Real-metric spacetime properties of 'rigor mortis' accelerations—Born's 'rigid motion' collinear scenarios—without Minkowski spacetime

Brian Coleman

BC Systems (Erlangen), Velchronos,
Moyard, County Galway H91HFH6, Ireland
** E-mail: bjc.sys@gmail*

An extended accelerating body under 'rigid motion' by definition has unvarying separation between its constituents in all comoving inertial frames. Tantamount to a nonuniform, dynamically changing *repulsive* gravitational field, how constituents' fixed ['proper'] own-accelerations must individually differ was established by *Woodhouse* in 2003 using Minkowski spacetime, by *Franklin* in 2010 using Lorentz transformations, and by the present author in 2013 using curiously undocumented yet simple inter-rocket radar period equations. A second 'pseudo-rigor mortis' *attractive* gravitational field scenario introduced in 2018 is now further considered. In both cases radar trajectories are shown to exhibit unchanging inverse square root of two geodesic curvature on corresponding real-metric spacetime surfaces of ubiquitously zero Gauss curvature.

Keywords: spacetime metric; own-surface; hemix; rigor mortis motion; radar paths; geodesics; Gauss curvature; gravitational fields.

1. Introduction

An important though 'one-off' continuous medium acceleration scenario was introduced in 1909/1910 relativity papers [1, 2] by Einstein's close colleague *Max Born*. Medium constituents—'increments' or 'particles'—undergo fixed but individually differing ['proper'] own-accelerations whereby medium-attached observers in ever changing comoving inertial frames would perceive each other's separation distances as remaining constant, a situation markedly different from the likewise germane *homogeneously accelerating* medium topic which emerged half a century later and subsequently became known as *Bell's string paradox*. What Born termed 'rigid motion' (*'starre Bewegung'*) in relativity is however better redesignated as *'rigor mortis acceleration'* due to conflicting connotations in differential geometry.

Until recently its exposition has been heavily obscured even in contemporary textbooks both by unnecessary mathematics as well as by misleading 'presentism' formulations. Yet it has remained a core topic in the somewhat muddled literature of an accelerating extended medium arguably for an odd reason: the scenario's expedient status as a 'cherry picked' (one might say) one-off showcase *coincidentally* compatible with the 1907 *Minkowski metric*, the latter being a still widely used yet—where an *extended* accelerating medium is concerned—*otherwise overgeneralised* axiom [3, 4, 5, 6, 7, 8)]. 'Idealised' by each medium's increment i having its own 'minuscule rocket' and being 'in the limit' of zero mass—instead of imagining a *comoving gravitational field* approach which, as outlined below, is problematic—Born's scenario in its traditional variant actually involves a mathematically exceedingly simple scaled *'rigor mortis' accelerating increments condition.*

In his widely used 2001/2006 book *Relativity, Special, General and Cosmological* ([9]§3.8, Fig. 3.3) *Wolfgang Rindler*, for example, presented a 'rigid motion' worldlines chart on the basis of the complex variables Minkowski metric[a], yet neither presented nor referred to the underlying accelerating increments condition. In the same context, this emerged implicitly in 2003 as *the rockets' distance separation* $c^2[\frac{1}{a'} - \frac{1}{a}]$, in a generally erudite book by *Nicholas Woodhouse* [11] which however required a six-page derivation likewise invoking a simplistic presentism: *"...the increase in separation is exactly matched by the Lorentz contraction"*. On the other hand, although his paper misrepresented the 'contraction' phenomenon by reiterating (like many other authors) Rindler's incorrect formula[b] for the expansion of *a uniformly accelerating* medium, *Jerrold Franklin* [12] in 2010 formally derived the explicit acceleration relationship as a Lorentz transformations solution *without* using Minkowski's metric (here scaled for a unit acceleration rear rocket):

$$\alpha_i = 1/(1 + l_i). \tag{1}$$

A more 'physical' radar intervals approach published in 2016 utilised the wholly neglected area of relativity radar physics whose straightforwardness has (to put it politely) 'escaped the notice' of most relativists. Applied to scaled radar equation (9) established in [6] ($\left[\alpha_f e^{\grave{\tau}} + 1 - \alpha_f(1 + L)\right] = 1/\left[\alpha_f e^{-\acute{\tau}} + 1 - \alpha_f(1 + L)\right]$), acceleration condition (1) directly yields an *unchanging* forward radar interval $\grave{\tau} - \acute{\tau} = 2ln(1/\alpha_f)$ which—in contrast to ever increasing radar intervals in the *uniform acceleration* case [6]—is characteristic of rigor mortis acceleration. This present paper, which also further elaborates upon a second *'pseudo-rigor mortis'* accelerating medium scenario whereby $\alpha_i = 1/(1 - l_i)$, adopts an even more direct approach based on the hitherto seemingly absent concept of *real-metric spacetime*.

