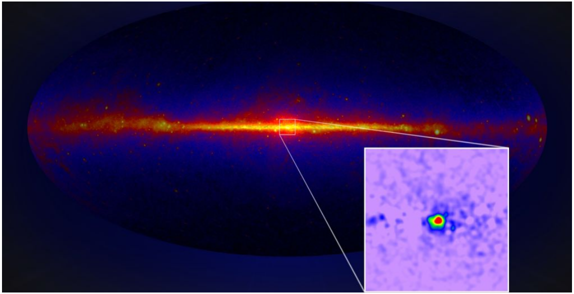
Characterizing Unresolved Point Sources with the Non Poissonian Template Fit



Ben Safdi

University of Michigan
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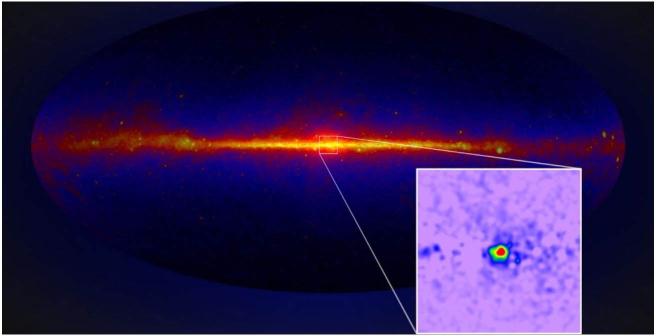
- My focus today: explain the NPTF as a method

Example Application: *Fermi* Galactic Center Excess

Galactic Center Excess in *Fermi* data



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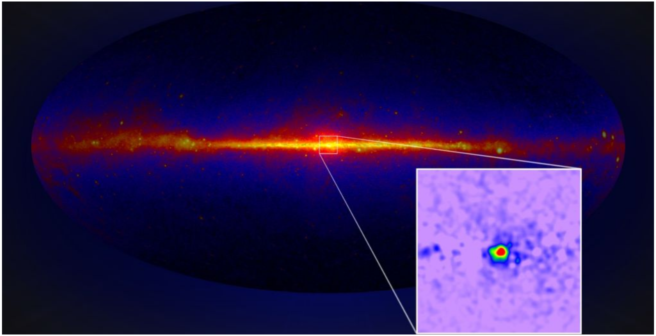

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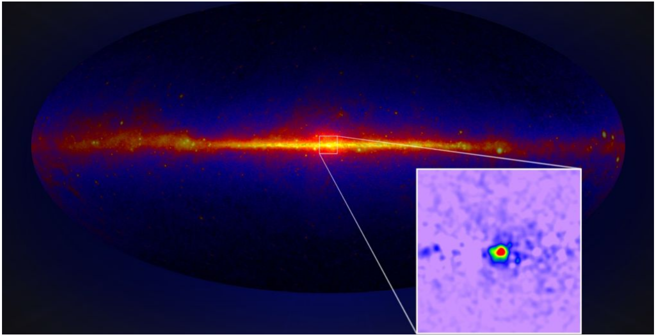

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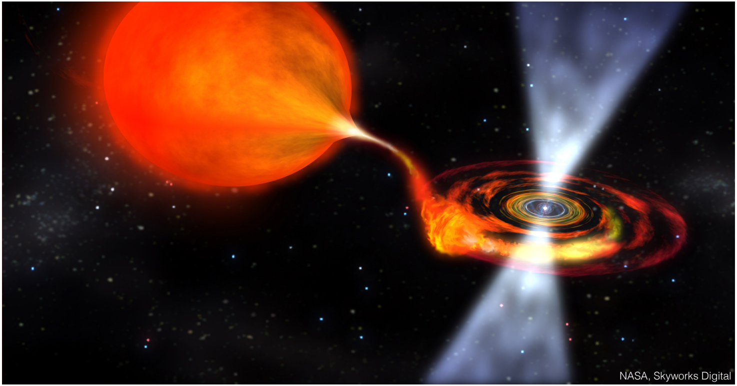
- Robust against mis-modeling cosmic-ray-induced emission (but see E. Carlson et. al. 2016)

