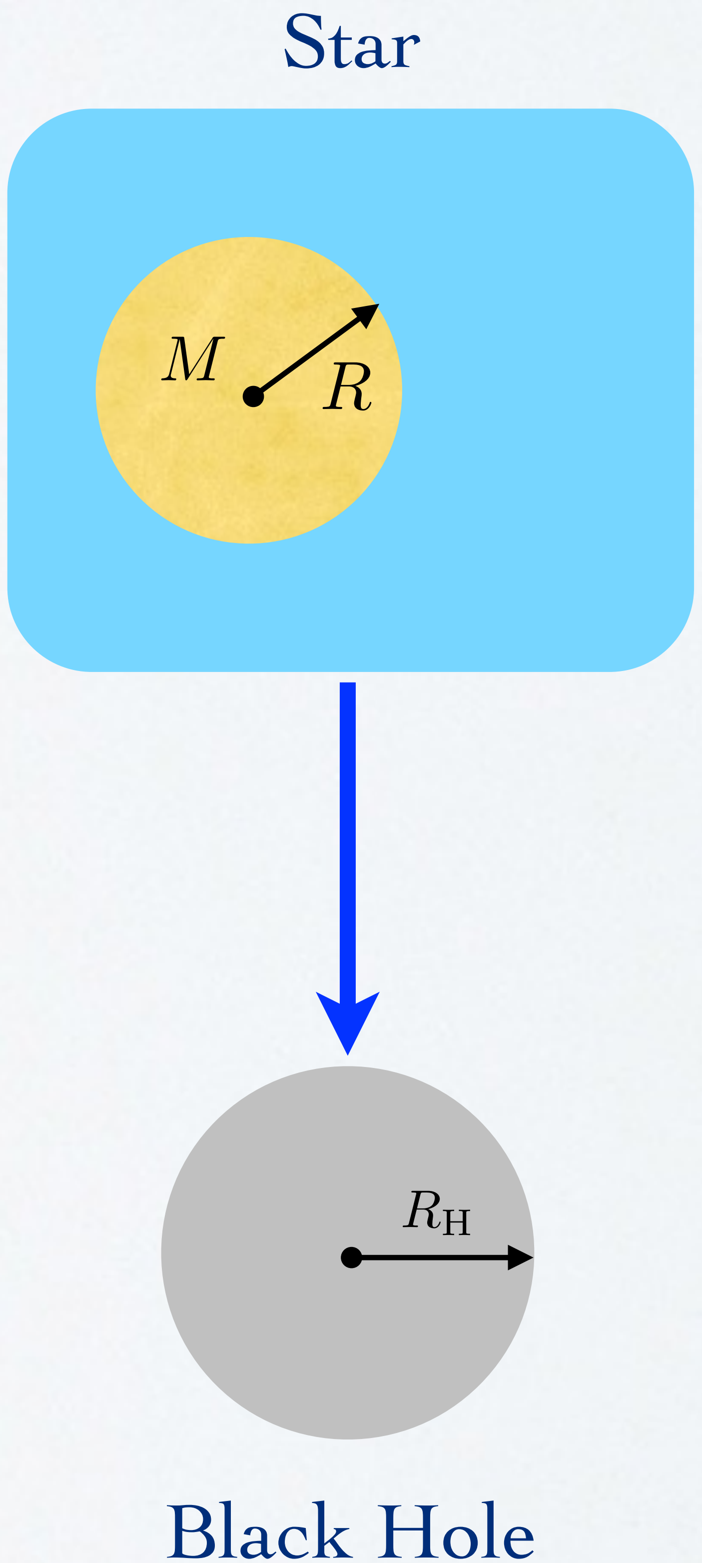

$H_{\rm B}(\infty) = \mu\, N_{\rm B} \simeq M$

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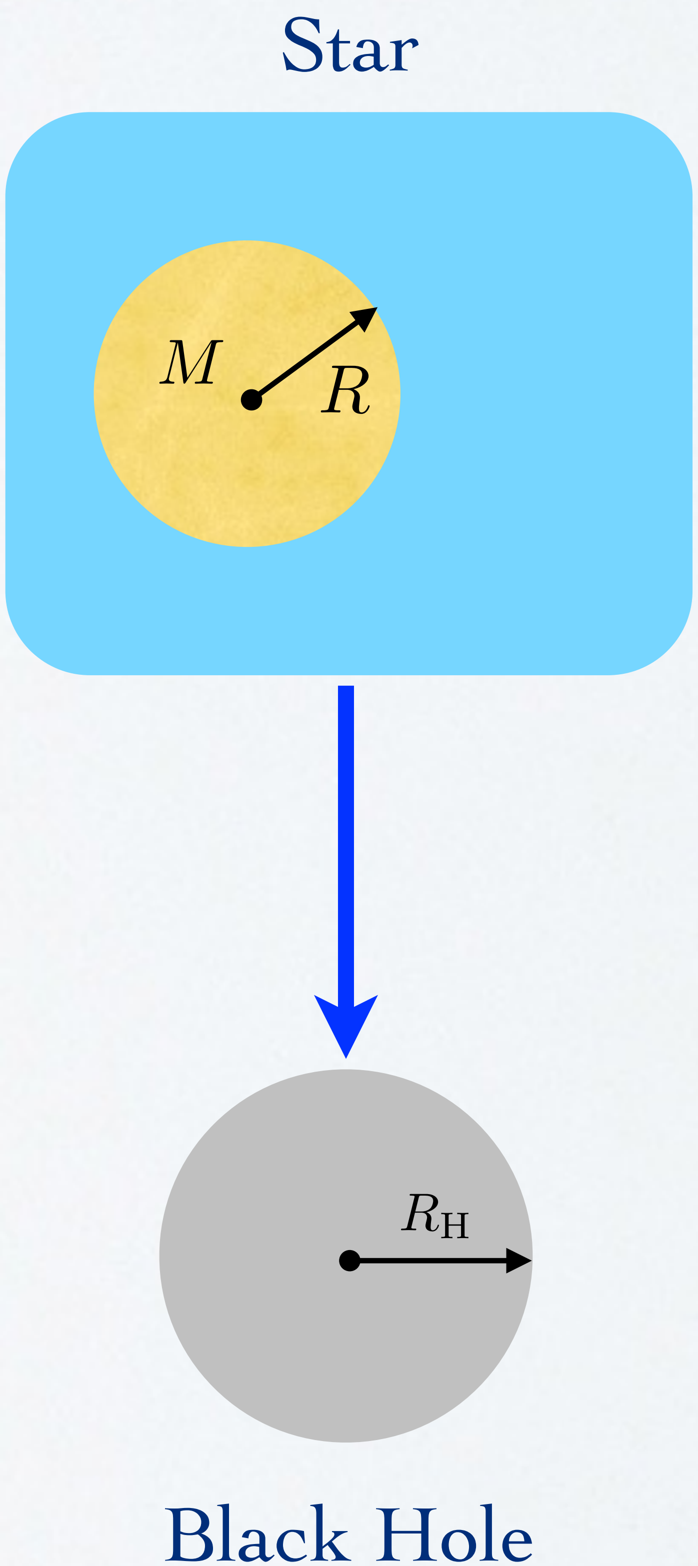

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$$H = M + K_{\rm B}(R) + U_{\rm BG}(R) + U_{\rm BB}(R) + U_{\rm GG}(R)$$

kinetic ($K_{\rm B}$, $U_{\rm BG}$) — pressure ($U_{\rm BB}$, $U_{\rm GG}$)

$$U_{\rm BG}(R) \simeq N_{\rm B}\,\mu\,\phi_{\rm N}(R) \simeq -N_{\rm B}\,\mu\,\frac{\ell_{\rm p}\,M}{m_{\rm p}\,R} = -\frac{M^2\,\ell_{\rm p}}{m_{\rm p}\,R}$$

