Muon-electron scattering at NLO in QED

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MUonE meeting

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with M. Alacevich, M. Chiesa, G. Montagna, O. Nicrosini, F. Piccinini

Literature

- Relevant literature:

  - ⋆ G. Abbiendi *et al.*,
    Measuring the leading hadronic contribution to the muon g-2 via μe scattering
    Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ex]

  - ⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
    A new approach to evaluate the leading hadronic corrections to the muon g-2
    Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

  - ↦ G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini,
    Matching perturbative and parton shower corrections to Bhabha process at flavour factories
    Nucl. Phys. B **758** (2006) 227 - hep-ph/0607181

  - ↦ M. Cacciari, G. Montagna, O. Nicrosini and F. Piccinini,
    SABSPV: A Monte Carlo integrator for small angle Bhabha scattering
    Comput. Phys. Commun. **90** (1995) 301 - hep-ph/9507245

New perspective: $\mu e \to \mu e$ scattering in fixed target experiment

↦ A 150 GeV high-intensity (∼1.3×10^7 μ's/s) muon beam is available at CERN NA

↦ Muon scattering on a low-Z target ($\mu e \to \mu e$) looks an ideal process

- ⋆ it is a pure t–channel process →

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha}\right|^2$$

- ⋆ Assuming a 150 GeV incident μ beam we have

$s \simeq 0.164\ \mathrm{GeV}^2$ $\quad -0.143 \lesssim t < 0\ \mathrm{GeV}^2$ $\quad 0 < x \lesssim 0.93$ **it spans the peak!**

Benefits:

- ⋆ the highly boosted kinematics allows to access a wide angular range in the CM
- ⋆ the same detector can be exploited for signal and normalization
- ⋆ the same process is used for signal and normalization:
  the region $x \lesssim 0.3$, where $\Delta\alpha_{\rm had}(t) < 10^{-5}$, can be used for normalization

μe scattering kinematics for leading order ($2 \to 2$, elastic process)

p_1, p_2 initial state μ and e $\qquad$ p_3, p_4 final state μ and e

In the lab

$$
\begin{aligned}
p_1 &= (E_\mu^{beam}, 0, 0, p) \\
p_2 &= (m_e, 0, 0, 0) \\
p_3 &= p_1 + p_2 - p_4 \\
p_4 &= (E_e, p_e \sin\theta_e, 0, p_e \cos\theta_e)
\end{aligned}
$$

In the center of mass

$$
\begin{aligned}
p_1 &= (E_{CM}^\mu, 0, 0, p_{CM}) \\
p_2 &= (E_{CM}^e, 0, 0, -p_{CM}) \\
p_3 &= (E_{CM}^\mu, p_{CM}\sin\theta, 0, p_{CM}\cos\theta) \\
p_4 &= (E_{CM}^e, -p_{CM}\sin\theta, 0, -p_{CM}\cos\theta)
\end{aligned}
$$

Invariants:

$$
\begin{aligned}
s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\
&= m_e^2 + m_\mu^2 + 2E_{CM}^\mu E_{CM}^e + 2p_{CM}^2 \\
&= m_e^2 + m_\mu^2 + 2E_\mu^{beam} m_e
\end{aligned}
$$

$$
\begin{aligned}
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\
&= -2p_{CM}^2(1 - \cos\theta) \\
&= 2m_e^2 - 2E_e m_e
\end{aligned}
$$

$$
p_{CM} = \frac{1}{2}\sqrt{\frac{\lambda(s, m_\mu^2, m_e^2)}{s}}
$$

$$
t = m_\mu^2 \frac{x^2}{x-1} \propto E_e
$$

$$
E_e = m_e \frac{1 + r^2\cos^2\theta_e}{1 - r^2\cos^2\theta_e}
$$

$$
r \equiv \frac{\sqrt{\left(E_\mu^{beam}\right)^2 - m_\mu^2}}{E_\mu^{beam} + m_e}
$$

Next-to-leading order calculation

- The μe cross section and distributions must be known as precisely as possible
  $\mapsto$ radiative corrections (RCs) are mandatory

⋆ First step are QED $\mathcal{O}(\alpha)$ (i.e. QED NLO, next-to-leading order) RCs