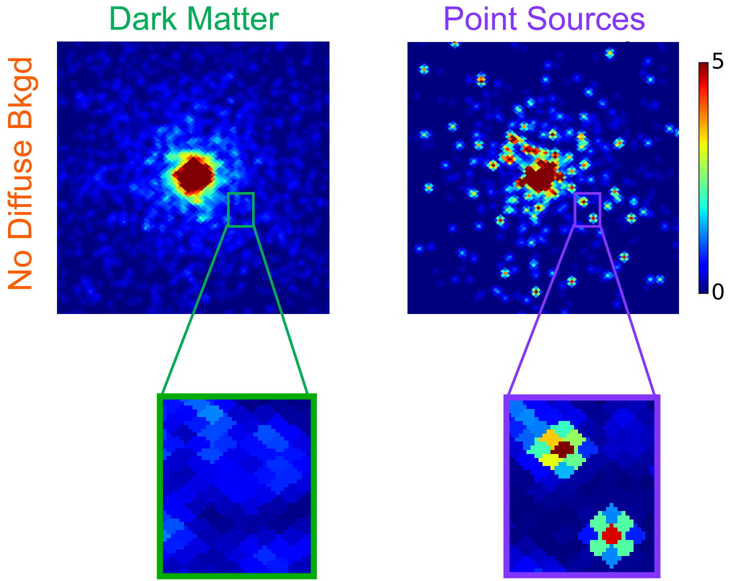
Dark Matter or dim Point Sources?



- Spherically symmetric population of millisecond pulsars
- ApJ 812 (2015): T. Brandt, B. Kocsis—Globular cluster model + MSP luminosity
- May appear more PS-like than DM annihilation

The Non-Poissonian Template Fit

Photon Statistics: DM vs. Point Sources



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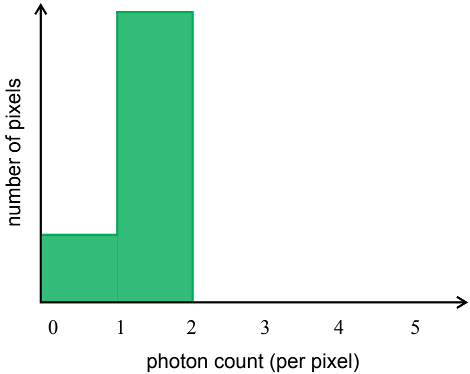

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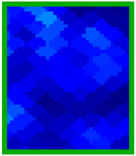

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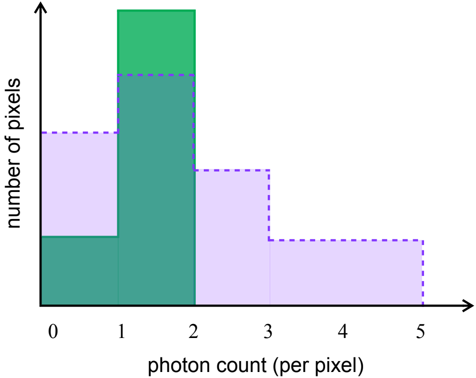

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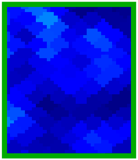
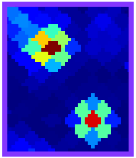


Point Sources



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  - F is average flux (photons / cm^2 / s)
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- Calculate the $p_k^{(p)}$ with probability generating functions (Malyshev & Hogg 2011) + recursion relations (Lee, Lisanti, **B.S.** 2014)

Non-Poissonian template fit (NPTF)

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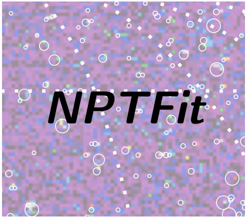
- The likelihood function:

$$p(d|\theta, \mathcal{M}) = \prod_{\text{pixels } p} p_{n_p}^{(p)}(\theta)$$

The NPTF Code Package (`NPTFit`)



- N. Rodd, **B.S.**, S. Mishra-Sharma, Astron. J. 2017
- Open source: `https://github.com/bsafdi/NPTFit`
- Extensive documentation: `https://nptfit.readthedocs.io`
- Fast and semi-analytic evaluation of $p_{n_p}^{(p)}(\theta)$ and $p(d|\theta, \mathcal{M})$ in `c++`
  - any PSF, variety of dN/dS characterizations, arbitrary number of PS templates.
- Python interface
- Bayesian (`Multinest`) and Frequentist (`Minuit`) options

