Towards Improved Overclosure Bounds for Dark Matter Models

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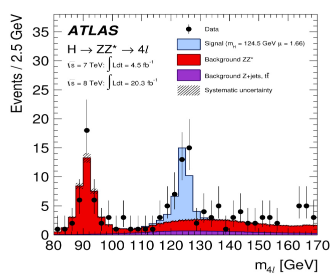
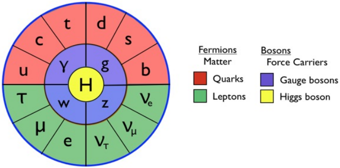
Outline

The Standard Model today...

- Discovery of the Higgs boson:
  ⇒ last great success
- The picture might appear complete
- Quite a few fundamental particles
  ⇒ explain a lot of phenomena

G. Aad et al. (ATLAS coll.), Phys. Rev. D 90 052004 (2014)





Particles of the Standard Model

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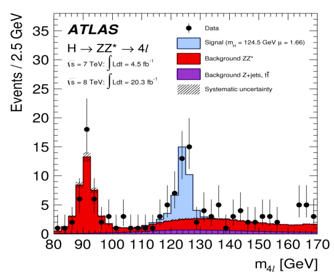
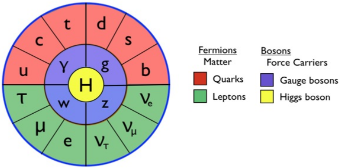




Particles of the Standard Model

However...

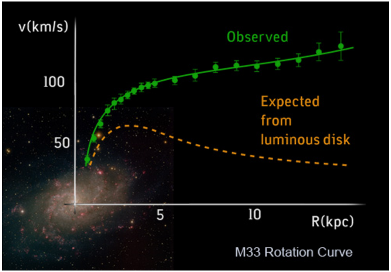
- m_h is UV sensitive
- neutrino masses ...
- dark matter
- baryon asymmetry in the universe

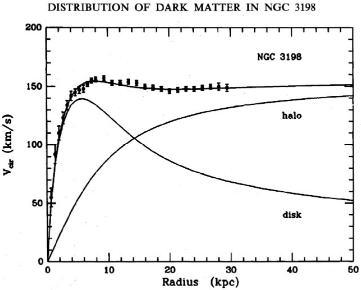
Evidence for Dark Matter I

- We can infer the existence of dark matter from its gravitational effects

At different scales

1. Star-velocity distribution in a galaxy V. Rubin and W. Ford (1970)
2. Galaxy-velocity distribution in a cluster of galaxies F. Zwicky (1937)
3. Strong and weak gravitational lensing J. K. Adelman-McCarthy et al. (2005)





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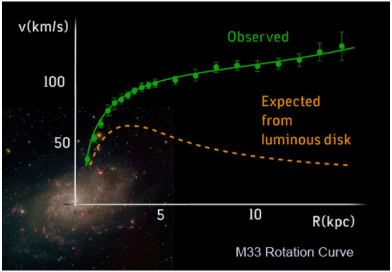
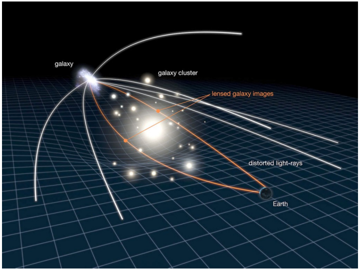
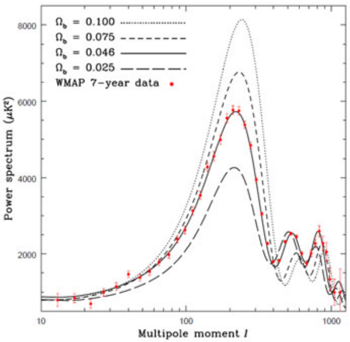




Evidence for Dark Matter II

Even at cosmological scales

- clear evidence from the Cosmic Microwave Background P.A.R. Ade et al. 1502.01589
- early universe before decoupling: baryon-photon fluid oscillations



- Ω_m, Ω_b and photons
- dynamics of the fluid: gravitational collapse vs expansion due to pressure

$$\Omega_{dm} h^2 = 0.1186 \pm 0.0020$$

$$\Omega_b h^2 = 0.02226 \pm 0.00023$$

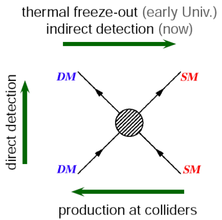
- Ω_b consistent with BBN predictions!
- only with dark matter structure formation could occur

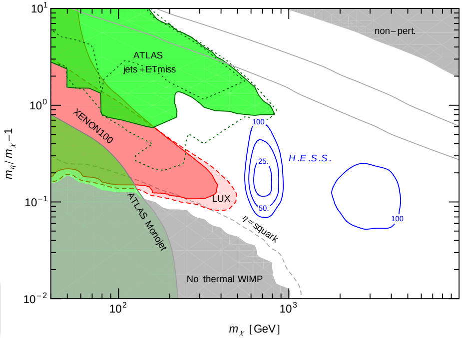
Weakly interacting massive particles

- Many candidates: axions, sterile neutrinos, composite dark matter ... G. Gelmini 1502.01320

WIMPs are attractive for some reasons

- arise to solve problems within particle physics realm (SUSY, extra dimensions...)
- relic abundance from freeze-out ($\Omega_{\rm DM} h^2$ today)
- testable experimentally with direct, indirect and collider searches





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How reliable is the curve obtained from the cosmological relic abundance?

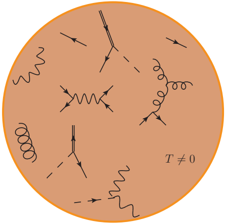
Wimp relic density

The thermal history in brief... P. Gondolo and G. Gelmini (1991)

- χ participates in weak interactions: equilibirum abundance in the early universe

$$\chi\chi \leftrightarrow f\,\bar{f}$$

- Massive particle, introduce a scale besides the **temperature T**
- Recombination $f\,\bar{f} \to \chi\chi$ is Boltzmann suppressed at $T < M$ ($n_{\rm F,B} \sim e^{-M/T}$)
- Eventually the DM pairs do not annihilate any more: freeze-out abundance



Boltzmann Equation

- n_χ total number density of DM particles
- annihilation and creation processes
- expanding background

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

- Kinetic equilibrium is assumed!

