An improved analytical representation of the postmerger and ringdown

Gunnar Riemenschneider[1,2], Alessandro Nagar[2,3,4], Gregorio Carullo[5], Giulia Pagano[5,6], Geraint Pratten[7], Walter Del Pozzo[5,6] and John Veitch[8,9].

[1] University of Torino; [2] INFN sez. di Torino; [3] Centro Fermi; [4] IHES; [5] University of Pisa; [6] INFN sez. di Pisa; [7] Universitat de les Illes Balears; [8] University of Birmingham; [9] University of Glasgow

gunnarmarkus.riemenschneider@unito.it

1. Overview

Analytical waveform models informed by (or calibrated to) Numerical Relativity (NR) simulations are of paramount importance for the detection and parameter estimation of coalescing binary black holes (BBHs). The precise knowledge of merger and ringdown part specifically relies only on state-of-the-art NR simulations. An accurate effective description of this part of the waveform, through suitably designed fitting strategies, can serve both as: (i) to complete waveform models, like the Effective-One-Body based ones `SEOBNRv4` [1] or `TEOBResumS` [2] or (ii) as stand-alone templates [3, 4, 1] that can be used in searches or parameter-estimation of high-mass binaries such as GW150914 or GW170729. Here we discuss a specific multipolar postmerger-ringdown waveform model informed by spin-aligned NR simulations spanning from the equal-mass to the extreme-mass-ratio limit case. This model is one of the building blocks of `TEOBResumS` [2]. In the following we use only geometrical units with $G = c = 1$. Below, $h_{\ell m}$ refers to the usual spin-weighted, spherical-harmonic modes of the complex waveform $h = h_+ - ih_\times$. The masses are indicated with m_i and the dimensionless spins as $\chi_i \equiv S_i/m_i^2$, with $i = 1, 2$; $M \equiv m_1 + m_2$ is the total mass of the system and $\nu \equiv m_1 m_2/M^2$ is the symmetric mass ratio. Finally M_{BH} refers to the mass of the remnant black hole (BH).

2. The analytical setup

The postpeak template is defined, following Ref. [4], on the QNM-rescaled ringdown waveform $\bar{h}_{\ell m}(\tau) \equiv (\mathcal{R}/M)e^{\sigma_1^{\ell m}\tau + i\phi_0} h_{\ell m}(\tau)/\nu$, where $\tau = (t - t_{\rm peak})/M_{\rm BH}$, $\sigma_1^{\ell m} \equiv \alpha_1^{\ell m} + i\omega_1^{\ell m}$ is the (dimensionless, $M_{\rm BH}$ rescaled) complex frequency of the fundamental (positive frequency, $\omega_1 > 0$) QNM of the (ℓ, m) mode and ϕ_0 the phase at peak. $\bar{h}_{\ell m}(\tau)$ is decomposed into phase and amplitude

$$\bar{h}_{\ell m}(\tau) = A_{\bar{h}_{\ell m}}(\tau)e^{i\phi_{\bar{h}_{\ell m}}(\tau)}. \tag{1}$$

The amplitude and phase are fit using the following ansätze

$$A_{\bar{h}_{\ell m}}(\tau) = c_{1\,\ell m}^A \tanh\left(c_{2\,\ell m}^A \tau + c_{3\,\ell m}^A\right) + c_{4\,\ell m}^A, \tag{2}$$

$$\phi_{\bar{h}_{\ell m}}(\tau) = -c_{1\,\ell m}^\phi \ln\left(\frac{1 + c_{3\,\ell m}^\phi e^{-c_{2\,\ell m}^\phi \tau} + c_{4\,\ell m}^\phi e^{-2c_{2\,\ell m}^\phi \tau}}{1 + c_{3\,\ell m}^\phi + c_{4\,\ell m}^\phi}\right) \tag{3}$$

As in Ref. [3], only three of the eight coefficients $(c_{3\,\ell m}^A, c_{3\,\ell m}^\phi, c_{4\,\ell m}^\phi)$ are independent and need to be fitted. The remaining 5 are imposed through

$$c_{2\,\ell m}^A = \frac{1}{2}\alpha_{21}^{\ell m}, \tag{4}$$

$$c_{4\,\ell m}^A = \frac{1}{\nu}A_{\ell m}^{\rm peak} - c_{1\,\ell m}^A \tanh\left(c_{3\,\ell m}^A\right), \tag{5}$$