1.1. *The simple nonaccelerating own-surface*

Discarding relativity literature's traditional pseudo-euclidean treatments of spacetime theory, we start with the simplest imaginable example of an extended object's spacetime 'real-metric' own-surface: a rectangular lattice representing a *nonaccelerating* one-spatial dimensional medium such as a long 'floating' spacestation moving at constant velocity v in a 'home' inertial reference frame $\underline{H}$. As shown in Fig. 1's zero acceleration case, horizontal *'increment curves'* trace identical own-times τ of medium increments i identified by their unchanging distance $\lambda = l_i$ from the rear end in each comoving frame $\underline{C}$. Vertical lines—*'medium curves'*—represent the whole medium at regular equal own-time intervals. The chart also shows *radar trajectories* emitted and reflected at regular time intervals between the front and rear ends, which are diagonal lines corresponding to scaled unit limit speed in the unchanging comoving frame $\underline{C}$. Radar periods of course remain identical.

[a] A historical account of Minkowski's contributions to relativity theory appears in [10].
[b] $L\gamma$ in fact constitutes accelerating rockets' *asynchronous spatial dispersal* in comoving frames.[7]

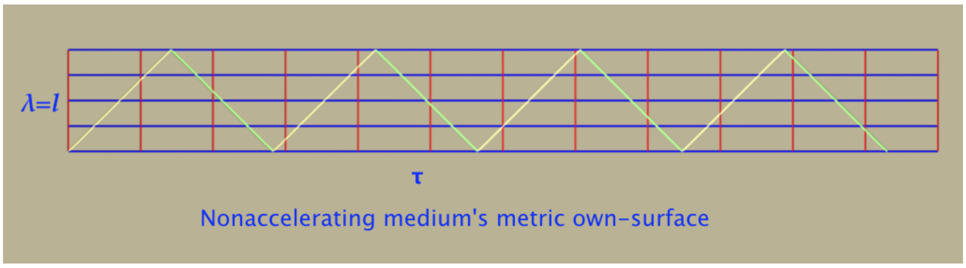

Fig. 1. A nonaccelerating medium's INTRINSIC metric own-surface with radar trajectories

2. A 'hybrid' accelerating medium category

As already described in [6], our scenario involves two rockets launched a distance $L \leq 1$ apart, with home frame observer clocks and rocket clocks synchronised. Scaling times by α_r/c and lengths by α_r/c^2 so that both rear rocket own-thrust α_r and limit cosmic speed c equal *one*, we label the *'reference' rear rocket's own-time* as $\underline{\tau}$. An extended medium between the rockets comprises increments i with launch separation l_i from the unit thrust rear rocket, each assumed to have its own 'minuscule rocket' as well as being 'in the limit' of zero mass. Hence no inter-increment forces or delays are entailed. The familiar equations relating home frame time and distance of a fixed own-thrust α rocket with own-time τ, are (dropping i subscripts)):

$$\alpha t = \sinh \alpha\tau, \quad v = \tanh \alpha\tau, \quad \gamma = \cosh \alpha\tau, \quad \alpha x = \cosh \alpha\tau - 1 \qquad \text{and} \tag{2}$$

$$\text{THE HYPERBOLIC WORLD-LINE EQUATION: } (\alpha x + 1)^2 - (\alpha t)^2 = 1. \tag{3}$$

2.1. *A shortcut to the rigor mortis condition*

On a rigor mortis intrinsic own-surface as in Fig. 2, increment curve path lengths will trace 'own-clock-time' τ progressions of individual increments. These will be crossed by medium curves where each point's curved length λ from the rear reflects the point's 'own-length' from the rear end. From equation (2)ii, the home frame velocity of each such medium curve's increment with a fixed own-thrust α, relates as $v = \tanh(\alpha\tau)$. Therefore, for any set velocity, increment curve lengths tracing own-times $\tau = \tanh^{-1}(v)/\alpha$ are inversely proportional to their respective α values.

By definition, during rigor mortis acceleration, a medium's incrment separations $\Delta\lambda = \Delta l$ remain fixed as v increases. Hence rigor mortis own-surface increment curves must remain equally spread i.e. *they must be parallel*. Fig. 2's 'exploded' extract shows a 'miniscule triangle with medium curve segment $\Delta\lambda$, increment curve segment $\Delta\tau$ and radar trajectory curve segment Δs.

