# Hydrodynamic simulations of a capillary plasma discharge

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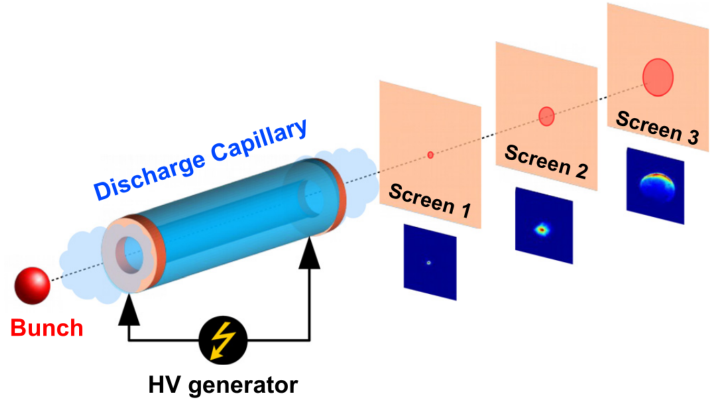
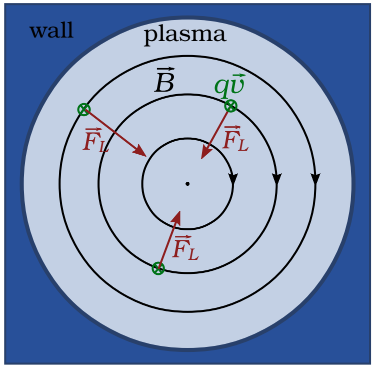
# Contents

What is an active plasma lens (APL)*?

**It is a device which can focus an electron beam, thanks to an azimuthal magnetic field generated by a discharge current induced in a gas-filled capillary**




Scheme of principle of an active plasma lens [*courtesy of R. Pompili*]



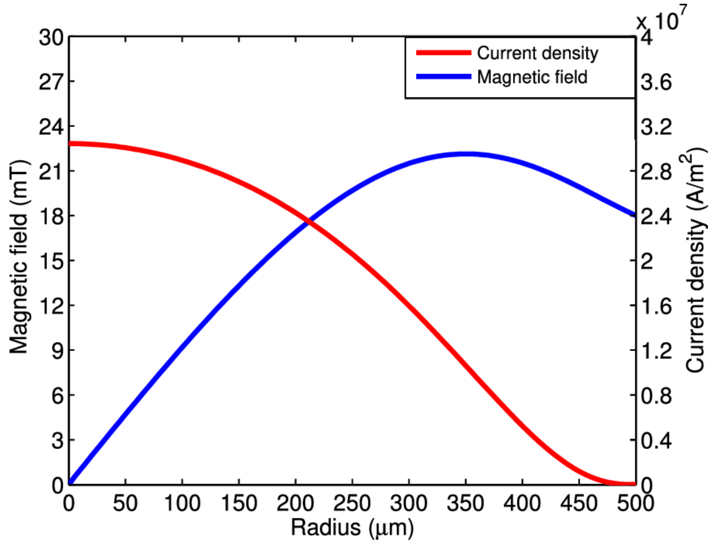

Schematic transverse view of an active plasma lens

- Interesting features:
  - Magnetic field gradients even higher than those achievable in permanent magnet quadrupoles, up to several kT/m
  - Compact and capable of focusing a beam in both transverse planes
  - The focusing strength, K, scales as 1/γ

*W.K.H. Panofsky and W.R. Baker, Rev. Sci. Instr. 21, 445 (1950)

# 1. Motivation: Problems related to APLs

- Magnetic field may have a non linear radial dependence, which produces **aberrations,** causing an *increase of both emittance growth and minimum achievable spot size*.
  - Current density may concentrate on the capillary axis, where the resistivity is lower ( $\eta_{plasma} \propto T^{-3/2}$ )
  - Non satisfactory transverse profile of $B_\theta$

