BHs in an effective field theory extension of GR

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General Relativity is consistent with many observations: (classical tests, GPS, black holes, gravitational waves, expanding universe, etc…)

One may think that only GR is enough

However…
However…

- GR predicts curvature singularity
- quantum gravity

→ GR seems to be some effective theory of more fundamental theory

- dark energy
- strange matter in GR

Try to consider modification of GR
Usually, many people study modified gravities by adding other physical degrees of freedom: scalar tensor, Horndeski, etc…

today’s talk
We also want to consider (effective) theories not adding other physical degrees of freedom (purely gravitational theory) and, BHs in this theory
Effective field theory (EFT) is originally introduced by Weinberg to describe low energy theory of particle physics.

This approach is very powerful for many physical systems, even for classical field theory.
EFT extension of GR

Assumptions:
- lowest order is vacuum GR \( R_{\mu\nu} = 0 \)
- only graviton
- correction terms are only made from curvature tensor
- covariance, causality, etc…

We want to apply EFT to gravitational field for compact objects
Candidates for EFT

\[ S = \int dx^4 \sqrt{|g|} 2M_{\text{pl}}^2(R + \text{[corrections]}) \]

\[ \mathcal{O}((R_{\mu\nu\rho\sigma})^2) \] terms always becomes surface term if the lowest order is Ricci flat

\[ \mathcal{O}((R_{\mu\nu\rho\sigma})^3) \] terms violates causality

[Camanho, Edelstein, Maldacena and Zhiboedov, 2016]
(However, also mentioned the possibility that the theory becomes healthy by adding higher spin particles)
\[ S = \int dx^4 \sqrt{|g|} 2M_{\text{pl}}^2 \left[ R - \frac{\mathcal{C}^2}{\Lambda^6} - \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} - \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda^6} \right] \]

\[ \mathcal{C} := R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \quad \tilde{\mathcal{C}} := R_{\alpha\beta\gamma\delta} \tilde{R}^{\alpha\beta\gamma\delta} \]

\[ (\tilde{R}^{\alpha\beta\gamma\delta} := \epsilon^{\alpha\beta}_{\mu\nu} R^{\mu\nu\gamma\delta}, \quad \epsilon_{0123} = \sqrt{-g}) \]

\[ R^{\mu\alpha} - \frac{1}{2} g^{\mu\alpha} R = \frac{1}{\Lambda^6} \left( 8 R^{\mu\nu\alpha\beta} \nabla_\nu \nabla_\beta \mathcal{C} + \frac{g^{\mu\alpha}}{2} \mathcal{C}^2 \right) \]

\[ + \frac{1}{\tilde{\Lambda}^6} \left( 8 \tilde{R}^{\mu\rho\alpha\nu} \nabla_\rho \nabla_\nu \tilde{\mathcal{C}} + \frac{1}{2} g^{\mu\alpha} \tilde{\mathcal{C}}^2 \right) \]

\[ + \frac{1}{\Lambda^6} \left( 4 \tilde{R}^{\mu\rho\alpha\nu} \nabla_\rho \nabla_\nu C + 4 R^{\mu\rho\alpha\nu} \nabla_\rho \nabla_\nu \tilde{\mathcal{C}} + \frac{g^{\mu\alpha}}{2} \tilde{\mathcal{C}} \mathcal{C} \right) \]

We used \( R_{\mu\nu} = 0 \) for RHS
EFT action makes sense only if solution is close to GR sol., then ghost does not appear

(see also arXiv:1808.07897 by Allwright and Lehner
arXiv:1404.2236 by Burgess and Williams)
Black hole solutions

Dimensionless parameters

\[(\epsilon_1, \epsilon_2, \epsilon_3) := \left(\frac{1}{M^6 \Lambda^6}, \frac{1}{M^6 \tilde{\Lambda}^6}, \frac{1}{M^6 \Lambda^6}\right)\]

\[M : \text{mass of BH}\]

Spherically symmetric spacetime

\[ds^2 = -f_t(r)dt^2 + \frac{1}{f_r(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\]