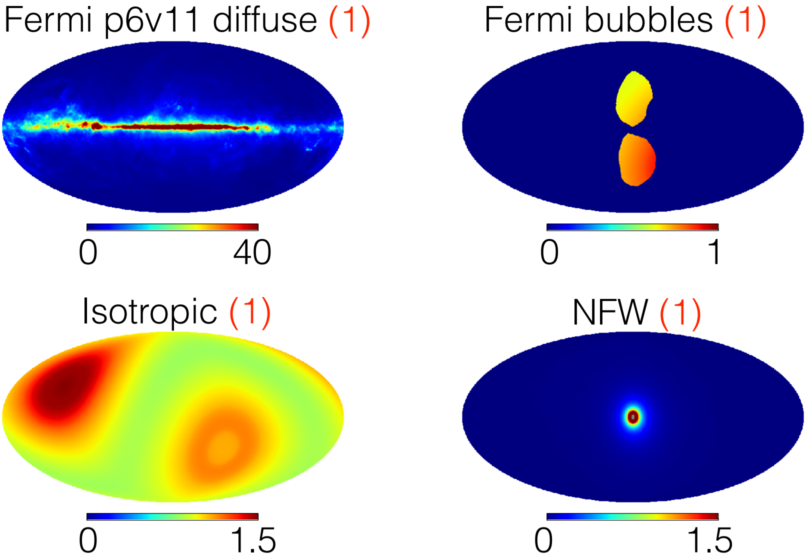
NPTF in practice: the Galactic Center Excess

Fermi data

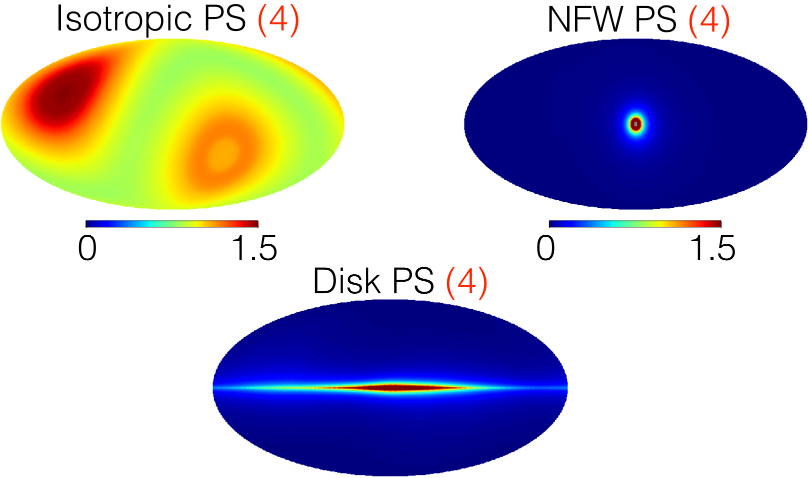
Fermi (NASA)

- Pass 8 data:
  Ultracleanveto class, top quartile by PSF (August 4, 2008—June 3, 2015)
- Energy range: ∼2–12 GeV

The models: Poissonian templates

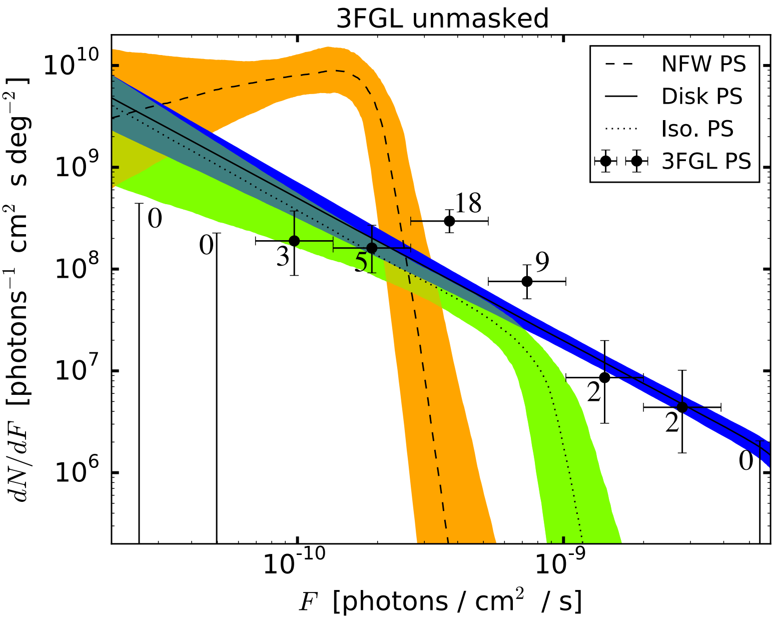


The models: Non-Poissonian templates



- Disk: $n \propto \exp\left(-R/\,5\text{ kpc}\right)\exp\left(-|z|/\,0.3\text{ kpc}\right)$