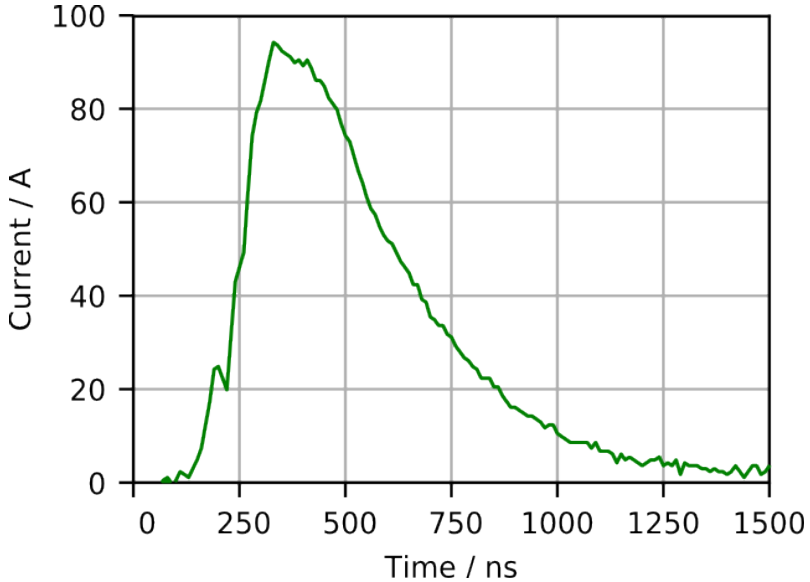


[reprinted from Appl. Phys. Lett. 110, 104101 (2017)]

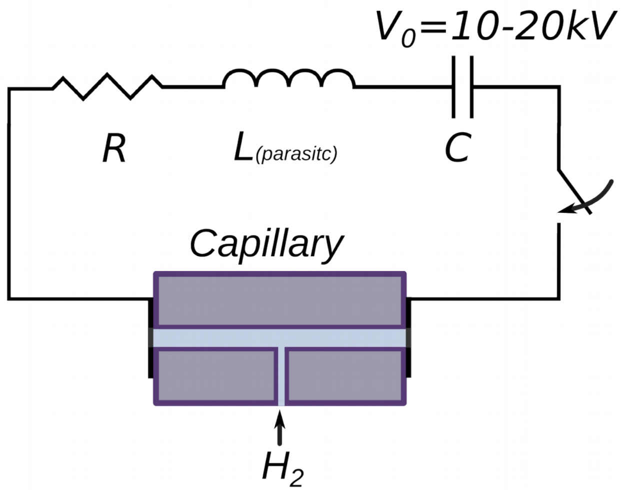
- Undesired **passive plasma lensing** may occur.
  - If the electron density is in certain ranges, beam emittance grows significantly
  - This is very likely to happen on the plasma plums coming from the capillary extremities

- Made of printed plastic or sapphire (with external support of printed plastic)
- Filled with hydrogen (≈$10^{-6}$-$10^{-7}$g/cm$^3$)
- Typical dimensions:
  - Diameter of the aperture: 1-2 mm
  - Length: 1-3 cm

Capillary, made of printed plastic




Example of time profile of the current during a discharge




Scheme of the electric circuit powering the gas discharge.

# 3. Simulations: model for 2D simulations

- A fluid model is employed $\leftarrow$ *Kinetic approach is computationally impractical*

- The plasma is studied as a hot gas which evolves because of the thermal pressure (**hydrodynamic** approximation)
- Main source of heat is the **ohmic dissipation**
- A divergence-free current density is computed with a time-varying static potential (**static current flow** case)

$$\begin{cases} \frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\vec{v}) = 0 \\ \frac{\partial}{\partial t}(\rho\vec{v}) + \nabla\cdot(\rho\vec{v}\vec{v}) = -\nabla p \\ \frac{\partial}{\partial t}E + \nabla\cdot(E\vec{v}) = -\nabla\cdot(p\vec{v}) + \nabla\cdot(\kappa\nabla T) + \eta\|\vec{J}\|^2 \\ \nabla\cdot\vec{J} = 0,\ \vec{J} = -\frac{\nabla V}{\eta} \end{cases}$$

