BHs in an effective field theory extension of GR

PRL**121**, 251105 (2018) [arXiv:1808.08962]

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20^{th} Feb 2019 @ Rome

Introduction

General Relativity is consistent with many observations: (classical tests, GPS, black holes, gravitational waves, expanding universe, etc…)

One may think that only GR is enough

However…

However…

- GR predicts curvature singularity
- quantum gravity

→ GR seems to be some effective theory of more fundamental theory

- dark energy

strange matter in GR

Try to consider modification of GR

Usually, many people study modified gravities by adding other physical degrees of freedom: scalar tensor, Horndeski, etc…

today's talk

We also want to consider (effective) theories not adding other physical degrees of freedom (purely gravitational theory)

and, BHs in this theory

EFT

Effective field theory (EFT) is originally introduced by Weinberg to describe low energy theory of particle physics

This approach is very powerful for many physical systems, even for classical field theory

EFT extension of GR

Assumptions:

- lowest order is vacuum GR $(R_{\mu\nu} = 0)$
- only graviton
- correction terms are only made from curvature tensor
- covariance, causality, etc…

We want to apply EFT to gravitational field for compact objects

Candidates for EFT

$$S=\int dx^4\sqrt{|g|}2M_{\rm pl}^2(R+[\text{corrections}])$$

$\mathcal{O}((R_{\mu\nu\rho\sigma})^2)$ terms always becomes surface term if the lowest order is Ricci flat

$\mathcal{O}((R_{\mu\nu\rho\sigma})^3)$ terms violates causality
[Camanho, Edelstein, Maldacena and Zhiboedov, 2016]
(However, also mentioned the possibility that the theory becomes healthy by adding higher spin particles)

$\mathcal{O}((R_{\mu\nu\rho\sigma})^4)$ corrections [Endlich+ 2017]

$$S = \int dx^4 \sqrt{|g|} 2M_{\rm pl}^2 \left[R - \frac{\mathcal{C}^2}{\Lambda^6} - \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} - \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda_{-}^6} \right]$$

$$\mathcal{C} := R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \quad \tilde{\mathcal{C}} := R_{\alpha\beta\gamma\delta} \tilde{R}^{\alpha\beta\gamma\delta}$$

$$(\tilde{R}^{\alpha\beta\gamma\delta} := \epsilon^{\alpha\beta}{}_{\mu\nu} R^{\mu\nu\gamma\delta}, \quad \epsilon_{0123} = \sqrt{-g})$$

$$R^{\mu\alpha} - \frac{1}{2} g^{\mu\alpha} R = \frac{1}{\Lambda^6} \left(8 R^{\mu\nu\alpha\beta} \nabla_\nu \nabla_\beta \mathcal{C} + \frac{g^{\mu\alpha}}{2} \mathcal{C}^2 \right)$$

$$+ \frac{1}{\tilde{\Lambda}^6} \left(8 \tilde{R}^{\mu\rho\alpha\nu} \nabla_\rho \nabla_\nu \tilde{\mathcal{C}} + \frac{1}{2} g^{\mu\alpha} \tilde{\mathcal{C}}^2 \right)$$

$$+ \frac{1}{\Lambda_{-}^6} \left(4 \tilde{R}^{\mu\rho\alpha\nu} \nabla_\rho \nabla_\nu \mathcal{C} + 4 R^{\mu\rho\alpha\nu} \nabla_\rho \nabla_\nu \tilde{\mathcal{C}} + \frac{g^{\mu\alpha}}{2} \tilde{\mathcal{C}}\mathcal{C} \right)$$

We used $R_{\mu\nu} = 0$ for RHS

- EFT action makes sense only if solution is close to GR sol., then ghost does not appear

(see also arXiv:1808.07897 by Allwright and Lehner
arXiv:1404.2236 by Burgess and Williams)

Black hole solutions

Dimension less parameters

$$(\epsilon_1, \epsilon_2, \epsilon_3) := \left(\frac{1}{M^6 \Lambda^6}, \frac{1}{M^6 \tilde{\Lambda}^6}, \frac{1}{M^6 \Lambda_-^6} \right)$$

M : mass of BH

Spherically symmetric spacetime

$$ds^2 = -f_t(r) dt^2 + \frac{1}{f_r(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

