Weak field and full GR cosmological simulations

Marco Bruni
Institute of Cosmology and Gravitation, Portsmouth, UK

EPS Conference on Gravitation, *19/02/2019*

Outline

- standard ΛCDM cosmology and a basic question
- non-linear Post-Friedmann ΛCDM: a weak-field/ post-Newtonian type approximation scheme for cosmology
- Post-Friedmann application: cosmological frame dragging from Newtonian N-body simulations
- full Numerical Relativity cosmological simulations
- Conclusions and Outlook

Credits: first part

- Irene Milillo, Daniele Bertacca, MB and Andrea Maselli, *The missing link: a nonlinear post-Friedmann framework for small and large scales* [arXiv: 1502.02985], Physical Review D, **92**, 023519 (2015)
- MB, Dan B. Thomas and David Wands, *Computing General Relativistic effects from Newtonian N-body simulations: Frame dragging in the post-Friedmann approach*, Physical Review D, **89**, (2014) 044010 [arXiv:1306.1562]
- Dan B. Thomas, MB, Kazuya Koyama, Baojiu Li and Gong-bo Zhao *f(R) gravity on non-linear scales: The post-Friedmann expansion and the vector potential*, JCAP, 1507 (2015) 07, 051 [arXiv:1503.07204]
- C. Rumpf, E. Villa, D. Bertacca and M. Bruni, *Lagrangian theory for cosmic structure for- mation with vorticity: Newtonian and post-Friedmann approximations*, Phys. Rev. D **94** (2016) 083515 [arXiv:1607.05226]
- A. Maselli, B. Bruni & D. Thomas, Interacting vacuum-energy in a Post-Friedmann expanding Universe (to be submitted)

Credits: second part

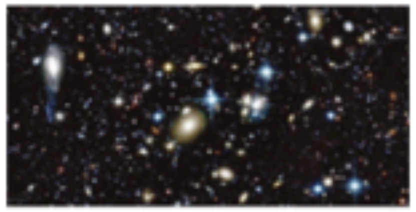
Featured in Physics | Editors' Suggestion

2 citations

Departures from the Friedmann-Lemaitre-Robertston-Walker Cosmological Model in an Inhomogeneous Universe: A Numerical Examination

John T. Giblin, Jr., James B. Mertens, and Glenn D. Starkman

Phys. Rev. Lett. **116**, 251301 (2016) – Published 24 June 2016



Cosmologists have begun using fully relativistic models to understand the effects of inhomogeneous matter distribution on the evolution of the Universe.

Show Abstract +

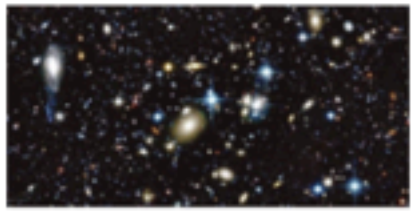
Featured in Physics | Editors' Suggestion

1 citation

Effects of Nonlinear Inhomogeneity on the Cosmic Expansion with Numerical Relativity

Eloisa Bentivegna and Marco Bruni

Phys. Rev. Lett. **116**, 251302 (2016) – Published 24 June 2016

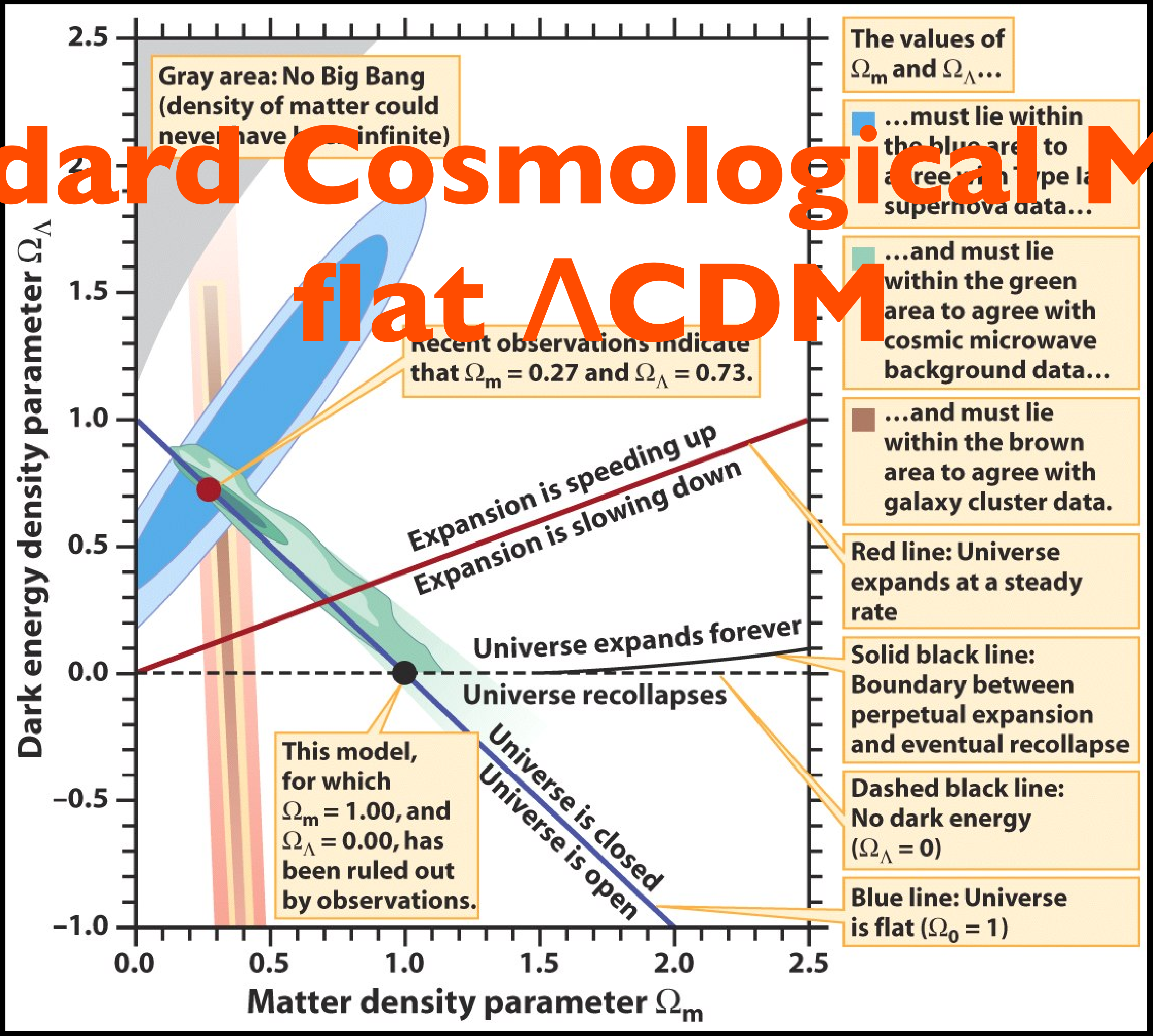


Cosmologists have begun using fully relativistic models to understand the effects of inhomogeneous matter distribution on the evolution of the Universe.