Newton

R.C., A. Giugno, A. Giusti, PLB *763* (2016) *337*

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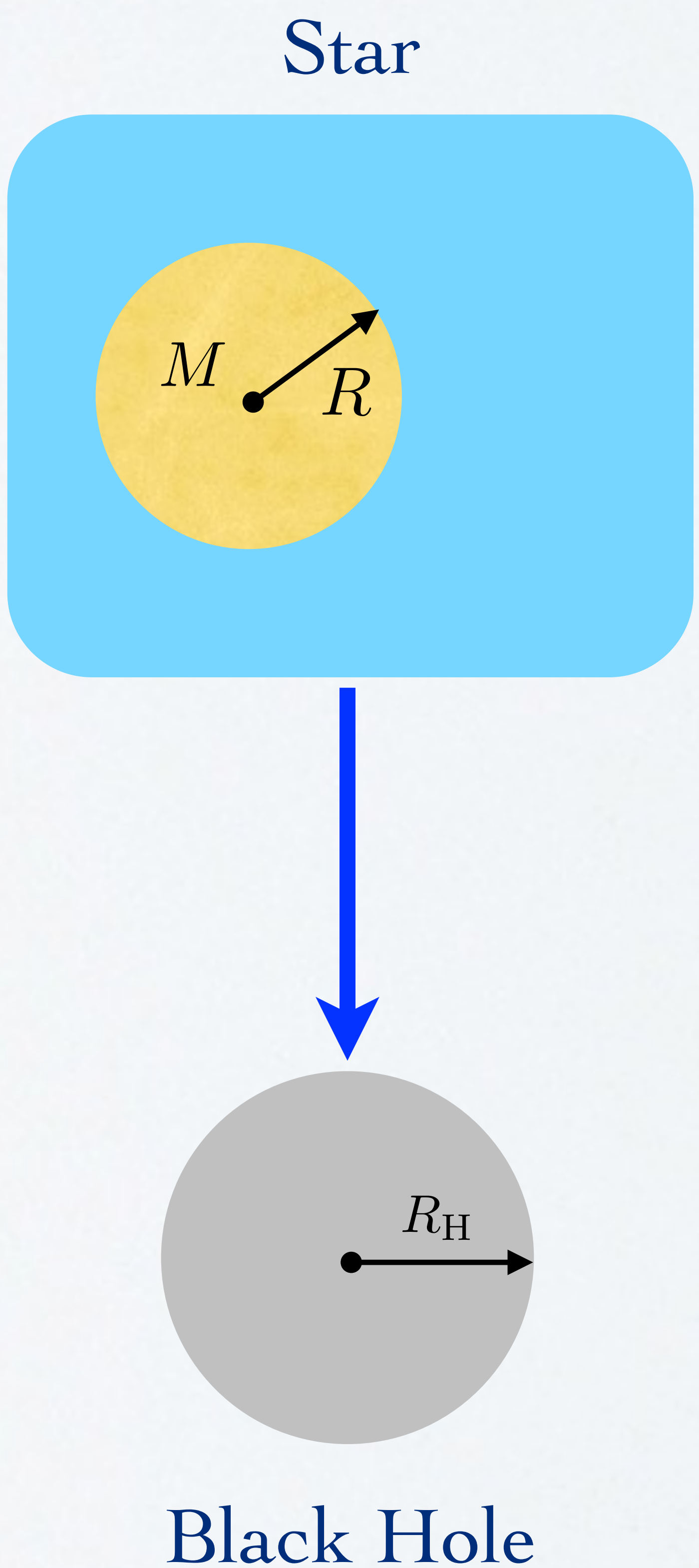

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Newton

$$U_{\rm BG}(R) \simeq N_{\rm G}\,\epsilon_{\rm G}(R)$$

Coherent state

$$N_{\rm G} \simeq \frac{M^2}{m_{\rm p}^2} \sim \frac{R_{\rm H}^2}{\ell_{\rm p}^2}$$

$$\epsilon_{\rm G} \simeq -\frac{\ell_{\rm p}}{R}\,m_{\rm p}$$

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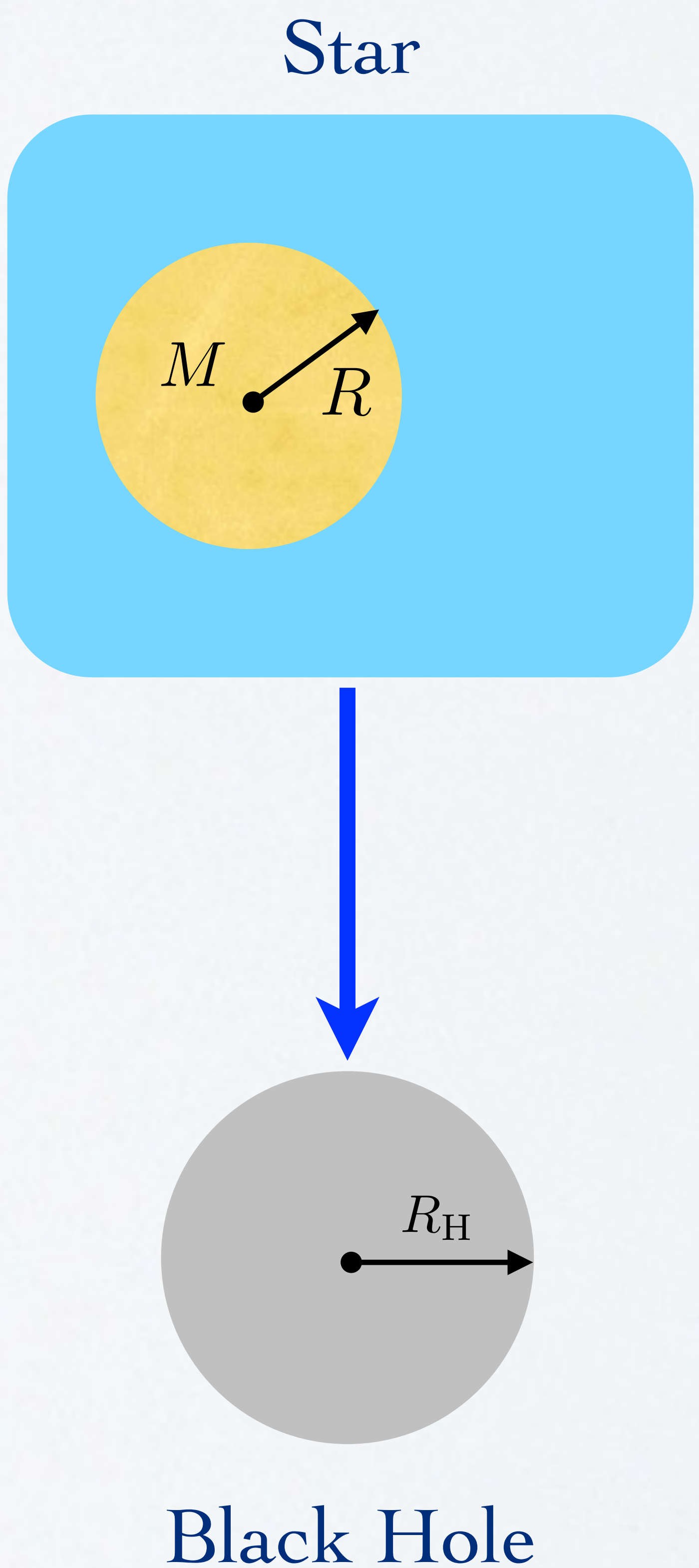

