GRAVITATIONAL WAVE EXPERIMENTS
1. DETECTORS

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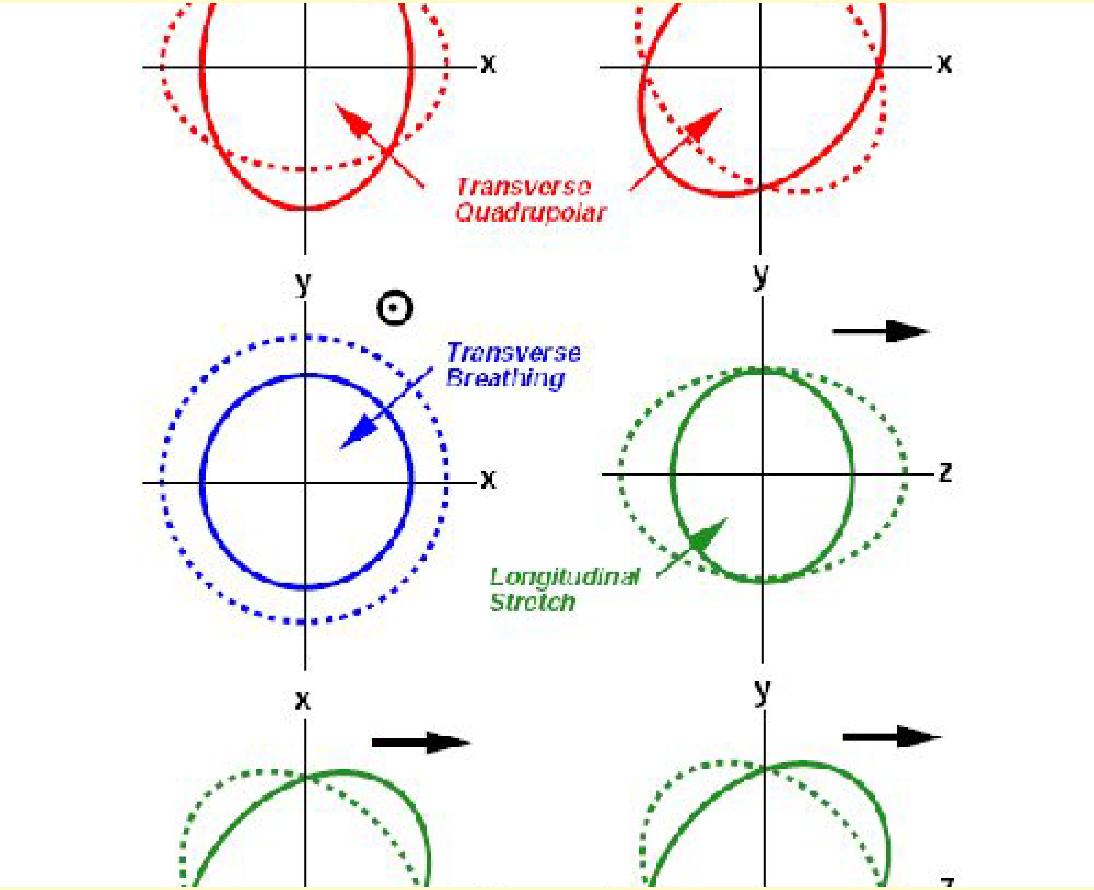
XIX FRASCATI SPRING SCHOOL "BRUNO TOUSCHEK" in
Nuclear, Subnuclear and Astroparticle Physics

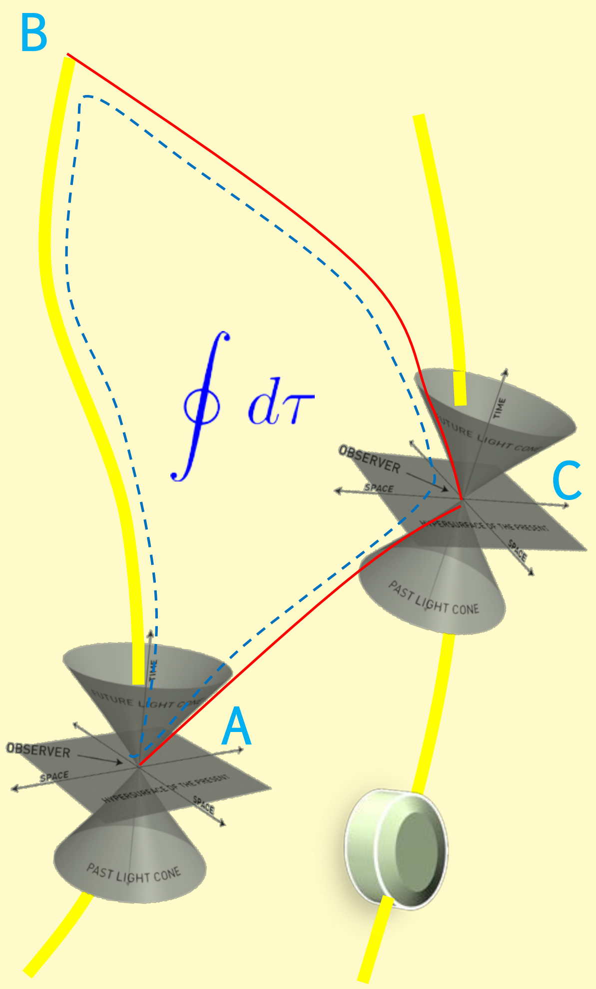
GW DETECTORS

Coupling between detector and signal



The basic observable for interferometric detection

- The round trip time of a light signal measured with the clock of an observer
- This does not depend on the chosen reference system
- However, the way in which the effect is described depends from it:
  - in the previous video, the round trip time changes because the round trip length does;
  - there can be other pictures, more useful for certain purposes



Round trip time evaluation

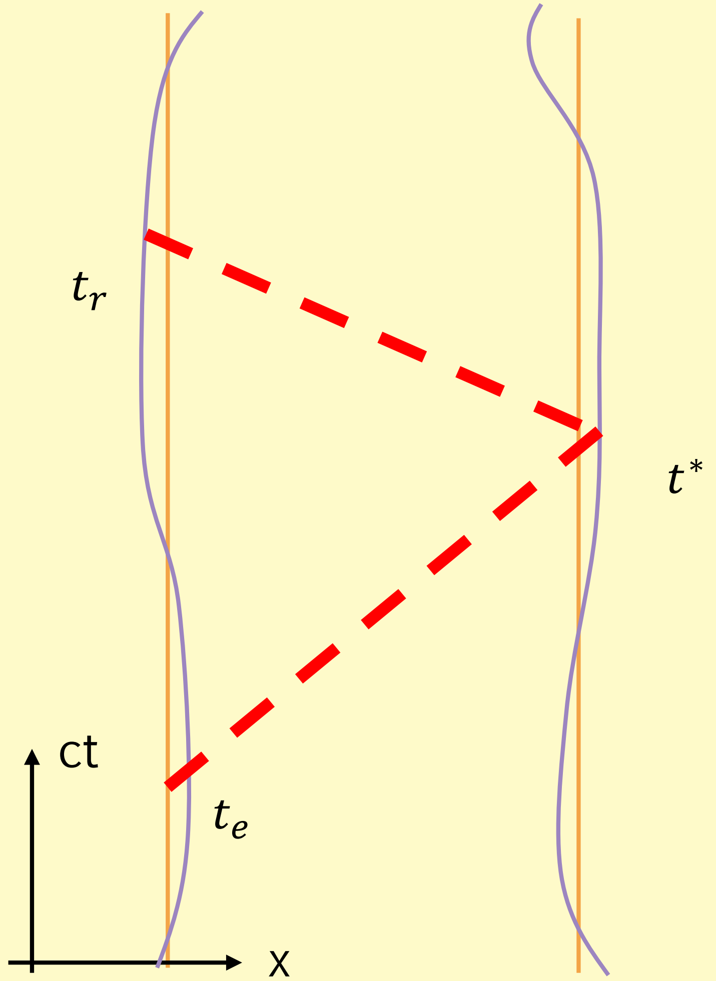
- We will assume a free fall motion of the observer and of the reflecting mirror, so for both we have

$$\frac{du^\mu}{d\tau} = -\Gamma^\mu_{\nu\rho} u^\nu u^\rho$$

- But remember that $\Gamma^\mu_{\nu\rho} = O(h)$ if $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, so

$$\frac{d}{d\tau}\delta u^\mu = -\Gamma^\mu_{00} = -\frac{1}{2}\eta^{\mu\lambda}\left(2h_{\lambda 0,0} - h_{00,\lambda}\right)$$

- Generally speaking, observer and mirror coordinates changes owing to the effect of gravitational waves
- But note that in TT gauge $\delta u^\mu = const$



$u^\mu_{obs} = \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix} + \delta u^\mu_{obs}$ $\quad u^\mu_{mir} = \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix} + \delta u^\mu_{mir}$

TT gauge

- Mirror and observer coordinates are fixed. For the light signal we get

$$d\tau^2 = -c^2 dt^2 + dx^k dx^k + h_{ij}^{TT} dx^i dx^j = 0$$

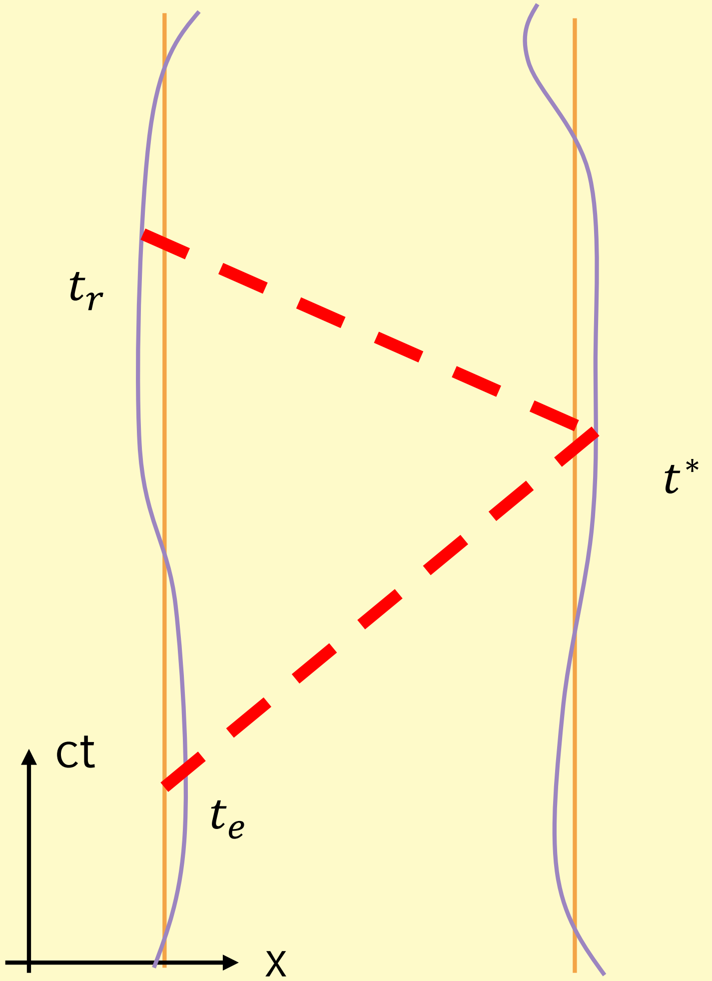
- From which $cdt = \pm dx \left(1 + \frac{1}{2} h_{xx}^{TT}\right) \qquad dx = \pm cdt \left(1 - \frac{1}{2} h_{xx}^{TT}\right)$

- Let's take for simplicity a plane wave propagating along the z direction. In this case $h_{xx}^{TT} = h_+ \cos \omega t$ and

$$\int_0^L dx = c \int_{t_e}^{t^*} \left(1 - \frac{h_+}{2} \cos \omega t\right) dt = c(t^* - t_e) - \frac{h_+}{2} \frac{\sin \omega t^* - \sin \omega t_e}{\omega / c}$$

$$\int_L^0 dx = -c \int_{t^*}^{t_r} \left(1 - \frac{h_+}{2} \cos \omega t\right) dt = -c(t_r - t^*) + \frac{h_+}{2} \frac{\sin \omega t_r - \sin \omega t^*}{\omega / c}$$

$$2L = c(t_r - t_e) - \frac{h_+}{2} \frac{\sin \omega t_r - \sin \omega t_e}{\omega / c}$$



$u^\mu_{obs} = \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix} + \delta u^\mu_{obs} \qquad u^\mu_{mir} = \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix} + \delta u^\mu_{mir}$

Iterative solution O(h)

$$2L = c(t_r - t_e) - \frac{h_+}{2}\frac{\sin\omega t_r - \sin\omega t_e}{\omega/c}$$

$$t_r = t_e + \frac{2L}{c} + O(h)$$

$$t_r - t_e = \frac{2L}{c} + \frac{h_+}{2c}\frac{\sin\omega\left(t_e + \frac{2L}{c}\right) - \sin\omega t_e}{\omega/c}$$

$$t_r - t_e = \frac{2L}{c} + \frac{L}{c}h_{xx}\left(t_e + \frac{L}{c}\right)\frac{\sin\omega L/c}{\omega L/c}$$

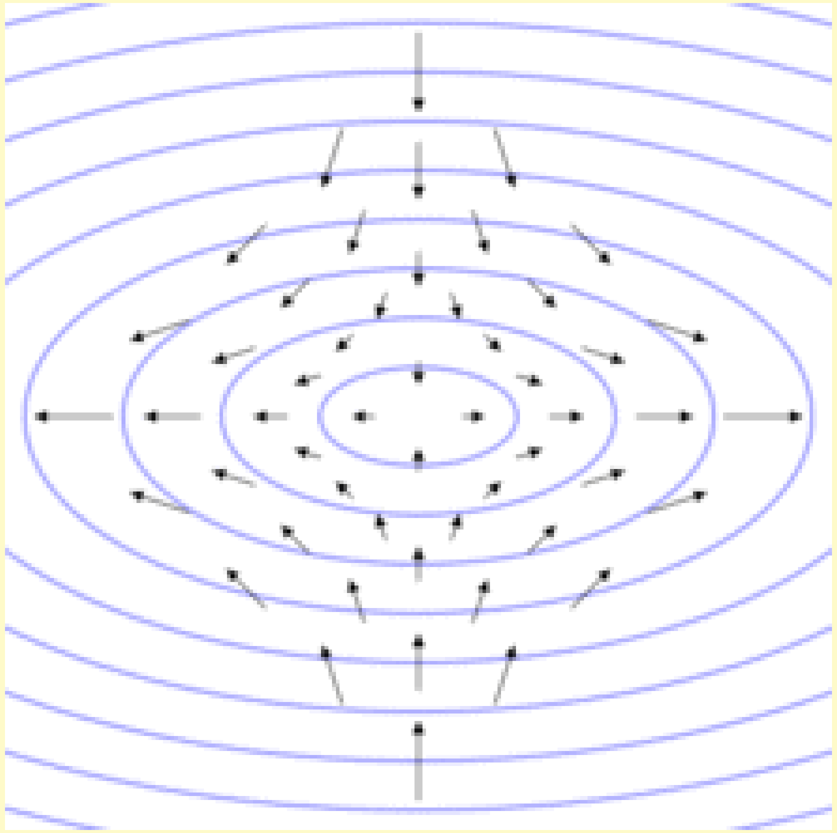
- The GW strain is encoded in the time dependence of the round trip time
- Note that there is a high frequency cut off, which is effective when $\lambda < L$
- By repeating these steps for a round trip along the y direction we find a similar expression, with h_{xx} -> h_{yy}
- Note that only the + polarization is coupled to these round trips

$$\frac{D^2\xi^\mu}{d\tau^2} = R^\mu{}_{\nu\rho\sigma} u^\nu u^\rho \xi^\sigma$$

Description in the proper detector frame (LLF)

- When the wavelength of the GW is large compared with the detector size, we can directly use the geodesic deviation equation to evaluate the round trip time
- In this reference frame the propagation of the light is NOT affected by the gravitational wave
- On the other hand, mirror moves accordingly with

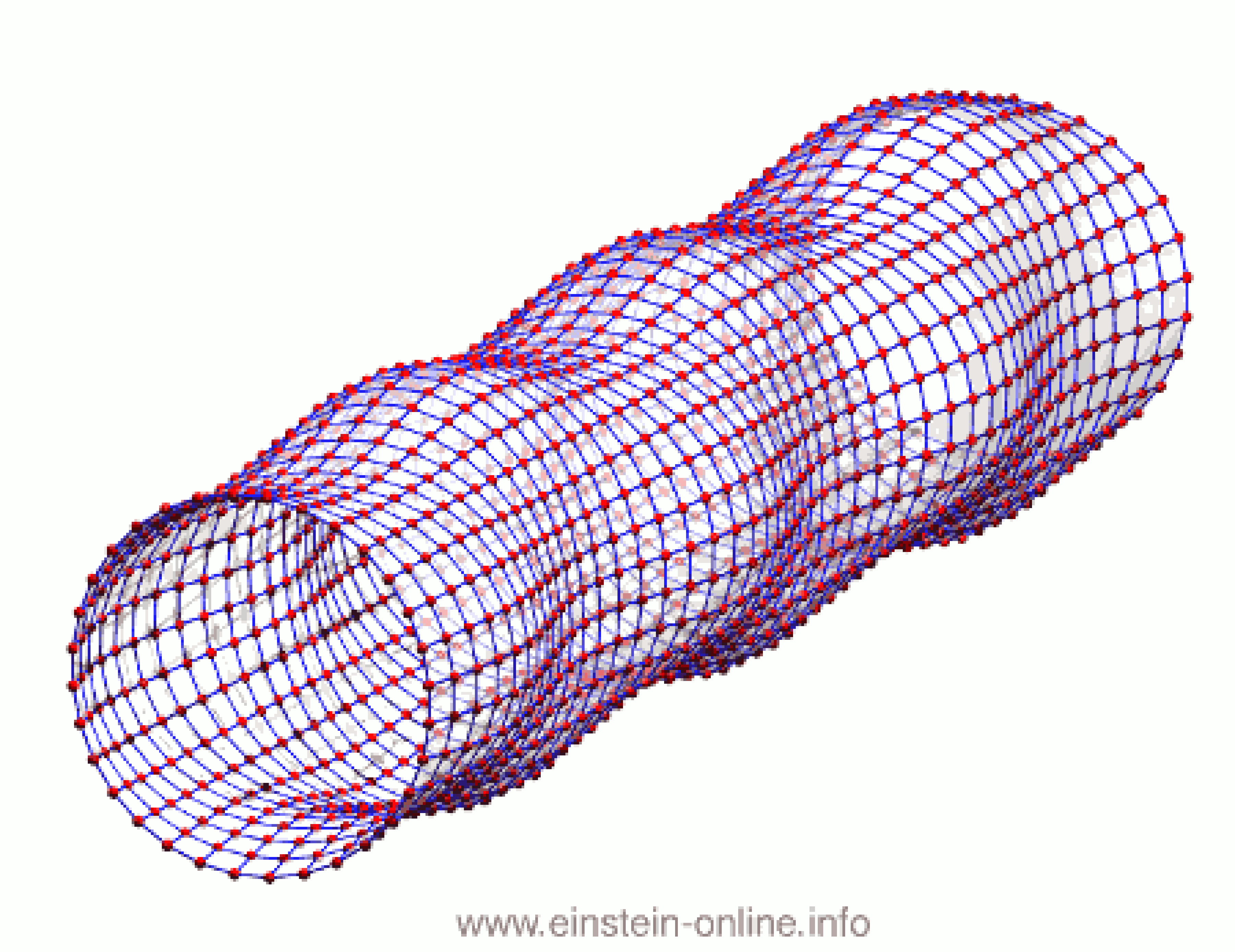
$$\frac{d^2}{dt^2} x_M(t) = \frac{1}{2}\ddot{h}_{xx} x_M(t) \simeq \frac{1}{2}\ddot{h}_{xx} L$$



