Muon beam energy

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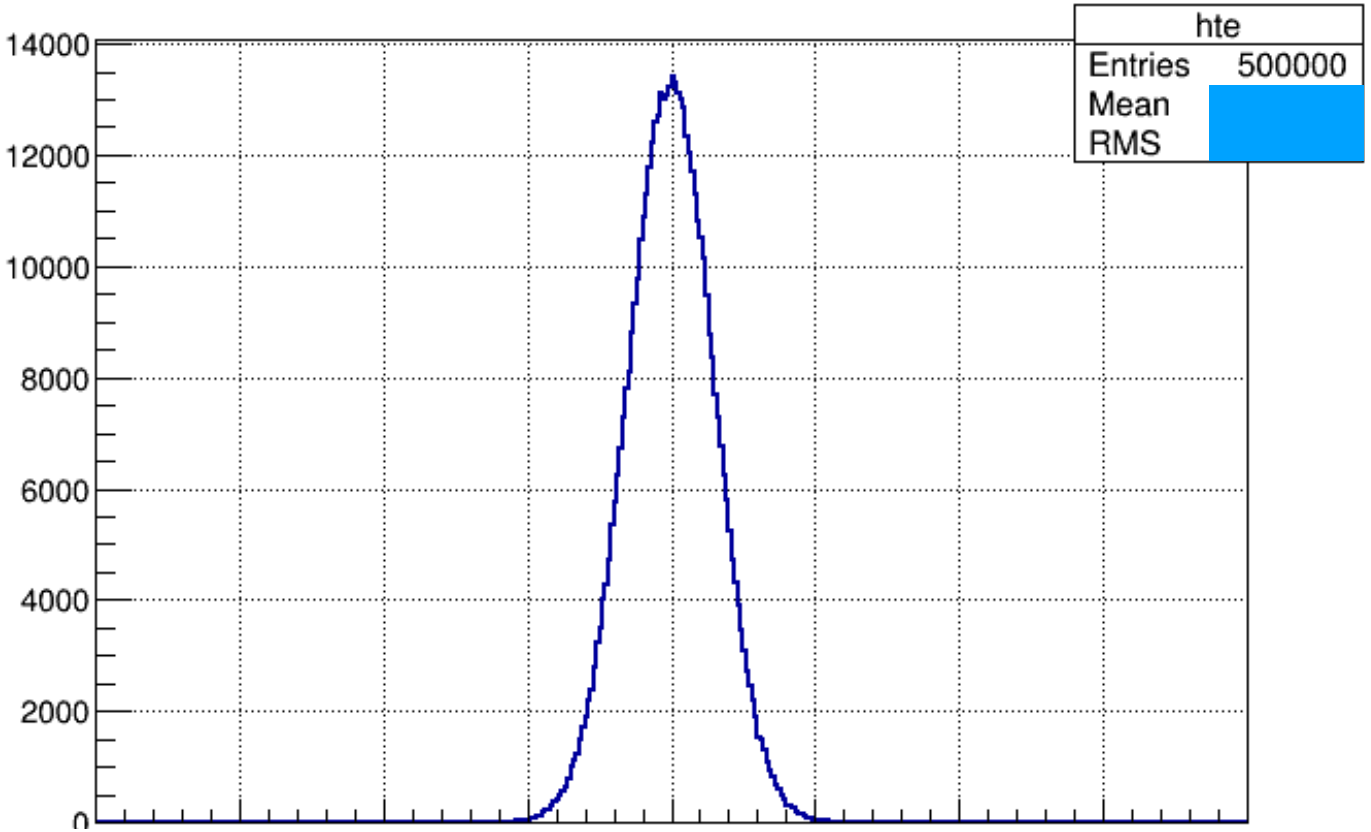
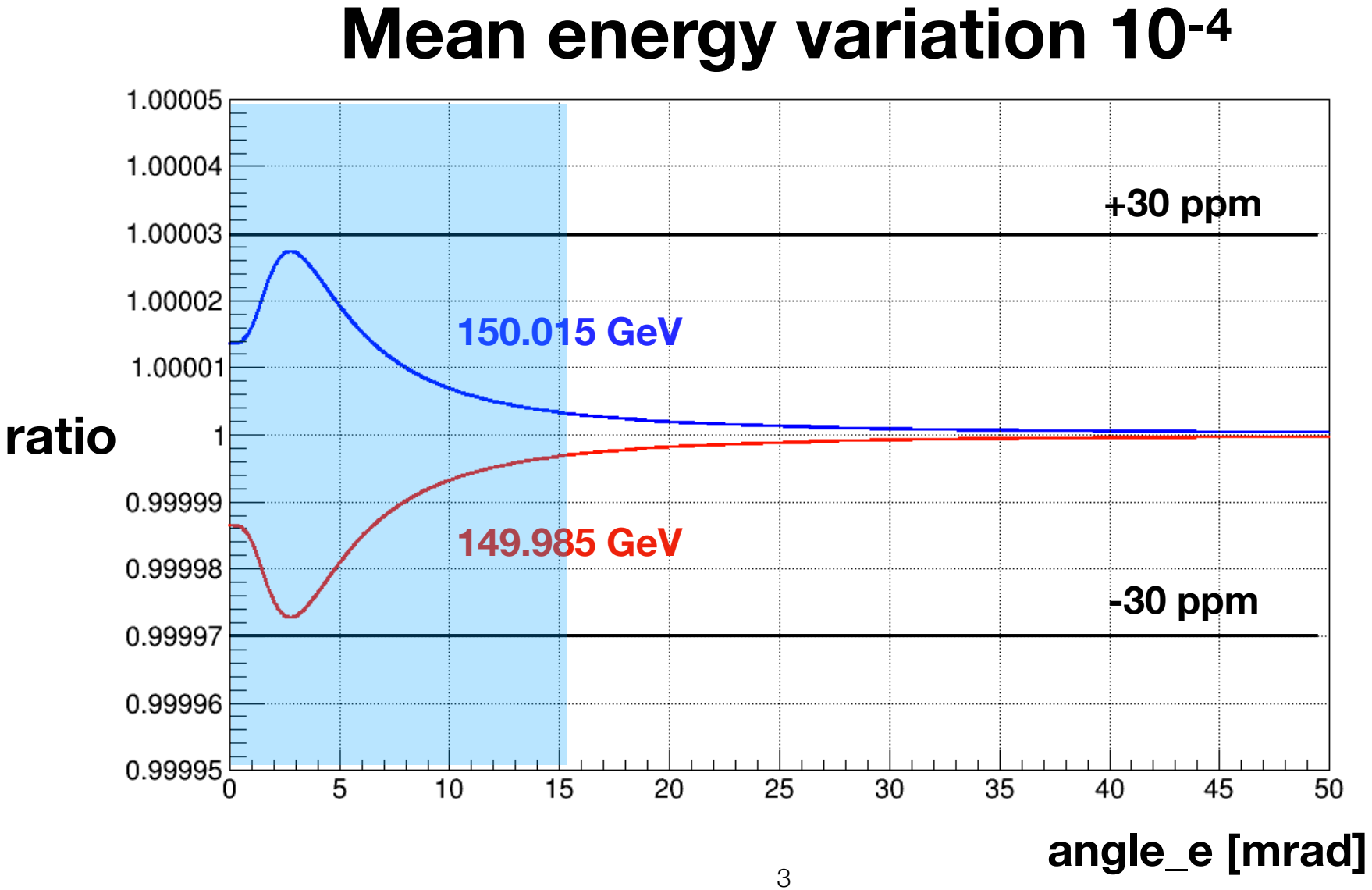
The problem

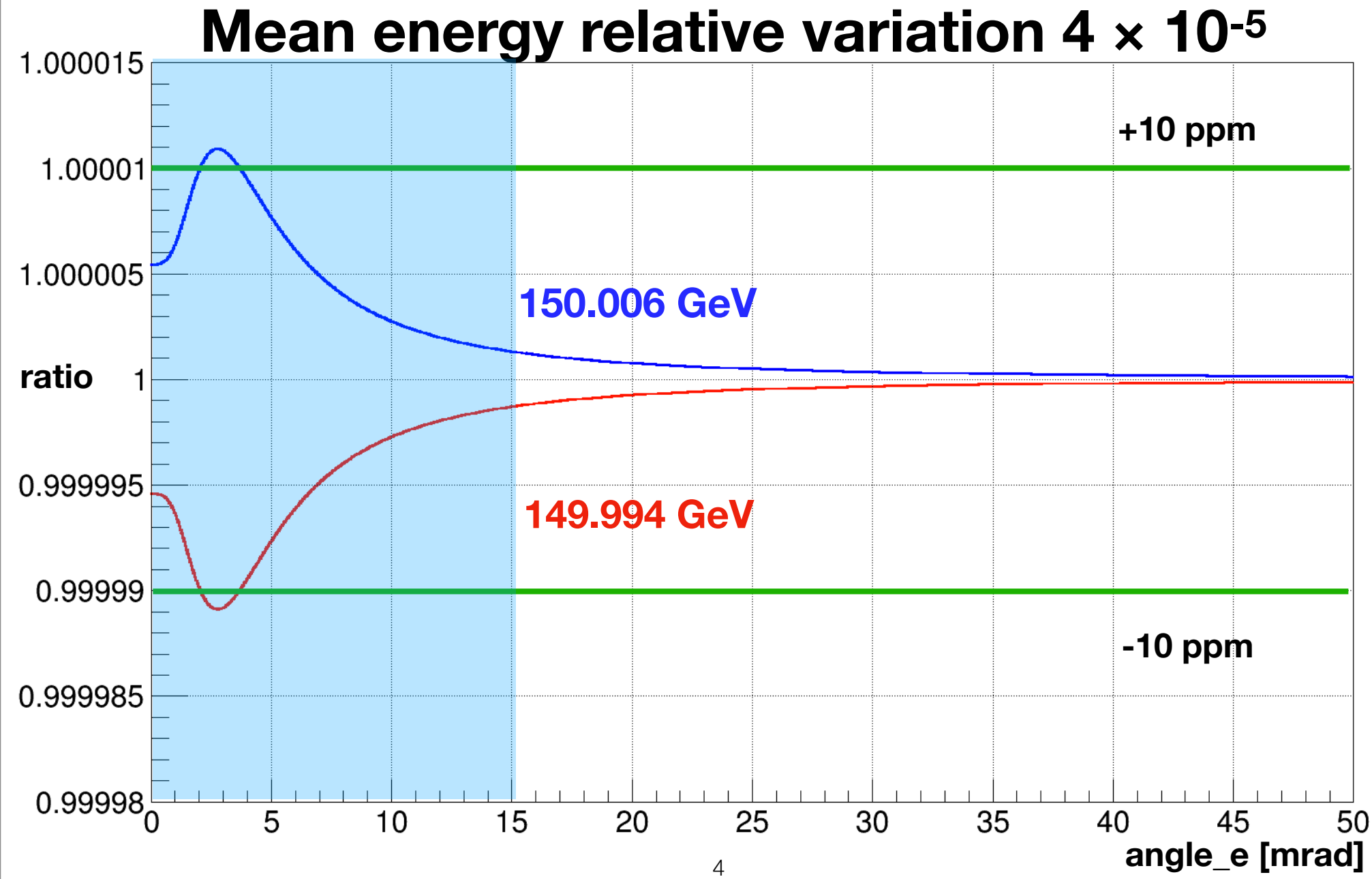
Assume we know the resolution of the beam energy to 1%:
The aim to define the energy scale to measure the elastic scattering at the required precision