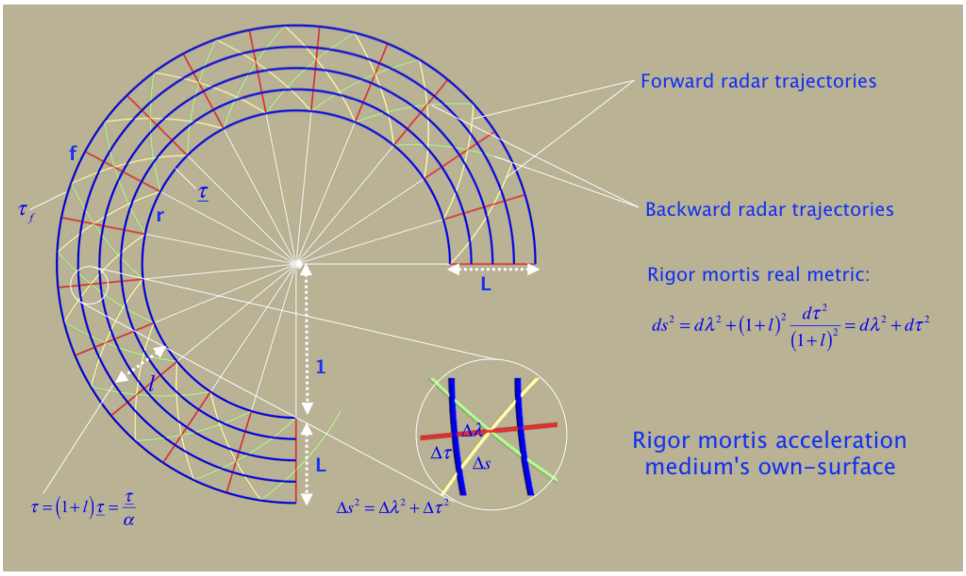

Fig. 2. Rigor Mortis intrinsic own-surface with radar trajectories

Both forward and reverse radar trajectories have unit limit speed $\frac{d\lambda}{d\tau} = 1$ in each momentary inertial frame and accordingly, in the limit, they must cross both medium curves and increment curves *at 45 degrees*. Hence the latter are *at right angles*. By symmetry therefore, *fixed length medium curves must be straight lines*. These conditions *alone* determine that the rigor mortis medium's own-surface must be *a circular strip* as shown. If rear increment own-acceleration $\alpha_r = 1$, then $\Delta\tau.\alpha = \Delta\underline{\tau}.1$ and from the geometry:

$$\frac{1+l}{1} = \frac{\Delta\tau}{\Delta\underline{\tau}} = \frac{1}{\alpha}.$$

This exactly matches rigor mortis condition (1).

3. Characteristics of rigor mortis accelerations

Fig. 2's own-surface is shown for rear rocket own-time $0 \leq \underline{\tau} \leq 3\pi/2$. Increment curves (in blue) appear as *circular arc segments of radius* $(1+l) = 1/\alpha$ which metrically correspond to increments' *differing* elapsed own-time since launch:

$$\tau = \underline{\tau}/\alpha = \underline{\tau}(1+l). \tag{4}$$

These are crossed by straight radial medium curves (red) spread along the surface at rear increment own-time intervals $\Delta\underline{\tau} = 3\pi/32$. Slower accelerating increments nearer the front f need greater clock own-times τ than those nearer rear increment r at the same shared home frame velocity $v = \tanh\underline{\tau} = \tanh(\tau\alpha) = \tanh(\tau/(1+l))$, and so be relatively stationary to one another in each respective comoving frame. The own-surface fixed velocity straight line loci also metrically reflect the perceived unchanging total length of the medium in each co-moving frame $\underline{C}$.

3.1. *Home frame spacetime charts and a radar interval*

Each own-surface point represents *an event* characterised by an increment's medium curve *distance* $\lambda(=l)$ from the rear, as well as by *a specific increment own-time* τ corresponding to the circular arc path length progression. As discussed above:

$$\text{'RIGOR MORTIS' RADAR INTERVAL } \grave{\underline{\tau}} - \acute{\underline{\tau}} = 2\ln(1/\alpha_f). \tag{5}$$

As already detailed in [6], shared velocity events are simultaneous in comoving frames i.e the associated event time disparities ($\Delta\mathfrak{T}$) are zero. A rigor mortis medium's curve thus has the same unchanging total 'own-length' L, as in the zero acceleration scenario.