Annihilation cross section

- Early universe thermodynamics: particles in a hot plasma

$$\boxed{f_B^{\rm eq}(E) = \frac{1}{e^{E/T}-1}\,, \quad f_F^{\rm eq}(E) = \frac{1}{e^{E/T}+1}}$$

- particle number density $n_\chi^{\rm eq} = g_\chi \int_{\mathbf{p}} n_F(E) \to g_\chi \left(\frac{MT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{M}{T}}$ $\left(n_i^{\rm eq} \approx g_i \frac{T^3}{\pi^2}\right)$
- **kinetic equilibrium**: momenta distribution, e.g. $\chi f \to \chi f$

$$p \sim T\,, \quad p \sim \sqrt{MT} \approx M\sqrt{\frac{T}{M}} \equiv Mv \qquad \boxed{f_i(E) = f_i^{\rm eq}(E)\frac{n_i}{n_i^{\rm eq}}}$$

- **chemical equilibrium**: detailed balance of a reaction, e. g. $\chi\chi \leftrightarrow f\bar{f}$

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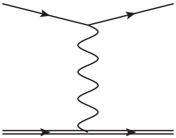

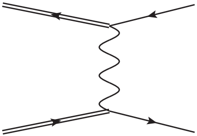

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- thermally averaged cross section

$$\langle \sigma v\rangle = \frac{\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} \sigma v\, e^{-E_1/T} e^{-E_2/T}}{\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} e^{-E_1/T} e^{-E_2/T}}\,, \quad v = |\mathbf{v}_1 - \mathbf{v}_2|\,, \quad \frac{d\sigma}{d\Omega} = \frac{1}{4M^2 v}|\mathcal{M}|^2 \frac{1}{32\pi}$$

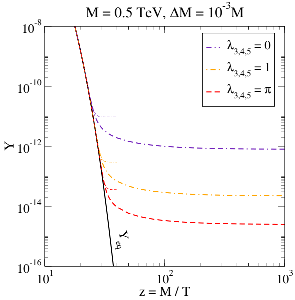
The overclosure bound from relic density

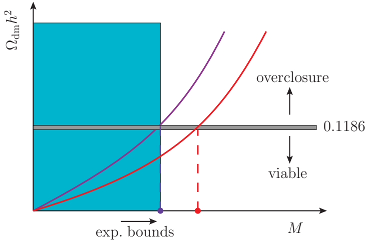
- $\langle \sigma v \rangle$: input from particle physics and thermal averaged, with $v \sim \sqrt{T/M} < 1$

$$\langle \sigma v \rangle \approx \langle a + bv^2 + \dots \rangle = a + \frac{3}{2} b \frac{T}{M} + \dots \Rightarrow \boxed{\langle \sigma v \rangle \approx \frac{\alpha^2}{M^2}}$$

- new variables $Y_\chi = n_\chi / s$ and $z = M/T$ $\Rightarrow$ connect to the observed abundance

$$Y_{\rm phys} = Y(z_{\rm final}) : \quad \Omega_{\rm dm} = \frac{M Y_{\rm phys} s(T_0)}{\rho_{\rm cr}(T_0)} \Rightarrow \Omega_{\rm dm} h^2 = \frac{M}{\rm GeV} \frac{Y_{\rm phys}}{3.645 \times 10^{-9}}$$



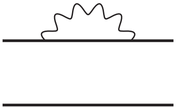


Wimp in a thermal bath

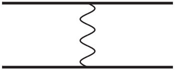
What are the in-medium effects?

- at $T > 160$ GeV the electroweak symmetry is restored $\langle \phi \rangle = 0$
- χ are **non-relativistic**: have time to undergo several interactions

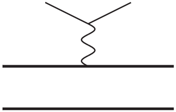
A) Mass correction



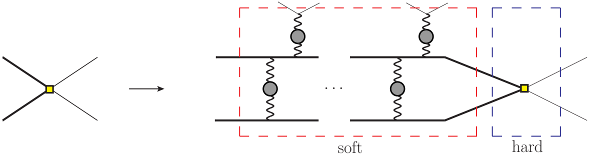
B) Sommerfeld effect and bound states



C) Interaction rate

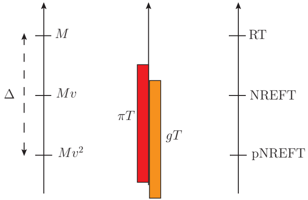


- How does all this reflect into the $\chi\chi$ annihilation?



Non-relativistic and thermal scales I

- Non-relativistic scales: $M \gg Mv \gg Mv^2$ (Coulomb potential $v \sim \alpha$)
- Thermal scales: πT and $m_D \approx \alpha^{1/2} T$, if weakly-coupled plasma $\pi T \gg m_D$



NON-RELATIVISTIC AND THERMAL SCALES I

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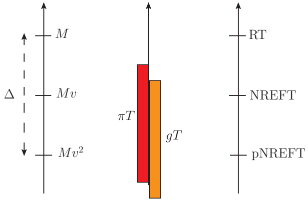
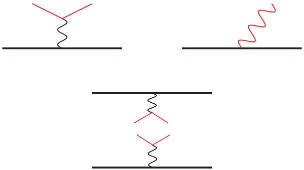




1. **Thermal widths**: the heavy particle is constantly kicked by plasma constituents

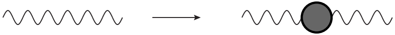
M. Laine, O. Philipsen, P. Romatschke and M. Tassler hep-ph/0611300; N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky 0804.0993;

N. Brambilla, M. A. Escobedo, J. Ghiglieri and A. Vairo :1109.5826 and 1303.6097

$$\Gamma_{GD} \sim \alpha^3 T\,, \quad \Gamma_{LD} \sim \alpha T\,, \quad \Gamma_{LD}^{\rm pair} \sim \alpha^2 T^3 r^2 \sim \begin{cases} \Gamma \sim \frac{\alpha^2 T^3}{M^2 v^2} \sim \alpha^2 \frac{T^2}{M}\,, & v \sim \sqrt{T/M} \\ \Gamma \sim \frac{\alpha^2 T^3}{M^2 v^2} \sim \frac{T^3}{M^2}\,, & v \sim \alpha \end{cases}$$

Non-relativistic and thermal scales II

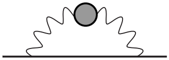
2. **Thermal masses**: gauge-boson-like exchange $m_D \sim \alpha^{1/2} T$
   $\rightarrow$ define the distance at which gauge exchange varies



$m \qquad \longrightarrow \qquad m + m_D$

- the heavy dark matter particles experience thermal mass shifts:

$\delta M_{\text{th}} \sim \alpha T^2/M$



$\delta M_{\text{th}} \sim -\alpha m_D/2 \sim -\alpha^{3/2} T$

- Salpeter correction in nuclear theory: annihilation rate is enhanced

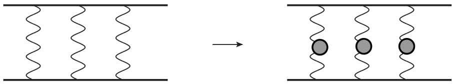
$$\gamma \sim e^{-2M/T} \rightarrow \gamma \sim e^{-2M/T} e^{\alpha m_D/T}$$

Non-relativistic and thermal scales III

3. **Sommerfeld effect**: distortion of the wave function of the annihilating pair ($m=0$) J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami hep-ph/0610249; J.L. Feng, M. Kaplinghat and H.-B. Yu 1005.4678

$$S_{\rm att.} = \left(\frac{\pi\alpha}{v}\right)\frac{1}{1-\exp(-\frac{\pi\alpha}{v})}\,, \quad S_{\rm rep.} = \left(\frac{\pi\alpha}{v}\right)\frac{1}{\exp(\frac{\pi\alpha}{v})-1}$$

$\rightarrow$ how thermal effects change this effect?