Mean Energy uncertainty $\sigma_{LO}(E_\mu)$ relative variation

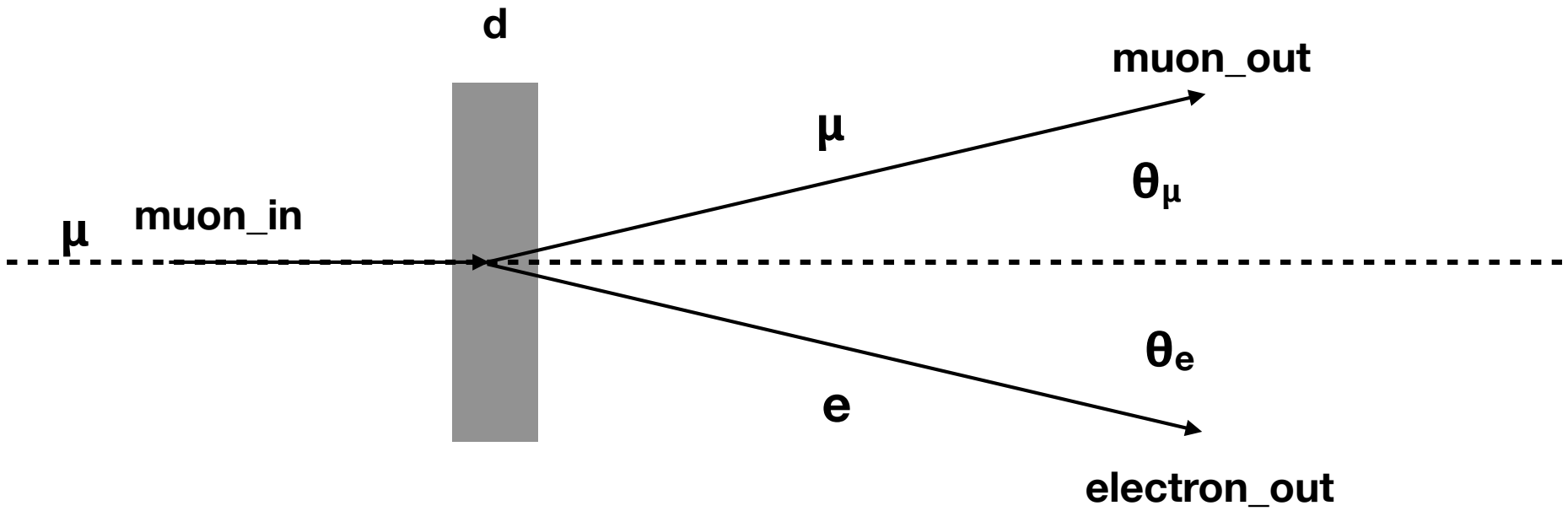


$\sigma_{LO}(E_\mu)$ relative variation



Measurement Technique

Equal angles condition $\theta_\mu = \theta_e$



You can easily check that :

E' ~ E/2 = 75 GeV

$\theta_\mu = \theta_e$ ~ 2.5 mrad

Small MSC effects 0.025 mrad

Equal angles condition

The equal angle condition, angle_e = angle_μ, implies:

$$E'_e = \frac{(E_\mu + m_e)^2 - m_\mu^2 + m_e^2}{2(E_\mu + m_e)}$$

$$E'_\mu = \frac{(E_\mu + m_e)^2 + m_\mu^2 - m_e^2}{2(E_\mu + m_e)}$$

$$E'_e = m_e \frac{1 + r^2 cos^2\theta_e}{1 - r^2 cos^2\theta_e}$$

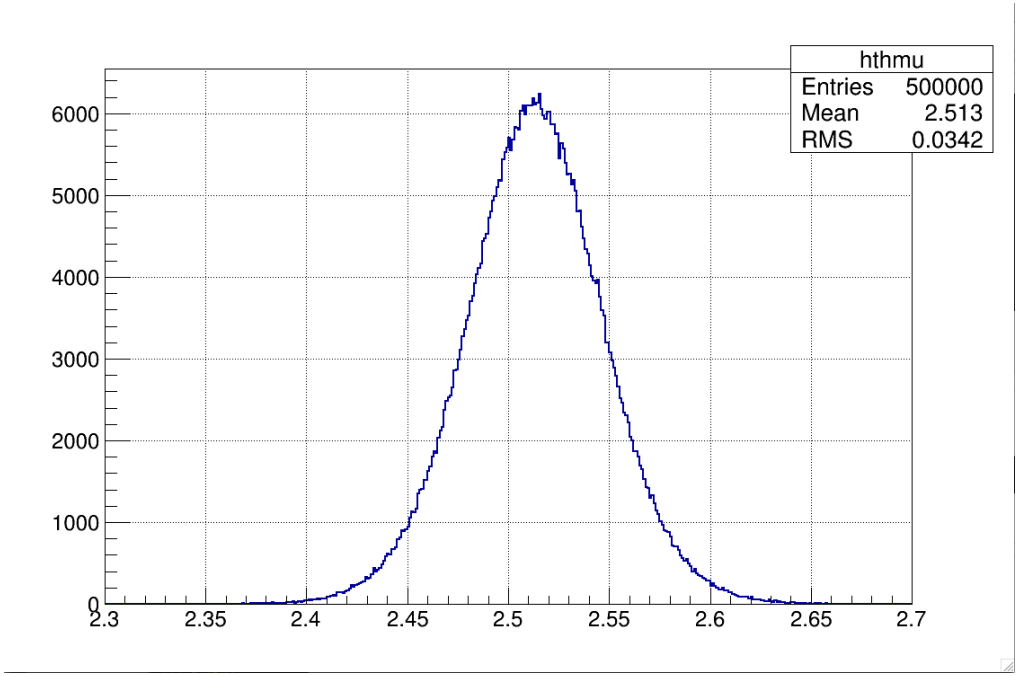
$$r^2 \equiv \frac{E_\mu^2 - m_\mu^2}{(E_\mu + m_e)^2}$$

$$cos^2\theta_e = \frac{x - 1}{x + 1} \times \frac{1}{r^2}$$

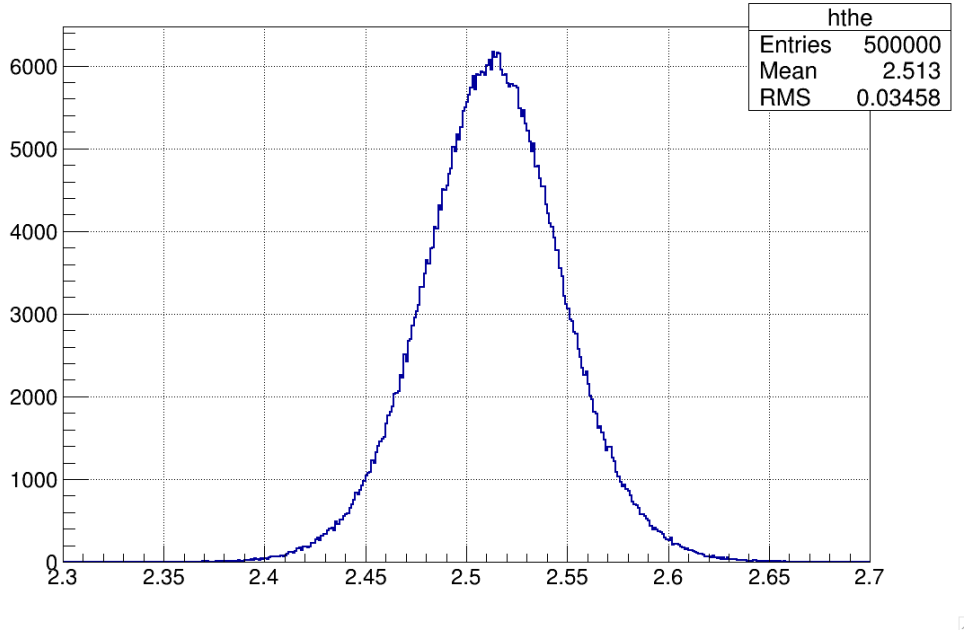
$$x \equiv \frac{(E_\mu + m_e)^2 - m_\mu^2 + m_e^2}{2m_e(E_\mu + m_e)}$$

Angular distributions

muon beam energy 150 GeV and energy resolution 1%



angle_μ [mrad]



angle_e [mrad]

