15 March 2018

1) The JUNO sensitivity to MH is null

(when the $\Delta\chi^2$ estimator is used)

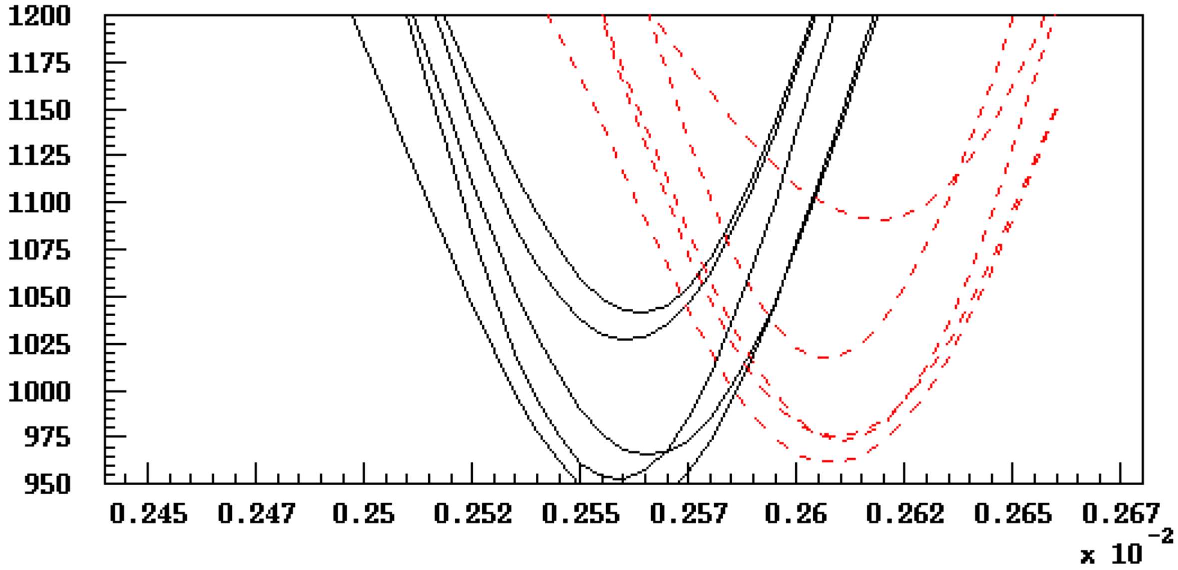
2) The extended-F estimator

(when any couple of Δm^2_{atm} are used)

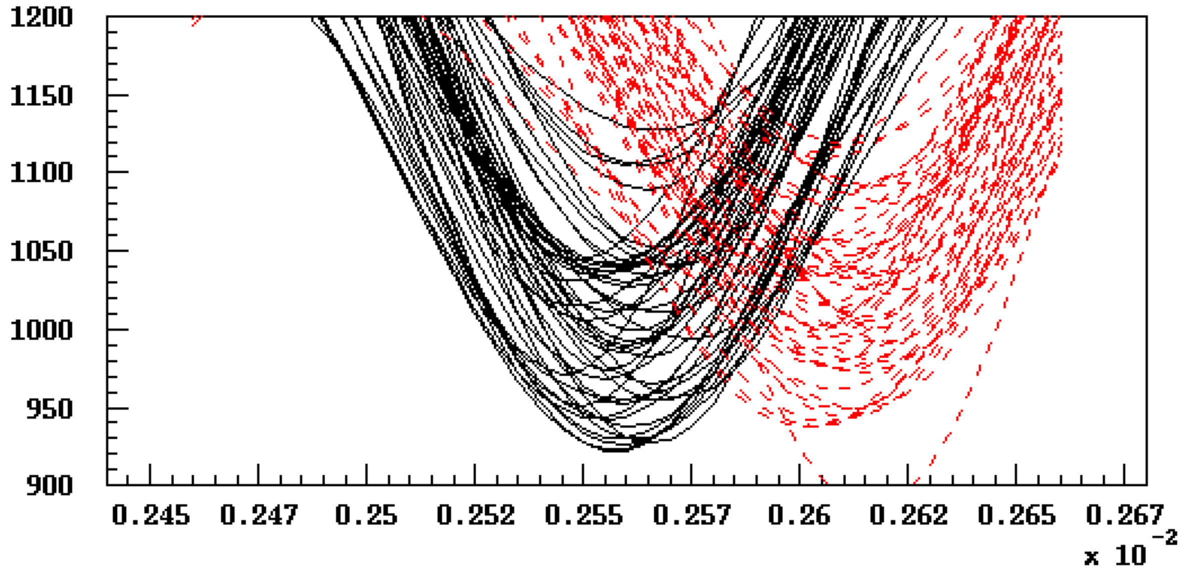
Luca Stanco, JUNO-Italy meeting, Frascati, 15 March 2018

Reproduce JUNO evaluation for MH doing an event-by-event simulation

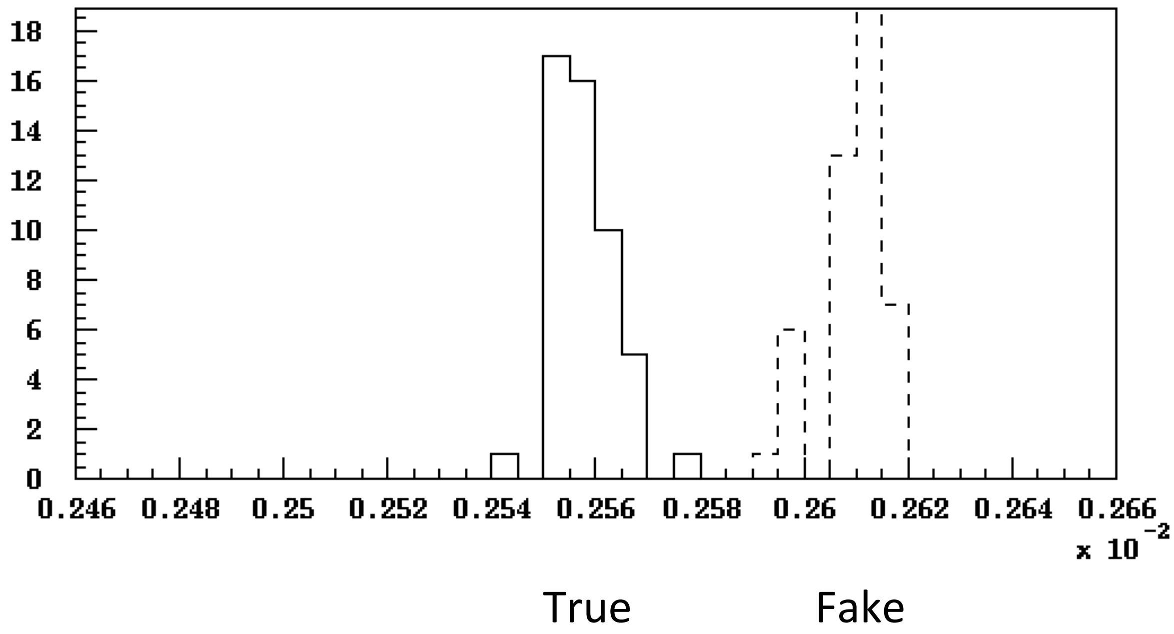
5 toys



50 toys

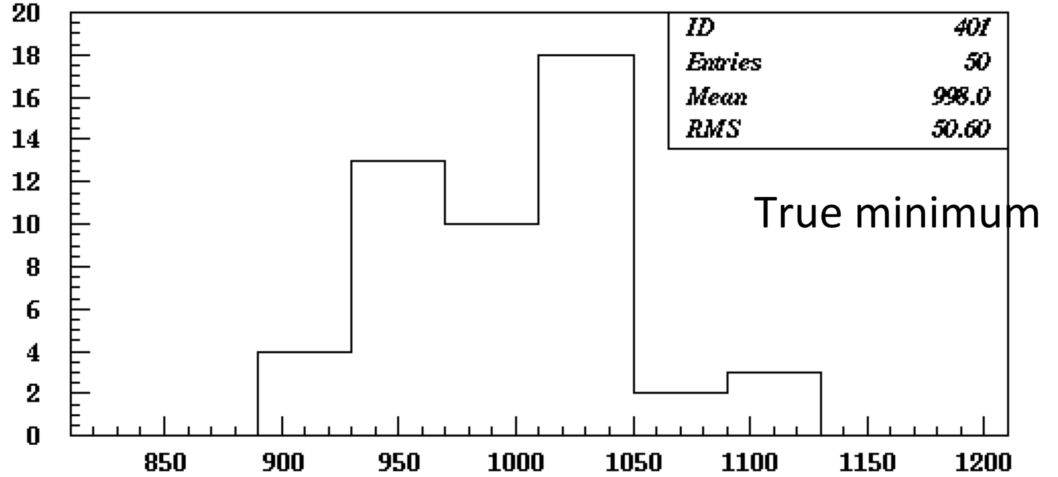


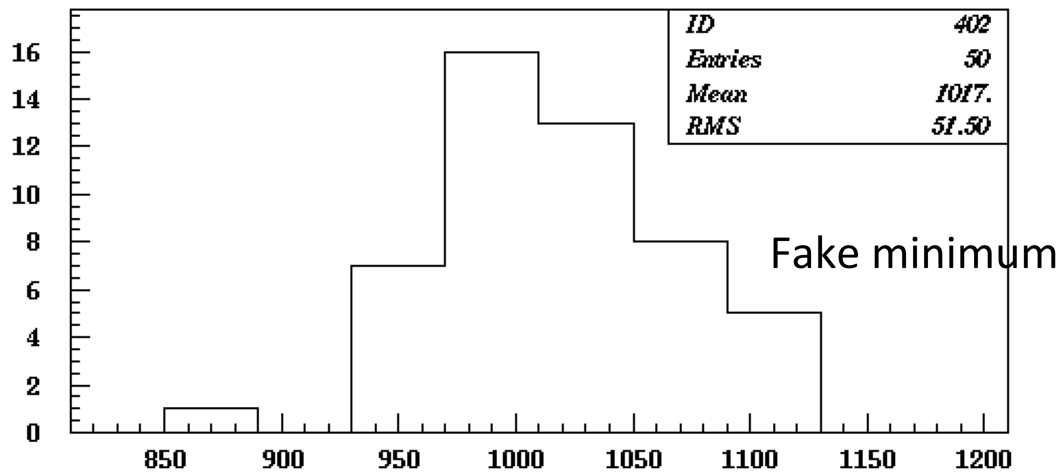
The two minima in Δm^2_{atm}



They are ok: centered at the right (expected values), good resolution

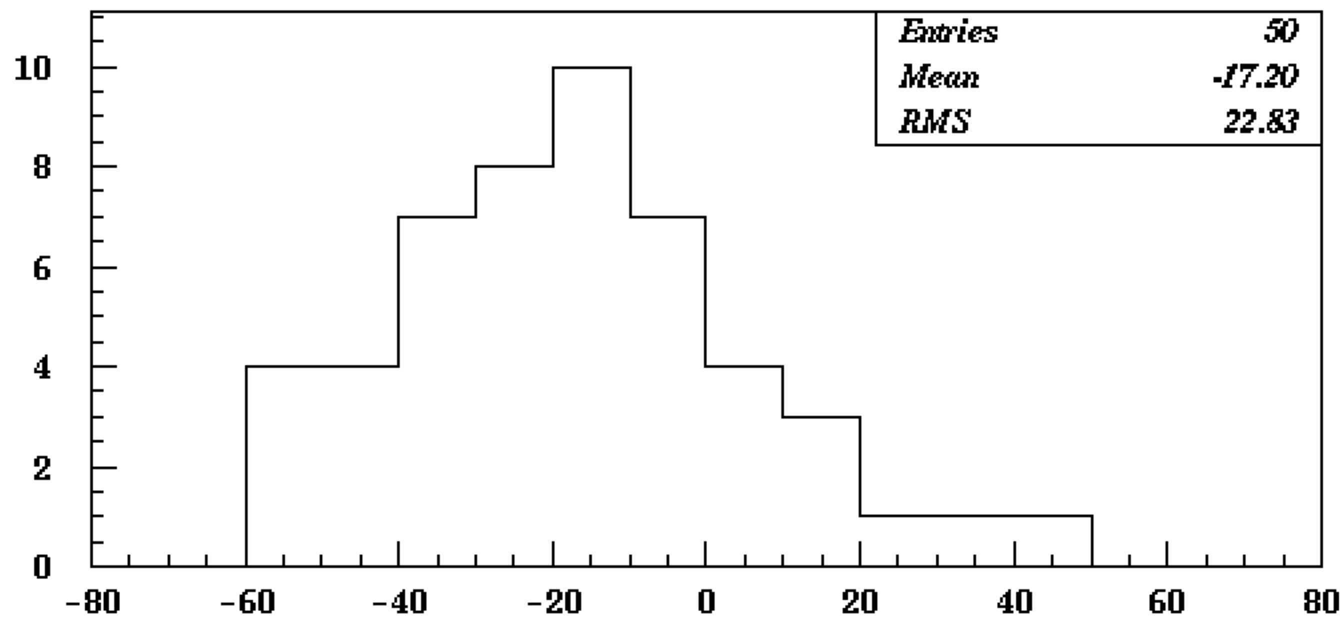
The two χ^2 minima seem more problematic