...and similarly for the other positions

This gives an intuitive description in term of

«Newtonian» forces.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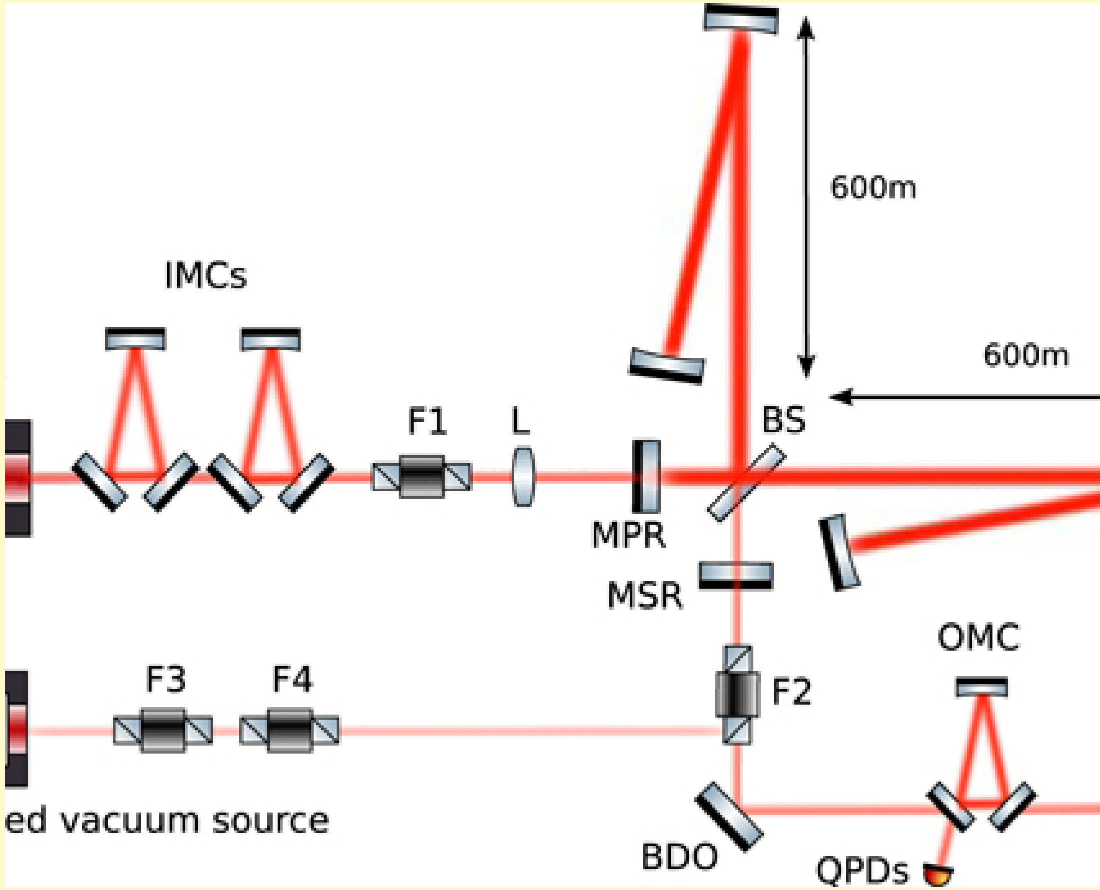
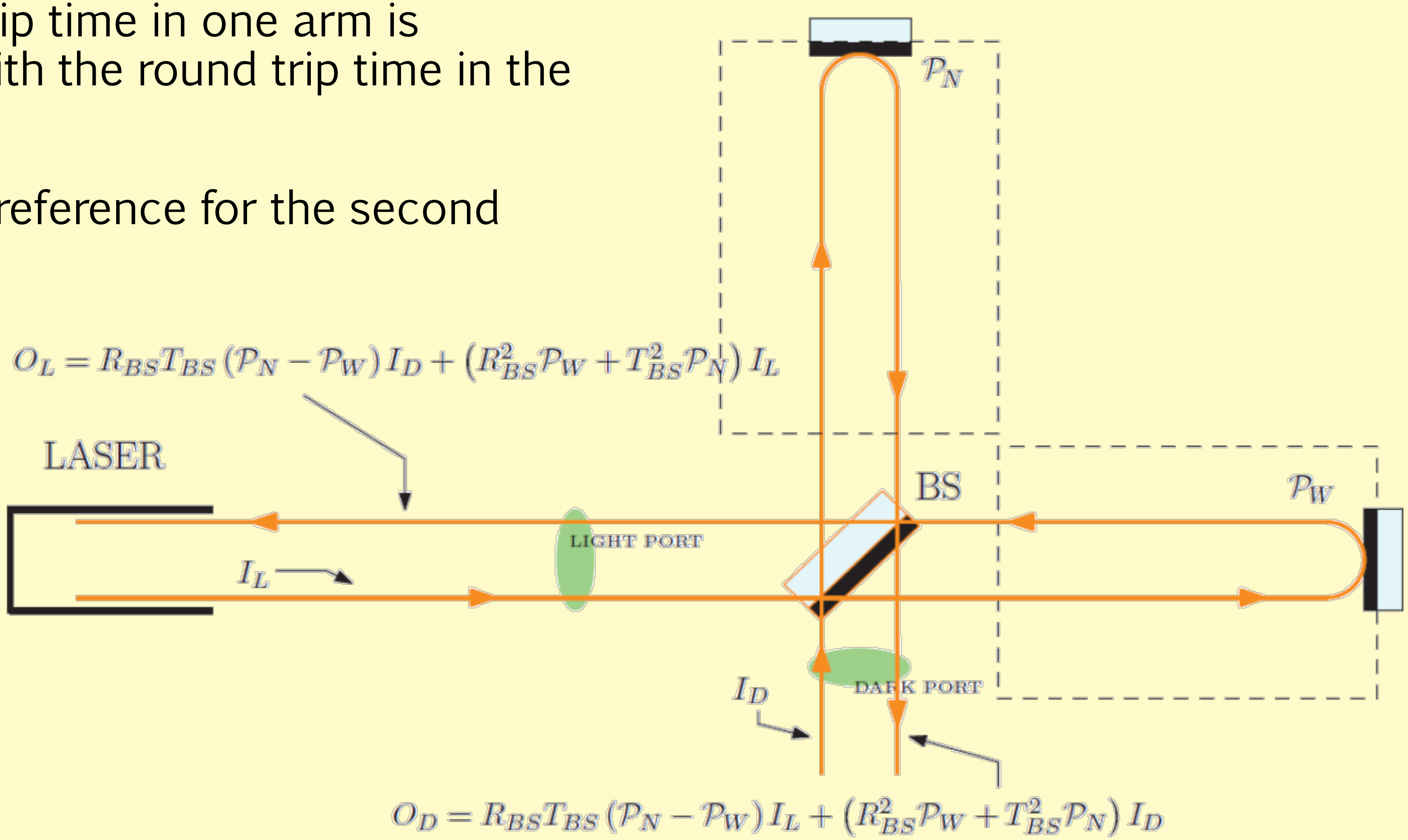
GW DETECTORS

Optical schemes

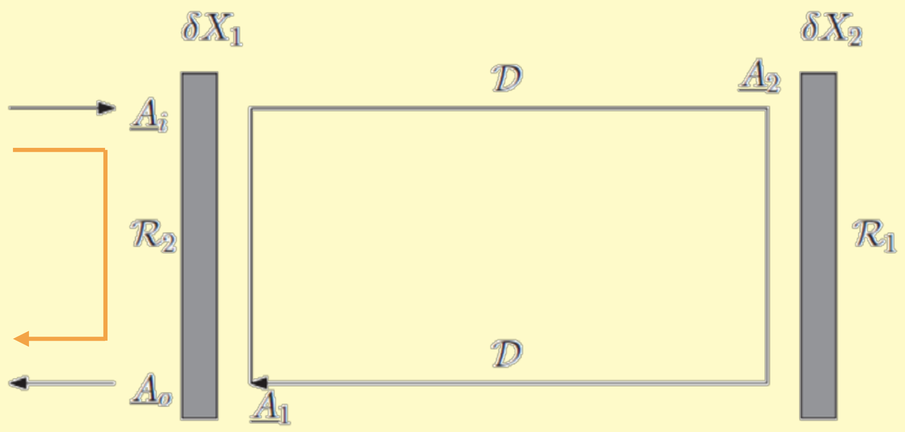


How the round trip time is really measured? Interferometer

- The round trip time in one arm is compared with the round trip time in the second
- First RT is a reference for the second



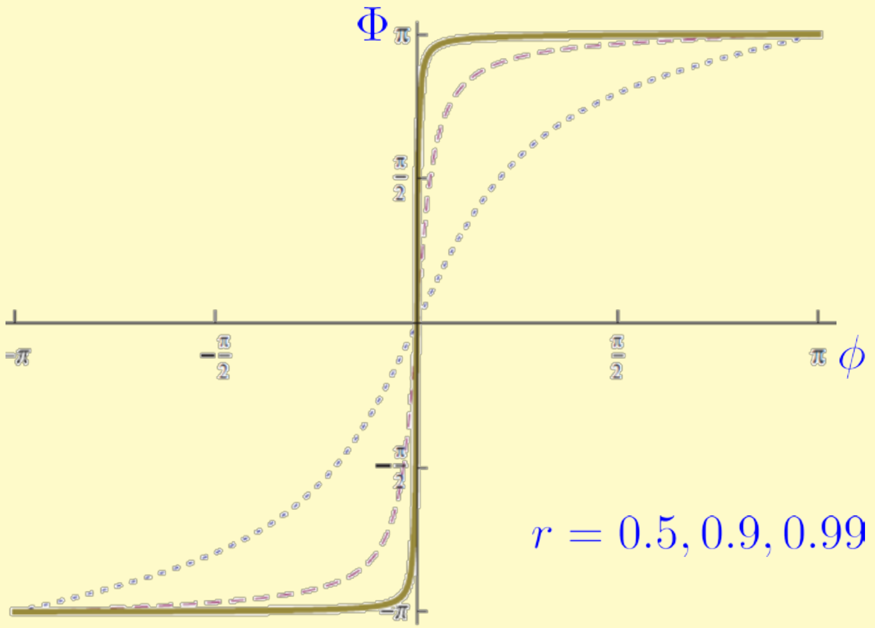
How the round trip time is really measured? Cavity



$$A_o = -rA_i + t^2\left(e^{i\phi}A_i + re^{2i\phi}A_i + r^2e^{3i\phi}A_i + \cdots\right)$$

$$A_o = \frac{e^{i\phi} - r}{1 - re^{i\phi}}A_i = e^{i\Phi}A_i$$

- Here the output field get a phase shift



- To measure the phase we need a reference: we can modulate the input field to obtain «promptly reflected» sidebands

Advanced Virgo Optical Scheme

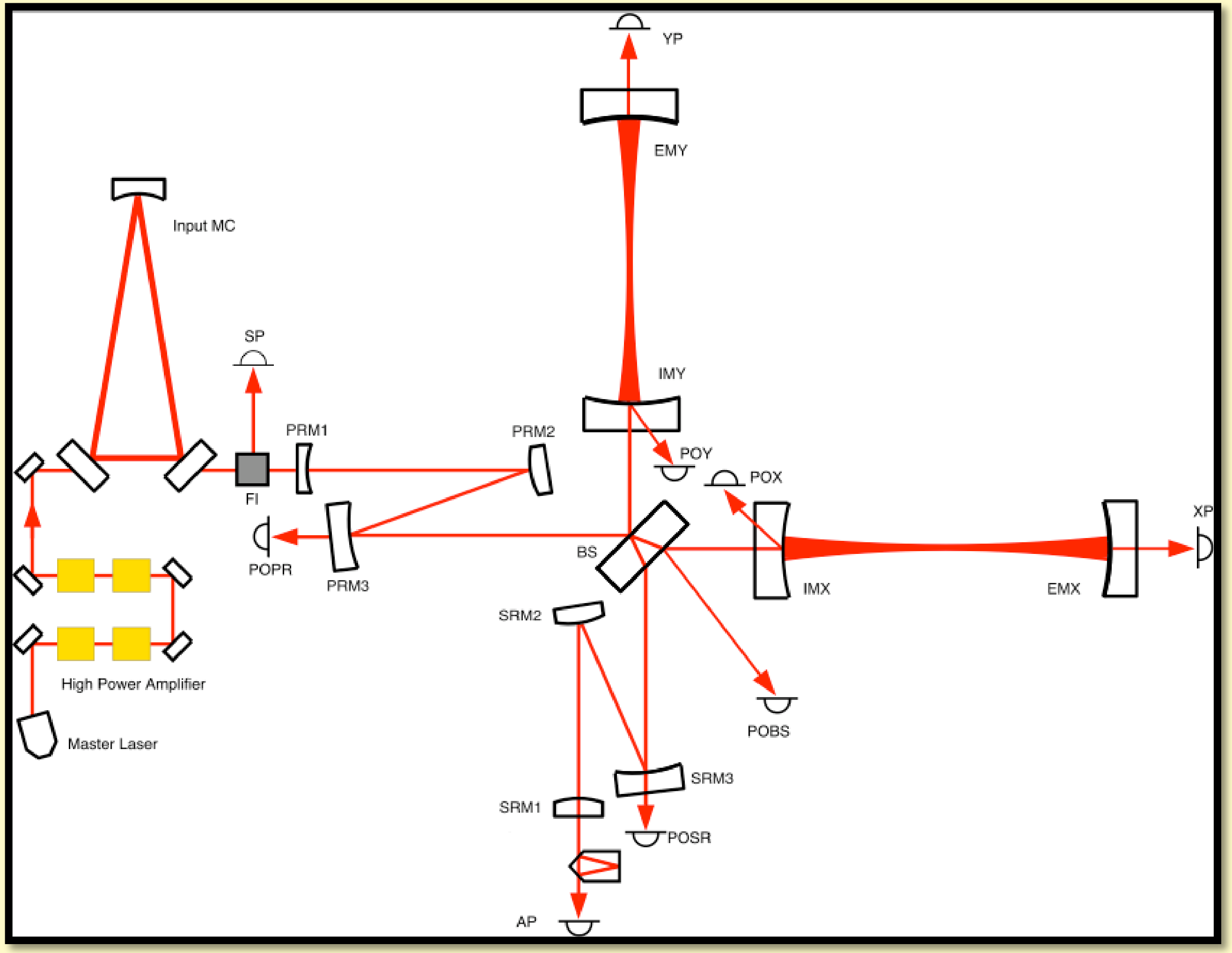
- Arm with Michelson resonant cavities
- Power recycling
- Signal recycling
- Observed:

$$h(t) \equiv D^{ij} h_{ij}^{TT}$$

where

$$D^{ij} = u^i u^j - v^i v^j$$

is the «detector tensor»