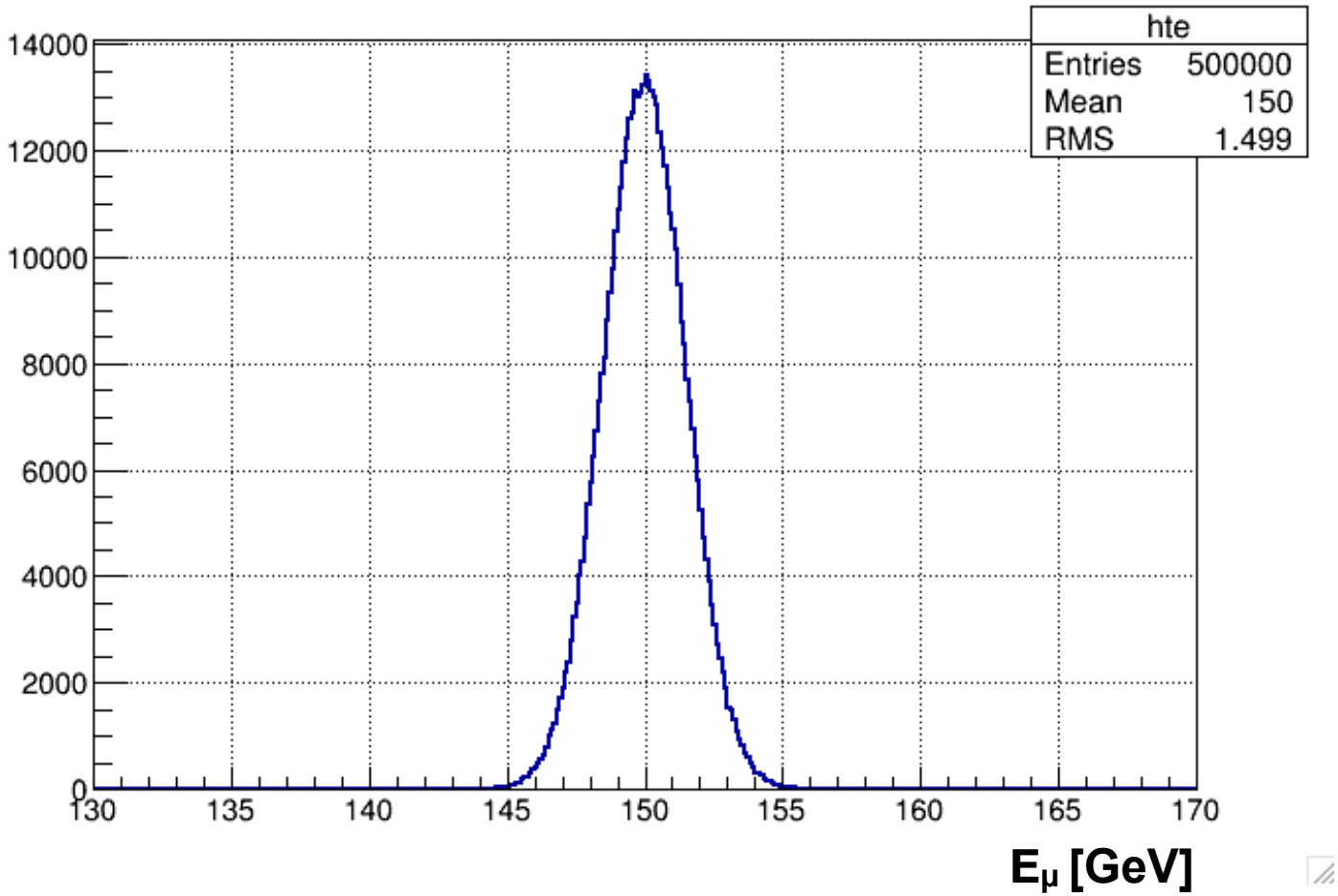
The angular resolution depends on:

- **Muon beam energy fluctuations**
- **Detector angular resolution: 0.02 mrad**
- **MSC effects ~ sqrt(d/E'). d: uniform bewteen 0 and the target thickness. At 75 GeV the expected MSC effect is 0.025 mrad.**

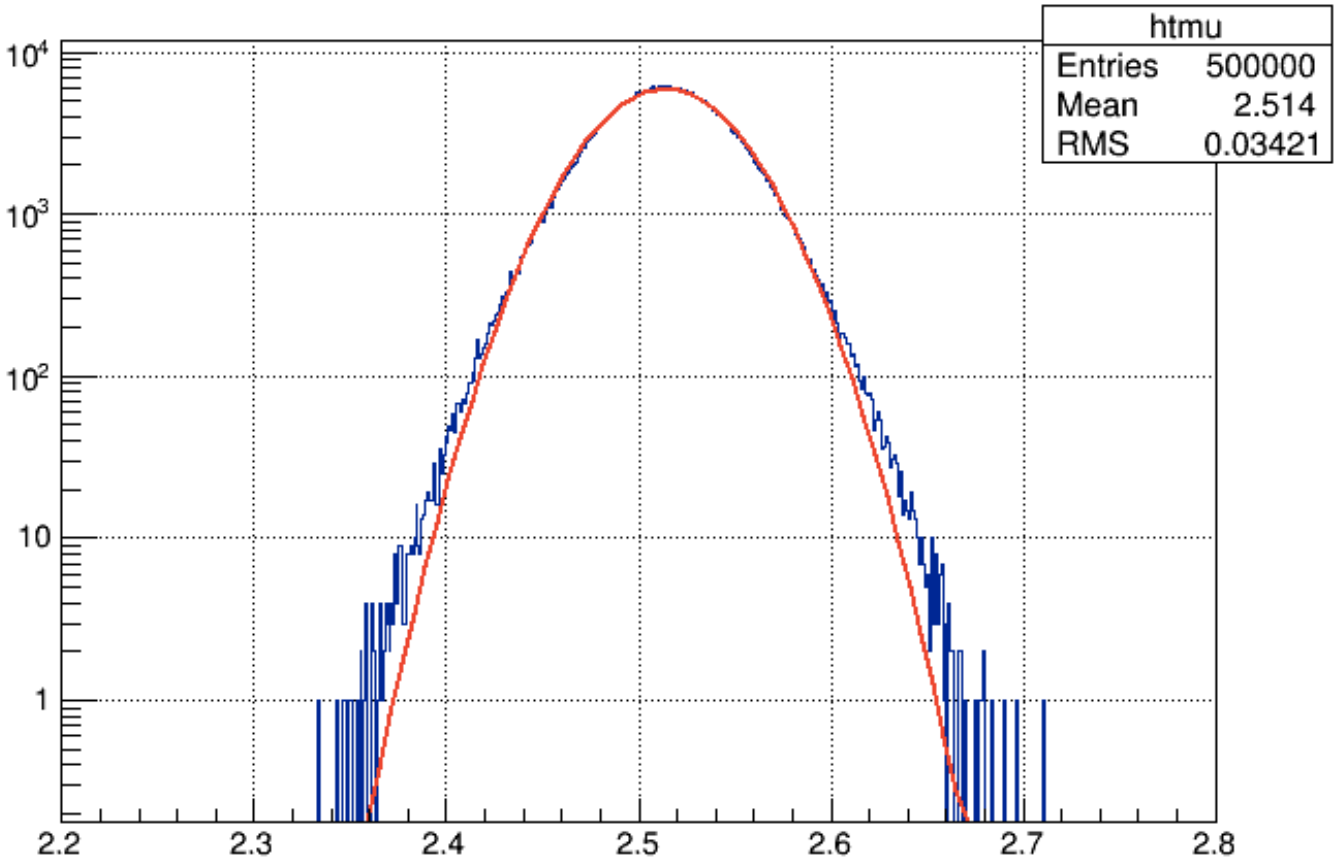
Muon beam energy

Mean energy 150 GeV, energy resolution 1%

Data set: 5×10^5 events



Angular distribution



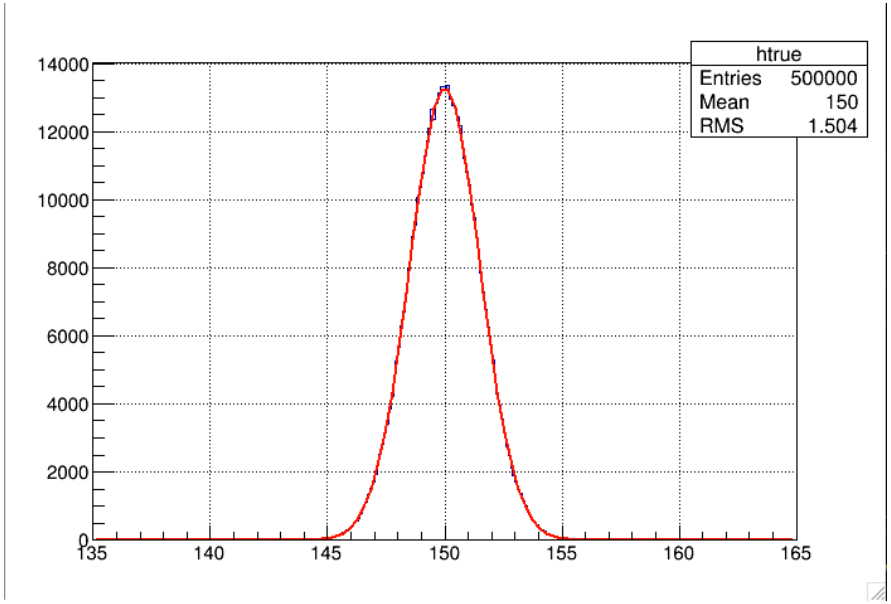

```
root [8] htmu->Fit("gaus")
Info in <TCanvas::MakeDefCanvas>:  created default TCanvas with name c1
 FCN=1765.64 FROM MIGRAD    STATUS=CONVERGED      55 CALLS          56 TOTAL
                     EDM=7.02086e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                                   STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  Constant     5.86916e+03   1.06231e+01   1.70649e-01  -1.32099e-05
   2  Mean         2.51377e+00   4.79833e-05   1.19866e-06  -1.74538e-01
   3  Sigma        3.38665e-02   3.81519e-05   5.99876e-06  -3.50680e-01
```