The two χ^2 minima are correlated: the central value seems ok

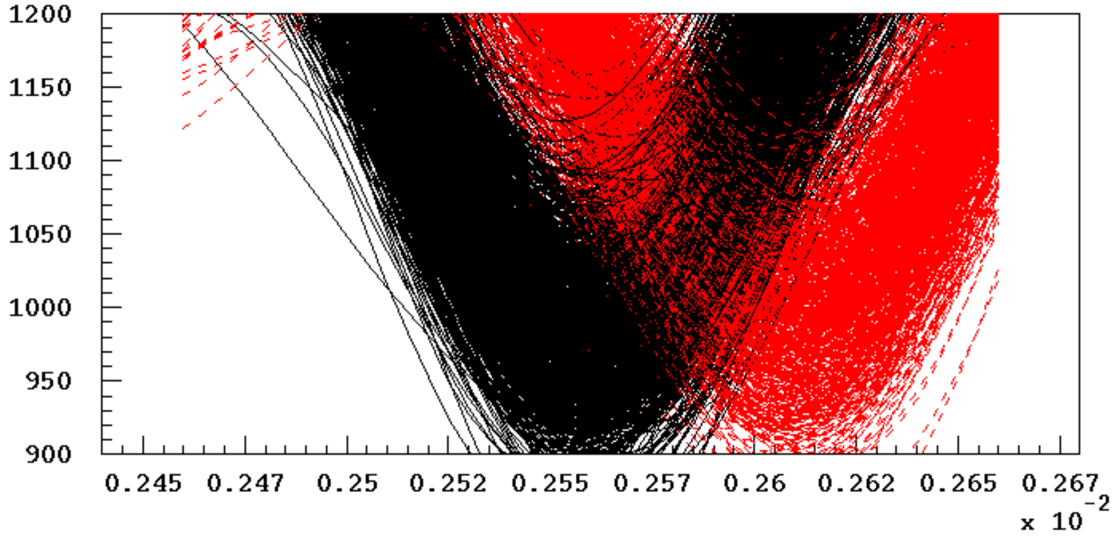
$$\Delta\chi^2_{MH} = \chi^2_{\min}(NH) - \chi^2_{\min}(IH)$$



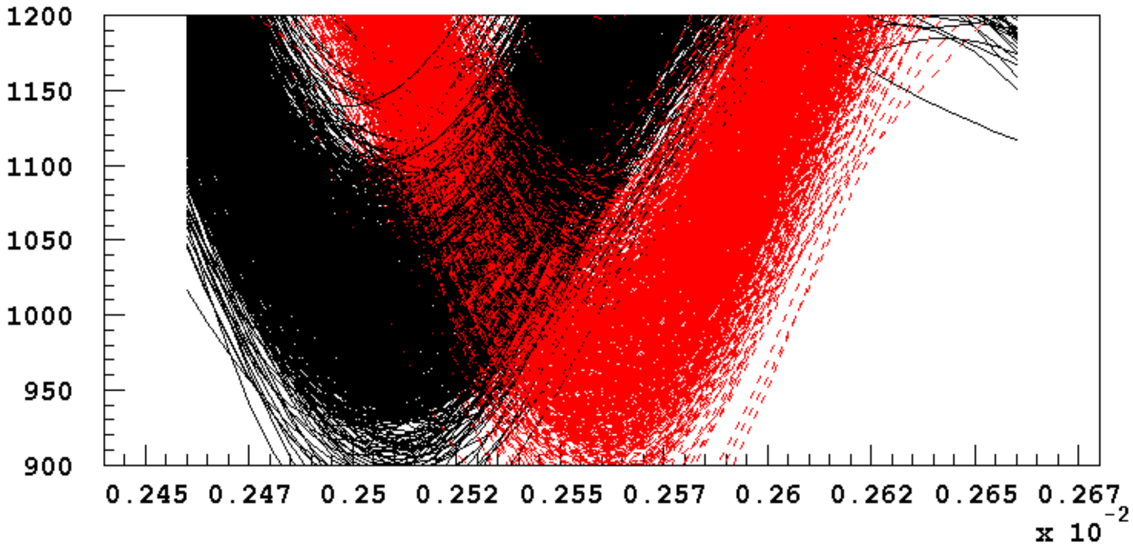
A heavy simulation is needed !

(outcome of the full-immersion MH meeting on February 28th)

Study of the $\chi 2$

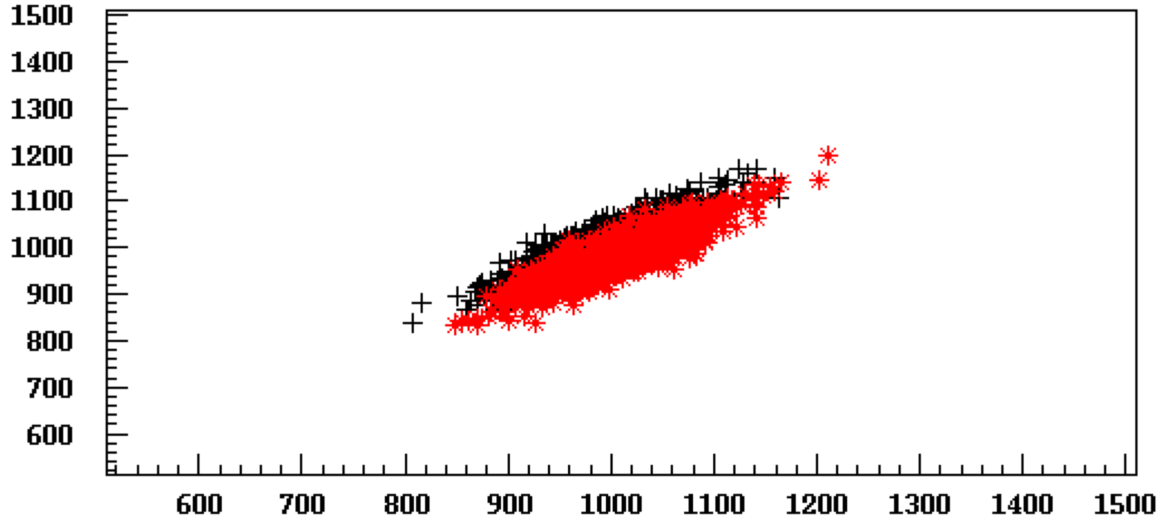


NH generation, 1000 toys



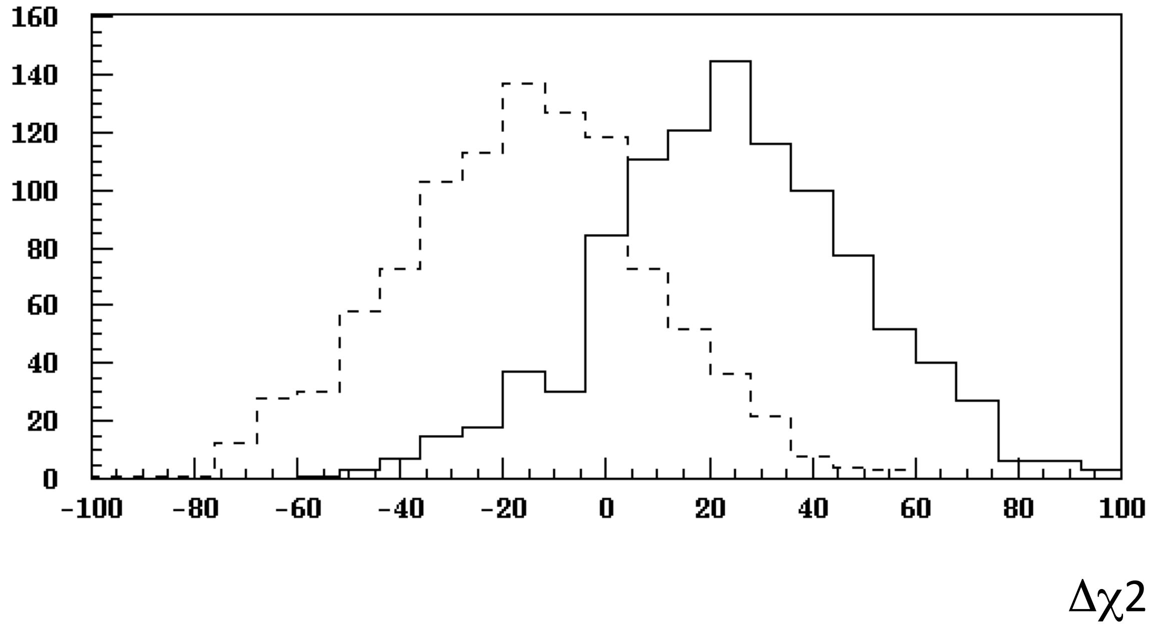
IH generation, 1000 toys

Several days of a server CPU system



1500
1400
1300
1200
1100
1000
900
800
700
600
600
700
800
900
1000
1100
1200
1300
1400
1500

??



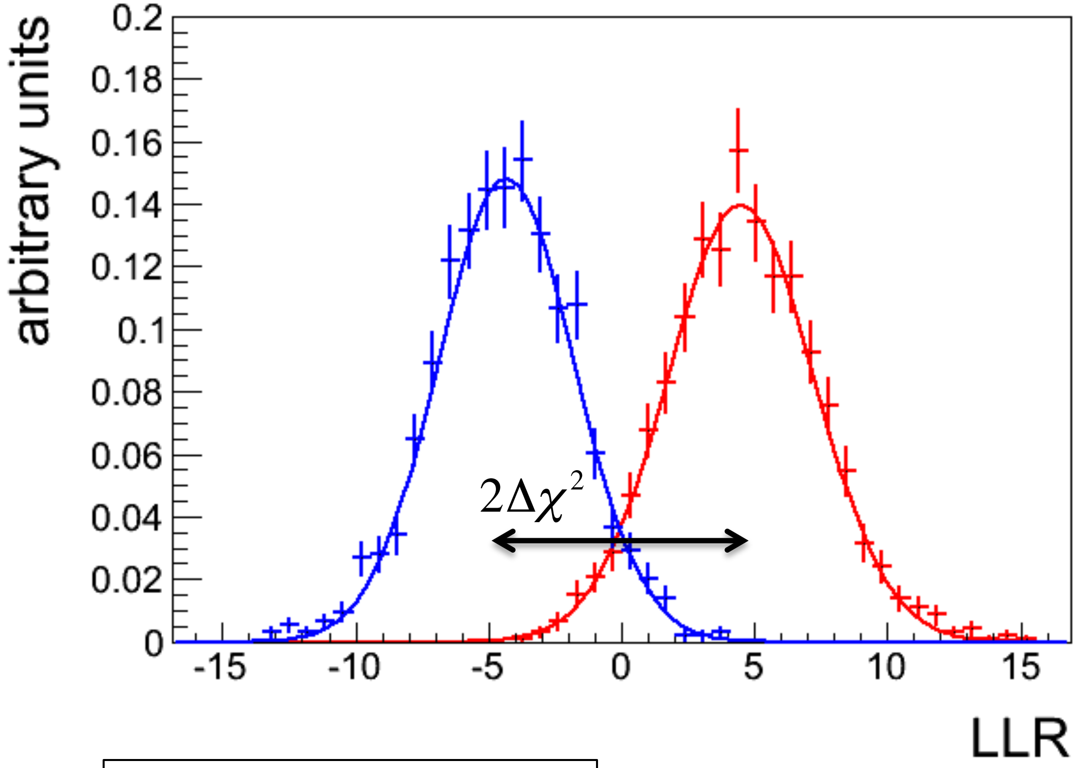
160
140
120
100
80
60
40
20
0
-100
-80
-60
-40
-20
0
20
40
60
80
100
Δχ2