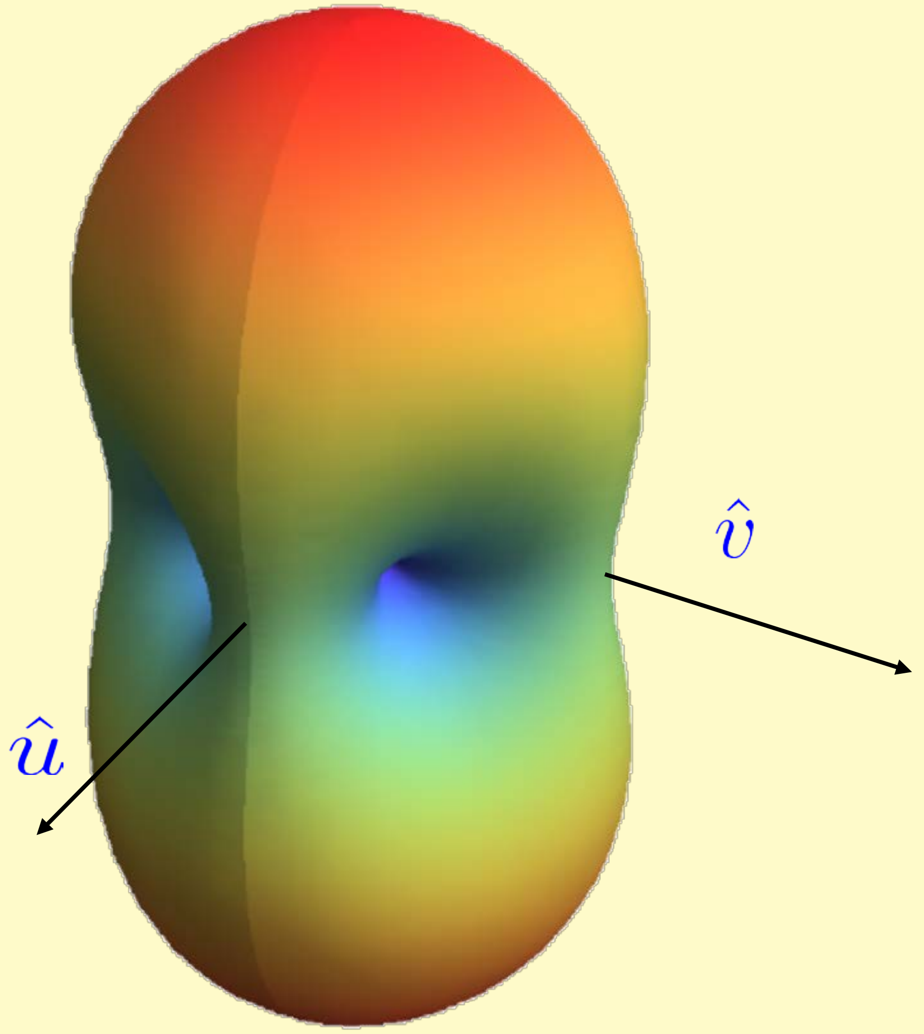


Detector tensor and angular sensitivity pattern

- We can represent as a function of the GW's direction of propagation

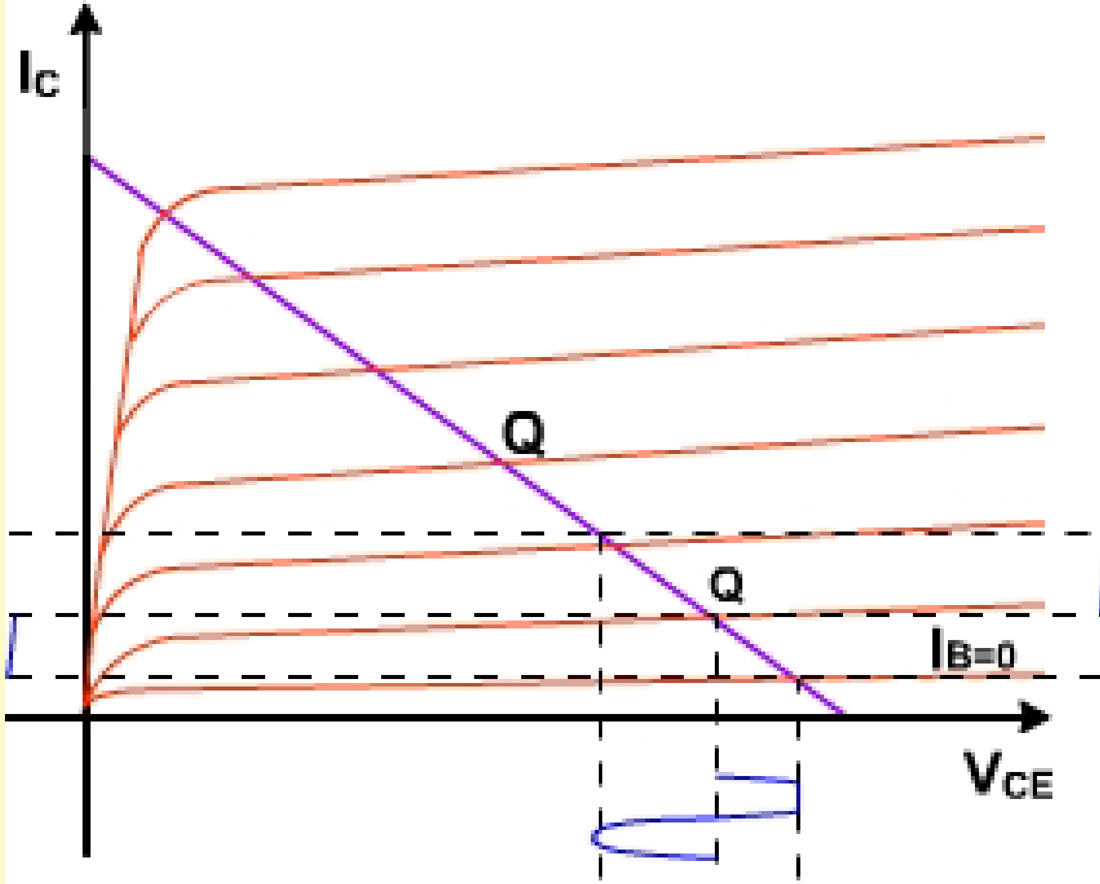
$$\Gamma(\hat{n}) = D^{ij}\left[\cos\psi\,\epsilon^{+}_{ij}(\hat{n}) + \sin\psi\,\epsilon^{\times}_{ij}(\hat{n})\right]$$

- For each $\hat{n}$ there is an optimal polarization angle ψ_{opt}
- $\psi_{opt} + \pi/2$ gives a decoupled polarization
- This gives the directionality of an interferometric detector



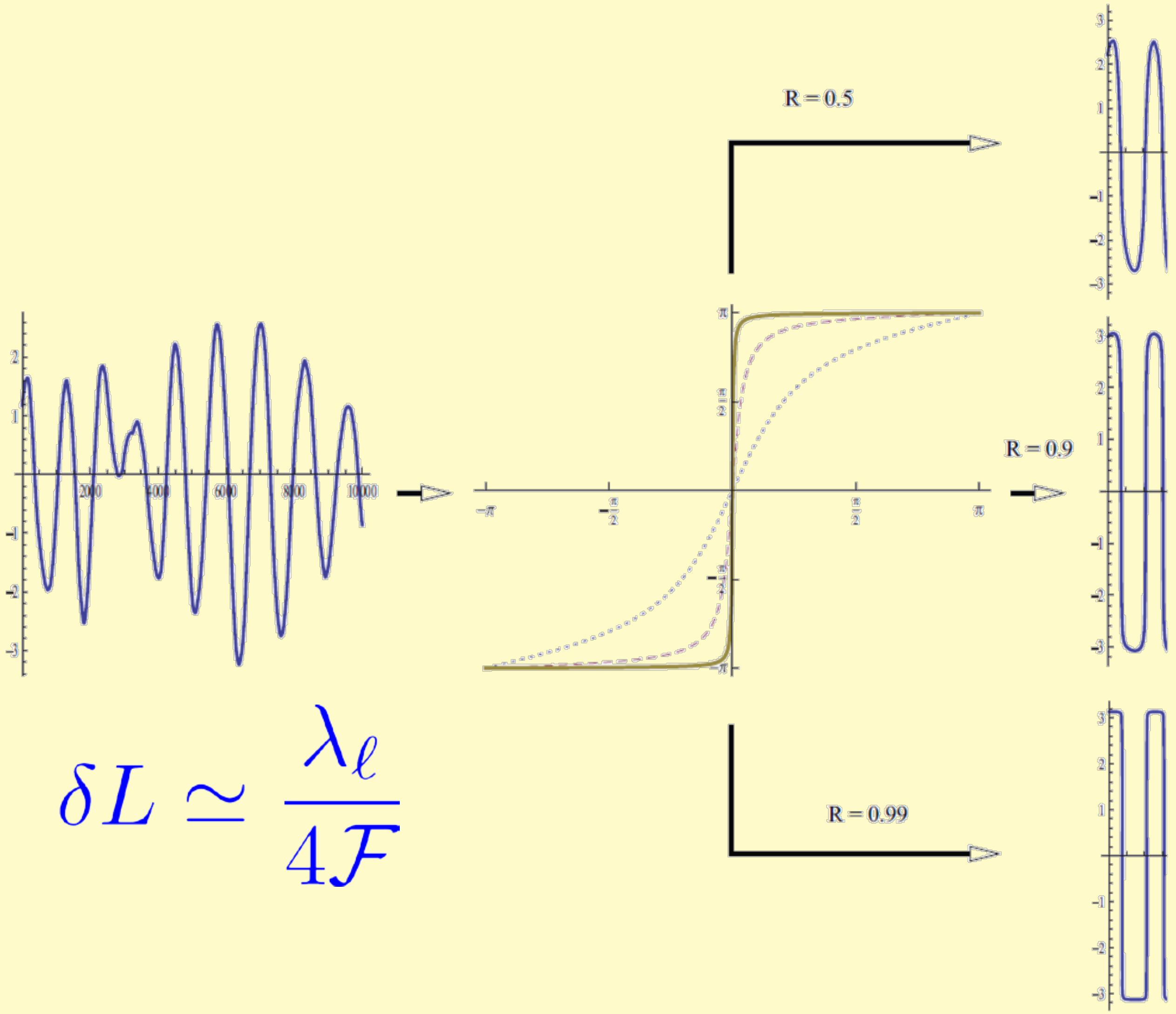
GW DETECTORS

The working point



Locking

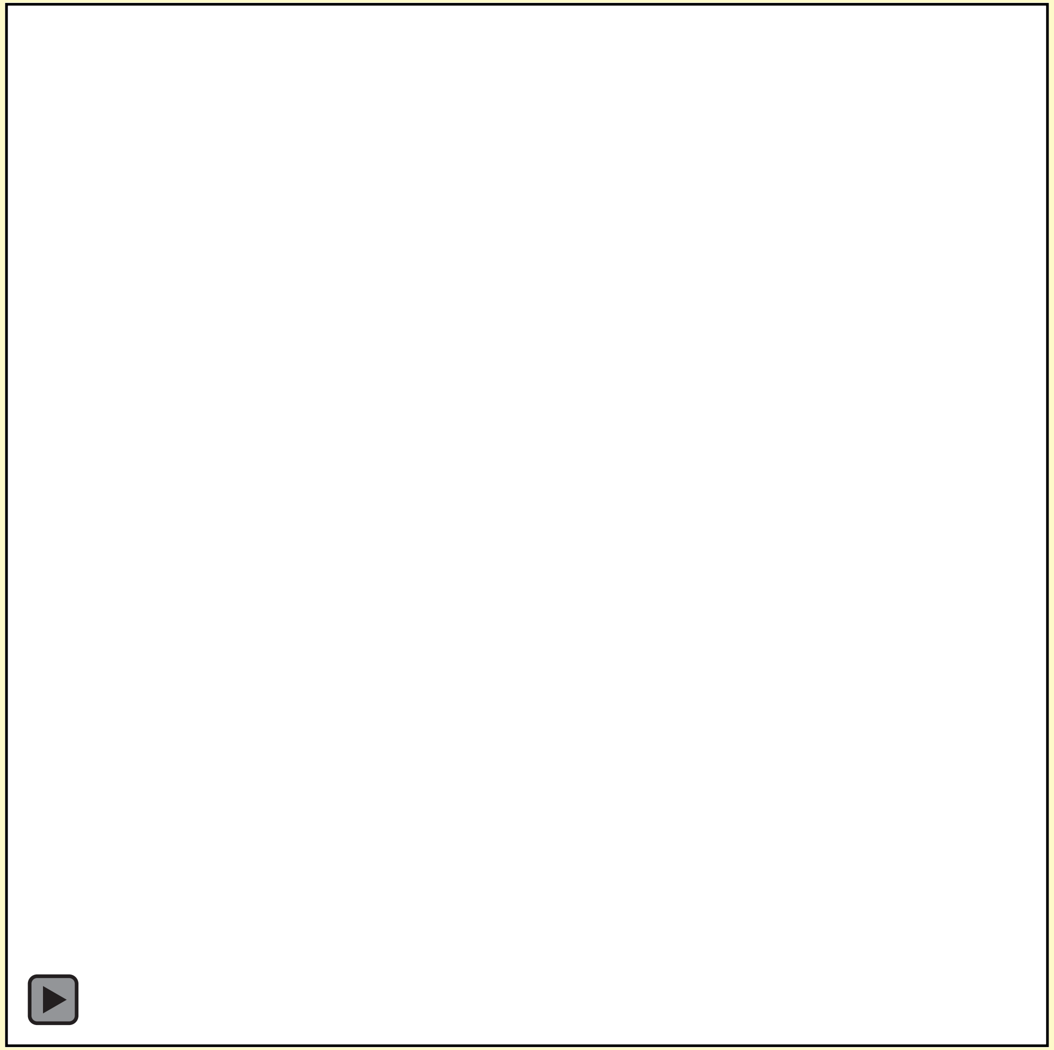
- The phase shift of a cavity is a very sensitive function of the cavity length only in a small region around the cavity resonance
- This is especially true when the finesse of the cavity is high
- We must maintain the cavity length in this small region if we want a sensitive detector
- This must be done with an active control strategy



$$\delta L \simeq \frac{\lambda_\ell}{4\mathcal{F}}$$

Guided lock

- Several strategies are studied and implemented, for example
  - Trial and error
  - Artificial reduction of finesse
  - Multistep procedures
- An interesting example: «Guided Lock» strategies
  - The idea: apply the control force also when the cavity is out of the working point, using information about the dynamics

