Signatures (?) of Dark Matter from galaxy dynamics

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GSSI – 11 aprile 2018
(L'Aquila)

1. Empirics of DM on galactic scales

2. Common misconceptions (and why, after all, we think DM exists)

3. Introduction to (classical) MOND

4. Problems of MOND

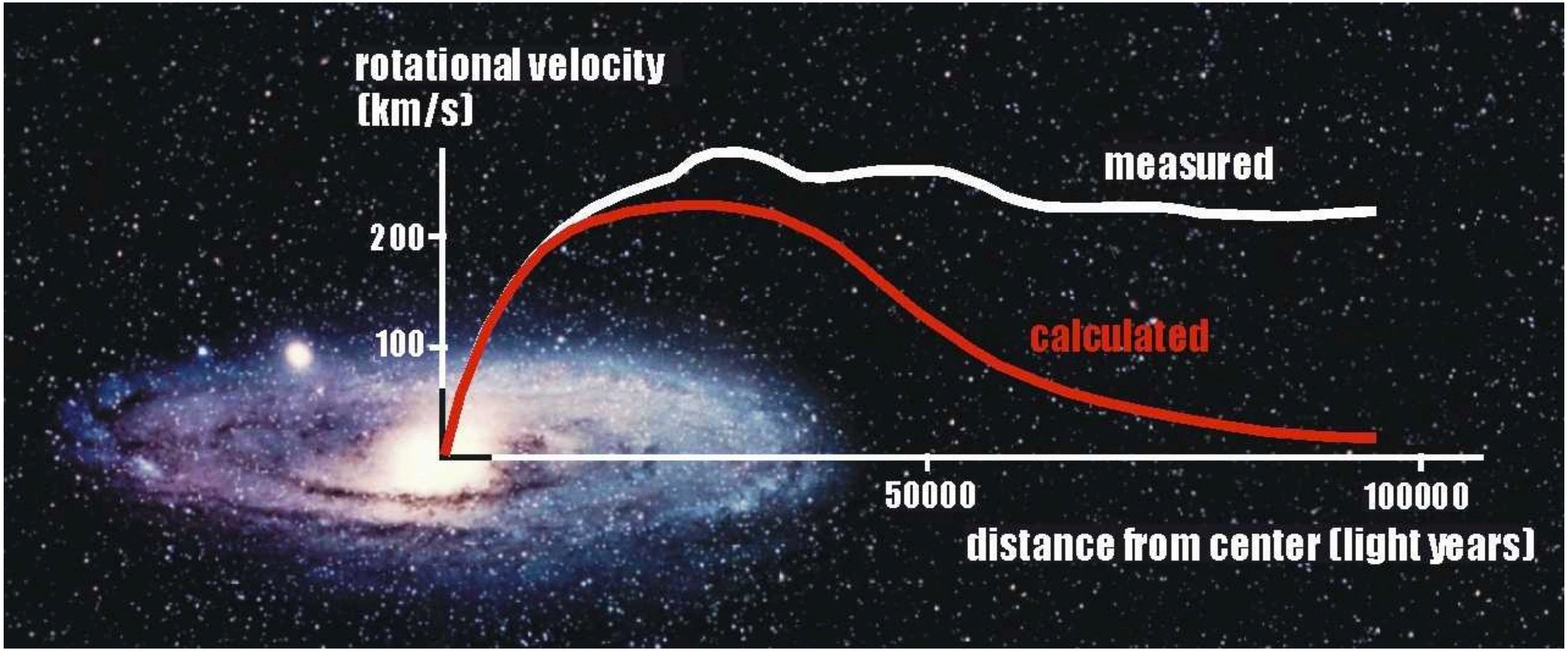
1. DM: EMPIRICS

1.a) Flat rotation curves of disk galaxies

1.b) Velocity dispersion profiles in ETGs

1.c) Hot gaseous atmospheres of ETGs

1a) FLAT ROTATION CURVES (HI) IN DISK GALAXIES



Common (textbook) ``explanation''

Spherical symmetry (+ II Newton theorem)

ISOTHERMAL DM HALO

$$\rho_{DM}(r) = \frac{v_{circ}^2}{4\pi G r^2}$$

But things are (much) more complicated …

"Effect" without a cause: flat rotation curve without DM

Relaxing the assumption of SPHERICAL SYMMETRY

$$\Phi(R,0) = \int_0^\infty \mathrm{d}k\, S_0(k) J_0(kR).$$

$$v_\mathrm{c}^2(R) = R\frac{\partial \Phi}{\partial R} = -R\int_0^\infty \mathrm{d}k\, k S_0(k) J_1(kR).$$

where

$$S_m(k) = -2\pi G \int_0^\infty \mathrm{d}R\, R J_m(kR) \Sigma_m(R).$$

Fourier-Hankel transform of disk density

Mestel disk

$$\Sigma(R) = \frac{A}{R}$$

Perfectly flat rotation curve

A purely stellar Mestel disk DOES NOT REQUIRE DM to show a flat rotation curve

NB: isothermal sphere in projection: Mestel disk!

Power-law ellipsoidal distributions

$$m^2 \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{a^2(1-\epsilon)^2} + \frac{z^2}{a^2(1-\eta)^2}, \quad \text{(B1)}$$

$a \geq b \geq c > 0$, $b/a \equiv 1 - \epsilon$ and $c/a \equiv 1 - \eta$. It is then an elementary exercise to show that under these assumptions the density can be written in full generality as

$$\rho(\mathbf{x}) = \frac{M}{4\pi a^3} \frac{\tilde{\rho}(m)}{(1-\epsilon)(1-\eta)}, \quad \int_0^\infty \tilde{\rho}(m) m^2 dm = 1, \quad \text{(B2)}$$

$$\rho_* = \frac{\rho_{\mathrm{n}}}{q_* m_*^\gamma}, \quad 0 < \gamma < 3.$$

Classical result on potential theory (e.g., see Chandrasekhar EFE)

$$\Phi(\mathbf{x}) = -\pi abcG \int_0^\infty \frac{\Delta\Psi(\mathbf{x};\epsilon,\eta)}{\sqrt{(a^2+\tau)(b^2+\tau)(c^2+\tau)}} d\tau,$$

where

$$\Delta\Psi(\mathbf{x};\epsilon,\eta) = 2\int_{m(\mathbf{x};\tau)}^\infty \rho(m) m dm,$$

with

$$m^2(\mathbf{x};\tau) \equiv \frac{x^2}{a^2+\tau} + \frac{y^2}{b^2+\tau} + \frac{z^2}{z^2+\tau};$$

For gamma=2 flat rotation curve independent of flattening

For axial ratio = 1 => isothermal sphere

NB: Face-on projection of the gamma=2 ellipsoid
is (again) the Mestel disk!

Ciotti-Bertin (2005) power-law tori

A&A 437, 419–427 (2005)
DOI: 10.1051/0004-6361:20042123

Astronomy & Astrophysics

A simple method to construct exact density-potential pairs from a homeoidal expansion★

L. Ciotti[1] and G. Bertin[2]

By using an expansion method, ordering arguments, and linearity of the Laplacian,
we can produce quite ``remarkable'' density-potential pair.
A surprisingly simple (and previously unknown) torus

$$\varrho = \frac{\tilde{R}^2}{\tilde{r}^\alpha}, \quad (\alpha > 0).$$

$$\Phi = \begin{cases} -\dfrac{\tilde{r}^{2-\alpha}}{(\alpha-2)(7-\alpha)}\left[\dfrac{4\tilde{r}^2}{(\alpha-4)(5-\alpha)} + \tilde{R}^2\right], & (\alpha \neq 4), \\ \dfrac{1}{3}\left(2\ln\tilde{r} - \dfrac{1}{2}\dfrac{\tilde{R}^2}{\tilde{r}^2}\right), & (\alpha = 4). \end{cases} \quad (19)$$

$$v_c^2 = \frac{(-\alpha^2 + 9\alpha - 16)\tilde{R}^{4-\alpha}}{(\alpha-2)(5-\alpha)(7-\alpha)},$$

alpha=4 => constant circular velocity!

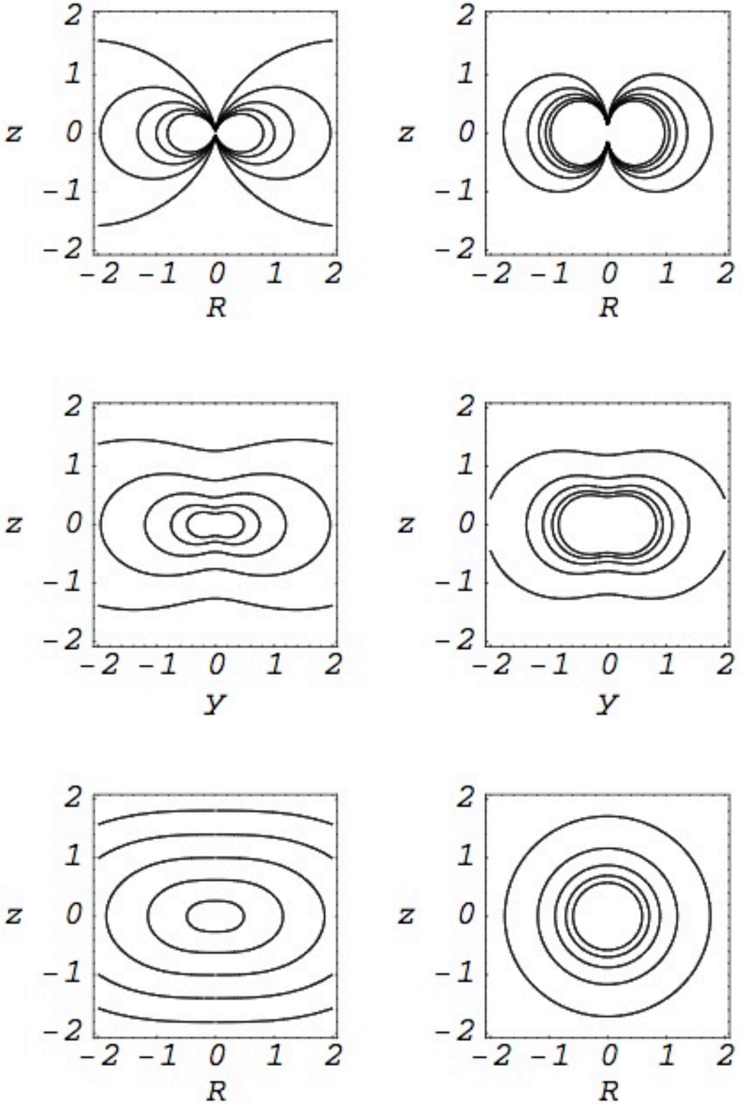


Fig. 1. Isodensity (*top*), constant projected density (*middle*), and isopotential (*bottom*) contours (in arbitrary units) for the toroidal model described by Eq. (18) with $\alpha = 3.1$ (*left*) and $\alpha = 4.9$ (*right*). The lower value of α is near the critical value for which the surface brightness is everywhere infinite, while the upper value is near the critical value for which the central mass diverges. The coordinates R, y, and z are normalized to the scale-length a. Note how a strongly non-spherical density distribution, such as that represented on the top right, produces a nearly spherical potential (*bottom right*).