In principle

$$n\sigma = \sqrt{\chi^2_{\min}(NH) - \chi^2_{\min}(NH)}$$

➔ 50% prob to reject the wrong MO with nσ

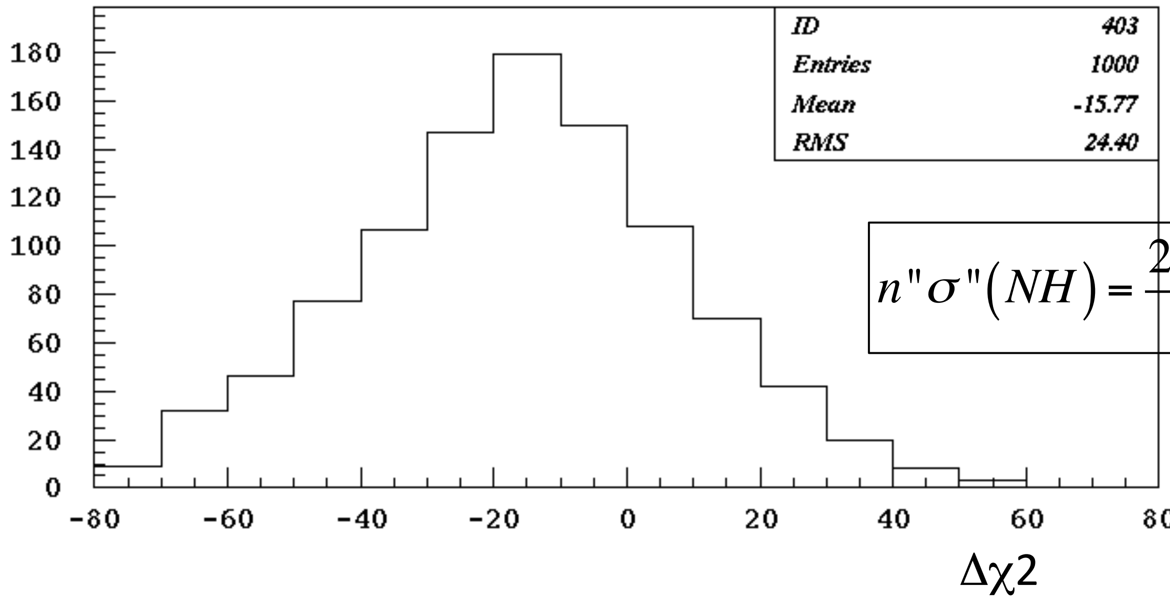
median sensitvity



$$n"\sigma" = \frac{2\Delta\chi^2}{2\sqrt{\Delta\chi^2}} = \sqrt{\Delta\chi^2}$$

But…

General result: sigma of each Gaussian = 2 √(Δχ²) (arXiv:1210.8141)

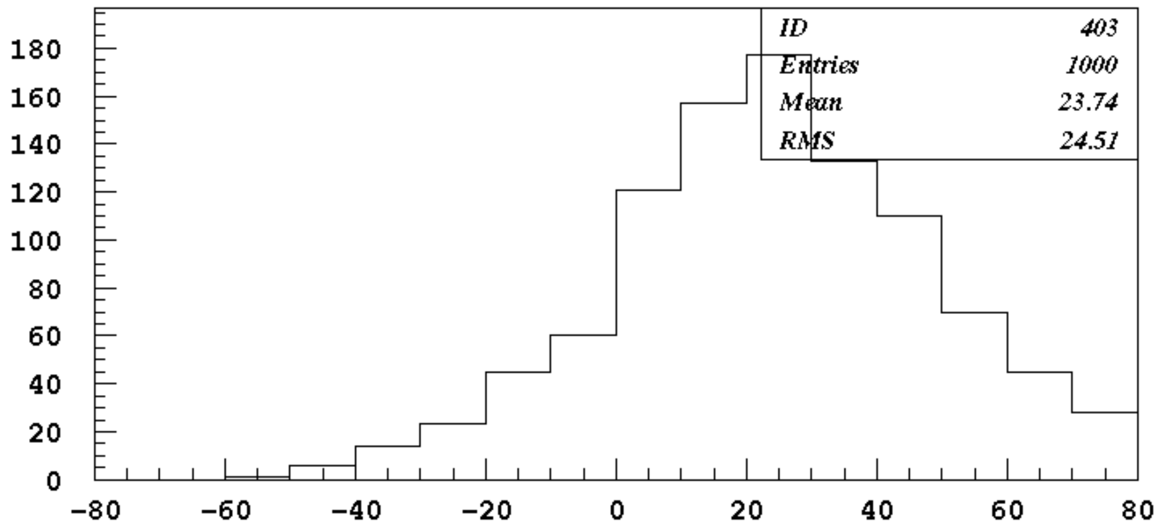


NH generation, 1000 toys

$$n"\sigma"(NH) = \frac{23.74+15.77}{24.40} = 1.62$$

The central values are not symmetric

The dispersion of each “Gauss” is not 2sqrt(Δχ2): They are much larger !



IH generation, 1000 toys

worse gaussianity fit for IH

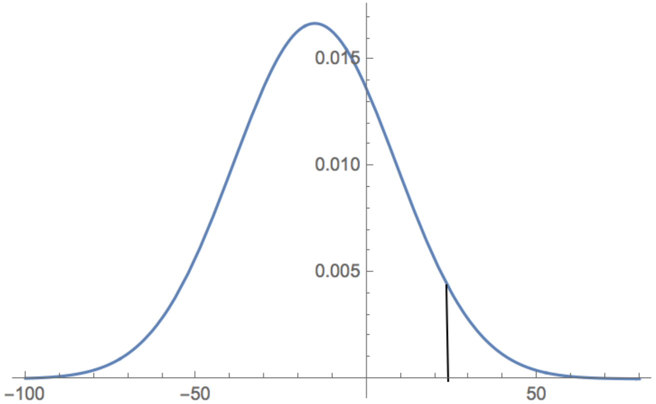
$$n"\sigma"(IH) = \frac{23.74+15.77}{24.51} = 1.61$$

p-values from the two Gaussian fits

$$\mu_{NH} = -15.21 \pm 0.77$$
$$\sigma_{NH} = 23.83 \pm 0.53$$
$$\mu_{IH} = 24.85 \pm 0.80$$
$$\sigma_{IH} = 24.54 \pm 0.68$$

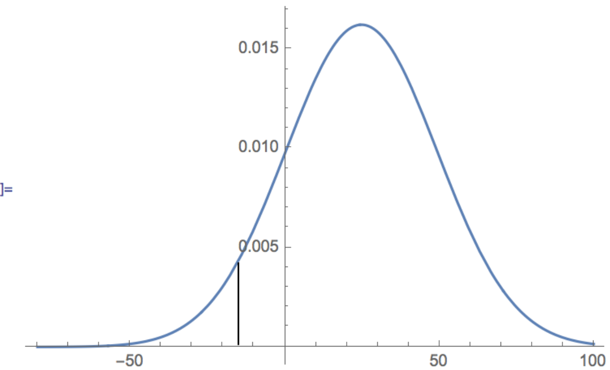
$$p-val_{NH} = \int_{\mu_{IH}}^{+\infty} G_{NH} = 4.6\%$$

$$p-val_{IH} = \int_{-\infty}^{\mu_{NH}} G_{IH} = 5.1\%$$



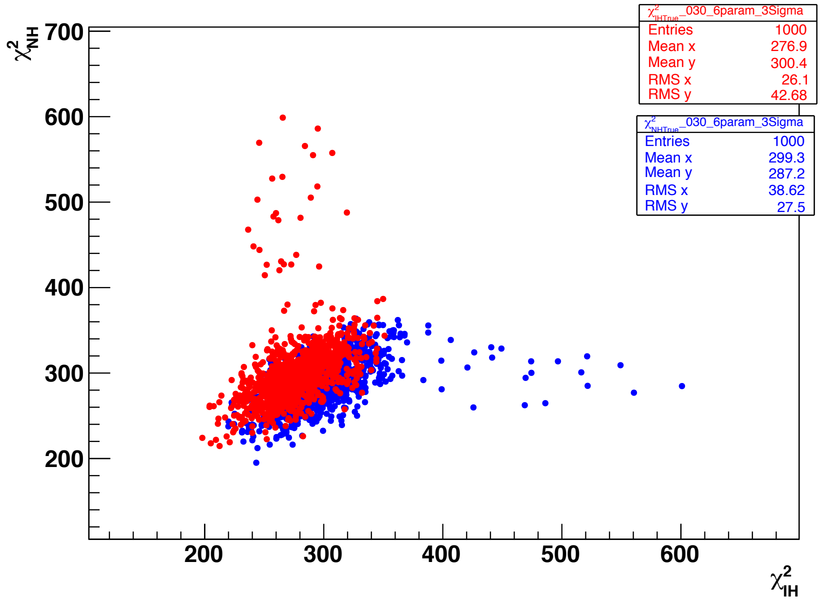

```
In[4]:= CDF[NormalDistribution[-15.21, 23.83], 24.85]
Out[4]= 0.953626

In[17]:= CDF[NormalDistribution[0, 1], 1.68]
Out[17]= 0.953521
```

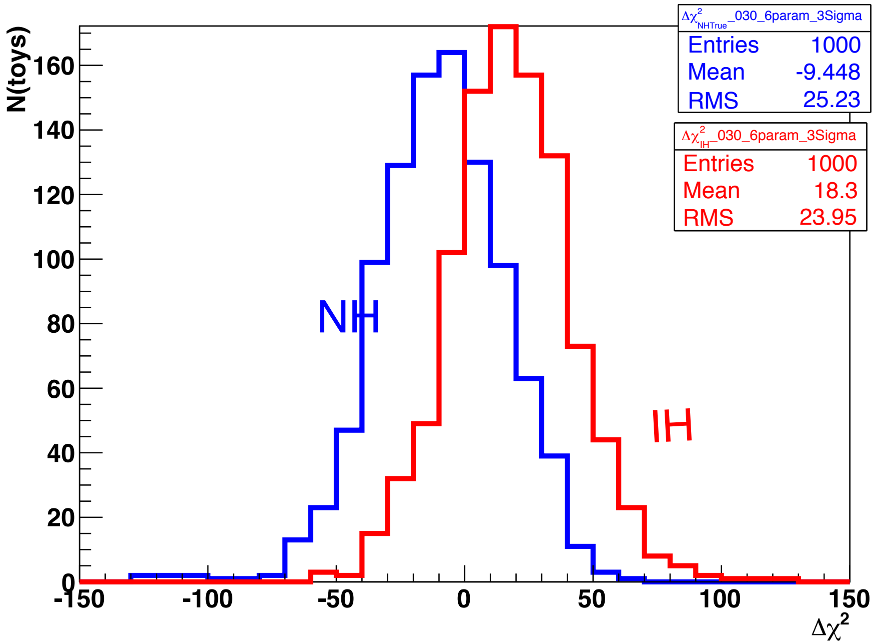
```
In[20]:= Plot[PDF[NormalDistribution[24.85, 24.54], x], {x, -80, 100}]
Out[20]=
```