Numerical Solution

- Given the data sample $\{(\boldsymbol{\theta}_e, \boldsymbol{\theta}_\mu)_k\}$
- Determine $\{(E_\mu, \boldsymbol{\theta}^*)_k\}$ that minimizes the chi2 for the true values $\{(\boldsymbol{\vartheta}_e, \boldsymbol{\vartheta}_\mu)_k\}$
- The chi2 a bivariate gaussian pdf
  $\boldsymbol{\chi}^2 \sim [(\boldsymbol{\theta}_\mu - \boldsymbol{\vartheta}_\mu)/\boldsymbol{\sigma}_\mu]^2 + [(\boldsymbol{\theta}_e - \boldsymbol{\vartheta}_e)/\boldsymbol{\sigma}_e]^2 - 2\boldsymbol{\rho}(\boldsymbol{\theta}_\mu - \boldsymbol{\vartheta}_\mu)/\boldsymbol{\sigma}_\mu\,(\boldsymbol{\theta}_e - \boldsymbol{\vartheta}_e)/\boldsymbol{\sigma}_e$
- $(\boldsymbol{\chi}^2_k)_{min} \rightarrow \{(E_\mu, \boldsymbol{\theta}^*)_k\}$

E_μ = 150.000 GeV, 1%

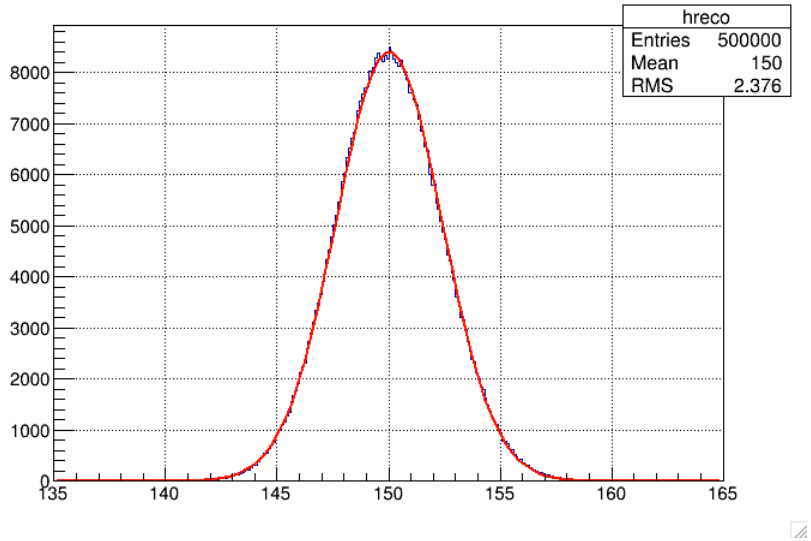
Beam Energy



root [8] htrue->Fit("gaus")
FCN=104.765 FROM MIGRAD STATUS=CONVERGED 49 CALLS 50 TOTAL
EDM=3.39896e-08 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER			STEP	FIRST
NO.	NAME	VALUE	ERROR	SIZE	DERIVATIVE
1	Constant	1.32641e+04	2.29710e+01	9.42060e-02	-3.89288e-07
2	Mean	1.50001e+02	2.12684e-03	7.15262e-05	1.22427e-01
3	Sigma	1.50354e+00	1.50298e-03	1.36629e-06	-5.10040e-02

Reconstructed Energy
Gaussian resolution × 2, no MSC

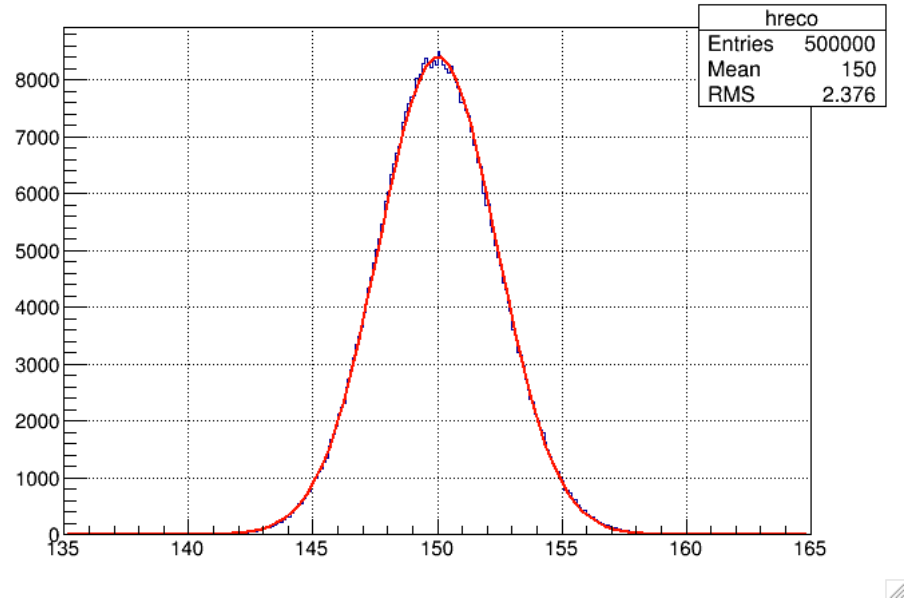


root [4] hreco->Fit("gaus")
FCN=420.25 FROM MIGRAD STATUS=CONVERGED 60 CALLS 61 TOTAL
EDM=2.97167e-08 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER			STEP	FIRST
NO.	NAME	VALUE	ERROR	SIZE	DERIVATIVE
1	Constant	8.40670e+03	1.45468e+01	1.19203e-01	1.81767e-05
2	Mean	1.50021e+02	3.36485e-03	7.15355e-05	-2.89061e-03
3	Sigma	2.37083e+00	2.36286e-03	2.71368e-06	7.99378e-01

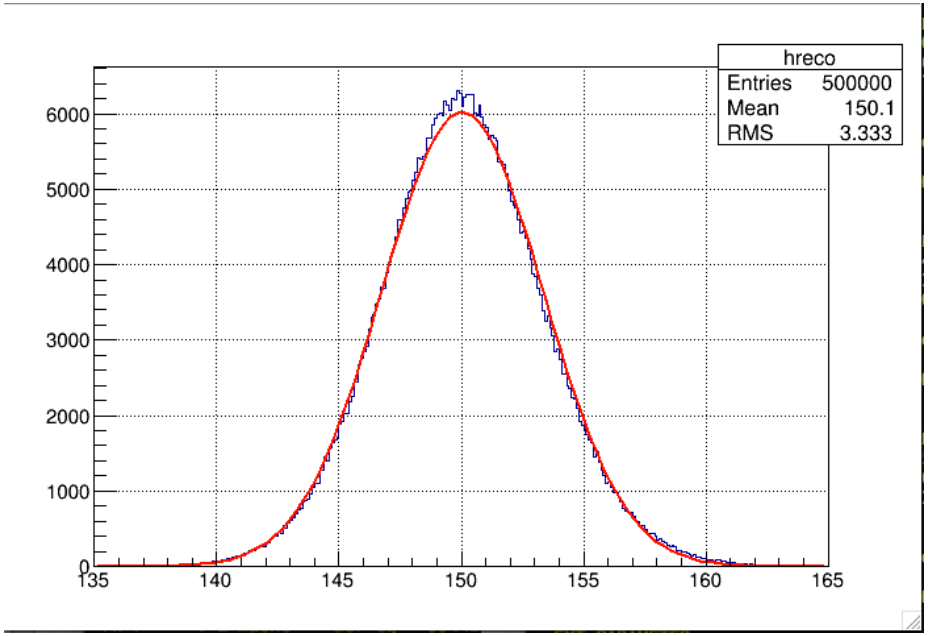
E_μ = 150.000 GeV, 1% (2)