The $\ell = 0^\circ$ excess: source-count function



eNPTF: Adding in Energy dependence (in progress)

The Energy-Dependent Non-Poissonian Template Fit (eNPTF)

- M. Buschmann (UM), N. Rodd (MIT/Berkeley), **B.S.** (UM) (to appear 2018)

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- The likelihood function:

$$p(d|\theta, \mathcal{M}) = \prod_{\text{pixels } p} p^{(p)}_{n_p^{(1)}, n_p^{(2)}, \ldots, n_p^{(N_e)}}(\theta)$$

- Keeps correlation between photons in same pixel but different energy bins, allows for non-trivial spectra in $\mathcal{M}$.

The eNPTF

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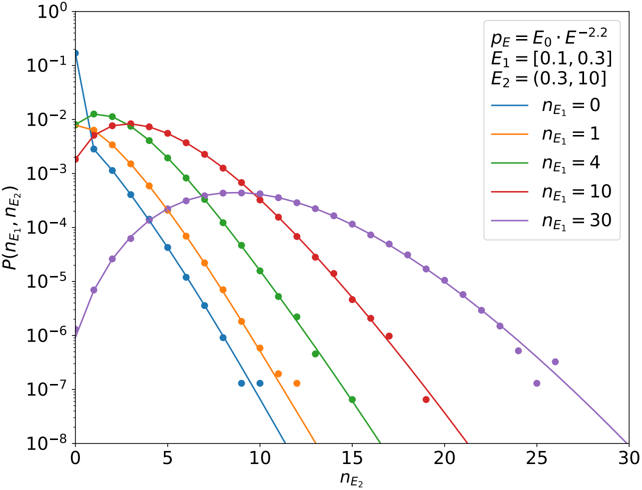

The eNPTF example: The GCE with Monte Carlo

- 3 energy bins $[2.0, 5.0, 12.6, 31.7]$ GeV
- known 3FGL sources masked
- Simulated components:
  - Diffuse emission
  - Known PSs
  - Unresolved disk and isotropic PSs
  - Spherical PS population with spectrum of GCE
- Templates:
  - Diffuse emission (3)
  - Known PSs (3)
  - Spherical PS population with spectrum of GCE (4 + 2)
    - dN/dF is broken power-law
- Spectral definition: $f_1 = N_2/N_1$, $f_2 = N_3/N_1$

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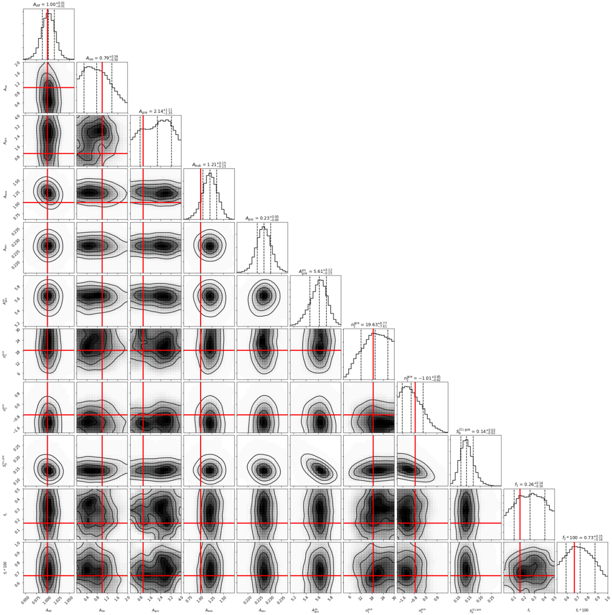

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- eNPTF to appear soon (2018) and be more powerful
- Warning: often more progress made by incorporating other data sets (*e.g.*, for the EGB knowledge of source locations or for extragalactic DM annihilation using locations and attributes of clusters)

Questions?