Since $\tilde{\mathcal{C}} = 0, \ \tilde{R}^{\mu\rho\alpha\nu} \nabla_\rho \nabla_\nu \mathcal{C} = 0$

only ϵ_1 correction term does not vanish

We consider a solution close to Schwarzschild black hole

$$f_t = 1 - \frac{2M}{r} + \epsilon_1 \delta f_t \qquad f_r = 1 - \frac{2M}{r} + \epsilon_1 \delta f_r$$

$$\delta f_t = -\frac{1024 M^9}{r^9} + \frac{1408 M^{10}}{r^{10}}$$

$$\delta f_r = -\frac{4608 M^9}{r^9} + \frac{8576 M^{10}}{r^{10}}$$

Horizon: $f_t|_{r=r_H} = \mathcal{O}(\epsilon_1^2)$

$$r_H = 2M\,(1 + 5\epsilon_1/8) + \mathcal{O}(\epsilon_1^2)$$

Gravitational perturbation

gravitational perturbation $h_{\mu\nu}$
around spherically sym BH $g_{\mu\nu}^{\mathrm{BH}}$

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{BH}} + h_{\mu\nu}$$

$$g_{\mu\nu}^{\mathrm{GR}} + \epsilon_i g_{\mu\nu}^{(\epsilon_i)} \qquad h_{\mu\nu}^{\mathrm{GR}} + \epsilon_i h_{\mu\nu}^{(\epsilon_i)}$$

Thanks to the spherical symmetry,
$h_{\mu\nu}$ can be expanded by $Y_{\ell m}(\theta,\phi)$

Odd and even parity perturbations:

$$h^{(-)}_{\mu\nu}dx^\mu dx^\nu = 2e^{-i\omega t}\sin\theta\partial_\theta Y_{\ell 0}d\phi\,(h_0 dt + h_1 dr)$$

$$h^{(+)}_{\mu\nu}dx^\mu dx^\nu = e^{-i\omega t}Y_{\ell 0}\Big(f_t H_0 dt^2 + 2H_1 dt dr$$
$$+f_r^{-1}H_2 dr^2 + r^2 K(d\theta^2 + \sin^2\theta d\phi^2)\Big)$$

($h_{\mu\nu}$ contains ϵ_i corrections)

Master equations

$(\epsilon_1, -)$ case

$$\Psi_-^{\epsilon_1} = \frac{i\sqrt{f_t^{\epsilon_1} f_r^{\epsilon_1}}h_1}{\omega r}\left(1 + \frac{1152M^8\epsilon_1(13M - 7r)}{r^9}\right)$$

$$\frac{d^2\Psi_-^{\epsilon_1}}{dr_*^2} + \left(\omega^2 - \sqrt{f_t^{\epsilon_1} f_r^{\epsilon_1}}\left(V_-^{\mathrm{GR}} + \epsilon_1 V_-^{\epsilon_1}\right)\right)\Psi_-^{\epsilon_1} = 0$$

$$V_-^{\epsilon_1} = -\frac{256M^8}{r^{12}} \times \Big(15561M^2 + Mr(146\ell(\ell+1)$$
$$-13509) + 9r^2\left(324 - 8\ell(\ell+1) + 7r^2\omega^2\right)\Big)$$

$$dr/dr_* = \sqrt{f_t^{\epsilon_i} f_r^{\epsilon_i}}$$

Still Schrödinger form!

Remarks

- Master eqs become Schrödinger form for $(\epsilon_1, \pm), (\epsilon_2, \pm)$ cases

- Love numbers are not zero

- Due to the parity violating term, odd and even modes are coupled for ϵ_3

$$\frac{d^2\tilde{\Psi}_-^{\epsilon_3}}{dr_*^2} + (\omega^2 - fV_-^{\mathrm{GR}})\tilde{\Psi}_-^{\epsilon_3} - \epsilon_3 f V^{\epsilon_3}\tilde{\Psi}_+^{\epsilon_3} = 0$$

$$\frac{d^2\tilde{\Psi}_+^{\epsilon_3}}{dr_*^2} + (\omega^2 - fV_+^{\mathrm{GR}})\tilde{\Psi}_+^{\epsilon_3} - \epsilon_3 f V^{\epsilon_3}\tilde{\Psi}_-^{\epsilon_3} = 0$$