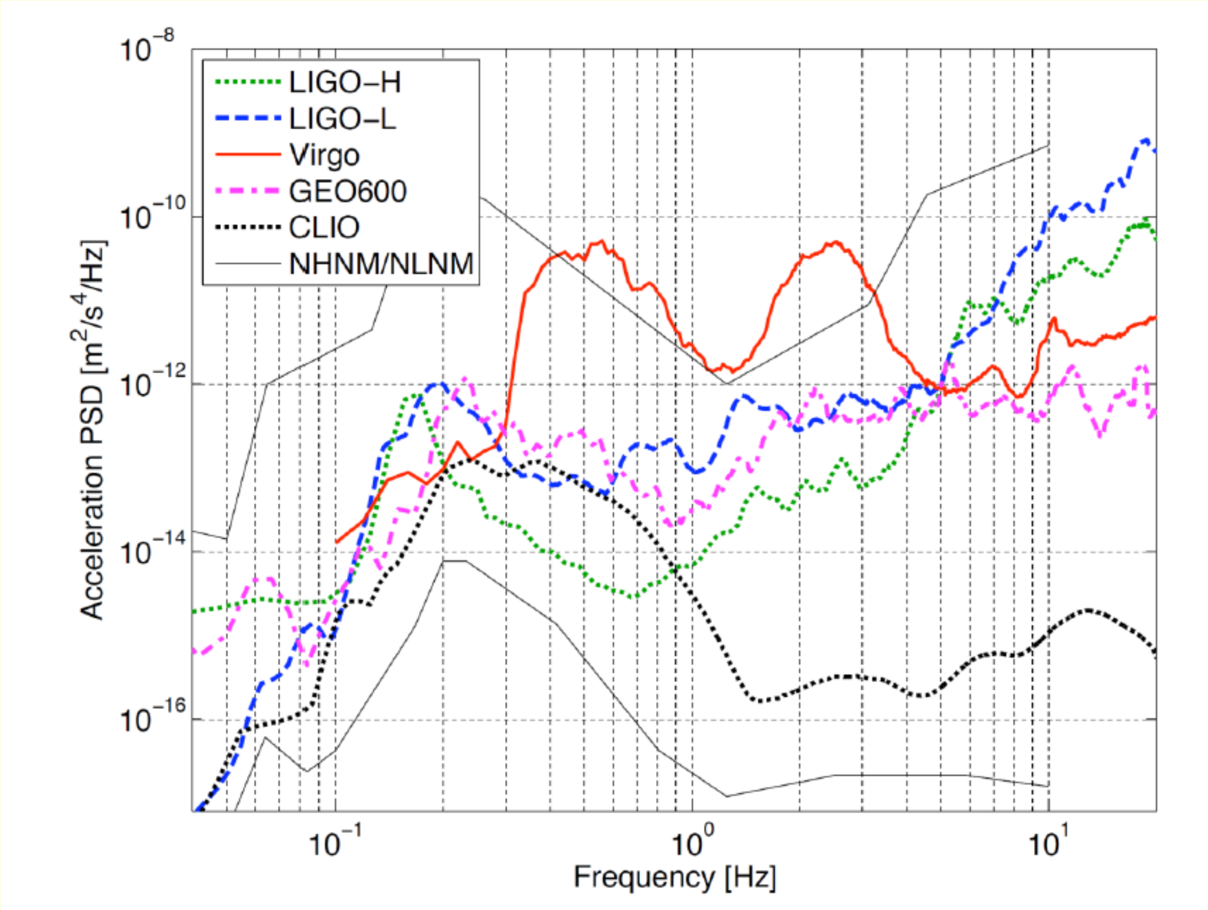


FUNDAMENTAL NOISES

Seismic noise

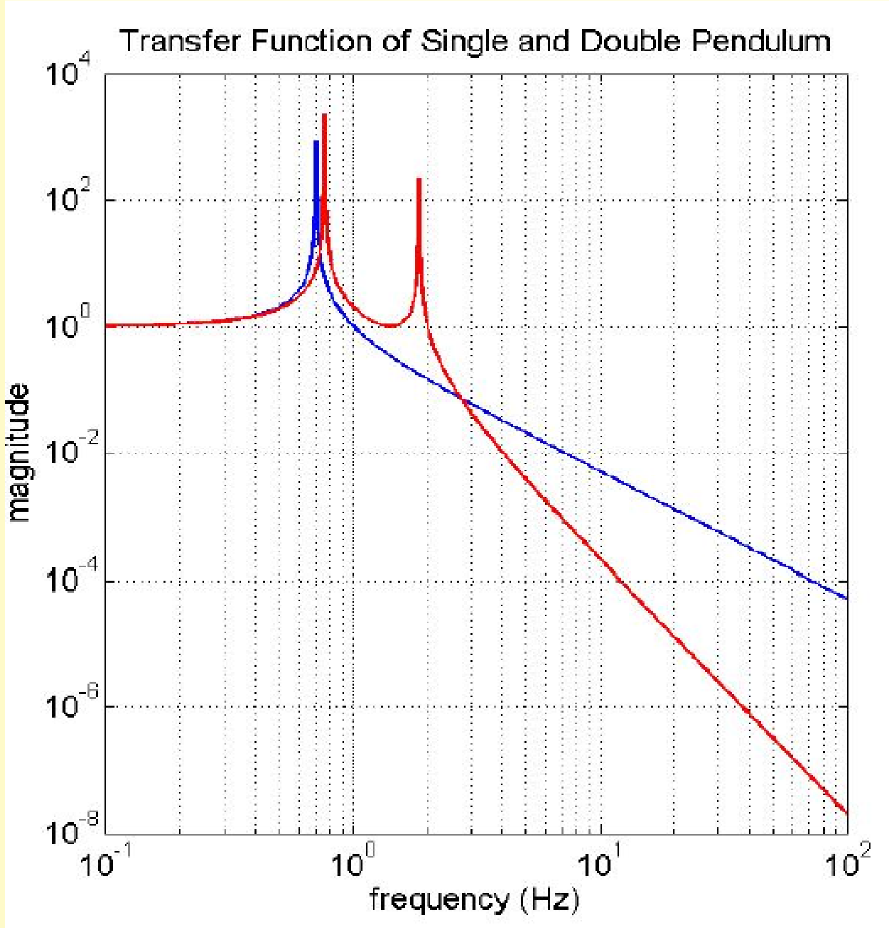
Seismic noise

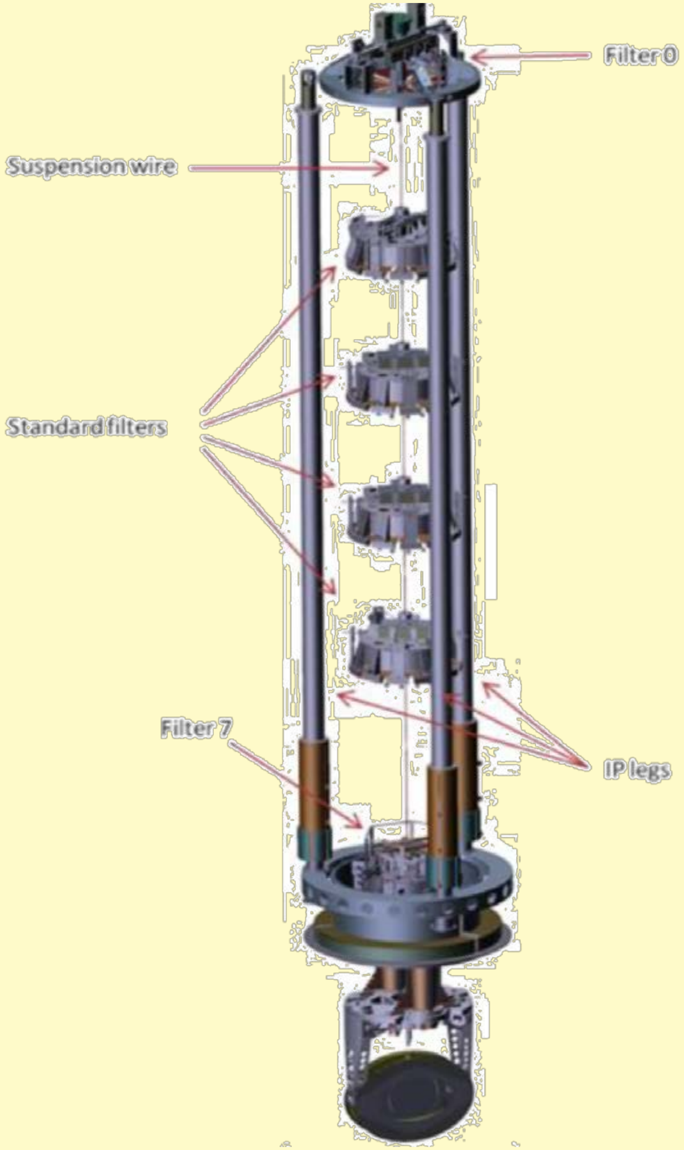
- Seismic noise influences the final sensitivity of the detector
  - $f < 1Hz$: microseismic noise. Natural sources (non-cultural and non local) depending on oceanic and large scale meterological conditions;
  - $f \simeq 1Hz$: wind effects and local meterological conditions;
  - $f > 1Hz$: mainly antropogenic noise
- Seismic noise is reduced at the required level using a mirror suspension system, which allows to extend the detection band below 100 Hz
- In VIRGO this is implemented by an hybrid passive/active system called Super-Attenuator



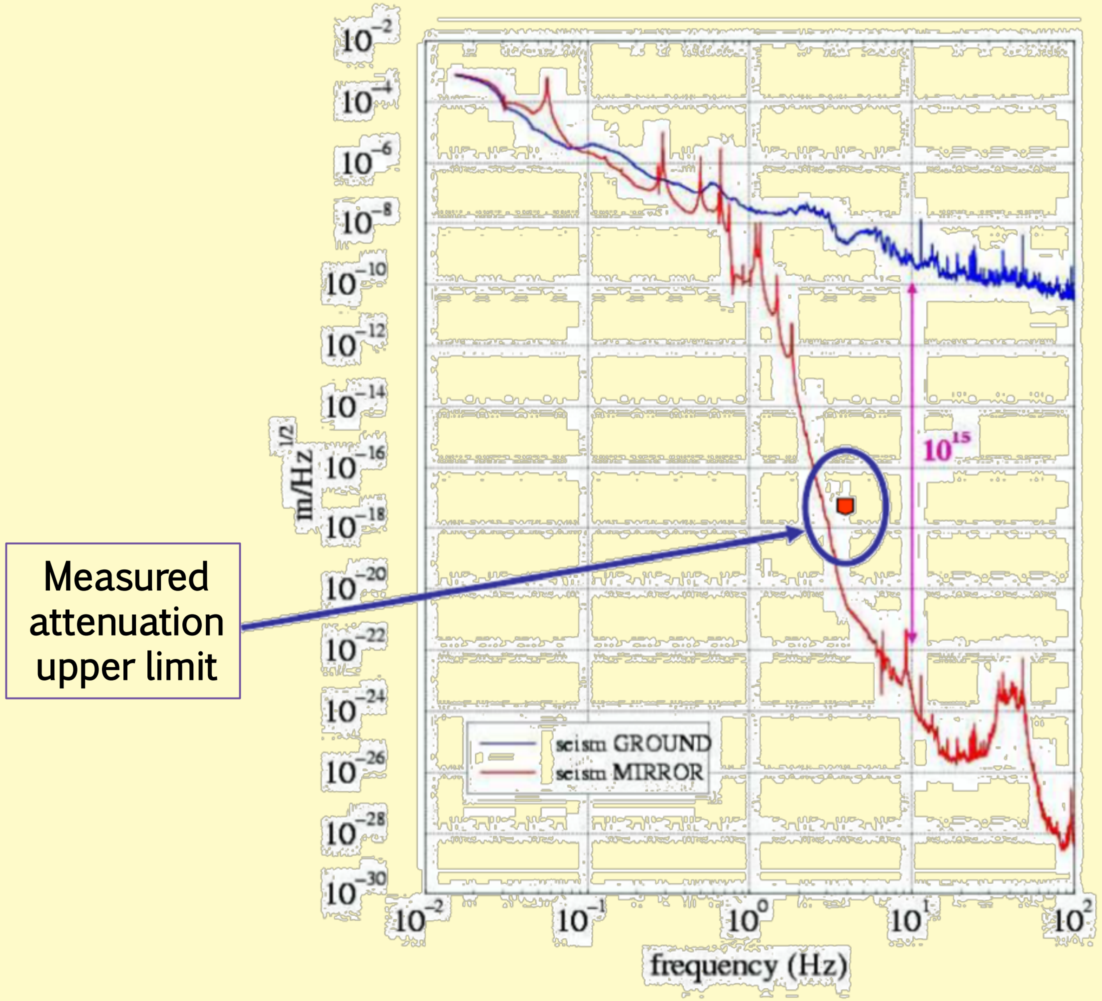
Seismic suspension

- Based on the working principle of a multistage pendulum:
  - Inverted Pendulum
  - N filters
  - Payload or last stage



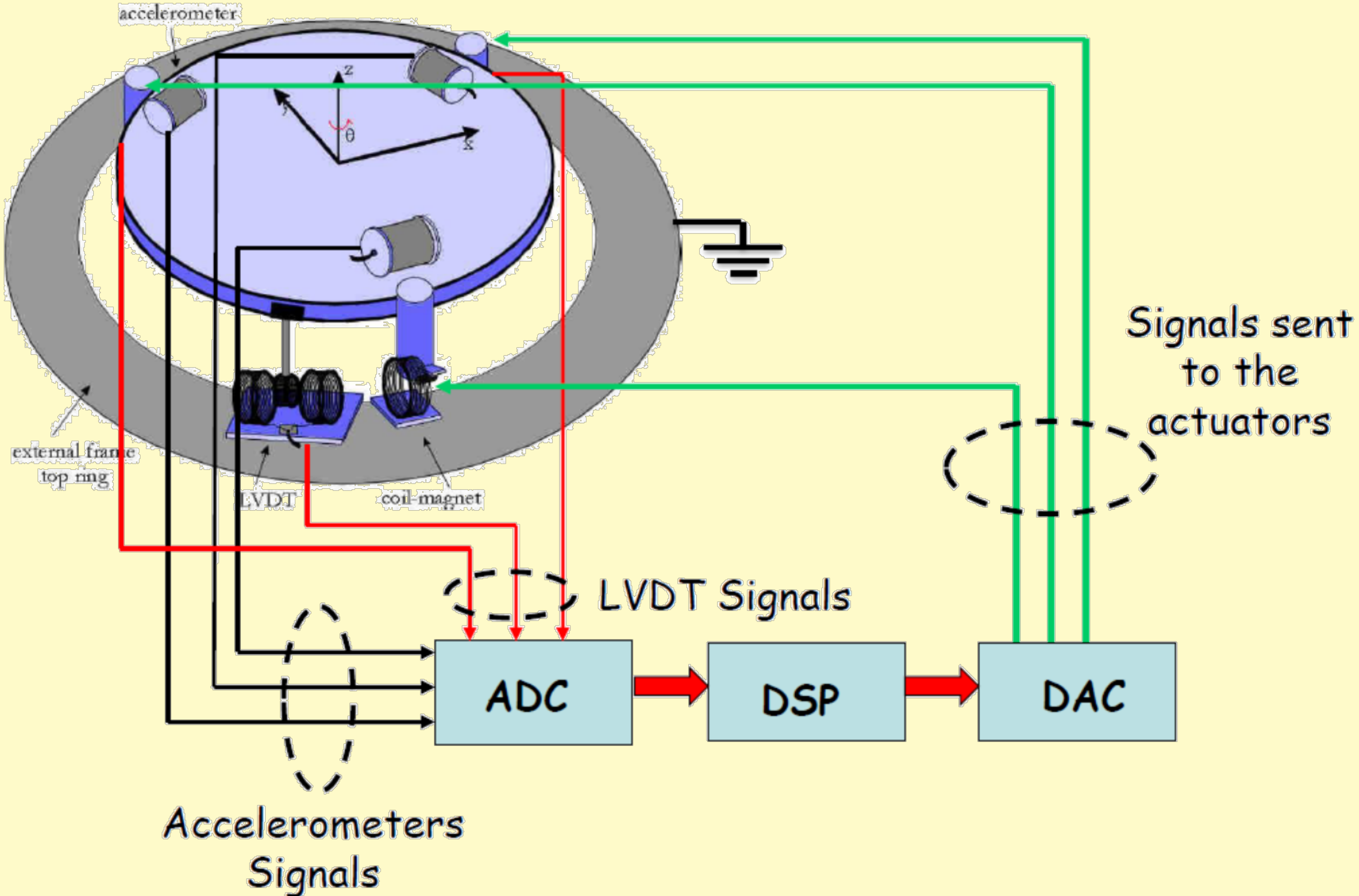


Active control

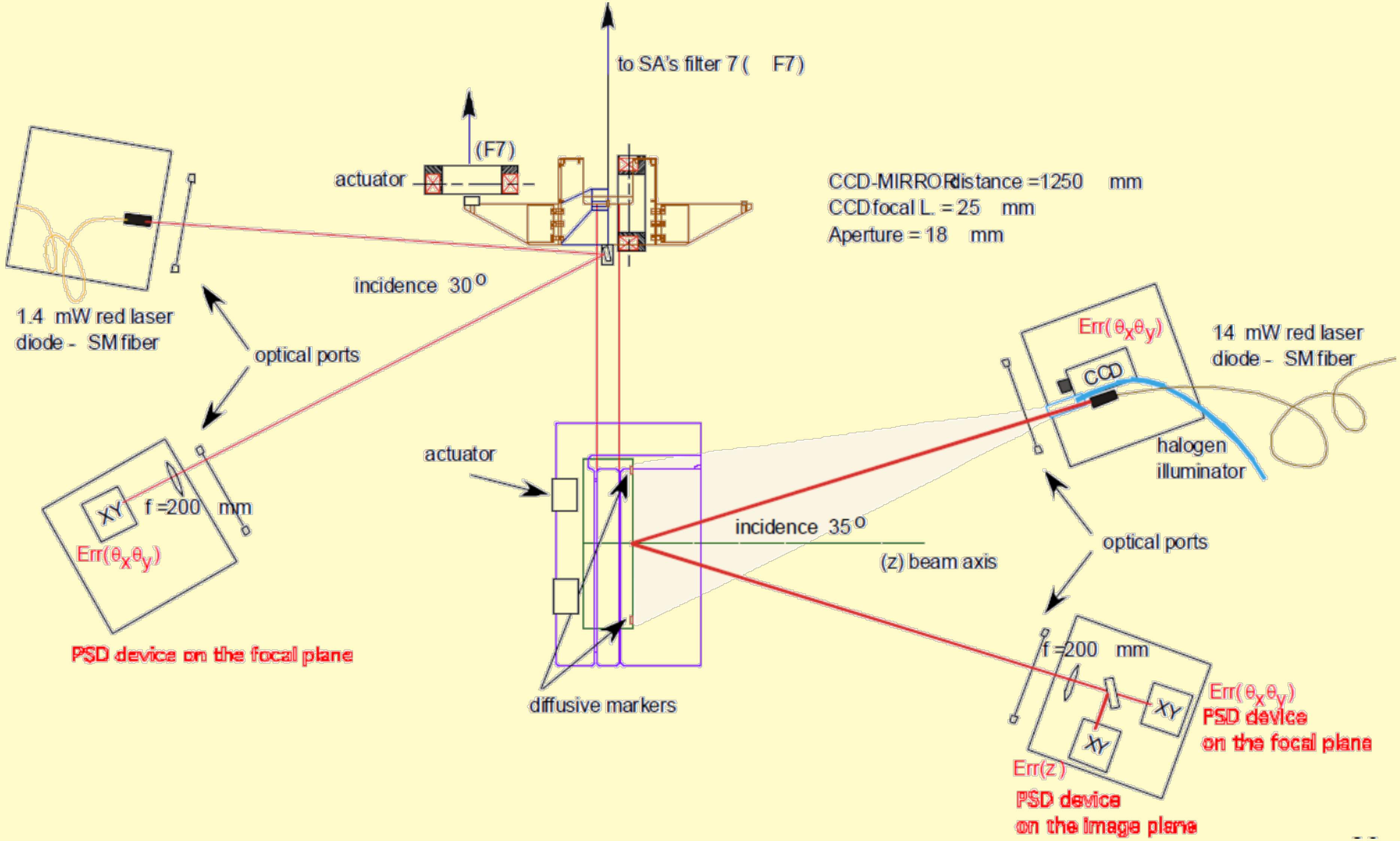


- Passive attenuation do not work below few Hz
- In the range $0.04Hz < f < 2Hz$ the chain resonance modes induces tens of microns swings
- This is much larger than the linear working point region: we need active control:
  - Inertial Damping on Inverted Pendulum (tidal strain and drifts)
  - Local Control on Marionette (angular mirror displacements reduced down to a fraction of μrad
  - Local Damping on Reference Mass