3.2. *Rigor mortis own-surface's real metric*

The rear rocket's increment curve is a unit radius *planar* circular arc $[\cos\underline{\tau}, \sin\underline{\tau}, 0]$ (expressed as a 3D curve with $z = 0$). For convenience we use both *cylindrical* coordinates[c] $r = 1 + l$, $\theta = \tau = \underline{\tau}/\alpha = \underline{\tau}(1 + l)$, $z = 0$, and *cartesian* coordinates $x = r\cos\theta$, $y = r\sin\theta$, $z = 0$. Hence RIGOR MORTIS PLANAR OWN-SURFACE

$$\mathfrak{RM}_P(\underline{\tau}, l) =_{Cyl} [1 + l, \underline{\tau}(1 + l), 0] =_{Cyl} [1 + l, \tau, 0] =_{xyz} [\cos\tau, \sin\tau, 0]\,(1+l);\ \ 0 \le l \le L. \tag{6}$$

A surface's 'differential metric' relates any two minimally apart event points—in the limit. Therefore $dr = dl$ and $d\theta = d\tau$ and a surface's metric interval is $ds^2 = dr^2 + r^2 d\theta^2$ i.e. $ds^2 = dl^2 + d\tau^2$. Hence, since in this rigor mortis case $\lambda = l$:

$$\text{RIGOR MORTIS OWN-SURFACE METRIC} \qquad ds^2_{\mathfrak{RM}} = d\tau^2 + d\lambda^2. \tag{7}$$

3.3. *'Pseudo-rigor mortis' acceleration*

Adopting the alternate *pseudo rigor mortis condition* $\alpha_i = 1/(1 - l_i)$ results in a (perhaps hitherto unknown ?) case which 'mirrors' the rigor mortis scenario whose own-surface is shown in Fig. 3. Rear and front rocket distance 'dispersions' between events occurring at respective identical home frame velocities, always equal the fixed launch length. As also explained in [6] however, such events do not simultaneously share comoving frames due to the associated time disparity $\Delta\mathfrak{T}$ which, curiously, is *exactly twice* the shared velocity home frame time disparity ΔT.

PSUEDO RIGOR MORTIS PLANAR OWN-SURFACE

$$\mathfrak{PRM}_P(\underline{\tau}, l) =_{Cyl} [1 - l, \underline{\tau}(1 - l), 0] =_{Cyl} [1 - l, \tau, 0] =_{xyz} [\cos\tau, \sin\tau, 0]\,(1-l);\ \ 0 \le l \le L. \tag{8}$$

In this case $\lambda = -l$, $dr = -dl$ and $d\theta = d\tau/(1 - l)$ and the surface's metric interval is $ds^2 = dr^2 + r^2 d\theta^2 = dl^2 + d\tau^2$. Hence, identical to equation (7):

$$\text{PSEUDO RIGOR MORTIS OWN-SURFACE METRIC} \qquad ds^2_{\mathfrak{PRM}} = d\tau^2 + d\lambda^2. \tag{9}$$

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[c] r here denotes a *cylindrical radius coordinate*—not to be confused with the same letter's usage as *a subscript denoting the rear rocket increment.*