Reconstructed Energy
Gaussian resolution × 2, no MSC

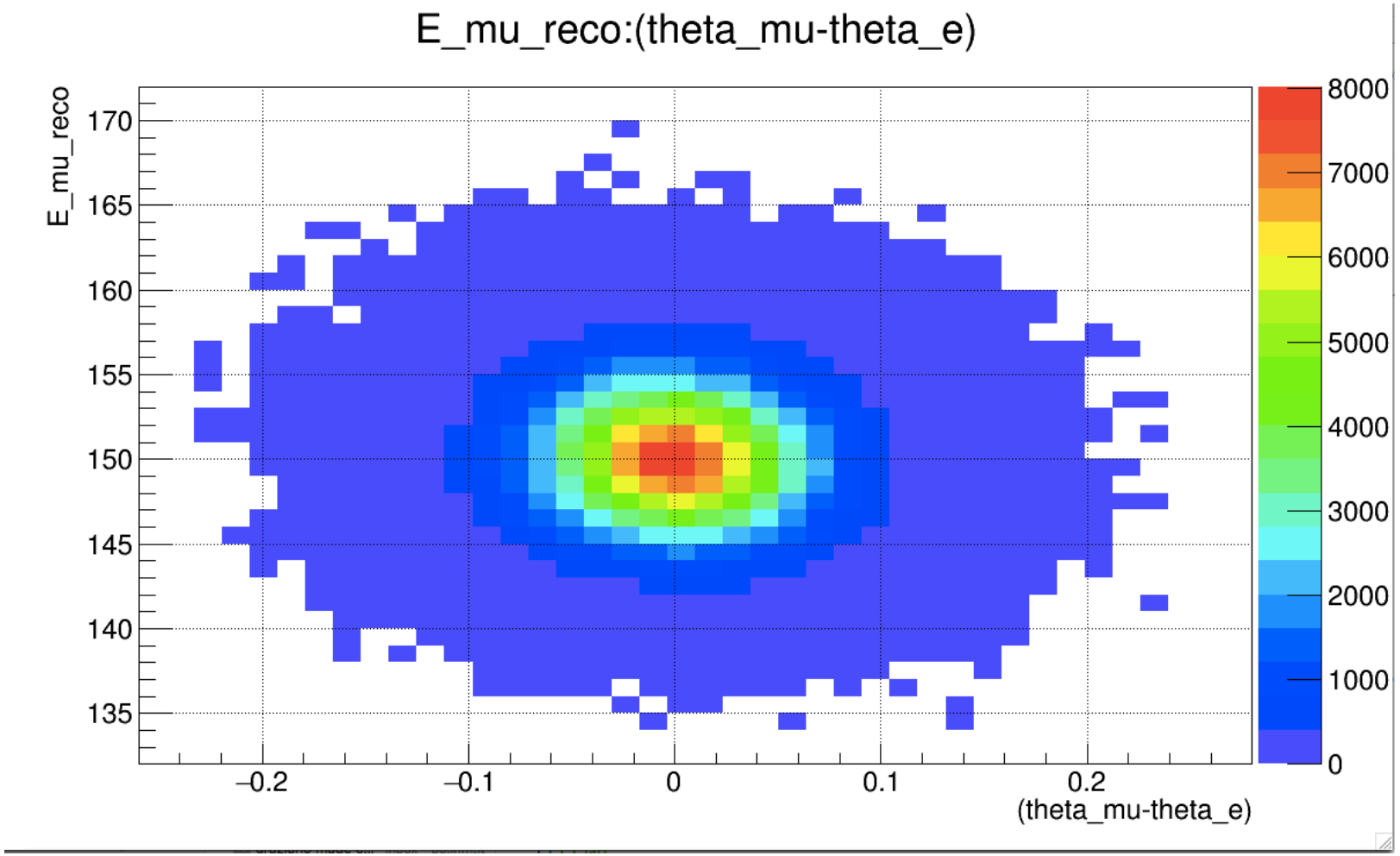


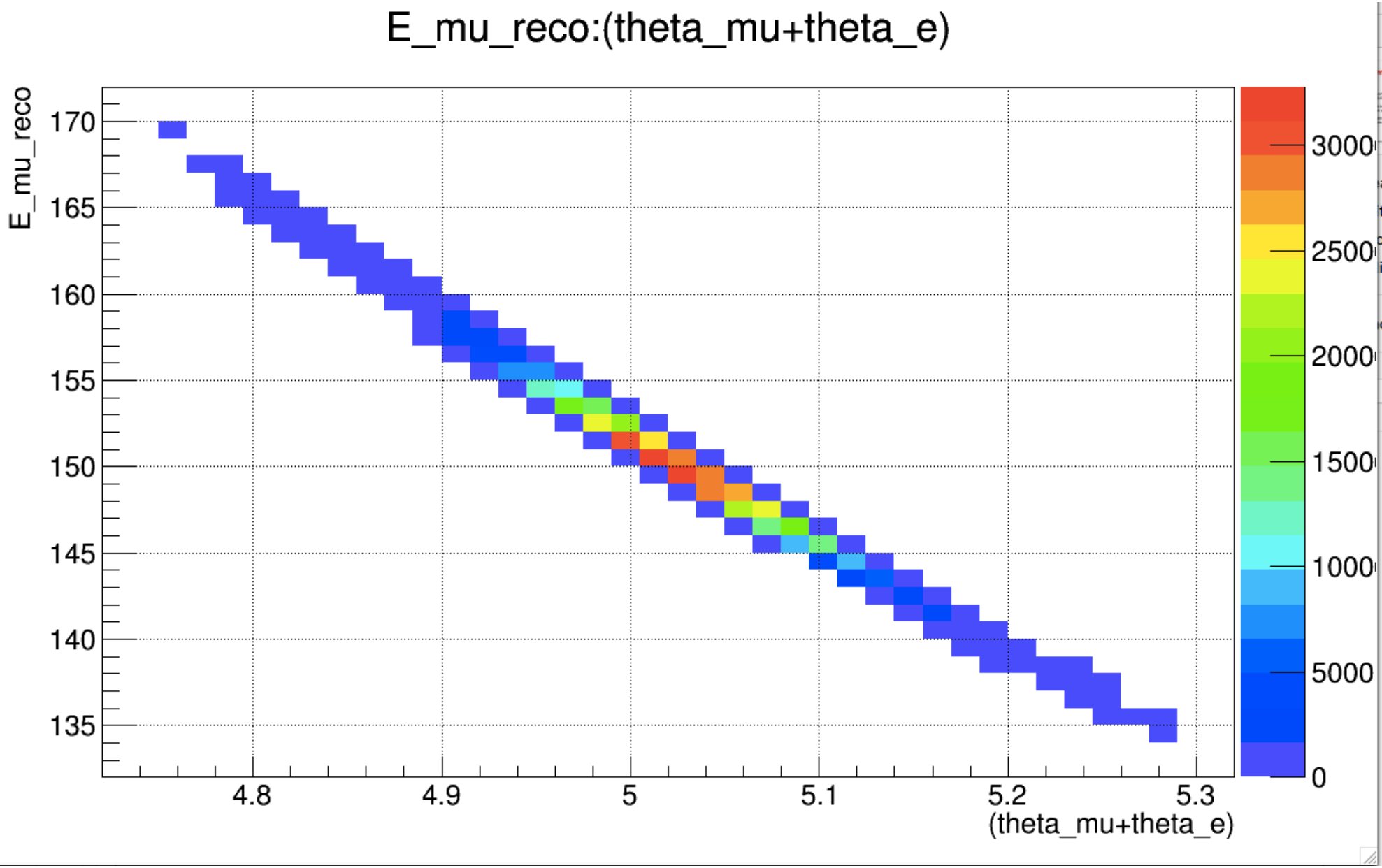

```
root [4] hreco->Fit("gaus")
 FCN=420.25 FROM MIGRAD    STATUS=CONVERGED      60 CALLS          61 TOTAL
                     EDM=2.97167e-08    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant    8.40670e+03   1.45468e+01   1.19203e-01   1.81767e-05
  2  Mean        1.50021e+02   3.36485e-03   7.15355e-05  -2.89061e-03
  3  Sigma       2.37083e+00   2.36286e-03   2.71368e-06   7.99378e-01
```

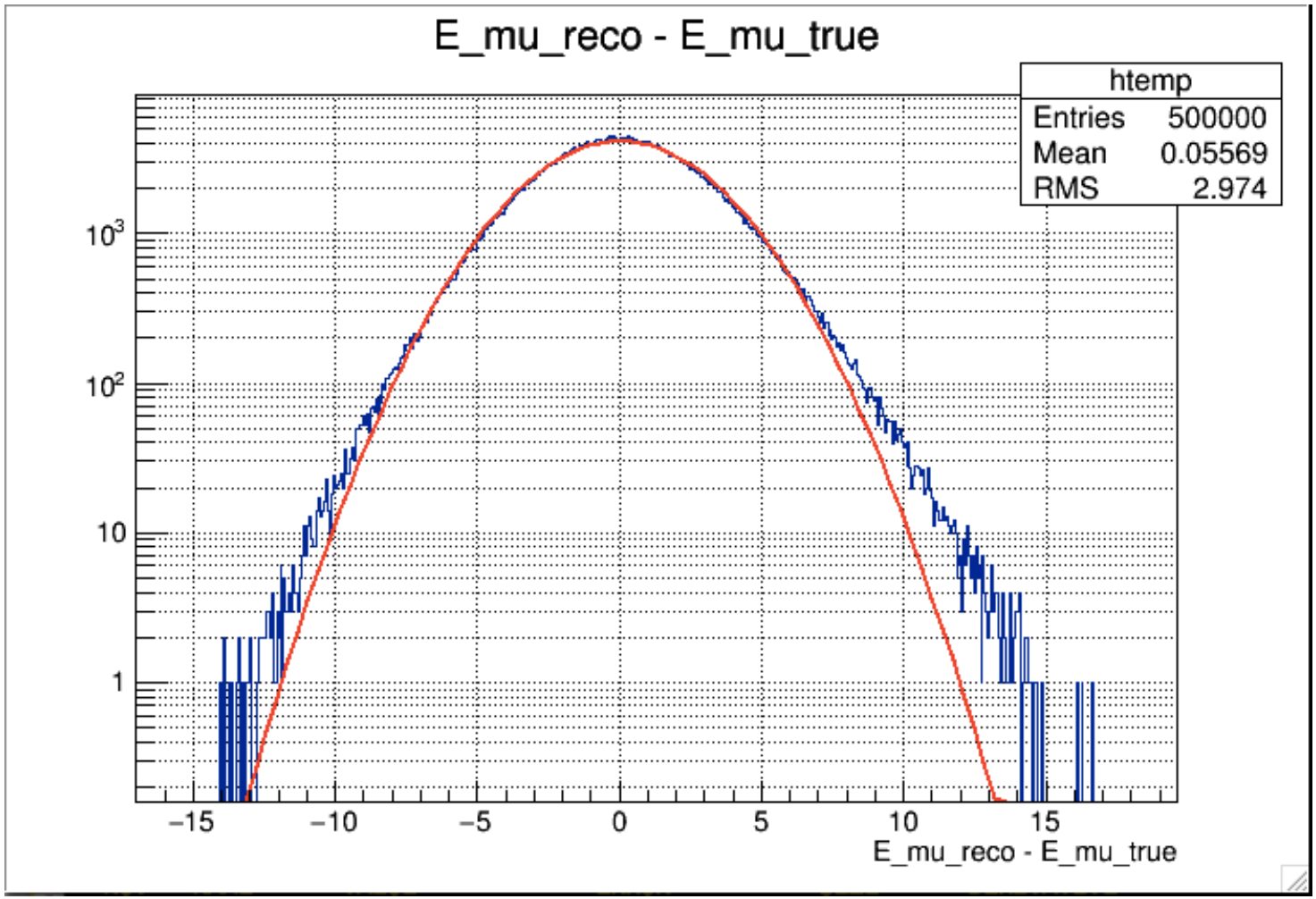
Reconstructed Energy
Detector resolution and MSC




```
FCN=1558.63 FROM MIGRAD    STATUS=CONVERGED      67 CALLS          68 TOTAL
                     EDM=1.72479e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant    6.01998e+03   1.08044e+01   1.64429e-01  -6.27233e-05
  2  Mean        1.50039e+02   4.69433e-03   9.02286e-05  -1.75306e-02
  3  Sigma       3.30304e+00   3.64094e-03   5.55220e-06  -1.74515e+00
```