density

Face-on projection of the alpha=4 torus: Mestel disk, <u>again</u>!

potential

We can conjecture that all systems than in projection are the Mestel disk have an equatorial constant circular velocity

From the astrophysical point of view, we have SEVERAL examples of systems showing that

IN PRINCIPLE

a flat rotation curve is

NOT

``per se'' the FINAL PROOF of the presence of DM

<u>A more realistic case: exponential disks and Bessel functions</u>

In the optical region a PURE exponential stellar disk produces an ALMOST FLAT rot. curve: NO NEED of DM from STELLAR DYNAMICS in GALACTIC DISKS (Kalnajis 1976, and Vera Rubin ``claim'')

COMPELLING need of DM
ONLY in the external regions from HI observations
AND dynamical arguments as follows

Consequences

A flat rotation curve in the optical disk is NOT indication of DM

HOWEVER: a stellar/gaseous disk is not the solution
(Ostriker-Peebles stability against bar instability)

For the same reason, ALSO a DM disk is excluded

In the very external regions: monopole dominated
(spherical) potential

ALMOST ISOTHERMAL & SPHEROIDAL DM HALO

1b) ETGs VELOCITY DISPERSION PROFILES

In presence of a "isothermal DM sphere" a power-law stellar density with *isotropic* velocity dispersion has a flat velocity dispersion profile (from Jeans equations)

$$\rho_*(r)\sigma_r^2(r) = \frac{G}{r^2 + r_a^2} \int_r^{\infty} \rho_*(r) M_T(r) \left(1 + \frac{r_a^2}{r^2}\right) dr$$

$$\rho_*[r] = \frac{A}{r^a} \quad \Rightarrow \quad \sigma_r^2[r] = \frac{v_{halo,circ}^2}{a}$$

Several two-component models are available in the literature

THE ASTROPHYSICAL JOURNAL, 471:68–81, 1996 November 1

THE ANALYTICAL DISTRIBUTION FUNCTION OF ANISOTROPIC TWO-COMPONENT HERNQUIST MODELS

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Received 1996 March 25; accepted 1996 May 28

Astron. Astrophys. 321, 724–732 (1997)

ASTRONOMY AND ASTROPHYSICS

Stellar systems following the $R^{1/m}$ luminosity law

II. Anisotropy, velocity profiles, and the fundamental plane of elliptical galaxies

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THE ASTROPHYSICAL JOURNAL, 520:574–591, 1999 August 1

MODELING ELLIPTICAL GALAXIES: PHASE-SPACE CONSTRAINTS ON TWO-COMPONENT (γ_1, γ_2) MODELS

L. CIOTTI[1,2]

Received 1998 October 20; accepted 1999 March 11

Mon. Not. R. Astron. Soc. **393,** 491–500 (2009) doi:10.1111/j.1365-2966.2008.14009.x

Two-component galaxies with flat rotation curve

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··· also in non-spherical cases

Mon. Not. R. Astron. Soc. **279**, 240–248 (1996)

The energetics of flat and rotating early-type galaxies and their X-ray luminosity

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Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **448**, 2921–2933 (2015) doi:10.1093/mnras/stv202

Miyamoto–Nagai discs embedded in the Binney logarithmic potential: analytical solution of the two-integrals Jeans equations

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2926 *C. O. Smet, S. Posacki and L. Ciotti*

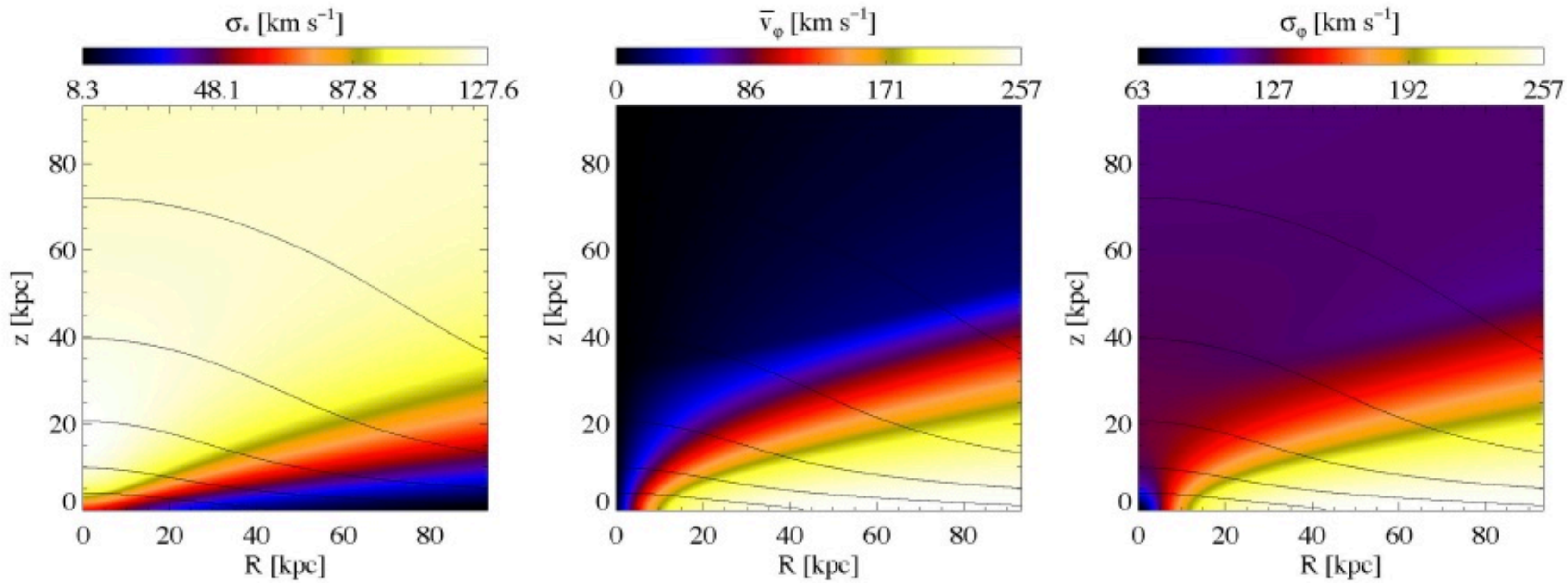

Figure 2. Two-dimensional maps in the meridional plane of the vertical and radial velocity dispersion $\sigma_* = \sqrt{\sigma_{**}^2 + \sigma_{*h}^2}$ (left-hand panel), of the ordered azimuthal velocity $\overline{v}_\varphi$ in the isotropic case ($k = 1$, central panel), and of the azimuthal velocity dispersion σ_φ in the fully velocity dispersion supported case ($k = 0$, right-hand panel). The structural parameters of the model are $M_* = 10^{11}\,\mathrm{M}_\odot$, $b = 2$ kpc, $s = 10$, $v_h = 250\,\mathrm{km\,s^{-1}}$, $R_h = 5b$, and $q = 0.7$. Solid lines represent isodensity contours of the stellar distribution.

··· the message here is for Particle Physicists:
be aware that the
dynamical modeling and the understanding of
Dynamics in Astrophysics is
by far deeper than you will ever imagine!

MASS ANISOTROPY DEGENERACY

TANGENTIAL anisotropy of the velocity dispersion tensor leads to HIGH values of velocity dispersion in the external regions:

if observations interpreted with isotropic models, artificially high values of the dynamical mass-to-light ratios (that can be erroneously interpreted as ``evidences'' of DM)

Analogous (but opposite) effect in the CENTRAL REGIONS due to
RADIAL ANISOTROPY

“Cause” without effect: ``low’’ central velocity dispersion with DM!

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS 473, 5476–5491 (2018) doi:10.1093/mnras/stx2771

Advance Access publication 2017 October 25

Two-component Jaffe models with a central black hole – I. The spherical case

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Accepted 2017 October 23. Received 2017 October 23; in original form 2017 July 21

A family of fully analytical (Jeans & Phase Space) two component galaxy models with central BH

$$\rho_g(r) = \frac{\mathcal{R}\xi\rho_n}{s^2(\xi+s)^2}$$

$$\xi \equiv \frac{r_g}{r_*}, \quad \mathcal{R} \equiv \frac{M_g}{M_*} = \mathcal{R}_{DM} + 1$$

$$\sigma_r^2(0) = \frac{\Psi_n \mathcal{R}}{2\xi}$$

You can add DM mass with NO changes in the central vel. dispersion!

Alma Mater Studiorum
Università degli Studi di Bologna

SCUOLA DI SCIENZE
Dipartimento di Fisica e Astronomia
Corso di Laurea Magistrale in Astrofisica e Cosmologia

Dark matter and stellar populations in the central region of early-type galaxies

Tesi di Laurea Magistrale

Candidato:
Caterina Caravita

Relatore:
Chiar.mo Prof. Luca Ciotti

Corelatore:
Chiar.ma Prof.ssa Silvia Pellegrini

The only proper way to look at DM halos in ETGs is by using multi-component phase-space distribution functions, build line profiles, check for phase-space consistency:

HIGHLY NON TRIVIAL

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **401**, 1091–1098 (2010) doi:10.1111/j.1365-2966.2009.15697.x

Consistency criteria for generalized Cuddeford systems

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Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **408,** 1070–1074 (2010) doi:10.1111/j.1365-2966.2010.17184.x

How general is the global density slope–anisotropy inequality?