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- Point sources:

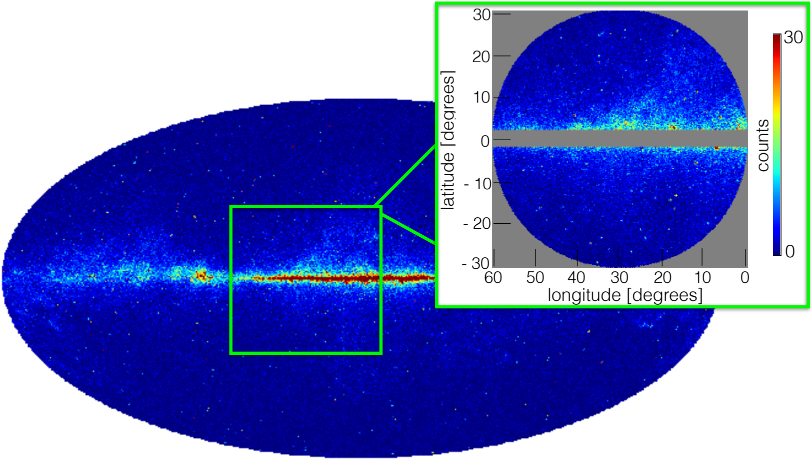
$$P^{(p)}(t) = \exp\left[\sum_{m=1}^{\infty} x_m^p (t^m - 1)\right], \quad x_m^p = \int dS \, \frac{dN^{(p)}}{dS} \frac{S^m}{m!} e^{-S}$$

  - Flux to Counts: $\dfrac{dN^{(p)}}{dS} = \dfrac{1}{\mathcal{E}^{(p)}} \dfrac{dN^{(p)}}{dF}$
  - S is average number of photon counts
  - $\mathcal{E}^{(p)}$ is the exposure map

See Lee, Lisanti, **BS** 2014 for recusion relations

Check 1: the $\ell = 30^\circ$ excess

Mask 4° around plane, out to 30° around $\ell = 30^\circ$



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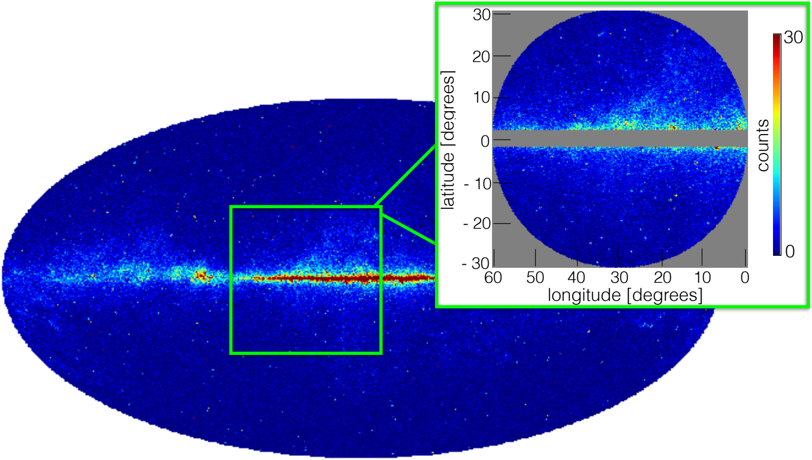
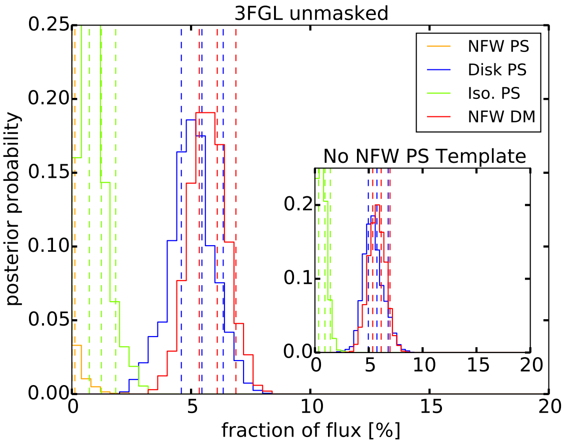


- Plots normalized for region within 10° of ROI center ($b \geq 4^\circ$).

