A new measurement of the Earth’s gravitomagnetic field a century after the formulation of the Lense-Thirring effect

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Summary

- The Lense-Thirring effect and gravitomagnetism
- The LARASE experiment and its goals
- The LASSOS model for the spin
- Thermal effects and their modelling
- Model for the Earth’s gravitational field
- Precise orbit determination
- A new measurement of the Lense-Thirring effect
- Conclusions and future work
The Lense-Thirring effect and gravitomagnetism

Gravitomagnetism is a peculiarity of Einstein's theory of General Relativity

- It is strongly connected to the concepts of inertia (how it originates) and rotation (apparent forces like gravitational forces)
- “Inertia here arises from mass-energy there”, represents a link to Mach’s ideas...
- Gravitomagnetism has no classical (Newtonian) gravitational counterpart, but it has a strong analogy with magnetism

Gravitoelectromagnetic fields \((\vec{E}_G, \vec{B}_G)\) →

\[
\begin{align*}
\vec{V} \cdot \vec{E}_G &= -4\pi G \rho \\
\vec{V} \cdot \vec{B}_G &= 0 \\
\vec{V} \times \vec{E}_G &= -\frac{1}{2c} \frac{\partial}{\partial t} \vec{B}_G \\
\vec{V} \times \vec{B}_G &= \frac{2}{c} \frac{\partial}{\partial t} \vec{E}_G - \frac{8\pi G}{c} \vec{j} \\
\vec{E}_G &= -\vec{V} \Phi - \frac{1}{2c} \frac{\partial}{\partial t} \vec{A} \\
\vec{B}_G &= \vec{V} \times \vec{A} \\
\frac{1}{c} \frac{\partial}{\partial t} \Phi + \frac{1}{2} \vec{V} \cdot \vec{A} &= 0
\end{align*}
\]

\(\rho = \) mass-charge density
\(j = \) mass-current density
The Lense-Thirring effect and gravitomagnetism

Gravito-electromagnetism: linearized theory of General Relativity (GR)

In the Weak-Field and Slow-Motion (WFSM) limit of the theory of GR, Einstein’s equations reduce to a form quite similar to those of electromagnetism. Following this approach we have:

- gravitoelectric field produced by masses, analogous to the electric field produced by charges
- gravitomagnetic field produced by mass currents, analogous to the magnetic field produced by electric currents.

\[ G_{\alpha\beta} = 8\pi \frac{G}{c^4} T_{\alpha\beta} \]

\[ \bar{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2} \eta_{\alpha\beta} h \]

\[ \eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \]

\[ \Phi = -\frac{GM_{\odot}}{R_{\odot}} \]

Gravitoelectric potential

Gravitomagnetic potential

\[ A^l = \frac{G}{c} \frac{J^k x^l}{r^3} \varepsilon_{nk} \]
The Lense-Thirring effect and gravitomagnetism

Formal analogy with electrodynamics: linearized theory of General Relativity (WFSM limit)

Classical Electrodynamics:
\[ A(r) \]
\[ B(r) \]
\[ J \]
\[ \mu \]
\[ \rho \]
\[ \mathbf{J} \]
\[ \mathbf{B} \]

Classical Geometrodynamics (WFSM):
\[ G = c = 1 \]
\[ B_G(r) \]

Solution:
\[ \Delta \mathbf{A} = -4\pi \hat{\mathbf{J}}_n \]
\[ \mathbf{A}(\mathbf{r}) = \int \frac{\mathbf{J}_n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' \]
\[ \Delta \mathbf{h} = 16\pi \hat{\mathbf{J}}_m \]
\[ \mathbf{h}(\mathbf{r}) = -4\int \frac{\mathbf{J}_m(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' \]
\[ \mathbf{J} = \int \mathbf{r} \wedge \mathbf{J}_m(\mathbf{r}') d^3r' \]
\[ \tilde{\mathbf{h}}(\mathbf{r}) = -2\hat{\mathbf{J}}_n \frac{\mathbf{r} \wedge \mathbf{r}}{r^3} \]
\[ \tilde{\mathbf{B}}_g = \tilde{\mathbf{h}} \frac{3(\mathbf{r} \wedge \hat{\mathbf{r}}) - \hat{\mathbf{J}}}{r^3} \]
\[ \mathbf{F} = m\mathbf{\ddot{r}} = q(\mathbf{E} + \mathbf{\hat{r}} \wedge \mathbf{B}) \]

This phenomenon is known as dragging of gyroscopes or dragging of inertial frames.

Therefore, mass currents (as the rotating Earth) drag gyroscopes and change the orientation of their axes.
The Lense-Thirring effect and gravitomagnetism

The so-called Lense-Thirring effect (1918) is a consequence of the Gravitomagnetic field of the Earth produced by its rotation, i.e. by its Angular Momentum:

\[
\begin{align*}
\frac{d\Omega}{dt}_{\text{sec}} &= \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1 - e^2)^{3/2}} \\
\frac{d\omega}{dt}_{\text{sec}} &= -\frac{6G}{c^2 a^3} \frac{J_\oplus}{(1 - e^2)^{3/2}} \cos i
\end{align*}
\]

These are the results of the frame–dragging effect or Lense–Thirring effect:

moving masses (i.e., mass–currents) are rotationally dragged by the angular momentum of the primary body (mass–current)

Lense-Thirring, Phys. Z, 19, 1918
The LARASE experiment and its goals

The LAranged Satellites Experiment (LARASE) goals:

• The main goal is to provide accurate measurements for the gravitational interaction in the weak-field and slow-motion limit of General Relativity by means of a very precise laser tracking of geodetic satellites orbiting around the Earth (the two LAGEOS and LARES).