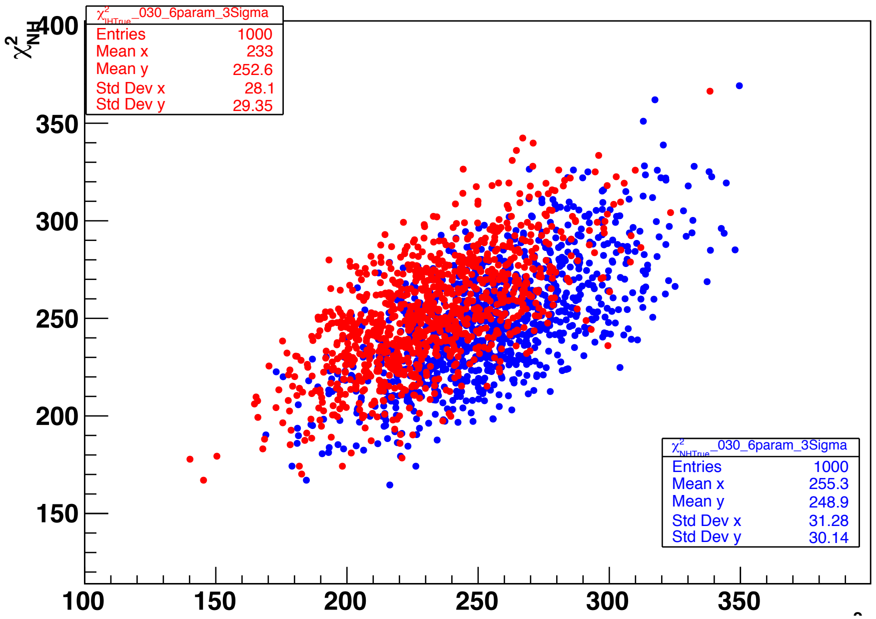

```
In[21]:= CDF[NormalDistribution[24.85, 24.54], -15.21]
Out[21]= 0.0512937

In[25]:= CDF[NormalDistribution[0, 1], -1.63]
Out[25]= 0.0515507
```

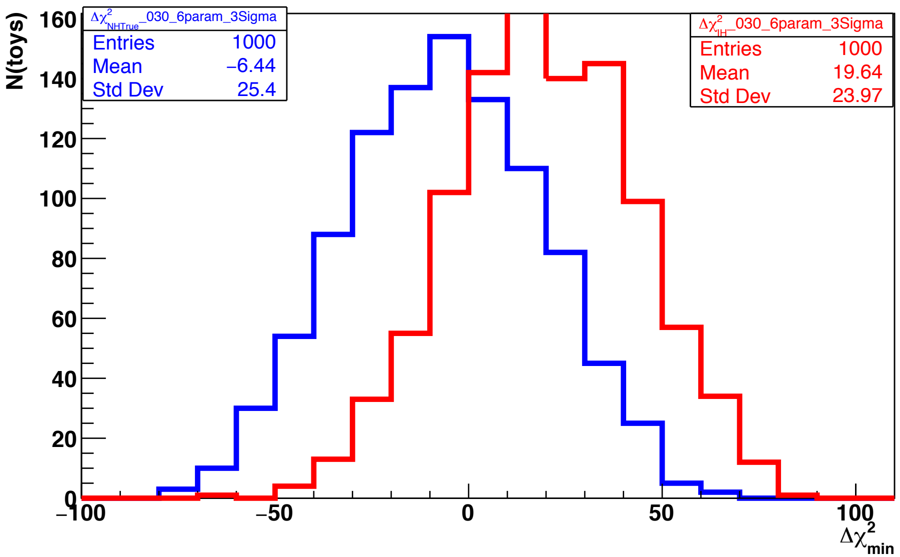
Similar results (actually worse) with an independent program, making the full minimization on $\chi 2$ (no scan). By Fatma.





And with a scan à la JUNO (by Fatma)





Note: the usual expression is usually due to not plausible assumptions

From 1210.8141 plausibly this sentence is wrong as the "error of the error" is forgotten

Due to fluctuation, N_i^{data} may deviate from their mean values $\overline{N}_i$, and the $(\Delta\chi^2)_{\mathrm{min}}$ has the statistical uncertainty. It is plausibly assumed that these fluctuation of N_i^{data} follow the Gaussian distributions with the variance $\sqrt{\overline{N}_i}$. The uncertainty of the $(\Delta\chi^2)_{\mathrm{min}}$ can

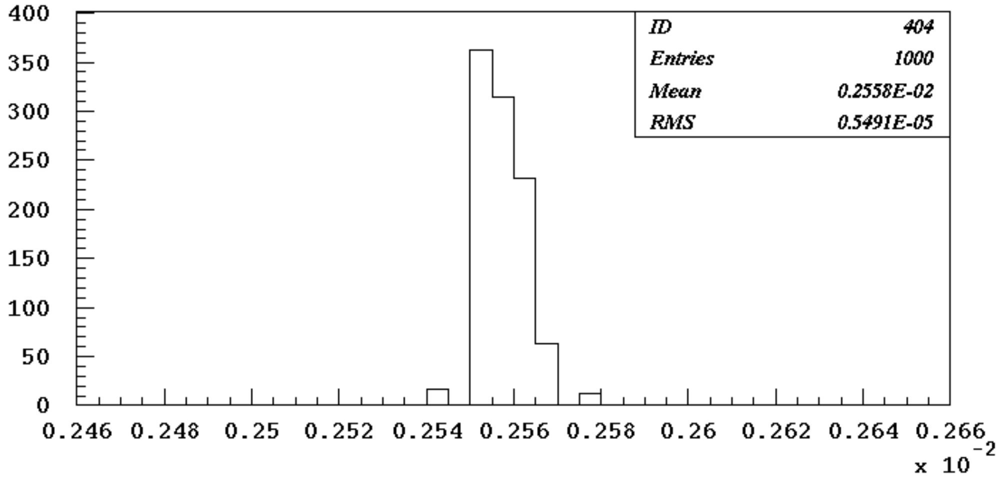
From 1305.5150 plausibly this equivalence is very approximate:
(and justified by the next paper)

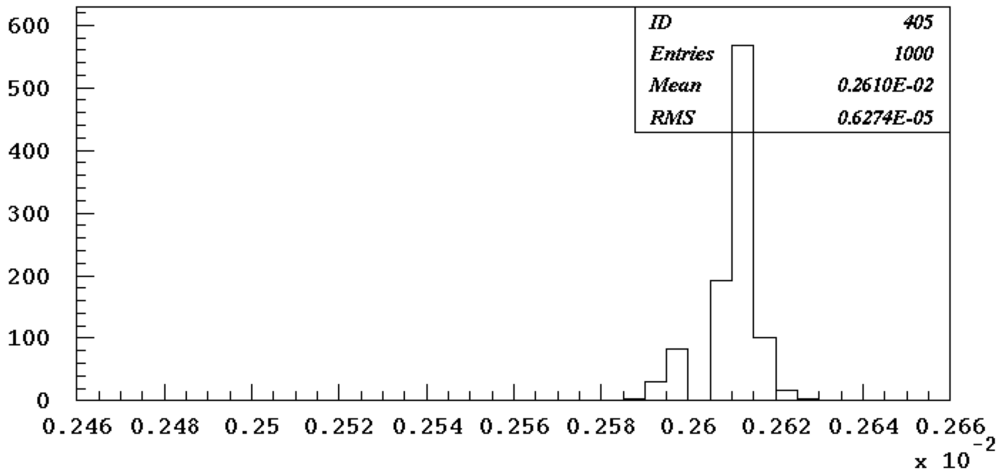
$$\sum_i \frac{2\left(y_i^N - y_i\right)}{\sigma_i} \approx \sqrt{\sum_i \frac{4\left(y_i^N - y_i\right)^2}{\sigma_i^2}}$$

From 1210.3651 plausibly this equivalence is very approximate for many reasons:

$$\sum_i \left(\frac{\left(\mu_i^{NH} - \mu_i^{IH}\right)^2}{\mu_i^{NH}} + \frac{\left(\mu_i^{NH} - \mu_i^{IH}\right)^3}{\left(\mu_i^{NH}\right)^2} \right) \approx \sum_i \frac{\left(\mu_i^{NH} - \mu_i^{IH}\right)^2}{\mu_i^{NH}}$$

(the most relevant one being the same of the first paper)