4. **Bound state**: if they exist, they have binding energies $|\Delta E| \sim \alpha^2 M$

B. von Harling and K. Petraki 1407.7874; S.P. Liew and F. Luo 1611.08133; A. Mitridate, M. Redi, J. Smirnov and A. Strumia 1702.01141

$$\gamma \sim e^{-2M/T} \rightarrow \gamma \sim e^{-2M/T} e^{\alpha^2 M/T}$$

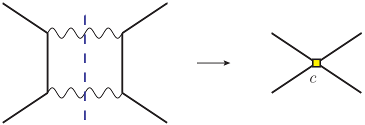
$\rightarrow$ of $\mathcal{O}(1)$ for $T \sim \alpha^2 M$: really important if bound states exist at freeze-out!

ANNIHILATION RATE

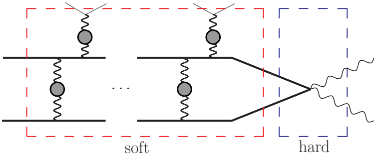
- Annihilation of a heavy pair: DM-DM, with energies $\sim 2M$ (forget about T)

$$\mathcal{O} = i\frac{c}{M^2}\phi^\dagger\phi^\dagger\phi\phi\,,\; c \approx \alpha^2 \quad \text{(inclusive s-wave annihilation)}$$

G. T. Bodwin, E. Braaten and G. P. Lepage hep-ph/9407339



- $M \gg T \Rightarrow \Delta x \sim \frac{1}{k} \sim \frac{1}{M} \ll \frac{1}{T}$ local and insensitive to the thermal scales



- we want to "thermal-average"

$$\langle \phi^\dagger\phi^\dagger\phi\phi\rangle_T$$

Beyond the free case: the spectral function

Compare Boltzmann equation with linear response theory

$$(\partial_t + 3H)n = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2) \quad \text{and} \quad (\partial_t + 3H)n = -\Gamma_{\rm chem}(n - n_{\rm eq})$$

$$\langle \sigma v \rangle \equiv \frac{\Gamma_{\rm chem}}{2 n_{\rm eq}} \Rightarrow \langle \sigma v \rangle = \frac{4}{n_{\rm eq}^2} \frac{c}{M^2} \gamma \quad \text{where } \gamma = \langle \phi^\dagger \phi^\dagger \phi \phi \rangle_T$$

D. Bodeker and M. Laine 1205.4987; S. Kim and M. Laine 1602.08105; S. Kim and M. Laine 1609.00474

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- thermal expectation value of the operators that annihilate/create a DM-DM pair

$$\gamma = \frac{1}{Z} \sum_{m,n} e^{-E_m/T} \langle m | \phi^\dagger \phi^\dagger | n \rangle \langle n | \phi \phi | m \rangle$$

- any correlator in equilibrium can be expressed in term of the *spectral function*

$$\rho(\omega, \mathbf{k}) = \int_{-\infty}^{\infty} dt \int_{\mathbf{r}} e^{i\omega t - i\mathbf{k}\cdot\mathbf{r}} \langle \left[(\phi^\dagger \phi^\dagger)(t, \mathbf{r}), (\phi\phi)(0, \mathbf{0}) \right] \rangle_T$$

$$\gamma = \int_{2M-\Lambda}^{\infty} \frac{d\omega}{\pi} e^{-\frac{\omega}{T}} \int_{\mathbf{k}} \rho(\omega, \mathbf{k}) + \mathcal{O}(e^{-4M/T}), \ \alpha^2 M \ll \Lambda \sim M$$

From ρ to a Schrödinger equation

- non-relativistic dynamics:

$$E_m \equiv \omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + V(r)$$

- the spectral function $\rho(E', \mathbf{0})$ is obtained from

$$\left[H - i\Gamma - E'\right] G(E'; \mathbf{r}, \mathbf{r'}) = N\delta^3(\mathbf{r} - \mathbf{r'}) \qquad \lim_{\mathbf{r},\mathbf{r'} \to 0} \mathrm{Im}\, G(E'; \mathbf{r}, \mathbf{r'}) = \rho(E')$$

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$$\Gamma \to 0 \; : \qquad \lim_{T\to 0} \rho(E') = N \sum_m |\psi(\mathbf{0})|^2 \pi \delta(E_m - E')$$

- E_m are the s-wave energy eigenvalues of $H = -\frac{\nabla^2}{M} + V(r)$

- In the free case $V(r) \to 0$

$$\rho_{\text{free}}(E') = N \frac{M^{\frac{3}{2}} \theta(E') \sqrt{E'}}{4\pi}$$

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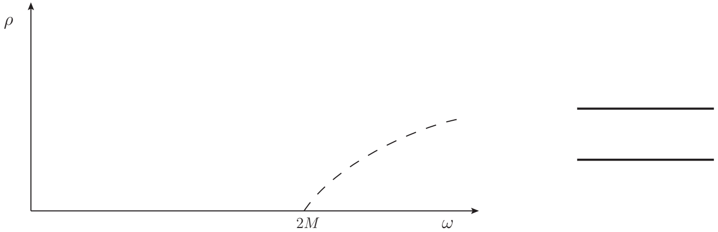
- the spectral function $\rho(E', \mathbf{0})$ is obtained from

$$\left[H - i\Gamma - E'\right] G(E'; \mathbf{r}, \mathbf{r'}) = N\delta^3(\mathbf{r} - \mathbf{r'}) \qquad \lim_{\mathbf{r},\mathbf{r'} \to 0} \mathrm{Im}\, G(E'; \mathbf{r}, \mathbf{r'}) = \rho(E')$$

$$\gamma \approx \left(\frac{MT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{2M}{T}} \int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{-\frac{E'}{T}} \rho(E') \to \gamma_{\text{free}} = \frac{n_{eq}^2}{4} \Rightarrow \langle \sigma v \rangle = \frac{c}{M^2}$$



From ρ to a Schrödinger equation

- non-relativistic dynamics:

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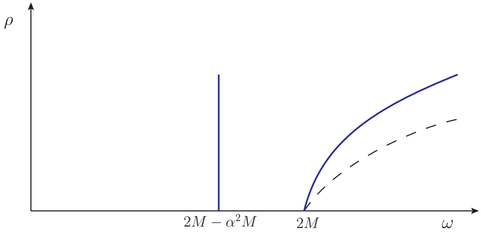