• Beside the quality of the tracking observations, guaranteed by the powerful Satellite Laser Ranging (SLR) technique of the International Laser Ranging Service (ILRS), also the quality of the dynamical models implemented in the Precise Orbit Determination (POD) software plays a fundamental role in order to obtain precise and accurate measurements.

• The models have to account for the perturbations due to both gravitational and non-gravitational forces in such a way to reduce as much as possible the difference between the observed range, from the tracking, and the computed one, from the models.

• In particular, LARASE aims to improve the dynamical models of the current best laser-ranged satellites in order to perform a precise and accurate orbit determination, able to benefit also space geodesy and geophysics.
The LARASE experiment and its goals

**LAGEOS, LAGEOS II and LARES**

**orbit, size, mass and materials**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LARES</th>
<th>LAGEOS</th>
<th>LAGEOS II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a [km]</td>
<td>7 820</td>
<td>12 270</td>
<td>12 163</td>
</tr>
<tr>
<td>e [deg]</td>
<td>0.001</td>
<td>0.004</td>
<td>0.014</td>
</tr>
<tr>
<td>I [deg]</td>
<td>69.5</td>
<td>109.8</td>
<td>52.7</td>
</tr>
<tr>
<td>R [cm]</td>
<td>18.2</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>M [kg]</td>
<td>386.8</td>
<td>406.9</td>
<td>405.4</td>
</tr>
<tr>
<td>A/M [m²/kg]</td>
<td>2.69 · 10⁻⁴</td>
<td>6.94 · 10⁻⁴</td>
<td>6.97 · 10⁻⁴</td>
</tr>
</tbody>
</table>

\[
\frac{A}{M}_{\text{Lares}} \cong \frac{1}{2.6} \frac{A}{M}_{\text{Lageos}}
\]

**Materials**

- **LARES**
  - Material: Tungsten
  - Bin: 30 s

- **LAGEOS**
  - Material: Al/Brass/Be/Cu
  - Bin: 120 s
The LARASE experiment and its goals

The LARASE activities:

1. Review of the literature, technical notes and all the documentation (NASA, ALENIA, ASI) related with the structure of the satellites and their physical characteristics
2. A reconstruction of the internal and external structure of the satellites with finite elements techniques
3. New spin model for the two LAGEOS satellites and LARES accounting of their complex interaction with the Earth's magnetic field: LASSOS (LAraSE Satellites Spin Models Solutions)
4. New models for the thermal thrust perturbations, also with a Finite Element Model (FEM)
5. Impact of the neutral drag on the two LAGEOS satellites and on LARES
6. Precise Orbit Determination for the two LAGEOS satellites and for LARES
7. Solid and Ocean tides on the two LAGEOS satellites and on LARES
8. Gravitational perturbations with estimate of the spherical harmonics (SH) of low degree
9. Fundamental Physics measurements
The LARASE experiment and its goals

Some results: moments of inertia and internal structure

Table 1. Principal moments of inertia of LAGEOS, LAGEOS II and LARES in their flight arrangement.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Moments of inertia (kg m²)</th>
<th>$I_{zz}$</th>
<th>$I_{xx}$</th>
<th>$I_{yy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAGEOS</td>
<td></td>
<td>11.42 ± 0.03</td>
<td>10.96 ± 0.03</td>
<td>10.96 ± 0.03</td>
</tr>
<tr>
<td>LAGEOS II</td>
<td></td>
<td>11.45 ± 0.03</td>
<td>11.00 ± 0.03</td>
<td>11.00 ± 0.03</td>
</tr>
<tr>
<td>LARES</td>
<td></td>
<td>4.77 ± 0.03</td>
<td>4.77 ± 0.03</td>
<td>4.77 ± 0.03</td>
</tr>
</tbody>
</table>

- The core is made of BRASS
- The stud is made of BERYLLIUM and COPPER
The rotational dynamics of a satellite represents a very important issue that deeply impacts the goodness of the orbit modelling. Indeed, the modelling of several disturbing effects (like the thermal thrust ones) depends on the knowledge of the spin period and orientation in the inertial space:

1. Yarkovsky–Schach effect
2. Earth–Yarkovsky (Rubincam) effect
3. Asymmetric reflectivity from the satellite surface

Their modelling will greatly improve the POD of the two LAGEOS satellites avoiding the current (and significant) use of empirical accelerations during the data reduction.
The LASSOS model for the spin

Past Spin Models

The best spin models developed in the past are:

1. Bertotti and less (JGR 96 B2, 1991)
2. Habib et al. (PRD 50, 1994)
4. Andrés, 1997 (PhD Thesis) and LOSSAM

All of these studies, with the exception of Habib et al., attack and solve the problem of the evolution of the rotation of a satellite in a terrestrial inertial reference system, in the so-called rapid spin approximation and they introduce equations for the external torques that are averaged over time; their fit to the spin observations was good, especially in the case of the LOSSAM model for the LAGEOS II satellite. Habib et al. use a body-fixed reference system and non-averaged torques; their model does not fit so well the observations.
The LASSOS model for the spin