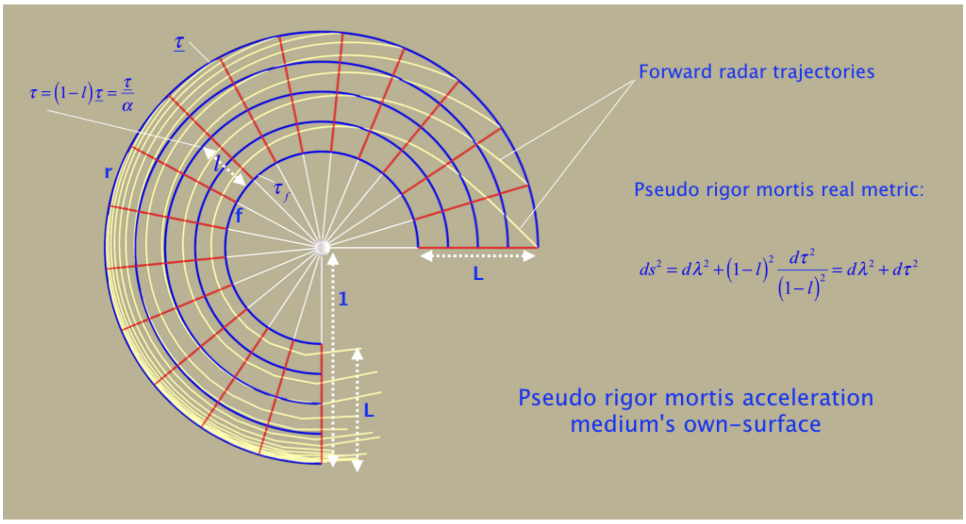

Fig. 3. Pseudo rigor mortis own-surface

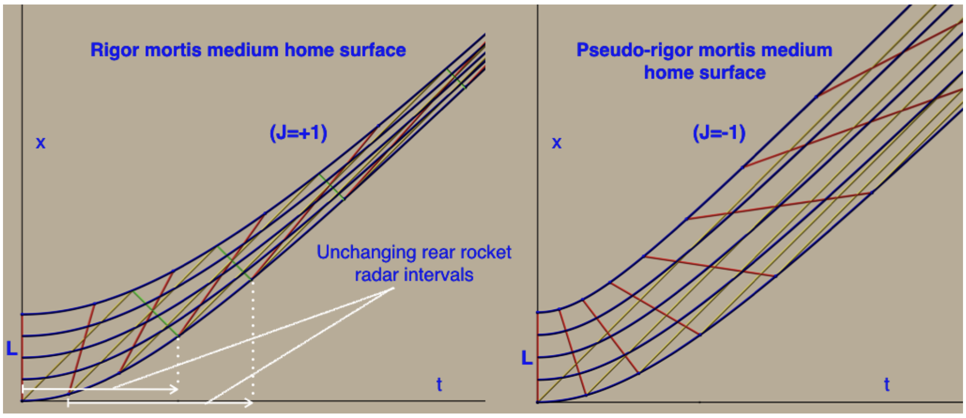

Fig. 4. Home frame world-surfaces' hyperbolic world-lines of rigor mortis/pseudo rigor mortis medium increments with diagonal radar trajectories and fixed-velocity loci.

Fig. 4 shows home frame world-surfaces' world-lines for the rear and front rockets and intermediate medium increments, as well as shared velocity medium lines, for the rigor mortis and pseudo rigor mortis cases. Also shown for the rigor mortis case are outgoing and reflection radar trajectories emitted from the rear rocket at regular rear rocket own-time intervals $\Delta\underline{\tau} = 3\pi/32$. Very notably, radar intervals in terms of the rear rocket's own-time clock periods *remain constant.* Notwithstanding their obvious straightforwardness, such spacetime chart radar paths have been hitherto wholly absent from relativity books and papers—a quite remarkable phenomenon.

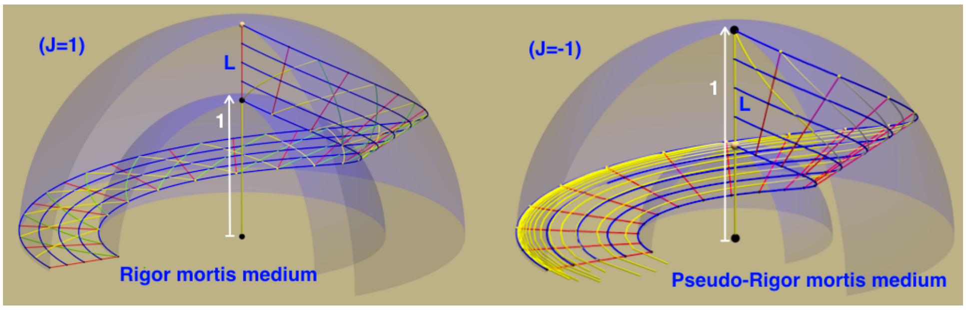

Fig. 5. Rigor mortis and pseudo-rigor mortis hemicoidal own-surfaces.

4. The 'hemicised' rigor mortis own-surface

Fig 2 and 3's intrinsic own-surfaces, which are ruled surfaces hosting radially distributed straight line medium curves, may be *isometrically reshaped* without altering its intrinsic metric properties, so as to also *extrinsically* reflect velocities and also allow increments' own-time τ to be extended indefinitely. If made of paper, the surface would not tear if so deformed. Circular increment curve arcs of Fig. 2 and Fig. 3's *planar* own-surfaces appear twisted in Figures 5 and 6 respective 'hemicised' rigor mortis own-surfaces—without being stretched or compressed—in the form of spherical *hemix* curves described in [7]. HEMIX $\mathfrak{H} =$

$$ {}_{R\theta\phi}[1,\tau,\phi] = {}_{r\theta z}[v,\tau,\frac{1}{\gamma}] = {}_{r\theta z}[\tanh\tau,\tau,1/\cosh\tau] = {}_{xyz}[\tanh\tau\cos\tau,\tanh\tau\sin\tau,1/\cosh\tau]. \tag{10}$$