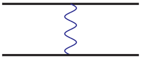

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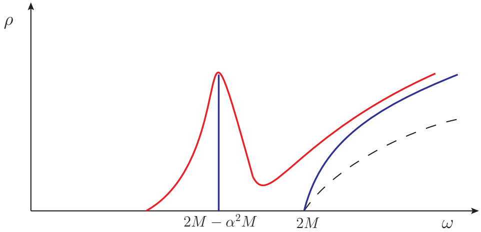
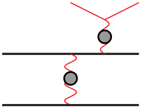


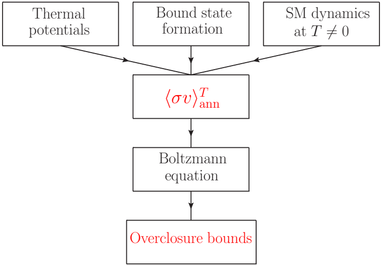


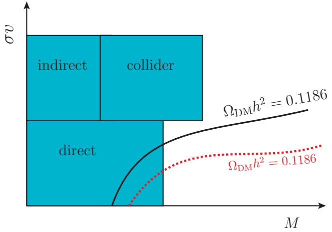
Summary of the theoretical framework

Relic density can be factorized in some steps

M. Laine and S. Kim 1609.00474

- Calculate the matching coefficients from the hard annihilation process, $E \sim 2M$
- Compute the static potentials and thermal widths induced by the particle exchanged by the heavy ones
- Extract the spectral function $\Rightarrow$ annihilation rate
- Solve the Boltzmann equation with the **thermal** cross section





The inert doublet model

- Supplement SM with χ SU(2) doublet, no coupling with fermions, unbroken vacuum
- We focus on the high-mass regime of the model: $M \gtrsim 530$ GeV
- Degenerate case: 4 states, $H_0, H_{\bar{0}}, H_{\pm}$ with the same mass

T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, 0903.4010

$$\begin{aligned} \mathcal{L}_\chi &= (D^\mu \chi)^\dagger (D_\mu \chi) - M^2 \chi^\dagger \chi \\ &- \left\{ \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 \, \phi^\dagger \phi \, \chi^\dagger \chi + \lambda_4 \, \phi^\dagger \chi \, \chi^\dagger \phi + \left[\frac{\lambda_5}{2} (\phi^\dagger \chi)^2 + h.c. \right] \right\} \end{aligned}$$

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- Annihilation happening at $T \sim M/20 ... M/10^4 \Rightarrow v \sim \sqrt{T/M} \ll 1$

$$\chi = \frac{1}{\sqrt{2M}} \left(C e^{-iMt} + D^\dagger e^{iMt} \right) , \quad \chi^\dagger = \frac{1}{\sqrt{2M}} \left(D e^{-iMt} + C^\dagger e^{iMt} \right)$$

$$\delta\mathcal{L}_{\text{NREFT}} = i \left(\frac{c_1}{M^2} \underbrace{C_p^\dagger D_p^\dagger D_q C_q}_{\equiv O_1} + \frac{c_2}{M^2} \underbrace{C_p^\dagger T^a_{pq} D_q^\dagger D_r T^a_{rs} C_s}_{\equiv O_2} + \frac{c_3}{M^2} \underbrace{D_p^\dagger D_q^\dagger D_p D_q}_{\equiv O_3} + \frac{c_4}{M^2} \underbrace{C_p^\dagger C_q^\dagger C_p C_q}_{\equiv O_4} \right)$$

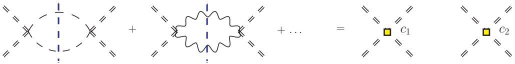
Matching the hard process

- Matching matrix elements of four-particle states: imaginary part of c_i

$$c_1 = \frac{g_1^4 + 3g_2^4 + 8\lambda_3^2 + 8\lambda_3\lambda_4 + 2\lambda_4^2}{256\pi}$$

$$c_2 = \frac{g_1^2 g_2^2 + \lambda_4^2}{32\pi}$$

$$c_3 = c_4 = \frac{\lambda_5^2}{128\pi}$$



Degenerate case: cross section with free-heavy scalar

- $\langle \sigma v \rangle = \frac{4}{n_{\rm eq}^2} \sum_i^4 c_i \gamma_i$ and with $N_1 = 2$, $N_2 = \frac{3}{2}$, $N_3 = N_4 = 6$ we obtain

$$\langle \sigma_{\rm eff}\, v \rangle^{(0)} = \frac{c_1}{2} + \frac{3c_2}{8} + \frac{3(c_3 + c_4)}{2}$$

- we redefined the $c_i \to c_i/M^2$

INCLUDING THE POTENTIALS

- the heavy scalars interact with gauge bosons

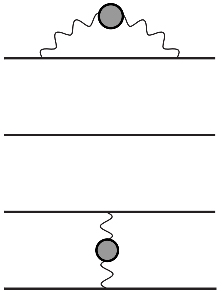
$$\left[-\frac{\nabla_r^2}{M} + \mathcal{V}_i(r) - E'\right] G_i(E';\mathbf{r},\mathbf{r}') = N_i\,\delta^{(3)}(\mathbf{r}-\mathbf{r}')\,, \quad \lim_{\mathbf{r}\to\vec{0}} \mathrm{Im}\, G_i(E';\mathbf{r},\mathbf{r}') = \rho_i(E')$$

ELECTROWEAK THERMAL POTENTIALS

$$\mathcal{V}_{\mathrm{W}}(r) \equiv \frac{g_2^2}{4}\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\, i\langle W_0^+ W_0^- \rangle_{\mathrm{T}}(0,k)\,,$$

$$\mathcal{V}_{\mathrm{A}}(r) \equiv \frac{g_2^2}{4}\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\, i\langle A_0^3 A_0^3 \rangle_{\mathrm{T}}(0,k)$$

$$\mathcal{V}_{\mathrm{B}}(r) \equiv \frac{g_1^2}{4}\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\, i\langle B_0 B_0 \rangle_{\mathrm{T}}(0,k)$$



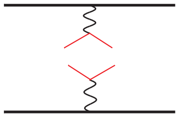
HTL PROPAGATORS FOR GAUGE BOSONS

- $m \ll \pi T$, it holds at $T < M \Rightarrow$ capture thermal effects

$$i\langle W_0^+ W_0^- \rangle_T = \frac{1}{\mathbf{k}^2 + m_{\widetilde{W}}^2} - \frac{i\pi T}{k} \frac{m_{\rm E2}^2}{(\mathbf{k}^2 + m_{\widetilde{W}})^2} \qquad \text{(static limit)}$$

- $m_{\widetilde{W}}^2 = m_W^2 + m_{\rm E2}^2$ and $m_{\widetilde{W}} = \frac{gv}{2}$

$$m_{\rm E1}^2 = \left(\frac{n_S}{6} + \frac{5n_G}{9}\right) g'^2 T^2, \quad m_{\rm E2}^2 = \left(\frac{2}{3} + \frac{n_S}{6} + \frac{n_G}{3}\right) g^2 T^2$$