The NLO cross section is split into two contributions,

$$\sigma_{NLO} = \sigma_{2\to2} + \sigma_{2\to3} = \sigma_{\mu e\to\mu e} + \sigma_{\mu e\to\mu e\gamma}$$

$\mapsto$ IR singularities are regularized with a vanishingly small photon mass $\boldsymbol{\lambda}$

$\mapsto$ $[2\to2]/[2\to3]$ phase space splitting at an arbitrarily small γ-energy cutoff $\boldsymbol{\omega_s}$

- $\mu e \to \mu e$

$$\sigma_{2\to2} = \sigma_{LO} + \sigma_{NLO}^{virtual} = \frac{1}{F}\int d\Phi_2(|\mathcal{A}_{LO}|^2 + 2\Re[\mathcal{A}_{LO}^* \times \mathcal{A}_{NLO}^{virtual}(\boldsymbol{\lambda})])$$

- $\mu e \to \mu e\gamma$

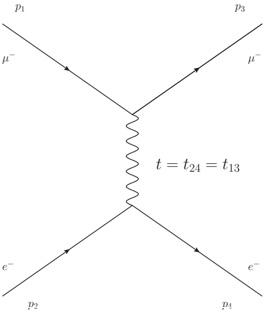
$$\sigma_{2\to3} = \frac{1}{F}\int_{\omega>\boldsymbol{\lambda}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2 = \frac{1}{F}\left(\int_{\boldsymbol{\lambda}<\omega<\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2 + \int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2\right)$$

$$= \Delta_s(\boldsymbol{\lambda},\boldsymbol{\omega_s})\int d\sigma_{LO} + \frac{1}{F}\int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}_{NLO}^{1\gamma}|^2$$

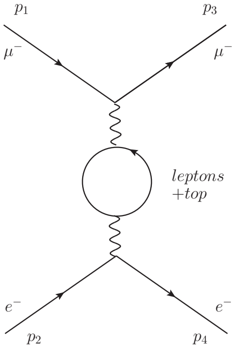
- the integration over the 2/3-particles phase space is done with MC techniques and fully-exclusive events are generated

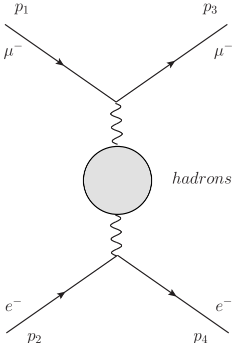
LO and NLO vacuum polarization diagrams

- $\mathcal{A}_{LO}$

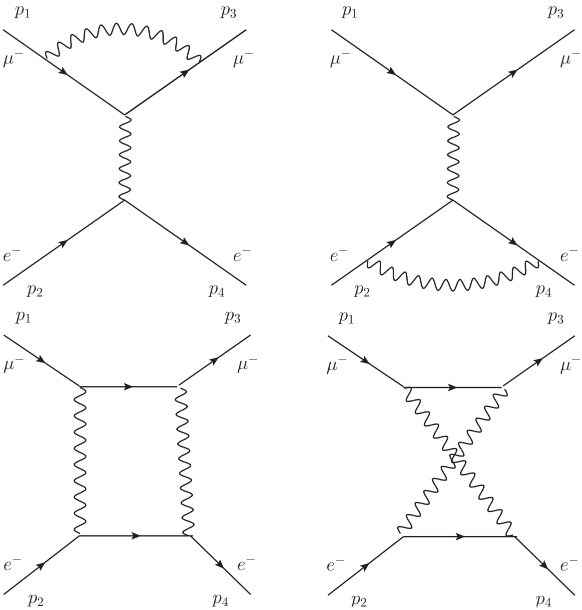


- $\mathcal{A}_{NLO}^{virtual}$



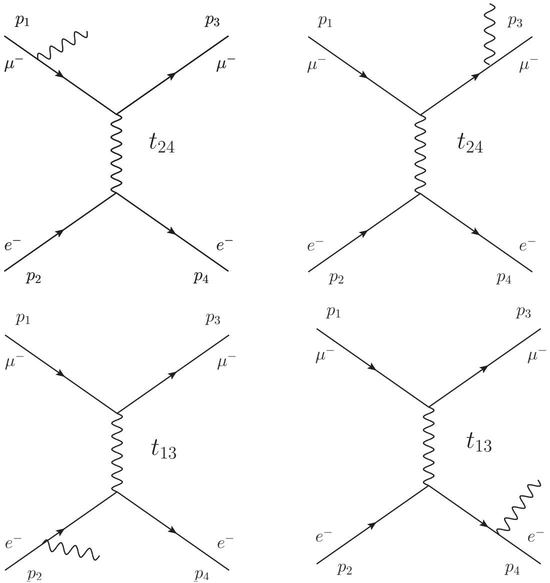


NLO virtual diagrams $\mathcal{A}_{NLO}^{virtual}$ (dependent on λ)



