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On some enigmatic properties of relaxation functions

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The Debye model of dielectric relaxation provides us with the simplest shape of spectral functions, i.e. relations connecting characteristics of polarized dielectric medium and the frequency of the polarizing electric field. An example of such a relation is $\chi(i\omega) = (1 + i\omega\tau_D)^{-1}$ which shows how the complex susceptibility $\chi(i\omega)$ depends on ω and on a single material parameter called the characteristic time τ_D. As expected, such a spectral function transformed to the time domain leads to the exponential decay law. In contemporary dielectric physics the Debye model finds only a little application and to fit the experimental data it is usually replaced by phenomenologically rooted modifications among which the Cole-Cole, Cole-Davidson, Havriliak-Negami and "excess wings" models play dominant role. The first three of them, like the Debye pattern, still depend on a single characteristic time τ^* but involve insertion of fractional powers $0 <, \beta < 1$ according to $\chi(i\omega) = (1 + (i\omega\tau_D)^{\alpha})^{-\beta}$. Meanwhile, the "excess wings" model introduces additional characteristic times, e.g. through the ansatz $\chi(i\omega) = (1 + (i\omega\tau_2)^{\alpha})/(1 + (i\omega\tau_2)^{\alpha} + i\omega\tau_2)$. All the above mentioned spectral functions, if treated as functions of a complex variable z, share a common behaviour –for suitably adjusted, but physically well justified values of parameters, they are analytic in the lower half of the complex plane and map it into the upper halfplane. On first sight this property seems negligible but appears to have important mathematical consequences. Functions which obey it have uniquely determined integral representations given as weighted sums of Debye's spectral functions (in mathematics equivalent to the so-called Stieltjes transforms of such weight functions) and if transformed to the time domain and taken for $t > 0$ are completely monotone, i.e. representable as Laplace transforms (exponential decay law) of nonnegative locally integrable functions. This suggests that widely accepted models of non-Debye dielectric relaxation phenomena may be obtained through summing up elementary Debye relaxations with different, continuously distributed characteristic times. In the talk we are going to provide arguments if this statement should be treated as a mathematical artefact or physically based conjecture.

[1] R. Garrappa, F. Mainardi, and G. Maione, Frac. Calc. and Appl. Anal. 19, 1105 (2016) [2] K. Górska, A. Horzela, Ł. Bratek, G. Dattoli, and K. A. Penson, J. Phys. A: Math. Theor. 51, 135202 (2018).

Summary

Topic

1. Diffusive dynamics

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