**LARASE Spin Model LASSOS (LArase Satellites Spin mOdel Solutions)**

We have deeply reviewed previous spin models, in particular we:

- first built our own spin model in the rapid spin approximation
- adopted non-averaged torques in the equations to describe the slow spin approximation: we solved the problem of a metallic sphere rotating in an alternate magnetic field
- introduced in the equations all known possible torques (like in LOSSAM model)
- solved the equations in a body-fixed reference system in order to better describe the misalignment between the symmetry axis and the spin
- included in the equations the terms due to the transversal asymmetry
- carefully studied the satellites moments of inertia
The LARASE experiment and its goals

LARASE Spin Model: results for LAGEOS II

LARase Satellites Spin mOdel Solutions (LASSOS)

Blue = LARASE model for the rapid-spin
Red = LARASE general model

Spin Orientation: $\alpha, \delta$
The LARASE experiment and its goals

LARASE Spin Model: results for LAGEOS II

LARase Satellites Spin mOdel Solutions (LASSOS)

Rotational Period: P

Blue = LARASE model for the rapid-spin
Red = LARASE general model
Thermal effects and their modelling

An intricate role, among the complex non-gravitational perturbations, is played by the subtle thermal thrust effects that arise from the radiation emitted from the satellite surface as consequence of the non uniform distribution of its temperature.

In the literature of the older LAGEOS satellite this problem was attacked since the early 80s’ of the past century to explain the (apparently) anomalous behavior of the along-track acceleration of the satellite, characterized by a complex pattern:

Rubincam, Afonso, Ries, Scharroo, Farinella, Metris, Vokrouhlicky, Slabinsky, Lucchesi, Andres, ...

represents a non exhaustive list of the researchers that have successfully worked on this very important issue.

Figure 2. LAGEOS 1 anomalous acceleration: observed data points (squares) are based on 15 day fits to laser data by the Center for Space Research, University of Texas at Austin. The vertical bars mark eclipse seasons. N at top of bar denotes season when satellite travels northward through earth shadow; S denotes season with southward travel.
Thermal effects and their modelling

The dynamical problem to solve is quite complex and should account of the following main aspects:

- **A deep physical characterization of the satellite**
  - emission and absorption coefficients, thermal conductivity, heat capacity, thermal inertia, ...
- **Rotational dynamics of the satellite**
  - Spin orientation and rate
- **Radiation sources**
  - Sun and Earth
We developed a simplified thermal model of the satellite based on
- the energy balance equation on its surface
- a linear approach for the distribution of the temperature with respect to its equilibrium (mean) temperature

A general thermal model based on
- a satellite (metallic structure) in thermal equilibrium
- the CCRs rings are at the same temperature of the satellite
- for each CCR the thermal exchange with the satellite is computed

\[ \frac{dQ_i}{dt} \cong \sum_j (P_j \cdot \varepsilon_j \sigma A_{ext,j} T_i^4) + \sum_k R_{i,k} (T_k^4 - T_i^4) + \sum_k C_{i,k} (T_k - T_i) + \cdots = H_i \frac{\partial T_i}{\partial t} \]
The main perturbations to be taken into account are:

- **The solar Yarkovsky-Schach effect**
  - an anisotropic emission of thermal radiation that arises from the temperature gradients across the surface produced by the solar heating and the thermal inertia of the various parts (mainly from the CCRs)
  - it produces long-term effects when the thermal radiation is modulated by the eclipses

- **The Earth Yarkovsky thermal (or Rubincam) effect**
  - the temperature gradients responsible of the anisotropic emission of thermal radiation are produced by the Earth’s infrared radiation
  - the bulk of the effect is due to the CCRs and their thermal inertia

- **The asymmetric reflectivity effect**

In the following only the Yarkovsky-Schach effect will be considered
Thermal effects and their modelling

In case of a simplified thermal model we can skip the details of a complete characterization of the satellite thermal behavior. What really matters are:

- The satellite mean temperature
  - $T_0$
- The temperature difference between the CCRs of the hemisphere facing the Sun with respect to those in the dark side
  - $\Delta T$
- The CCRs thermal inertia
  - $\tau$

- In the following the results for the LAGEOS II satellite are shown
- The LASSOS (LArase Satellites Spin mOdel Solutions) general spin model has been used
Thermal effects and their modelling

Analysis performed for the Yarkovsky-Schach effect:

• We run our routine over a 20 years time span from MJD 48932, i.e. Nov. 6th 1992, and we computed the effects on the orbit elements of LAGEOS II

• We compared the results with the residuals in the satellite orbit elements that we obtained from a POD with GEODYN II:
  o Background gravity model: EIGEN-GRACE02S
  o Arc length of 7 days
  o No empirical accelerations
  o Thermal effects (Yarkovsky Schach and Rubincam) not modelled
  o General relativity modelled with the exception of the Lense-Thirring effect
Orbit perturbation and comparison with the residuals: semi-major axis

\[ \frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} [T + e(T \cos f + R \sin f)] \]
Thermal effects and their modelling