Show Abstract +

Standard Cosmology

Standard Cosmological Model: flat ΛCDM



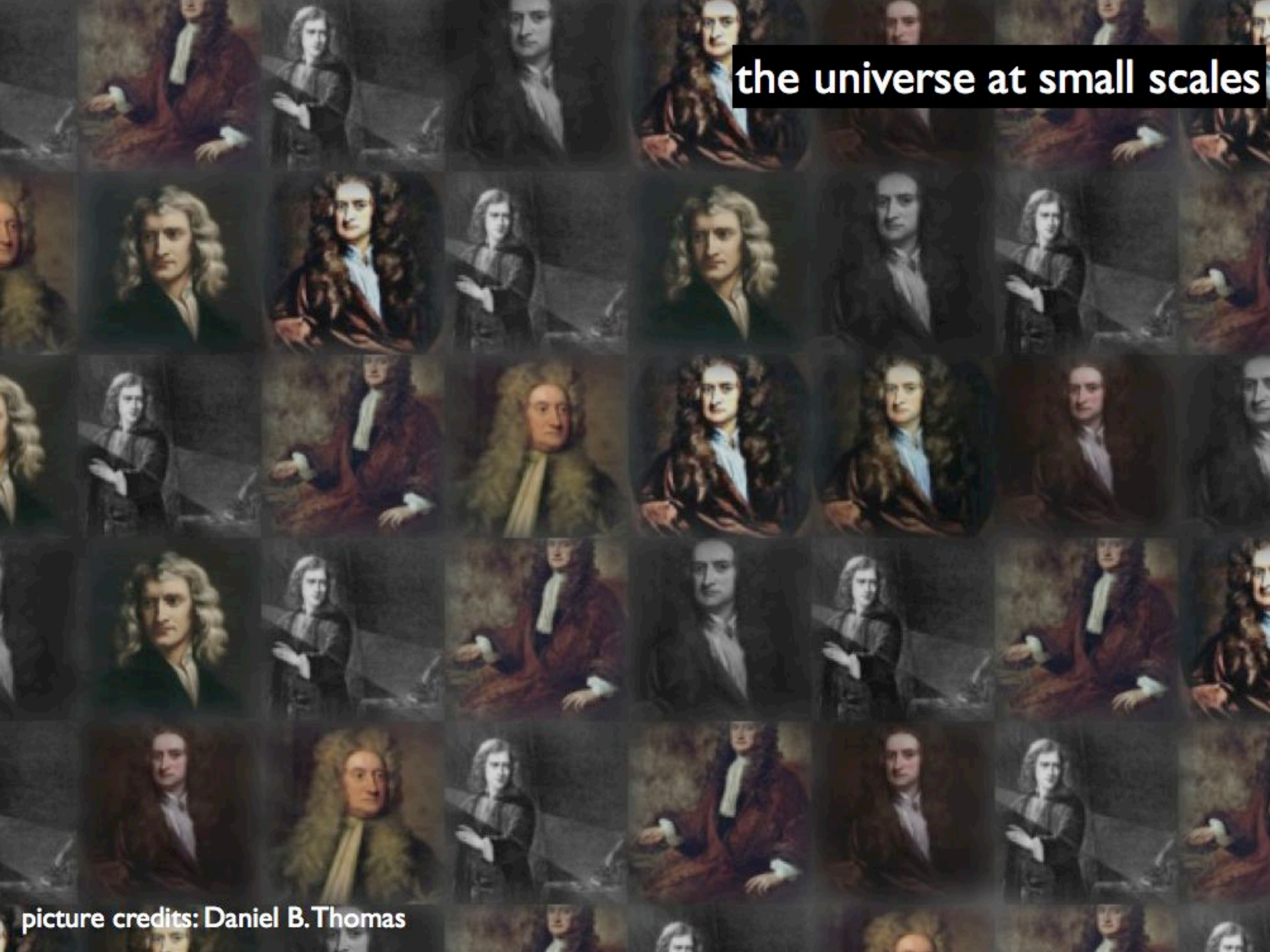
Standard ΛCDM Cosmology

- Recipe for modeling based on 3 main ingredients:
  1. Homogeneous isotropic background, FLRW models
  2. Relativistic Perturbations, good for large scales, e.g. CMB and LSS; I-order, II order, "gradient expansion" (aka long-wavelength approximation)
  3. Newtonian study of non-linear structure formation (N-body simulations or approx. techniques, e.g. 2LPT) at small scales
- on this basis, well supported by observations, the flat ΛCDM model has emerged as the Standard "Concordance" Model of cosmology.

the universe at very large scales: GR

picture credits: Daniel B. Thomas

the universe at small scales



picture credits: Daniel B. Thomas

Questions on ΛCDM

- Recipe for modelling based on 3 main ingredients:
  1. Homogeneous isotropic background, FRW models
  2. Relativistic Perturbations (e.g. CMB; linear, nonlinear)
  3. Newtonian study of non-linear structure formation (numerical simulations or approx. techniques)
- Is 3 enough? (more data, precision cosmology, observations and simulations covering large fraction of H^{-1}, etc...)
  - We need to bridge the gap between 2 and 3