Since \[\tilde{\mathcal{C}} = 0, \tilde{\mathcal{R}}^\mu_{\rho\alpha\nu} \nabla_\rho \nabla_\nu \mathcal{C} = 0\]
only \(\epsilon_1\) correction term does not vanish
We consider a solution close to Schwarzschild black hole

\[ f_t = 1 - \frac{2M}{r} + \epsilon_1 \delta f_t \quad f_r = 1 - \frac{2M}{r} + \epsilon_1 \delta f_r \]

\[ \delta f_t = -\frac{1024M^9}{r^9} + \frac{1408M^{10}}{r^{10}} \]

\[ \delta f_r = -\frac{4608M^9}{r^9} + \frac{8576M^{10}}{r^{10}} \]

Horizon: \( f_t \mid_{r=r_H} = \mathcal{O}(\epsilon_1^2) \)

\[ r_H = 2M \left(1 + 5\epsilon_1/8\right) + \mathcal{O}(\epsilon_1^2) \]
gravitational perturbation $h_{\mu\nu}$ around spherically sym BH $g_{\mu\nu}^{BH}$

$g_{\mu\nu} = g_{\mu\nu}^{BH} + h_{\mu\nu}$

$g_{\mu\nu}^{GR} + \epsilon_i g_{\mu\nu}^{(\epsilon_i)}$

$h_{\mu\nu}^{GR} + \epsilon_i h_{\mu\nu}^{(\epsilon_i)}$
Thanks to the spherical symmetry, \( h_{\mu\nu} \) can be expanded by \( Y_{\ell m}(\theta, \phi) \)

Odd and even parity perturbations:

\[
h_{(-)}^{\mu\nu} dx^\mu dx^\nu = 2e^{-i\omega t} \sin \theta \partial_\theta Y_{\ell 0} d\phi (h_0 dt + h_1 dr)
\]

\[
h_{(+)}^{\mu\nu} dx^\mu dx^\nu = e^{-i\omega t} Y_{\ell 0} \left( f_t H_0 dt^2 + 2H_1 dt dr \\
+ f_{r^{-1}} H_2 dr^2 + r^2 K (d\theta^2 + \sin^2 \theta d\phi^2) \right)
\]

(\( h_{\mu\nu} \) contains \( \epsilon_i \) corrections)
\( \Psi_{-}^{\epsilon_1} = \frac{i \sqrt{f_t^{\epsilon_1} f_r^{\epsilon_1}} h_1}{\omega r} \left( 1 + \frac{1152M^8 \epsilon_1 (13M - 7r)}{r^9} \right) \)

\[
\frac{d^2 \Psi_{-}^{\epsilon_1}}{dr_*^2} + \left( \omega^2 - \sqrt{f_t^{\epsilon_1} f_r^{\epsilon_1}} \left( V_{-}^{GR} + \epsilon_1 V_{-}^{\epsilon_1} \right) \right) \Psi_{-}^{\epsilon_1} = 0
\]

\[
V_{-}^{\epsilon_1} = -\frac{256M^8}{r^{12}} \times \left( 15561M^2 + Mr(146\ell(\ell + 1) - 13509) + 9r^2 (324 - 8\ell(\ell + 1) + 7r^2 \omega^2) \right)
\]

\[
dr/dr_* = \sqrt{f_t^{\epsilon_i} f_r^{\epsilon_i}}
\]

Still Schrödinger form!
Remarks

- Master eqs become Schrödinger form for \((\epsilon_1, \pm), (\epsilon_2, \pm)\) cases

- Love numbers are not zero

- Due to the parity violating term, odd and even modes are coupled for \(\epsilon_3\)

\[
\frac{d^2 \tilde{\Psi}^{\epsilon_3}_-}{dr_*^2} + (\omega^2 - fV_{\text{GR}}) \tilde{\Psi}^{\epsilon_3}_- - \epsilon_3 fV^{\epsilon_3} \tilde{\Psi}^{\epsilon_3}_+ = 0
\]

\[
\frac{d^2 \tilde{\Psi}^{\epsilon_3}_+}{dr_*^2} + (\omega^2 - fV^{\text{GR}}) \tilde{\Psi}^{\epsilon_3}_+ - \epsilon_3 fV^{\epsilon_3} \tilde{\Psi}^{\epsilon_3}_- = 0
\]
Quasi normal modes