Orbit perturbation and comparison with the residuals: eccentricity

\[
\frac{de}{dt} = \frac{\sqrt{1 - e^2}}{na} [R \sin f + T (\cos f + \cos u)]
\]
Orbit perturbation and comparison with the residuals: argument of pericenter

\[
\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{n_{ea}} \left[ -R \cos f + T \left( \sin f + \frac{1}{\sqrt{1-e^2}} \sin u \right) \right] - \frac{W}{n a^2 \sqrt{1-e^2}} \tan i \frac{1}{r} \sin(\omega + f)
\]
Thermal effects and their modelling

Preliminary comparison between the simplified and the general thermal model

Accelerations in Gauss co-moving frame
Thermal effects and their modelling

Preliminary comparison between the simplified and the general thermal model
Thermal effects and their modelling

Preliminary comparison between the simplified and the general thermal model

General model with all thermal effects
Thermal effects and their modelling

Preliminary comparison between the simplified and the general thermal model

![Graph showing thermal effects and their modelling]
Thermal effects and their modelling

Preliminary comparison between the simplified and the general thermal model
Model for the Earth’s gravitational field

The correct knowledge of the Earth’s gravitational field impacts significantly on the Lense-Thirring effect measurement:

\[
\langle \dot{\Omega}_{LT} \rangle_{\text{sec}} = \frac{2G}{c^2 a^3 (1 - e^2)^{3/2}} \frac{J_{\oplus}}{1} 
\]

\[
U = -\frac{GM_{\oplus}}{r}\sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \left(\frac{R_{\oplus}}{r}\right)^{\ell} P_{\ell m}(\sin \varphi) \left(C_{\ell m} \cos m \lambda + S_{\ell m} \sin m \lambda \right),
\]

\[
\langle \dot{\Omega}_{\text{class}} \rangle_{\text{sec}} = - \frac{3}{2} n \left(\frac{R_{\oplus}}{a}\right)^2 \frac{\cos i}{(1 - e^2)^2} \left\{-\sqrt{5} \bar{C}_{20}\right\} + \ldots
\]

with important (possible) systematic effects...

\[\ell = \text{even and } m = 0\]
The magnitude of the effect to be measured

\[ \langle \dot{\Omega}_{LT} \rangle_{sec} = \frac{2G}{c^2 a^3 (1 - e^2)^{3/2}} \]

\begin{align*}
G & \approx 6.670 \times 10^{-8} \, \text{cm}^3 \, \text{s}^{-2} \, \text{g}^{-1} \\
J_\oplus & \approx 5.861 \times 10^{40} \, \text{cm}^2 \, \text{g}^{-1} \, \text{s}^{-1} \\
c & \approx 2.9979250 \times 10^{10} \, \text{cm/s}
\end{align*}

30 mas \approx 1.8 \text{ m}

The effect on the orbit is quite small

Total precession:

\[ \dot{\Omega}_{\text{Lageos}}^{\text{Obser}} \approx +126^\circ / \text{yr} \quad \dot{\Omega}_{\text{LageosII}}^{\text{Obser}} \approx -231^\circ / \text{yr} \]
Model for the Earth’s gravitational field

By solving a linear system of three equations in three unknowns, we can solve for the relativistic precession while reducing the impact in the measurement of the non perfect knowledge of the Earth’s gravitational field:

\[
\begin{align*}
\dot{\Omega}_2^{L1} \delta \tilde{C}_{2,0} + \dot{\Omega}_4^{L1} \delta \tilde{C}_{4,0} + \dot{\Omega}_6^{L1} \delta \tilde{C}_{6,0} + \dot{\Omega}_{LT}^{L1} \mu + \cdots &= \delta \dot{\Omega}_{res}^{L1} \\
\dot{\Omega}_2^{L2} \delta \tilde{C}_{2,0} + \dot{\Omega}_4^{L2} \delta \tilde{C}_{4,0} + \dot{\Omega}_6^{L2} \delta \tilde{C}_{6,0} + \dot{\Omega}_{LT}^{L2} \mu + \cdots &= \delta \dot{\Omega}_{res}^{L2} \\
\dot{\Omega}_2^{LR} \delta \tilde{C}_{2,0} + \dot{\Omega}_4^{LR} \delta \tilde{C}_{4,0} + \dot{\Omega}_6^{LR} \delta \tilde{C}_{6,0} + \dot{\Omega}_{LT}^{LR} \mu + \cdots &= \delta \dot{\Omega}_{res}^{LR}
\end{align*}
\]

\[\dot{\Omega}^{comb} = \delta \dot{\Omega}_{res}^{L1} + k_1 \delta \dot{\Omega}_{res}^{L2} + k_2 \delta \dot{\Omega}_{res}^{LR}\]

- LT effect observable
- \(k_1\) and \(k_2\) are such that to cancel the unmodelled effects/errors of two even zonal harmonics (order \(m=0\)) of the Earth’s gravitational field

\[\langle \dot{\Omega}_{\text{class}} \rangle_{sec} = -\frac{3}{2} n \left( \frac{R_\oplus}{a} \right)^2 \frac{\cos i}{(1-e^2)^2} \left\{ -\sqrt{5} \delta \tilde{C}_{20} \right\} + \cdots\]
Model for the Earth’s gravitational field

In our analysis we taken into account the time dependency of the main even zonal harmonics on the basis of GRACE monthly solutions and not simply the constant values for these harmonics provided by their static solutions.