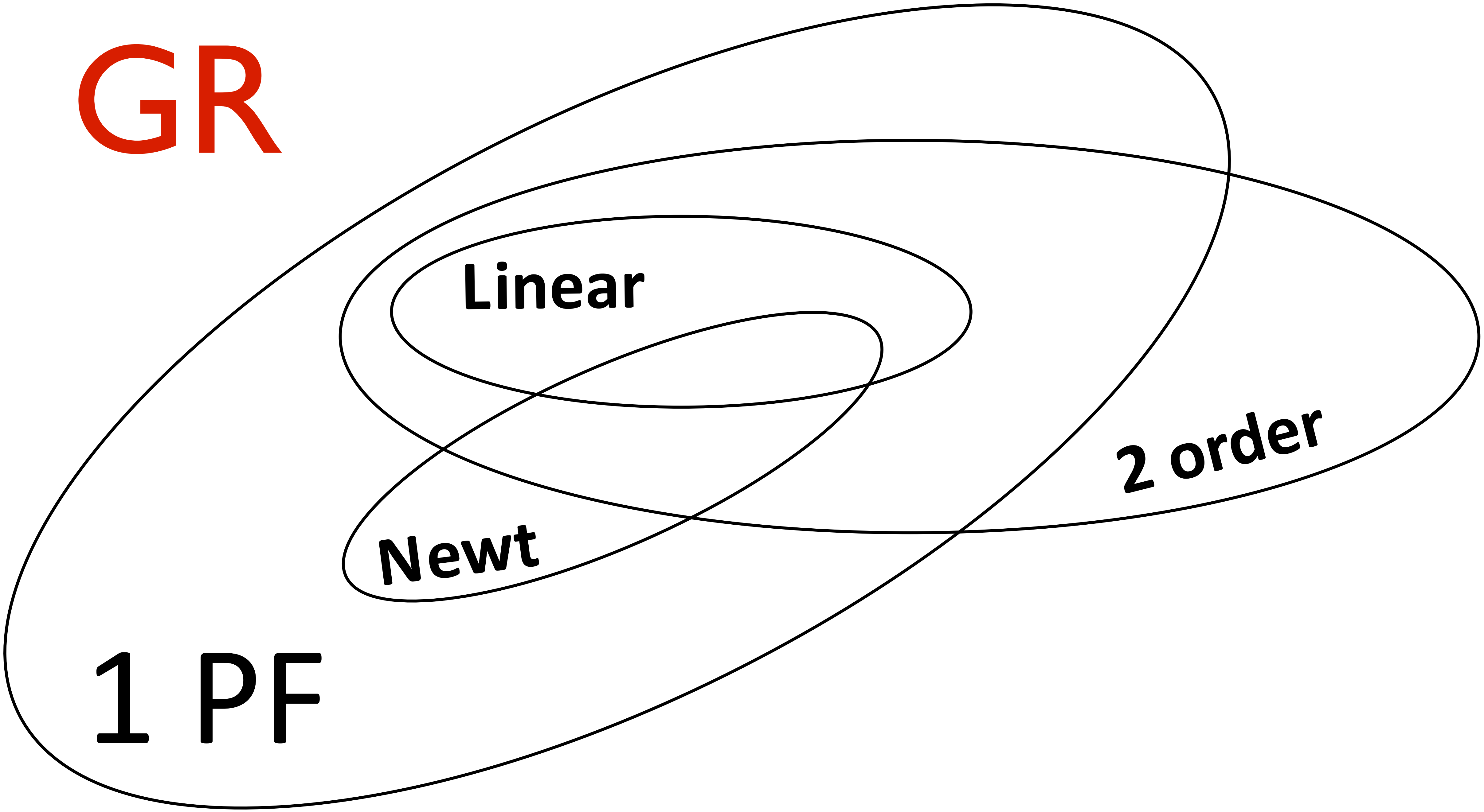
nonlinear post-Friedmann framework

nonlinear post-Friedmann framework

- GR, flat ΛCDM background
- fully non-linear density field
- post-F: weak-field + small peculiar velocities
  - start with a weak field approach on a FLRW background (similar to post-Minkowski), Hubble flow is not slow but peculiar velocities are small $\boxed{\dot{\vec{r}} = H\vec{r} + a\vec{v}}$
- post-F: non-linear framework including both Newtonian regime and first-order GR perturbations

post-Friedmann framework

- spaces of equations (not solutions!)



Newtonian ΛCDM, with a bonus

- insert leading order terms in E.M. conservation and Einstein equations
- subtract the background, getting usual Friedmann equations
- introduce usual density contrast by $\rho=\rho_b(1+\delta)$

from E.M. conservation:
Continuity & Euler equations

$$\dot{\delta} + \frac{v^i \delta_{,i}}{a} + \frac{v^i{}_{,i}}{a}(\delta + 1) = 0 \,,$$

$$\dot{v}_i + \frac{v^j v_{i,j}}{a} + \frac{\dot{a}}{a} v_i = \frac{1}{a} U_{N,i} \,.$$

Poisson

$$G^0{}_0 + \Lambda = \frac{8\pi G}{c^4} T^0{}_0 \quad \rightarrow \quad \frac{1}{c^2}\frac{1}{a^2}\nabla^2 V_N = -\frac{4\pi G}{c^2}\bar{\rho}\delta$$

Newtonian ΛCDM, with a bonus

what do we get from the ij and 0i Einstein equations?

$$\text{trace of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4}T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{2}{a^2}\nabla^2(V_N - U_N) = 0\,,$$ zero "Slip"

$$\text{traceless part of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4}T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{1}{a^2}\left[(V_N - U_N)_{,i}{}^{,j} - \frac{1}{3}\nabla^2(V_N - U_N)\delta^j_i\right] = 0$$

bonus

$$G^0{}_i = \frac{8\pi G}{c^4}T^0{}_i \;\rightarrow\; \frac{1}{c^3}\left[-\frac{1}{2a^2}\nabla^2 B^N_i + 2\frac{\dot a}{a^2}U_{N,i} + \frac{2}{a}\dot V_{N,i}\right] = \frac{8\pi G}{c^3}\bar\rho(1+\delta)v_i$$

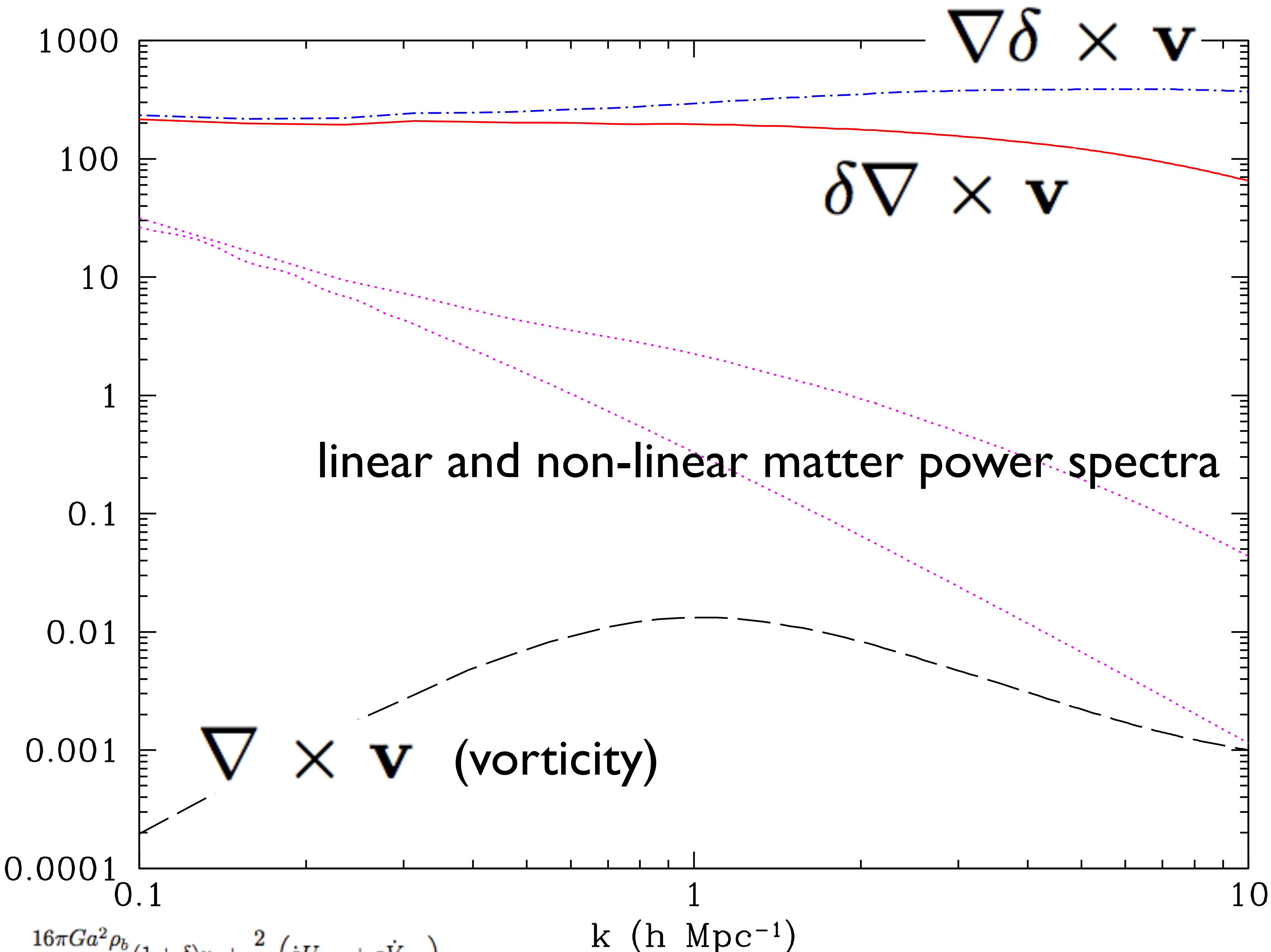
- Newtonian dynamics at leading order, with a bonus: the frame dragging potential B_i is not dynamical at this order, but cannot be set to zero: doing so would forces a constraint on Newtonian dynamics
- result entirely consistent with vector relativistic perturbation theory
- in a relativistic framework, gravitomagnetic effects cannot be set to zero even in the Newtonian regime, cf. Kofman & Pogosyan (1995), ApJ 442:

magnetic Weyl tensor at leading order

$$H_{ij} = \frac{1}{2c^3}\left[B^N_{\mu,\nu(i}\varepsilon_{j)}{}^{\mu\nu} + 2v_\mu(U_N + V_N)_{,\nu(i}\varepsilon_{j)}{}^{\mu\nu}\right]$$

nonlinear post-Friedmann framework: applications

power spectra: sources

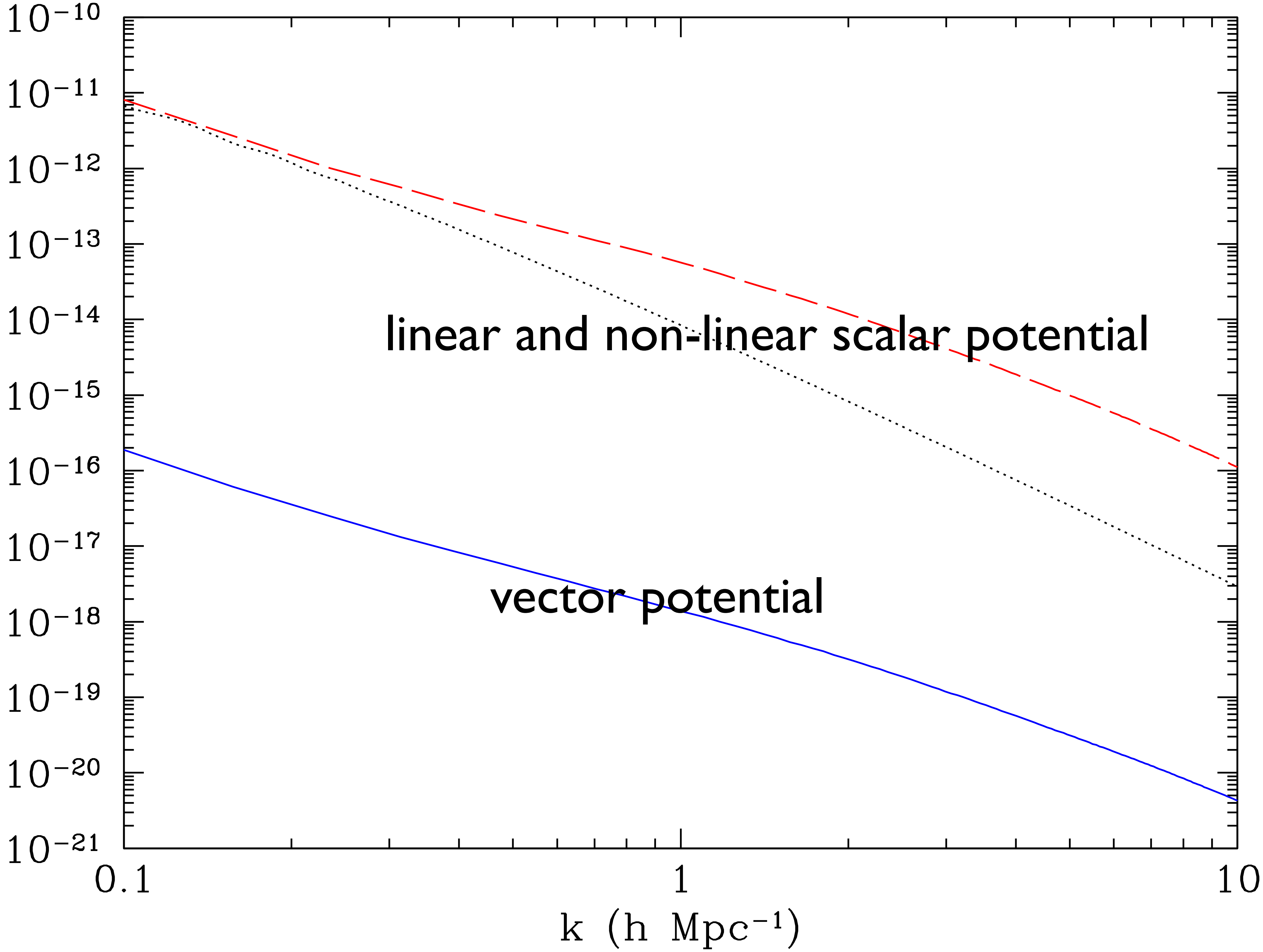


$$\frac{1}{c^3}\nabla^2 P_i^N = -\frac{16\pi G a^2 \rho_b}{c^3}(1+\delta)v_i + \frac{2}{c^3}\left(\dot{a}U_{N,i} + a\dot{V}_{N,i}\right)$$

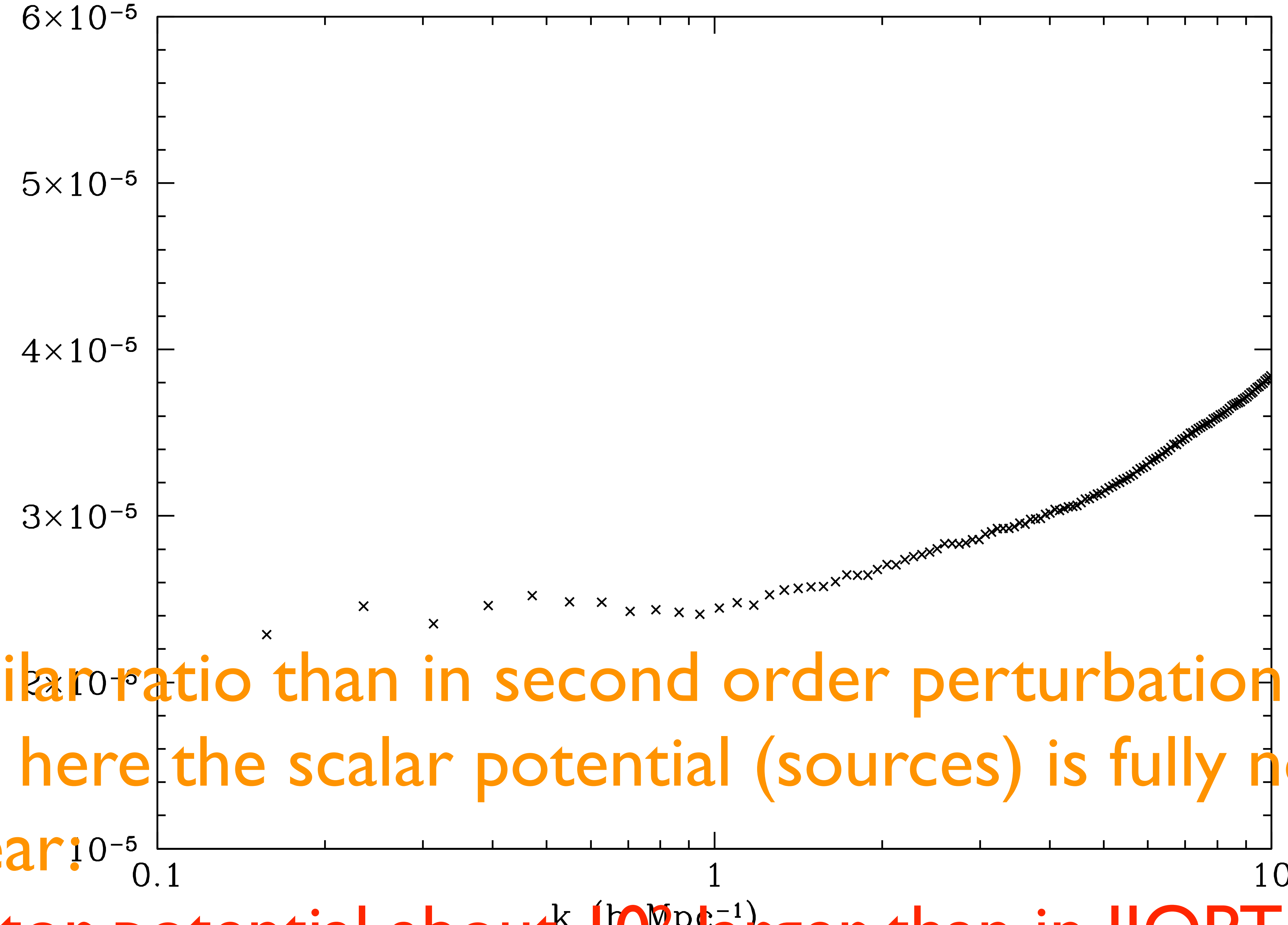
$$\nabla \times \nabla^2 \vec{P}^N = -\left(16\pi G \rho_b a^2\right) \nabla \times [(1+\delta)\vec{v}]$$