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Criterion 1: in all spherical systems whose density distribution is a separable function of radius and total potential, $\rho = A(r)B(\Psi_{\mathrm{T}})$, the global inequality $\gamma(r) \geq 2\beta(r) \quad \forall r$ holds $\Leftrightarrow$ $B(\Psi_{\mathrm{T}})$ is a monotonically increasing function of Ψ_{T}.

1c) HOT GASEOUS ATMOSPHERES OF ETGs

From Poincare' theorem on stratifications
Hydrostatic equilibrium => X-ray emission => total potential

$$M(r) = -r \frac{kT}{G\mu m_{\mathrm{p}}} \left(\frac{\mathrm{d}\,\log\rho}{\mathrm{d}\,\log r} + \frac{\mathrm{d}\,\log T}{\mathrm{d}\,\log r} \right)$$

IN PRINCIPLE, WE CAN RECONSTRUCT THE TOTAL POTENTIAL,
AND BY SUBTRACTION
OBTAIN DM AMOUNT AND DISTRIBUTION

··· HOWEVER, gas motions can mimic DM halo presence!

Mon. Not. R. Astron. Soc. **350,** 609–614 (2004) doi:10.1111/j.1365-2966.2004.07670.x

On the use of X-rays to determine dynamical properties of elliptical galaxies

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Mon. Not. R. Astron. Soc. **370,** 1797–1803 (2006) doi:10.1111/j.1365-2966.2006.10590.x

Reconciling optical and X-ray mass estimates: the case of the elliptical galaxy NGC 3379

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⋯ and gas IS flowing in ETGs!

THE ASTROPHYSICAL JOURNAL, 376:380–403, 1991 August 1

WINDS, OUTFLOWS, AND INFLOWS IN X-RAY ELLIPTICAL GALAXIES. I.

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Received 1990 July 6; accepted 1991 January 24

Astron. Astrophys. 333, 433–444 (1998)

ASTRONOMY AND ASTROPHYSICS

Decoupled hot gas flows in elliptical galaxies

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THE ASTROPHYSICAL JOURNAL, 494:535–545, 1998 February 20

DECOUPLED AND INHOMOGENEOUS GAS FLOWS IN S0 GALAXIES

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MNRAS **445**, 1351–1369 (2014) doi:10.1093/mnras/stu1834

The effects of galaxy shape and rotation on the X-ray haloes of early-type galaxies – II. Numerical simulations

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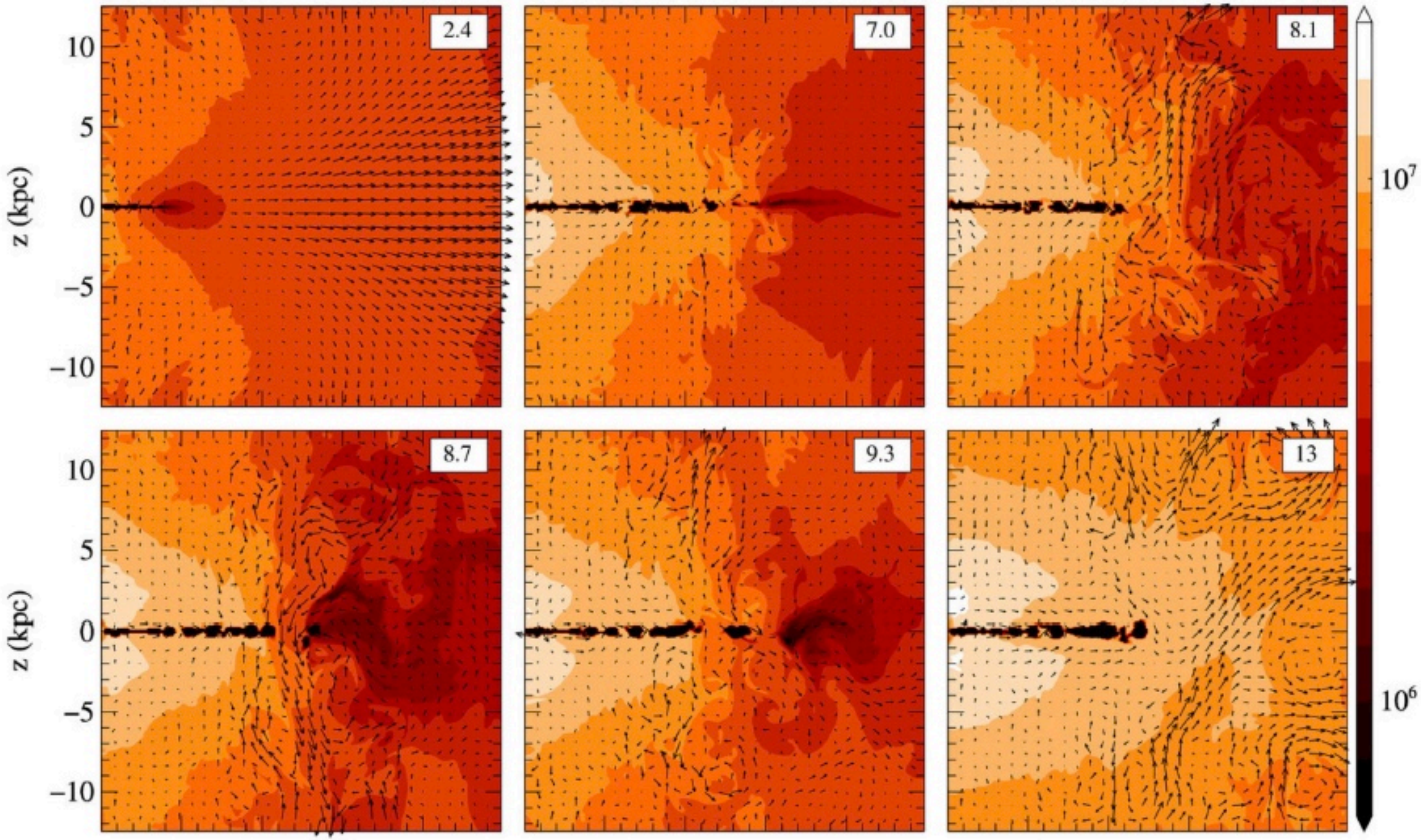

2.4
7.0
8.1
8.7
9.3
13
z (kpc)
z (kpc)
10
5
0
−5
−10
10^7
10^6

··· and we have also gas flows induced by AGN activity to complicate the situation

THE ASTROPHYSICAL JOURNAL, 487:L105–L108, 1997 October 1

COOLING FLOWS AND QUASARS: DIFFERENT ASPECTS OF THE SAME PHENOMENON? I. CONCEPTS

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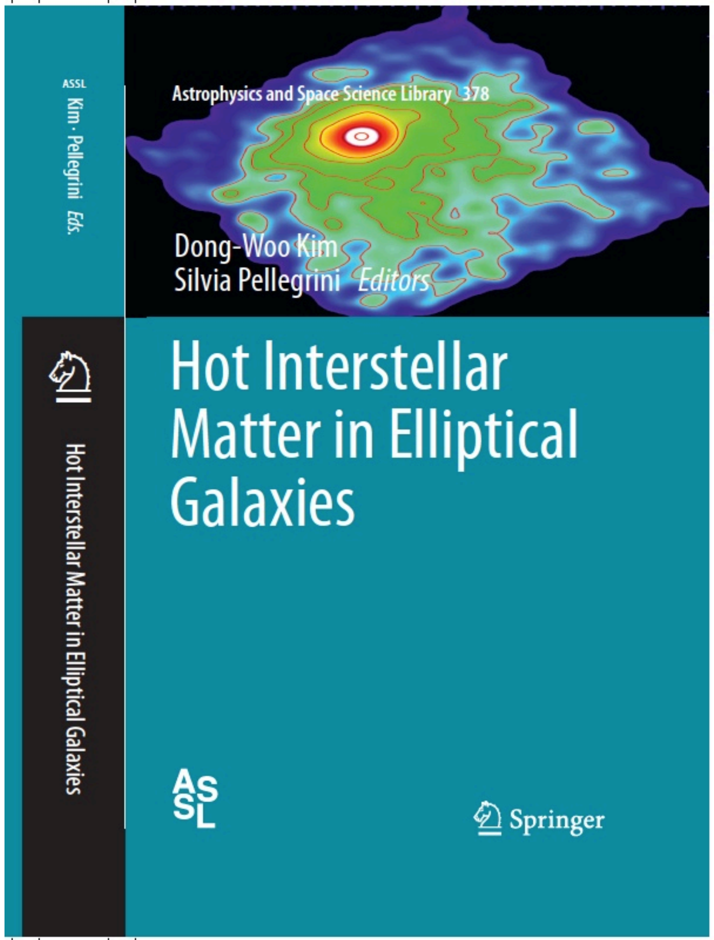
Received 1997 June 17; accepted 1997 July 22; published 1997 September 2



AGN feedback in elliptical galaxies: numerical simulations

Luca Ciotti and Jeremiah P. Ostriker

A chapter in this book

3.INTRODUCTION TO MOND

WHY SHOULD WE BOTHER
TO STUDY A NEW LAW
OF GRAVITY ?

DM “paradigm” : observational ``evidences’’ that if Newtonian gravity is correct then we need large amounts of DM

“Conspiracy” : curiously, DM seems to “know” very well how baryons are distributed. The standard example is the case of flat rotation curves in spiral galaxies

Unknown DM constituents

Can we avoid the use of DM by a modification of the Newton gravity law at low accelerations?

NB: it can be proved that a modification of gravity as a function of distance is INCONSISTENT with observations. More sophisticated modifications needed.