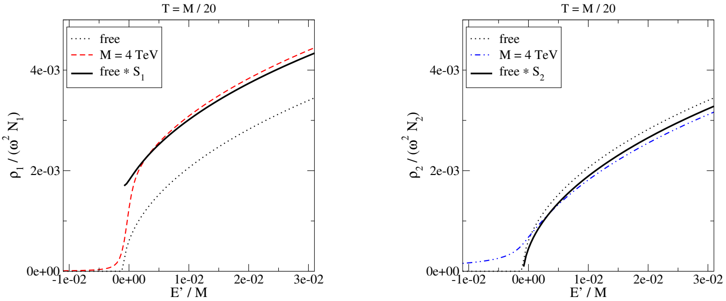
$$\mathcal{V}_{\rm W}(r) = \frac{g^2}{16\pi}\left[\frac{\exp(-m_{\widetilde{W}} r)}{r} - i\,\frac{T m_{\rm E2}^2 \phi(m_{\widetilde{W}} r)}{m_{\widetilde{W}}^2}\right]$$

$$\mathcal{V}_{\rm W}(0) = -\frac{g^2}{16\pi}\left(m_{\widetilde{W}} + i\,\frac{T m_{\rm E2}^2}{m_{\widetilde{W}}^2}\right) + \left.\frac{g^2 m_W}{16\pi}\right|_{T=0}$$

Results for the spectral functions

- the potential for the attractive channel reads

$$\mathcal{V}_1 = 2\mathcal{V}_{\rm W}(0) + \mathcal{V}_{\rm A}(0) + \mathcal{V}_{\rm B}(0) - 2\mathcal{V}_{\rm W}(r) - \mathcal{V}_{\rm A}(r) - \mathcal{V}_{\rm B}(r)$$

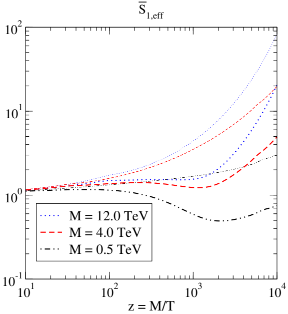


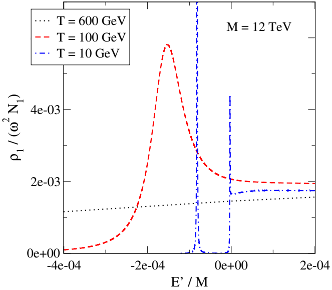
- there is no large deviation with respect to a $T = 0$ Sommerfeld factor
- no bound states around the freeze-out, non-zero tail in the repulsive channel

Average Sommerfeld factors

- $\bar{S}_i$: **distortion of the wave function, thermal widths, Salpeter correction, bound states**

$$\bar{S}_i \equiv \frac{e^{2\Delta M_T/T}}{N_i}\left(\frac{4\pi}{MT}\right)^{\frac{3}{2}}\int_{-\Lambda}^{\infty}\frac{\mathrm{d}E'}{\pi}e^{-E'/T}\rho_i(E')\,,\quad 2\Delta M_T \equiv \mathrm{Re}\left[2\mathcal{V}_W(0)+\mathcal{V}_A(0)+\mathcal{V}_B(0)\right]$$



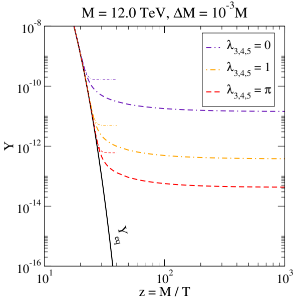


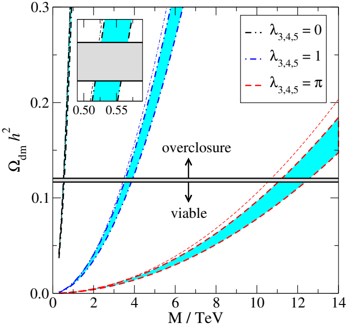
$$\langle \sigma v\rangle^{(0)} \to \langle \sigma v\rangle = \frac{c_1\bar{S}_1}{2}+\frac{3\,c_2\bar{S}_2}{8}+\frac{3\,(c_3+c_4)\bar{S}_3}{2}$$

Overclosure bound for IDM

Recall the Boltzmann equation...

$$Y'(z) = -\langle \sigma v \rangle M m_{\rm Pl} \frac{c(T)}{\sqrt{24\pi e(T)}} \frac{Y^2(z) - Y^2_{\rm eq}(z)}{z^2}\Bigg|_{T=M/z}$$





- $\lambda_i = 0$: $M < 519 \pm 4$GeV $\rightarrow M < 523 \pm 4$GeV or $M < 562 \pm 4$GeV
- $\lambda_i = \pi$: $M < 10.6 \pm 0.1$TeV $\rightarrow M < 11.1 \pm 0.1$TeV or $M < 12.1 \pm 0.1$TeV

Simplified models

To link effectively a BSM theory and dark matter

- SUSY, extra dimension... have a rather large parameter space
- Constraints are set on a simple model that captures the most relevant physics

A. De Simone and T. Jacques 1603.08002

Majorana fermion DM + Coloured mediator

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_\chi + \mathcal{L}_\eta + \mathcal{L}_{\rm int}$$

$$\mathcal{L}_\chi^{\rm M} = \frac{1}{2}\bar{\chi} i \partial\!\!\!/ \chi - \frac{M}{2}\bar{\chi}\chi \, , \quad \mathcal{L}_\eta = (D^\mu \eta)^\dagger (D_\mu \eta) - M_\eta^2 \eta^\dagger \eta - \lambda_2 \left(\eta^\dagger \eta\right)^2$$

$$\mathcal{L}_{\rm int} = -{\rm y}\, \eta^\dagger \bar{\chi} P_R q - {\rm y}^* \, \bar{q} P_L \chi \eta - \lambda_3 \eta^\dagger \eta H^\dagger H$$

M. Garny, A. Ibarra and S. Vogl 1503.01500

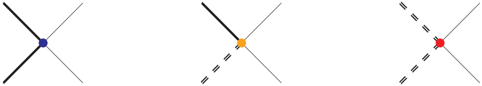
- the annihilation of $\chi\chi$ pairs is p-wave suppressed — J. Edsjö and P. Gondolo hep/ph-9704361

$\Rightarrow$ the role of the (co)annihilating η is important and driven by QCD

$$\langle \sigma v \rangle \approx \langle \sigma v \rangle_{\chi\chi} + e^{-\frac{\Delta M}{M}} \langle \sigma v \rangle_{\eta\chi} + e^{-2\frac{\Delta M}{M}} \langle \sigma v \rangle_{\eta\eta}$$

Strong interactions enter...