+ counterterms

NLO real diagrams $\mathcal{A}^{1\gamma}_{NLO}$



Method and cross-checks

- Calculation performed in the on-shell renormalization scheme
- Full mass dependency kept everywhere, fermions' helicities kept explicit
- Diagrams manipulated with the help of **FORM**, independently by at least two of us [perfect agreement]

  J. Vermaseren, **https://www.nikhef.nl/~form**
- 1-loop tensor coefficients and scalar 2-3-4 points functions evaluated with **LoopTools** and **Collier** libraries [perfect agreement]

  T. Hahn, **http://www.feynarts.de/looptools**

  A. Denner, S. Dittmaier, L. Hofer, **https://collier.hepforge.org**
- UV finiteness and λ independence verified with high numerical accuracy
- 3 body phase-space cross-checked with 3 independent implementations [perfect agreement]
- Comparison with past/present independent results *[in progress, all good so far]*

  ~~P. Van Nieuwenhuizen, Nucl. Phys. B **28** (1971) 429~~

  T. V. Kukhto, N. M. Shumeiko and S. I. Timoshin, J. Phys. G **13** (1987) 725

  D. Y. Bardin and L. Kalinovskaya, DESY-97-230, **hep-ph/9712310**

  N. Kaiser, J. Phys. G **37** (2010) 115005

  Fael, Passera *et al.*

Further cross-checks and (simple) simulation setup

- Good agreement with NLO Bhabha (only t-channel diagrams) if $m_\mu \to m_e$ as implemented in **BabaYaga@NLO** (within known approximations)
- Everything calculated in the center-of-mass frame, and boosted in the lab if needed
- Implemented in a MC event generator (*temporary* extension of **BabaYaga@NLO**)
- $\boldsymbol{\omega_s}$ independence

$$\sigma_{LO} = 245.039062\ \mu\text{b}$$

$$\sigma_{NLO}(\omega_s = 5\cdot 10^{-5}\times\sqrt{s}\equiv\omega_1) = 244.3957 \pm .0005\ \mu\text{b}$$

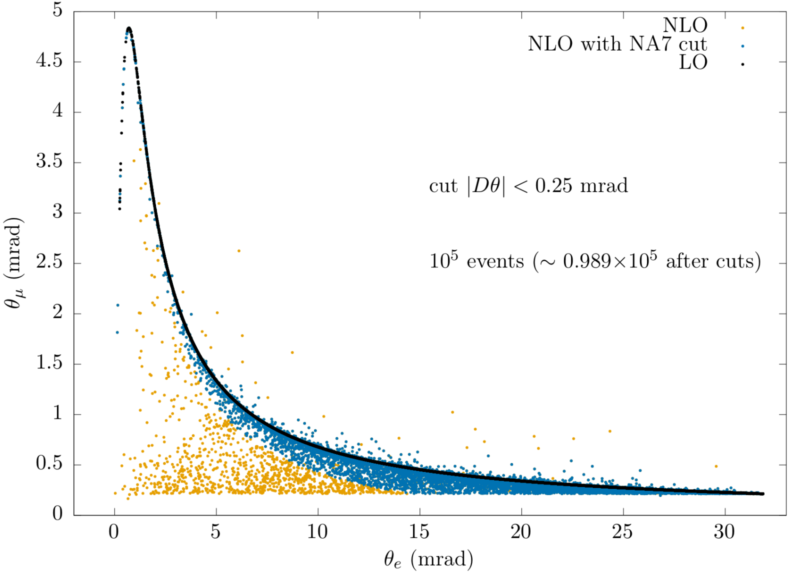
$$\sigma_{NLO}(\omega_s = 1\cdot 10^{-6}\times\sqrt{s}=\omega_1/50) = 244.3948 \pm .0007\ \mu\text{b}$$