↓

Milgrom & Bekenstein: NON-linear theory based on a Lagrangian density (that can be extended to have a Lorentz - covariant formulation)

$$\nabla \cdot \left[\mu \left(\frac{\|\nabla \phi\|}{a_0} \right) \nabla \phi \right] = 4\pi G \rho,$$

$$\mu(y) \sim \begin{cases} y & \text{for } y \ll 1, \\ 1 & \text{for } y \gg 1; \end{cases}$$

$$\mu(y) = \frac{y}{\sqrt{1+y^2}}$$

Test particles move according to $\mathbf{g} = -\nabla \phi,$

"Deep" MOND regime $\nabla \cdot (\|\nabla \phi\| \nabla \phi) = 4\pi G a_0 \rho.$

MOND field equation and Poisson $\nabla^2 \phi_{\rm N} = 4\pi G\rho$ ➔

$$\mu(\|\nabla\phi\|/a_0)\nabla\phi = \nabla\phi_{\rm N} + \mathbf{S}, \qquad \mathbf{S} = \mathrm{curl}\,\mathbf{h}$$

When S=0 simple relation with the Newtonian force field

$$g_{MOND}(r) = \frac{\sqrt{GM(r)a_0}}{r}$$

Flat rotation curve

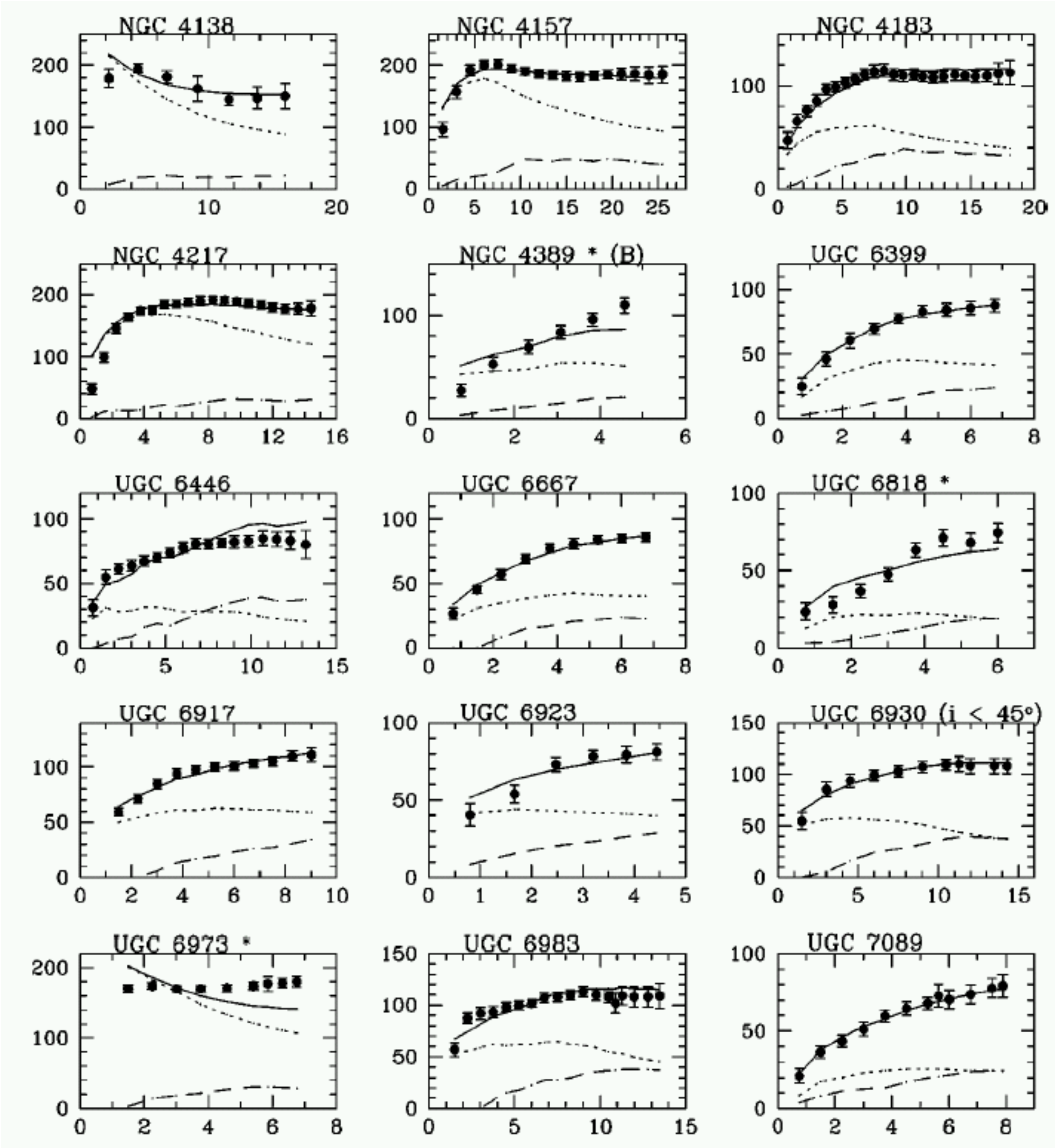
Unfortunately, this works only spherical-cylindrical-planar stratifications

The acceleration scale [the ONLY free parameter]
fixed by observation, e.g., by fitting a few, well observed R.Cs.

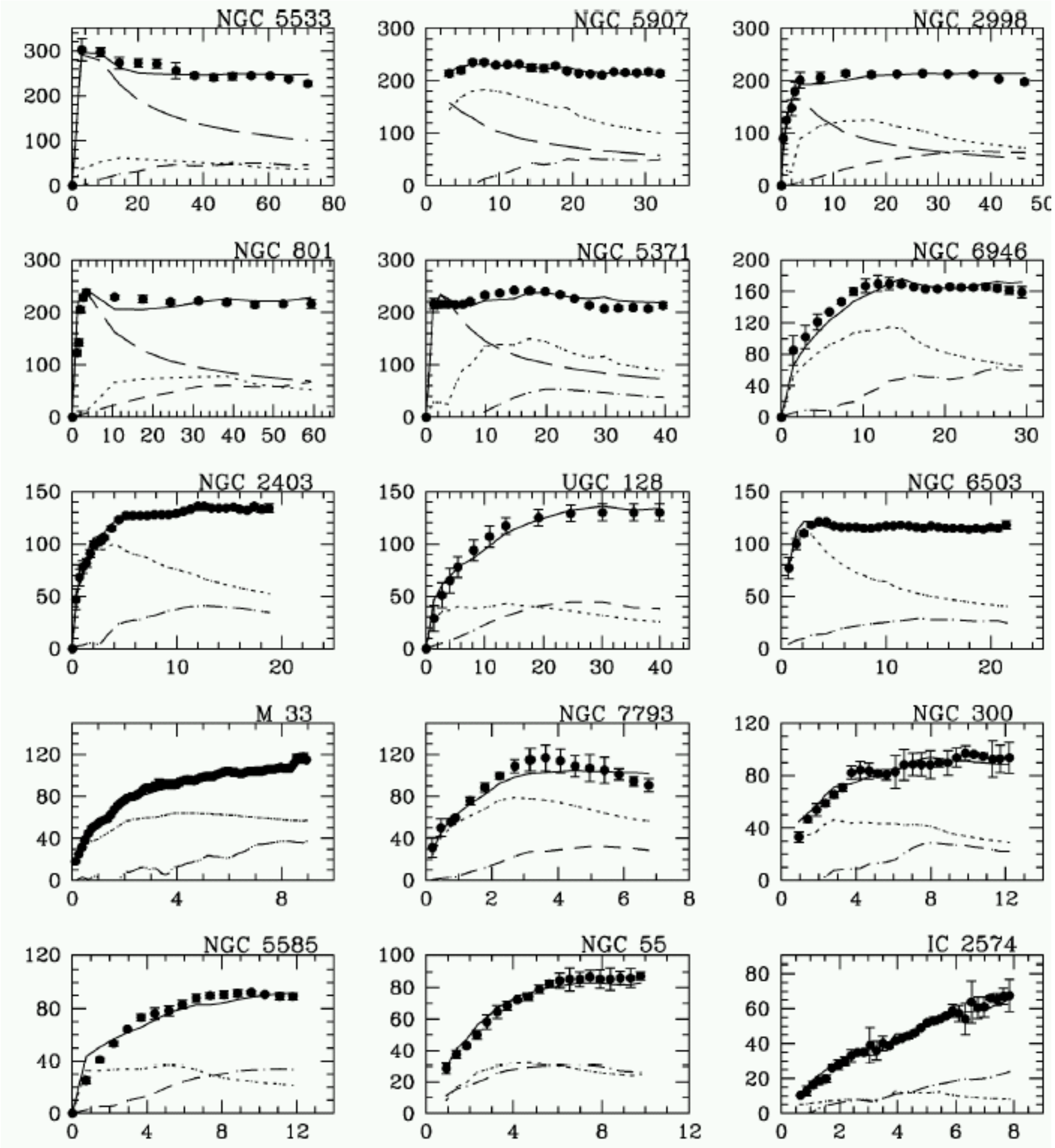
$$a_0 \cong 1.2 10^{-8} cm/s^2$$

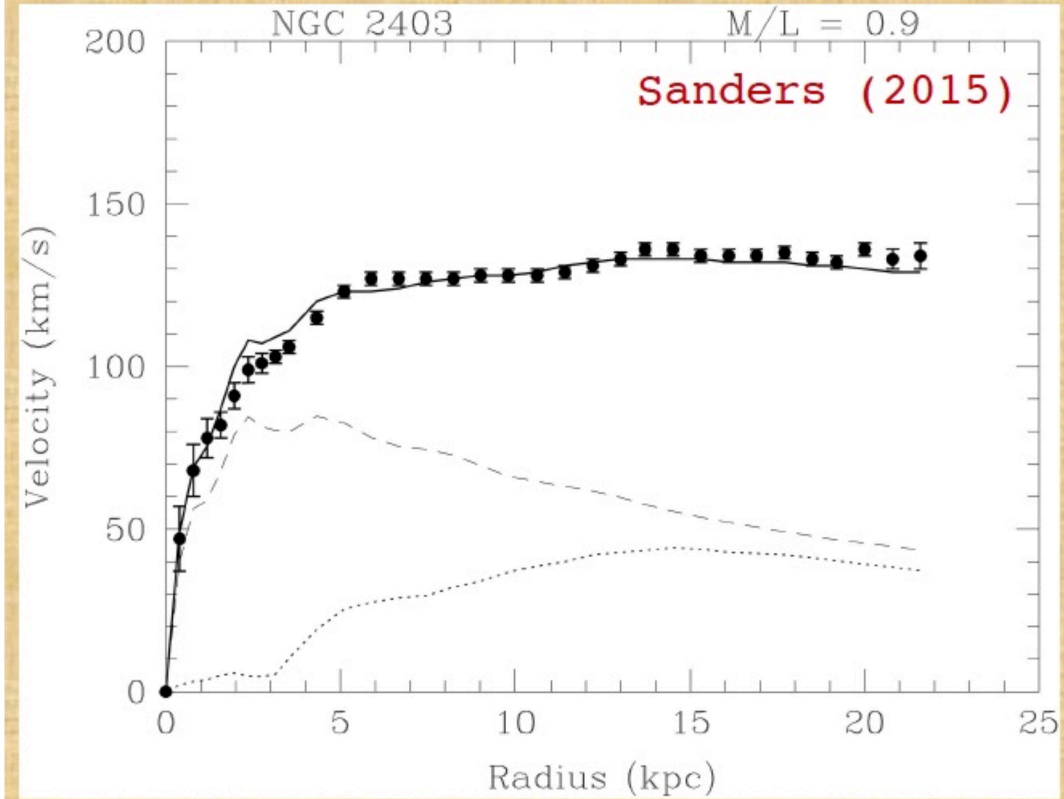
MOND "prediction": the dynamics of <u>low acceleration systems</u> should be described by their baryon distribution and the same value of a0

