Status of TEOBResumS

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Time-domain effective-one-body waveforms from coalescing compact binaries with nonprecessing spins, tides and self-spin effects
Complementary posters by: Gamba, Martinetti, Messina, Rettegno, Riemenschneider
What is TEOBResumS?

Effective-one-body (EOB)-based waveform model for spin-aligned coalescing binaries

- BBH sector improved by Numerical-Relativity information
- BNS (tidal) sector compatible with high-accuracy NR simulations. Spin-induced quadrupole moments included
- Public stand-alone C-implementation:
  
  git clone https://alex_nagar@bitbucket.org/eob_ihes/teobresums.git

Challenges:
- physical completeness
- accuracy
- efficiency (AR vs NR)
- $10^7$ templates needed for a single event
2-body problem in GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRinks with time

Waveform

Strong-field information

EOBNR models

Complementary route: IMRPhenom models

PN_glue_NR, EOB_glue_NR hybrids (glued waveforms) to build phenomenological templates [Khan et al., 2015]
Analytical Effective-One-Body approach

Provides a complete description of dynamics and radiation from relativistic binaries

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

(1) Replace two-body dynamics \((m_1, m_2)\) by dynamics of a particle
\((\mu = m_1 m_2/(m_1 + m_2))\) in an effective metric \(g_{\mu\nu}^{\text{eff}}(u)\), with

\[
u = GM/c^2 R, \quad M = m_1 + m_2
\]

(2) Systematically use RESUMMATION of PN expressions (both \(g_{\mu\nu}^{\text{eff}}\)
and \(\mathcal{F}_{RR}\)) based on various physical requirements

(3) Require continuous deformation w.r.t.
\(\nu = \mu/M = m_1 m_2/(m_1 + m_2)^2\) in the interval \(0 \leq \nu \leq \frac{1}{4}\)

\[
A_{\text{5PN}}^{\text{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32} \pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^\ln \ln u]u^5 + \nu[a_6^c(\nu) + a_6^\ln \ln u]u^6
\]

1PN 2PN 3PN 4PN 5PN
TEOBResumS: ringdown from NR

Damour&AN 2014: NR-informed phenomenological description of postmerger phase

1. Factorize the fundamental QNM, fit what remains
2. Global fit all over parameter space
3. Del Pozzo & AN, PRD 95 (2017) 124034

\[ h(\tau) = e^{\sigma_1 \tau - i\phi_0} \bar{h}(\tau) \]

\[ \bar{h}(\tau) \equiv A_\bar{h} e^{i\phi_{\bar{h}}(\tau)}. \]

\[ A_\bar{h}(\tau) = c_1^A \tanh(c_2^A \tau + c_3^A) + c_4^A, \]

\[ \phi_{\bar{h}}(\tau) = -c_1^\phi \ln \left( \frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi} \right) \]

\[ c_2^A = \frac{1}{2} \alpha_{21}, \]

\[ c_4^A = \hat{A}_{22}^{\text{mrg}} - c_1^A \tanh(c_3^A), \]

\[ c_1^A = \hat{A}_{22}^{\text{mrg}} \alpha_1 \frac{\cosh^2(c_3^A)}{c_2^A}, \]

\[ c_1^\phi = \Delta \omega \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi (c_3^\phi + 2c_4^\phi)}, \]

\[ c_2^\phi = \alpha_{21}, \quad \alpha_{21} = \alpha_2 - \alpha_1 \]

\[ \Delta \omega \equiv \omega_1 - M_{\text{BH}} \omega_{22}^{\text{mrg}} \]

- Extended to several modes
- (2,2); (2,1); (3,3); (3,2); (4,4); (4,3); (5,5)
- Usable as stand-alone ringdown template
- Specific fits for peak quantities
- NO mode-mixing (for the moment...)

See poster of G. Riemenschneider
TEOBResumS point-mass potential

\[ A(u; \nu, a^c_6) = P_5^1 [A_{5PN}^{\text{Taylor}} (u; \nu, a^c_6)] \]

From EOB/NR-fitting: \( a^c_6(\nu) = 3097.3 \nu^2 - 1330.6 \nu + 81.3804 \)

Nagar, Riemenschneider, Pratten 2017
Spinning BBHs

Spin-orbit & spin-spin couplings

(i) Spins aligned with $\mathbf{L}$: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with $\mathbf{L}$: attractive (faster) Shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)

$$\chi_{1,2} = \frac{c S_{1,2}}{G m_{1,2}^2}$$

EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

Damour&AN, PRD90 (2014), 024054 (Hamiltonian)

Damour&AN, PRD90 (2014), 044018 (Ringdown)