We fitted the first 15 even zonal harmonics from GRACE data with a linear trend, and we modelled them in our code as:

\[ \tilde{C}_{\ell,0}(t) = \tilde{C}_{\ell,0}(t_0) + \dot{\tilde{C}}_{\ell,0}(t - t_0) \]
Model for the Earth’s gravitational field

We estimate the even zonal harmonics of low degree with the LT effect: comparison with GRACE results
Model for the Earth’s gravitational field

We estimate the even zonal harmonics of low degree with the LT effect: comparison with GRACE results
Model for the Earth’s gravitational field

In our analysis we considered several solutions for the gravitational field of the Earth’s from GRACE and GOCE missions:

1. EIGEN-GRACE02S (2004)
2. GGM05S (2014)  (official field of the ILRS)
3. ITU_GRACE16 (2016)
4. Tonji-Grace02s (2017)
5. Tonji-Grace02k (2018)
6. GOSG01S (2018)

This allows us to better estimate and constrain systematics errors among the different solutions

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Rate [yr(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>(-4.8416528046720 \times 10^{-4})</td>
<td>(\simeq 0.0)</td>
</tr>
<tr>
<td>4.0</td>
<td>(+5.4021417522157 \times 10^{-7})</td>
<td>(-2.6790978790439 \times 10^{-11})</td>
</tr>
<tr>
<td>6.0</td>
<td>(-1.4992584301767 \times 10^{-7})</td>
<td>(-6.4611528233550 \times 10^{-12})</td>
</tr>
<tr>
<td>8.0</td>
<td>(+4.9478882967800 \times 10^{-8})</td>
<td>(\simeq 0.0)</td>
</tr>
<tr>
<td>10.0</td>
<td>(+5.3316662523196 \times 10^{-8})</td>
<td>(+3.8368765925296 \times 10^{-12})</td>
</tr>
</tbody>
</table>
Precise orbit determination (POD)

Analysis with **GEODYN II** over a time span of about 25.3 years (from October 30, 1992)

<table>
<thead>
<tr>
<th>Component</th>
<th>Models/Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geopotential (static part)</td>
<td>JGM–3; EGM–96; CHAMP; GRACE; GOCE</td>
</tr>
<tr>
<td>Geopotential (tides)</td>
<td>Ray GOT99.2</td>
</tr>
<tr>
<td>Lunisolar + Planetary Perturbations</td>
<td>JPL ephemerides DE–403</td>
</tr>
<tr>
<td>General relativistic corrections</td>
<td>PPN</td>
</tr>
<tr>
<td>Direct solar radiation pressure</td>
<td>Cannonball model</td>
</tr>
<tr>
<td>Albedo radiation pressure</td>
<td>Knocke–Rubincam model</td>
</tr>
<tr>
<td>Earth–Yarkovsky effect</td>
<td>Rubincam 1987 – 1990 model</td>
</tr>
<tr>
<td>Spin–axis evolution</td>
<td>Farinella et al., 1996 model, LARASE (2018) model</td>
</tr>
<tr>
<td>Stations position</td>
<td>ITRF2000; ITRF 2008; <strong>ITRF2014</strong></td>
</tr>
<tr>
<td>Ocean loading</td>
<td>Scherneck model (with GOT99.2 tides)</td>
</tr>
<tr>
<td>Polar motion</td>
<td>IERS (estimated)</td>
</tr>
<tr>
<td>Earth rotation</td>
<td>VLBI + GPS</td>
</tr>
</tbody>
</table>
Precise orbit determination (POD)

Range residuals of the three satellites (MJD=48925 - MJD=58165)

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Mean</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAGEOS</td>
<td>-0.60</td>
<td>5.9</td>
</tr>
<tr>
<td>LAGEOS II</td>
<td>-0.75</td>
<td>3.5</td>
</tr>
<tr>
<td>LARES</td>
<td>-0.02</td>
<td>4.1</td>
</tr>
</tbody>
</table>

Analysis 0001 with empirical accelerations
Analysis 0002 with no empirical accelerations

POD on a 25.3 yr timespan
Precise orbit determination (POD)

RMS of the three satellites (MJD=48925 - MJD=58165)

<table>
<thead>
<tr>
<th></th>
<th>[cm]</th>
<th>Mean</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAGEOS</td>
<td>2.3</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>LAGEOS II</td>
<td>1.5</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>LARES</td>
<td>3.3</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

Analysis 0001 with empirical accelerations
Analysis 0002 with no empirical accelerations

POD on a 25.3 yr timespan
A new measurement of the Lense-Thirring effect

New aspects with respect to previous measurement of the LT effect:

• We considered several models for the background gravitational field of the Earth
  ▪ This allows to highlight possible systematics among the different models

• For the first 10 even zonal harmonics we considered their explicit time dependency following the monthly solutions from GRACE measurements
  ▪ This has reduced the systematic error of the background gravitational field