The prescription is surprisingly successful



Ursa Major
Sanders & Verheijen


NGC 5533
NGC 5907
NGC 2998
NGC 801
NGC 5371
NGC 6946
NGC 2403
UGC 128
NGC 6503
M 33
NGC 7793
NGC 300
NGC 5585
NGC 55
IC 2574



- No dark matter
- Characteristic acceleration $a_0 \sim 10^{-10}$ m/s^2
- $a \gg a_0 \rightarrow a_{MOND} \sim a_{Newt}$
- $a \ll a_0 \rightarrow a_{MOND} \sim (a_{Newt} a_0)^{1/2}$

…the present situation

• MOND COULD BE AN ALTERNATIVE (?) TO DM

• INVESTIGATIONS LIMITED BY NON-LINEARITY AND FIELD S [analytical method to produce exact solutions Ciotti, Londrillo, & Nipoti (2006, ApJ, 640, 741), Ciotti, Zhao and de Zeeuw (MNRAS)]

• ATTEMPTS TO COMPUTE 2-body REL. TIME & DYN. FRICTION in MOND SUGGEST THAT THESE TIMES ARE SHORTER THAN IN NEWTONIAN GRAVITY (Ciotti & Binney, 2004, MNRAS, 351,285)

• N-BODY NUMERICAL SIMULATIONS CAN NOW BE DONE [Nipoti, Londrillo, & Ciotti (2006, ApJ)

4. TWO-BODY RELAXATION & DYNAMICAL FRICTION in MOND

4.1 Two-body relaxation time

Argument based on 2-body scattering
(wrong but instructive)

In deep MOND regime for nearly equal masses

$$F \simeq \frac{m_1 m_2}{\sqrt{m_1 + m_2}} \frac{\sqrt{G a_0}}{r} = \frac{G m_1 m_2}{r r_0}.$$

$$\Delta v_\perp \simeq \frac{2b}{v} \frac{F(b)}{m} = \frac{2Gm}{r_0 v}$$ ➔

Independent of *b*!

🡻

NO Coulomb log!

The previous argument is wrong, because MOND is a NON-linear theory.

In C&B04 we attempted a different approach based on perturbation of MOND field equation in a uniform background

We found

$$\frac{t_{2b}^{M}}{t_{2b}^{N}} = \frac{\sqrt{2}g_{N}^{2}}{g_{0}^{2}} = \frac{\sqrt{2}}{(1+\mathcal{R})^{2}}$$

where

$$\mathcal{R} \equiv \frac{M_{DM}}{M_{*}}$$

In the “equivalent” Newtonian system with DM

We also found that in MOND t_{fric} shorter by

$$(1+\mathcal{R})^2/\sqrt{2}$$

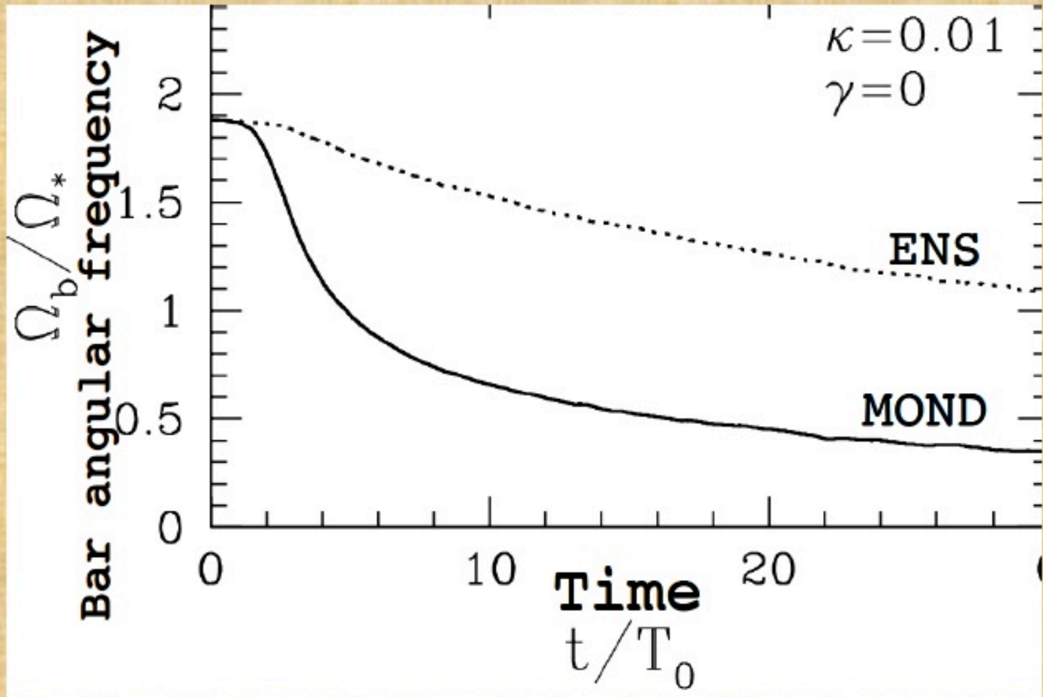
than in Newtonian system with same stellar mass & fixed DM field. Making DM field dynamical shortens Newtonian t_{fric} by

$$(1+\mathcal{R})$$

POSSIBLE PROBLEMS WITH
Galaxy groups
Dwarf galaxies

Dynamical friction

(Nipoti, Ciotti, Binney & Londrillo 2008)



- Spherical N-body systems (MOND and ENS)
- Small rigid bar rotating at the centre
- Measure slowing down of the bar

Large collision velocities in merging
(log nature of potential)

>>>Bullet Cluster<<<

4.2 Phase-mixing

First experiments on dissipationless collapses

(Nipoti, Londrillo, & Ciotti 2006, ApJ)

N-body particle-mesh code, based on our MOND potential solver (Ciotti, Londrillo, Nipoti 2006, ApJ, 640, 741)

TESTS OF NUMERICAL SIMULATIONS [1]

(potential solver)

Numerical recovering of the analytical, aspherical MOND density-potential pairs constructed by the CLN06 method (potential-deformation technique based on homeoidal expansion [Ciotti & Bertin 2005, A&A, 437, 419])

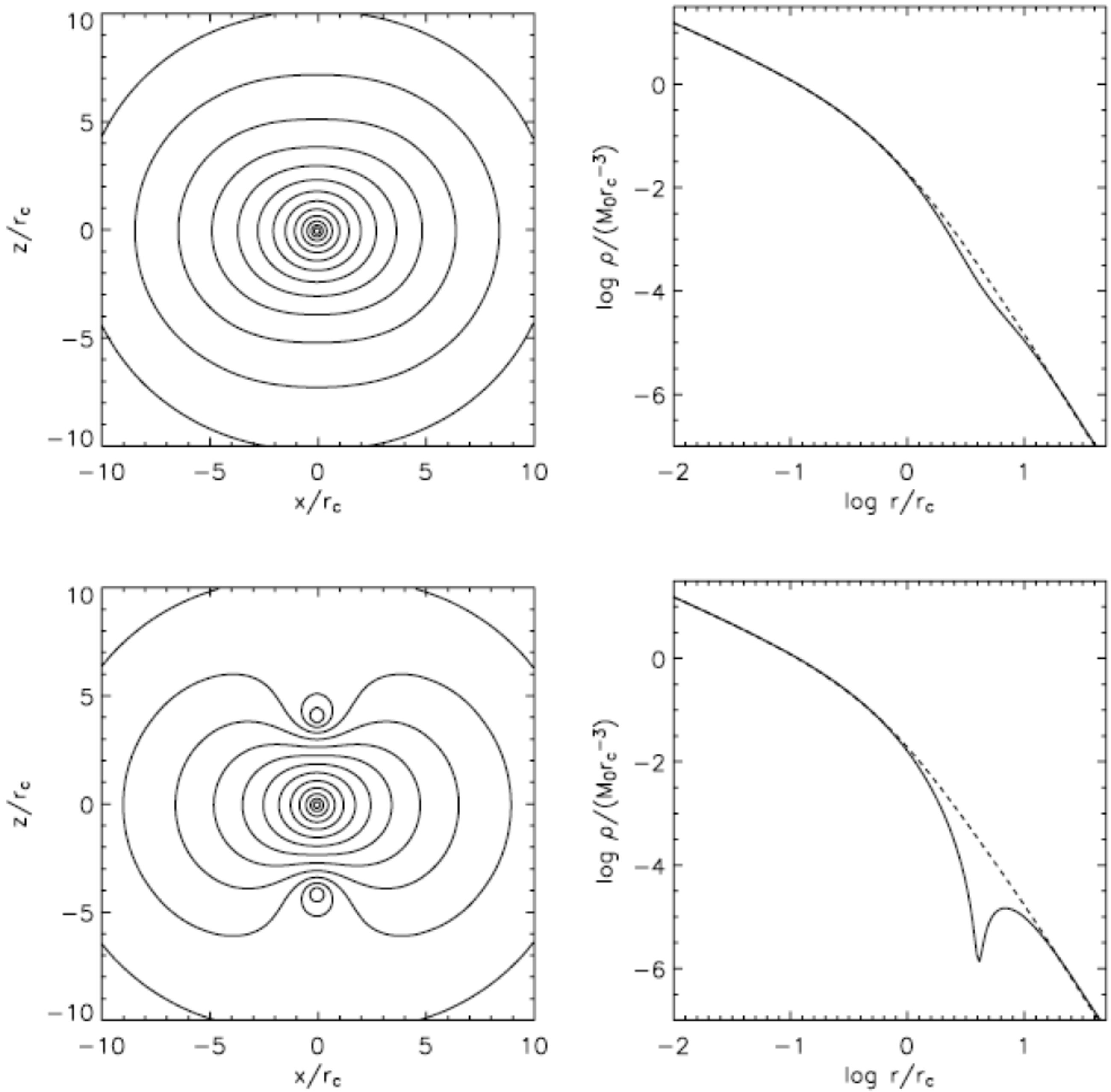


FIG. 2.—Isodensity contours (*left panels*) and density profiles (*right panels*) for two analytical dMOND axisymmetric ($\epsilon = 0$) Hernquist models with ϕ_1 as in eq. (21). The density profiles are taken along a radius in the equatorial plane (*dashed lines*) and along the symmetry axis z (*solid lines*). The model in the top panels has $\beta = 5$ and $\tilde{\eta} = 0.01$, while the model in the bottom panels has $\beta = 5$ and $\tilde{\eta} = 0.02$.