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Self-interaction

R.C., A. Giugno, A. Giusti, PLB *763* (2016) 337

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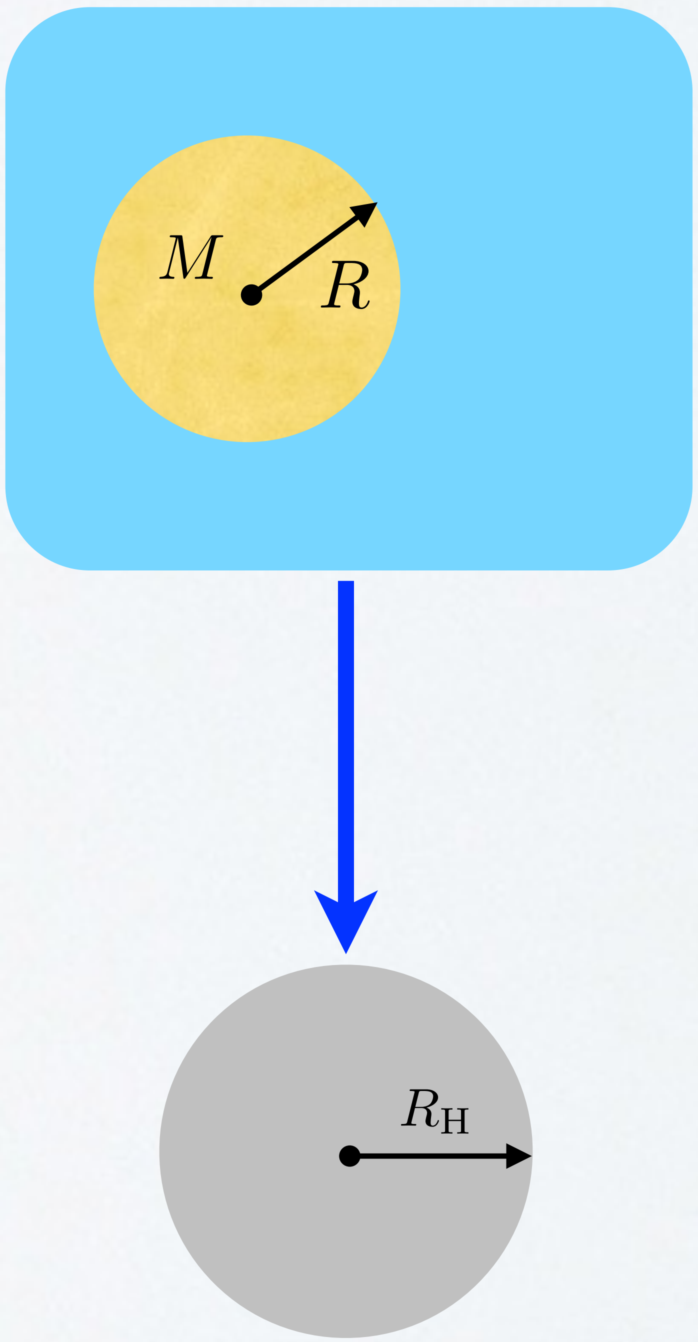

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Self-interaction

$$U_{\rm GG}(R_{\rm H}) \simeq -U_{\rm BG}(R_{\rm H}) \simeq M$$

$$R_{\rm H} \simeq \sqrt{N}\,\ell_{\rm p}$$

$$m \simeq \frac{m_{\rm p}}{\sqrt{N}}$$

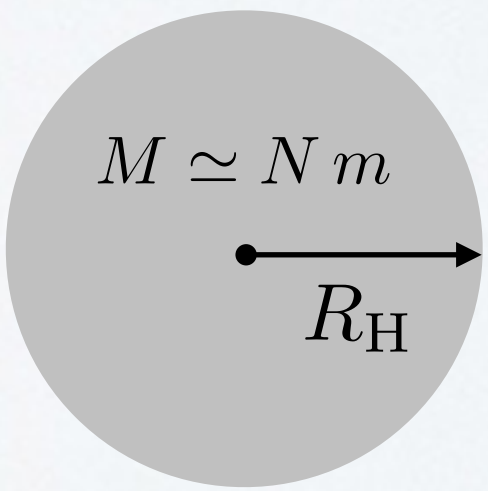
"maximal packing"

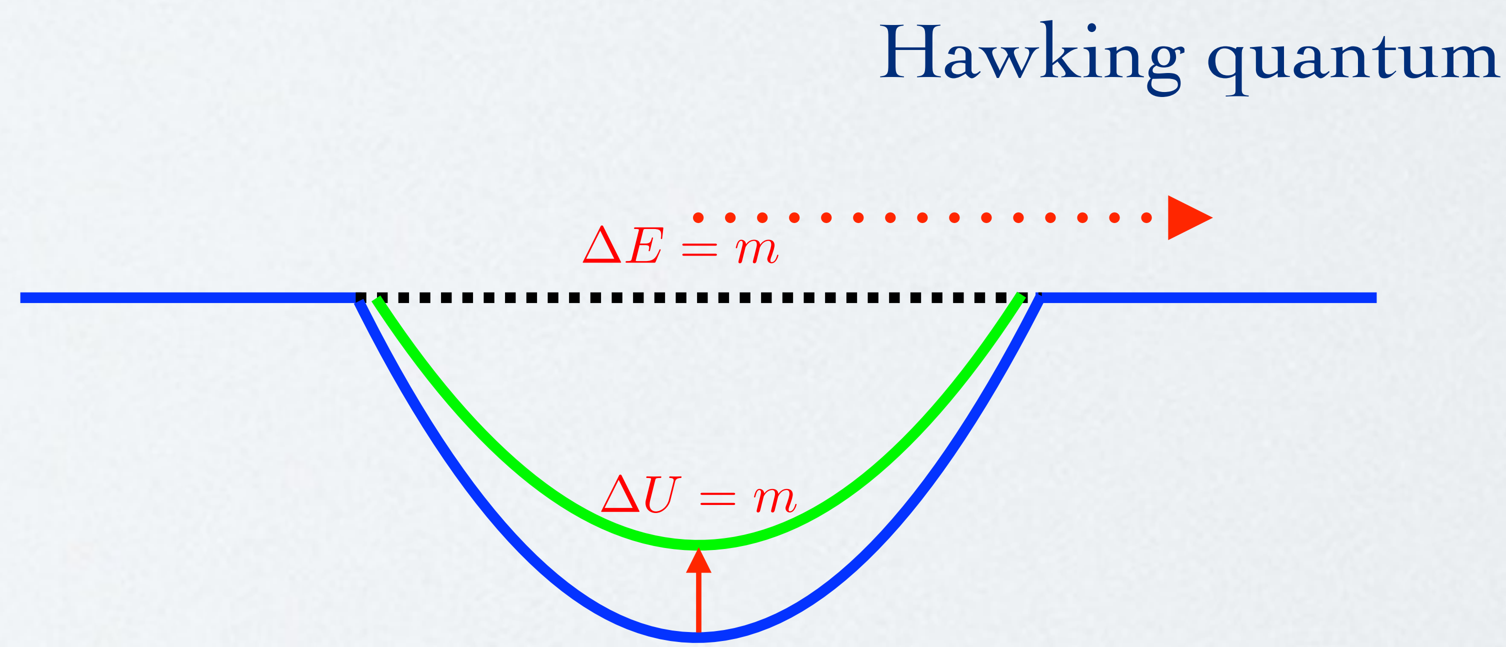
R.C., A. Giugno, A. Giusti, PLB 763 (2016) 337