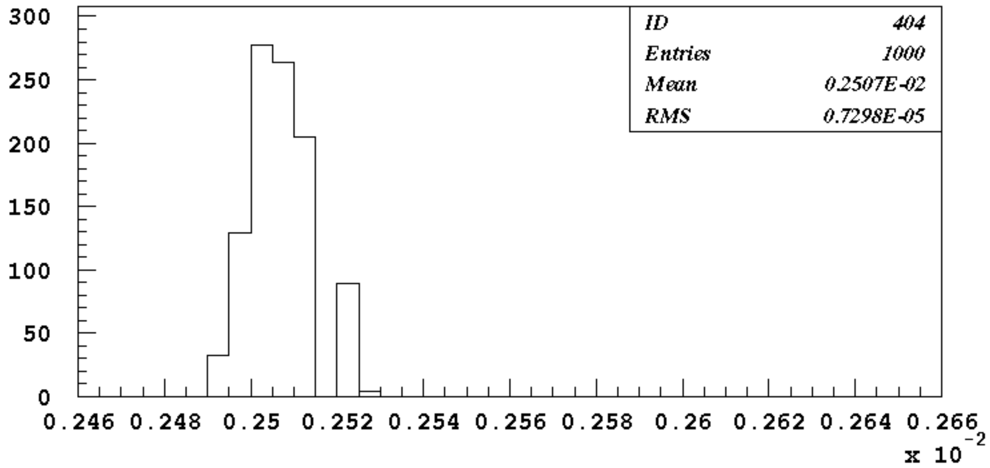


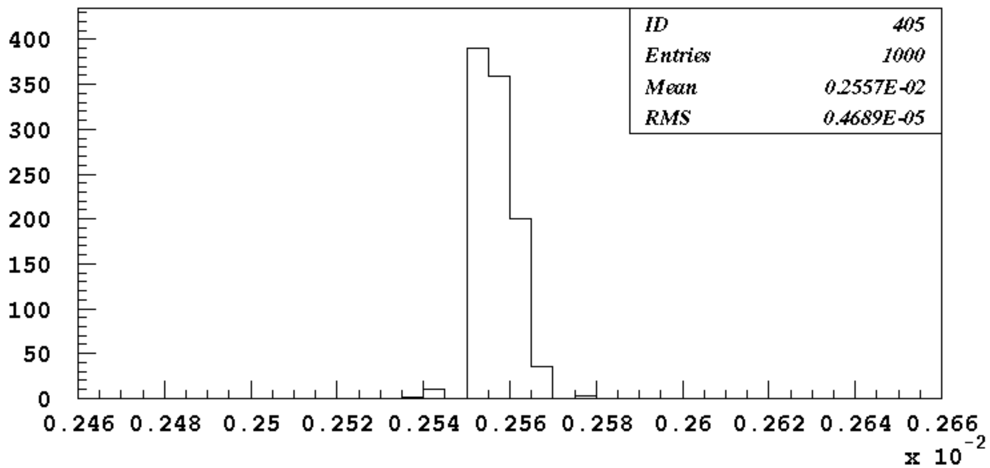
NH generation, 1000 toys

Resolution: ≈ 0.5%

Δm^2atm

IH generation, 1000 toys

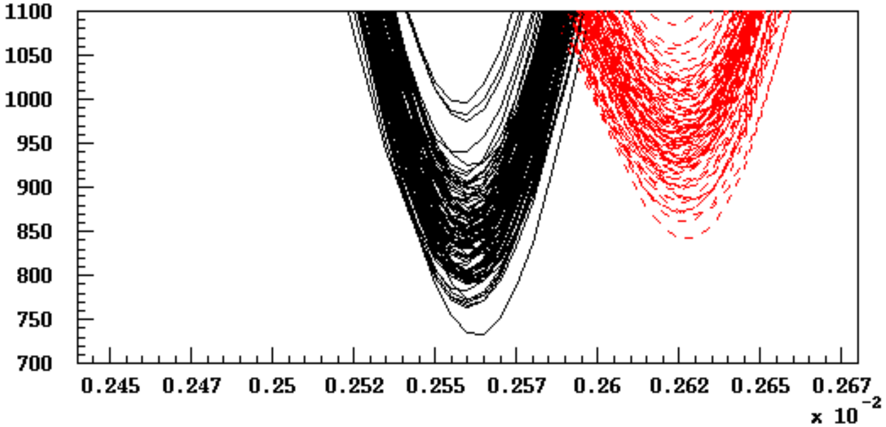


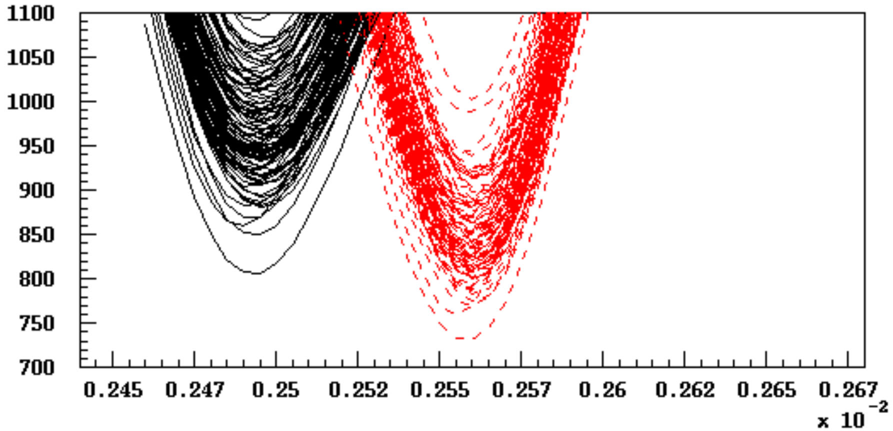


Δm^2atm

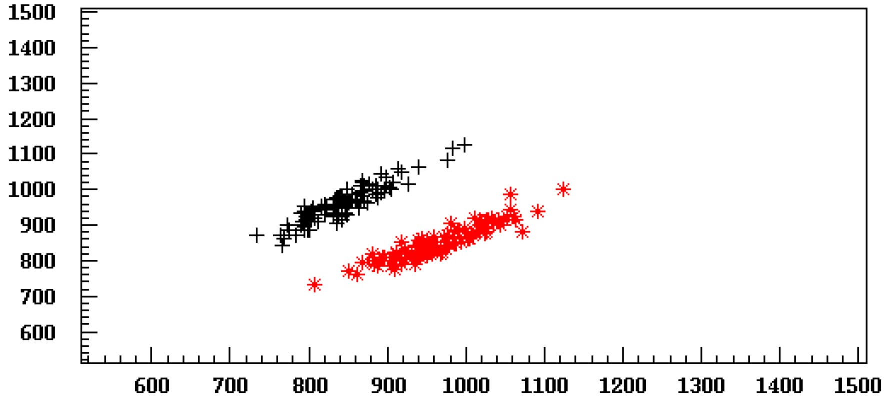
The origin of the very low performance: convolution with the energy resolution?

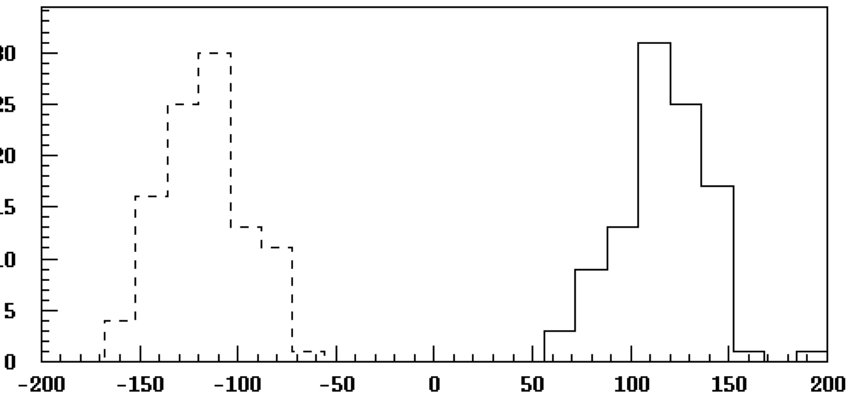
NH generation, 100 toys





IH generation, 100 toys





$\Delta\chi 2$

p-values from the two Gaussian fits

$$p-val_{NH} = \int_{\mu_{IH}}^{+\infty} G_{NH} = 0\%$$

$$p-val_{IH} = \int_{-\infty}^{\mu_{NH}} G_{IH} = 0\%$$

$$\left.\begin{aligned} \mu_{NH} &= -118.5 \pm 2.3 \\ \sigma_{NH} &= 21.9 \pm 1.9 \\ \mu_{IH} &= 116.1 \pm 2.7 \\ \sigma_{IH} &= 19.3 \pm 2.0 \end{aligned}\right\} \quad \sigma = 2\sqrt{\Delta\chi^2}$$

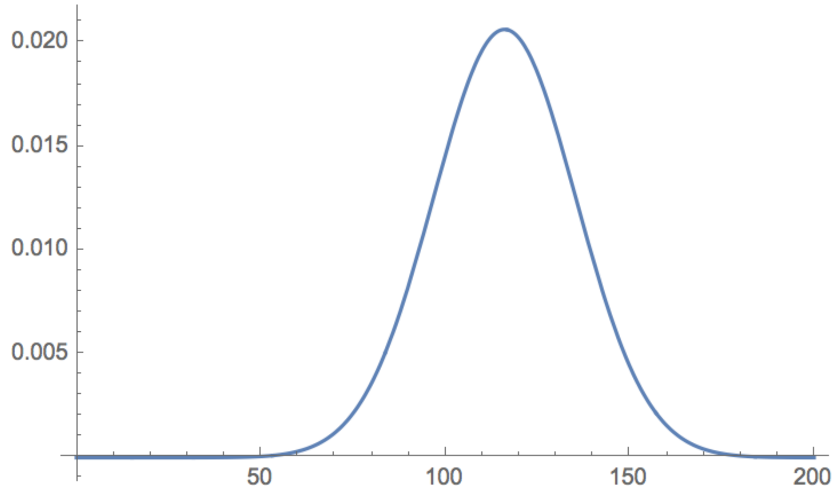
!!

21.9 vs 21.8 (NH)

19.3 vs 21.5 (IH)

```
In[6]:= Plot[PDF[NormalDistribution[116.1, 19.3], x], {x, 0, 200}]
```

Out[6]=




```
In[7]:= CDF[NormalDistribution[116.1, 19.3], -118.5]
```

Out[7]= 2.68359×10^{-34}

Conclusions for the $\Delta\chi 2$ study:

When the simulation is performed on a event-by-event basis
and not on a semi-analytical way
the significance drops drastically to less than 2 σ

The reason seems to stay in the convolution of the statistical fluctuation
and the energy resolution

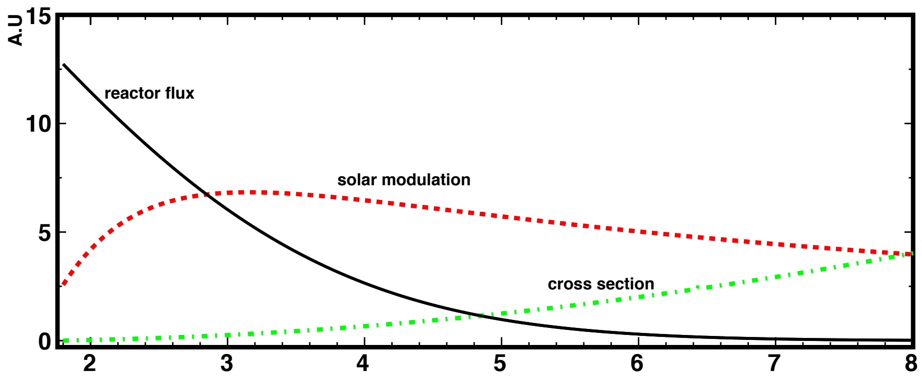
More studies are needed !!!

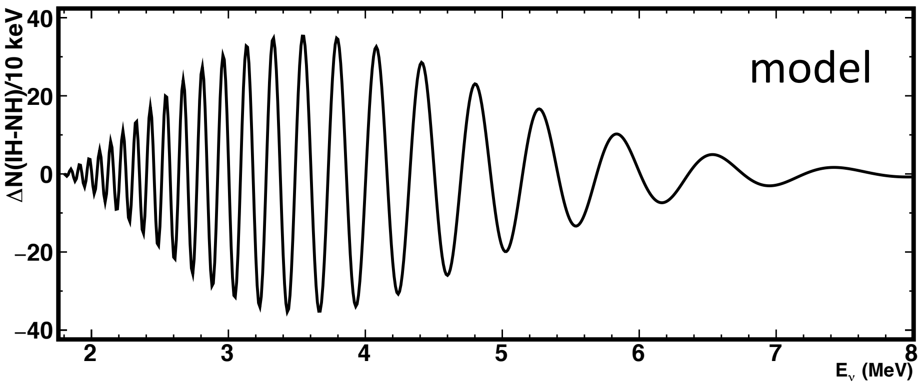
LS, G. Salamanna, A. Lokhov, C. Sirignano, F. Sawy, arXiv:1707.07651v3

New procedure, new estimator, which couples NH/IH, no fit

→ *a possible complementary analysis*

Amount of difference in events for 214 GW•year





$$\Delta N(E_\nu)_{IH-NH} = T \times \phi(E_\nu) \times \sigma_{\bar{\nu}_e p} \times \sin^2 2\theta_{13} \cos 2\theta_{12} \times \sin \frac{L \delta m^2_{sol}/2}{2E_\nu} \times \sin\left[\frac{L}{2E_\nu}\left(\Delta m^2_{atm} - \delta m^2_{sol}/2\right)\right]$$

New estimator: F

$$F = \sum_j \Delta_j^+ + \sum_j \Delta_j^-$$

$\Delta^+ = n_{obs} - n^{IH}_i$ when $n_{obs} > n^{IH}_i$ in I^+

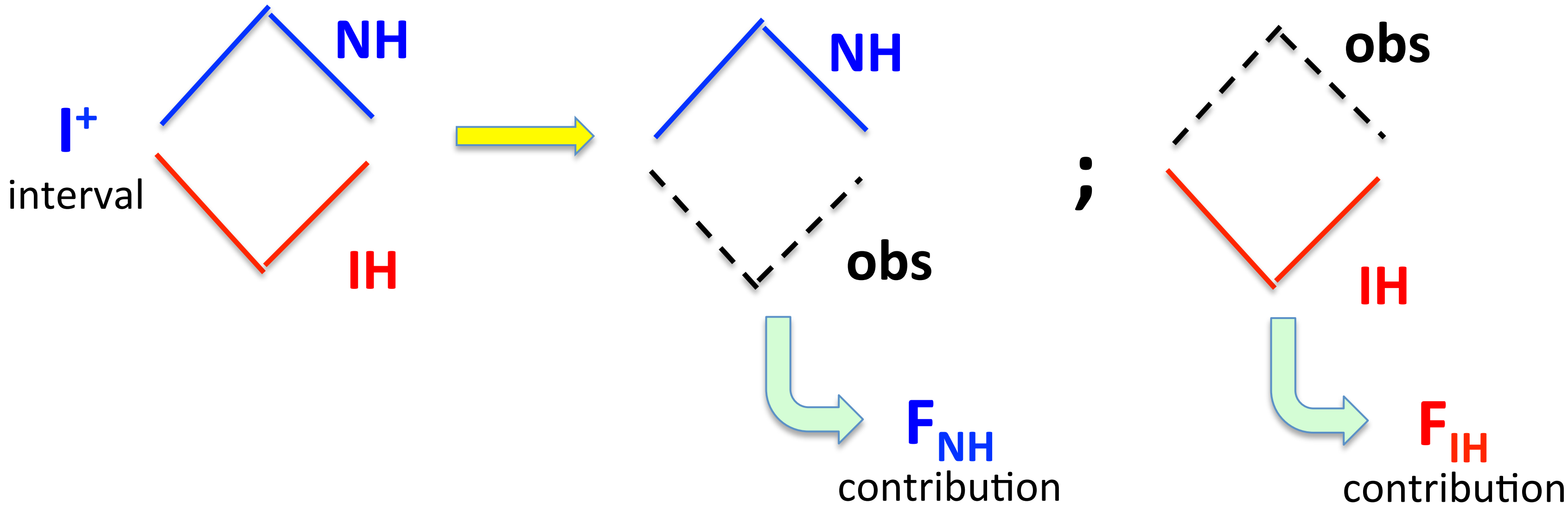
$\Delta^- = n^{IH}_i - n_{obs}$ when $n_{obs} < n^{IH}_i$ in I^-