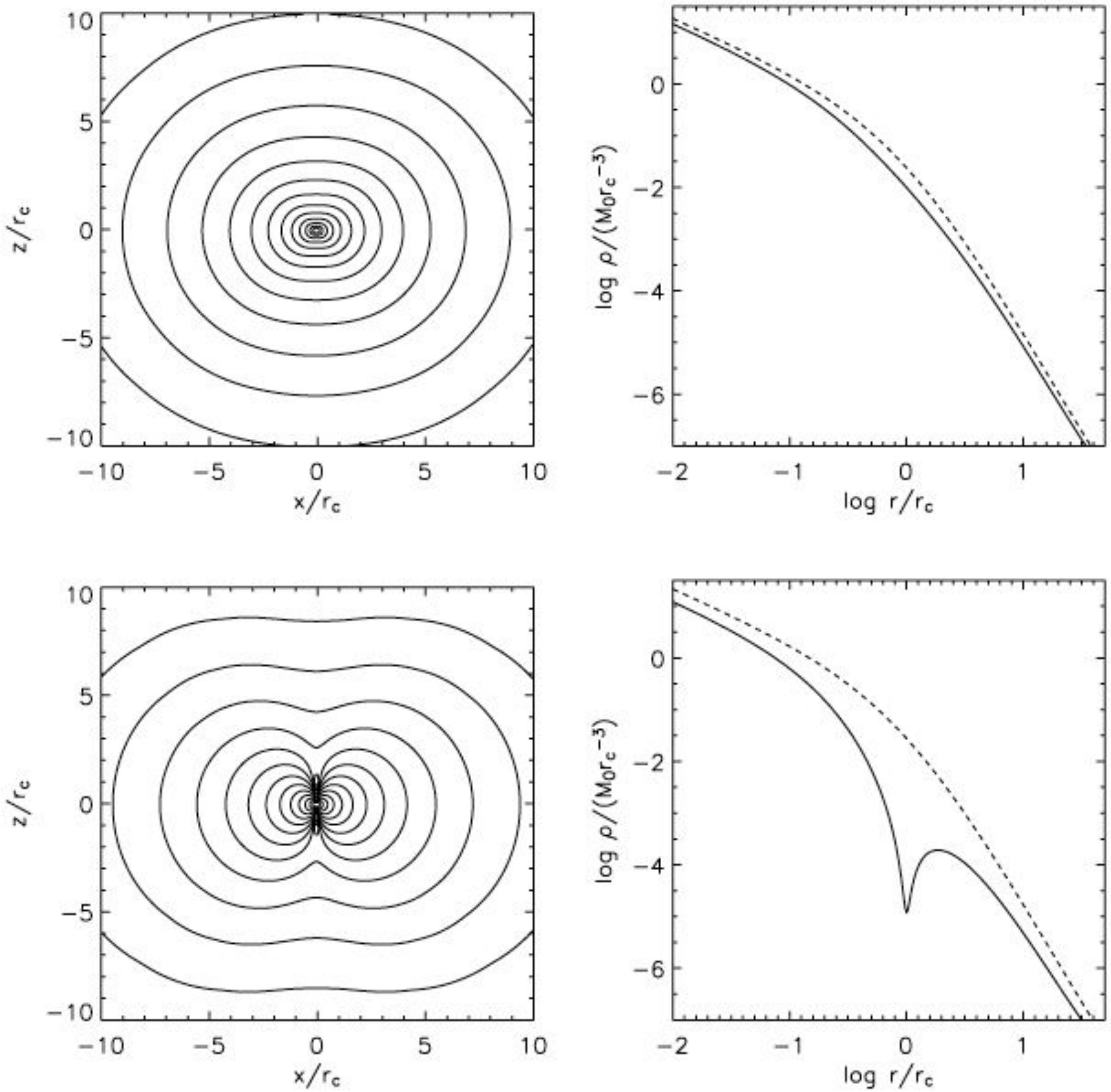
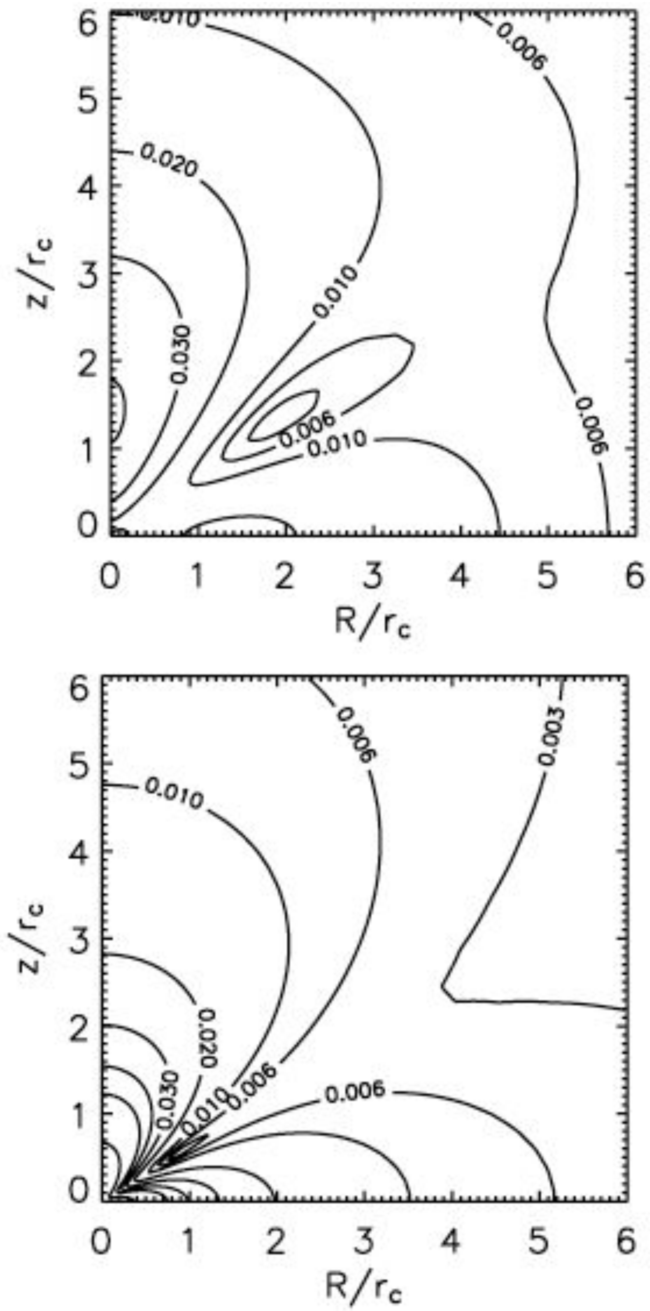


FIG. 3.—The same quantities as in Fig. 2 for two analytical dMOND axisymmetric ($\epsilon = 0$) Hernquist models with ϕ_1 as in eq. (24), with $\eta = 0.2$ (*top*) and $\eta = 0.4$ (*bottom*).



$$\left\|\vec{S}\right\| / \left\|\nabla \phi_N\right\|$$

Quite small… however, simulations with “forced” S=0 DO NOT conserve (as well known on theorerical grounds) LINEAR & ANGULAR MOMENTUM

TESTS OF NUMERICAL SIMULATIONS [2]

(conservation laws)

NB: total energy in MOND systems DIVERGES
however

in MOND as in Newtonian gravity for a system at equilibrium

VIRIAL THEOREM $$2K + W = 0,$$

$$W = \mathrm{Tr}\, W_{ij} \qquad W_{ij} \equiv - \int \rho(\mathbf{x}) x_i \frac{\partial \phi(\mathbf{x})}{\partial x_j} d^3\mathbf{x}$$

In addition

W is conserved in the limit of dMOND, being $W = -(2/3)\sqrt{Ga_0 M_*^3}$

for *all* systems of finite total mass M_*

Proof

$$W = -\frac{1}{4\pi G a_0} \int \mathcal{D}[\phi] \nabla \cdot (\|\nabla\phi\| \nabla\phi) \, d^3\mathbf{x}, \qquad \mathcal{D} \equiv < \mathbf{x}, \nabla >.$$

The integrand can be written as

$$\mathcal{D}[\phi] \nabla \cdot (\|\nabla\phi\| \nabla\phi) = \nabla \cdot (\mathcal{D}[\phi] \|\nabla\phi\| \nabla\phi) - \|\nabla\phi\| < \nabla\phi, \nabla\mathcal{D}[\phi] >$$