R.C., M. Lenzi, O. Micu, PRD 98 (2018) 104016 + work in progress

Corpuscular Black Holes

Dvali & Gomez: BH = BEC of self-sustained gravitons at critical point





$$R_{\mathrm{H}} \simeq \sqrt{N}\, \ell_{\mathrm{p}}$$

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$$N \gg 1$$

Dvali, Gomez, Fortsch. Phys. 61 (2013) 742

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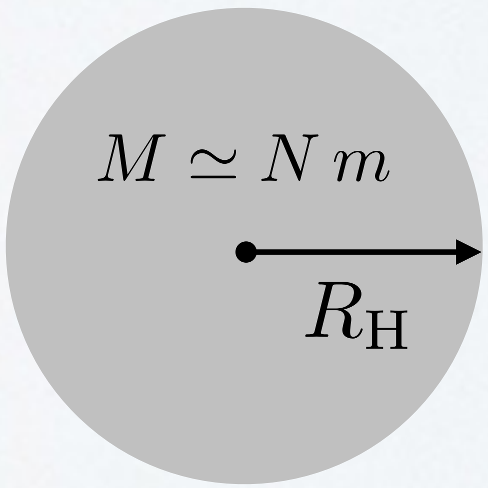

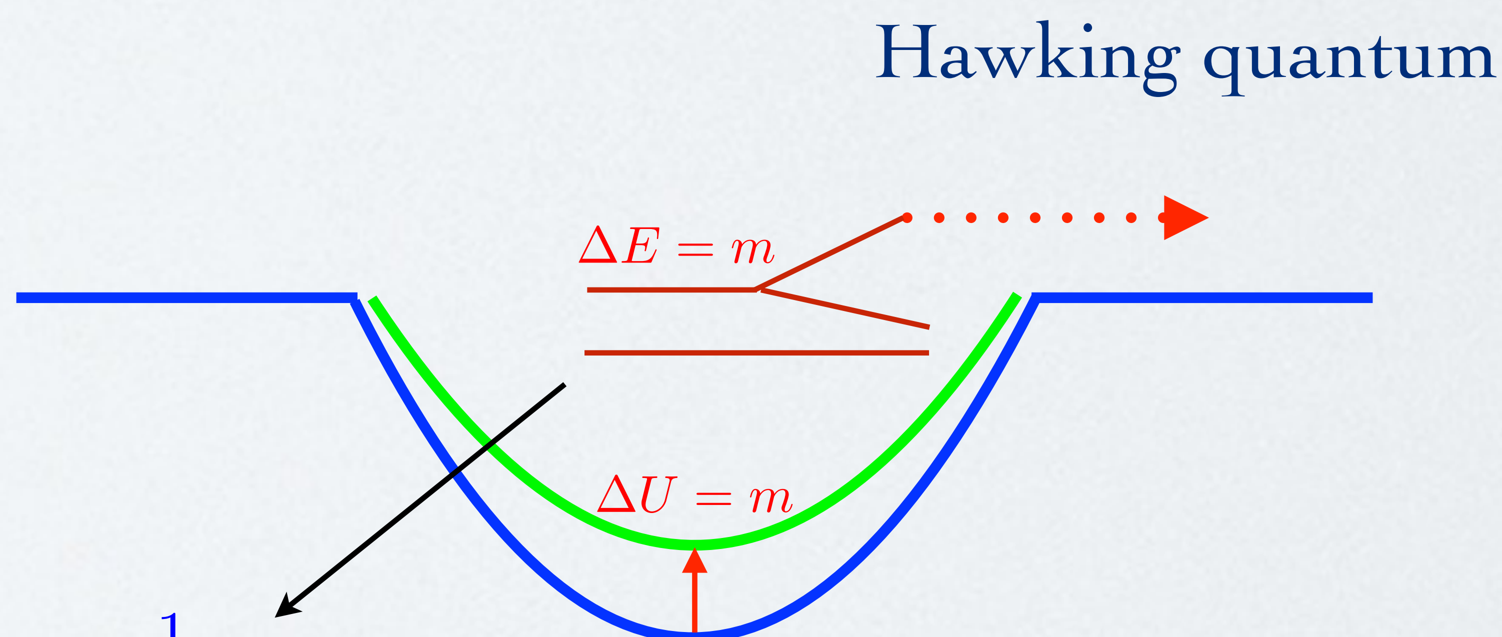

$$R_{\rm H} \simeq \sqrt{N}\, \ell_{\rm p}$$

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$$\Gamma \sim \frac{1}{N^2}\, N^2\, \frac{1}{\sqrt{N}\, \ell_{\rm p}}$$

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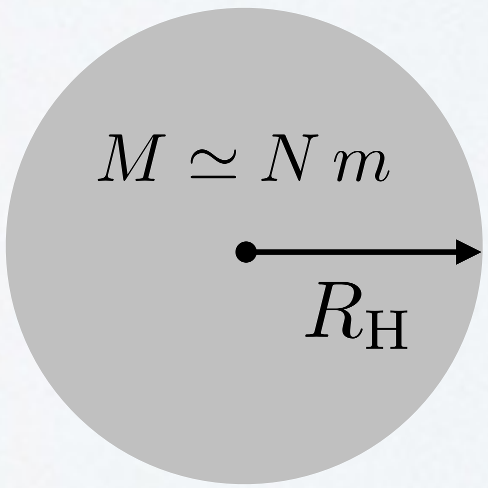

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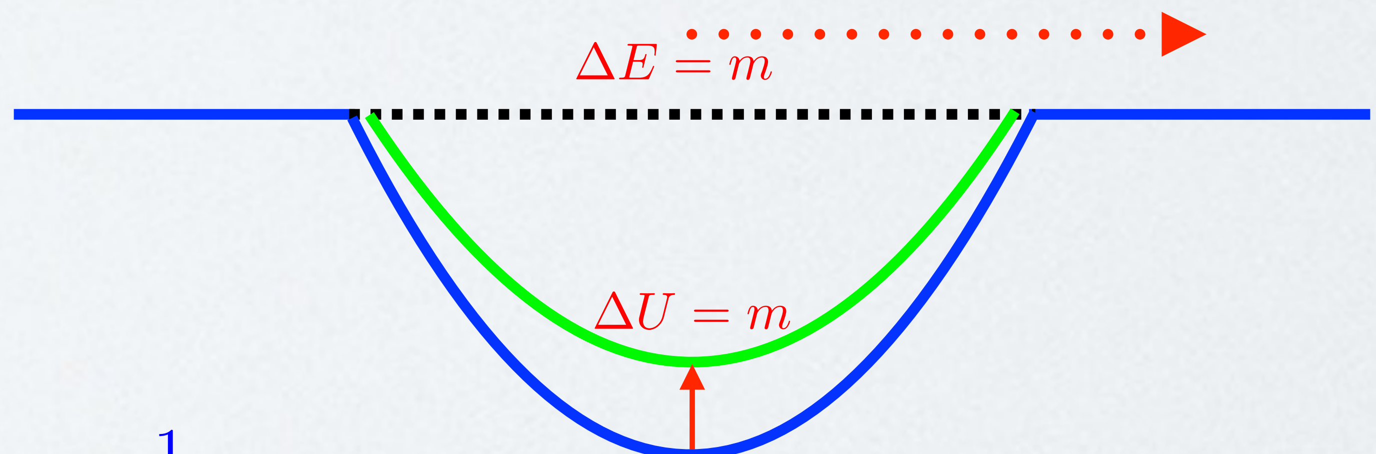
Hawking quantum



$$\Gamma \sim \frac{1}{N^2}\, N^2\, \frac{1}{\sqrt{N}\,\ell_{\mathrm{p}}}$$

$$\dot{M} \simeq m\,\frac{\dot{N}}{\sqrt{N}} \sim -m\,\frac{\Gamma}{\sqrt{N}} \sim -\frac{m_{\mathrm{p}}^3/\ell_{\mathrm{p}}}{M^2}$$

Hawking decay (for gravitons …)

Dvali, Gomez, Fortsch. Phys. 61 (2013) 742

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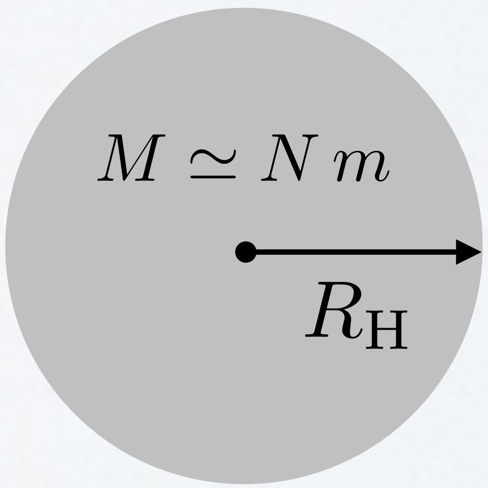

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Open problems:

1) Horizon (causal structure)?
2) Outer potential/metric?
3) Matter source inside?
4) Other Hawking particles?