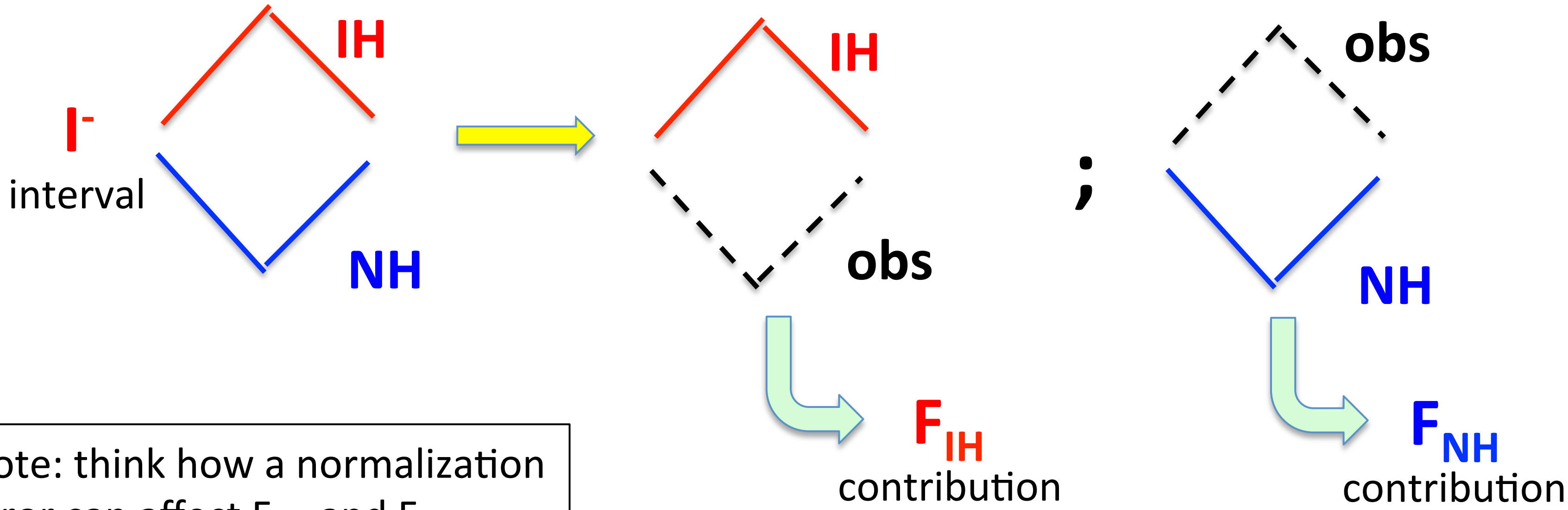
I^+ intervals when $n^{IH}_i < n^{NH}_i$

I^- intervals when $n^{IH}_i > n^{NH}_i$

F computation

emphasize the energy intervals where one of the two mass hierarchies is expected to produce more/less events than the opposite one.





Note: think how a normalization error can affect F_{NH} and F_{IH}

The F estimator and the $\Delta m_{31}^2\left(NH\right) = \Delta m_{23}^2\left(IH\right)$ issue

From a theoretical point of view the relations hold: $\Delta m_{31}^2(NH) = -\Delta m_{32}^2(IH)$,

("given the unique value of the lightest neutrino mass") $\Delta m_{32}^2(NH) = -\Delta m_{31}^2(IH)$

From the experimental point of view there are infinite solutions:

$1)\ m_1 = a, m_2 = b, m_3 = c$

$2)\ m_1 = a', m_2 = b', m_3 = c'$ either NH or IH

$3)\ m_1 = a'', m_2 = b'', m_3 = c''$

...

The evaluation of F is based on the construction of the $I_{\pm}$ intervals where the relation $\Delta m_{31}^2\left(NH\right) = \Delta m_{23}^2\left(IH\right)$ is used.

The construction brings to an optimal estimator.

In other words, we use a theoretical constraint to construct $I_{\pm}$

That is part of the algorithm, like the other algebraic elaborations

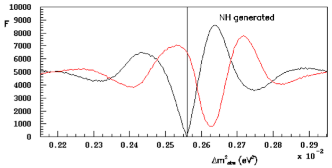
Does that matters ?

In principle what matter are the F distributions and the results

The F procedure

One single JUNO exp

One curve for NH → One best value for Δm^2 (NH)

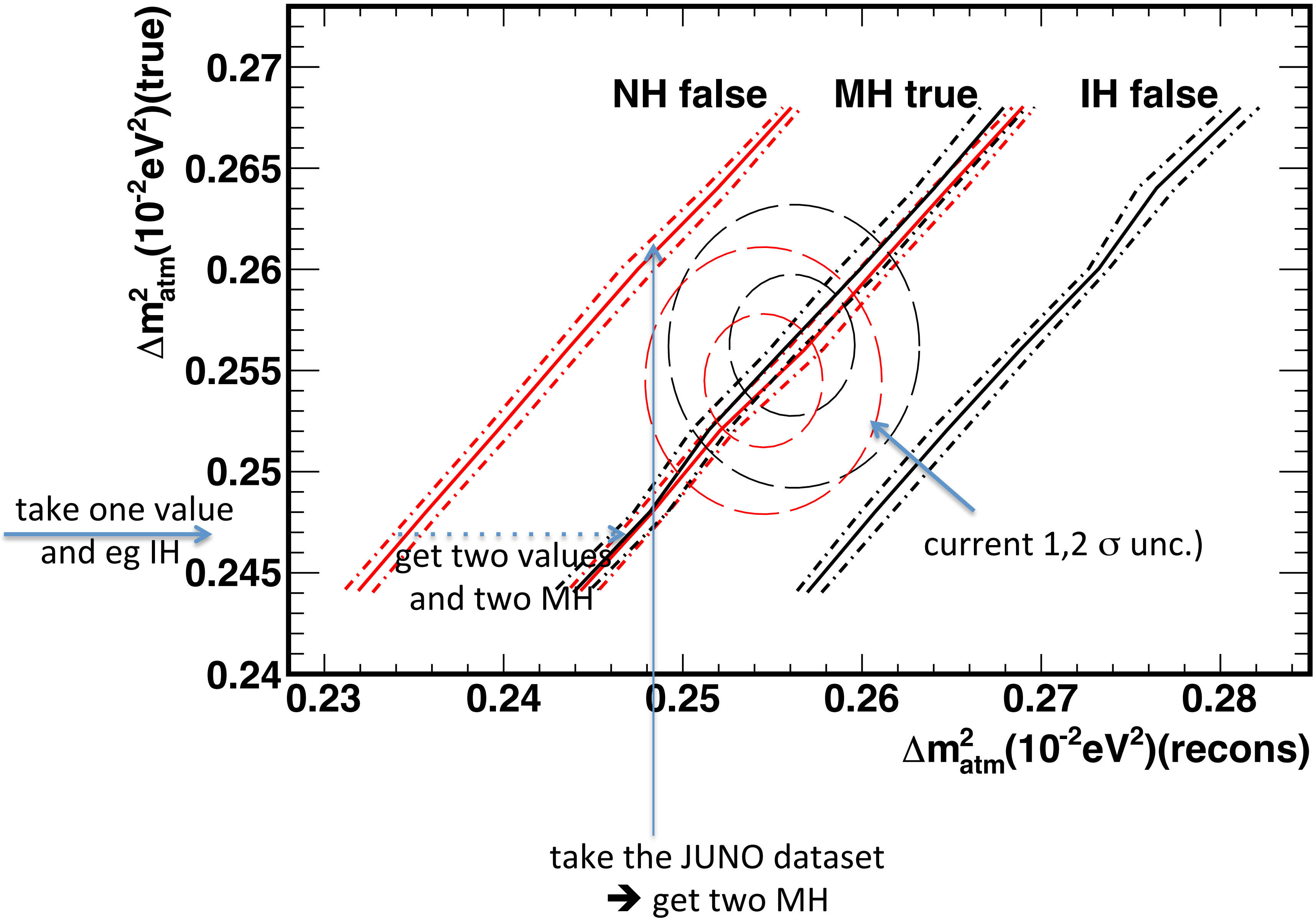


One curve for IH → One best value for Δm^2 (IH)

The "distance" of the two best values provides a significance > 5 σ

Technically: the "distance" in Δm^2 is equivalent to the "distance" in F at the same Δm^2

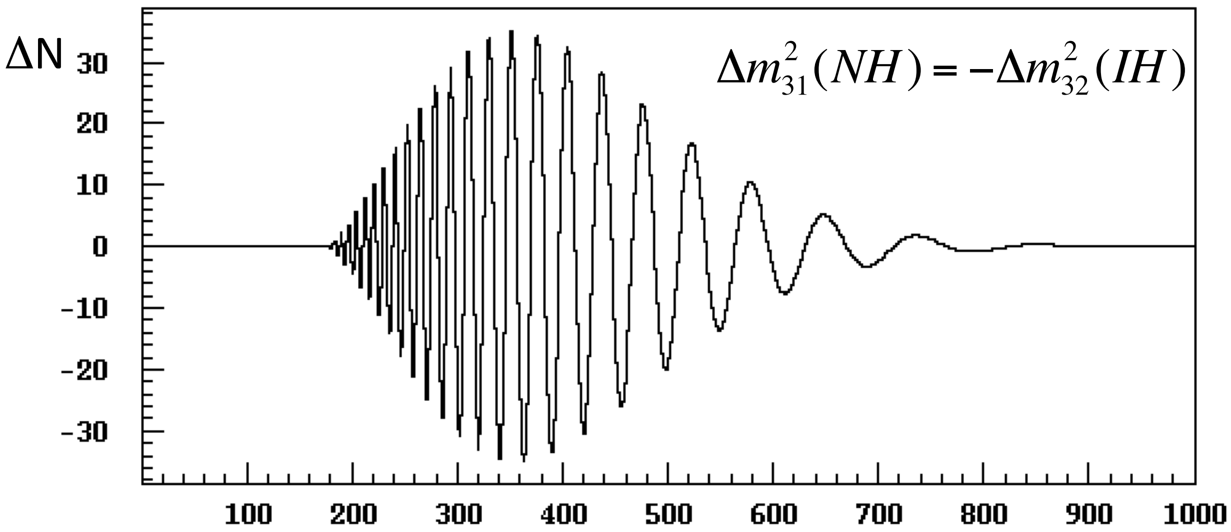
The key-picture

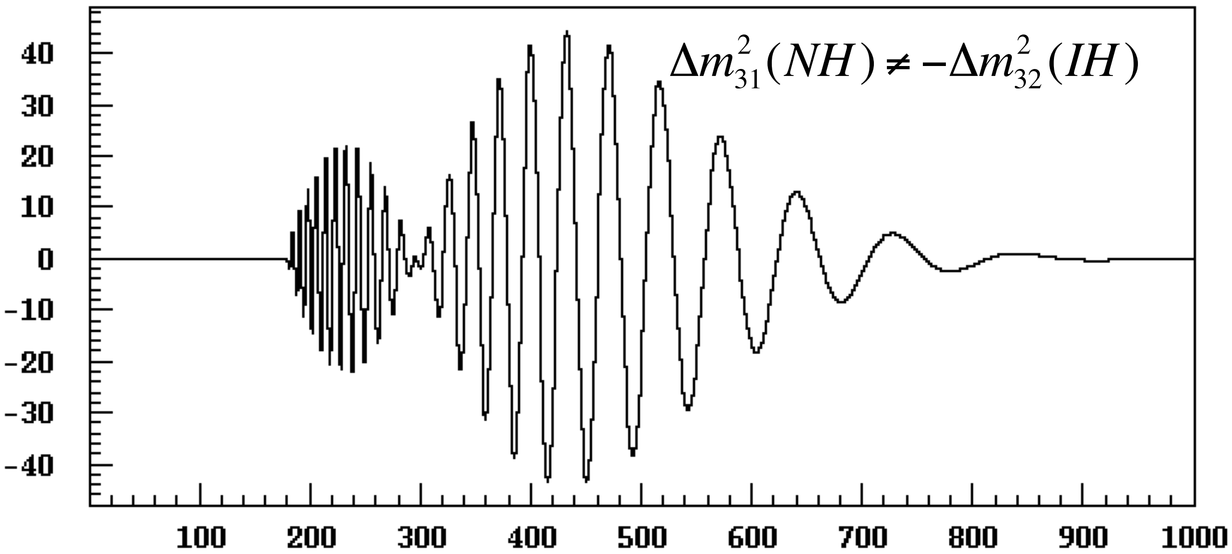


However, one may think that the definition of F could artificially enhance the significance.