• Together with the relativistic LT precession we estimated also some of the low-degree even zonal harmonics (\(\ell=\text{even} \) and \(m=0\)) of the background gravitational field
  ▪ This allows to estimate the direct correlation between the relativistic LT precession with the coefficients of the gravitational field
A new measurement of the Lense-Thirring effect

New aspects with respect to previous measurement of the LT effect:

• The relativistic LT precession has been measured both in the i) residuals of the rates of the combined nodes and in their ii) integration
  ▪ This is the first time that the measurement has been performed on the rate of the combined observables: case i)

• The measurement has been obtained both via linear fits and non-linear fits
  ▪ This is also the first time that a reliable measurement of the LT precession has been obtained by means of a simple linear fit
A new measurement of the Lense-Thirring effect

A new preliminary measurement of the LT effect

- We run GEODYN II over a time span of about 6.5 years (2359 days) from MJD 56023, i.e. April 6\textsuperscript{th} 2012, and we computed the effects on the orbit elements of LAGEOS, LAGESOS II and LARES:
  - Background gravity model: GGM05S + other fields from GRACE and GOCE
  - Arc length of 7 days
  - No empirical accelerations
  - Thermal effects (Yarkovsky Schach and Rubincam) not modelled
  - General relativity modelled with the exception of the Lense-Thirring effect

\[ \dot{\Omega}_{\text{comb}} = \dot{\Omega}_{\text{res}}^{L1} + k_1 \delta \dot{\Omega}_{\text{res}}^{L2} + k_2 \delta \dot{\Omega}_{\text{res}}^{LR} \]

- LT effect observable
- \( k_1 \) and \( k_2 \) are such that to cancel the unmodelled effects/errors of two even zonal harmonics (order \( m=0 \)) of the Earth’s gravitational field: \( C_{2,0} \) and \( C_{4,0} \)

\[ \dot{\Omega}_{\text{class}} = -\frac{3}{2} \frac{n}{\alpha} \left( \frac{R_\oplus}{a} \right)^2 \frac{\cos i}{(1-e^2)^2} J_2 + \cdots \]

30 mas \( \cong \) 1.8 m

**Table 1**

<table>
<thead>
<tr>
<th>Orbital element</th>
<th>LAGEOS</th>
<th>LAGESOS II</th>
<th>LARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega^{LT} )</td>
<td>30.67</td>
<td>31.50</td>
<td>118.48</td>
</tr>
</tbody>
</table>
A new measurement of the Lense-Thirring effect

Residuals in the right ascension of the ascending node rate of the satellites

These residuals are due to unmodeled:

- periodic effects
  - thermal thrust effects
  - asymmetric reflectivity
  - tides + gravitational field
- secular effect related with the Lense-Thirring precession

\[ \dot{\Omega} \]

<table>
<thead>
<tr>
<th>Thermal effects</th>
<th>LAGEOS</th>
<th>LAGEOS II</th>
<th>LARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{\Omega} )</td>
<td>1052</td>
<td>570</td>
<td>211</td>
</tr>
<tr>
<td>2( \dot{\Omega} )</td>
<td>526</td>
<td>285</td>
<td>105</td>
</tr>
<tr>
<td>( \dot{\lambda} )</td>
<td>365</td>
<td>365</td>
<td>365</td>
</tr>
<tr>
<td>2( \dot{\lambda} )</td>
<td>183</td>
<td>183</td>
<td>183</td>
</tr>
<tr>
<td>2(( \dot{\Omega} ) - ( \dot{\lambda} ))</td>
<td>280</td>
<td>111</td>
<td>67</td>
</tr>
<tr>
<td>( \dot{\Omega} + \dot{\lambda} )</td>
<td>271</td>
<td>953</td>
<td>497</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solid tides</th>
<th>LAGEOS</th>
<th>LAGEOS II</th>
<th>LARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>165.565</td>
<td>911</td>
<td>622</td>
<td>217</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ocean tides</th>
<th>LAGEOS</th>
<th>LAGEOS II</th>
<th>LARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>163.555</td>
<td>221</td>
<td>138</td>
<td>98</td>
</tr>
</tbody>
</table>

*Some of these spectral lines are also common to solid and ocean tides.*
A new measurement of the Lense-Thirring effect

Spectral analysis of the R.A. of the ascending node rate of the satellites and of their combination:

$$\dot{\Omega}^{\text{comb}} = \delta\Omega_{\text{res}}^{L1} + k_1 \delta\Omega_{\text{res}}^{L2} + k_2 \delta\Omega_{\text{res}}^{LR}$$

$$k_1 \approx 0.345$$
$$k_2 \approx 0.073$$
A new measurement of the Lense-Thirring effect

Combined residuals in the right ascension of the ascending node rate of the satellites and the of the combined nodes

\[ \dot{\Omega}^{\text{comb}} = \delta \dot{\Omega}_{\text{res}}^{L1} + k_1 \delta \dot{\Omega}_{\text{res}}^{L2} + k_2 \delta \dot{\Omega}_{\text{res}}^{LR} \]

\[ \dot{\Omega}_{\text{GR}}^{\text{comb}} = 50.17 \text{ mas/yr} \]
A new measurement of the Lense-Thirring effect