R.C., A. Orlandi, JHEP 08 (2013) 025

R.C., A. Giugno, O. Micu, A. Orlandi, PRD 90 (2014) 084040

R.C., A. Giugno, A. Orlandi, PRD 91 (2015) 124069

R.C., A. Giugno, O. Micu, A. Orlandi, Entropy 17 (2015) 6893

R.C., M. Lenzi, O. Micu, PRD 98 (2018) 104016 + work in progress

Corpuscular Black Holes

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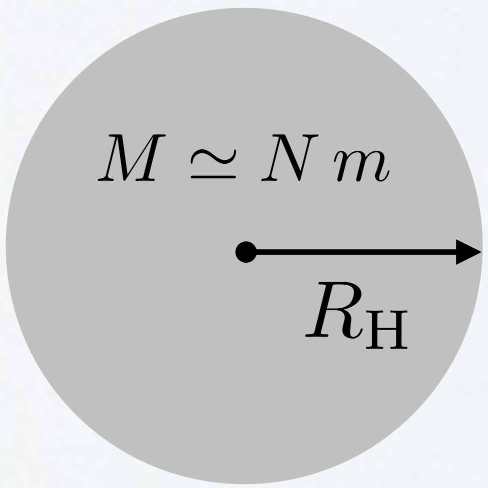

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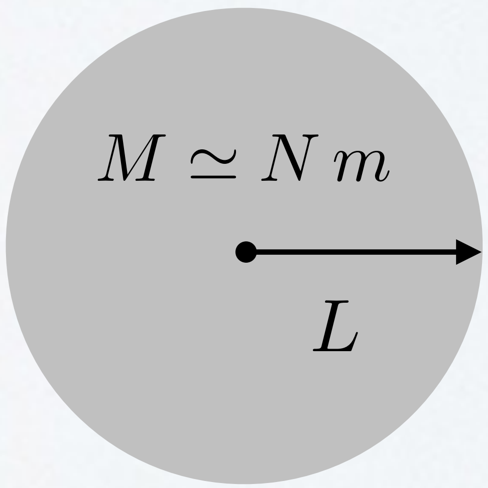
Work in progress (in the bootstrapped picture)

Corpuscular de Sitter

de Sitter = BEC of self-sustained gravitons at critical point



$L = H^{-1}$

$$H^2 = \frac{\dot{a}^2}{a^2} \simeq G_{\rm N}\, \rho$$

inflaton

G. Dvali, A. Gomez, JCAP 1401 (2014) 023

Corpuscular de Sitter

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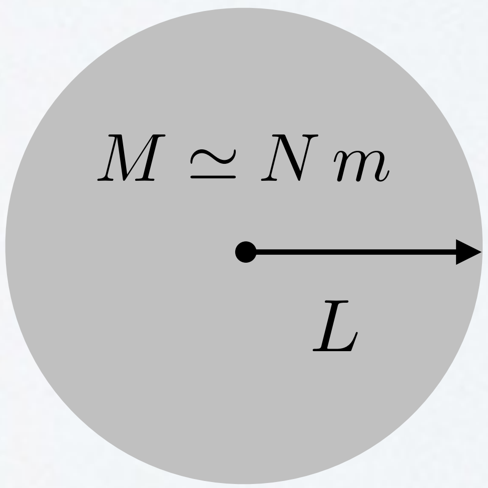

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inflaton

$$L^3\, H^2 = L \simeq G_{\rm N}\, M$$

$$L \simeq \sqrt{N}\, \ell_{\rm p}$$

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G. Dvali, A. Gomez, JCAP 1401 (2014) 023

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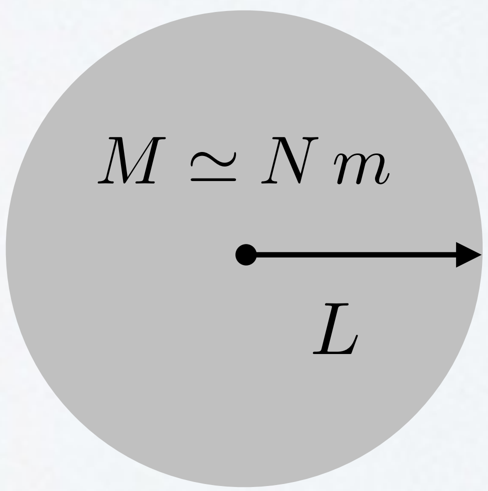