⋆ Simulation setup for $\mu^- e^- \to \mu^- e^- \ (\gamma)$

- $E_\mu^{beam} = 150$ GeV $\to$ $\sqrt{s} \simeq 0.4055$ GeV
- $t_{min} = -\lambda(s, m_\mu^2, m_e^2)/s \simeq -0.1429$ GeV^2
- $t_{max} \simeq -1.021\cdot 10^{-3}$ GeV^2 (i.e. $E_e > 1$ GeV in the lab)

$\mapsto$ **Cuts:** $t_{min} \le t_{24} \le t_{max}$ $t_{min} \le t_{13} \le t_{max}$

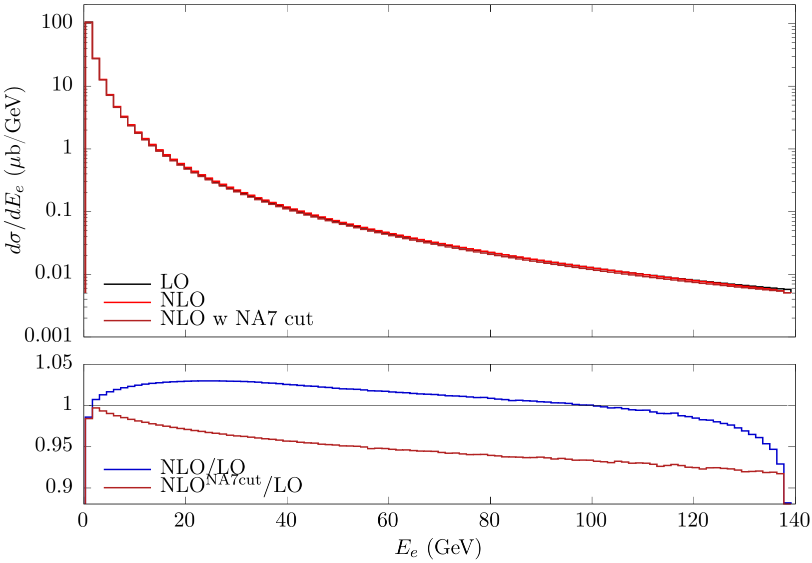
μ-e angle correlation in the lab



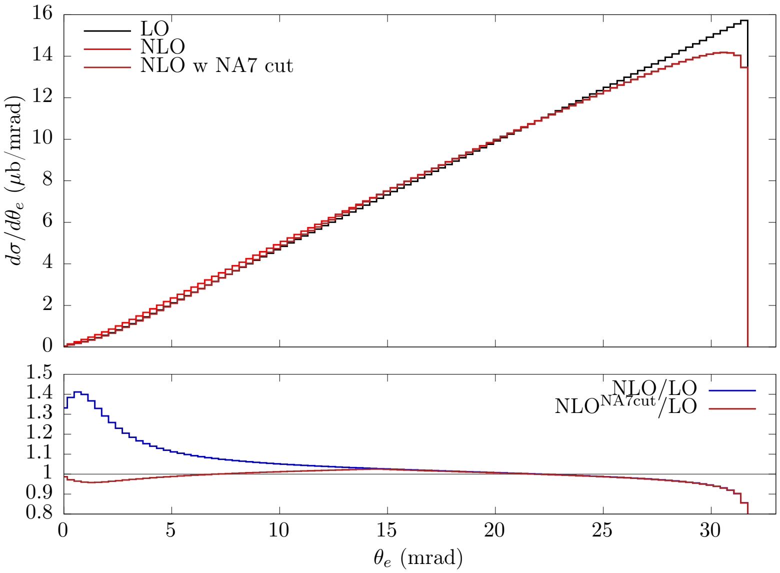
- NA7 cut: $D\theta \simeq$ distance in the θ_μ-θ_e plane < 0.25 mrad