Correlations between the estimated quantities:
A new measurement of the Lense-Thirring effect

Combined residuals in the right ascension of the ascending node rate of the satellites

\[ \dot{\Omega}_{comb} = \delta\dot{\Omega}_{res}^L + k_1 \delta\dot{\Omega}_{res}^L + k_2 \delta\dot{\Omega}_{res}^L \]

\[ \dot{\Omega}_{GR}^{comb} = 50.17 \text{ mas/yr} \]

\[ \mu = \frac{\dot{\Omega}_{comb}}{\dot{\Omega}_{GR}} = \begin{cases} 1 & \text{In General Relativity} \\ 0 & \text{In Newtonian physics} \end{cases} \]

From the mean value:

\[ \mu_{Fit} - 1 = -19 \times 10^{-3} \pm 7 \times 10^{-3} \pm \delta \mu \pm \delta \mu_{sys} \]

From the slope:

\[ \mu_{Fit} - 1 = 2 \times 10^{-3} \pm 7 \times 10^{-3} \pm \delta \mu_{sys} \]
A new measurement of the Lense-Thirring effect

A very preliminary estimate of the systematics

\[ \Omega_{\text{comb}} = \delta \Omega_{r\text{es}}^L + k_1 \delta \Omega_{r\text{es}}^L + k_2 \delta \Omega_{r\text{es}}^L \]

\[ \dot{\Omega}_{GR}^{\text{comb}} = 50.17 \text{ mas/yr} \]

<table>
<thead>
<tr>
<th></th>
<th>$\delta \mu$ [%]</th>
<th>$\delta \mu$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perturbations</td>
<td>non.-int res.</td>
<td>int. res.</td>
</tr>
<tr>
<td>Gravitational field</td>
<td>2.20</td>
<td>0.74</td>
</tr>
<tr>
<td>Periodic effects</td>
<td>3.00 (7.00)</td>
<td>0.29 (0.54)</td>
</tr>
<tr>
<td>de Sitter</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>RSS</td>
<td>3.73 (7.34)</td>
<td>0.85 (0.96)</td>
</tr>
</tbody>
</table>

From the mean value:

\[ \mu_{\text{Fit}} - 1 = -19 \times 10^{-3} \pm \delta \mu \pm 70 \times 10^{-3} \]

From the slope:

\[ \mu_{\text{Fit}} - 1 = 2 \times 10^{-3} \pm 7 \times 10^{-3} \pm 10 \times 10^{-3} \]
Conclusions and future work

The activities of LARASE proceeds in terms of:

• development of new reliable models
  ✓ for the (subtle) non-gravitational perturbations (Spin and Thermal Thrust effects)
  ✓ as well as (in part) for the gravitational ones

• precise orbit determination (POD)
  ✓ tracking data, models, stations, reference frames, ...

• precise and accurate measurements of the gravitational interaction in the weak-field and slow-motion limit of General Relativity
  ✓ Lense-Thirring and other effects ...
Conclusions and future work

• in the centennial of the Lense-Thirring effect, we presented a new precise measurement for this relativistic precession on the combined orbits of the LAGEOS, LAGEOS II and LARES satellites: $\sim 0.2\%$

• next goal is to provide a careful evaluation of the systematic errors of the measurement: $\sim 1-2\%$ (work in progress)

• the Lense-Thirring effect represents a weak manifestation of Mach’s Principle and it proves that mass-currents in general relativity contribute to the curvature of space-time
Testing the gravitational interaction in the field of the Earth via satellite laser ranging and the Laser Ranged Satellites Experiment (LARASE)

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R Peron\textsuperscript{1,3}, G Pucacco\textsuperscript{3,4} and M Visco\textsuperscript{1,3}

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LARASE website
http://larase.roma2.infn.it
Many thanks for your kind attention
The LARASE collaboration

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- ISTI/CNR, Pisa: L. Anselmo, C. Pardini
- Dep. Physics, Tor Vergata-Roma: M. Bassan, G. Pucacco
Neutral drag perturbation for LARES
Comparison SATRAP - GEODYN

Decay of the semimajor axis of LARES on a timespan of 5.8 and the solar activity

\[ \frac{da}{dt} \approx -2.44 \text{ mm/d} \]

\[ \langle T \rangle \approx -1.289 \times 10^{-11} \text{ m/s}^2 \]
Comparison SATRAP - GEODYN

Gauss accelerations for LARES obtained by SATRAP

\[
\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} [T + e(T \cos f + R \sin f)] \\
\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na} [R \sin f + T(\cos f + \cos u)]
\]
Comparison SATRAP - GEODYN

GEODYN residuals for the semi-major axis and eccentricity of LARES compared with their predictions for the neutral drag perturbation obtained with SATRAP and the application of Gauss equations.

\[
\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} [T + e(T\cos f + R\sin f)]
\]

\[
\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na} [R\sin f + T(\cos f + \cos u)]
\]

![Graphs showing comparison between SATRAP and GEODYN residuals for semi-major axis and eccentricity](image-url)
Constraints on $1/r^2$
Yukawa-like long range interaction

**Post data reduction analysis:** 13-yr analysis of the LAGEOS II orbit (FIT)

We obtained \( b \approx 3294.6 \text{ mas/yr} \), very close to the prediction of GR.