$\rho$: mass density
$\vec{v}$: fluid velocity
$p$: thermal pressure
$E$: total energy density
$\kappa$: thermal conductivity
$T$: plasma temperature
$\vec{J}$: current density
$V$: electric potential
$\eta$: electrical resistivity

Further remarks:
- Lorentz force on the fluid is neglected
- No self consistent magnetic field is present $\rightarrow$ skin effect cannot be observed

# 3. Simulations: model for 2D simulations

Additional details:

- An ionization model exploiting a **local thermodynamic equilibrium** approximation is suitable (we use Saha's equation):

$$z(z,r,t) = f(T(z,r,t), \rho(z,r,t)), \quad z\text{: ionization degree}$$

- We include the ionization energy contribution to the internal energy:

$$E_{\mathrm{int}} = \frac{3}{2}\frac{\rho}{m_{\mathrm{p}}}(1+z)k_{\mathrm{B}}T + \frac{\rho}{m_{\mathrm{p}}}\chi z$$

- Electron scattering with neutral H is included in computation of $\kappa$ and $\eta$:

$$\kappa = \kappa(n_e, T, \nu_{ei} + \nu_{en}) \qquad \nu_{\mathrm{en}} = n_{\mathrm{n}}\sigma_{\mathrm{en}}\langle v_{\mathrm{e}}\rangle$$

$$\eta = \eta(n_e, T, \nu_{ei} + \nu_{en}) \qquad \sigma_{\mathrm{en}} \approx 1.4 \cdot 10^{-15}\mathrm{cm}^2$$

$n_{\mathrm{e,n}}$: electron,neutral H density

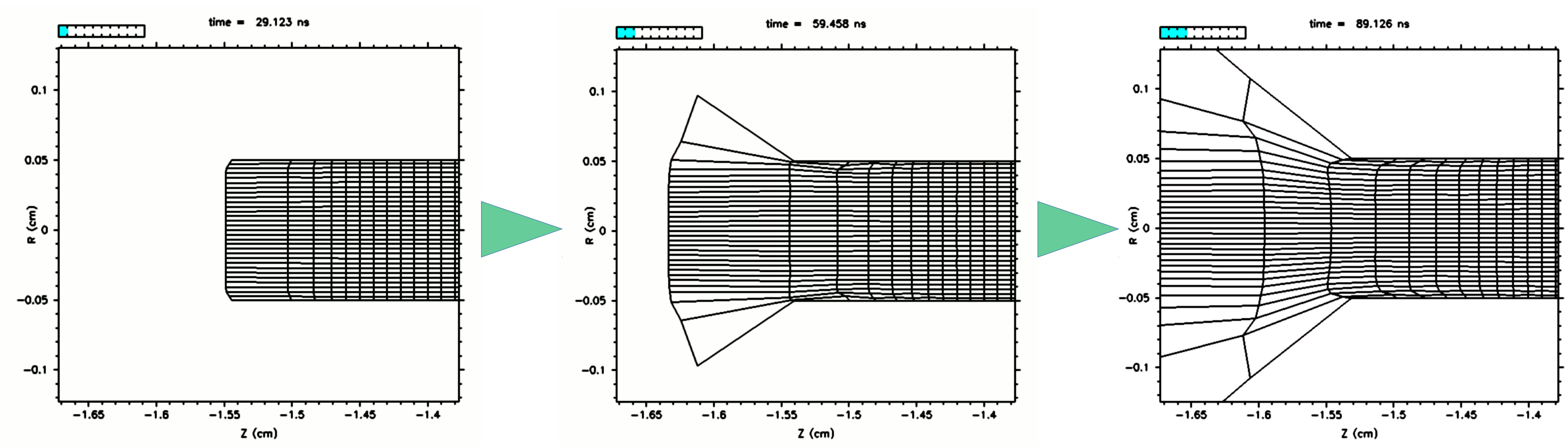
$\nu_{\mathrm{ei,en}}$: e-i,e-n collision freq