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Newtonian: R

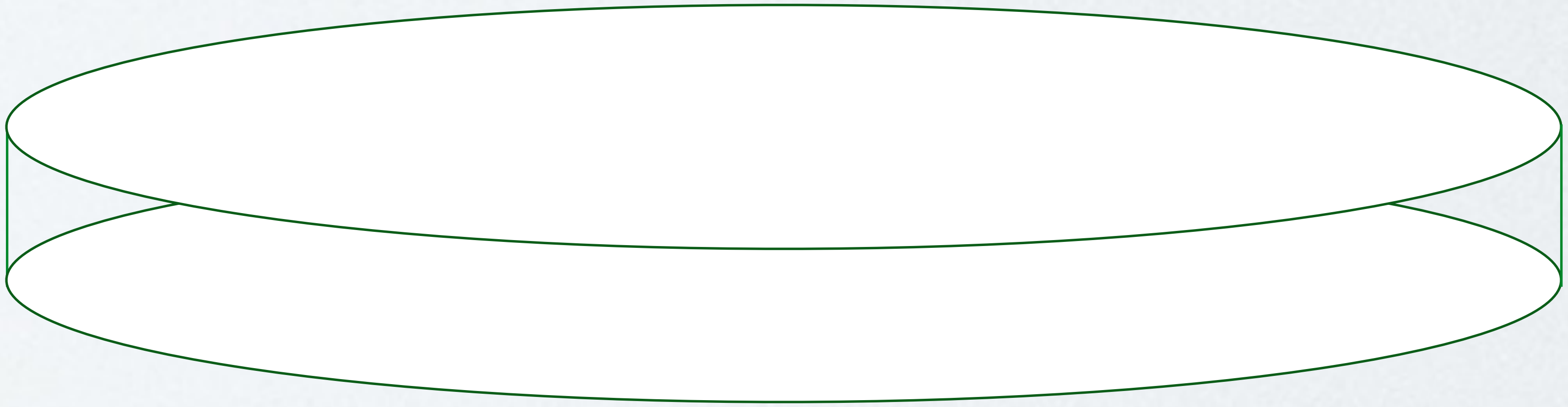
R^2 Post-Newtonian

Starobinsky's inflation

R.C., A. Giugno, A. Giusti, PRD 97 (2018) 024041

CosmoDEC

de Sitter = BEC of self-sustained gravitons at critical point



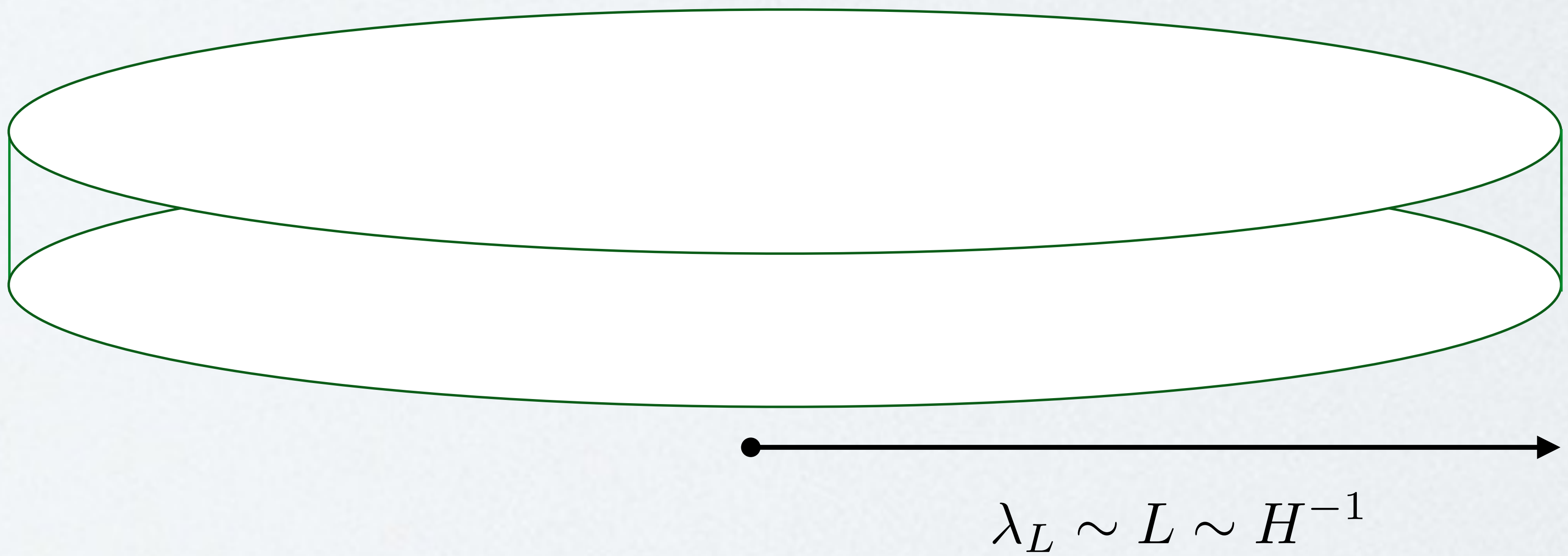
M. Cadoni, R.C., A. Giusti, W. Mück, M. Tuveri, PLB *776* (2018) 242
M. Cadoni, R.C., A. Giusti, M. Tuveri, PRD 97 (2018) 044047

CosmoDEC

de Sitter = BEC of self-sustained gravitons at critical point

Holographic

$L \simeq \sqrt{N}\, \ell_{\mathrm{p}}$



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CosmoDEC

de Sitter + clumped matter

Holographic

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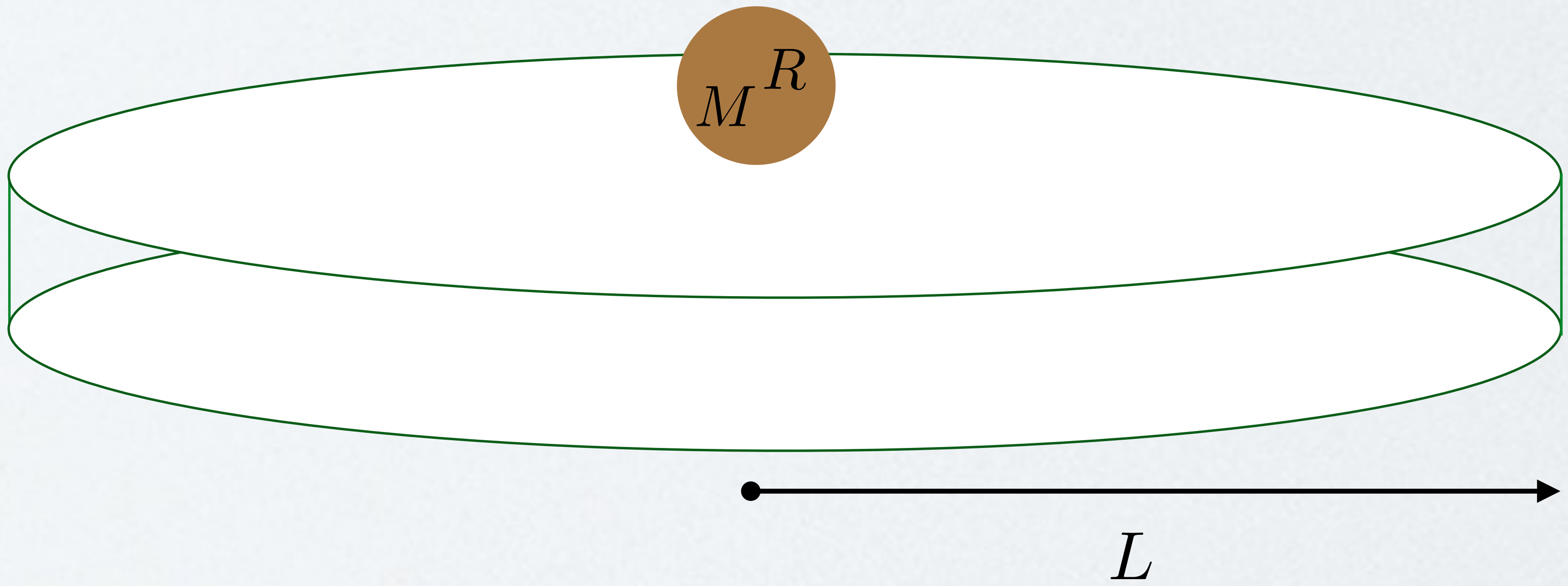