$$\nabla \times [(1+\delta)\vec{v}] = (\nabla\delta) \times \vec{v} + (1+\delta)\, \nabla \times \vec{v}$$

scalar and vector potentials



ratio of the potentials



similar ratio than in second order perturbation theory but here the scalar potential (sources) is fully non-linear:

vector potential about 10^2 larger than in IIOPT

cf. Lu, Ananda, Clarkson & Maartens (2009)

post-F vector potential in f(R)

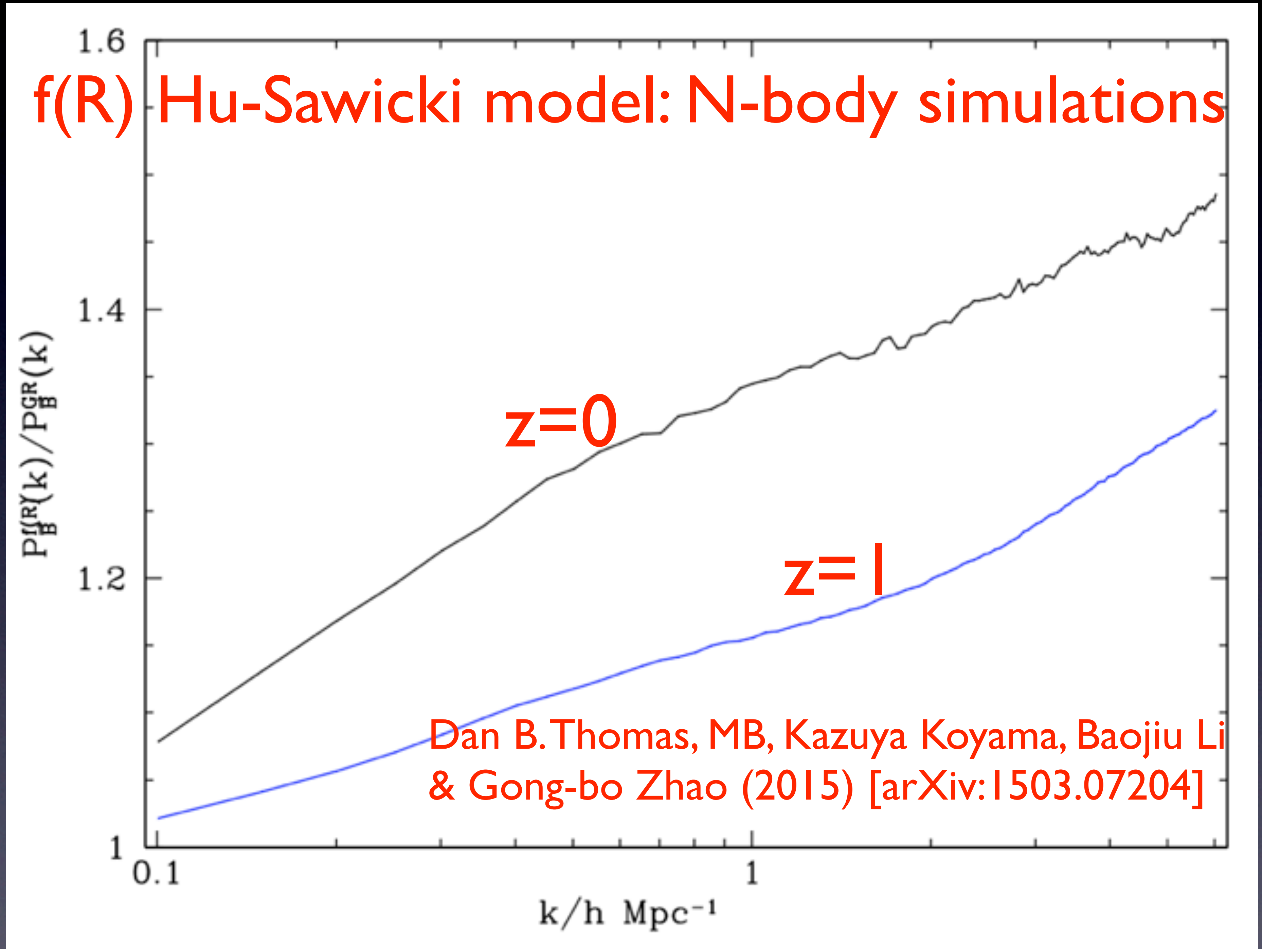


FIG. 3: The ratio of the vector potential power spectrum in $f(R)$ gravity to that in GR, for $|f_{R_0}| = 10^{-5}$. The blue curve shows the ratio at redshift one, and the black curve shows the ratio at redshift zero.

take-home message #1

- GR effects in structure formation do exists at small nonlinear scales
- gravito-magnetic effect at leading order
- are they measurable? (work in progress)
- Expect larger effects in any Dark Energy scenario

Full GR
Numerical Relativity
Simulations

Eloisa Bentivegna & MB, PRL 116, 251302 (2016)

cf. J.T. Giblin Jr., J.B. Mertens & G.D. Starkman, PRL 2016, 251301 (2016)

Density perturbations in EdS and top-hat

$$a(t) = a_i \left(\frac{t}{t_i}\right)^{2/3} ,$$

$$\delta(t) = \delta_+ a(t) + \delta_- a(t)^{-3/2}$$

- top-hat turnaround and collapse time:

characterized by the value of δ at these events:

$$\delta_T = 1.06 \qquad \delta_c = 1.696$$

Full GR
Numerical Relativity
Simulations

Eloisa Bentivegna & MB, PRL 116, 251302 (2016)
cf. Giblin, Mertens & Starkman, PRL 116, 251301 (2016)

+ Macpherson, Lasky & Price, PRD 95, 064028 (2017)

Assumptions and procedure

- Initial conditions: a small δ 10^{-2}-10^{-6} on EdS background

$$\rho_i = \bar{\rho}_i \left(1 + \delta_i \sum_{j=1}^{3} \sin \frac{2\pi x^j}{L}\right)$$

