Modified Gravitational Waves Across Galaxies from Macroscopic Gravity

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Introduction

We analyze the propagation of gravitational waves in a medium containing bounded subsystems (molecules), able to induce significant Macroscopic Gravity effects. We establish a precise constitutive relation between the average quadrupole and the amplitudes of a vacuum gravitational wave, via the geodesic deviation equation. Then we determine the modified equation for the wave inside the medium and the associated dispersion relation. A phenomenological analysis shows that anomalous polarizations of the wave emerge with an appreciable experimental detectability if the medium is identified with a typical galaxy. Both the modified dispersion relation (wave velocity less than the speed of light) and anomalous oscillations modes could be detectable by the incoming LISA or pulsar timing arrays experiments, having the appropriate size to see the concerned wavelengths (larger than the molecular size) and the appropriate sensitivity to detect the expected deviation from vacuum General Relativity.

Constitutive Relation

1. $T_{\mu\nu}$ of a set of point like masses, small velocities and weak field. The point particles are grouped into molecules.

2. By applying Kaufman averaging method (1), it has been shown (2) that

$$\langle T_{\mu\nu}\rangle = T^{(f)}_{\mu\nu} + \frac{c^2}{2}\, Q_{\mu\rho\nu\sigma,}{}^{\rho\sigma}$$

$T^{(f)}_{\mu\nu}$: set of free particles (molecules' centers of mass)
$Q_{\mu\rho\nu\sigma}$: quadrupole polarization tensor, describes the molecular structure.
The quadrupole tensor possesses the same symmetries of the Riemann, *i.e.* is determined by twenty free components.

3. Twenty new variables → field equation is underdetermined.
A constitutive relation is needed.

4. We have calculated a constitutive relation in the context of Linearized Gravity, i.e. a relation between the quadrupole tensor and the amplitudes of an incoming gravitational wave, assuming the molecule to be a sphere of constant mass density and the wavelength of the incoming radiation greater than the molecular size:

$$Q_{i0j0} = \epsilon_g \left(\frac{2}{c^2}\phi_{,ij} + \frac{1}{2}\bar{h}_{ij,00}\right)$$

$$Q_{0ijk} = Q_{ijkl} = 0.$$

$\bar{h}_{\mu\nu}$: small deviation from Minkowski flat spacetime, $\partial^\mu \bar{h}_{\mu\nu} = 0$.
ϕ: Newtonian potential generated by the molecule itself.
ϵ_g: gravitational dielectric constant.

$$\epsilon_g = \frac{c^2 N L^5}{4G}$$

N: density of molecules inside the medium.
L: typical size of the molecule.

Modified wave equation and Dispersion relation

Wave equation inside the medium and dispersion relation associated:

$$\Box \bar{h}_{\mu\nu} = -\frac{1}{m^2}\partial_0^4 \bar{h}_{\mu\nu} \qquad m^2 = \frac{c^2}{4\pi G\epsilon_g} \qquad \rightarrow \qquad \omega(k) = c\sqrt{\sqrt{m^2k^2 + \frac{m^4}{4}} - \frac{m^2}{2}}.$$

$\omega(k) \in \mathbb{R}$ for any value of the wave number k: **DISPERSION ONLY**

Group velocity:

$$v_g(k) = \frac{m^2 k c}{2\sqrt{\left(\sqrt{m^2k^2+\frac{m^4}{4}} - \frac{m^2}{2}\right)\left(m^2k^2 + \frac{m^4}{4}\right)}} \xrightarrow[k \ll m]{} c\left(1 - \frac{3k^2}{2m^2}\right).$$

$$v_g(k) \xrightarrow[k\to 0]{} c \qquad v_g(k) \xrightarrow[k\to\infty]{} 0 \qquad v_g(k) \xrightarrow[m^2\to\infty]{} c \ \ \forall k$$

Polarizations

Inside the medium, GWs are no longer solutions of d'Alembert equation $\longrightarrow$ TT Gauge is no longer achievable after the activation of Hilbert gauge $\longrightarrow$ GWs are described by six degrees of freedom. Direction of propagation coincident with the z axis $\longrightarrow$ $\bar{h}_{00}$, $\bar{h}_{01}$, $\bar{h}_{02}$, $\bar{h}_{11}$, $\bar{h}_{12}$, $\bar{h}_{22}$ are independent fields.

1. $\bar{h}_{01}$ is a cross mode in the xz plane.
2. $\bar{h}_{02}$ is a cross mode in the yz plane.
3. $\bar{h}_{12}$ is a cross mode in the xy plane.
4. $\bar{h}_{11}$ is the superposition of a plus mode in the xy plane, an asymmetric plus (Fig. 1) mode in the yz plane and an asymmetric breathing (Fig. 2) mode in the xz plane.
5. $\bar{h}_{22}$ is the superposition of a plus mode in the xy plane, an asymmetric plus mode in the xz plane and an asymmetric breathing mode in the yz plane.
6. $\bar{h}_{00}$ is a breathing mode in the xy plane and an asymmetric breathing both in the xz and yz planes.