Therefore, it is worth to study the generic case

The extended F-estimator



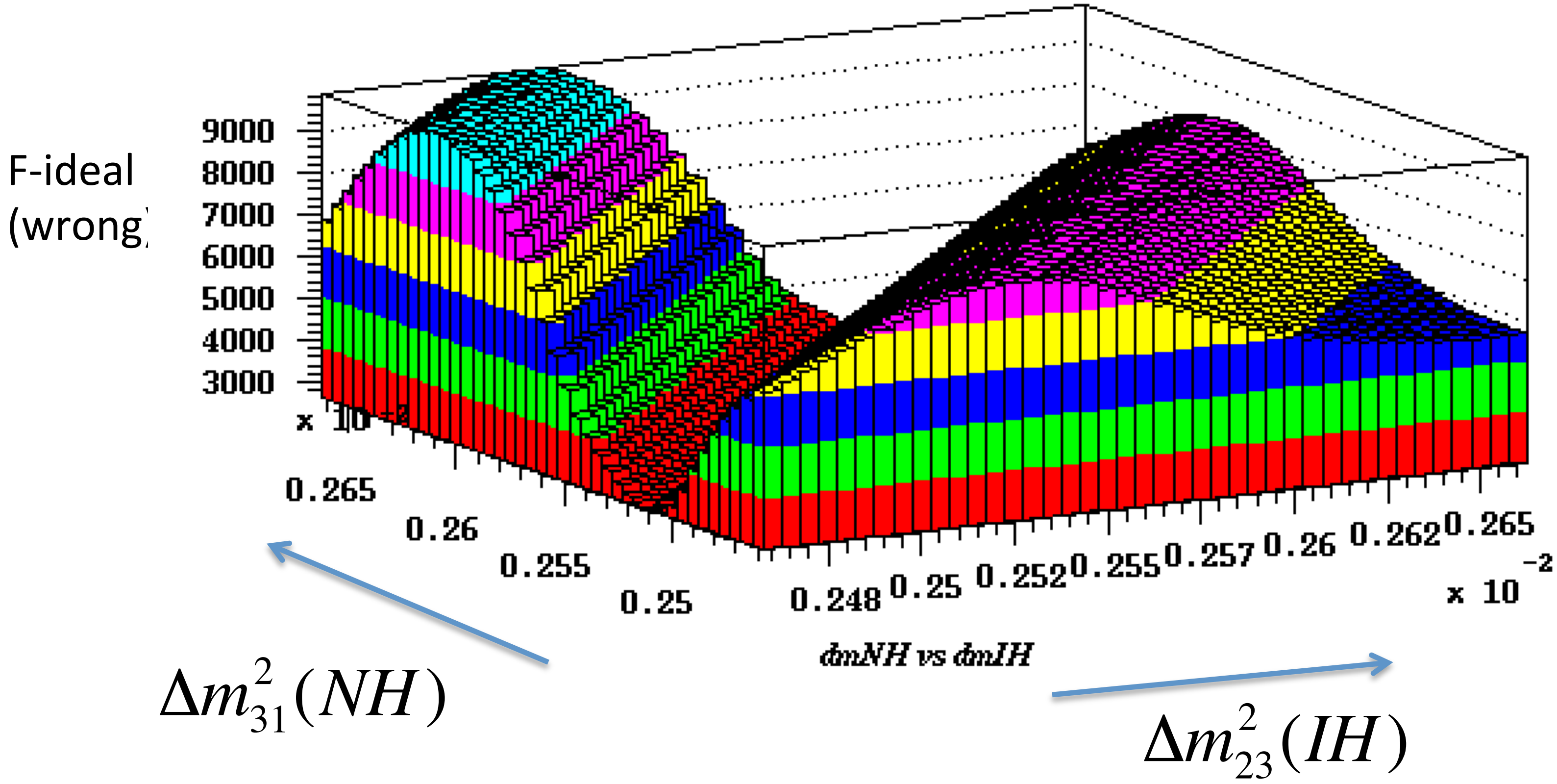


F-ideal(True)=0
F-ideal(Wrong)=Int |ΔN|

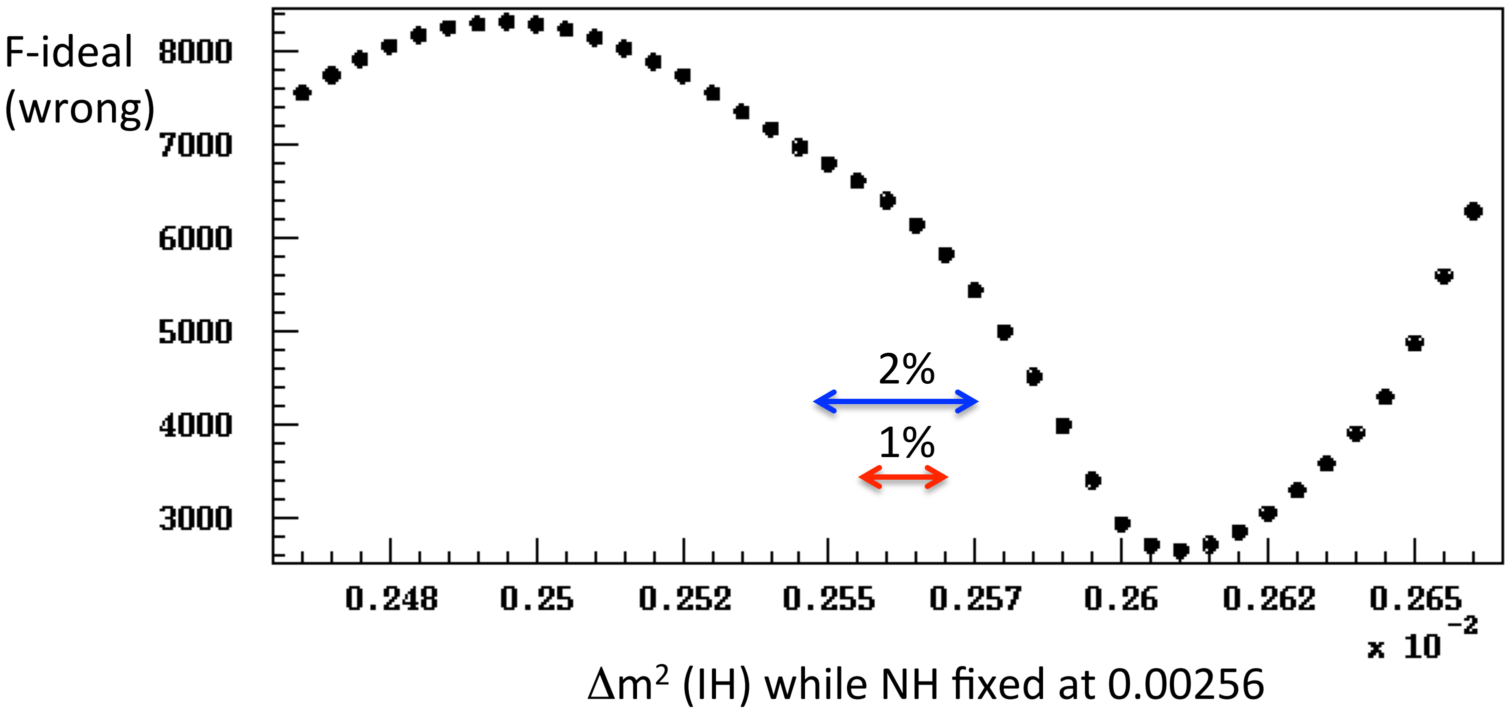
The F algorithm can be extended to any choice of Δm^2_{atm} different for NH and IH

Infinite models

How features and Sensitivities change ?

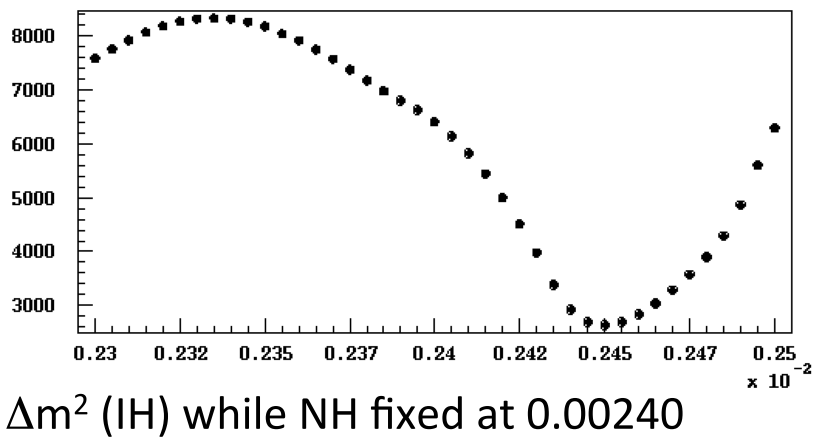


F-ideal
(wrong)
9000
8000
7000
6000
5000
4000
3000
x 10
0.265
0.26
0.255
0.25
0.248
0.25
0.252
0.255
0.257
0.26
0.262
0.265
x 10
-2
dmNH vs dmIH
$\Delta m^2_{31}(NH)$
$\Delta m^2_{23}(IH)$



3% uncert. corresponds to the solar mass

The shape does not change for different values of NH atmospheric mass



What matters is the relative difference between the two atmospheric masses

These behaviours are artificial. What is relevant:

1) the determination of the true atmospheric mass at the F (true) minimum
2) the distance between the two minima F(true) and F(wrong)

Further:

3) the significance level

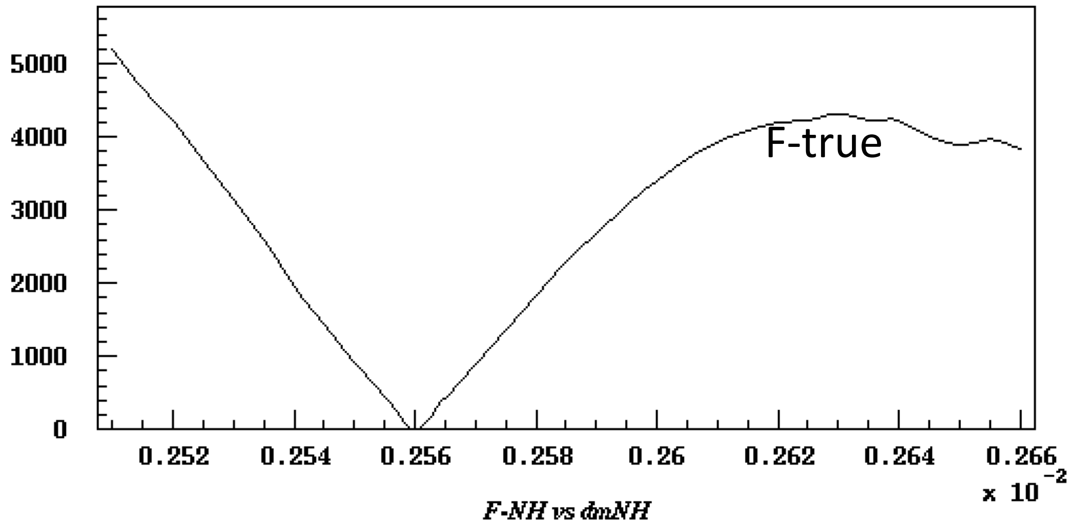
Let us overkill the result: pick up the worst model, ie

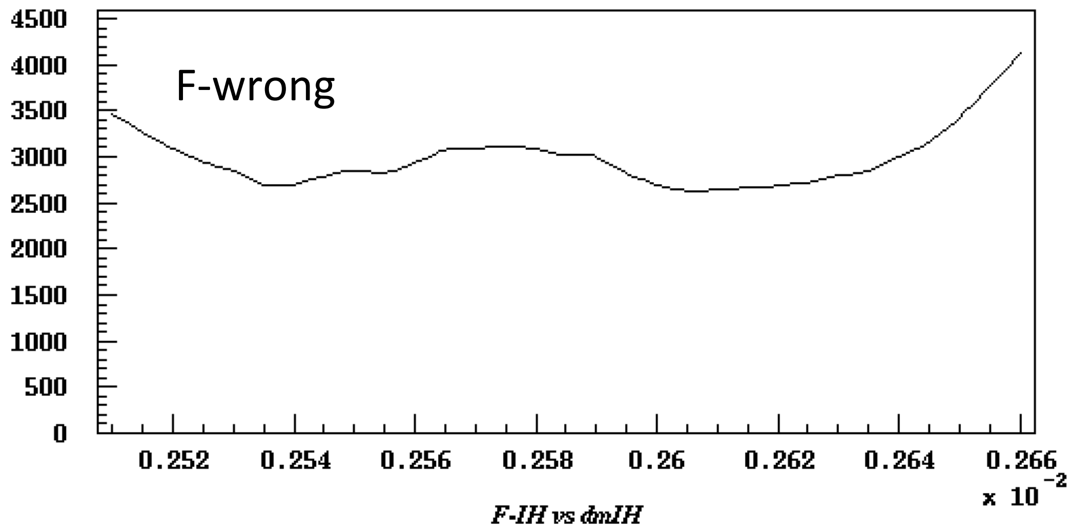
$$\Delta m_{31}^2(NH) = \left|\Delta m_{32}^2(IH)\right| - 0.00005\ eV^2$$