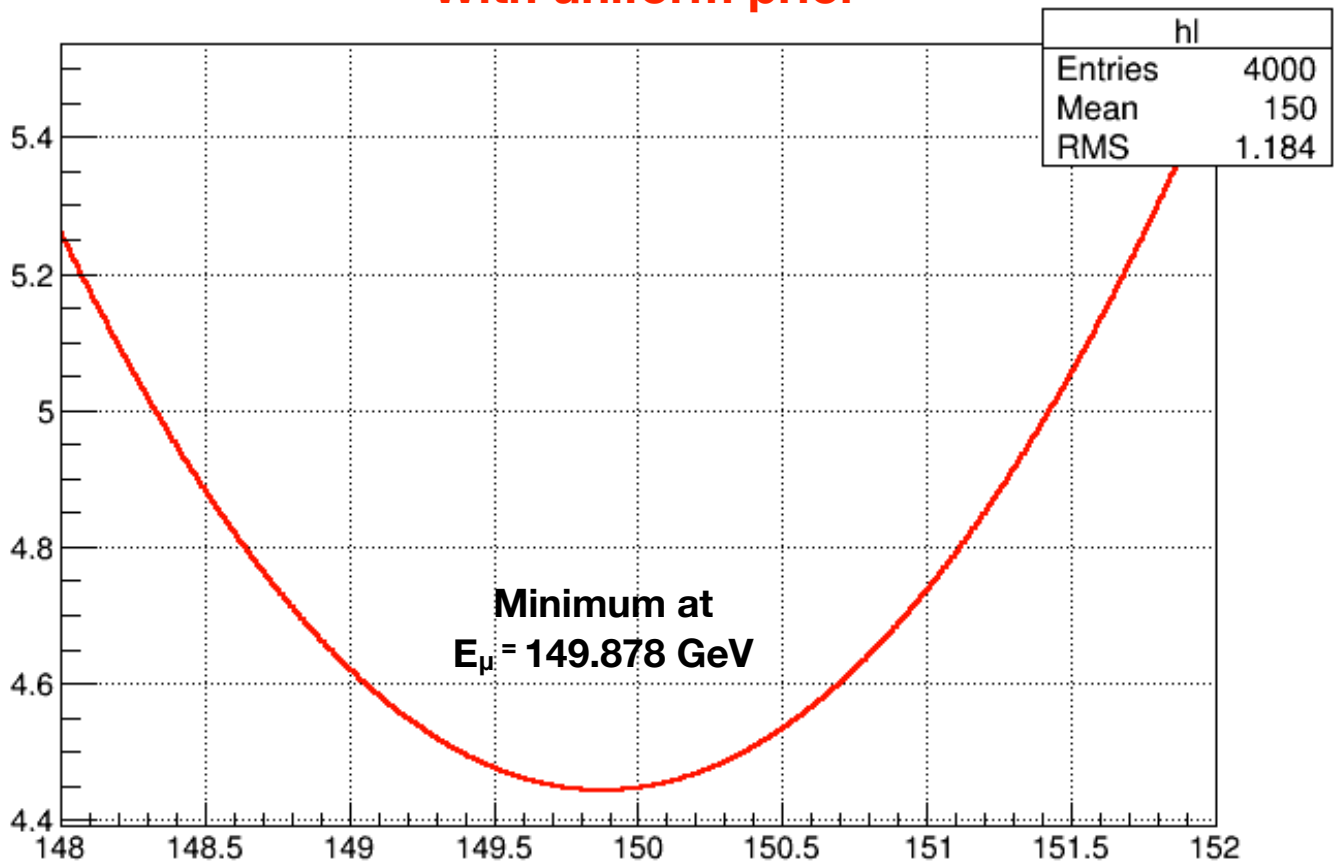

E_mu_reco:(theta_mu-theta_e)
E_mu_reco
170
165
160
155
150
145
140
135
−0.2
−0.1
0
0.1
0.2
(theta_mu-theta_e)
8000
7000
6000
5000
4000
3000
2000
1000
0


E_mu_reco:(theta_mu+theta_e)
E_mu_reco
170
165
160
155
150
145
140
135
4.8
4.9
5
5.1
5.2
5.3
(theta_mu+theta_e)
3000
2500
2000
1500
1000
5000
0


E_mu_reco - E_mu_true
htemp
Entries 500000
Mean 0.05569
RMS 2.974
10^3
10^2
10
1
−15
−10
−5
0
5
10
15
E_mu_reco - E_mu_true

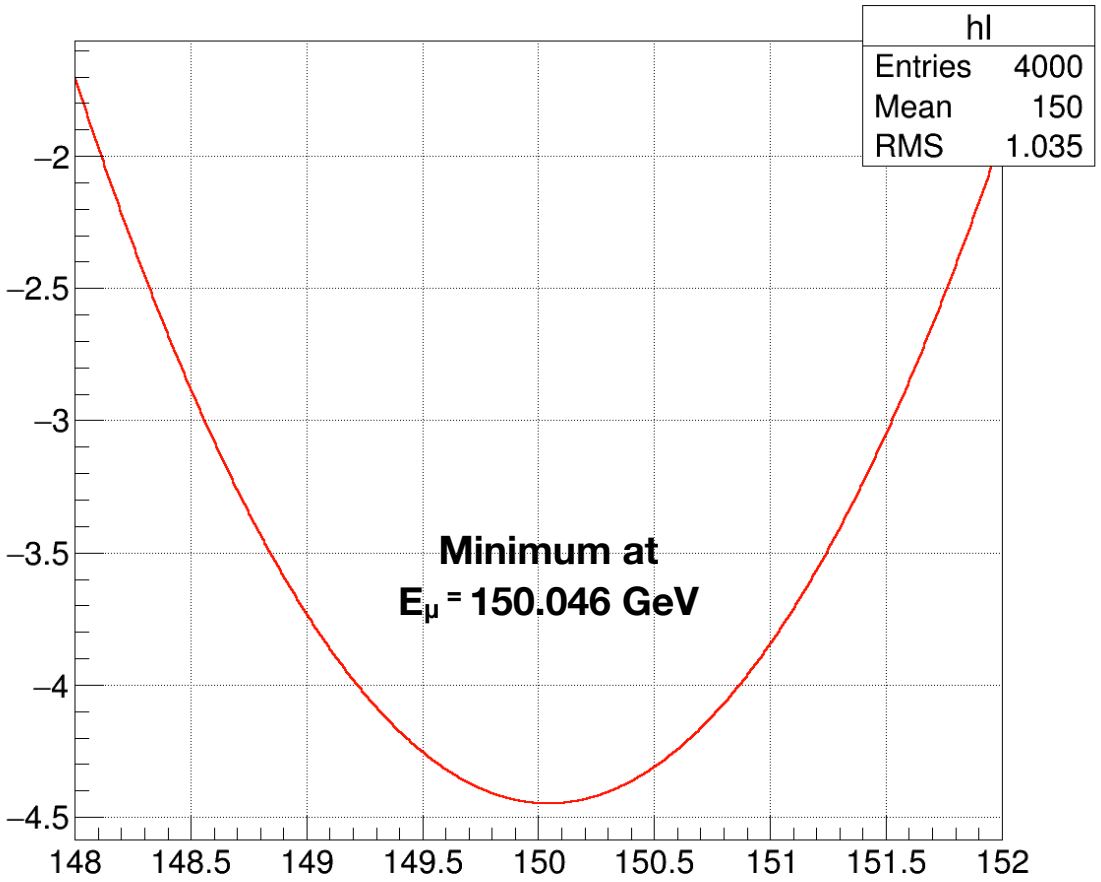
Bayesian

With uniform prior



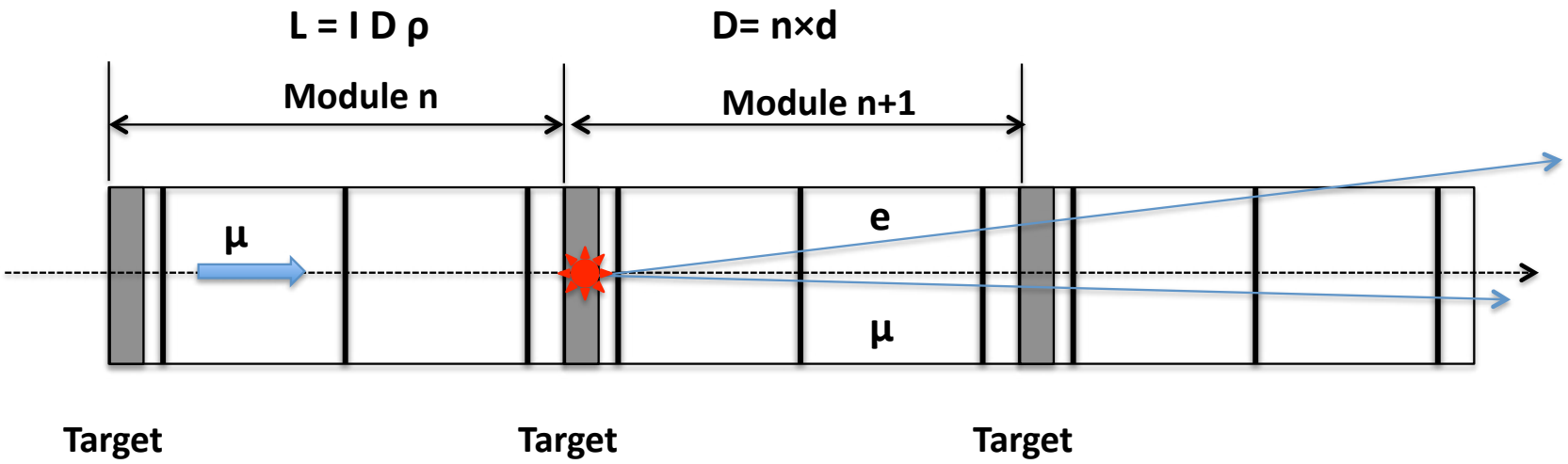
Bayesian (2)