Each hemix's path length, an increment's traced own-time τ, is proportional to its traversed 'equatorial' longitude. Each hemix increment curve representing an arbitrary fixed acceleration $\alpha < 1$ lies on a hemisphere of radius $1/\alpha = 1 + l$. In both charts fixed-velocity loci sharing comoving inertial frames appear as constant length L straight lines radially distributed at colatitude angle $\phi = \sin^{-1} v$ and spread along the own-surfaces at rear rocket own-time intervals $\Delta\underline{\tau} = 3\pi/32$ for $0 \le \underline{\tau} \le 3\pi/2$. Respective fixed velocity v medium curves, which are straight lines, remain likewise unchanged both in length and as well as shared longitude angle θ. Each such medium curve is rotated so that its inline radius forms a *colatitude angle* ϕ with the vertical axis, where comoving velocity $\sin\phi = v = \tanh\frac{\tau}{1+l} = \tanh\tau_{r}$. *Intrinsically* therefore *it is still the same own-surface*, in spite of being transformed and re-embedded in a three dimensional mathematical space.

RIGOR MORTIS MEDIUM'S OWN-SURFACE $\Upsilon_{RM} = [\tanh\underline{\tau}\cos\underline{\tau},\tanh\underline{\tau}\sin\underline{\tau},1/\cosh\underline{\tau}](1{+}l). \quad (11)$

PSEUDO-RIGOR MORTIS OWN-SURFACE $\Upsilon_{PRM} = [\tanh\underline{\tau}\cos\underline{\tau},\tanh\underline{\tau}\sin\underline{\tau},1/\cosh\underline{\tau}](1{-}l). \quad (12)$

Figure 6 shows emitted and reflected radar trajectories still manifesting unchanging radar emission-return own-time periods in terms of $\Delta\tau$ intervals. Notably, this own-surface tends *asymptotically* to become 'equatorial' as $\underline{\tau} = \tau/(1+l)$ approaches infinity i.e. it may progress endlessly. This metric applies to both the planar own-surface as well as the 'hemicised' own-surface. Since all distances and angles remain unchanged, both surfaces are intrinsically the same i.e. they are isometric.

5. The Minkowski metric' one-off compatibilities

If we replace the *positive* sign in our visualisable *real variables* rigor mortis and pseudo rigor mortis own-surface metric equations (7) and (9)—which are identical—by a *negative* sign, we obtain the *equivalent* yet non-visualisable complex variable

$$\text{Minkowski's pseudo-euclidean metric} \qquad ds^2_{\mathfrak{RM}} = d\tau^2 - d\lambda^2. \tag{13}$$

It turns out that, contrary to widespread consensus, these two rigor mortis scenarios are the sole *extended* accelerating medium cases where Minkowski spacetime may be considered valid.[6]

6. Migrants on a rigor mortis gravity train

Let us imagine a series of 'compartment rockets' constantly accelerating differently in accordance with the rigor mortis criterion and somehow joined together with connecting chambers allowing passengers to migrate between rockets. The passengers of each compartment age relatively in accordance with their respective own-acceleration which depends on its relative distance from the rearmost rocket: $\frac{\tau}{\underline{\tau}} = \frac{1}{\alpha} = \frac{1+l}{1}$. Their respective own-time clocks, synchronised at launch, would increasingly diverge. Notably, the physical basis behind this is their differently experienced gravity-like acceleration. Of course other than sensing such differing accelerations, passengers would not notice any difference in his or her individual biological feeling of 'getting older'.

Although a rigor mortis medium's increments after launch no longer share simultaneities in the home frame due their 'contrived' acceleration differences, they are simultaneously relatively stationary in each comoving frame and medium length remains unchanged. Paradoxically, *in each such comoving frame all increment clock own-times simultaneously differ*. Our 'compound spaceship' could house a colony where every member of the community would continue to remain stationary at unchanging distances relative to everybody else. Passengers could choose to move towards the rear rocket r or the front rocket f, depending on whether they prefer to age either more slowly or more quickly compared with other passengers. The dark green and cyan paths trajectories on the hemicised own-surface represent passengers choosing to age at a different rate relative to other passengers by moving towards the more slowly accelerating front inhabitants ages or towards the more rapidly accelerating rear rocket whose inhabitants age more slowly.