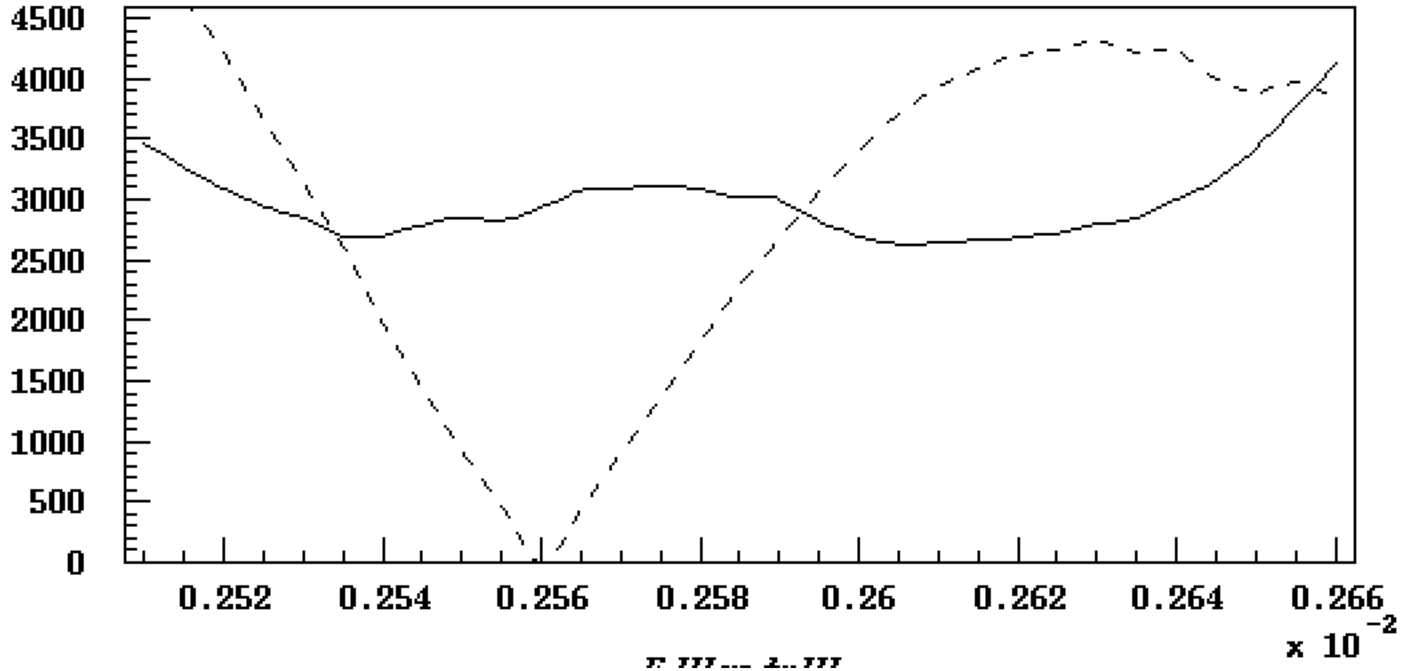
(from 6396 to 2637 counts)

One ideal toy, generated with NH, Δm^2=0.00256 eV^2

looked with the worst extended F estimator



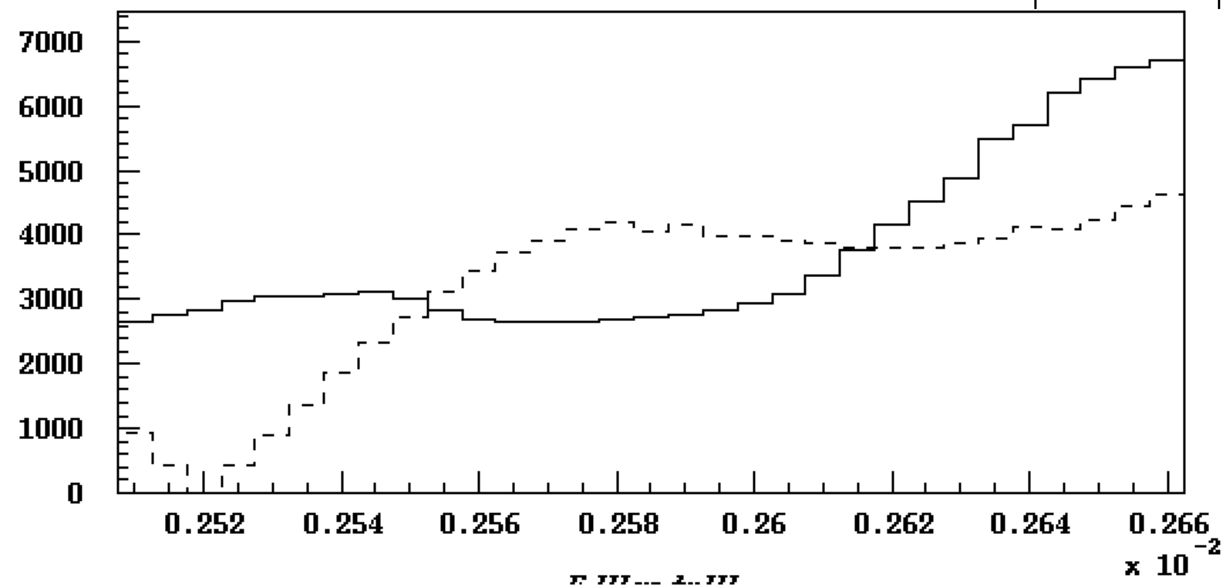




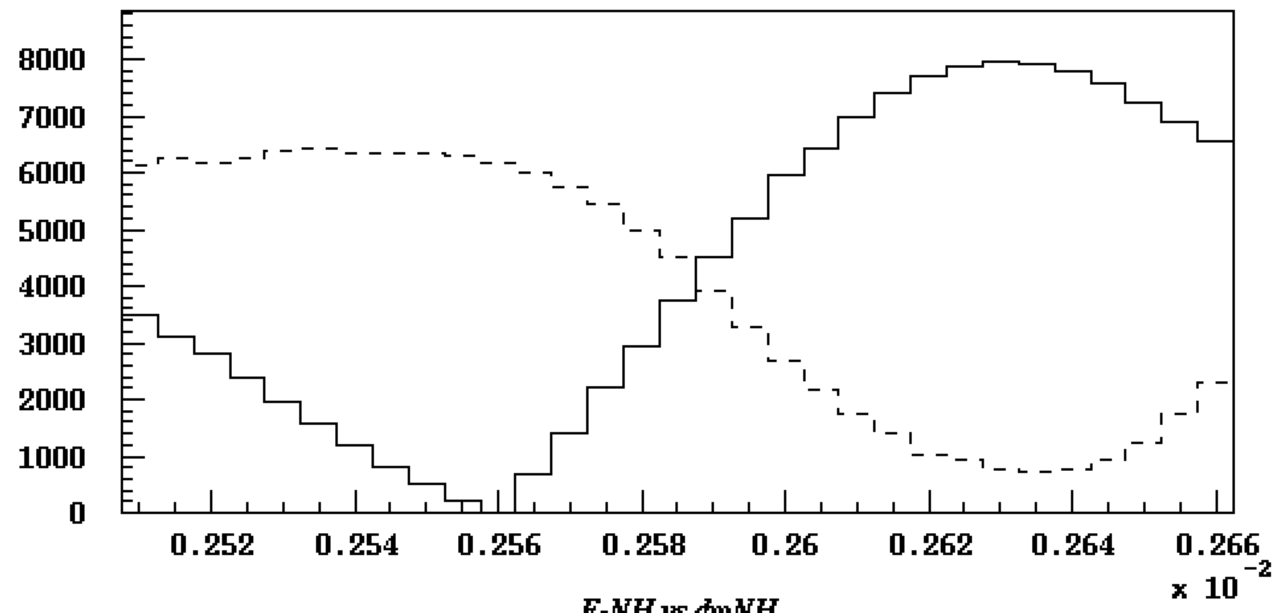
- The true value is correctly found
- The minima are closer (about $12*10^{-5}$ eV^2 - Δ) and fuzzy
- The sensitivity (following our method) on the true minimum should decrease
- A relevant absolute sensitivity seems available

More studies are needed !

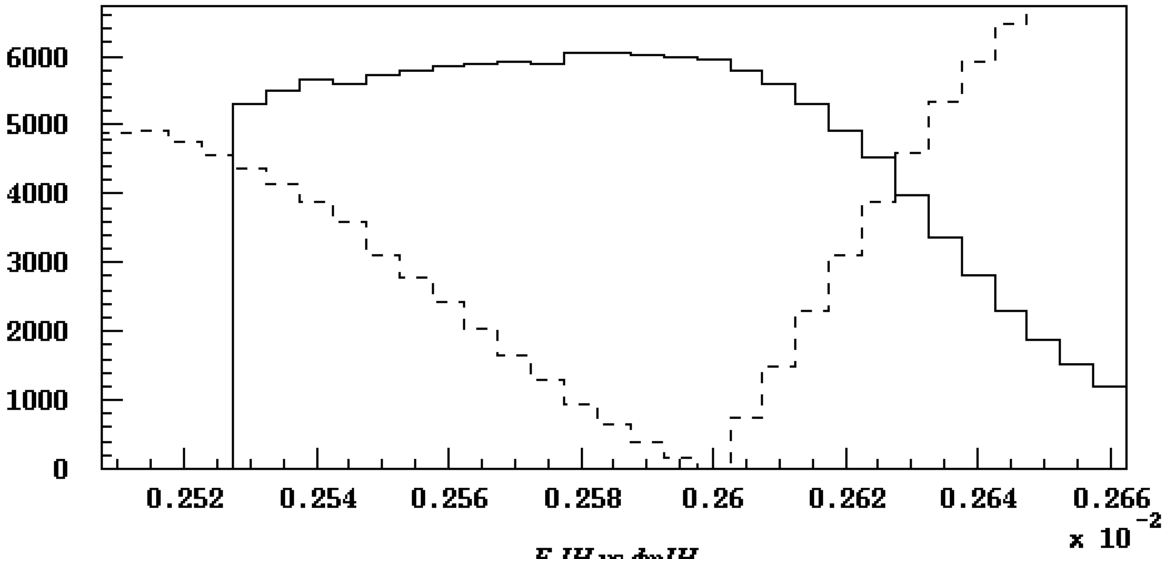
NH generated at 0.00252 and extended-F with $\Delta m_{31}^2(NH) = \left|\Delta m_{32}^2(IH)\right| - 0.00005\ eV^2$



NH generated at 0.00256 and extended-F with $\Delta m_{31}^2(NH) = \left|\Delta m_{32}^2(IH)\right| - 0.00002\ eV^2$



NH generated at 0.00260 and extended-F with $\Delta m_{31}^2(NH) = \left|\Delta m_{32}^2(IH)\right| + 0.00003\, eV^2$



Conclusions for the extended F study:

Modeling the F in a different way is possible.
It turns out to be a 1-D configuration model.

The true value is always identified but the sensitivity and distance of the degenerate solutions in terms of atmospheric masses change.
Different procedures to evaluate the sensitivity should be envisaged for specific single models.

One should keep in mind that the original F is the only optimized estimator as demonstrated by Alexey Lokhov.

The only "reasonable reason" to use an extended-F model is that due to a strong indication that there are two possible solutions, one for NH and one for IH, from the global fits, with two different atmospheric masses.