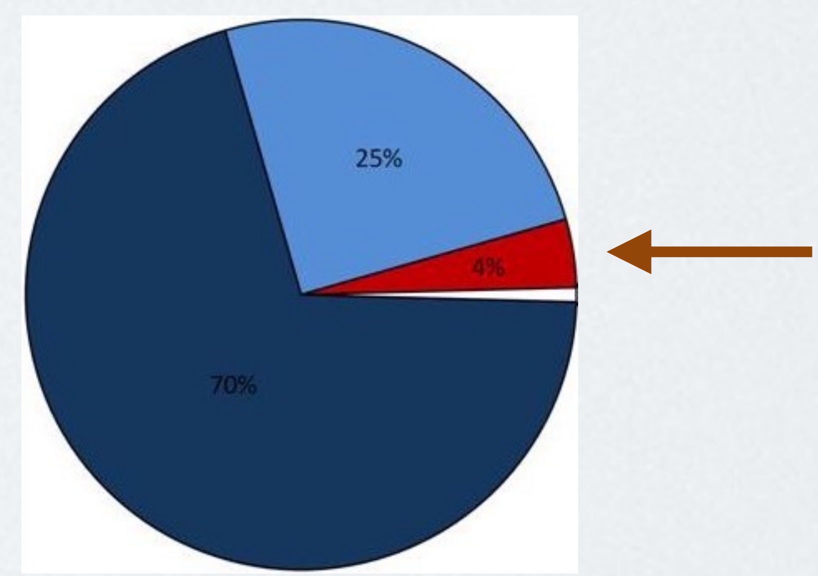

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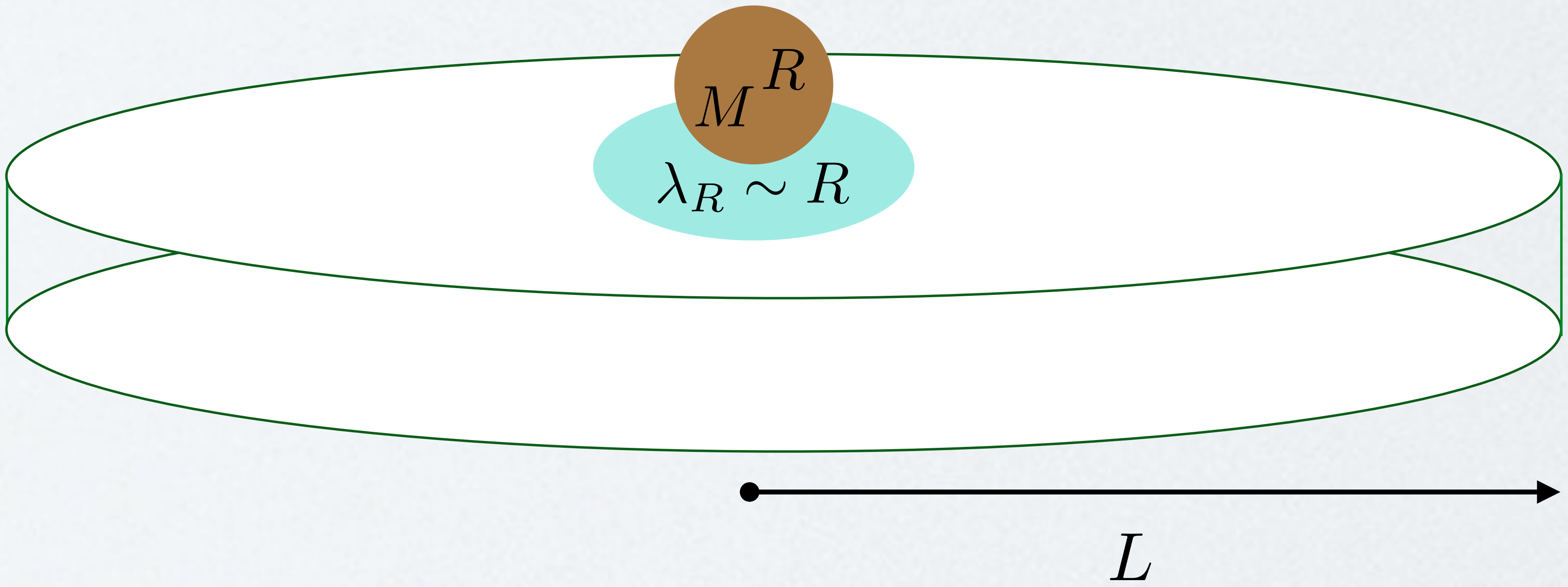
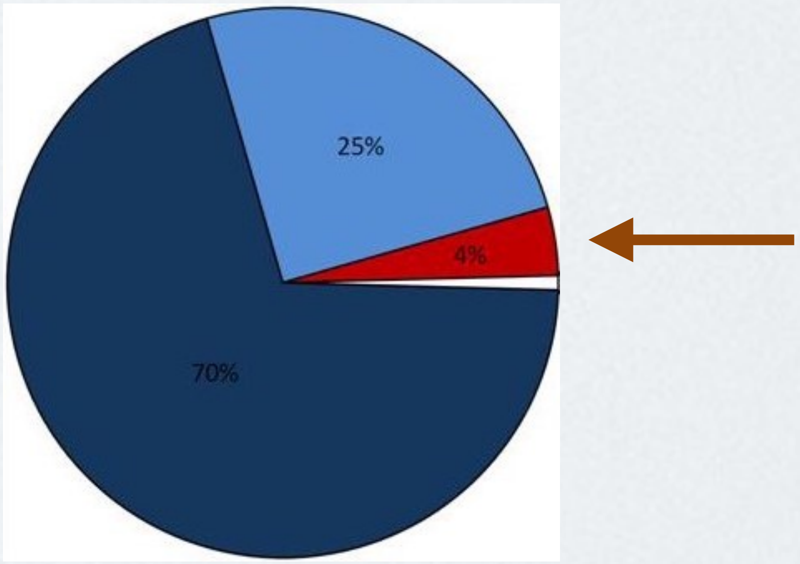


Holographic

$$R \simeq \sqrt{N_R}\,\ell_{\rm p}$$



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CosmoDEC

de Sitter + clumped matter = Dark Matter

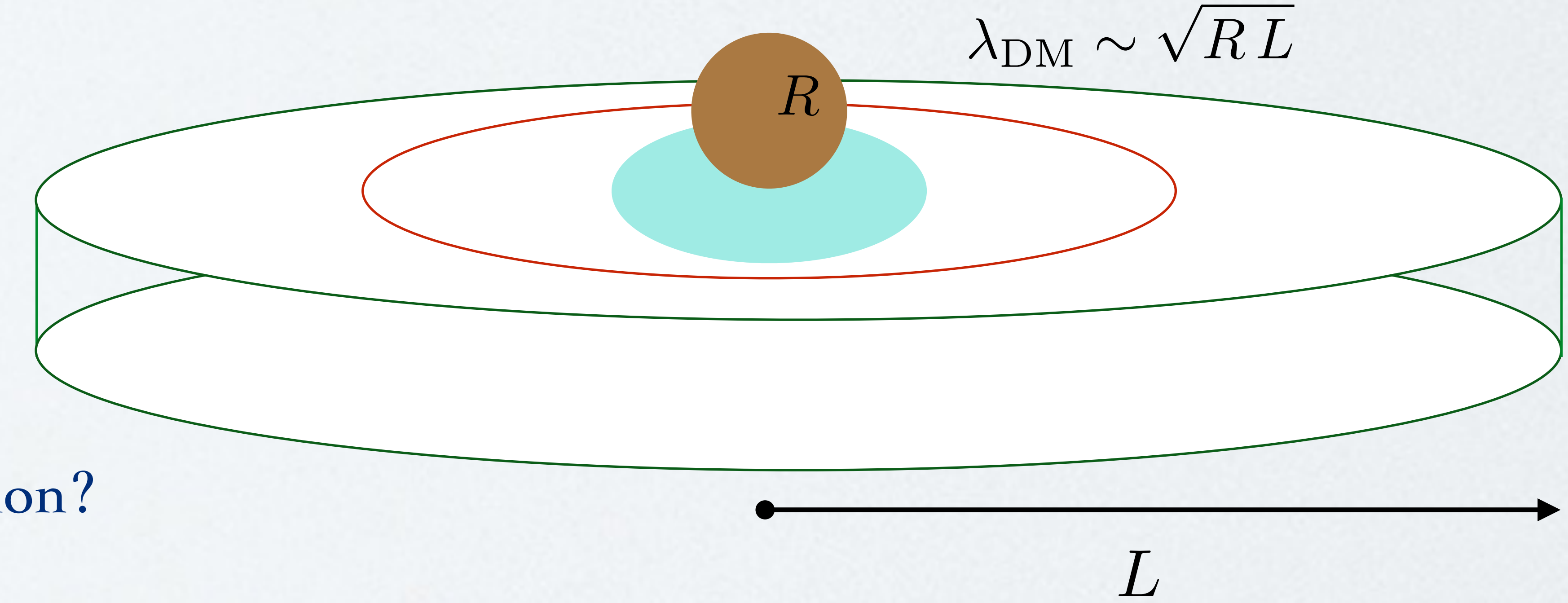
Holographic

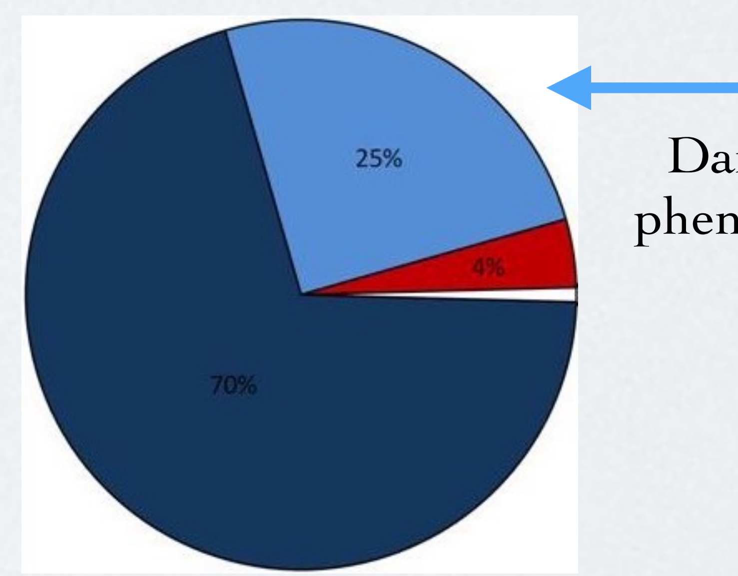
$L \simeq \sqrt{N}\,\ell_{\rm p}$

Backreaction?