Since the master eqs for $\epsilon_1, \epsilon_2$ are Schrödinger form, we can use existing codes to calculate QNMs numerically

$$\text{Re}[\omega] = \text{Re}[\omega_{GR}](1 + \epsilon_i \delta_{Re})$$
$$\text{Im}[\omega] = \text{Im}[\omega_{GR}](1 + \epsilon_i \delta_{Im})$$

$\epsilon_1, \ell = 2$ case:

$$\delta^+_{Re} = 0.45 \quad \delta^-_{Re} = 0.22$$
$$\delta^+_{Im} = -2.75 \quad \delta^-_{Im} = -0.64$$

$\pm$ modes have different spectrum
Constraints on parameters

At this stage, we only have very rough constraints on $\epsilon_1$ from LIGO observation

$$\frac{\omega - \omega_{GR}}{\omega_{GR}} \lesssim 1$$

$$\approx \epsilon_i$$

$$= 1/(M^6 \Lambda^6)$$

$$M \sim 30M_\odot \implies 1/\Lambda \lesssim 100\text{km}$$

PN analysis is important for stronger constraint
Slowly rotating BHs

We obtained slow rot BHs for $\varepsilon_1, \varepsilon_2$ cases upto $O(\chi^4)$

We found $\mathbb{Z}_2$ - symmetry violating solution for $\varepsilon_3$ case

\[
\delta g_{\mu\nu} dx^\mu dx^\nu = \varepsilon_3 \chi \left( \frac{73728 M^9}{r^9} dr^2 + \frac{256 M^9 (243 M - 160 r)}{5 r^8} \right)
\]

\[
(d\theta^2 + \sin^2 \theta d\phi^2) \cos \theta
\]

This is due to the parity violating term

(cf:arXiv:1901.01315, Cano, Ruipérez)
Summary

- We discussed BHs in an EFT extension of GR with $\mathcal{O}((R_{\mu\nu\rho\sigma})^4)$ terms.
- Derived spherically sym BHs and master eqs for gravitational perturbation.
  Master eqs are 2\textsuperscript{nd} order differential eqs.
  Non-zero Love number.
- Derived slow rot BHs.
  $\mathbb{Z}_2$ sym violating sol for $\epsilon_3$. 

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Future works

- $O((R_{\mu \nu \rho \sigma})^3)$ correction terms
- rapidly rotating BHs
  (cf: arXiv:1901.01315, Cano and Ruipérez they calculated 14th spin parameter)
- Similar analysis for scalar tensor theory
- QNM for $\epsilon_3$ (ongoing work)
- PN analysis (by Senateore et al)
Solve eqs perturbatively

Solve eqs order by order

\[ S = \int dx^4 \sqrt{|g|} \left( R + \epsilon(R^4_{\mu\nu\rho\sigma}) \right) \]

\[ \Longrightarrow G_{\mu\nu} = \epsilon X_{\mu\nu} \]

We assume \( g_{\mu\nu} = g^{(GR)}_{\mu\nu} + \epsilon h_{\mu\nu} \)

\[ G^{(GR)}_{\mu\nu} + \epsilon \delta G_{\mu\nu} = \epsilon X_{\mu\nu} \]

As far as we solve order by order, # of DOF does not increase 22/19
2nd order perturbations

\[ g_{\mu\nu} = g^{\text{Sch}}_{\mu\nu} + \epsilon g^{(1,0)}_{\mu\nu} + \epsilon^2 g^{(2,0)}_{\mu\nu} + ... \\
+ h^{(0,1)}_{\mu\nu} + \epsilon h^{(1,1)}_{\mu\nu} + \epsilon^2 h^{(2,1)}_{\mu\nu} + ... \\
+ h^{(0,2)}_{\mu\nu} + ... \]

Mathematically, we can treat \( O(h^2) \), \( O(\epsilon h) \), \( O(\epsilon^2) \) independently.
A toy model