The discrepancy is just **0.01%**.

From a sensitivity analysis, with constraints on some of the parameters that enter into the least squares fit, we obtained an upper bound of **0.2%**.

\[
\Delta \omega^{\text{FIT}} = a + b \cdot t + c \left( t - t_0 \right)^2 + \sum_{i=1}^{n} D_i \sin \left( \frac{2 \cdot \pi}{P_i} \cdot t + \Phi_i \right)
\]

\[
\Delta \omega_{\text{fit}}^{\text{LII}} \approx 3294.56 \text{ mas/yr}
\]

\[
\varepsilon = 1 - (0.12 \pm 2.10) \cdot 10^{-3} \pm 2.5 \cdot 10^{-2}
\]
Yukawa-like long range interaction

The measurement of the pericenter advance

Why measuring the shift of the argument of pericenter?

- These very weak NLRI are usually described by means of a Yukawa–like potential with strength \( \alpha \) and range \( \lambda \):

\[
V_{yuk} = -\alpha \frac{G_\infty M_1}{r} e^{-r/\lambda}
\]

\[
\alpha = \frac{1}{G_\infty \left( \frac{K_1}{M_1} \cdot \frac{K_2}{M_2} \right)}
\]

\[
\lambda = \frac{\hbar}{\mu c}
\]

- This Yukawa–like parameterization seems general (at the lowest order interaction and non-relativistic limit):

- scalar field with the exchange of a spin–0 light boson;
- tensor field with the exchange of a spin–2 light boson;
- vector field with the exchange of a spin–1 light boson;

\( M_1 \) = Mass of the primary source;
\( M_2 \) = Mass of the secondary source;
\( G_\infty \) = Newtonian gravitational constant;
\( r \) = Distance;
\( \alpha \) = Strength of the interaction;  \( K_1,K_2 \) = Coupling strengths;
\( \lambda \) = Range of the interaction;  \( \mu \) = Mass of the light-boson;
\( \hbar \) = Reduced Planck constant;  \( c \) = Speed of light
Yukawa-like long range interaction

Summary of the constraints obtained

| TABLE XVIII. Summary of the results obtained in the present work; together with the measurement error budget, the constraints on fundamental physics are listed and compared with the literature. |
| Parameter | Values and uncertainties (this study) | Uncertainties (literature) | Remarks |
| $\varepsilon_{\omega} - 1$ | $-1.2 \times 10^{-4} \pm 2.10 \times 10^{-3} \pm 2.54 \times 10^{-2}$ | | Error budget of the perigee precession measurement in the field of the Earth |
| $\frac{[2+\gamma-\beta]}{\beta} - 1$ | $-1.2 \times 10^{-4} \pm 2.10 \times 10^{-3} \pm 2.54 \times 10^{-2}$ | $\pm (1.0 \times 10^{-3}) \pm (2 \times 10^{-2})^a$ | Constraint on the combination of PPN parameters |
| $|\alpha|$ | $\lesssim 0.5 \pm 8.0 \pm 101 \times 10^{-12}$ | $\pm 1 \times 10^{-8b}$ | Constraint on a possible (Yukawa-like) NLR1 |
| $C_{\Phi \text{LAGEOS}}$ | $\lesssim (0.003 \text{ km})^4 \pm (0.036 \text{ km})^4 \pm (0.092 \text{ km})^4$ | $\pm (0.16 \text{ km})^4c \pm (0.087 \text{ km})^4d$ | Constraint on a possible NSGT |
| $\frac{1}{2t_1 + t_2}$ | $\lesssim 3.5 \times 10^{-4} \pm 6.2 \times 10^{-3} \pm 7.49 \times 10^{-2}$ | $3 \times 10^{-3e}$ | Constraint on torsion |

$^a$From the preliminary estimate of the systematic errors of [166] for the perihelion precession of Mercury.
$^c$From [5] and based on a partial estimate for the systematic errors.
$^d$From [7] and based on the analysis of the systematic errors only.
$^e$From [168] with no estimate for the systematic errors.

Lucchesi, Peron, Phy. Rev. D, 89, 2014
Yukawa-like long range interaction

Constraints on a long-range force: Yukawa like interaction

\[ |\alpha| \approx |(0.5 \pm 8) \cdot 10^{-12} \pm 101 \cdot 10^{-12}| \]

The region above each curve is ruled out at the 95.5% confidence level.


Previous limits with LAGEOS’s:

\[ |\alpha| < 10^{-5} \pm 10^{-8} \]
Yukawa-like long range interaction

Constraints on a long-range force: Yukawa like interaction

\[ |\alpha| \approx |(0.5 \pm 8) \cdot 10^{-12} \pm 101 \cdot 10^{-12}| \]


Previous limits with LAGEOS’s:

\[ |\alpha| < 10^{-5} \pm 10^{-8} \]
Yukawa-like long range interaction

Constraints on a long-range force: Yukawa like interaction

$$|\alpha| \cong |(0.5 \pm 8) \cdot 10^{-12} \pm 101 \cdot 10^{-12}|$$

The region above each curve is ruled out at the 95.5% confidence level.


Previous limits with LAGEOS’s:

$$|\alpha| < 10^{-5} \pm 10^{-8}$$