Let us focus on the term

$$\nabla \mathcal{D}[\phi] = \nabla \phi + \mathcal{D}[\nabla \phi]$$

so that

$$\|\nabla \phi\| < \nabla \phi, \nabla \mathcal{D}[\phi] >=$$

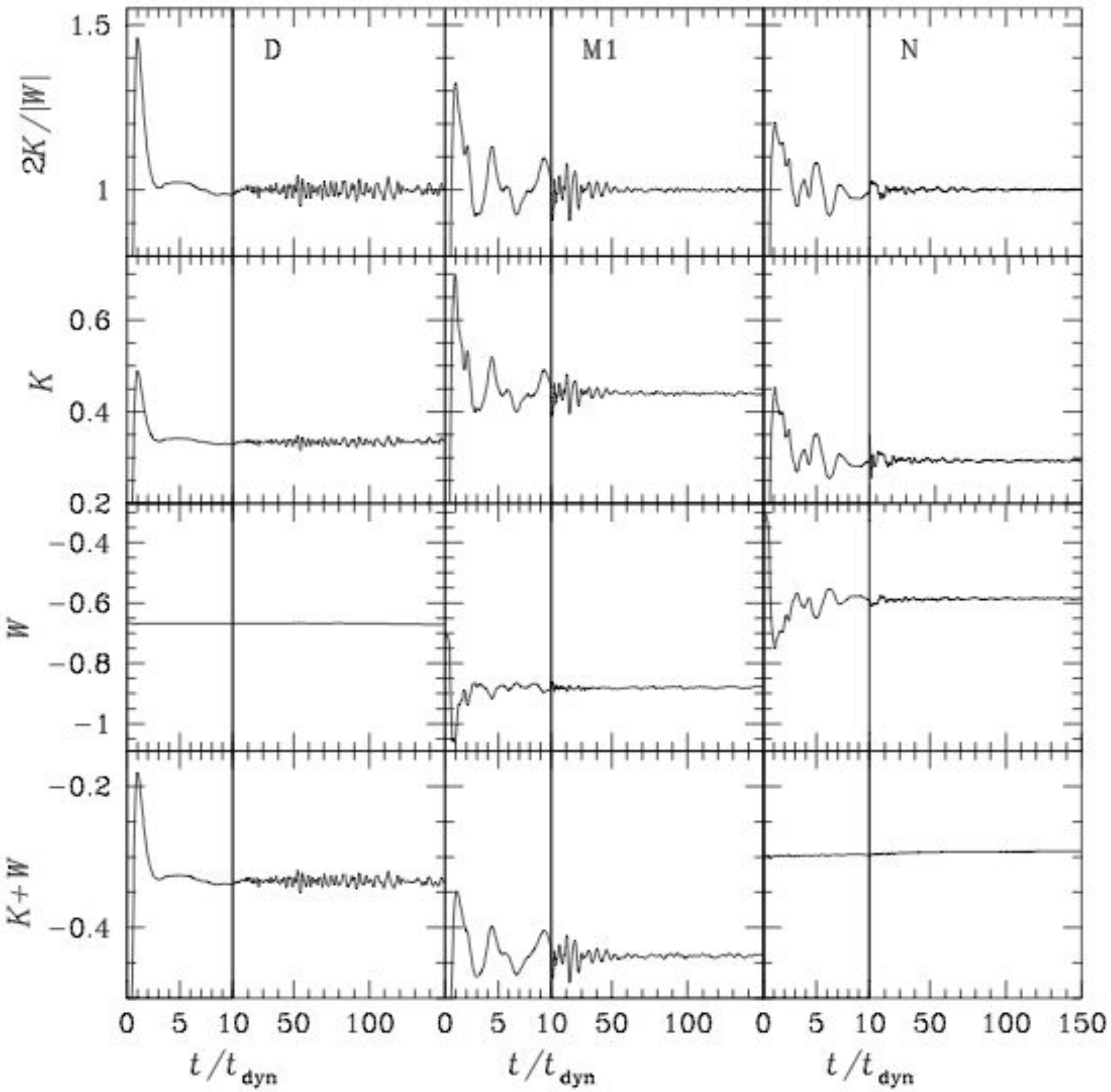
$$\|\nabla \phi\|^3 + \|\nabla \phi\| < \nabla \phi, \mathcal{D}[\nabla \phi] >= \|\nabla \phi\|^3 + \frac{\mathcal{D}\left[\|\nabla \phi\|^3\right]}{3} = \frac{\nabla \cdot \left(\mathbf{x}\|\nabla \phi\|^3\right)}{3}.$$

and finally

$$\boxed{\mathcal{D}[\phi]\nabla \cdot (\|\nabla \phi\| \nabla \phi) = \nabla \cdot \left(\mathcal{D}[\phi]\|\nabla \phi\| \nabla \phi - \frac{\mathbf{x}\|\nabla \phi\|^3}{3}\right)}$$

$$\mathbf{g} = -\nabla\phi \sim -\frac{\sqrt{GM_* a_0}}{r}\hat{\mathbf{e}}_r \qquad \text{for } r \to \infty$$

$$W = -\frac{1}{4\pi G a_0} \lim_{r\to\infty} \int_{4\pi} \frac{2}{3} r^3 g^2 d\Omega = -\frac{2}{3}\sqrt{G a_0 M_*^3}.$$