Quasi normal modes

Since the master eqs for ϵ_1, ϵ_2 are Schrödinger form, we can use existing codes to calculate QNMs numerically

$$\mathrm{Re}[\omega] = \mathrm{Re}[\omega_{\mathrm{GR}}](1 + \epsilon_i \delta_{\mathrm{Re}})$$

$$\mathrm{Im}[\omega] = \mathrm{Im}[\omega_{\mathrm{GR}}](1 + \epsilon_i \delta_{\mathrm{Im}})$$

$\epsilon_1, \ell = 2$ case:

$$\delta^{+}_{\mathrm{Re}} = 0.45 \qquad \delta^{-}_{\mathrm{Re}} = 0.22$$

$$\delta^{+}_{\mathrm{Im}} = -2.75 \qquad \delta^{-}_{\mathrm{Im}} = -0.64$$

$\pm$ modes have different spectrum

Constraints on parameters

At this stage, we only have very rough constraints on ϵ_1 from LIGO observation

$$
\begin{aligned}
\frac{\omega - \omega_{\mathrm{GR}}}{\omega_{\mathrm{GR}}} &\lesssim 1 \\
&\simeq \epsilon_i \\
&= 1/(M^6 \Lambda^6)
\end{aligned}
$$

$$M \sim 30 M_\odot \quad \Longrightarrow \quad 1/\Lambda \lesssim 100\mathrm{km}$$

PN analysis is important for stronger constraint

Slowly rotating BHs

We obtained slow rot BHs for ϵ_1, ϵ_2 cases upto $\mathcal{O}(\chi^4)$

We found $\mathbb{Z}_2$ - symmetry violating solution for ϵ_3 case

$$\delta g_{\mu\nu} dx^\mu dx^\nu = \epsilon_3 \chi \left(\frac{73728 M^9}{r^9} dr^2 + \frac{256 M^9 (243M - 160r)}{5r^8} (d\theta^2 + \sin^2\theta d\phi^2) \right) \cos\theta$$

This is due to the parity violating term

(cf:arXiv:1901.01315, Cano, Ruipérez)

Summary

- We discussed BHs in an EFT extension of GR with $\mathcal{O}((R_{\mu\nu\rho\sigma})^4)$ terms
- Derived spherically sym BHs and master eqs for gravitational perturbation

Master eqs are 2nd order differential eqs

Non-zero Love number

- Derived slow rot BHs

$\mathbb{Z}_2$ sym violating sol for ϵ_3

Future works

- $\mathcal{O}((R_{\mu\nu\rho\sigma})^3)$ correction terms
- rapidly rotating BHs
  (cf: arXiv:1901.01315, Cano and Ruipérez
  they calculated 14th spin parameter)
- Similar analysis for scalar tensor theory
- QNM for ϵ_3 (ongoing work)
- PN analysis (by Senatore et al)

Solve eqs perturbatively

Solve eqs order by order

$$S=\int dx^4\sqrt{|g|}(R+\epsilon(R^4_{\mu\nu\rho\sigma}))$$

$$\Longrightarrow \quad G_{\mu\nu}=\epsilon X_{\mu\nu}$$

We assume $g_{\mu\nu}=g^{(\mathrm{GR})}_{\mu\nu}+\epsilon h_{\mu\nu}$

$$G^{(\mathrm{GR})}_{\mu\nu}+\underline{\epsilon\delta G_{\mu\nu}}=\underline{\epsilon X_{\mu\nu}}$$

As far as we solve order by order,
# of DOF does not increase

2^{nd} order perturbations

$$g_{\mu\nu} = g_{\mu\nu}^{\text{Sch}} + \epsilon g_{\mu\nu}^{(1,0)} + \underline{\epsilon^2 g_{\mu\nu}^{(2,0)}} + \dots$$

$$+ h_{\mu\nu}^{(0,1)} + \underline{\epsilon h_{\mu\nu}^{(1,1)}} + \epsilon^2 h_{\mu\nu}^{(2,1)} + \dots$$

$$+ \underline{h_{\mu\nu}^{(0,2)}} + \dots$$

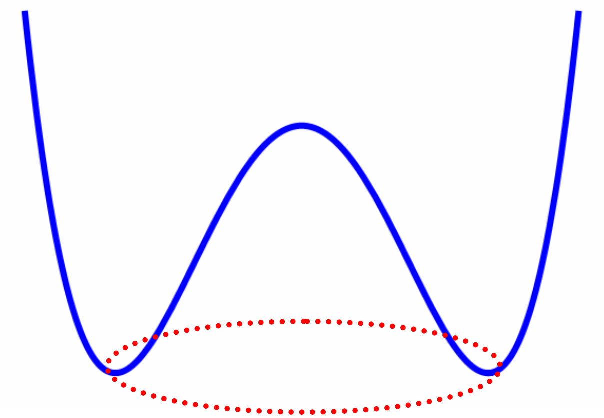
Mathematically, we can treat $O(h^2), O(\epsilon h), O(\epsilon^2)$ independently