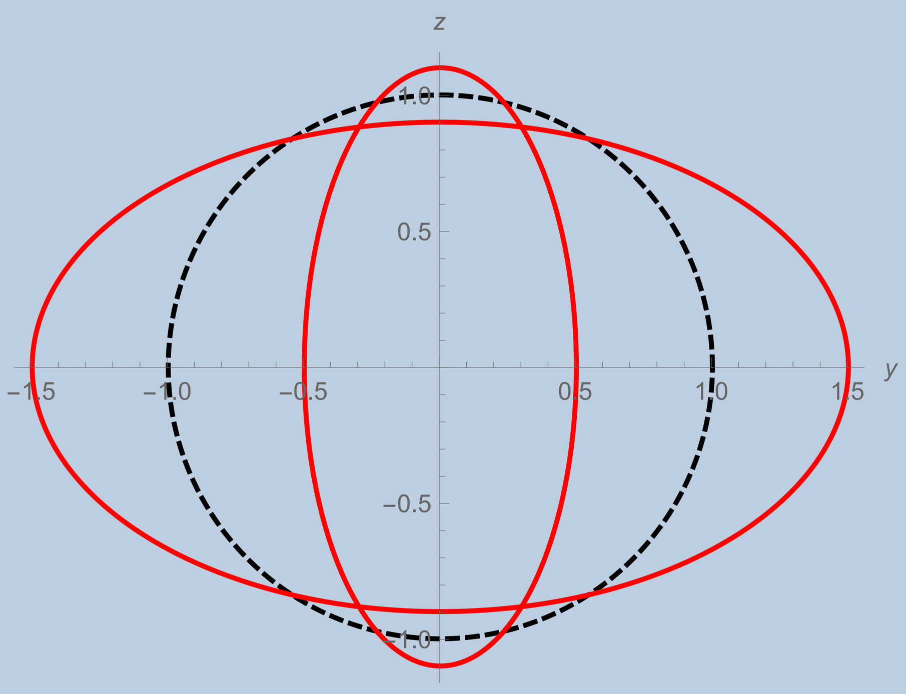

Figure 1: Asymmetric Plus.
$\delta y_{MAX} = 0.5 \qquad \delta z_{MAX} = 0.1$

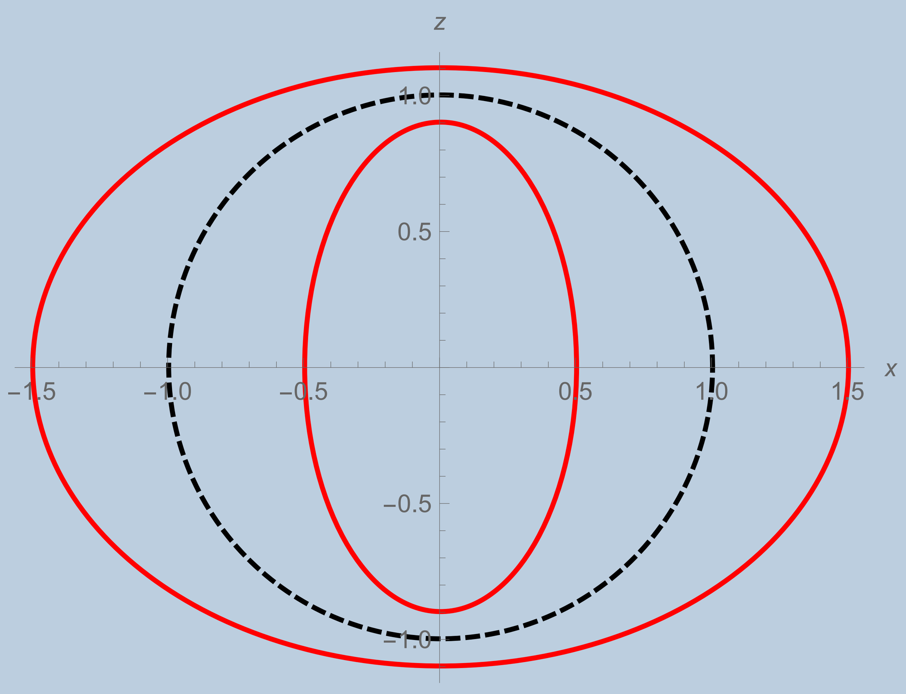

Figure 2: Asymmetric Breathing.
$\delta x_{MAX} = 0.5 \qquad \delta z_{MAX} = 0.1$

Two models of macroscopic medium

1. **Binary stars**

$L = 10^{12}\, m$, $N^{(1)} = 1\, pc^{-3}$, $N^{(2)} = 10^2\, pc^{-3}$, $N^{(3)} = 10^5\, pc^{-3}$

$\epsilon_g^{(1)} = 10^{39}\, kg\, m$, $\epsilon_g^{(2)} = 10^{41}\, kg\, m$, $\epsilon_g^{(3)} = 10^{44}\, kg\, m$

$k = 10^{-13}\, m^{-1}$, $v_g^{(1)} = c\,(1 - 10^{-11})$, $v_g^{(2)} = c\,(1 - 10^{-9})$, $v_g^{(3)} = c\,(1 - 10^{-6})$

2. **Open clusters**

$L = 10^{17}\, m$, $N = 10^{-8}\, pc^{-3}$

$\epsilon_g = 10^{54}\, kg\, m$

$k = 10^{-18}\, m^{-1}$, $v_g = c\,(1 - 10^{-5})$

References

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