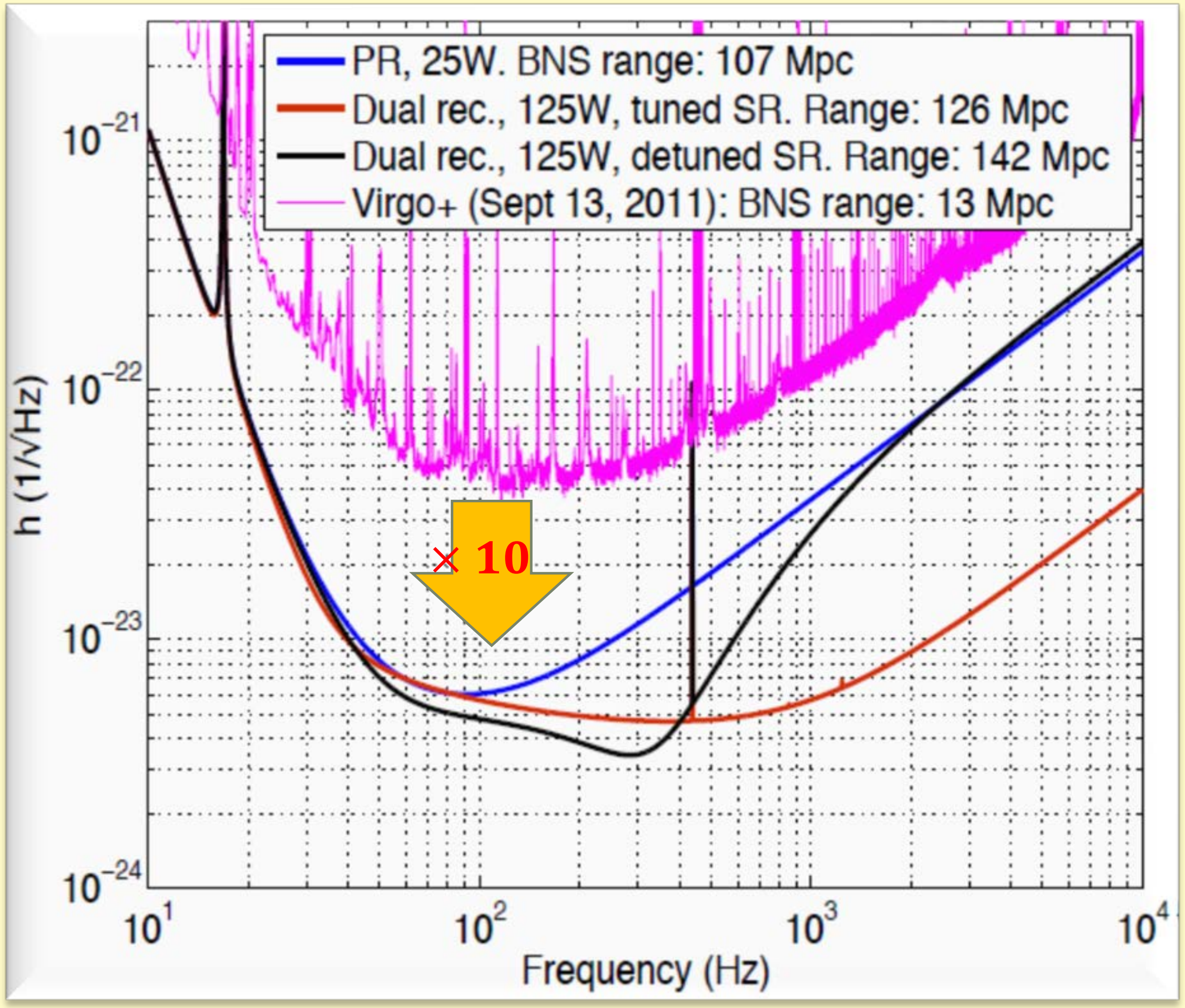
Inertial Damping



Local control

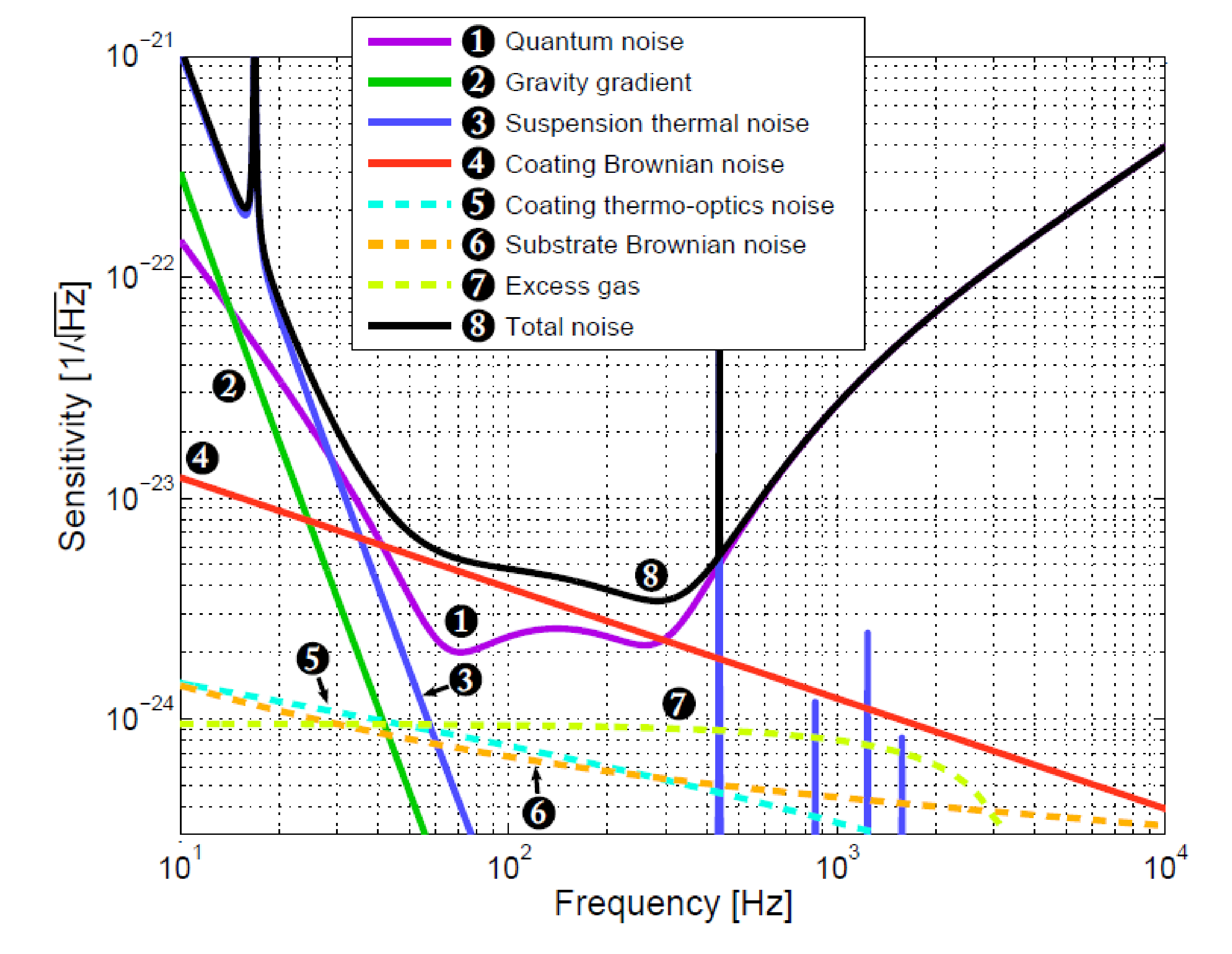


Advanced detectors: a sensitivity jump



- Larger beams
- Heavier mirrors: 42 kg (× 2)
- New payload
- Higher quality optics
- Larger finesse: $\mathcal{F} \simeq 450$ (× 3)
- Improved thermal control of aberrations
- Improved vacuum
- 200W fiber laser
- Signal recycling

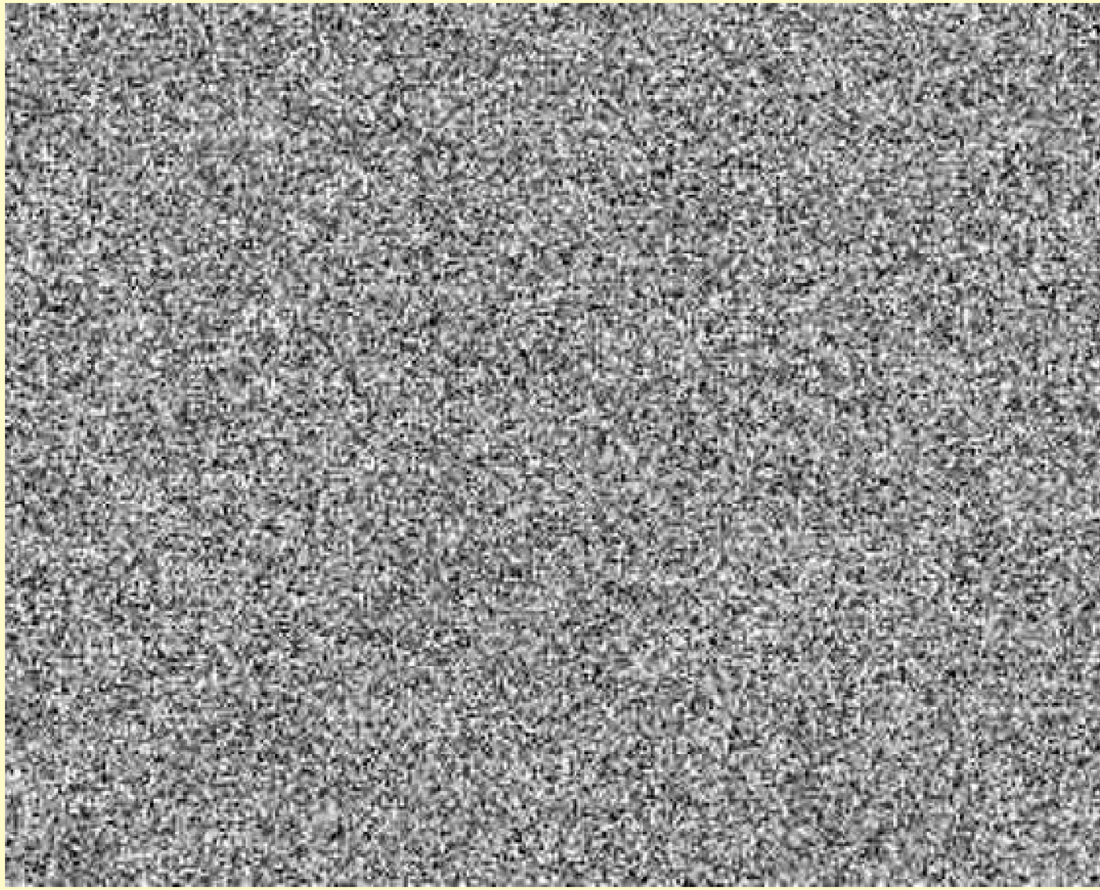
A fight with the noise



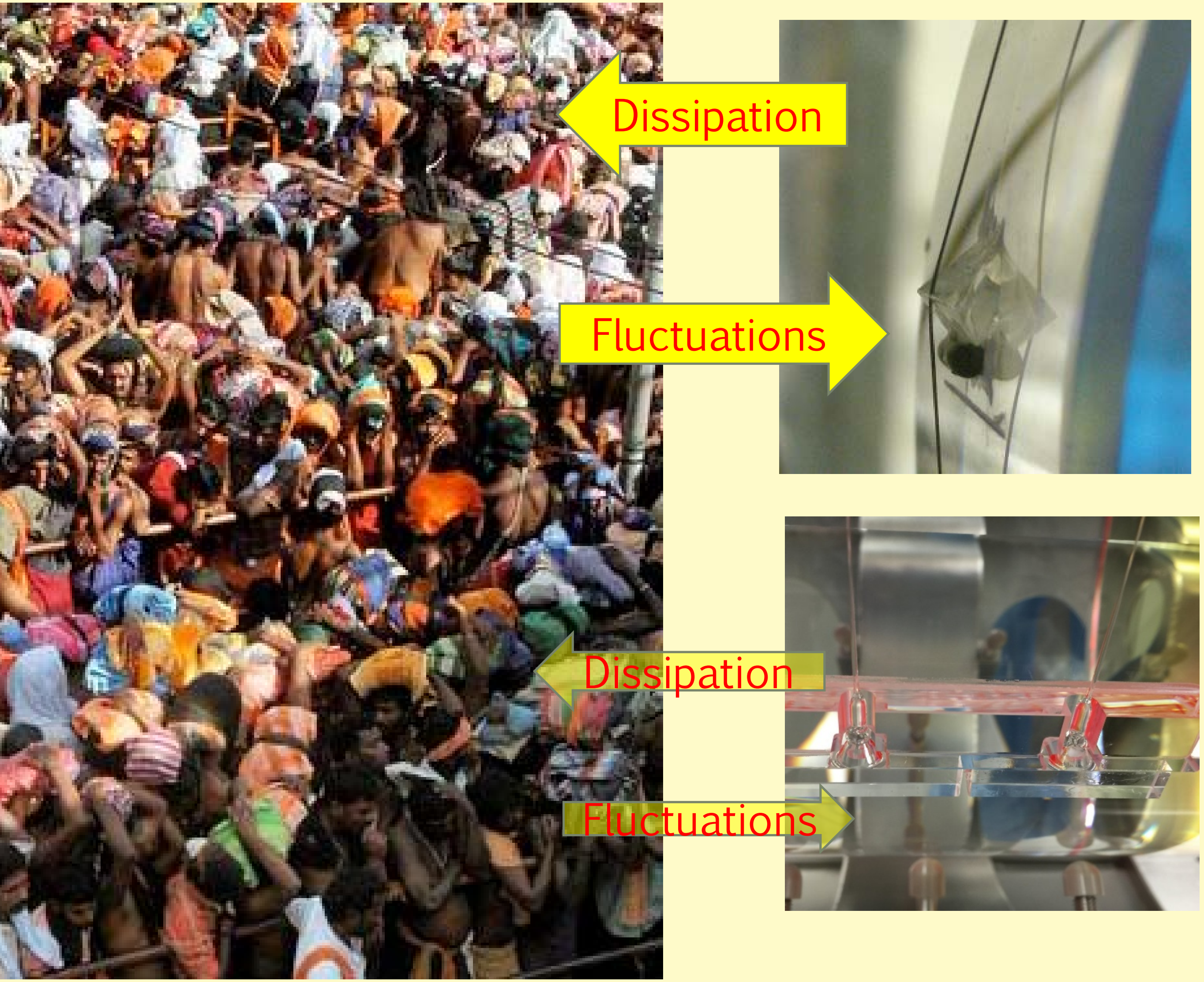
- Fundamental noises
  - Seismic
    - Direct
    - Newtonian
  - Thermal
    - Suspension
    - Mirror Coating
    - Mirror Bulk
  - Quantum
    - Shot noise
    - Radiation pressure
  - ....
- Technical noises
  - Laser frequency & intensity
  - Scattered light
  - Residual gas
  - Length and alignment control systems
  - Magnetic actuation
  - Acoustic couplings
  - Nonlinear couplings (up-conversion)

FUNDAMENTAL NOISES

Thermal noise

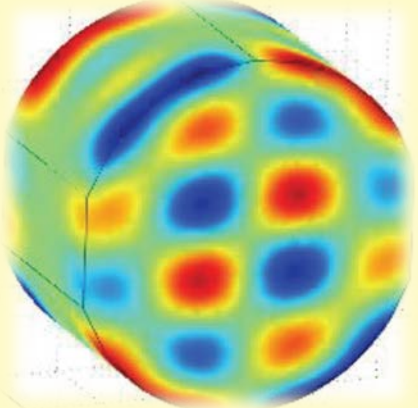


Thermal noise

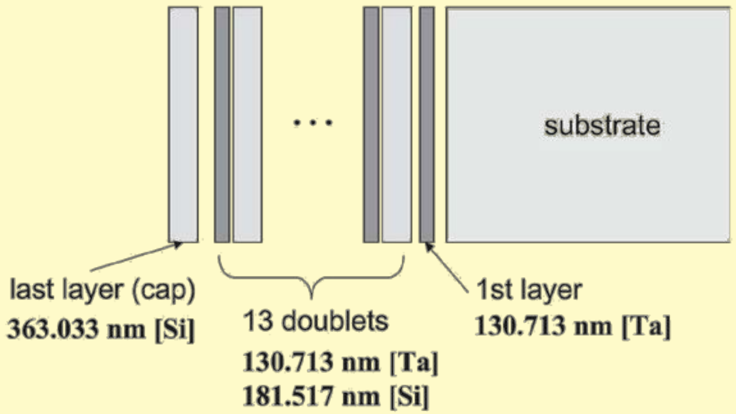


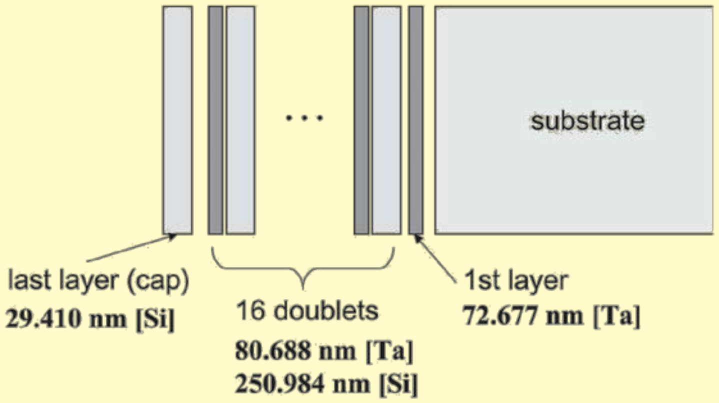
- Dominated by thermal fluctuations of mirrors and suspensions
- Handles:
  - Larger beam spot (statistical effect)
  - Fused silica fiber suspensions (low losses)
  - Improved mirror coatings (low losses)
  - Cryogeny (not foreseen in LIGO & Virgo)

$$S_F^{(m)} = 4k_B T \frac{\omega^{(m)}}{Q^{(m)}}$$



Coating thermal noise



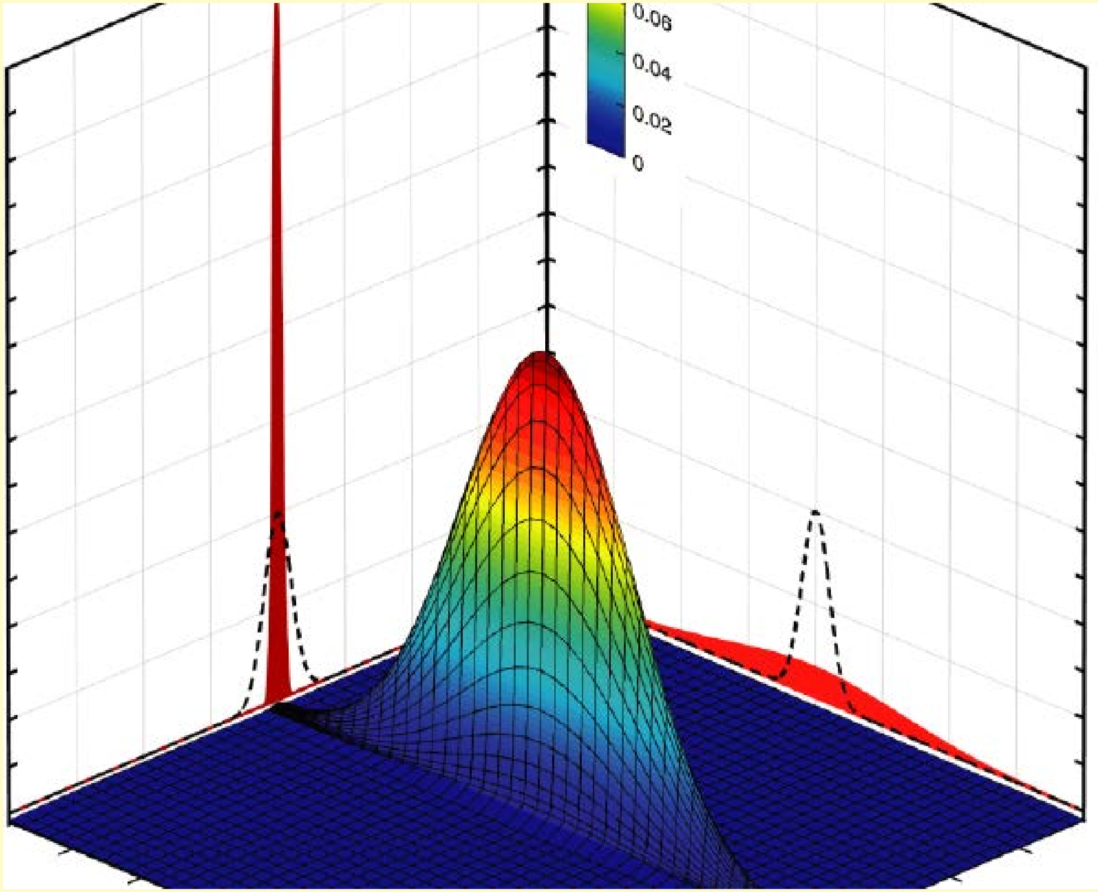


Opt. Express 23, 10938-10956 (2015)

- A difficult problem
- Constraint: good optical properties of materials
- Complex theoretical modelization of dissipation mechanisms
- Phenomenological approach: parameter optimization (genetic algorithms, ….)
- Experimental approach: test new materials (new kind of dopings)
- Currently the limit in the intermediate frequency region