$$c_{1\,\ell m}^A = \frac{1}{\nu}A_{\ell m}^{\rm peak}\alpha_1^{\ell m}\frac{\cosh^2\left(c_{3\,\ell m}^A\right)}{c_{2\,\ell m}^A}, \tag{6}$$

$$c_{1\,\ell m}^\phi = \Delta\omega_{\ell m}\frac{1 + c_{3\,\ell m}^\phi + c_{4\,\ell m}^\phi}{c_{2\,\ell m}^\phi\left(c_{3\,\ell m}^\phi + 2c_{4\,\ell m}^\phi\right)}, \tag{7}$$

$$c_{2\,\ell m}^\phi = \alpha_{21}^{\ell m}, \tag{8}$$

with $\Delta\omega_{\ell m} \equiv \omega_1^{\ell m} - M_{BH}\omega_{\ell m}^{\rm peak}$ and $\alpha_{21}^{\ell m} \equiv \alpha_2^{\ell m} - \alpha_1^{\ell m}$. The fit is done over a integer or half-integer multiple of the damping time of fundamental QNM, $\tau_1^{\ell m} = 1/\alpha_1^{\ell m}$ after the peak of each mode. The time difference between the peaks of the individual modes and the peak of the $(2,2)$ mode are imposed.

3. The primary and global fits

The model can be separated into 3 parts: (i) The information about peak: amplitude, frequency and time-shift with respect to the peak of the $(2,2)$ mode. (ii) The QNM frequencies and inverse damping times, fitted as functions of the spin of the remnant BH, determined using the fits presented in [5] and informed by [6]; and (iii) the effective posmerger parameters, fitted to NR, as discussed in the previous section, up to mass ratios of $m_1/m_2 = 18$. The modes, with $\ell \leq 4$, available in the non-spinning (spinning) case are shown in Fig. 1(Fig. 2). The amplitude is fitted with an accuracy of 10% all over. In the non-spinning(spinning) case the phase is fitted with errors always smaller then 0.1rad(0.3rad with the exception of one outlier at 0.6rad).

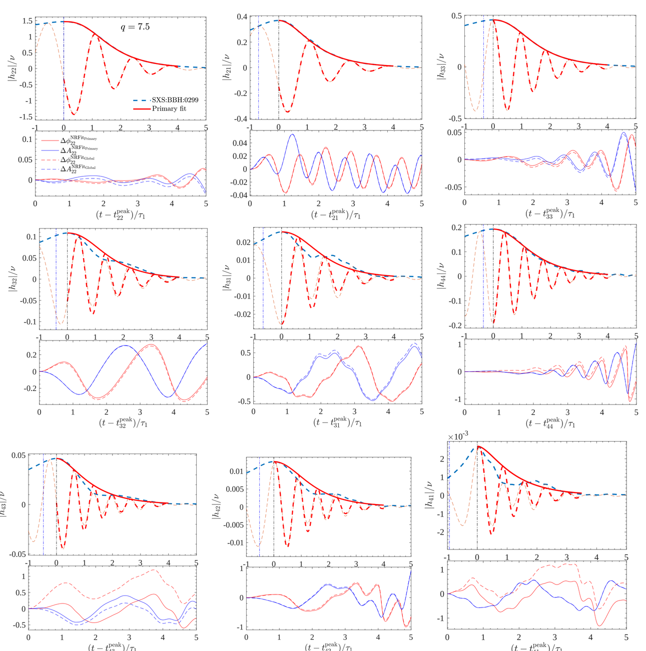

Figure 1: *Performance of primary postpeak fit and of the global interpolating fit on dataset SXS:BBH:0299 with mass ratio $q = 7.5$. We consider the modes $(\ell, m) = (2,1), (3,3), (3,2), (3,1), (4,4), (4,3), (4,2), (4,1)$. Vertical lines in black (blue) mark $t_{\rm peak}^{\ell m}$ ($t_{22}^{\rm peak}$).*