With gaussian prior at the nominal value



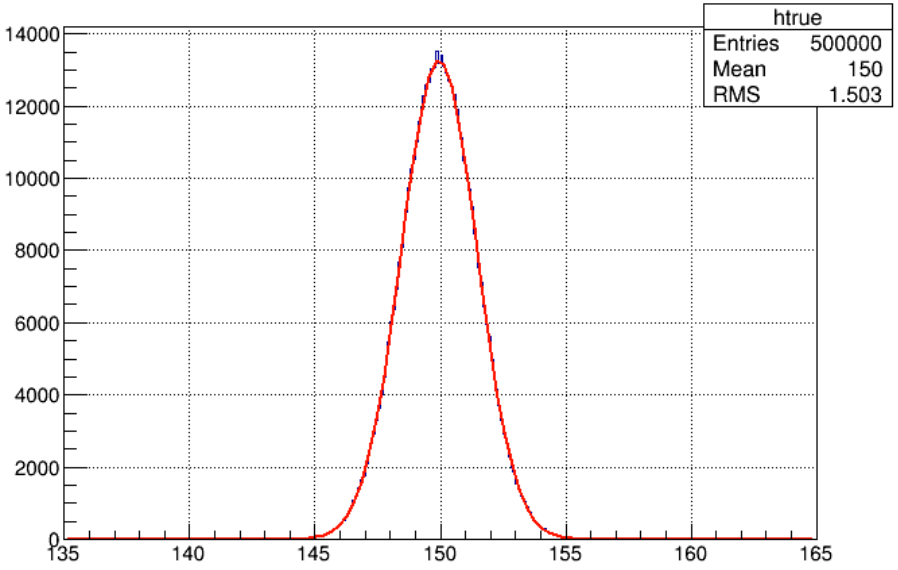
Measuring E_μ at various depth

Assume the Mean energy at the given depth
E_μ - 50 MeV



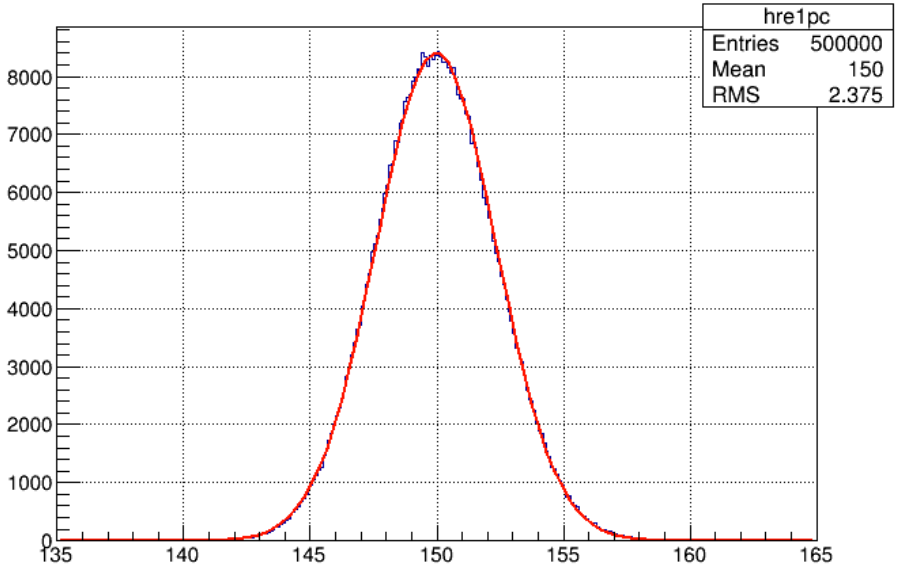
E_μ = 149.950 GeV, 1%

Beam Energy




```
root [9] htrue->Fit("gaus")
 FCN=114.149 FROM MIGRAD    STATUS=CONVERGED      47 CALLS          48 TOTAL
                     EDM=7.25783e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant    1.32709e+04   2.29738e+01   9.83477e-02  -5.73638e-06
  2  Mean        1.49951e+02   2.12569e-03   7.15023e-05  -2.27052e-02
  3  Sigma       1.50274e+00   1.50033e-03   1.42437e-06  -3.01507e-01
```

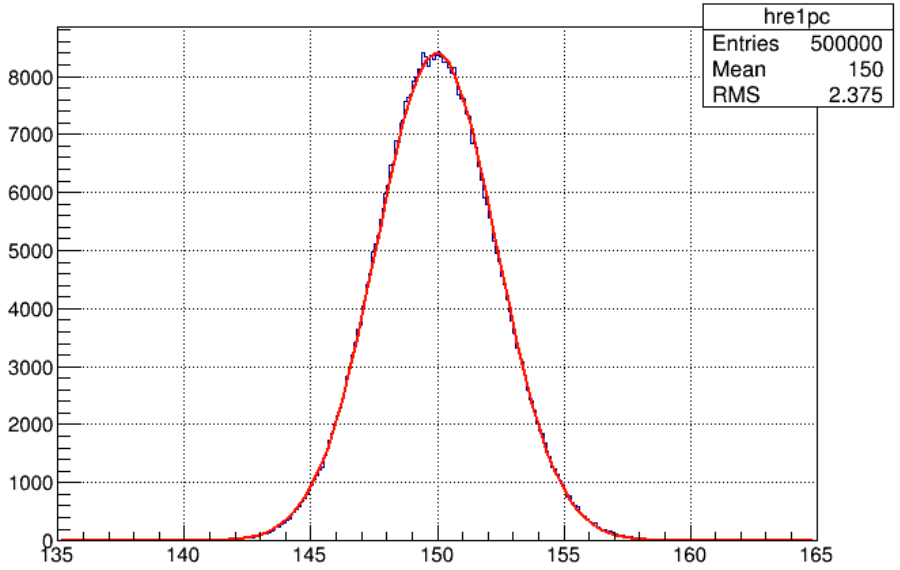
Reconstructed Energy
Gaussian resolution × 2, no MSC




```
root [3] hre1pc->Fit("gaus")
 FCN=411.943 FROM MIGRAD    STATUS=CONVERGED      60 CALLS          61 TOTAL
                     EDM=2.14264e-10    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant    8.40861e+03   1.45635e+01   1.18037e-01  -9.35036e-07
  2  Mean        1.49970e+02   3.36328e-03   7.15113e-05   1.46729e-03
  3  Sigma       2.37034e+00   2.36921e-03   2.69248e-06   2.56528e-02
```

E_μ = 149.950 GeV, 1% (2)

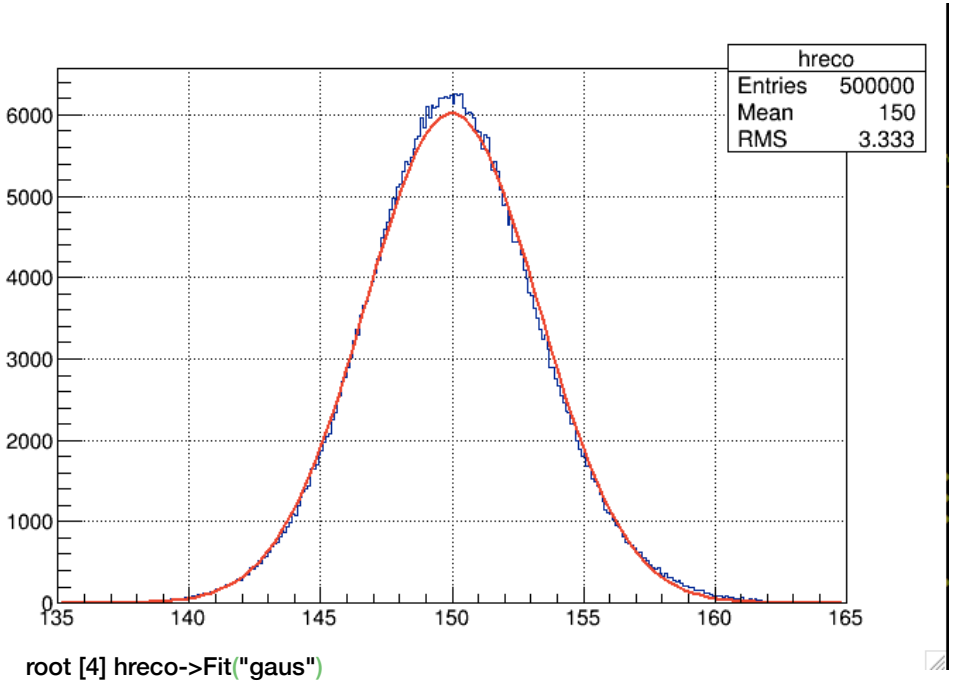
Reconstructed Energy
Gaussian resolution × 2, no MSC



root [3] hre1pc->Fit("gaus")
FCN=411.943 FROM MIGRAD STATUS=CONVERGED 60 CALLS 61 TOTAL
EDM=2.14264e-10 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER NO.	NAME	VALUE	ERROR	STEP SIZE	FIRST DERIVATIVE
1	Constant	8.40861e+03	1.45635e+01	1.18037e-01	-9.35036e-07
2	Mean	1.49970e+02	3.36328e-03	7.15113e-05	1.46729e-03
3	Sigma	2.37034e+00	2.36921e-03	2.69248e-06	2.56528e-02

Reconstructed Energy
Detector resolution and MSC

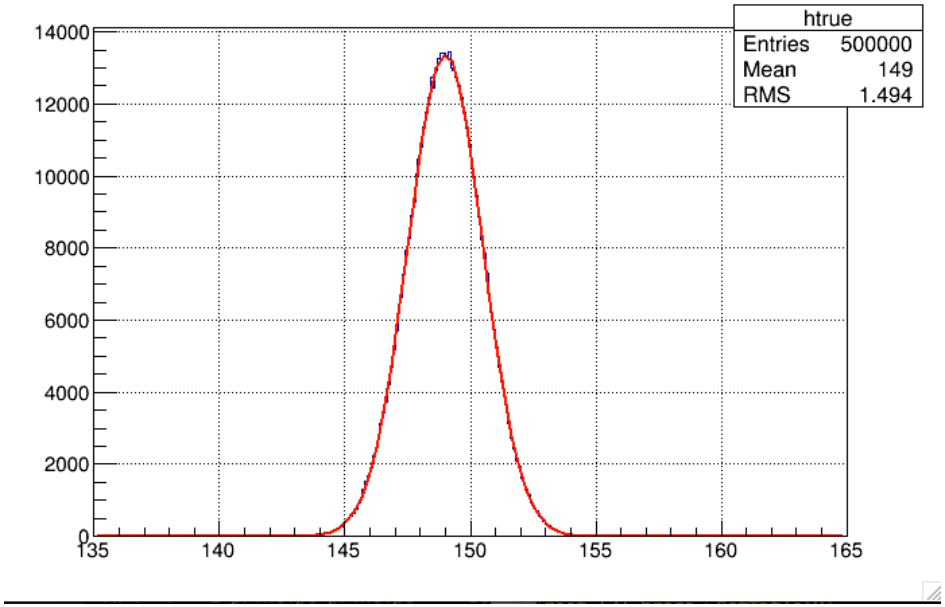


root [4] hreco->Fit("gaus")
FCN=1549.16 FROM MIGRAD STATUS=CONVERGED 70 CALLS 71 TOTAL
EDM=5.46475e-09 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER NO.	NAME	VALUE	ERROR	STEP SIZE	FIRST DERIVATIVE
1	Constant	6.02162e+03	1.08039e+01	1.63972e-01	-6.63720e-07
2	Mean	1.49989e+02	4.69318e-03	8.99305e-05	-2.23058e-02
3	Sigma	3.30221e+00	3.63708e-03	5.53245e-06	1.97076e-02