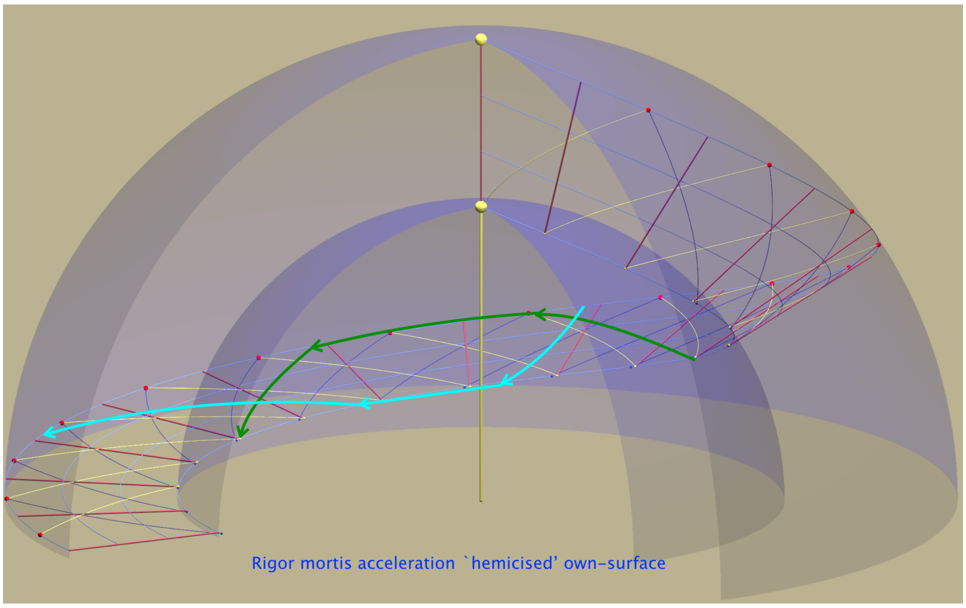

Fig. 6. The 'hemicised' rigor mortis own-surface

If instead of our rigor mortis gravity train compartments each independently producing its own specific thrust, it were somehow possible to arrange a gravitational field whereby gravity pull across the medium occurred in accordance with the equation $g = \frac{1}{1+l}$, *then the same effects would be achieved.* Such a gravity field would need to be not only repulsive[d] and nonuniform, but would also have consistently co-move with the medium itself. The same applies to the pseudo rigor mortis case which however would entail an attractive gravitational field.

7. Rigor mortis own-surface's Gauss and geodesics curvatures

Both rigor mortis own-surfaces have *zero Gauss curvature* throughout by virtue of Figures 2 and 3 planar forms. As easily shown, radar paths have constant nonzero geodesic curvature equal to $1/\sqrt{2}$.

Postscript: A fast track genesis of the geodesic equation

The following demonstrates that geodesic curvature maths, at least where single spatial dimension relativity is concerned, is not at all as complex as is generally suggested in the literature. For a path length λ curve $\boldsymbol{\rho} = \boldsymbol{\Omega}(q^1(\lambda), q^2(\lambda))$, $\boldsymbol{\rho}_\lambda \triangleq d\boldsymbol{\rho}/d\lambda$: $\boldsymbol{\rho}_\lambda \cdot \boldsymbol{\rho}_\lambda = 1$; $\boldsymbol{\rho}_\lambda \cdot \boldsymbol{\rho}_{\lambda\lambda} = 0$. Therefore unit vectors $\boldsymbol{\rho}_\lambda$, $\boldsymbol{N}$ and $\boldsymbol{N} \times \boldsymbol{\rho}_\lambda$ are orthogonal. Hence $\boldsymbol{\rho}_{\lambda\lambda}$ lies in the plane spanned by unit vectors $\boldsymbol{N}$ and $\boldsymbol{N} \times \boldsymbol{\rho}_\lambda$.

[d]Since the front increments' own-acceleration is less than that of rear increments.