A toy model

We assume that we know UV complete theory as

$$S = \int dx^4 (-\partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|))$$

$$V = \frac{\lambda}{2}\left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$



We want to see
low energy effective theory

$$\Phi = \frac{v}{\sqrt{2}}(1+\rho)e^{i\theta}$$ two real fields: ρ, θ

$$\frac{S}{v^2} = \int d^4x \left(-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1+\rho)^2(\partial\theta)^2 - V(\rho) \right)$$

$$V = \frac{M^2}{2}\left(\rho^2 + \rho^3 + \frac{1}{4}\rho^4\right) \qquad M^2 := \lambda v^2$$

$$\Longrightarrow \begin{cases} \Box\rho - (1+\rho)(\partial\theta)^2 - V' = 0 \\ \partial_\mu[(1+\rho)^2\partial^\mu\theta] = 0 \end{cases}$$

At low energy scale,

$$\rho = \frac{\rho^{(1)}}{M^2} + \frac{\rho^{(2)}}{M^4} + \frac{\rho^{(3)}}{M^6} + \cdots$$

Solve EOM order by order

$$\Box\rho - (1+\rho)\underline{(\partial\theta)^2} - V' = 0$$

$$V' = M^2(\underline{\rho} + 3\rho/2 + \rho^2/2)$$

$$\Longrightarrow \quad \rho^{(1)} = -(\partial\theta)^2$$

Similarly, we can obtain $\rho^{(2)}$

$$\rho = \frac{\rho^{(1)}}{M^2} + \frac{\rho^{(2)}}{M^4} + \frac{\rho^{(3)}}{M^6} + \cdots$$

$$= -\frac{(\partial\theta)^2}{M^2} - \frac{(\partial\theta)^4 + 2\Box(\partial\theta)^2}{2M^4} + \cdots$$

$$\frac{S}{v^2} = \int d^4x \left(-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1+\rho)^2(\partial\theta)^2 - V(\rho) \right)$$

$$= \int d^4x \Bigg(-\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4$$

$$-\frac{2}{M^4}(\partial_\mu\partial_\nu\theta)(\partial^\mu\partial^\alpha\theta)(\partial^\nu\theta)(\partial_\alpha\theta) + \cdots \Bigg)$$

Low energy effective action

What we did here?

$$\frac{S}{v^2} = \int d^4x \left(-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1+\rho)^2(\partial\theta)^2 - V(\rho) \right)$$

At low energy, ρ is like a Lagrange multiplier

We can substitute EOM of ρ to the action

$$\frac{S}{v^2} = \int d^4x \Big(-\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 - \frac{2}{M^4}\underline{(\partial_\mu\partial_\nu\theta)(\partial^\mu\partial^\alpha\theta)(\partial^\nu\theta)(\partial_\alpha\theta)} + \cdots \Big)$$

This theory apparently has a ghost as a new degree of freedom

$$\Box\theta = \frac{2}{M^2}\left[(\partial_\nu\theta)(\partial^\nu\theta)\Box\theta + 2(\partial_\mu\partial_\nu\theta)(\partial^\nu\theta)(\partial^\mu\theta)\right]$$
$$+\frac{4}{M^4}\Big[(\partial^\nu\partial^\sigma\Box\theta)(\partial_\nu\theta)(\partial_\sigma\theta) + (\partial^\sigma\Box\theta)(\Box\theta)(\partial_\sigma\theta)$$
$$+(\partial^\sigma\Box\theta)(\partial^\nu\theta)(\partial_\nu\partial_\sigma\theta) + (\partial^\mu\partial^\sigma\theta)(\Box\theta)(\partial_\mu\partial_\sigma\theta)$$
$$+2(\partial^\nu\partial^\mu\partial^\sigma\theta)(\partial_\mu\partial_\nu\theta)(\partial_\sigma\theta)\Big] + \cdots$$
$$= \frac{1}{M^2}S_1 + \frac{1}{M^4}S_2 + \cdots$$