We assume that we know UV complete theory as

\[ S = \int dx^4 (-\partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|)) \]

\[ V = \frac{\lambda}{2} \left( |\Phi|^2 - \frac{v^2}{2} \right)^2 \]

We want to see low energy effective theory 25/19
\[ \Phi = \frac{v}{\sqrt{2}} (1 + \rho) e^{i\theta} \quad \text{two real fields: } \rho, \theta \]

\[ \frac{S}{v^2} = \int d^4x \left( -\frac{1}{2} (\partial \rho)^2 - \frac{1}{2} (1 + \rho)^2 (\partial \theta)^2 - V(\rho) \right) \]

\[ V = \frac{M^2}{2} \left( \rho^3 + \rho^4 + \frac{1}{4} \rho^4 \right) \quad M^2 := \lambda v^2 \]

\[ \Longrightarrow \begin{cases} 
\Box \rho - (1 + \rho) (\partial \theta)^2 - V' = 0 \\
\partial_\mu [(1 + \rho)^2 \partial^\mu \theta] = 0 
\end{cases} \]
At low energy scale, 

\[ \rho = \frac{\rho^{(1)}}{M^2} + \frac{\rho^{(2)}}{M^4} + \frac{\rho^{(3)}}{M^6} + \cdots \]

Solve EOM order by order

\[ \Box \rho - (1 + \rho)(\partial \theta)^2 - V' = 0 \]

\[ V' = M^2(\rho + 3\rho/2 + \rho^2/2) \]

\[ \implies \rho^{(1)} = -(\partial \theta)^2 \]
Similarly, we can obtain $\rho^{(2)}$

$$\rho = \frac{\rho^{(1)}}{M^2} + \frac{\rho^{(2)}}{M^4} + \frac{\rho^{(3)}}{M^6} + \cdots$$

$$= -\frac{(\partial \theta)^2}{M^2} - \frac{(\partial \theta)^4}{2M^4} + \cdots$$

$$S_{v^2} = \int d^4x \left( -\frac{1}{2} (\partial \rho)^2 - \frac{1}{2} (1 + \rho)^2 (\partial \theta)^2 - V(\rho) \right)$$

$$= \int d^4x \left( -\frac{1}{2} (\partial \theta)^2 + \frac{1}{2M^2} (\partial \theta)^4 \right.$$

$$- \frac{2}{M^4} (\partial_\mu \partial_\nu \theta)(\partial^\mu \partial^\alpha \theta)(\partial^\nu \theta)(\partial^\alpha \theta) + \cdots \right)$$

Low energy effective action 28/19
What we did here?

\[
\frac{S}{v^2} = \int d^4x \left( -\frac{1}{2} (\partial \rho)^2 - \frac{1}{2} (1 + \rho)^2 (\partial \theta)^2 - V(\rho) \right)
\]

At **low energy**, \(\rho\) is like a Lagrange multiplier

We can substitute EOM of \(\rho\) to the action
This theory apparently has a ghost as a new degree of freedom

\[ \square \theta = \frac{2}{M^2} \left[ (\partial_\nu \theta) (\partial^\nu \theta) \square \theta + 2 (\partial_\mu \partial_\nu \theta) (\partial^\nu \theta) (\partial^\mu \theta) \right] + \frac{4}{M^4} \left[ (\partial^\nu \partial^\sigma \square \theta) (\partial_\nu \theta) (\partial_\sigma \theta) + (\partial^\sigma \square \theta) (\square \theta) (\partial_\sigma \theta) + (\partial^\sigma \square \theta) (\partial^\nu \theta) (\partial_\nu \partial_\sigma \theta) + (\partial^\mu \partial^\sigma \theta) (\square \theta) (\partial_\mu \partial_\sigma \theta) + 2 (\partial^\nu \partial^\mu \partial^\sigma \theta) (\partial_\mu \partial_\nu \theta) (\partial_\sigma \theta) \right] + \cdots \]

\[ = \frac{1}{M^2} S_1 + \frac{1}{M^4} S_2 + \cdots \]
However, at low energy scale