The $\ell = 30^\circ$ excess: no evidence for spherical PSs

- NFW DM, NFW PS templates centered around $\ell = 30^\circ$
- Disk template centered around $\ell = 0^\circ$



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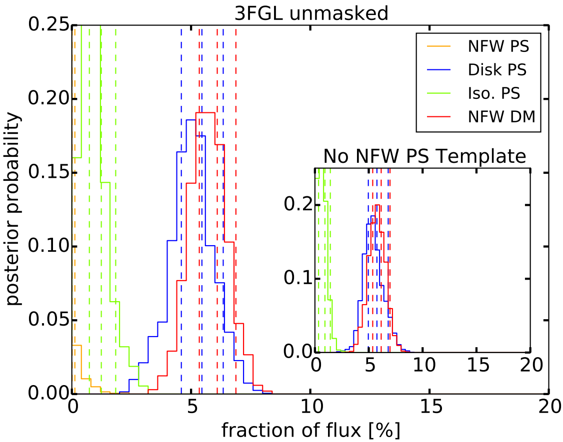

- Bayes factor $\sim$ 0.1

ROI: the $\ell = 0^\circ$ excess

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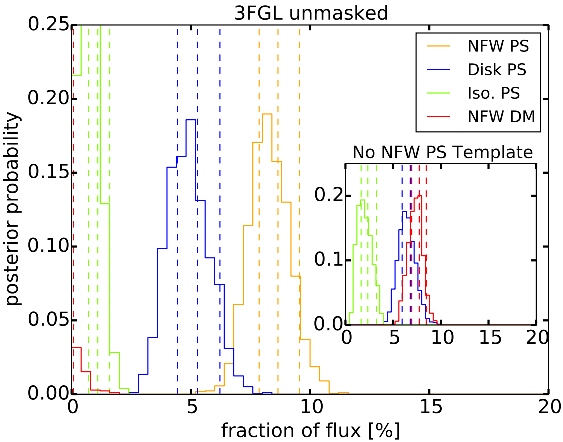

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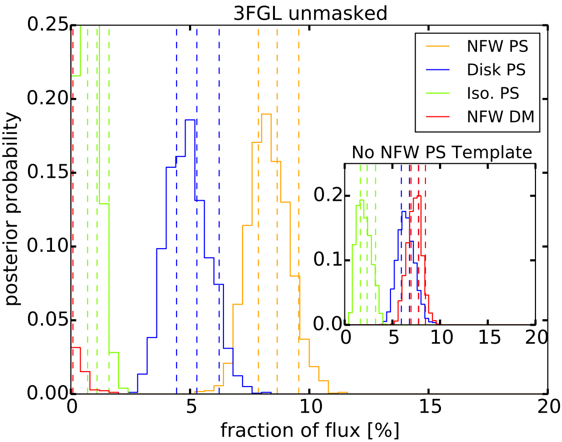
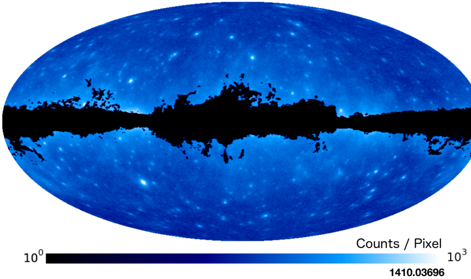
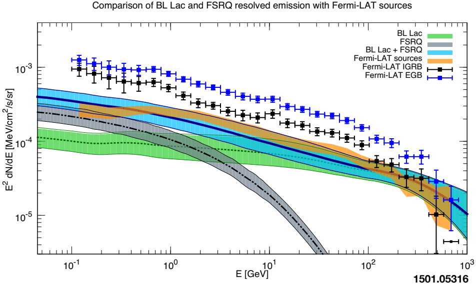


- Bayes factor $\sim 10^9$ (3FGL unmasked), 10^4 (3FGL masked)