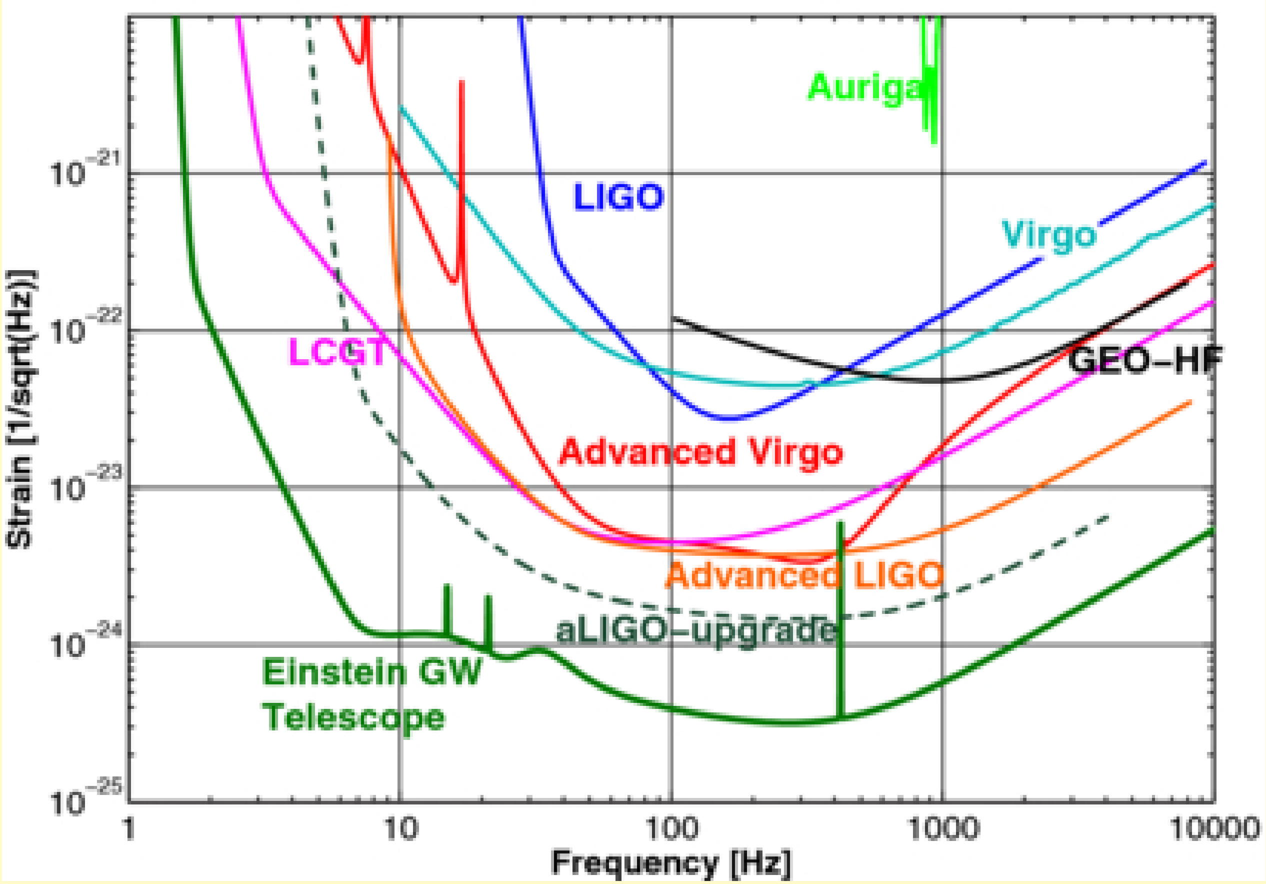
FUNDAMENTAL NOISES

Quantum noise



Quantum Noise

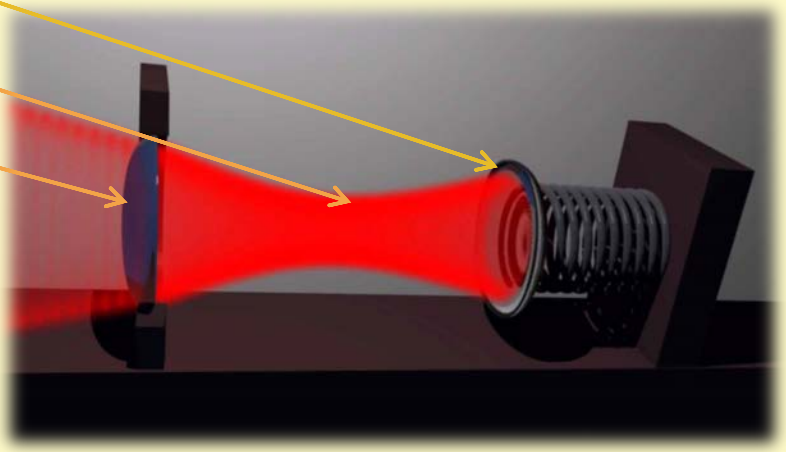
- There is a fundamental limit on *continuous* position measurements of quantum nature (the Standard Quantum Limit)
- How can we deal with it?



The basic building block: a resonant cavity

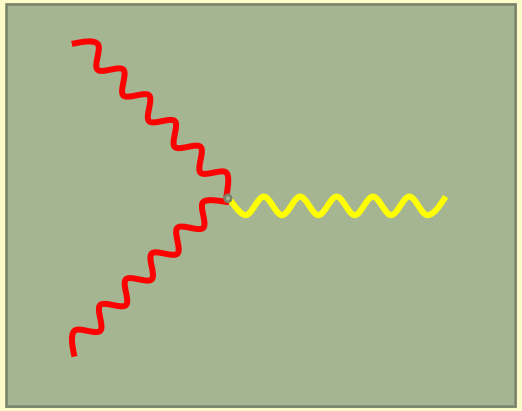
- Basic Hamiltonian of the system:

$$H = \text{ext. couplings} + \hbar\omega_c\, \hat{a}^{+}\, \hat{a} + \hbar\Omega_m\, \hat{b}^{+}\, \hat{b}$$



- The cavity frequency depends on its length....

$$\omega_c = \omega_{c,0} + \left(\frac{\partial \omega_c}{\partial x}\right)_0 x + \cdots$$

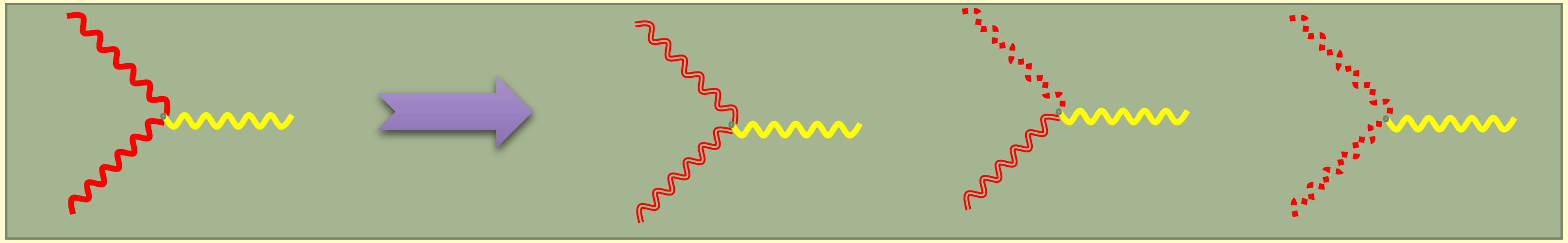


- ...and we get

$$H = H_0 - \hbar g_0 \hat{a}^{+}\, \hat{a}\left(\hat{b} + \hat{b}^{+}\right)$$

Linearized equations

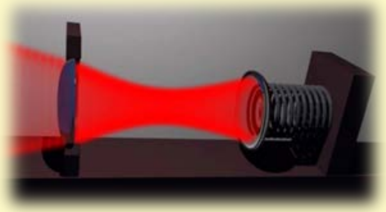
- A very good approximation in the regime we are interested to: $\hat{a} = \alpha + \delta\hat{a}$



- .... keeping only linear and quadratic terms in the frame rotating with the laser frequency....

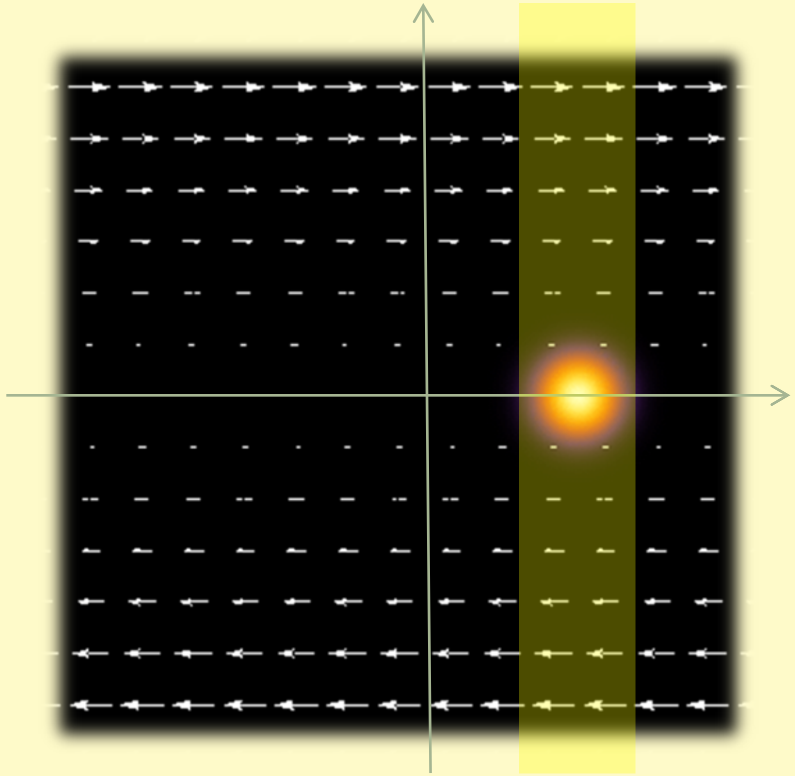
$$H = -\hbar\Delta\delta\hat{a}^{+}\delta\hat{a} + \hbar\Omega_m\hat{b}^{+}\hat{b}$$
$$-\hbar g_0(\alpha\delta\hat{a}^{+} + \alpha^{*}\delta\hat{a})\left(\hat{b}^{+} + \hat{b}\right) + \cdots$$
$$\Delta = \omega_\ell - \omega_{cav}$$

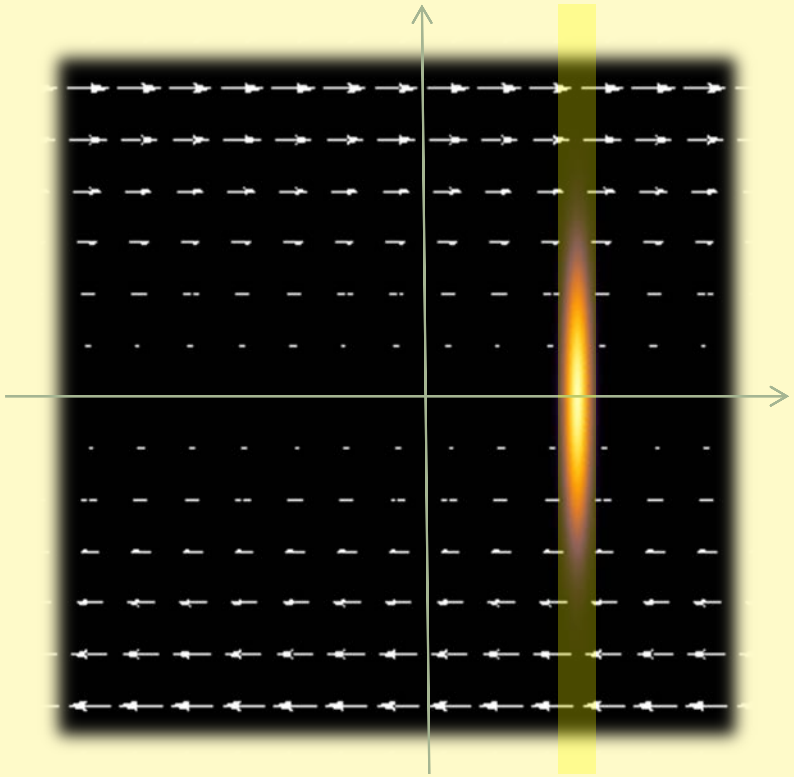
- Motion dependent phase shift ($\Delta = 0$)
- Squeezing, anti-damping, amplification ($\Delta = \Omega_m$)
- Cooling ($\Delta = -\Omega_m$)

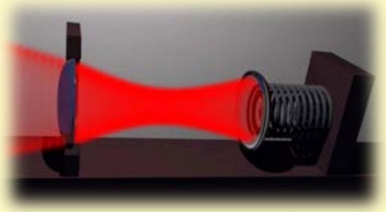


Repeated measurements

- If we measure the position of a free mass with a given accuracy, we get an indeterminacy on velocity
- The position indeterminacy grows with time
- We cannot predict exactly where the mass will be after some time







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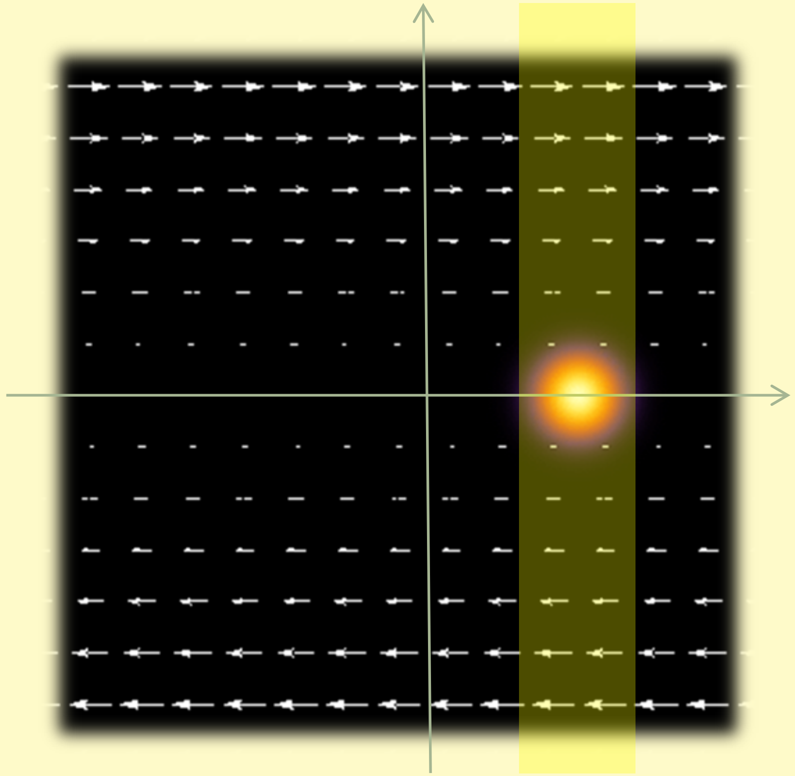
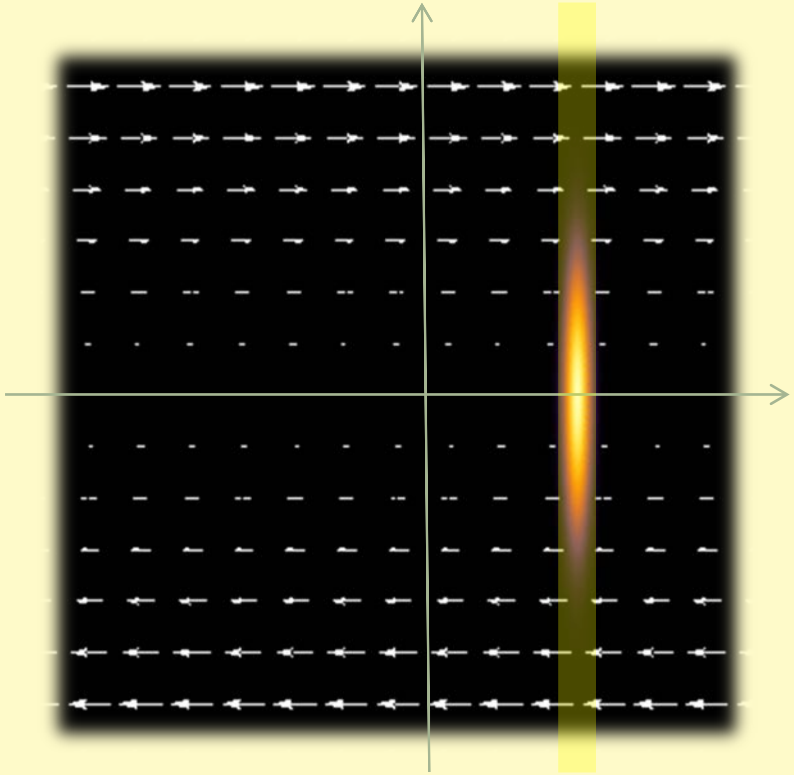
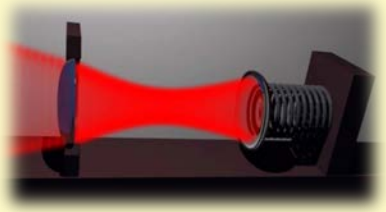
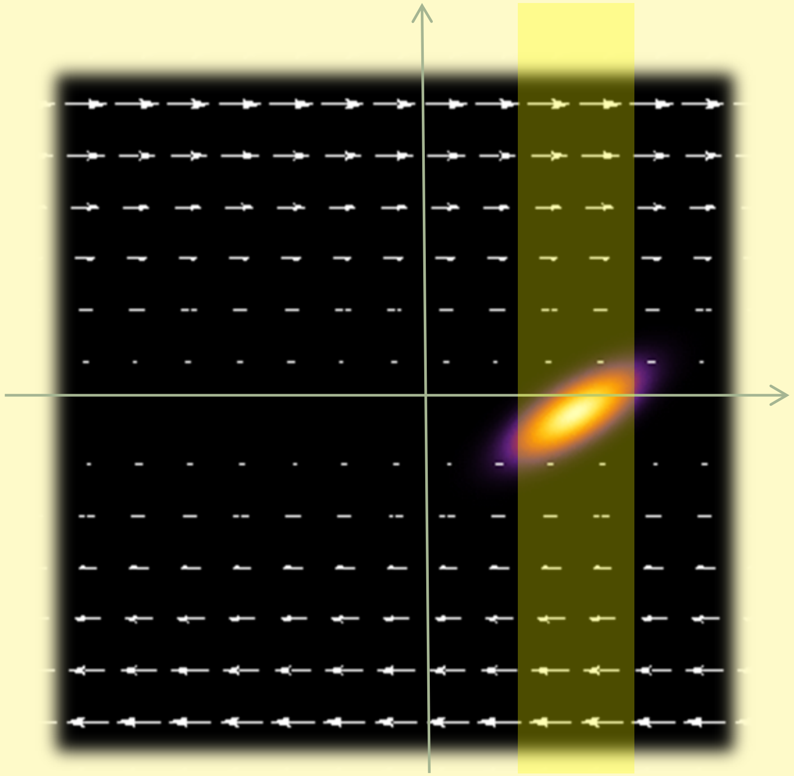


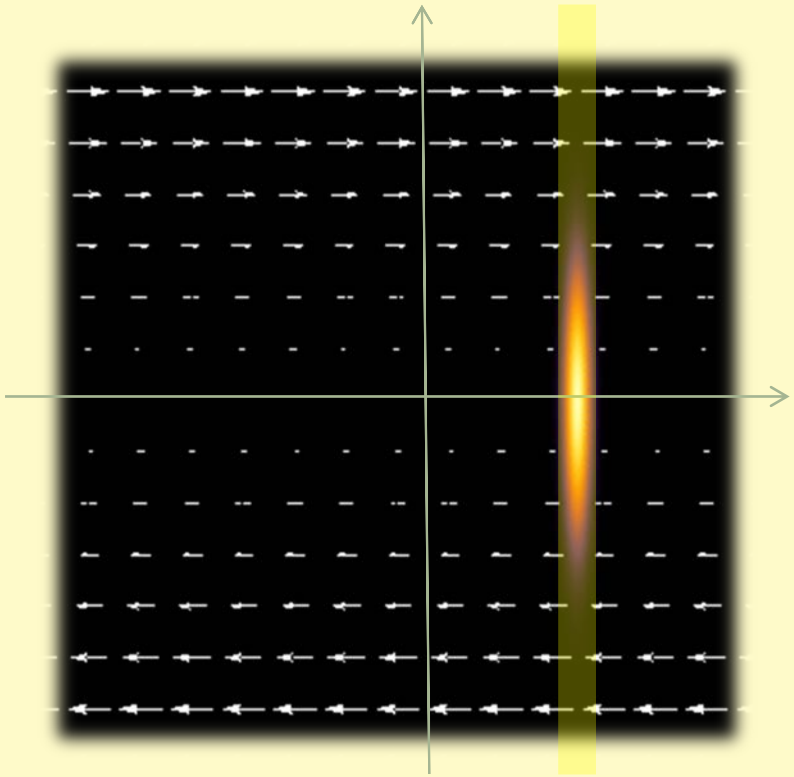




Repeated measurements

- If we measure the position of a free mass with a given accuracy, we get an indeterminacy on velocity
- The position indeterminacy grows with time
- We cannot predict exactly where the mass will be after some time
- Best compromise: Standard Quantum Limit





Is SQL currently relevant?