AN, Damour, Reisswig & Pollney, PRD 93 (2016), 044046

AN, Riemenschneider & Platten, PRD2017

AN, Bernuzzi, Del Pozzo et al., PRD98.104052

$$\hat{H}_{\text{eff}} = \frac{g_{S}}{r^3} \mathbf{L} \cdot \mathbf{S} + \frac{g_{S^*}}{r^3} \mathbf{L} \cdot \mathbf{S}^* + \sqrt{A(1 + \gamma^{ij} p_i p_j + Q_4(p))}$$
TEOBResumS: spin-aligned BBH

- Spin-orbit parameter informed by 30 BBH NR simulations
- BEST faithfulness with all NR available (200 simulations)
- Robust and simple
- Tides and spin-induced moment included (BNS)
- ONLY publicly available stand-alone EOB code

\[ \tilde{F}(M) \equiv 1 - F = 1 - \max_{\mathbf{\ell}_0, \phi_0} \frac{\langle h_{22}^{\text{EOB}}, h_{22}^{\text{NR}} \rangle}{\|h_{22}^{\text{EOB}}\| \|h_{22}^{\text{NR}}\|}, \]

Nagar, Bernuzzi, Del Pozzo et al., PRD98.104052

effective NNNLO spin-orbit “function”
\[ c_3(\tilde{a}_A, \tilde{a}_B, \nu) = p_0 \frac{1 + n_1 \tilde{a}_0 + n_2 \tilde{a}_0^2}{1 + d_1 \tilde{a}_0} + (p_1 \nu + p_2 \nu^2 + p_3 \nu^3) \tilde{a}_0 \sqrt{1 - 4\nu} \]
\[ + p_4 (\tilde{a}_A - \tilde{a}_B) \nu^2, \]  
(17)

\[ \tilde{a}_{1,2} = X_{1,2} \chi_{1,2} \]

\[ X_{1,2} = \frac{m_{1,2}}{M} \]

\[ \tilde{a}_0 = \frac{S + S_*}{M^2} = X_A \chi_A + X_B \chi_B = \tilde{a}_A + \tilde{a}_B \]

ONLY 2 EOBNR models
- TEOBResumS
- SEOBNRv4 (AEI)

Hamiltonian comparison: see poster of F. Martinetti
TABLE IV. Summary of the parameters that characterize GW150914 as found by cpnest and using TEOBResumS as template waveform, compared with the values found by the LVC collaboration [135]. We report the median value as well as the 90% credible interval. For the magnitude of the dimensionless spins $|\chi_A|$ and $|\chi_B|$ we also report the 90% upper bound. Note that we use the notation $\chi_{\text{eff}} \equiv \hat{a}_\text{o}$ for the effective spin, as introduced in Eq. (8).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TEOBResumS</th>
<th>LVC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M/M_\odot$</td>
<td>$73.6^{+5.7}_{-5.2}$</td>
<td>$70.6^{+4.6}_{-4.5}$</td>
</tr>
<tr>
<td>$M_\odot$</td>
<td>$70.0^{+5.0}_{-4.6}$</td>
<td>$67.4^{+4.1}_{-4.0}$</td>
</tr>
<tr>
<td>$</td>
<td>\chi_A</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>\chi_B</td>
<td>$</td>
</tr>
<tr>
<td>$d_L/\text{Mpc}$</td>
<td>$479^{+188}_{-235}$</td>
<td>$410^{+160}_{-180}$</td>
</tr>
</tbody>
</table>

Nagar, Bernuzzi, Del Pozzo et al., PRD98.104052
GW170814

$M_{\text{tot}}/M_\odot = 63.48^{+1.36}_{-1.31}$

$q = 0.85^{+0.09}_{-0.13}$

$\chi_{\text{eff}} = 0.14^{+0.07}_{-0.08}$

$M_\text{f}/M_\odot = 60.18^{+1.24}_{-1.68}$

$a_r = 0.73^{+0.02}_{-0.02}$
Neutron stars: tides & spin

**TEOBResumS today** [AN+, PRD98, 2018,104052]

- tidal effects + nonlinear-in-spin-effects ($S^2, S^3, S^4, \ldots$) [AN+, PRD99, 2019,044007]
- analytically very complete model (almost final)
- $l=3$ GSF-informed + gravitomagnetic tides [Akca+++, PRD, 2019, in press]
- checked with (state-of-the-art but short) NR simulations up to merger
- EFFICIENT due to the post-adiabatic approximation [AN & Rettegno PRD99, 2019 021501]
- no precession (yet!)

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No real need of EOB-surrogate!