Extragalactic Gamma Ray Background

Extragalactic Gamma-ray Background





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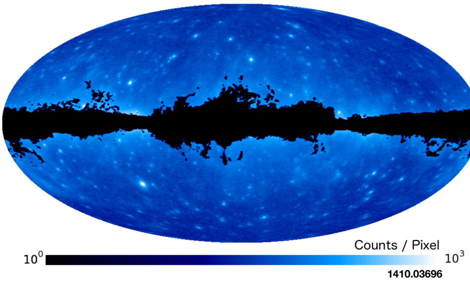
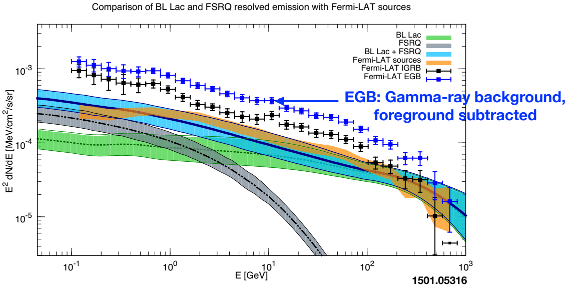
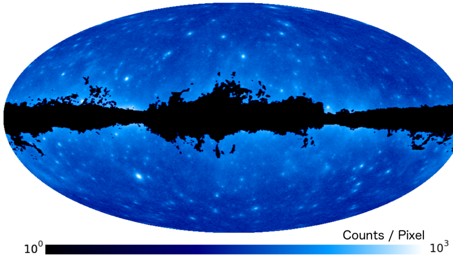
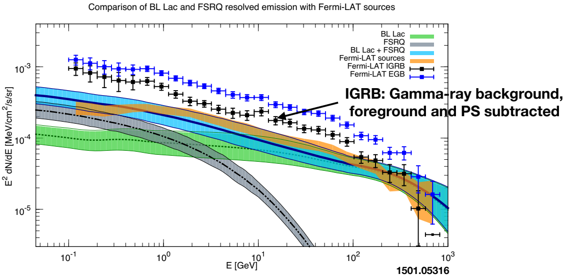




Extragalactic Gamma-ray Background

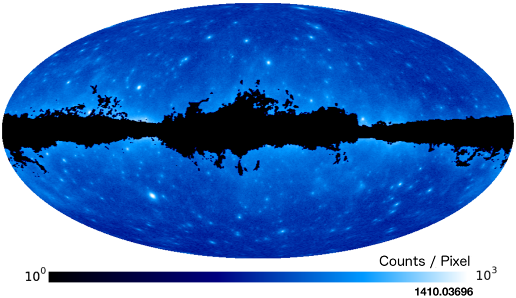


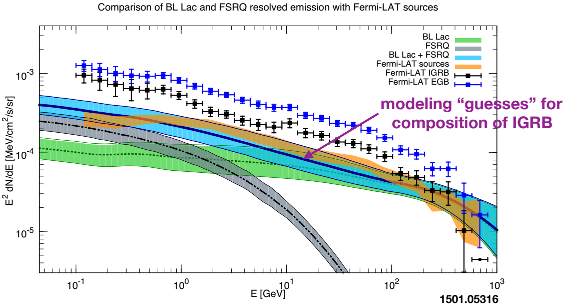
1410.03696



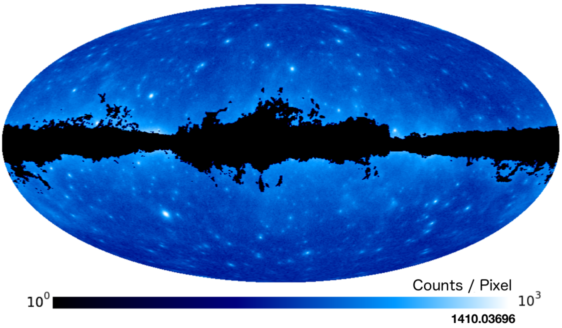
1501.05316

Extragalactic Gamma-ray Background





Why Understand the Extragalactic Gamma-ray Background?



- Constrain contributions from diffuse emission (dark matter)
- Probe source populations (BL Lac, FSRQs, MAGN, SFG)
- Implication for other messengers (*e.g.*, IceCube)

eNPTF details

$$
\begin{aligned}
|a| &= a_1 + \cdots + a_k \\
a + b &= (a_1 + b_1, \ldots, a_k + b_k) \\
a^b &= a_1^{b_1} \cdot \ldots \cdot a_k^{b_k} \\
a! &= \frac{|a|!}{a_1! \cdot \ldots \cdot a_k!}.
\end{aligned}
$$