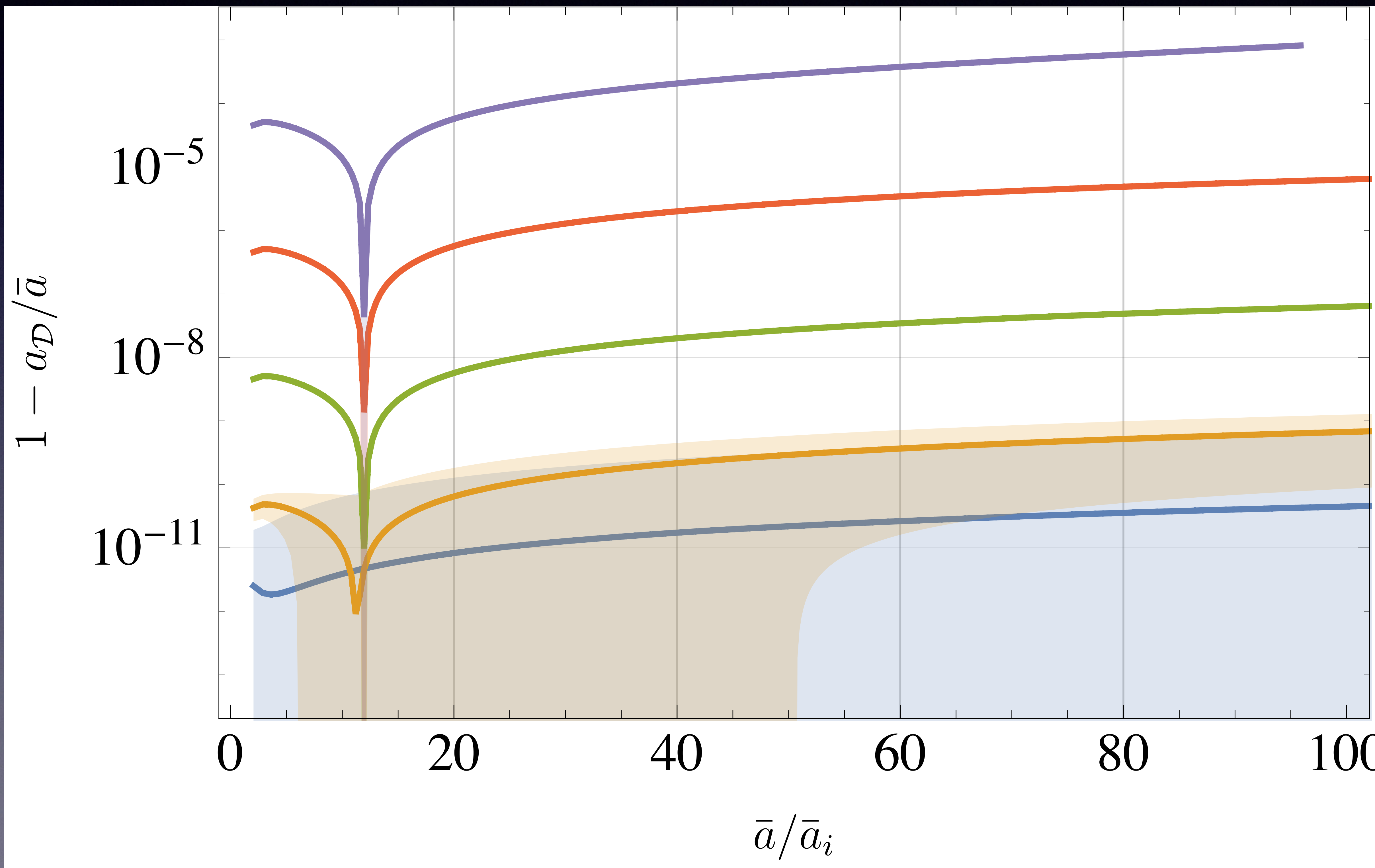
- synchronous-comoving gauge, irrotational fluid (Lagrangian approach)
- Integrate EFE using the Einstein Toolkit, freey available open source infrastructure for Numerical Relativity
- use a variant of BSSN formulation of EFE

Assumptions and procedure

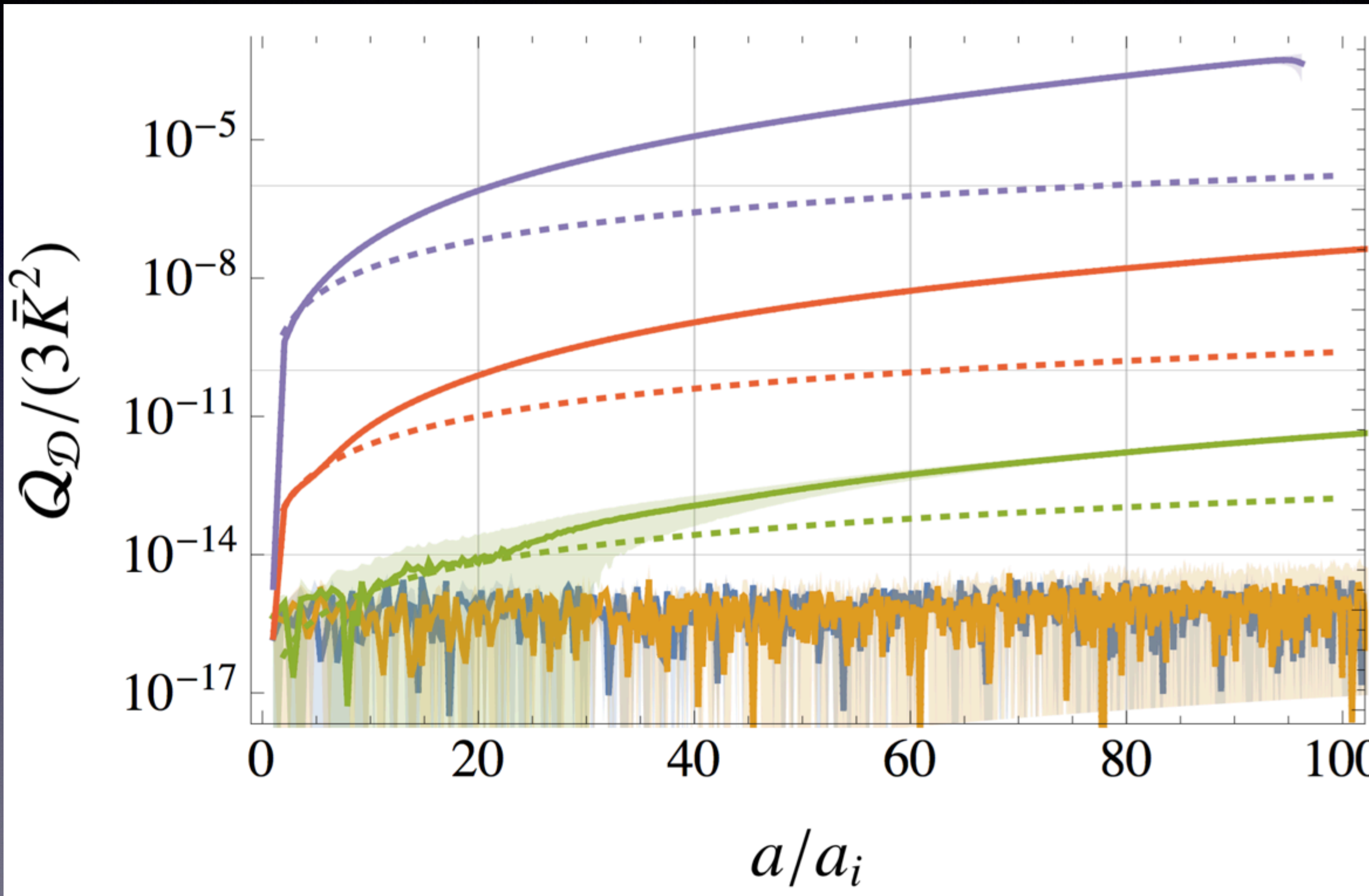
- solve initial constraint
- evolve EFE with periodic boundary conditions on comoving box of size L
- initial conditions: perturbations of EdS with $H_i^{-1}=L/4$
- domain discretised with 160^3 points
- compare average quantities and EdS evolution
- measure local quantities (expansion and density)