- Again $\eta = \frac{1}{\sqrt{2M}}\left(\phi e^{-iMt} + \varphi^\dagger e^{iMt}\right)$ and $\chi = (\psi e^{-iMt}, -i\sigma_2 \psi^* e^{iMt})$



$$\begin{aligned}
\mathcal{L}_{\text{abs}} &= i\left\{ c_1\, \psi_p^\dagger \psi_q^\dagger \psi_q \psi_p + c_2 \left(\psi_p^\dagger \phi_\alpha^\dagger \psi_p \phi_\alpha + \psi_p^\dagger \varphi_\alpha^\dagger \psi_p \varphi_\alpha \right) \right. \\
&+ \left. c_3\, \phi_\alpha^\dagger \varphi_\alpha^\dagger \varphi_\beta \phi_\beta + c_4\, \phi_\alpha^\dagger \varphi_\beta^\dagger\, \varphi_\gamma \phi_\delta\, T^a_{\alpha\beta} T^a_{\gamma\delta} + c_5 \left(\phi_\alpha^\dagger \phi_\beta^\dagger \phi_\beta \phi_\alpha + \varphi_\alpha^\dagger \varphi_\beta^\dagger \varphi_\beta \varphi_\alpha \right) \right\}
\end{aligned}$$

- the matching coefficients are

$$c_1 = 0\,, \quad c_2 = \frac{|y|^2(|h|^2 + g_s^2 C_F)}{128\pi M^2}\,,$$

$$c_3 = \frac{1}{32\pi M^2}\left(\lambda_3^2 + \frac{g_s^4 C_F}{N_c}\right), \quad c_4 = \frac{g_s^4(N_c^2 - 4)}{64\pi M^2 N_c}\,, \quad c_5 = \frac{|y|^4}{128\pi M^2}\,.$$

Rates and annihilation cross section

- we have an in-vacuum and thermal mass splitting

$$\Delta M_T \equiv \Delta M + \frac{(g_s^2 C_F + \lambda_3) T^2}{12M} - \frac{g_s^2 C_F m_D}{8\pi}$$

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$$V \equiv \frac{g_s^2}{2} \int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \left[\frac{1}{\mathbf{k}^2 + m_D^2} - i\frac{\pi T}{k} \frac{m_D^2}{(\mathbf{k}^2 + m_D^2)^2} \right] , \quad m_D = g_s \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$$

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- the thermally modified Sommerfeld factors are

$$\bar{S}_i = \left(\frac{4\pi}{MT} \right)^{\frac{3}{2}} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi} \, e^{[\mathrm{Re}\mathcal{V}_i(\infty) - E']/T} \, \frac{\rho_i(E')}{N_i}$$

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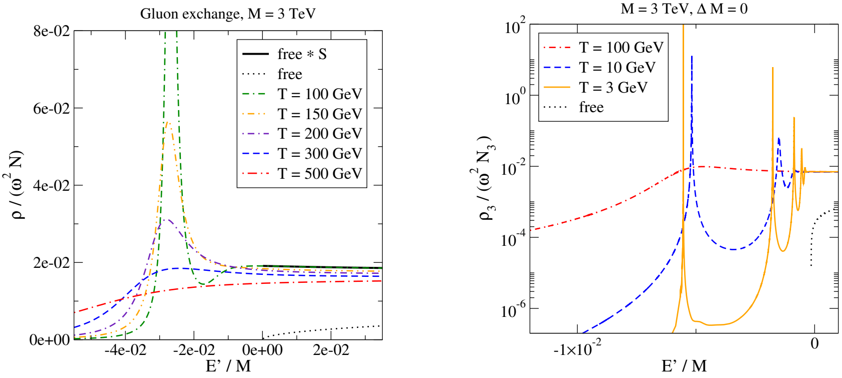
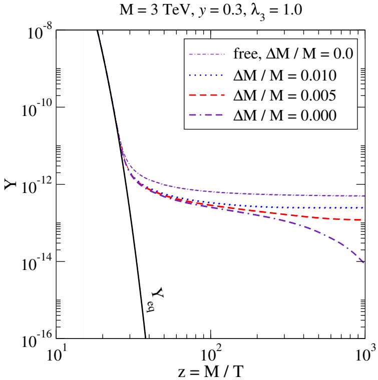
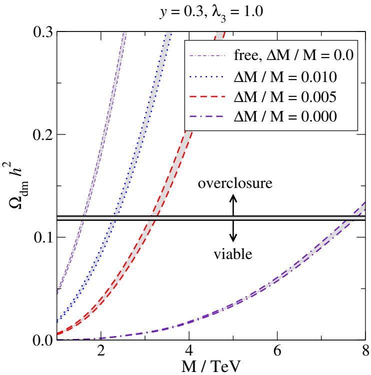
$$\langle \sigma_{\mathrm{eff}} v \rangle = \frac{2c_1 + 4c_2 N_c \, e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c + 1)] N_c \, e^{-2\Delta M_T/T}}{\left(1 + N_c \, e^{-\Delta M_T/T}\right)^2}$$

Bound states and thermal widths

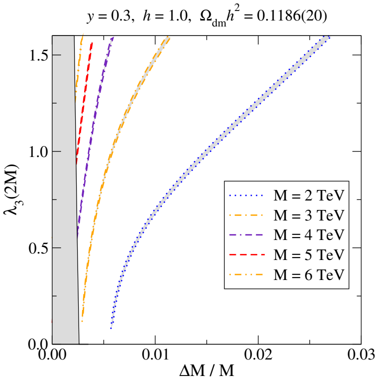


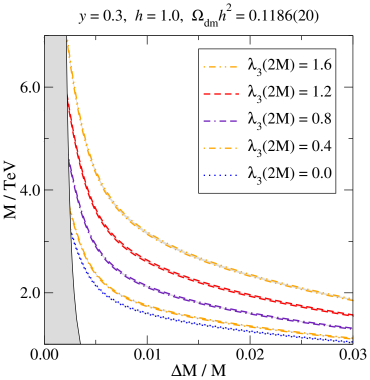
- bound states already start to form at $z \sim 15$ and visible at $z \sim 20$
- at high temperatures: reduced Sommerfeld effect with respect to a massless gluon





- a blind $\Delta M = 0$ brings to very large masses M
- **however the splitting cannot be arbitrary small**!
  if $2\Delta M - |E_1| < 0$ the lightest two-particle states are $(\eta^\dagger\eta)$
  $\Rightarrow$ $(\chi\chi)$ rapidly convert into $(\eta^\dagger\eta)$ that are short lived and promptly decay





- gray bands implement the constraint $2\Delta M - |E_1| > 0$
- the model can be phenomenologically viable up to $M \sim 5...6$ TeV
- y and h have a small impact on $\Omega_{\rm dm}$, whereas λ_3 enters the very efficient singlet channel thorough $c_3 = (\lambda_3^2 + g_s^2 C_F/N_c)/(32\pi^2 M^2)$
- Note: a $\lambda_3 \neq 0$ is always generated at high scale (from RGEs)