- The counts originate from different templates, so if $\beta_{p,l} = (n_{p,l,1}, \ldots, n_{p,l,k})$ is the distribution of counts in the l-th template, than $\sum_l \beta_{p,l} = \alpha_p$ holds.
- For a single template l and k different bins, the probability to see $\beta_{p,l}$ counts in pixel p :

$$
p^{(p)}_{\beta_{p,l}}(\vec{\theta}) = p^{(p)}_{|\beta_{p,l}|}(\vec{\theta})\, \beta_{p,l}!\, \lambda_l(\vec{\theta})^{\beta_{p,l}}.
$$

eNPTF details

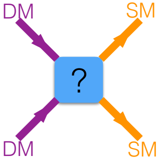
- Here, each $\lambda_{l,m}(\vec{\theta})$ in $\lambda_l(\vec{\theta}) = (\lambda_{l,1}(\vec{\theta}), \ldots, \lambda_{l,k}(\vec{\theta}))$ corresponds to the probability that a count in template l contributes to the m-th bin,
- Sum over all permutations to get probabilities

$$p^{(p)}_{\alpha_p}(\vec{\theta}) = \sum_{\sum_l \beta_{p,l} = \alpha_p} \prod_l p^{(p)}_{\beta_{p,l}}(\vec{\theta}).$$

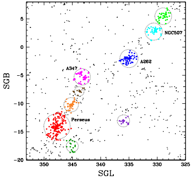
DM annihilation with group catalogs

- 1. DM model + group catalog $\rightarrow$ gamma-ray flux map

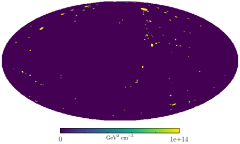
Model / EFT



Group catalogs



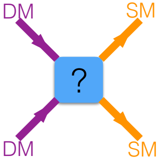
Flux map



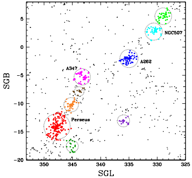
DM annihilation with group catalogs

- 1. DM model + group catalog $\to$ gamma-ray flux map

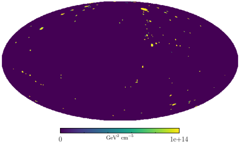
Model / EFT



Group catalogs

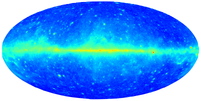


Flux map

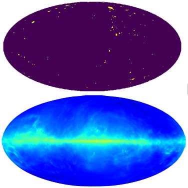


- 2. How do we search for that flux in *Fermi* data?

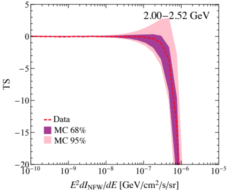
Fermi Data



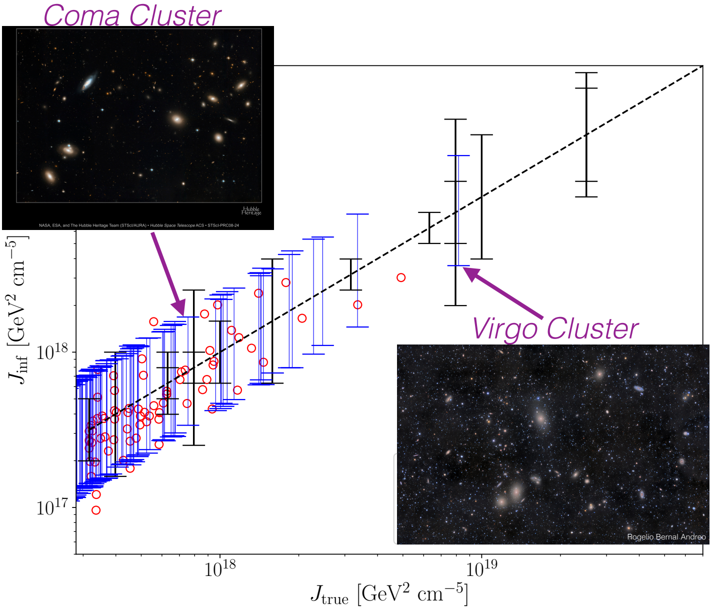
Background + Signal Templates



Likelihood Profiles



DarkSky N-body simulation vs real data



Real Data limit consistent with DarkSky

- Remove handful of halos with large cosmic-ray emission ($\text{TS} > 5$, $\sigma_A v > 10 \times$ best indiv. limit)