first goal:
backreaction

average expansion

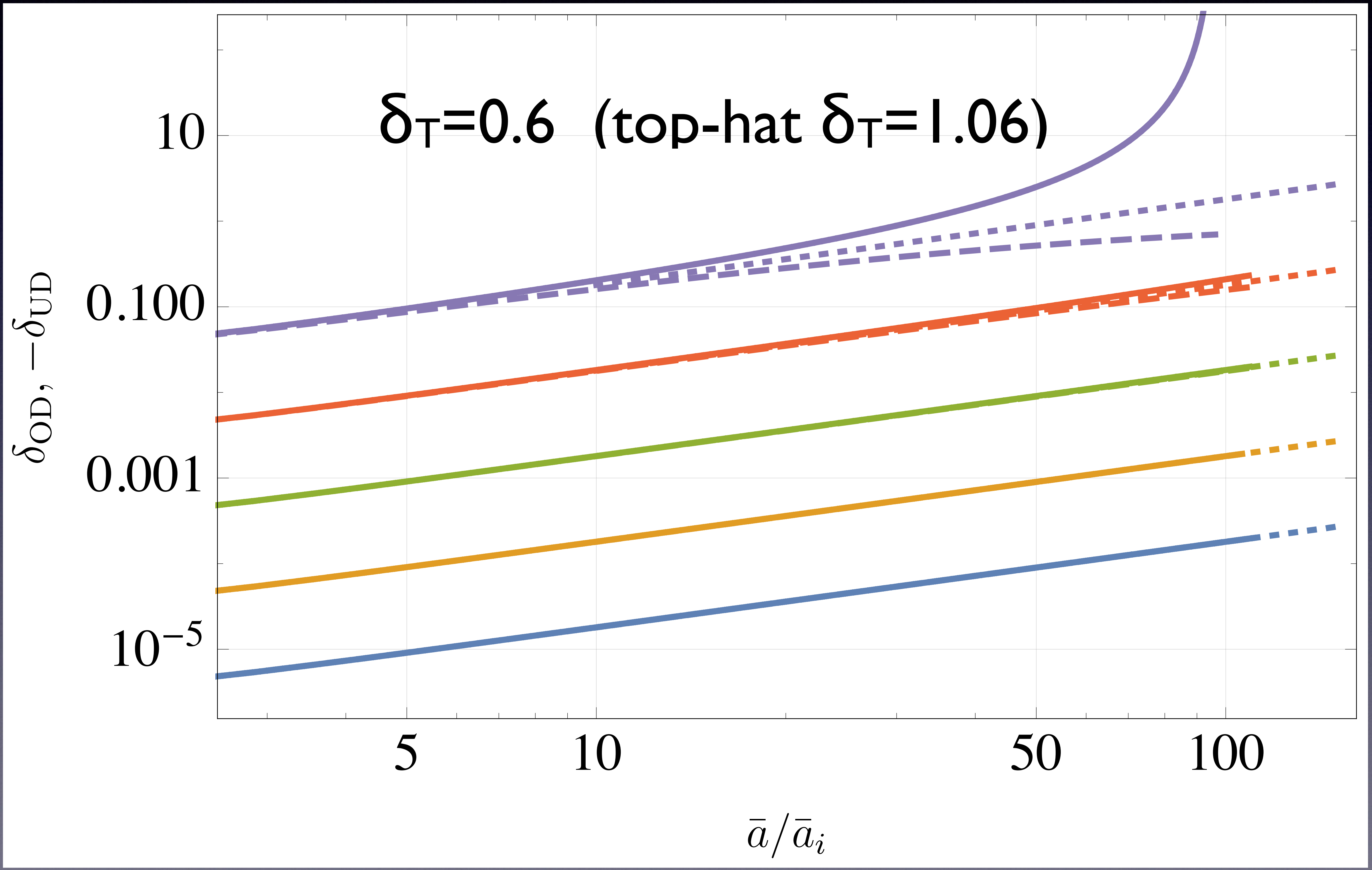


backreaction: Ω_Q

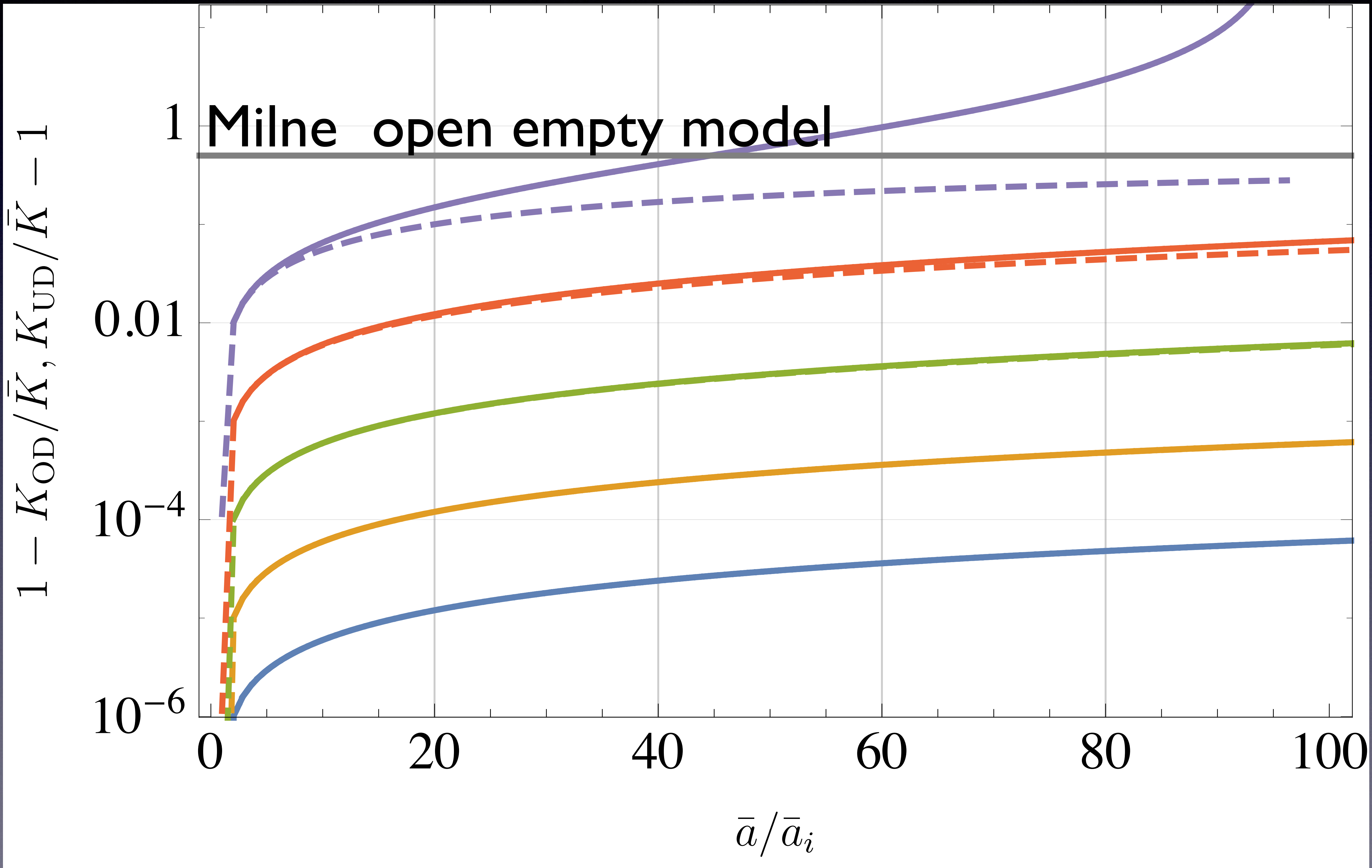


second goal:
peaks, collapse and voids

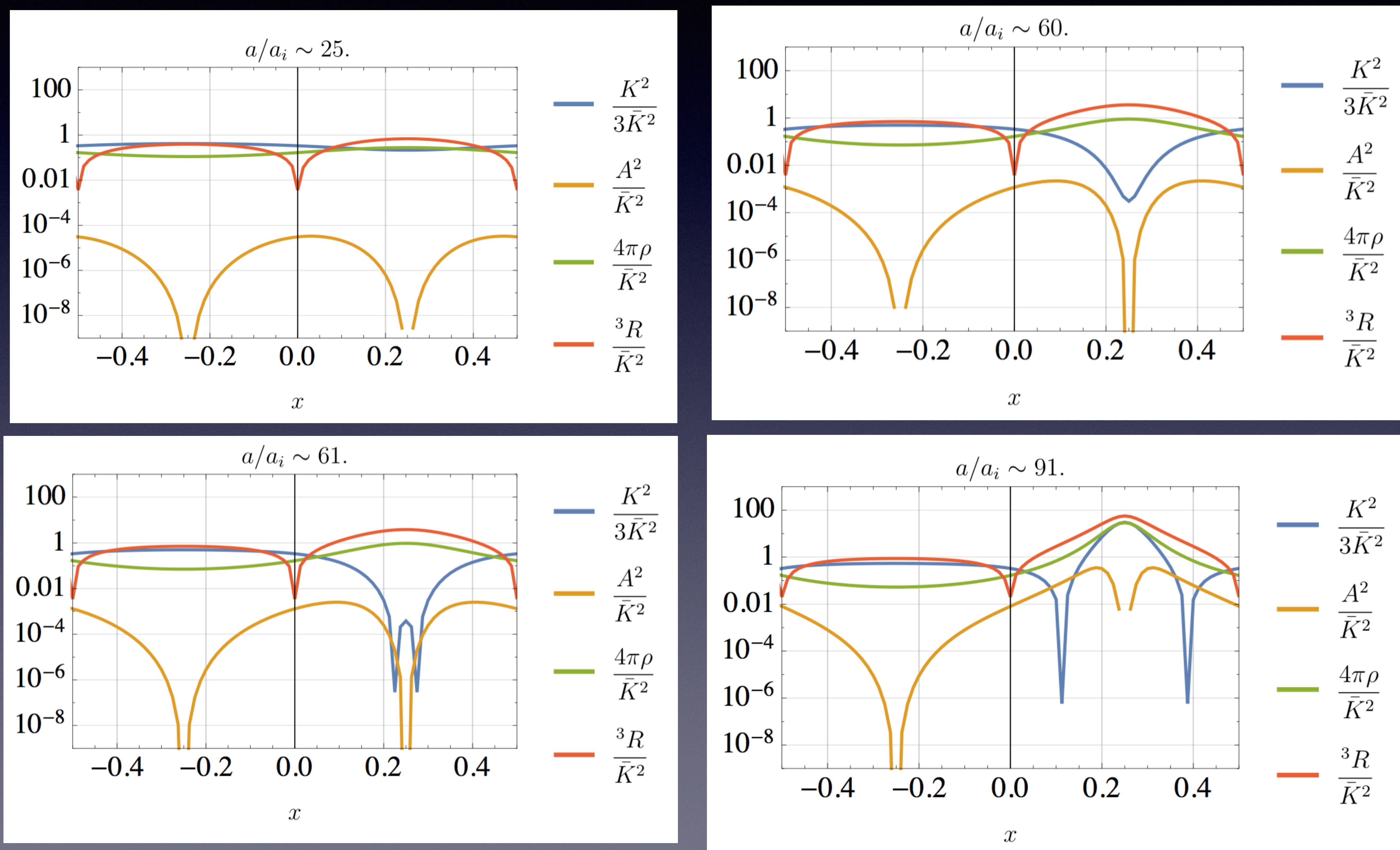
over and under densities



local expansion of peaks and voids



local contribution to Raychaudhuri equation



take-home message #2

- dynamical back-reaction is probably small on the largest scales
- more work needed to understand small scales effect and apparent discrepancies between results in different gauges
- light-cone effects and observational back-reaction needs to be studied in full GR
- models with Dark Energy needs full GR investigation

recent progress

- Bentivegna, *An automatically generated code for relativistic inhomogeneous cosmologies,* [arXiv:1610.05198]; Bentivegna et al, *Light propagation through black-hole lattices,*[arXiv:1611.09275]
- Giblin, Mertens, & Starkman, *Observable Deviations from Homogeneity in an Inhomogeneous Universe* [arXiv:1608.04403]; *A cosmologically motivated reference formulation of numerical relativity* [arXiv:1704:04307]
- Macpherson, Lasky & Price, *Inhomogeneous Cosmology with Numerical Relativity,* Macpherson, PRD 95, 064028 (2017)
- Daverio et al. *A numerical relativity scheme for cosmological simulations* [arXiv:1611.03437]