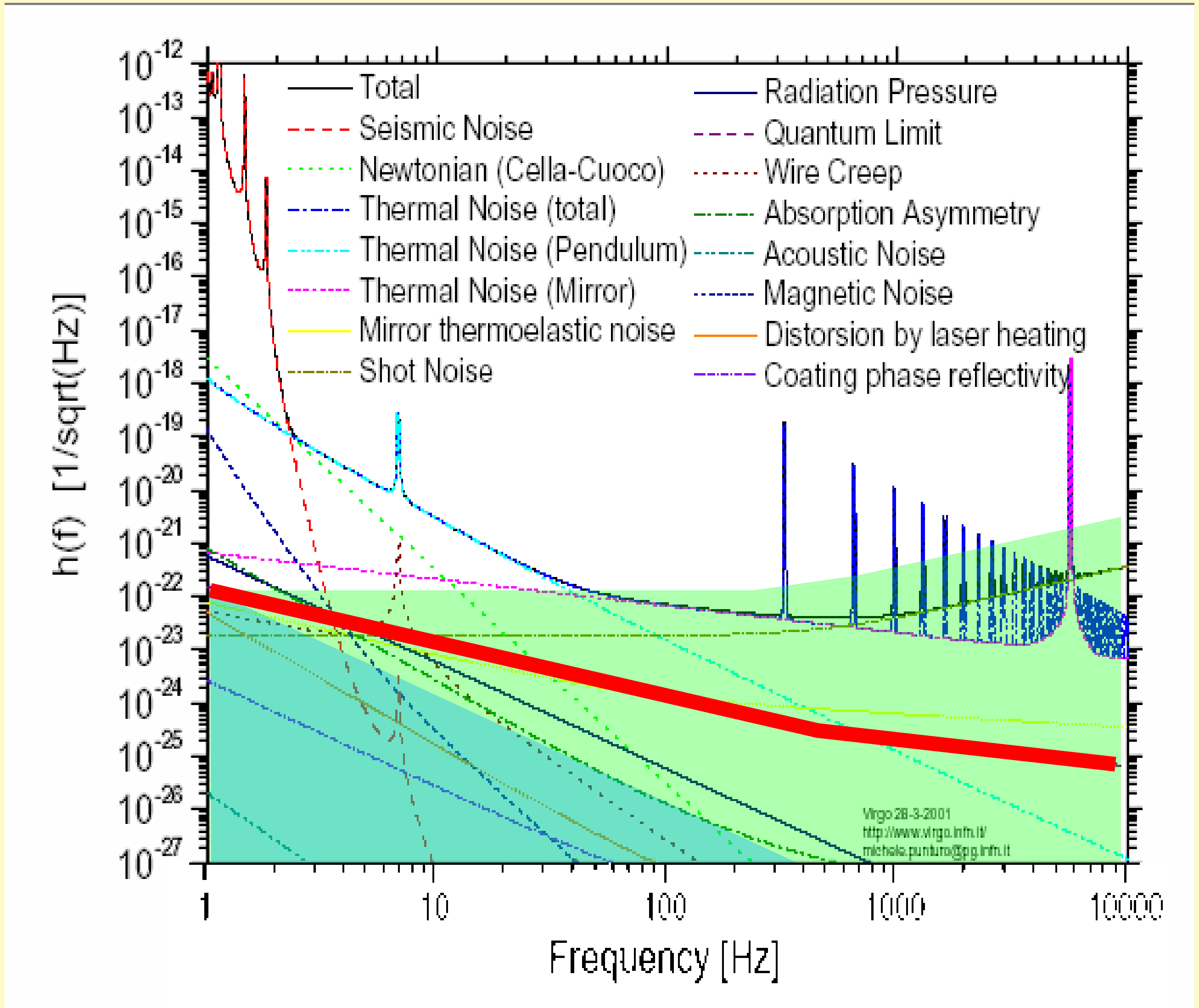
$$S_h = \frac{2}{\omega L}\sqrt{\frac{\hbar}{m}}$$

Shot noise

$$S_h = \frac{\lambda}{4\pi L F}\sqrt{\frac{h\nu\left[1+\left(\frac{F\omega L}{c}\right)^2\right]}{W}}$$

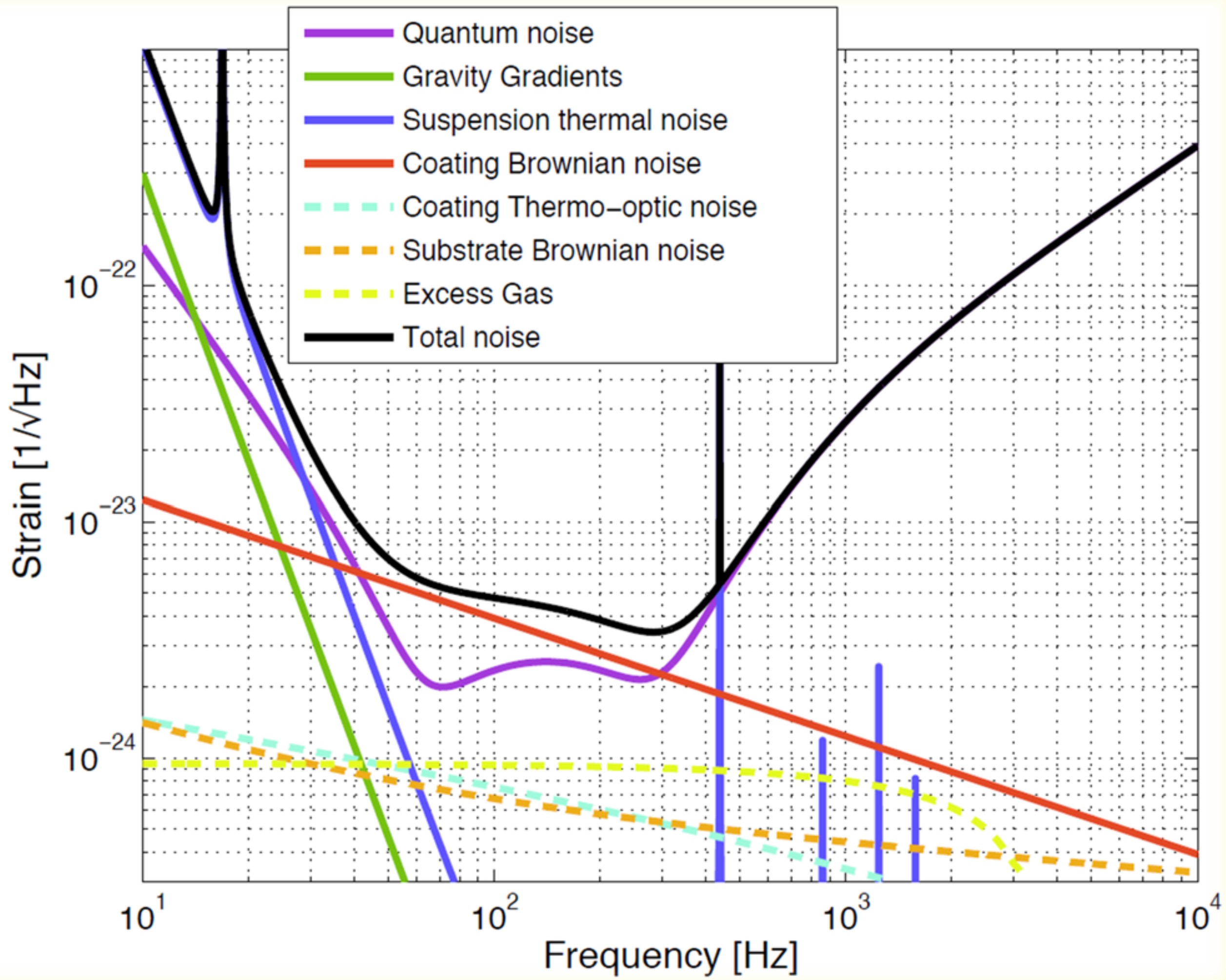
Radiation pressure

$$S_h = \frac{F}{LcM\omega^2}\sqrt{\frac{Wh\nu}{1+\left(\frac{F\omega L}{c}\right)^2}}$$



Current and future interferometers

- QN will be one of the main noise sources
- Design sensitivity of Advanced LIGO
- Fundamental problem for third generation detectors (Einstein Telescope)

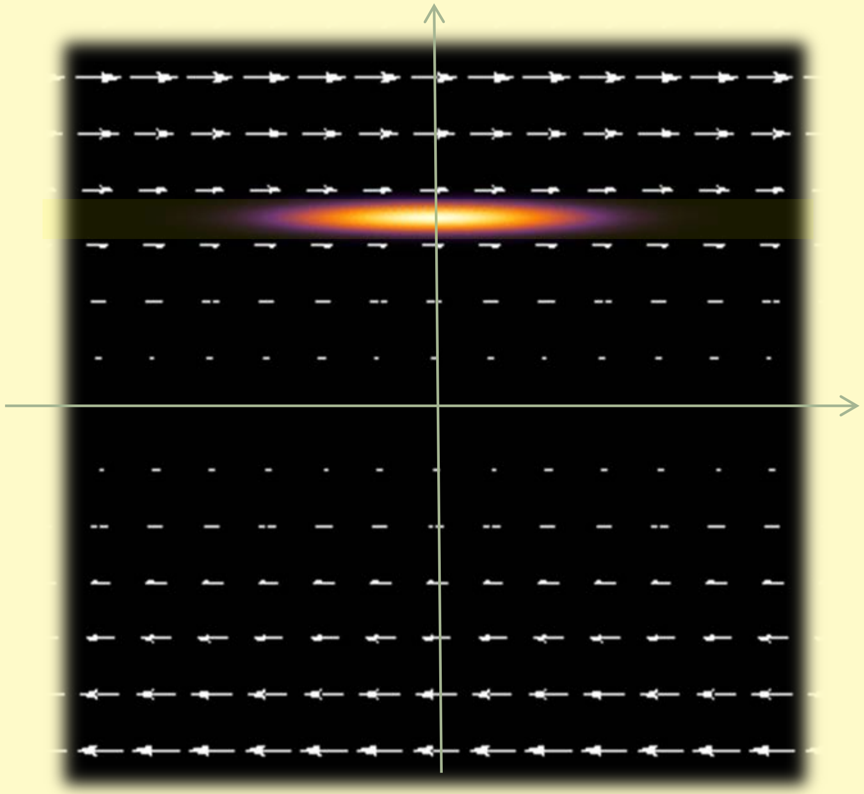


Can the SQL be evaded?

The SQL is not a fundamental limit:

- We are NOT really interested in measuring the positions of the mirrors
- Instead, we are interested in monitoring the (classical) GW strain which act on them

Simple example: velocity measurement



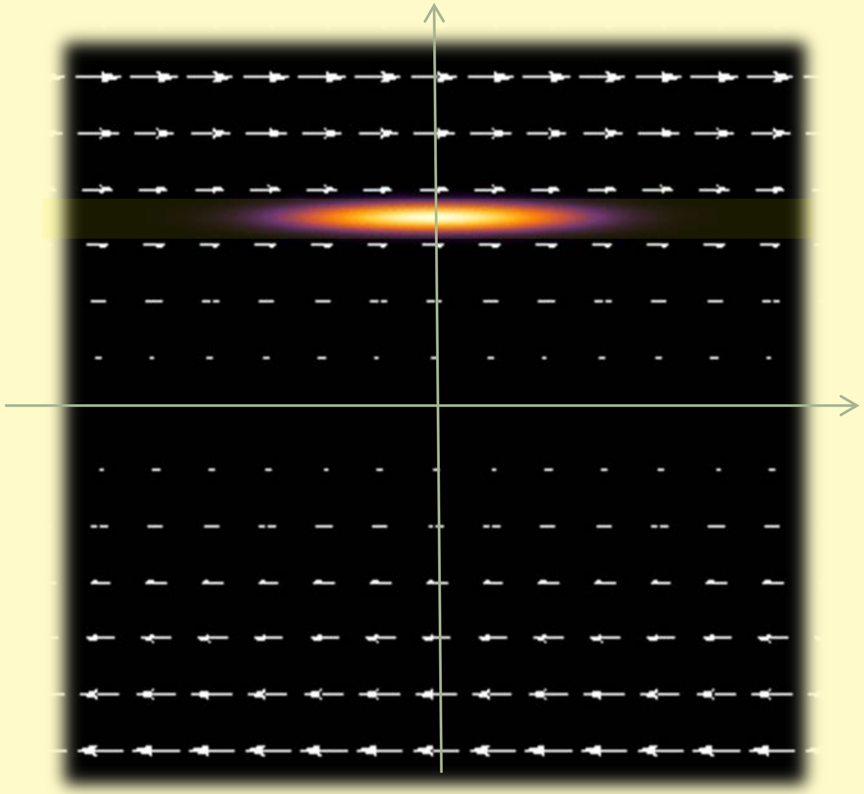
- No back action of the measurement of the velocity
- More generally, there is no SQL for a conserved quantity
- Can we do this?

Can the SQL be evaded?

The SQL is not a fundamental limit:

- We are NOT really interested in measuring the positions of the mirrors
- Instead, we are interested in monitoring the (classical) GW strain which act on them

Simple example: velocity measurement



- No back action of the measurement of the velocity
- More generally, there is no SQL for a conserved quantity
- Can we do this?

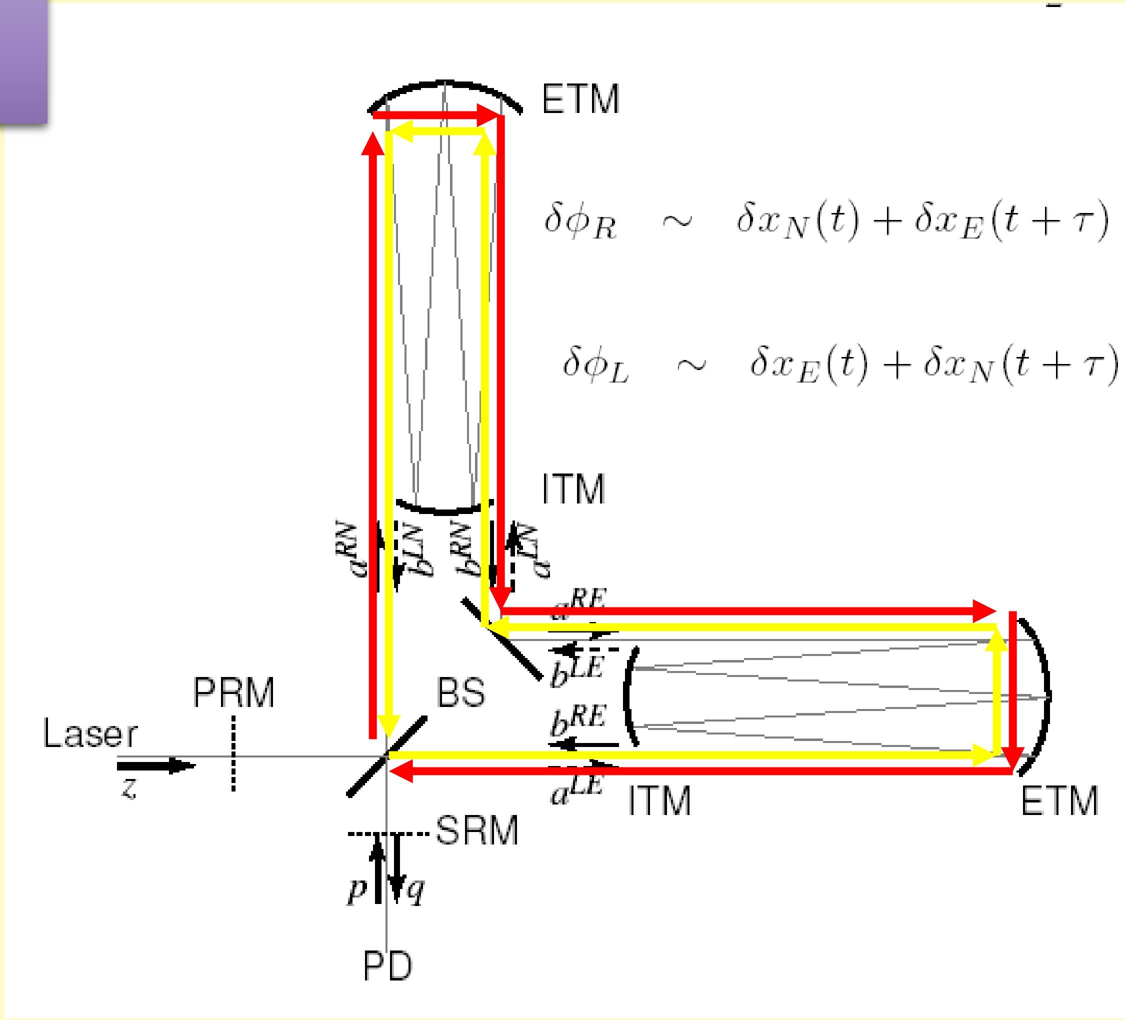
The Speed Meter

Sagnac interferometer: a way to apply these concepts directly.