$\langle v_{\mathrm{e}}\rangle$: avg. electron velocity

- Magnetization of electrons (and ions) is neglected

Additional details:

- The capillary has a circular cross section → **2D axial symmetry** is employed
- **Lagrangian approach** (the mesh moves to follow the matter):
  - **advantage**: no need to mesh the whole domain, as it would be in Eulerian approach
  - **disadvantage**: *grid pathologies* have to be controlled with a rezoning scheme

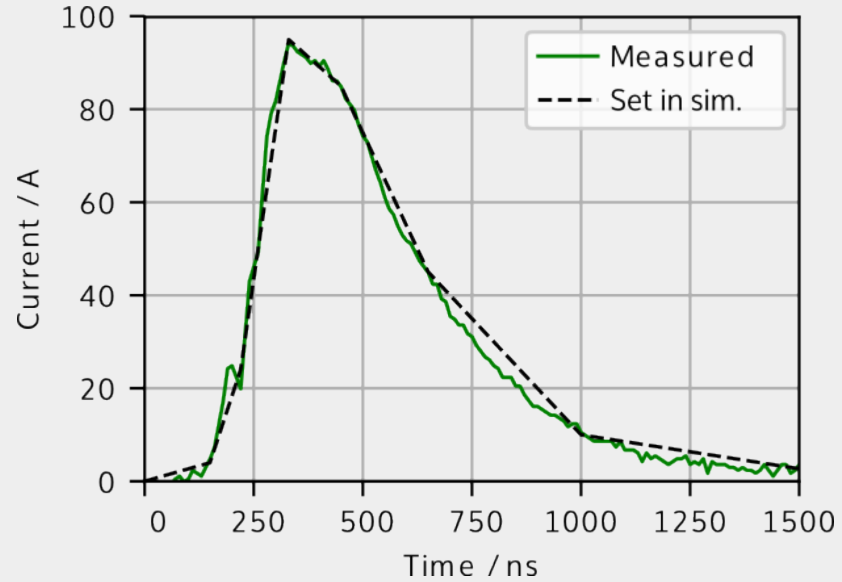



Typical evolution of the mesh in a hydrodynamic simulation (with DUED)

# 3. Simulations: preliminary results of 2D simulations

A suitably modified, reduced version of the code **DUED***, that implements the above model, has been used to simulate the discharge in a realistic capillary