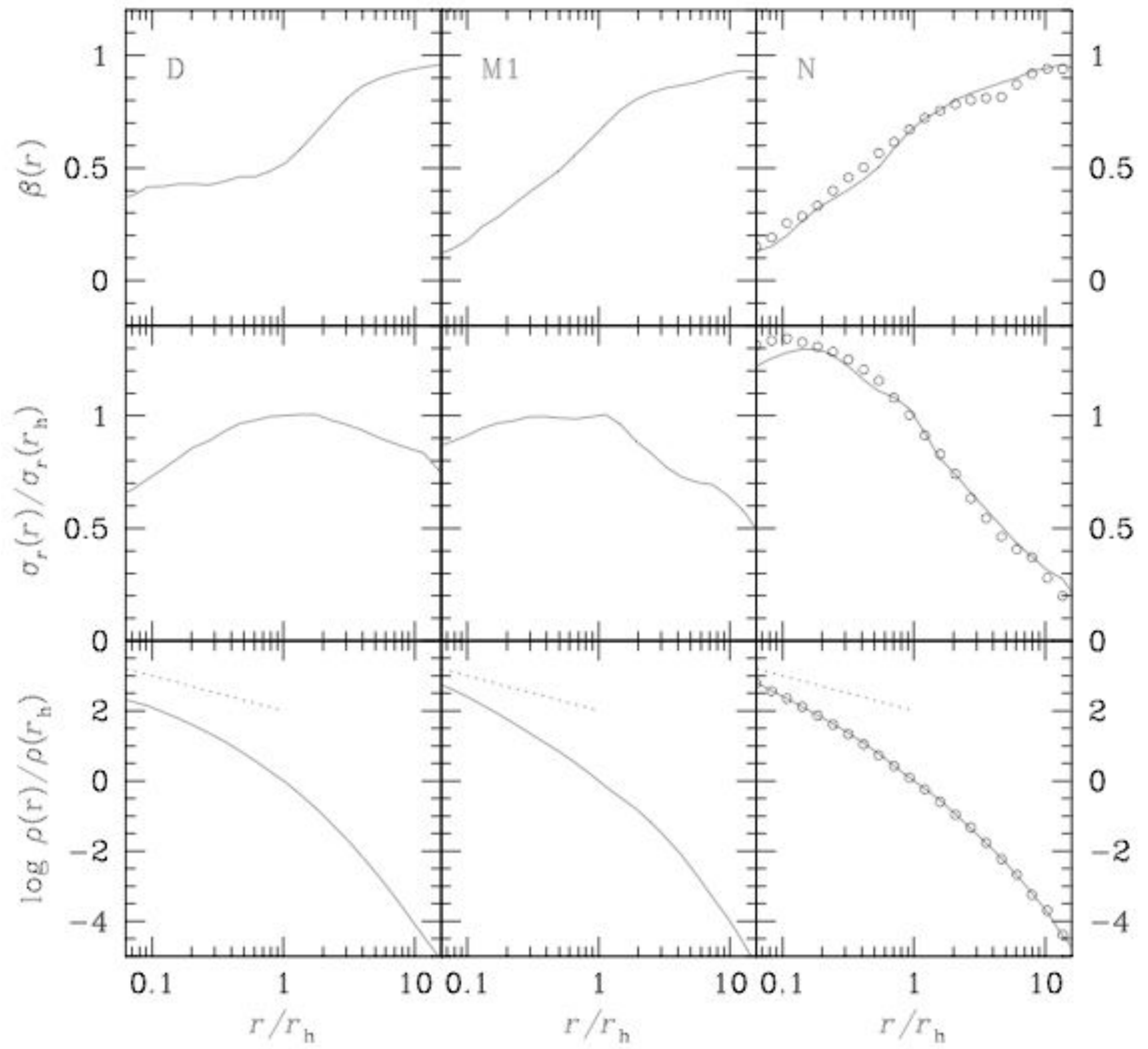


VIRIAL

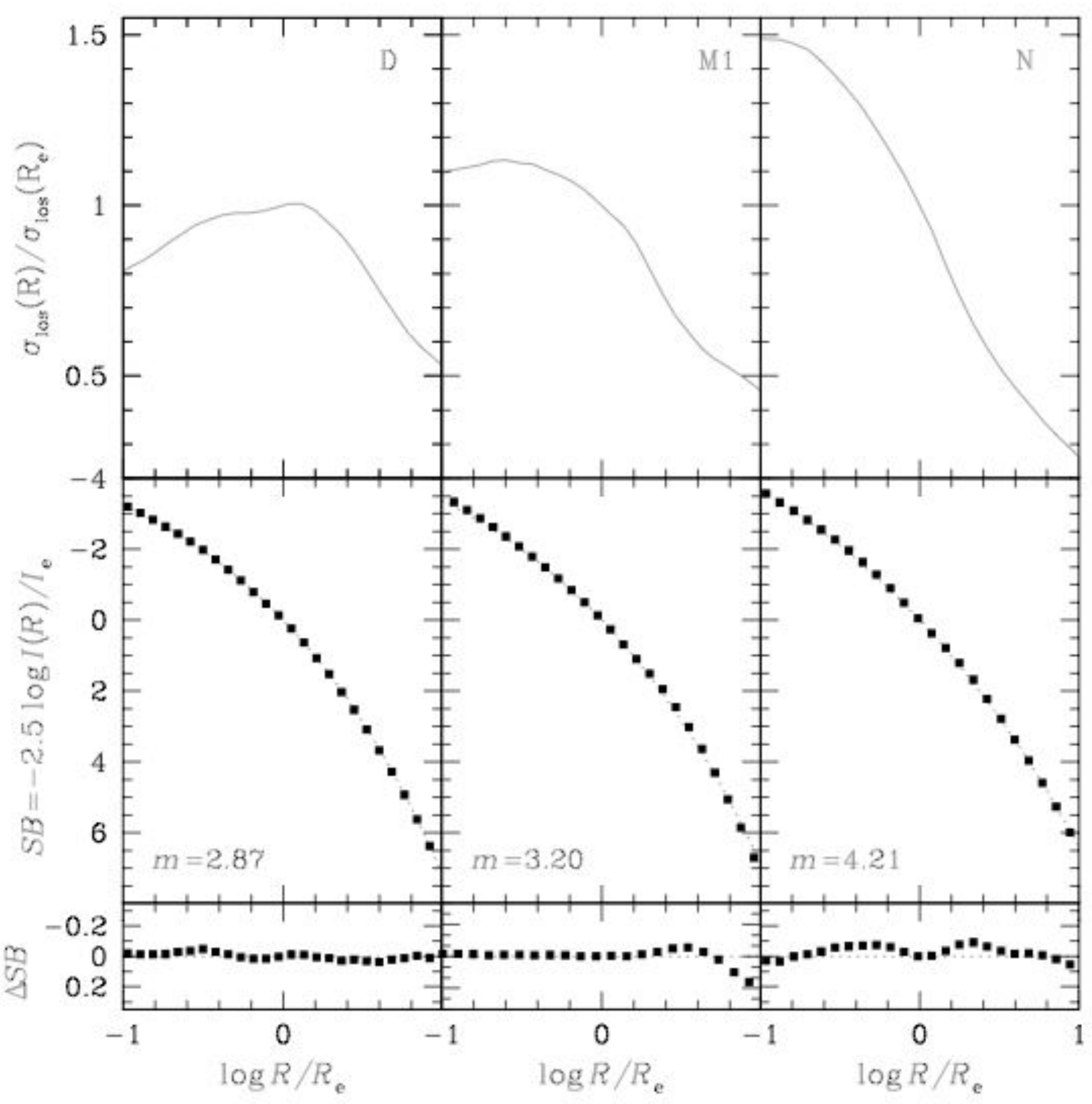
W conservation

MOND
“energy”

Internal structure & dynamics



Projected properties



<u>Sersic</u>

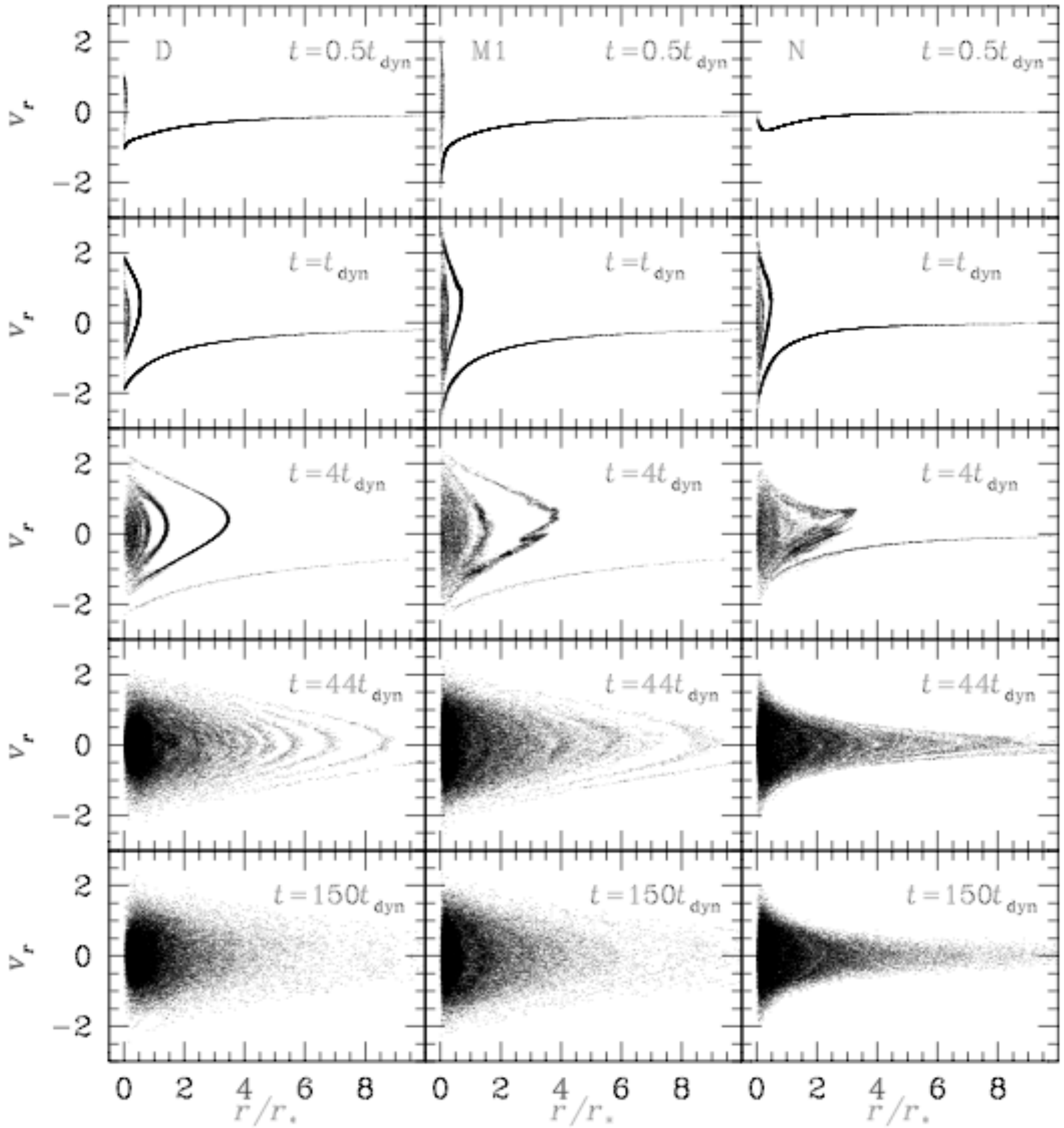
$$I(R) = I_0 \exp[-b(m)(R/R_e)^{1/m}]$$

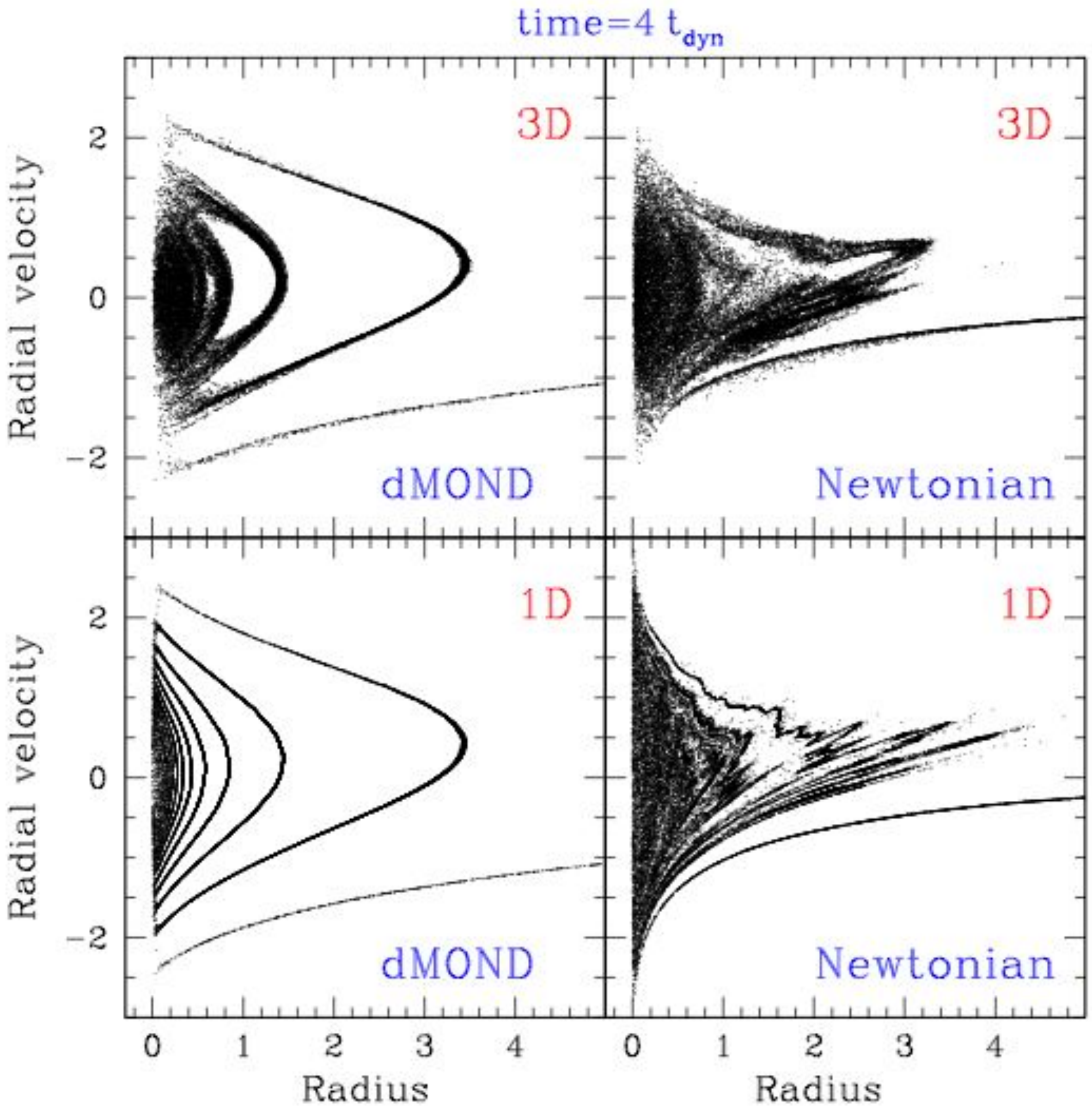
b(m)~2m-1/3+O(1/m)

(Ciotti&Bertin 1999)

Mixing properties

TIME ↓





Frozen angles