- East & wojtak, *Comparing Fully General Relativistic and Newtonian Calculations of Structure Formation* [arXiv:1711.06681]
- Giblin et al, *The Limited Accuracy of Linearized Gravity* [arXiv:1810.05203]

recent progress

- N-body with weak field GR:
  - Adamek et al., *g-evolution: a cosmological N-body code based on General Relativity* [arXiv:1604.06065]
- Initial conditions for Newtonian N-body evolution:
  - Chisari & Zaldarriaga (2011), Green & Wald (2012)
  - Fidler et al. *Relativistic initial conditions for N-body simulations* [arXiv:1702.03221], [arXiv:1606.05588], [arXiv:1505.04756]
  - Adamek et al, *The effect of early radiation in N-body simulations of cosmic structure formation* [arXiv:1703.08585]

Conclusions

- post-F: framework including Newtonian and 1 GR order
  - Frame dragging small, but further work needed, e.g. lensing
  - Adamek et al.: consistent results, plus $\Phi=\Psi$ at leading order
- Full GR Numerical Relativity simulations:
  - within the fluid assumption (stop before shall crossing), backreaction is small and the box expands like EdS
  - peaks collapse much faster than standard Top-Hat
  - voids expand up to 28% faster than average (background)
  - other work fully consistent with our general conclusions