\[ \theta = \theta^{(0)} + \frac{\theta^{(1)}}{M^2} + \frac{\theta^{(2)}}{M^4} + \cdots \]

\[ \square \theta^{(0)} = 0 \quad \text{(EOM: } \square \theta = \frac{1}{M^2} S_1 + \frac{1}{M^4} S_2 + \cdots \text{)} \]

\[ \frac{1}{M^2} \square \theta^{(1)} = \frac{1}{M^2} S_1 (\theta^{(0)} + \theta^{(1)}/M^2) \]

\[ \frac{1}{M^4} \square \theta^{(2)} = \frac{1}{M^2} S_1 (\theta^{(1)}/M^2) + \frac{1}{M^4} S_2 (\theta^{(0)}) \]

The ghost does not appear for \( \theta = \theta^{(0)} + [\text{corrections}] \)
What we should learn from this:

- EFT apparently can have ghost (higher derivative terms) even if UV complete theory is healthy

- EFT action makes sense around 0th order solution, then ghost does not appear

(see also arXiv:1808.07897 by Allwright and Lehner
arXiv:1404.2236 by Burgess and Williams)
gravitational perturbation $h_{\mu\nu}$
around spherically sym BH $g_{\mu\nu}^{BH}$

$$g_{\mu\nu} = g_{\mu\nu}^{BH} + h_{\mu\nu}$$

$g_{\mu\nu}^{GR} + \epsilon_i g_{\mu\nu}^{(\epsilon_i)}$

$h_{\mu\nu}^{GR} + \epsilon_i h_{\mu\nu}^{(\epsilon_i)}$

For not very small epsilon,

$\mathcal{O}(\epsilon h)$ terms $\gg \mathcal{O}(h^2)$ terms

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GR case (Schwarzschild case)

\[ \Psi_{-}^{GR} = \frac{i f h_1(r)}{r \omega} \quad \Psi_{+}^{GR} = \frac{1}{\lambda r + 6M} \left[ -r^2 K + \frac{i f r H_1}{\omega} \right] \]

\[ \frac{d^2 \Psi_{\pm}^{GR}}{dr_*^2} + \left( \omega^2 - f V_{\pm}^{GR} \right) \Psi_{\pm}^{GR} = 0 \]

\[ V_{-}^{GR} = \frac{(\lambda + 2)}{r^2} - \frac{6M}{r^3} \]

\[ V_{+}^{GR} = \frac{1}{r^3(\lambda r + 6M)^2} \times \left[ 36\lambda M^2 r + 6\lambda^2 Mr^2 + \lambda^2(\lambda + 2)r^3 + 72M^3 \right] \]

Same form as 1-dim Schrödinger eq

(Other metric components are written by \( \Psi_{\pm}^{GR} \))
Strategy to obtain master eq

RHS of EOM $G_{\mu\nu} = \epsilon_i S_{\mu\nu}^i$ contains higher derivative
Calculation seems to be difficult

We assume that perturbed eqs reduce to a single master eq by $\Psi_{\pm}^\epsilon = \Psi_{\pm}^{GR} + \mathcal{O}(\epsilon_i)$
Then RHS becomes

\[ [\text{RHS}] = \epsilon_i S_{i\mu\nu}^i (h_{\mu\nu}) \]
\[ = \epsilon_i S_{i\mu\nu}^i (h_{\mu\nu}^{GR} + \epsilon_i h_{\mu\nu}^{(\epsilon_i)}) \]
\[ = \epsilon_i S_{i\mu\nu}^i (\Psi_{\pm}^{GR}) \text{ (we know the relation with } h_{\mu\nu}^{GR} \text{ and } \Psi_{\pm}^{GR}) \]
\[ = \epsilon_i S_{i\mu\nu}^i (\Psi_{\pm}^{\epsilon_i}) \text{ (} \Psi_{\pm}^{\epsilon_i} = \Psi_{\pm}^{GR} + \mathcal{O}(\epsilon_i) \text{)} \]
\[ \text{RHS} = \epsilon_i S^i_{\mu\nu}(\Psi^{\epsilon_i}_{\pm}) \]