### Table

<table>
<thead>
<tr>
<th>$f_0$ [Hz]</th>
<th>$r_0$</th>
<th>$r_{\text{min}}$</th>
<th>$N_r$</th>
<th>$\Delta r$</th>
<th>$\tau_{\text{SPA}}$ [sec]</th>
<th>$\tau_{\text{ODE}}$ [sec]</th>
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<tbody>
<tr>
<td>20</td>
<td>112.81</td>
<td>12</td>
<td>500</td>
<td>0.20</td>
<td>0.03</td>
<td>0.53</td>
</tr>
<tr>
<td>10</td>
<td>179.02</td>
<td>12</td>
<td>830</td>
<td>0.20</td>
<td>0.05</td>
<td>1.1</td>
</tr>
</tbody>
</table>

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see poster of P. Rettegno
TABLE III. Equal-mass BNS configurations considered in this work. From left to right the column reports: the EOS, the gravitational mass of each star, the compactness, the quadrupolar dimensionless Love numbers, the leading-order tidal coupling function, the quadrupolar "tidal deformability" for each object, the corresponding value of the quadrupolar "tidal deformability" for each object, and the gravitational mass of each object.

<table>
<thead>
<tr>
<th>name</th>
<th>EOS</th>
<th>$M_{A,B}[M_\odot]$</th>
<th>$C_{A,B}$</th>
<th>$k_2^{A,B}$</th>
<th>$\kappa_T$</th>
<th>$\Lambda_{2}^{A,B}$</th>
<th>$\chi_{A,B}$</th>
<th>$C_{Q,A,QB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAM:0095</td>
<td>SLy</td>
<td>1.35</td>
<td>0.17</td>
<td>0.093</td>
<td>73.51</td>
<td>392</td>
<td>0.0</td>
<td>5.491</td>
</tr>
<tr>
<td>BAM:0039</td>
<td>H4</td>
<td>1.37</td>
<td>0.149</td>
<td>0.114</td>
<td>191.34</td>
<td>1020.5</td>
<td>0.141</td>
<td>7.396</td>
</tr>
<tr>
<td>BAM:0064</td>
<td>MS1b</td>
<td>1.35</td>
<td>0.142</td>
<td>0.134</td>
<td>289.67</td>
<td>1545</td>
<td>0.0</td>
<td>8.396</td>
</tr>
</tbody>
</table>
GW170817- Parameter Estimation (LVC)

- Only existing EOB model independent from existing waveform models in LIGO/Virgo
- PE of the binary neutron star GW170817: arXiv:1811.12907 (GWTC-1)

\[ \tilde{\Lambda} = \frac{16 (m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{13 M^5}. \]

see also poster of R. Gamba on the impact of crust modelization arXiv:1902.04616
Tides (for GW170817)

TEOBResumS: GSF-improved tides
SEOBNRv4T: “dynamical” tides (f-mode coupling)\cite{Hinderer+2016}
Spin-orbit & self-spin effects (EOS-dependent)

FIG. 19. The match computed between SEOBNRv4T and TEOBResumS. The match values are color-coded. Based on 17300 randomly chosen points. The plot highlights the high compatibility between the two models.

Excellent compatibility between the two models
Higher modes

Improved resummation of waveform amplitudes [Messina+ 2018]

NR completion merger-bringdown: (2,2), (3,3),(3,2),(4,4),(4,3),(5,5)

Nonspinning\((3,1),(4,2),(4,1)\)

FIG. 10. Unfaithfulness between EOBiResumMultipoles and SXS simulations between mass ratios \(q \in \{1, 10\}\) using the zero-detuned high power Advanced LIGO design sensitivity PSD. We show the minimum and maximum unfaithfulness over all angles \((\theta, \varphi)\), demonstrating that the worst case performance is always below 3% for binaries with a total mass \(M < 200M_\odot\).

see posters of F. Messina-G. Riemenschneider
Use of TEOBResumS

- Use as a benchmark for testing the accuracy of phenomenological tidal models, e.g. IMRPhenomP_NRTidal [Dietrich et al. 2018]

- Used to test high-PN (5.5PN) approximants and to identify biases in tidal parameters due to the inaccurate point-mass baseline. [Dudi, Messina, Nagar & Bernuzzi in prep.]

see poster of F. Messina
Conclusions

What exists

- C version stand-alone: complete and working. Dimensionless units. Including both the PA implementation and the NQC iteration.

- BBH (merger+ringdown), BNS & BHNS (up to merger)

  git clone https://alex_nagar@bitbucket.org/eob_ihes/teobresums.git

What is in progress/to be released

- BBH sector: Higher modes: (2,1),(3,3),(3,2),(4,4),(4,3),(5,5): completed with NR-informed peak and postpeak part. The others are “bare” (but still have a peak). Even more modes available for the non spinning case, e.g. (3,1), (4,1), (4,2).

- BNS sector: improved with more EOS-dependent information and higher modes available. l=3 GSF-resummed tidal potential; included up to NNLO self-spin effects. Not negligible also for small spins. Current approximates underestimate these effects. EOB-controlled PN approximant to improve current PhenomPv2NRTidal model [AN+, PRD99, 2019,044007]

- LAL (LVC) version of the model in progress and will be released soon.