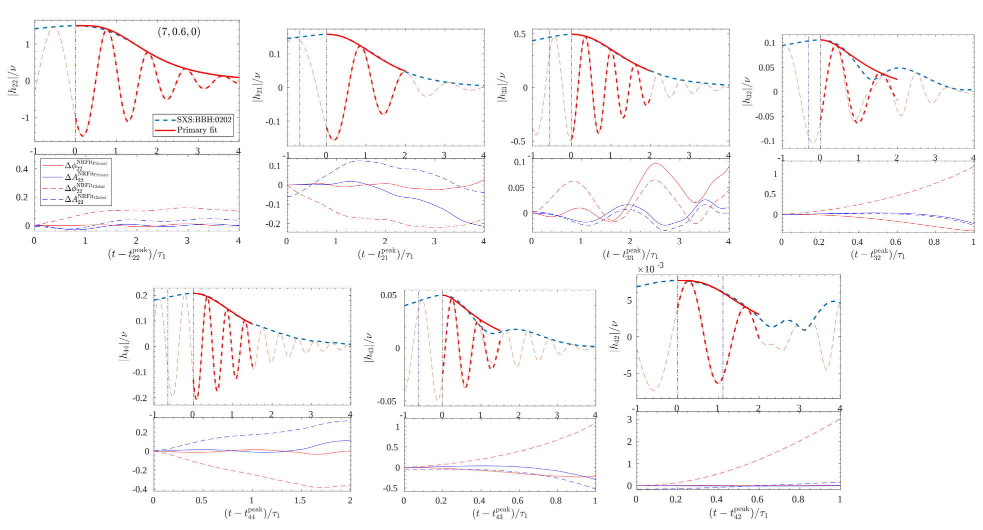

Figure 2: *The performance shown at the example of the waveform SXS:BBH:0202, with $(q, \chi_1, \chi_2) = (7, 0.6, 0)$. We show the $(\ell, m) = (2,2), (2,1), (3,3), (3,2), (4,4), (4,3), (4,2)$. Vertical lines in black (blue) mark $t_{\rm peak}^{\ell m}$ ($t_{22}^{\rm peak}$).*

4. The Numerical Relativity completion of `TEOBResumS`: merger and ringdown

The waveform of the $(2,2)$-mode of the `TEOBResumS` model is completed by the postmerger-ringdown template [2]. For all subdominant modes it is done in the same way. Thus the higher-modes-complete postmerger-ringdown is important to generate full waveforms. In Fig. 3 a full multipolar comparison is shown.

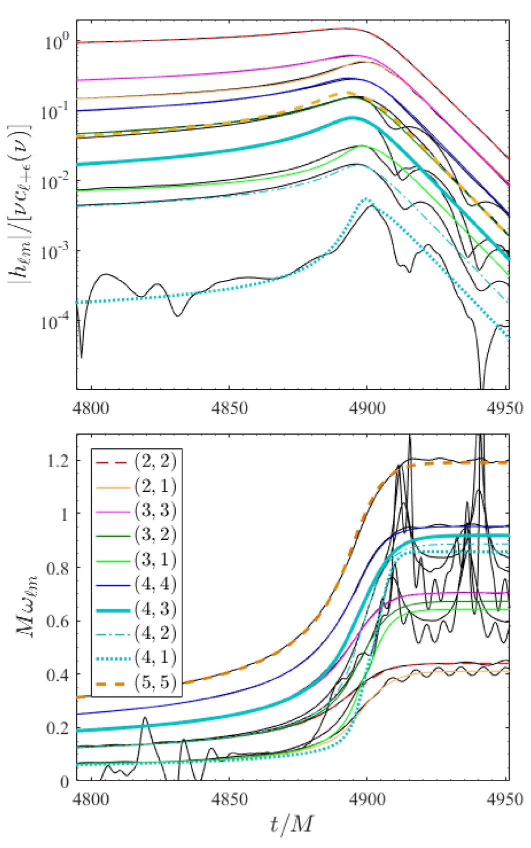

Figure 3: *The multipolar comparison of the `TEOBResumS` model compared with the $q = 6$ waveform SXS:BBH:0166. The waveforms have been aligned around the merger. All the modes up to $\ell \leq 4$, with $m \geq 1$ and the $(5,5)$ mode are shown. The top figure shows the comparison of the amplitude normalized to the leading-order Newtonian contribution. The bottom panel is comparing the frequencies of the individual modes.*

5. Stand-alone application on GW150914

Fig. 4 shows the reconstructed whitened waveform which is the results of a preliminary study, exploring the application of the analytical template as a stand-alone model for analysis in the time-domain on GW150914 interferometric data. This analysis uses the $\ell = m = 2$, $\ell = m = 3$ and $\ell = m = 4$ modes from the peak of the individual modes onward. The peak time of the individual modes, relative to the peak of the $\ell = m = 2$ mode, is obtained from a fit to NR data, while the peak of the l = m = 2 mode is measured directly from the data. Figure 4 refers to a preliminary investigation. Once developed in full, this model can potentially be a useful tool for analyzing high-SNR BBH signals . The most promising signals will be originating from intermediate mass BBH.