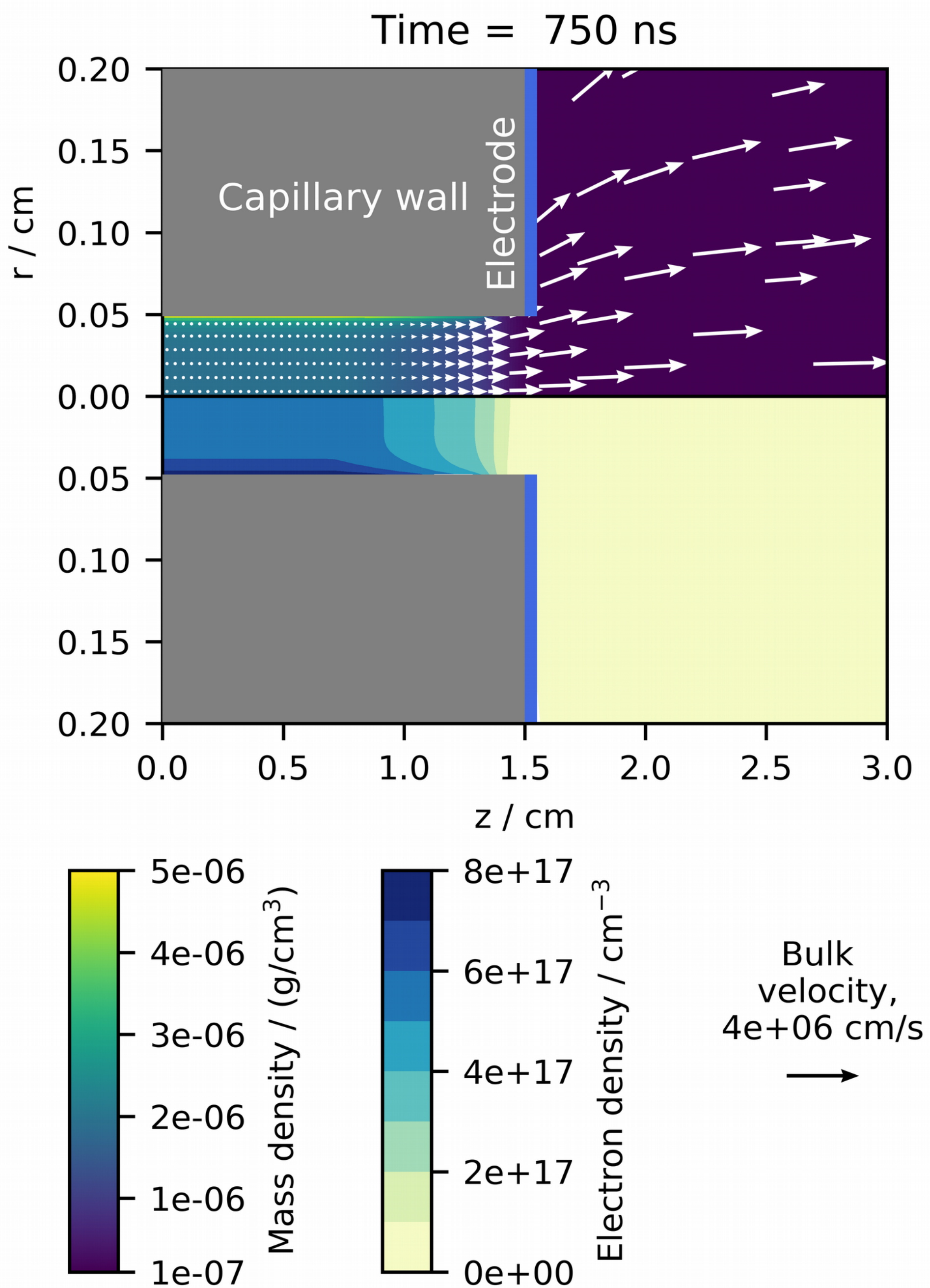
*Discharge configuration:*

| | |
|---|---|
| Capillary diameter | 1 mm |
| Capillary length | 3 cm |
| Initial gas density | $2.5 \cdot 10^{-6}$ g/cm$^3$ ($n_H = 1.5 \cdot 10^{18}$ cm$^{-3}$) |
| Initial gas temperature | 4000 K (0.34 eV) |
| Current profile |  |