RHS still contains higher derivative of \( \Psi^{\epsilon_i}_{\pm} \)

but we can use

\[ \frac{d^2 \Psi^{GR}_{\pm}}{dr_*^2} + (\omega^2 - fV^{GR}_{\pm}) \Psi^{GR}_{\pm} = 0 \]

to replace higher derivative to lower derivative

\( \text{cf. } \Psi^{\epsilon_i}_{\pm} = \Psi^{GR}_{\pm} + O(\epsilon_i) \)

Fortunately, all higher order terms of \( \omega \) cancel (I don’t know the physical reason…)

EOM is still at most second order

Not very difficult to find
Remarks

Master eqs become Schrödinger form for \((\epsilon_1, \pm), (\epsilon_2, \pm)\) cases
(see arXiv:1808.08962 for details)

The systems are stable for \(\epsilon_1 \geq 0, \epsilon_2 \geq 0\)
This parameter region is same as that from the causality constraint

Love numbers are not zero (this affects the form of GW in inspiral phase)
Quasi normal modes

Since the master eqs for $\epsilon_1, \epsilon_2$ are Schrödinger form, we can use existing codes to calculate QNMs numerically

$$\text{Re}[\omega] = \text{Re}[\omega_{\text{GR}}](1 + \epsilon_i \delta_{\text{Re}})$$
$$\text{Im}[\omega] = \text{Im}[\omega_{\text{GR}}](1 + \epsilon_i \delta_{\text{Im}})$$

$\epsilon_1$ case: $\delta_{\ell} = (\delta_{\text{Re}}, \delta_{\text{Im}})|_{\ell}$

$\delta_{\ell=2}^{\text{even}} = (0.45, -2.75)$, $\delta_{\ell=2}^{\text{odd}} = (0.22, -0.64)$

$\delta_{\ell=3}^{\text{even}} = (1.07, -6.42)$, $\delta_{\ell=3}^{\text{odd}} = -(0.0099, 0.44)$

$\delta_{\ell=4}^{\text{even}} = (1.76, -11.43)$, $\delta_{\ell=4}^{\text{odd}} = -(0.048, 0.19)$
Slowly rotating BHs

\[ g_{\mu\nu} = g_{\mu\nu}^{GR} + \epsilon_i \delta g_{\mu\nu} \]

\[ g_{\mu\nu}^{GR} = g_{\mu\nu}^{Sch} + \chi g_{\mu\nu}^{(1,0)} + \chi^2 g_{\mu\nu}^{(2,0)} + \cdots \]

\[ \chi = a/M \]

can be obtained from Kerr metric

\[ \delta g_{\mu\nu} = g_{\mu\nu}^{(0,1)} + \chi g_{\mu\nu}^{(1,1)} + \chi^2 g_{\mu\nu}^{(2,1)} + \cdots \]

from EOM for EFT
Slowly rotating BHs

We take the metric anzats
\[ ds^2 = -f(r)(1 + h(r, \theta))dt^2 + \frac{1 + m(r, \theta)}{f(r)} dr^2 \]
\[ + r^2 (1 + k(r, \theta)) d\theta^2 \]
\[ + r^2 \sin^2 \theta (1 + p(r, \theta))(d\phi - \Omega(r, \theta) dt)^2 \]
\[ f = 1 - \frac{2M}{r} \]

\( \mathcal{O}(\epsilon_i^0) \): \( h, m, k, p, \Omega \) are determined from Kerr

\( \mathcal{O}(\epsilon_i) \): \( h, m, k, p, \Omega \) are determined from EOM for EFT

(also expand \( h, m, k, p, \Omega \) in power of \( \cos \theta \))
Future works

- $\mathcal{O}((R_{\mu\nu\rho\sigma})^3)$ correction terms
- Understand the reason why the master eqs are still 2nd order differential eqs
- rapidly rotating BHs
  (cf: arXiv:1901.01315, Cano and Ruipérez they calculated 14th spin parameter)
- Similar analysis for scalar tensor theory
- QNM for $\epsilon_3$ (on-going work)
- PN analysis (by Senatore et al)