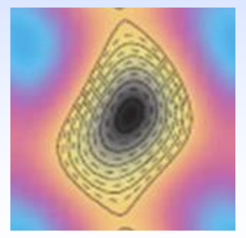
On the Modified Gravity and Common Nature of Dark Sector



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Abstract

We show that (Eur Phys J C, 2018), the gravity can be considered as defined not by one but two fundamental constants which enables us to explain quantitatively both dark energy (the cosmological constant) in GR equations and dark matter in weak-field limit simultaneously. Then, in order to throw more light on the nature of the constants appearing here, we generalize the Newton theorem on the 'sphere-point mass' equivalency to arbitrary dimensions. We also turn into gravitational lensing, where this additional term predict a critical value for the involved weak-field parameter. If this value will be established at future observations, this will mark the first discrepancy with GR of the conventional weak-field limit, directly linked to the nature of the dark sector of the Universe.

Introduction

Newton in *Principia* had proved that at the universal gravitational law r^-2 a spherically symmetric body affects external objects gravitationally as though all of its mass is concentrated at its center.

The most general function for force satisfying Newton's theorem is written as:

$$F = \frac{C_1}{r^2} + C_2 r$$

Identifying the Constants

While C_1 is related to G , it is possible to find a correspondence between C_2 and Λ .
Considering the Einstein's field equation (GR) together with cosmological constant:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The weak filed limit is written as:

$$g_{00} = 1 + \frac{2\Phi}{c^2} = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3}$$

However, in order to identify C_2 as Λ, we need to fix it's sign and numerical value.

Dark Sector and Constants

In the GR equations, Λ is considered to describe the accelerated expansion of the Universe.
According to Planck data, it's value is:

$$\Lambda = 1.11 \times 10^{-52}\ m^{-2}$$

The Λ -modified Newtonian gravity can be used to describe the Dark Matter.
In such case, the virial theorem will be:

$$\Lambda = \frac{3\,\sigma^2}{2\ R^2 c^2}$$

For galaxy groups of the Hercules-Bootes region, the value of Λ will be:

Galaxy group	$\sigma(km/s^{-1})$	$R_h(kpc)$	$\Lambda(m^{-2})$
NGC4736	50	338	3.84E-52
NGC4866	58	168	2.09E-51
NGC5005	114	224	4.55E-51
NGC5117	27	424	7.12E-53
NGC5353	195	455	3.23E-51
NGC5375	47	66	8.91E-51
NGC5582	106	93	2.28E-50
NGC5600	81	275	1.52E-51
UGC9389	45	204	8.55E-52
PGC55227	14	17	1.19E-50
NGC5961	63	86	9.43E-51
NGC5962	97	60	4.59E-50
NGC5970	92	141	7.48E-51
UGC10043	67	65	1.87E-50
NGC6181	53	196	1.28E-51
UGC10445	23	230	1.76E-52
NGC6574	15	70	8.07E-52
Average			8.24E-51
St.deviation			1.15E-50

On the other hand, considering the Λ -modified Newtonian gravity, it turns out that the second point of Shell theorem is not satisfied. Thus, we have non-force free property inside the shell which is essential to determine the disks' properties according to galactic halos.

Isometry Groups

Since, Λ was introduced not by Newton's theorem but according to conservation of the energy-momentum tensor and General Covariance principle. So it seems quite reasonable that, to make as more powerful justification, we try to infer Newton's theorem based on the above relativistic consideration.

Depending on Λ's sign—positive, negative or zero—one has three different vacuum solutions (three different asymptotic limits) for the field equations as:

Sign	Spacetime	Isometry Group	Curvature
$\Lambda > 0$	de Sitter (dS)	O(1, 4)	+
$\Lambda = 0$	Minkowski (M)	IO(1, 3)	0
$\Lambda < 0$	Anti de Sitter (AdS)	O(2, 3)	−

The interesting feature of all these 4D maximally symmetric Lorentzian geometries is that for all of them the stabilizer subgroup of isometry group is the Lorentz group O(1, 3).

$$dS = \frac{O(1,4)}{O(1,3)},\ M = \frac{IO(1,3)}{O(1,3)},\ AdS = \frac{O(2,3)}{O(1,3)}$$

In the non-relativistic limit we have:

$$O(1,4)\ \rightarrow \big(O(3) \times\ O(1,1)\big) \ltimes R^6$$
$$IO(1,3) \rightarrow (O(3) \times R) \ltimes R^6$$
$$O(2,3) \rightarrow \big(O(3) \times O(2)\big) \ltimes R^6$$

For all of them, the spatial part admits Euclidean algebra $E(3) = O(3) \ltimes R^3$, which means that the group $O(3)$, is the stabilizer subgroup.

The same analysis can be generalized to arbitrary dimensions. In this case the most general form of the potential Φ satisfying the Newton's theorem ($S^{d-1}\sim point$) is written as:

$$\Phi = \frac{C_1}{(d-2)r^{d-2}} + \frac{C_2 r^2}{2d}$$

Where the first constant can be identified as the Newton's gravitational constant the second one is the cosmological constant. Consequently the group theoretical analysis justifies that at each point of spatial geometry we have the exact $O(d)$ symmetry.

Observational Implications

The use of the strong gravity lens ESO 325-G004 demonstrates the efficiency of the lens studies to constrain the weak-field GR in the intergalactic scales.
Based on the analysis of the data of ESO325-G004:

$$\gamma = 0.97 \pm 0.09$$

Considering Λ in our equations, the γ parameter will be:

$$\gamma = 1 - \frac{\Lambda\ c^4 r^4}{12\ \ G^2 M^2}$$

Where we have a critical parameter for ESO325-G004, γ =0.998.

Conclusion:

1- Gravity can be defined as not one but two constants, "G" and "Λ" which appear readily in the modified weak-field limit of General Relativity.

2- Both constants jointly are able to explain quantitatively the dark energy and the dark matter, which hence appear as gravity effects.

3- Λ (the cosmological constant) is dimension-independent and matter-uncoupled. Hence, Λ can be considered as even more universal than G (constant of gravity) .

4- The critical parameter, γ, if established at future observations, will mark the first discrepancy with General Relativity of conventional weak-field limit.

Bibliography

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