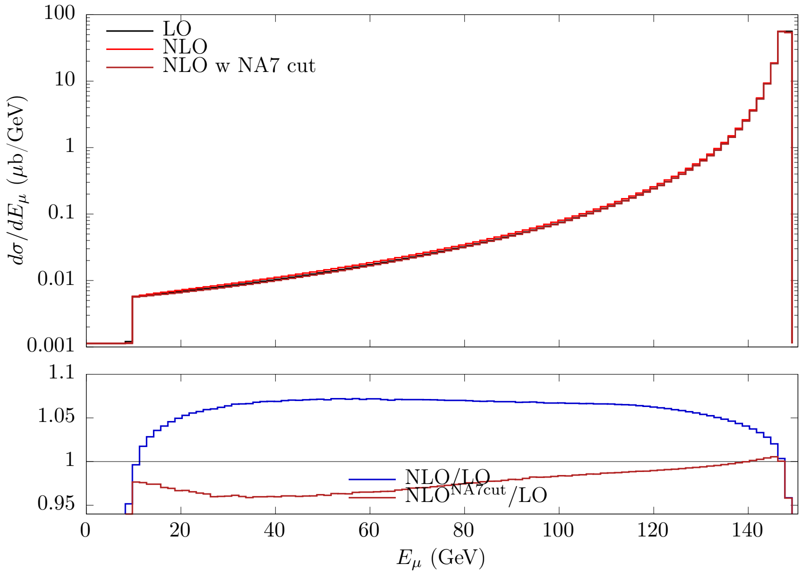
NA7 collaboration, NPB 277, 168 (1986)

E_e distribution and corrections in the lab

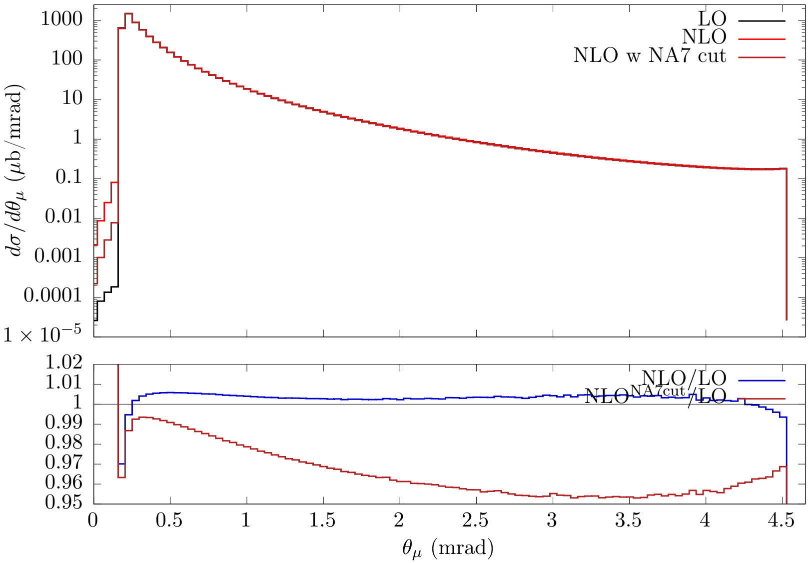


θ_e distribution and corrections in the lab




LO
NLO
NLO w NA7 cut
100
10
1
0.1
0.01
0.001
$d\sigma/dE_\mu$ (μb/GeV)
1.1
1.05
1
0.95
NLO/LO
NLO$^{\text{NA7cut}}$/LO
0
20
40
60
80
100
120
140
E_μ (GeV)

θ_μ distribution and corrections in the lab



Conclusions and Outlook

- ⋆ (At least) one working QED NLO calculation is available in an event generator for our μe proposal
- ⋆ The size of QED NLO RCs on the considered observables lies in the 1-5% range *(except in the low θ_e and θ_μ distributions)*, **within the loose cuts applied here**
- ⋆ The size of QED NLO RCs **strongly depends on the applied cuts**
- ⋆ The "Bhabha experience" at LEP & flavour factories compels to include also higher-orders (h.o., beyond NLO) RCs to reach high theoretical accuracy
- ⋆ Exact NNLO corrections are needed to reduce the theoretical uncertainty at the required level
- ⋆ A QED Parton Shower approach (matched to NLO) could be used to resum h.o. (multiple-photon emission effects) preserving fully exclusive generation
  - $\mapsto$ needs to be re-thought for the inclusion of (muon) mass effects
  - $\mapsto$ needs to be extended to be matched to exact NNLO corrections
- ⋆ A lot of work is in progress, i.e. study of the impact of the full NLO electro-weak RCs