E_μ = 149.000 GeV, 1%

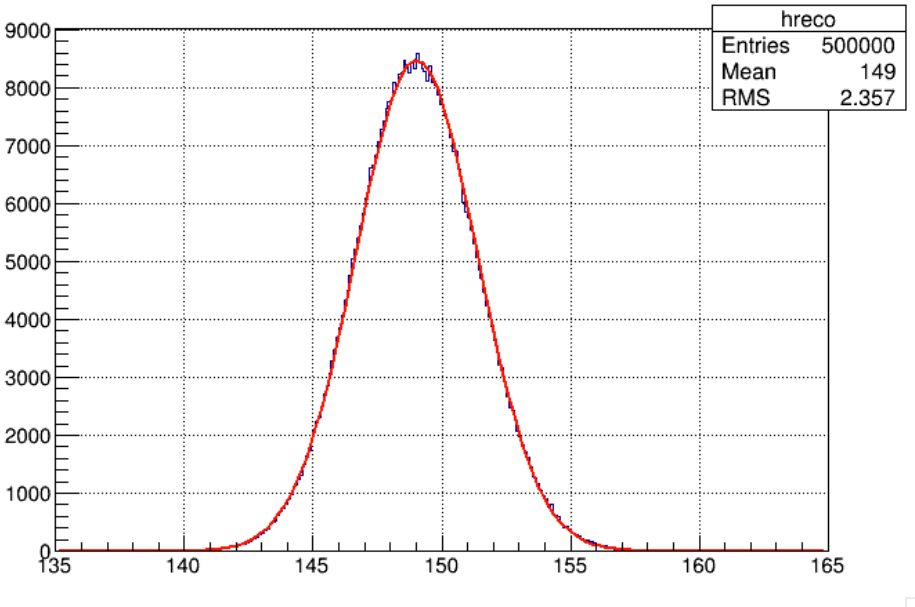
Beam Energy



root [5] htrue->Fit("gaus")
FCN=98.274 FROM MIGRAD STATUS=CONVERGED 59 CALLS 60 TOTAL
EDM=6.08977e-09 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER			STEP	FIRST
NO.	NAME	VALUE	ERROR	SIZE	DERIVATIVE
1	Constant	1.33538e+04	2.31245e+01	9.18855e-02	-3.84170e-07
2	Mean	1.49001e+02	2.11255e-03	7.10493e-05	5.20919e-02
3	Sigma	1.49347e+00	1.49256e-03	1.32350e-06	-2.80793e-02

Reconstructed Energy
Gaussian resolution × 2, no MSC

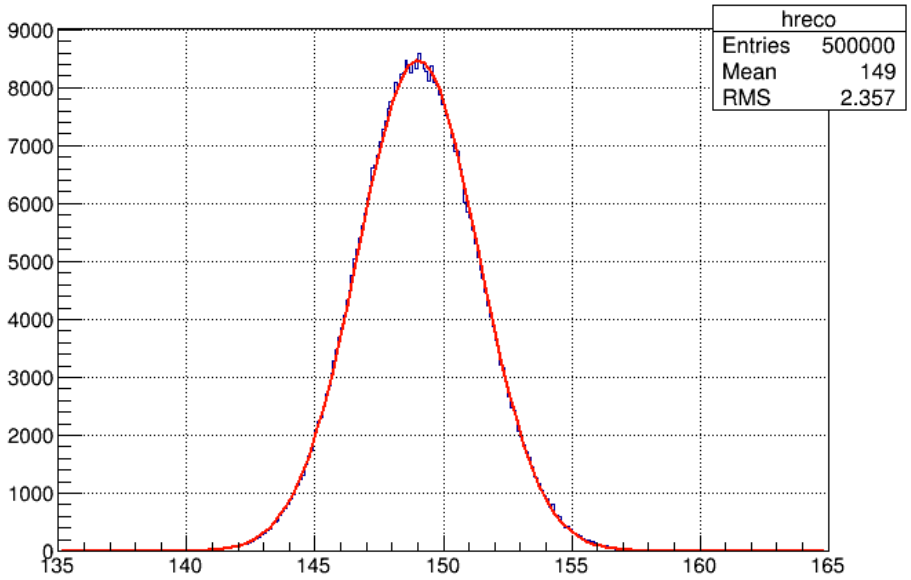


root [6] hreco->Fit("gaus")
FCN=404.88 FROM MIGRAD STATUS=CONVERGED 48 CALLS 49 TOTAL
EDM=2.95229e-08 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER			STEP	FIRST
NO.	NAME	VALUE	ERROR	SIZE	DERIVATIVE
1	Constant	8.47435e+03	1.46727e+01	1.17954e-01	1.89082e-05
2	Mean	1.49020e+02	3.33768e-03	7.10584e-05	9.43017e-03
3	Sigma	2.35199e+00	2.34860e-03	2.66788e-06	7.19651e-01

E_μ = 149.000 GeV, 1% (2)

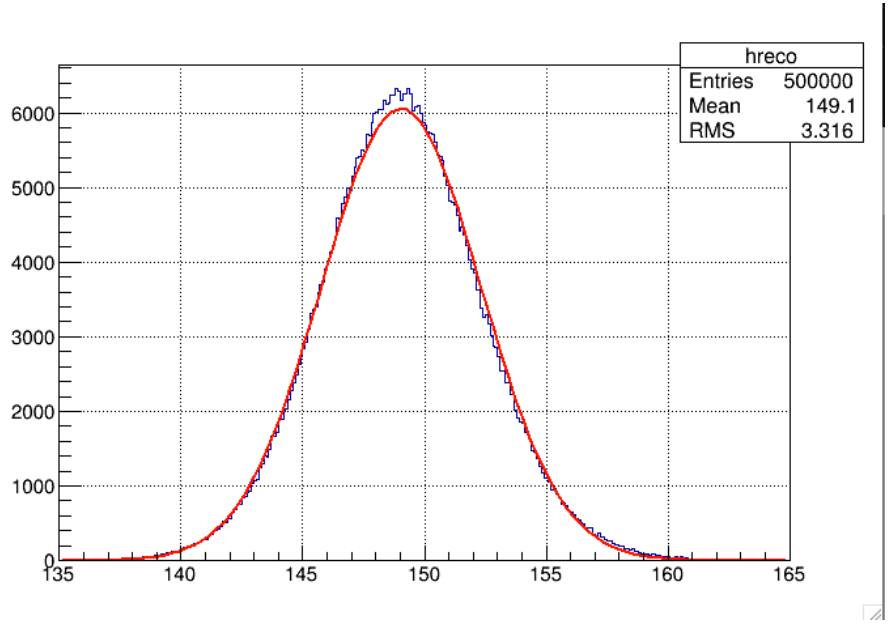
Reconstructed Energy
Gaussian resolution × 2, no MSC



root [6] hreco->Fit("gaus")
FCN=404.88 FROM MIGRAD STATUS=CONVERGED 48 CALLS 49 TOTAL
EDM=2.95229e-08 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER NO.	NAME	VALUE	ERROR	STEP SIZE	FIRST DERIVATIVE
1	Constant	8.47435e+03	1.46727e+01	1.17954e-01	1.89082e-05
2	Mean	1.49020e+02	3.33768e-03	7.10584e-05	9.43017e-03
3	Sigma	2.35199e+00	2.34860e-03	2.66788e-06	7.19651e-01

Reconstructed Energy
Detector resolution and MSC



root [2] hreco->Fit("gaus")
FCN=1577.16 FROM MIGRAD STATUS=CONVERGED 67 CALLS 68 TOTAL
EDM=2.13748e-07 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER NO.	NAME	VALUE	ERROR	STEP SIZE	FIRST DERIVATIVE
1	Constant	6.05340e+03	1.08652e+01	1.66324e-01	-6.78394e-05
2	Mean	**1.49039e+02**	4.66852e-03	9.02636e-05	-2.06646e-02
3	Sigma	3.28473e+00	3.62158e-03	5.58491e-06	-1.99024e+00

(

Reconstructed muon energy

```
root [16] .x fitDoubleGaussian.C(hre)
 entries 500000
 FCN=275.587 FROM MIGRAD    STATUS=CONVERGED     394 CALLS         395 TOTAL
                     EDM=1.19806e-07    STRATEGY= 1  ERROR MATRIX UNCERTAINTY   0.6 per cent
  EXT PARAMETER                                   STEP         FIRST
  NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
   1  norm        4.99742e+04   7.07746e+01   4.16709e-02   5.31997e-06
   2  frac        6.02001e-01   3.26601e-02  -1.01973e-04   3.53645e-02
   3  mu1         1.49758e+02   1.80780e-02  -4.27289e-05   3.31089e-02
   4  mu2         1.50362e+02   3.57675e-02  -4.53428e-05   2.31539e-02
   5  sigma1      2.87491e+00   3.11672e-02  -8.73417e-05  -1.91934e-02
   6  sigma2      3.87940e+00   3.88049e-02  -1.21684e-04  -3.42089e-02
chi2  275.587
NF    307
Probability 0.900884
```