However, at low energy scale

$$\theta = \theta^{(0)} + \frac{\theta^{(1)}}{M^2} + \frac{\theta^{(2)}}{M^4} + \cdots$$

$\Box\theta^{(0)} = 0$ (EOM: $\Box\theta = \frac{1}{M^2}S_1 + \frac{1}{M^4}S_2 + \cdots$)

$$\frac{1}{M^2}\Box\theta^{(1)} = \frac{1}{M^2}S_1(\theta^{(0)} + \cancel{\theta^{(1)}/M^2})$$

$$\frac{1}{M^4}\Box\theta^{(2)} = \frac{1}{M^2}S_1(\theta^{(1)}/M^2) + \frac{1}{M^4}S_2(\theta^{(0)})$$

The ghost does not appear

for $\theta = \theta^{(0)} + [\mathrm{corrections}]$

What we should learn from this:

- EFT apparently can have ghost (higher derivative terms) even if UV complete theory is healthy

- EFT action makes sense around 0^{th} order solution, then ghost does not appear

(see also arXiv:1808.07897 by Allwright and Lehner
arXiv:1404.2236 by Burgess and Williams)

Gravitational perturbation

gravitational perturbation $h_{\mu\nu}$
around spherically sym BH $g_{\mu\nu}^{\mathrm{BH}}$

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{BH}} + h_{\mu\nu}$$

$$g_{\mu\nu}^{\mathrm{GR}} + \epsilon_i g_{\mu\nu}^{(\epsilon_i)} \qquad h_{\mu\nu}^{\mathrm{GR}} + \epsilon_i h_{\mu\nu}^{(\epsilon_i)}$$

For not very small epsilon,

$$\mathcal{O}(\epsilon h) \text{ terms} \gg \mathcal{O}(h^2) \text{ terms}$$

GR case (Schwarzschild case)

$$\Psi_-^{\rm GR} = \frac{ifh_1(r)}{r\omega} \qquad \Psi_+^{\rm GR} = \frac{1}{\lambda r + 6M}\left[-r^2 K + \frac{ifrH_1}{\omega}\right]$$

$$\frac{d^2\Psi_\pm^{\rm GR}}{dr_*^2} + \left(\omega^2 - fV_\pm^{\rm GR}\right)\Psi_\pm^{\rm GR} = 0$$

$$dr/dr_* = f$$

$$f = 1 - 2M/r$$

$$V_-^{\rm GR} = \frac{(\lambda+2)}{r^2} - \frac{6M}{r^3}$$

$$\lambda = \ell^2 + \ell - 2$$

$$V_+^{\rm GR} = \frac{1}{r^3(\lambda r + 6M)^2} \times \Big[36\lambda M^2 r + 6\lambda^2 M r^2 + \lambda^2(\lambda+2)r^3 + 72M^3\Big]$$

Same form as 1-dim Schrödinger eq

(Other metric components are written by $\Psi_\pm^{\rm GR}$)

Strategy to obtain master eq

RHS of EOM $G_{\mu\nu} = \epsilon_i S^i_{\mu\nu}$ contains higher derivative
Calculation seems to be difficult

We assume that perturbed eqs reduce to a single master eq by $\Psi^{\epsilon_i}_{\pm} = \Psi^{\mathrm{GR}}_{\pm} + \mathcal{O}(\epsilon_i)$

Then RHS becomes

$$[\mathbf{RHS}] = \epsilon_i S^i_{\mu\nu}(h_{\mu\nu})$$
$$= \epsilon_i S^i_{\mu\nu}(h^{\mathrm{GR}}_{\mu\nu} + \epsilon_i h^{(\epsilon_i)}_{\mu\nu})$$
$$= \epsilon_i S^i_{\mu\nu}(\Psi^{\mathrm{GR}}_{\pm})$$