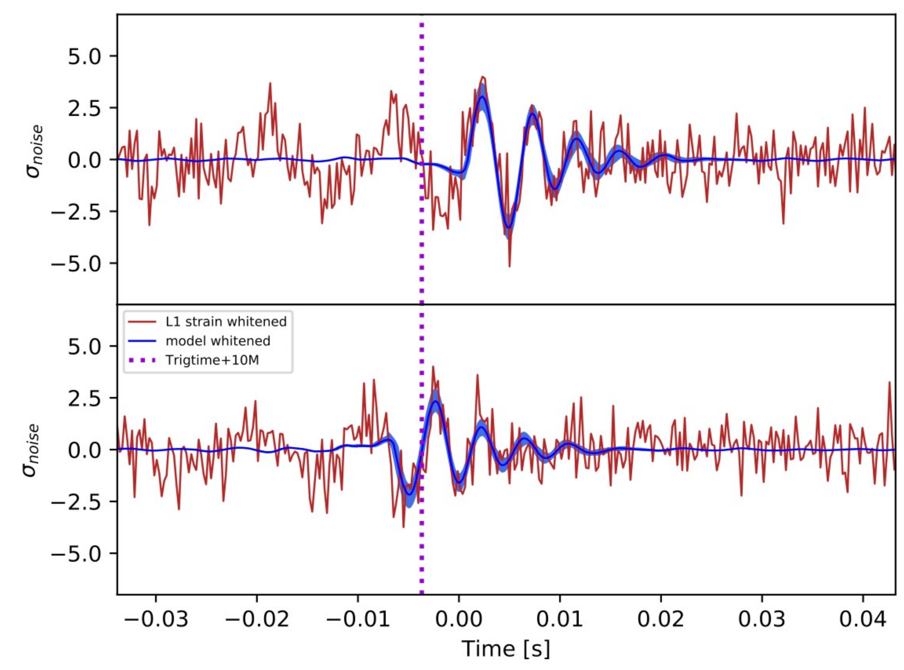

Figure 4: *The comparison of the reconstructed whitened waveform, obtained from a preliminary analysis, with the strain of GW150914, taken from the Gravitational Wave Open Science Center(https://www.gw-openscience.org/). The top panel shows the whitened strain seen in Hanford, while the bottom one shows the one observed in Livingston. The y-axis is given in standard deviations of the noise. The analysis is based on the nested sampling algorithm `CPNest` [7]*

6. Future work

By suitably factorizing and fitting NR ringdown waveforms, we could construct a robust and reliable post-merger ringdown model for spin aligned binaries that incorporates modes $(2,2)$, $(2,1)$, $(3,3)$, $(3,2)$ $(4,4)$, $(4,3)$, $(4,2)$. The $(3,1)$ and $(5,5)$ modes are currently well approximated simply by the nonspinning model. Such postmerger-ringdown model can be used both as stand-alone waveform template and for analytically reproducing the ringdon part of EOB-based waveform models like `TEOBiResumMultipolar`. Two obvious improvements are needed:

- The current template does not incorporate, for simplicity, the effect of mode mixing, that happens because for a given ℓ mode is a superposition of modes with various ℓ and different m's. A specific factorization of he NR ringdown is needed in this case
- The fitting template we use introduces, for the amplitude, unwanted features when the mass ratio is rather large, notably in the extreme-mass-ratio limit case. This prevents us from consistently extend the template in the region for $q \geq 18$ because of the appearence, at some stage, of such unwanted f properties. To build a postmerger waveform model that correctly incorporates the postmerger behavior requires a fitting template with additional analytical flexibility.

References

[1] Bohé A *et al.* 2017 *Phys. Rev.* **D95** 044028 (*Preprint* 1611.03703)

[2] Nagar A *et al.* 2018 *Phys. Rev.* **D98** 104052 (*Preprint* 1806.01772)

[3] Damour T and Nagar A 2014 *Phys.Rev.* **D90** 024054 (*Preprint* 1406.0401)

[4] Del Pozzo W and Nagar A 2017 *Phys. Rev.* **D95** 124034 (*Preprint* 1606.03952)

[5] Jiménez-Forteza X, Keitel D, Husa S, Hannam M, Khan S and Pürrer M 2017 *Phys. Rev.* **D95** 064024 (*Preprint* 1611.00332)

[6] Berti E, Cardoso V and Will C M 2006 *Phys. Rev.* **D73** 064030 (*Preprint* gr-qc/0512160)

[7] https://github.com/johnveitch/cpnest Parallel nested sampling algorithm in python

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