*Atzeni et al., Comput. Phys. Commun. 169 (2005) 153

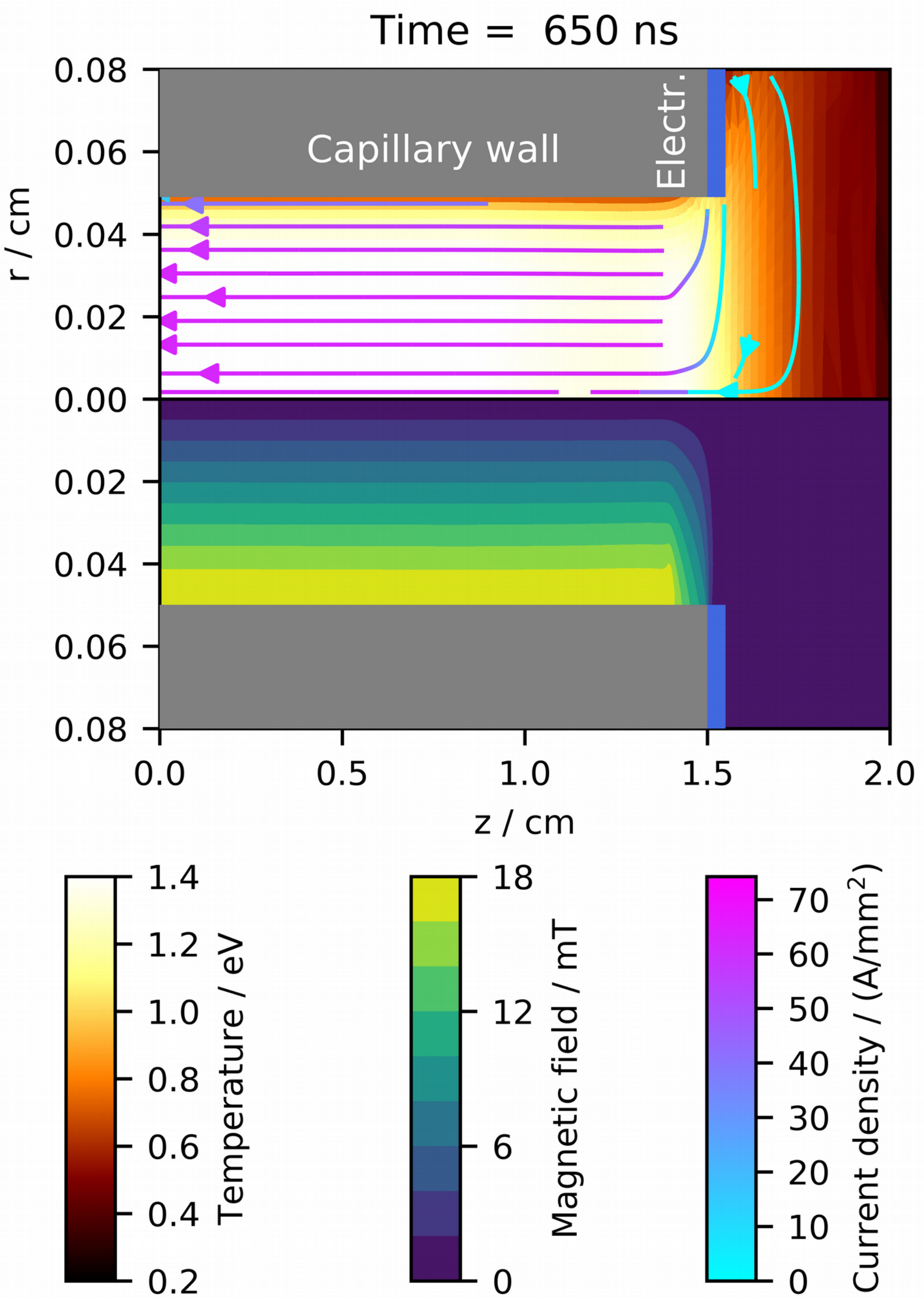
# 3. Simulations: preliminary results of 2D simulations




- The outflow of hot gas from the extremities of the capillary can be observed
- The plasma electron density ramps up from a value of less than $10^{17}$ cm $^{-3}$ near the capillary extremity, to $6 \cdot 10^{17}$ cm $^{-3}$ at the center of the capillary
- We will validate the simulation results by comparing the calculated electron density maps with longitudinally and time resolved measurements obtainable by means of the Stark Broadening technique*
- Computed electron density distribution can be used as realistic input for plasma-wakefield codes (e.g: Architect)

*F. Filippi et al, J. Instrum, 11(09), C09015 (2016)