(we know the relation with $h^{\mathrm{GR}}_{\mu\nu}$ and $\Psi^{\mathrm{GR}}_{\pm}$)

$$= \epsilon_i S^i_{\mu\nu}(\Psi^{\epsilon_i}_{\pm}) \quad (\Psi^{\epsilon_i}_{\pm} = \Psi^{\mathrm{GR}}_{\pm} + \mathcal{O}(\epsilon_i))$$

$$[\mathbf{RHS}] = \epsilon_i S^i_{\mu\nu}(\Psi^{\epsilon_i}_{\pm})$$

RHS still contains higher derivative of $\Psi^{\epsilon_i}_{\pm}$

but we can use $\frac{d^2\Psi^{\mathrm{GR}}_{\pm}}{dr_*^2} + \left(\omega^2 - fV^{\mathrm{GR}}_{\pm}\right)\Psi^{\mathrm{GR}}_{\pm} = 0$

to replace higher derivative to lower derivative (cf. $\Psi^{\epsilon_i}_{\pm} = \Psi^{\mathrm{GR}}_{\pm} + \mathcal{O}(\epsilon_i)$)

Fortunately, all higher order terms of ω cancel (I don't know the physical reason...)

EOM is still at most second order

Not very difficult to find

Remarks

Master eqs become Schrödinger form for $(\epsilon_1, \pm), (\epsilon_2, \pm)$ cases

(see arXiv:1808.08962 for details)

The systems are stable for $\epsilon_1 \geq 0, \epsilon_2 \geq 0$
This parameter region is same as that from the causality constraint

Love numbers are not zero (this affects the form of GW in inspiral phase)

Quasi normal modes

Since the master eqs for ϵ_1, ϵ_2 are Schrödinger form, we can use existing codes to calculate QNMs numerically

$$\mathbf{Re}[\omega] = \mathbf{Re}[\omega_{\mathbf{GR}}](1 + \epsilon_i \delta_{\mathbf{Re}})$$

$$\mathbf{Im}[\omega] = \mathbf{Im}[\omega_{\mathbf{GR}}](1 + \epsilon_i \delta_{\mathbf{Im}})$$

ϵ_1 case: $\delta_\ell = (\delta_{\mathbf{Re}}, \delta_{\mathbf{Im}})|_\ell$

$$\delta^{\mathbf{even}}_{\ell=2} = (0.45, -2.75),\ \delta^{\mathbf{odd}}_{\ell=2} = (0.22, -0.64)$$

$$\delta^{\mathbf{even}}_{\ell=3} = (1.07, -6.42),\ \delta^{\mathbf{odd}}_{\ell=3} = -(0.0099, 0.44)$$

$$\delta^{\mathbf{even}}_{\ell=4} = (1.76, -11.43),\ \delta^{\mathbf{odd}}_{\ell=4} = -(0.048, 0.19)$$

Slowly rotating BHs

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{GR}} + \epsilon_i \delta g_{\mu\nu}$$

$$g_{\mu\nu}^{\mathrm{GR}} = g_{\mu\nu}^{\mathrm{Sch}} + \underbrace{\chi g_{\mu\nu}^{(1,0)} + \chi^2 g_{\mu\nu}^{(2,0)} + \cdots}$$

$$\chi = a/M$$

can be obtained from Kerr metric

$$\delta g_{\mu\nu} = \underbrace{g_{\mu\nu}^{(0,1)} + \chi g_{\mu\nu}^{(1,1)} + \chi^2 g_{\mu\nu}^{(2,1)} + \cdots}$$

from EOM for EFT

Slowly rotating BHs

We take the metric anzats

$$ds^2 = -f(r)(1+h(r,\theta))dt^2 + \frac{1+m(r,\theta)}{f(r)}dr^2$$
$$+r^2(1+k(r,\theta))d\theta^2$$
$$+r^2\sin^2\theta(1+p(r,\theta))(d\phi - \Omega(r,\theta)dt)^2$$

$$f = 1 - \frac{2M}{r}$$

$\mathcal{O}(\epsilon_i^0)$: h, m, k, p, Ω are determined from Kerr

$\mathcal{O}(\epsilon_i)$: h, m, k, p, Ω are determined from EOM for EFT

(also expand h, m, k, p, Ω in power of $\cos\theta$)

Future works

- $\mathcal{O}((R_{\mu\nu\rho\sigma})^3)$ correction terms
- Understand the reason why the master eqs are still 2nd order differential eqs
- rapidly rotating BHs
  (cf: arXiv:1901.01315, Cano and Ruipérez they calculated 14th spin parameter)
- Similar analysis for scalar tensor theory
- QNM for ϵ_3 (on-going work)
- PN analysis (by Senatore et al)