Summary and Outlook

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- Simplified models with QCD (or strong) interactions: much larger effect
- The bounds is shifted from $M \sim 1.2$ TeV up to $M \sim 6$ TeV

Summary and Outlook

- Attempt to refine the calculation of the thermal freeze-out for WIMPs

- the freeze-out calculation is factorized into $\langle \sigma v \rangle \approx c_i \langle \mathcal{O}_i \rangle_T$
- $E \sim M$: matching coefficients at $T = 0 \rightarrow$ NREFTs
- $E \sim T, m_D$ solve a thermally modified Schrödinger equation for $\rho(E)$: distortion of the wave function, thermal widths, Salpeter correction, bound states

- Modest effect for the Inert Doublet Model: 1% to 20%
- **however** study the parameter space compatible with the new overclosure bounds

- Simplified models with QCD (or strong) interactions: much larger effect
- The bounds is shifted from $M \sim 1.2$ TeV up to $M \sim 6$ TeV

- **Outlook**: Address other models, study the impact of the Higgs (scalar) exchange, compare with bounds from experimental analysis

IDM mass ranges

- Low-mass regime: $M \lesssim M_W$
- Intermediate regime: $M_W \lesssim M \lesssim 535$ GeV, ruled out by XENON

  XENON100 Collaboration, E. Aprile et al. (2012), 1207.5988
- High-mass regime: $M \gtrsim 535$ GeV, unitary bound $\lambda_i \sim 4\pi \Rightarrow M \sim 58$ TeV

  T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, 0903.4010

Thermally average cross section and freeze-out

- thermally averaged cross section

$$\langle \sigma v \rangle = \frac{\int d^3p_1 d^3p_2 \; \sigma v \, e^{-E_1/T} e^{-E_1/T}}{\int d^3p_1 d^3p_2 e^{-E_1/T} e^{-E_1/T}}$$

- Freeze-out estimation

$$H \sim n\langle \sigma v \rangle \Rightarrow \frac{T^2}{m_{\rm Pl}} \sim \left(\frac{MT}{2\pi}\right) e^{-\frac{M}{T}} \frac{\alpha^2}{M^2}$$

- Thermal expectation value

$$\gamma = \frac{1}{\mathcal{Z}} e^{-E_m/T} \sum_m \langle m|\theta^\dagger \eta^\dagger \eta \theta|m\rangle$$

Sommerfeld factors at $T = 0$

- electroweak potentials: short distance part $r \ll m_{\widetilde{W}}$

$$\mathcal{V}_1(r) \simeq \frac{3g^2 + g'^2}{16\pi r}\,, \quad \mathcal{V}_2(r) \simeq \frac{g^2 - g'^2}{16\pi r}\,, \quad \mathcal{V}_3(r) \simeq \frac{g^2 + g'^2}{16\pi r}$$

- then we can use the standard form of the Sommerfeld factors

$$S_1 = \frac{X_1}{1 - e^{-X_1}}\,, \quad S_{2,3} = \frac{X_{2,3}}{e^{-X_{2,3}} - 1}$$

- where $X_i = \pi\alpha_i/v$ and $E' = 2\Delta M_T + Mv^2$

HTL approximation

- HTL is justified when the particle with which the gauge fields interact are ultrarelativistic, i.e. $m \ll \pi T$
- top and bottom common mass m_f, $W^{\pm}, Z, h$ with a common mass m_g

$$m_{\rm E1}^2 \simeq \frac{g'^2}{2}\left[\frac{49\,T^2}{18} + \frac{11\chi_F(m_f)}{3} + \chi_B(m_g)\right]$$

$$m_{\rm E2}^2 \simeq \frac{g'^2}{2}\left[\frac{3\,T^2}{2} + 3\chi_F(m_f) + 5\chi_B(m_g)\right]$$

- this is however a pure phenomenological recipe $m_b < \pi T < m_t$
- temperature dependent Higgs expectation value (it vanishes for $T \approx 160\,GeV$)

$$v_T^2 \equiv -\frac{m_\phi^2}{\lambda} \text{ for } m_\phi^2 < 0\,, \quad m_\phi^2 \equiv -\frac{m_h^2}{2} + \frac{(g'^2 + 3g^2 + 8\lambda + 4h_t^2)\,T^2}{16}$$

Low-temperature and mass splitting

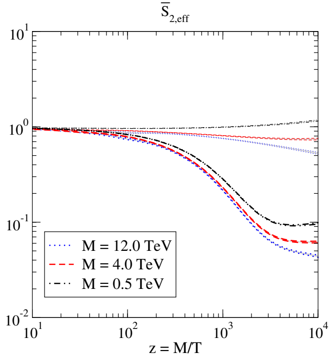
- the vacuum mass difference ΔM becomes important at very low temperature
- the effect is to reduce the importance of the coannihilating species
- it can be phenomenologically included via the substitution

$$\bar{S}_1 \;\rightarrow\; \bar{S}_{1,\text{eff}} \equiv \bar{S}_1 \left[\frac{1}{4} + \frac{3e^{-2\Delta M/T}}{4}\right]$$

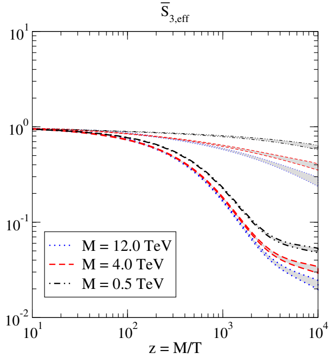
$$\bar{S}_{2,3,4} \;\rightarrow\; \bar{S}_{2,3,4,\text{eff}} \equiv \bar{S}_{2,3,4} \left[\frac{1}{12} + \frac{e^{-\Delta M/T}}{3} + \frac{7e^{-2\Delta M/T}}{12}\right]$$

- the appearance of $2\Delta M_T$ in $\bar{S}_i$ is due to

$$n_{eq} \approx 4 \left(\frac{MT}{2\pi}\right)^{\frac{3}{2}} e^{-(M+\Delta M_T)/T} \quad (1)$$



$\bar{S}_{2,eff}$
10^1
10^0
10^{-1}
10^{-2}
10^1
10^2
10^3
10^4
z = M/T
M = 12.0 TeV
M = 4.0 TeV
M = 0.5 TeV



$\bar{S}_{3,eff}$
10^1
10^0
10^{-1}
10^{-2}
10^1
10^2
10^3
10^4
z = M/T
M = 12.0 TeV
M = 4.0 TeV
M = 0.5 TeV