Holographic

$R \simeq \sqrt{N_R}\,\ell_{\rm p}$





Dark Matter phenomenology

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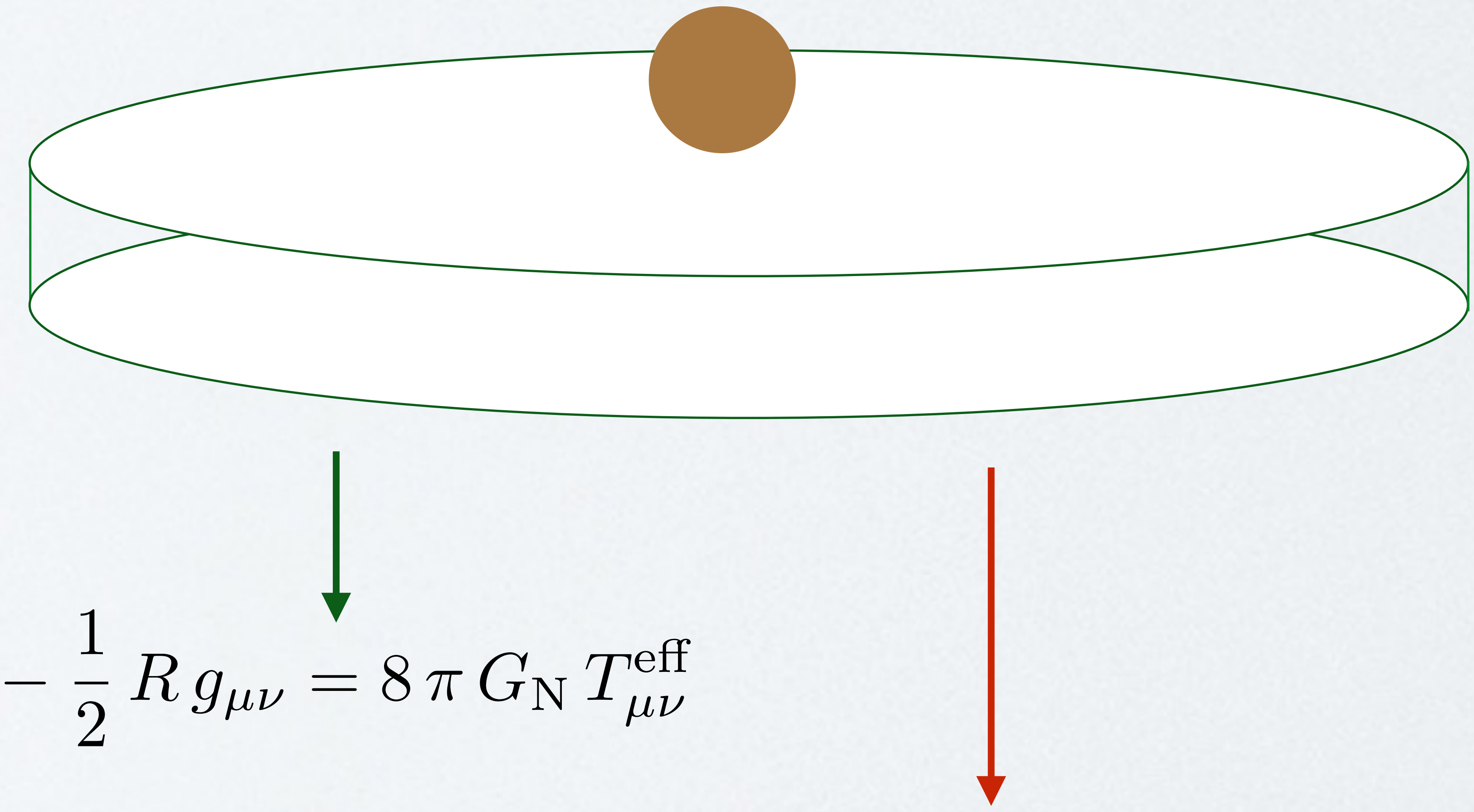
Outlook

Complete emergence of GR phenomenology and beyond

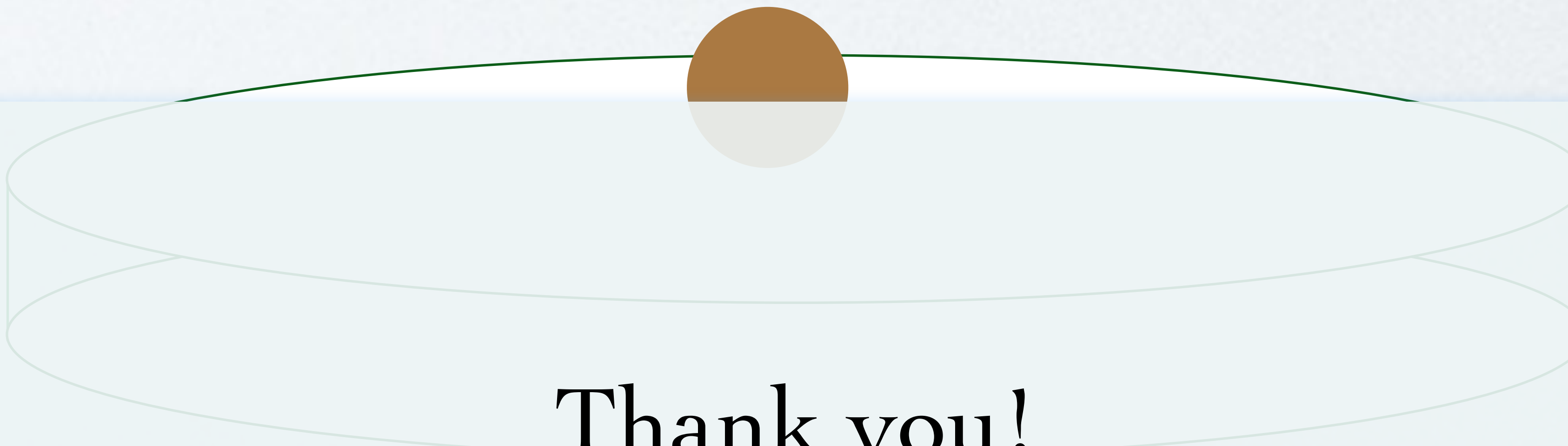


$$R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu} = 8\,\pi\,G_{\rm N}\,T^{\rm eff}_{\mu\nu}$$

$$\dot{M} \simeq m\,\frac{\dot{N}}{\sqrt{N}} \sim -m\,\frac{\Gamma}{\sqrt{N}} \sim -\frac{m_{\rm p}^3/\ell_{\rm p}}{M^2}$$

Outlook

Complete emergence of GR phenomenology and beyond



Thank you!

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