# 3. Simulations: preliminary results of 2D simulations

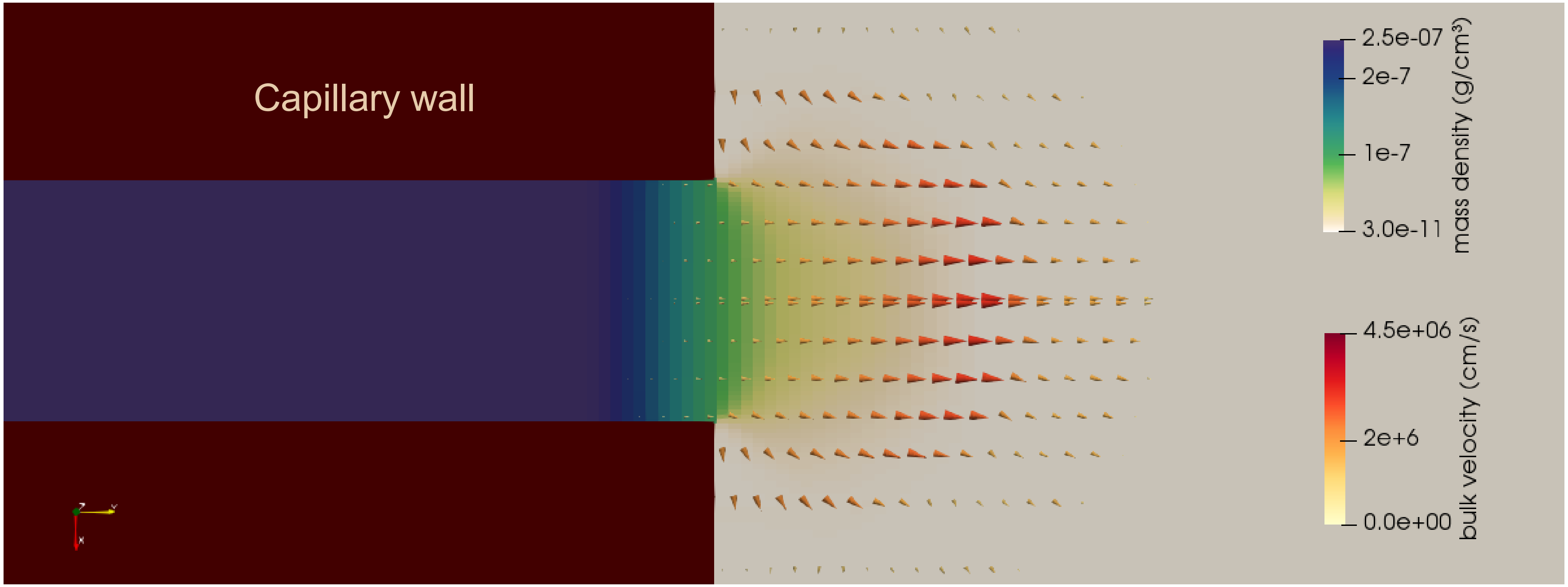



- It is possible to compute the magnetic field as post-processing
- Almost no magnetic field is present outside from the capillary
- The temperature reached by the plasma seems to be in qualitative agreement with what expected, also the current density intensifies where the temperature is higher

# 4. Upcoming

*Another computational environment* for simulating capillary discharges is under development

- Inside the MHD code **PLUTO*,** a new module is being implemented, to include an implicit method for thermal conduction and magnetic field diffusion
- Key advantage: Eulerian approach → no grid pathologies → more robustness with respect to geometry and B.C./I.C. changes
- Physics model: same as before, + Lorentz force, + Faraday law for magnetic field




*Mignone et al.,The Astrophysical Journal Supplement Series, Vol. 170, Iss. 1, pp. 228-242

# 5. Conclusions and outlook

- I have shown the working principle of an active plasma lens
  - Active plasma lenses are promising technologies
  - Non-uniform discharge current and plasma plums can worsen their performance
- The need for 2D simulations is being addressed
  - Hydrodynamic model with joule heating driven by a static current flow
  - Ionization degree computed with local thermodynamic equilibrium approximation
  - Lagrangian approach
  - Preliminary results available
- Future steps:
  - Final development of simulation environment with an Eulerian code
  - Validation of the results with experimental data
  - Optimization of the design parameters in order to improve the focal properties of the lens