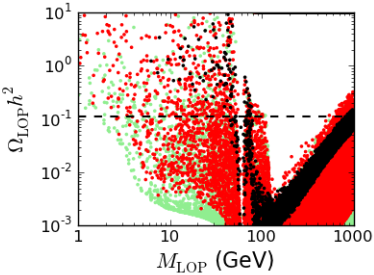
IDM scalar masses

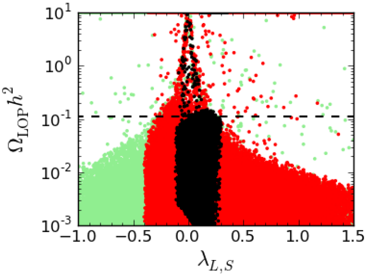
- with $v \equiv \langle \phi \rangle$

$$M_{H_0} = M^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)v^2 \,,$$
$$M_{H_{\bar{0}}} = M^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5)v^2 \,,$$
$$M_{H_0} = M^2 + \frac{1}{2}\lambda_3 v^2 \,,$$
$$\Delta M_{\rm SM} = \frac{g^2}{4\pi} M_W \sin^2\frac{\theta_W}{2}$$

- the different components can be non degenerate in mass

$$C = \begin{pmatrix} H_+ \\ \frac{H_0 - iH_{\bar{0}}}{\sqrt{2}} \end{pmatrix} \,, \quad D = \begin{pmatrix} H_- \\ \frac{H_0 + iH_{\bar{0}}}{\sqrt{2}} \end{pmatrix}$$

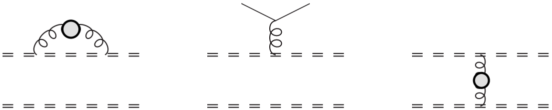




A. Goudelis, B. Herrmann and O. Stal 1303.3010

Scalar QCD potential

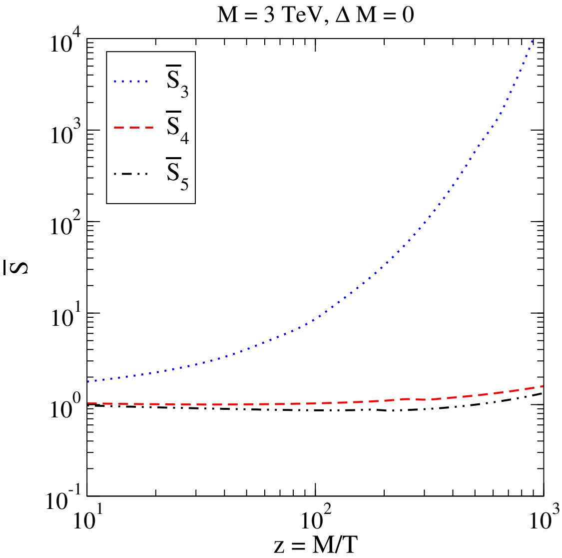
$$V(r) = \frac{g_s^2}{2} \begin{cases} \frac{\exp(-m_D r)}{4\pi r} - \frac{iT}{2\pi m_D r} \int_0^\infty \frac{dz \sin(z m_D r)}{(1+z^2)^2} \,, & r > 0 \\ -\frac{m_D}{4\pi} - \frac{iT}{4\pi} \,, & r = 0 \end{cases}$$



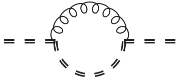
$$\mathcal{V}_1 = 0 \,, \quad \mathcal{V}_2 = C_F V(0) \,, \quad \mathcal{V}_3 = 2C_F[V(0) - V(r)]$$

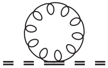
$$\mathcal{V}_4 = 2C_F V(0) + \frac{V(r)}{N_c} \,, \quad \mathcal{V}_4 = 2C_F V(0) + \frac{(N_c - 1) V(r)}{N_c}$$

Sommerfeld for scalar QCD

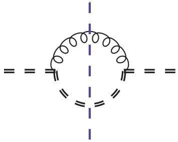


Rates I

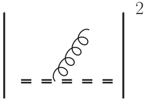




$$\frac{\mathrm{Re}\Pi_R}{2M_\eta} = \frac{g_s^2 C_F T^2}{12 M_\eta}$$



$\sim$

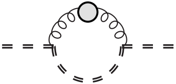


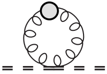
$$\frac{\mathrm{Im}\Pi_R}{2M_\eta} = 0$$

- $M_\eta + \Delta M$ and $\Delta M \ll \pi T \ll M_\eta$
- real part $\sim g_s^2 C_F \Delta M$ and imaginary part $\sim g_s^2 C_F |\Delta M| n_B(|\Delta M|) \sim g_s^2 C_F T$
- Resummed mass correction dominates over the unresummed when

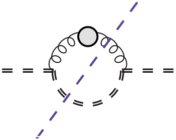
$$g_s^2 \frac{T^2}{M} \lesssim g_s^2 \underbrace{g_s T}_{m_D} \Rightarrow \frac{T}{M} \lesssim g_s$$

Rates II





$$\frac{\mathrm{Re}\Pi_R}{2M_\eta} = \frac{g_s^2 C_F T^2}{12 M_\eta} - \frac{g_s^2 C_F m_D}{8\pi}$$



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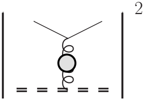
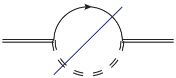
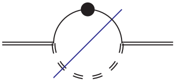


$$\frac{\mathrm{Im}\Pi_R}{2M_\eta} = -\frac{g_s^2 C_F T}{8\pi}$$

Rates III: equilibrium in the dark sector

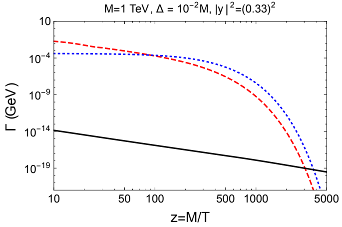




- $1 \to 2$ and $2 \to 2$ scattering

$$\Gamma_{1\to 2} = \frac{|y|^2 N_c M}{4\pi} \left(\frac{\Delta}{M}\right)^2 n_F(\Delta)$$

$$\Gamma_{2\to 2} = \frac{N_c |y|^2}{8M} \int \frac{d^3p}{(2\pi)^3} \frac{\pi m_q^2}{p(p^2+m_q^2)} n_F\left(\Delta + \frac{p^2}{2M}\right)$$



Gluodissociation in quarkonium

- $M \gg 1/r \gg T \gg \Delta V$, start with pNRQCD
- difference between the octet and singlet potential

$$\Delta V = \frac{1}{r}\left(\frac{\alpha_s}{2N_c} + C_F\alpha_s\right) = \frac{N_c\alpha_s}{2r} \sim M\alpha_s^2$$

- the thermal width is

$$\Gamma = \frac{4}{3}C_F\alpha_s r^2(\Delta V)^3 n_B(\Delta V) \approx \frac{1}{3}N_c^2 C_F\alpha_s^3 T$$

- at small distances the two contributions are

$$\Gamma_{\text{LD}} \sim g_s^2 C_F T m_D^2 r^2 \,, \quad \Gamma_{\text{GD}} \sim g_s^2 C_F T(\Delta E)^2 r^2$$