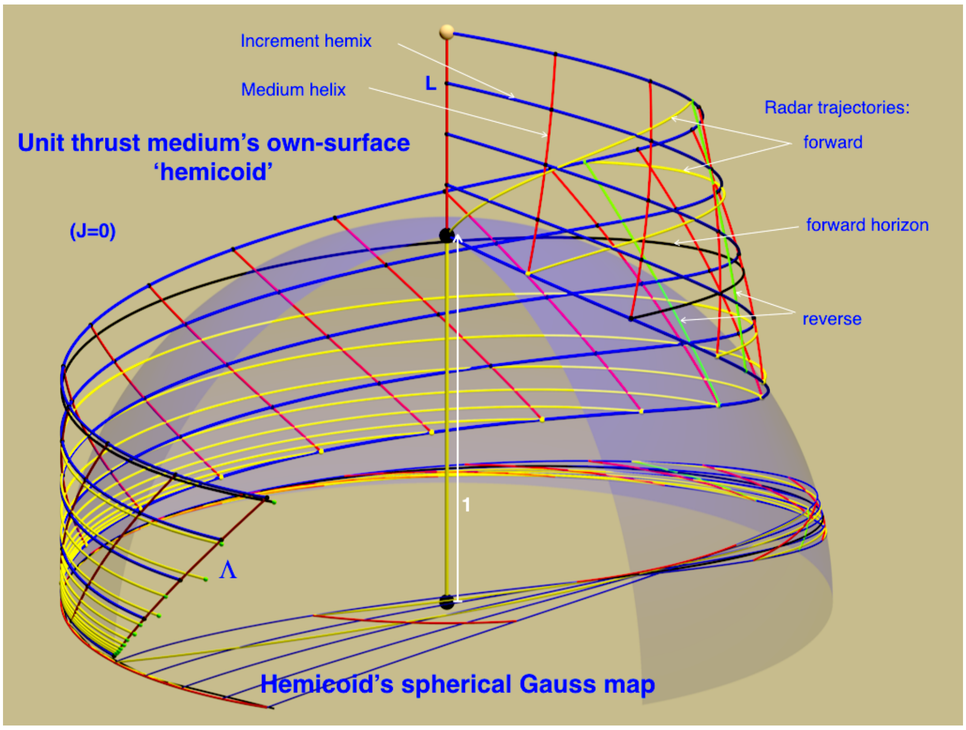

Fig. 7. Increment, medium and radar curves on unit thrust own-surface and its Gauss map

We may define the 2nd fundamental form coefficients B_{ij} and the Christoffel symbols Γ^r_{ij} as the manifold $\boldsymbol{\Omega}$'s second differential components along the normal and tangent vectors respectively i.e. $\boldsymbol{\Omega}_{ij} = B_{ij}\boldsymbol{N} + \Gamma^r_{ij}\boldsymbol{\Omega_r}$. Accordingly, with $G = |\boldsymbol{\Omega}_1 \times \boldsymbol{\Omega}_2|^2$ being the determinant of the first fundamental form coefficients:

$$\boldsymbol{\rho_{\lambda\lambda}} \triangleq \kappa_{\underline{n}}\boldsymbol{N} + \kappa_{\underline{g}}\boldsymbol{N} \times \boldsymbol{\rho_\lambda};\ k_{\underline{g}} = \boldsymbol{\rho_{\lambda\lambda}} \cdot (\boldsymbol{N} \times \boldsymbol{\rho_\lambda});\ \kappa_{\underline{n}} = \boldsymbol{\rho_{\lambda\lambda}} \cdot \boldsymbol{N}.$$

$$\boldsymbol{\rho_{\lambda\lambda}} = q^i_{,\lambda\lambda}\boldsymbol{\Omega_i} + q^i_{,\lambda}q^j_{,\lambda}(B_{ij}\boldsymbol{N} + \Gamma^r_{ij}\boldsymbol{\Omega_r}) = (q^r_{,\lambda\lambda} + q^i_{,\lambda}q^j_{,\lambda}\Gamma^r_{ij})\boldsymbol{\Omega_r} + q^i_{,\lambda}q^j_{,\lambda}B_{ij}\boldsymbol{N}. \quad (14)$$

$$k_{\underline{g}} = \boldsymbol{N}\cdot\boldsymbol{\rho_\lambda}\times\boldsymbol{\rho_{\lambda\lambda}} = q^i_{,\lambda}\left(q^r_{,\lambda\lambda} + q^i_{,\lambda}q^j_{,\lambda}\Gamma^r_{ij}\right)\boldsymbol{N}\cdot(\boldsymbol{\Omega_i}\times\boldsymbol{\Omega_r}) = \left(q^1_{,\lambda}(q^2_{,\lambda\lambda} + q^i_{,\lambda}q^j_{,\lambda}\Gamma^2_{ij}) - q^2_{,\lambda}(q^1_{,\lambda\lambda} + q^i_{,\lambda}q^j_{,\lambda}\Gamma^1_{ij})\right)\sqrt{\underline{G}}. \quad (15)$$

Special Note: The homogeneously accelerating medium case illustrated in Fig. 7 and dealt with in [8] in detail, will be further discussed in [13] (to be published shortly).

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