Basic idea: two beams which follows the same path, in opposite direction.

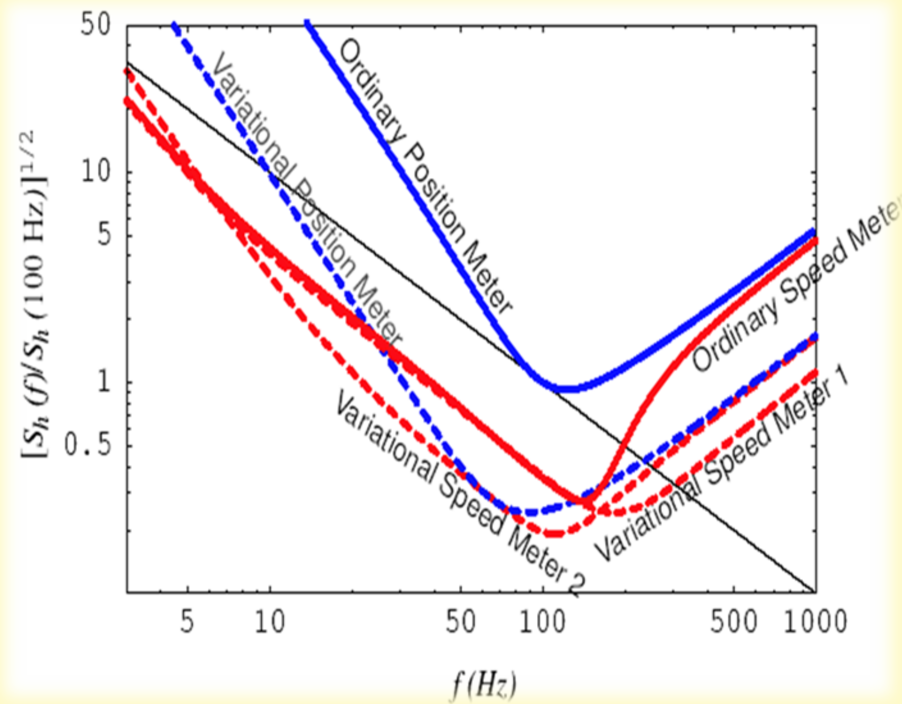
$$\delta\phi_R - \delta\phi_L \sim [\delta x_N(t) - \delta x_N(t+\tau)] - [\delta x_E(t) - \delta x_E(t+\tau)]$$

- If $\Omega \ll \tau^{-1}$ this is a measurement of mirrors' velocity.
- We should not be limited by any QN!

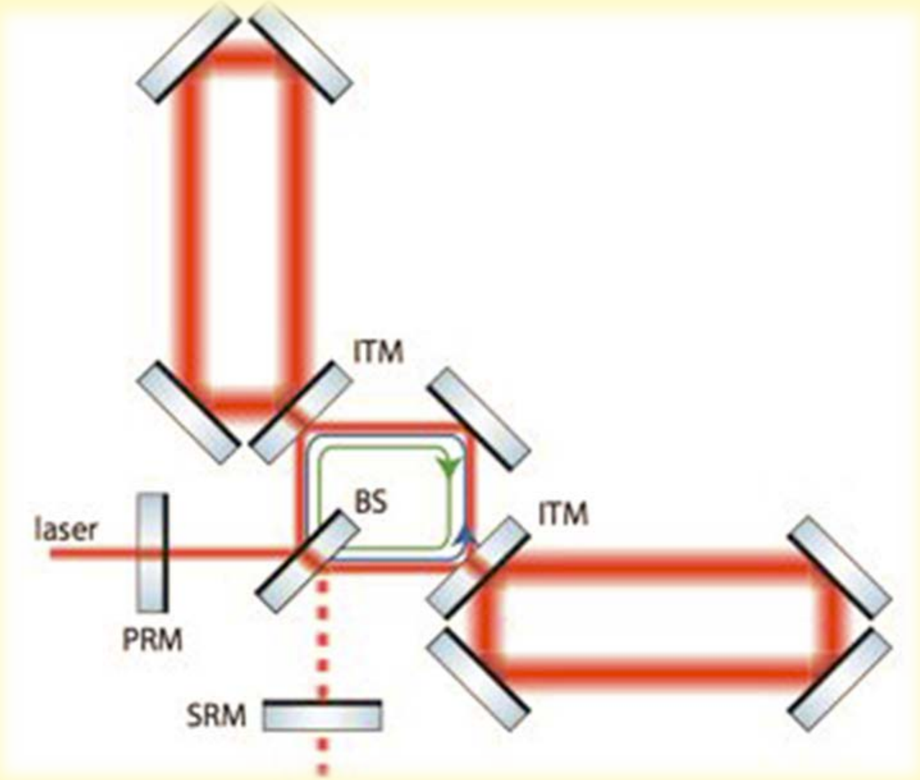


P. Purdue and Y. Chen, Phys. Rev. D ***66*** *(2002)*

Speed meter (continued)



An efficient implementation uses resonant cavities.

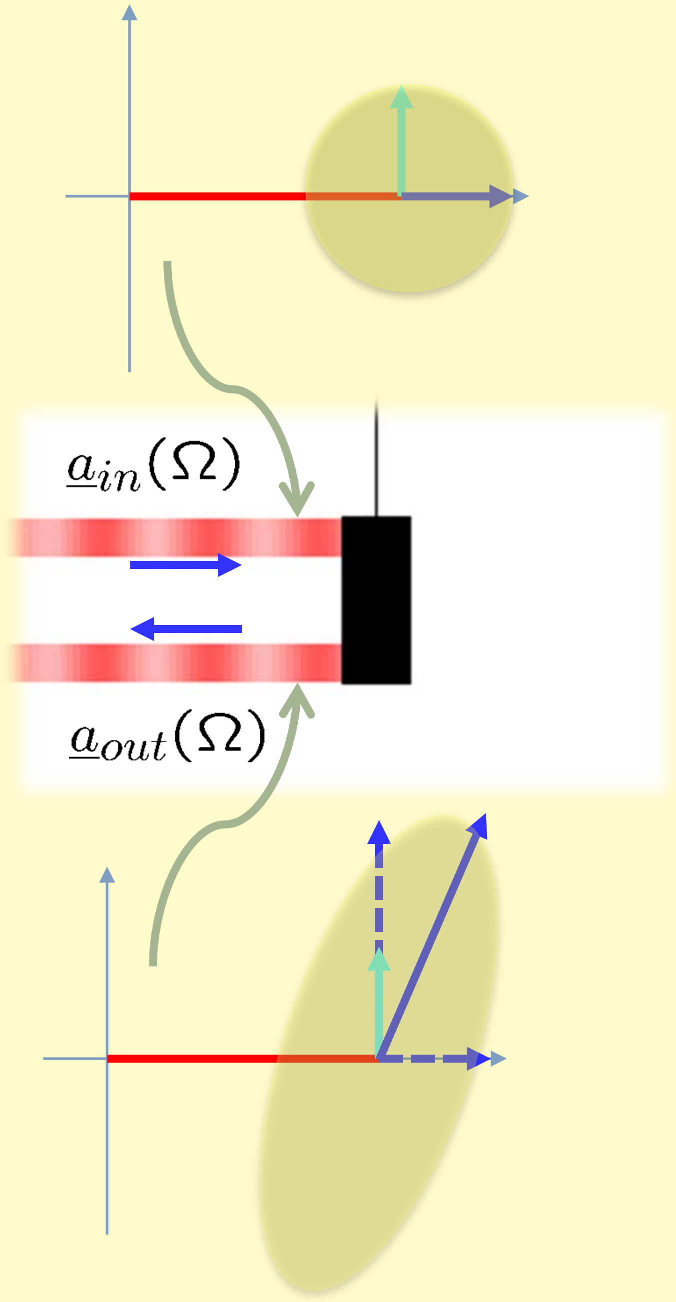


Can be used together with other techniques (talk about these in a moment...)

Image credit: H. Mueller-Ebhardt, PhD thesis

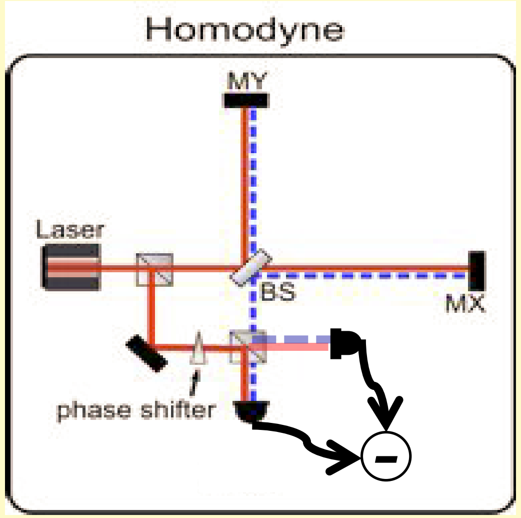
Ponderomotive squeezing

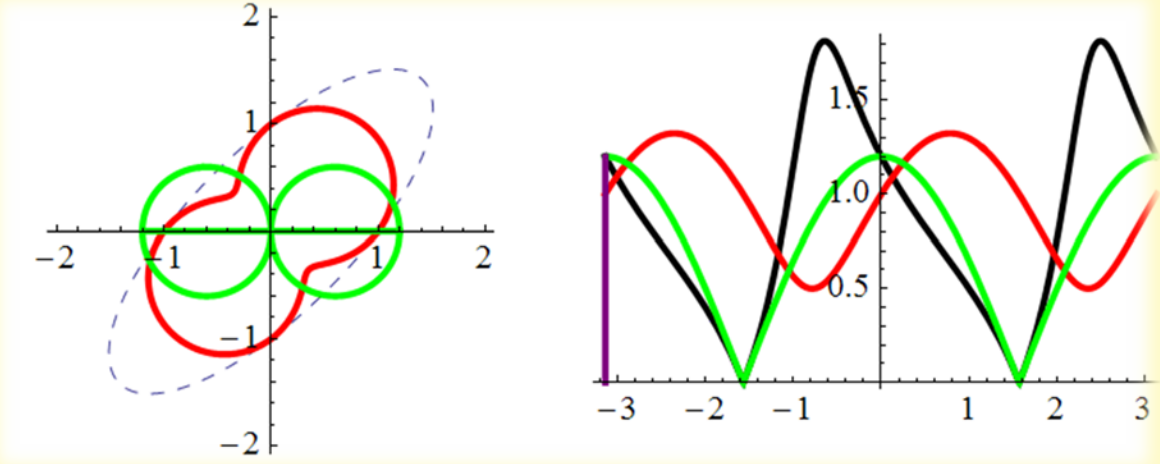
- In a coherent state phase and amplitude noise are uncorrelated
- Quasiprobability distribution: isotropic Gaussian
- Back action induced by radiation pressure generate a correlation between phase and amplitude fluctuations
- Fluctuations are increased along a direction, but decreased along another one: a squeezed state «Optomechanical Kerr effect»
- $\begin{pmatrix} \delta a_a \\ \delta a_p \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -\frac{2 I_\ell \omega_\ell}{m \Omega^2 c^2} & 1 \end{pmatrix} \begin{pmatrix} \delta a_a \\ \delta a_p \end{pmatrix}_{in}$



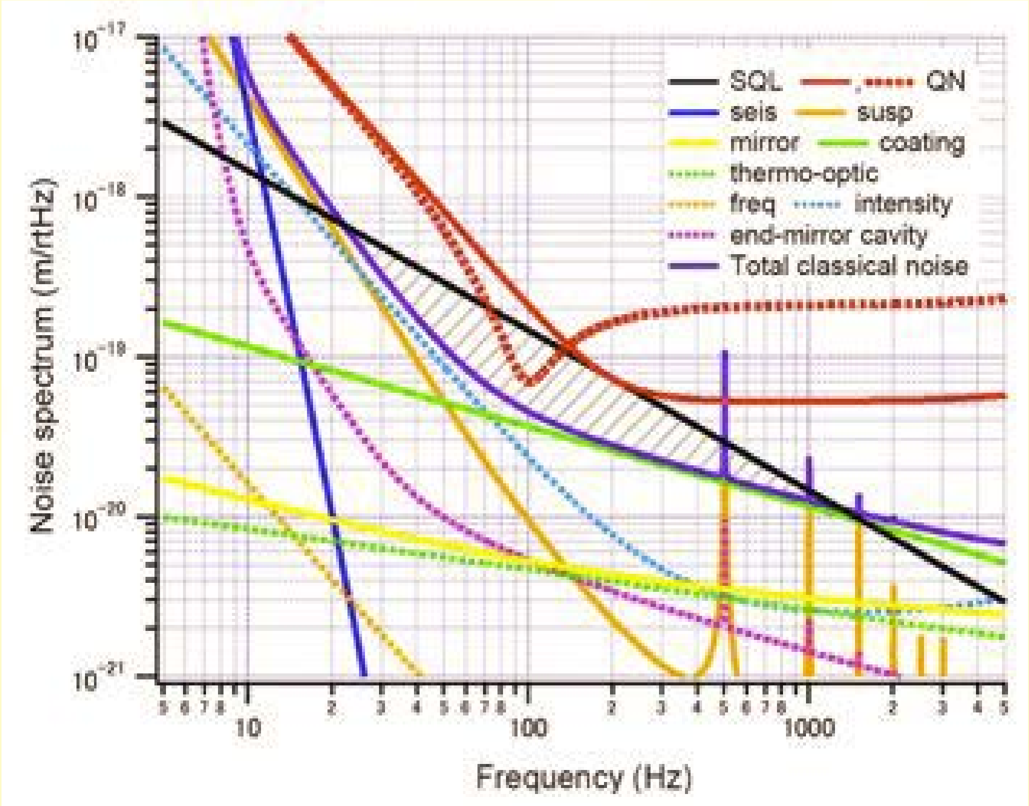
Reading the optimal quadrature

- The signal is encoded in the phase quadrature
- Without squeezing, this would be the optimal quantity to measure
- With squeezing, this is not so



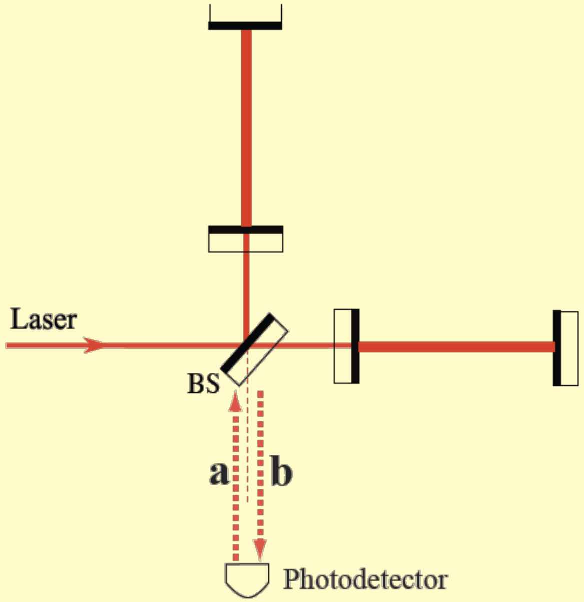


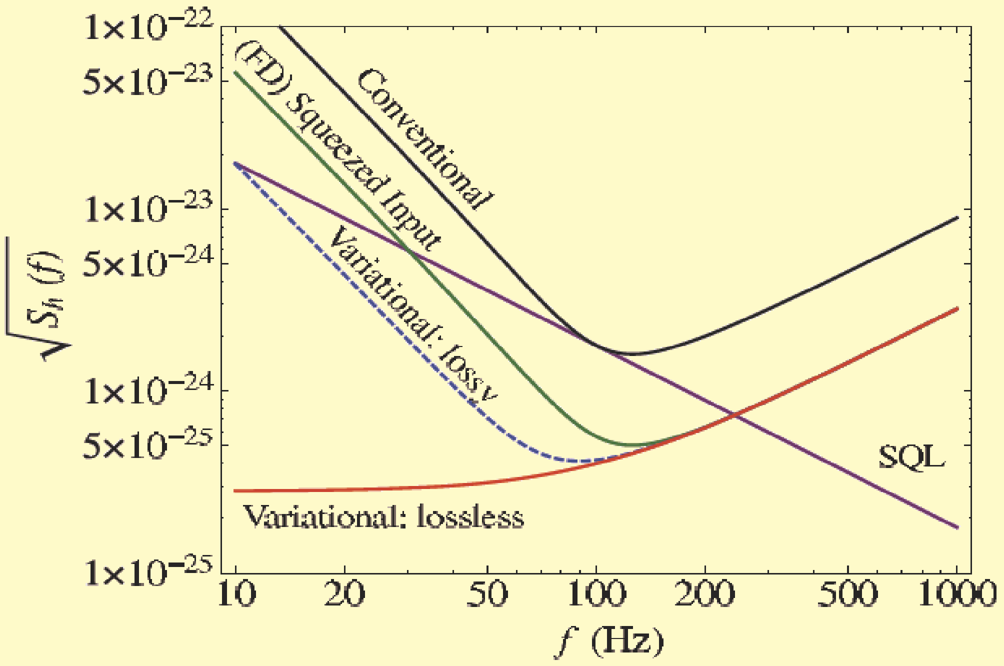
Using variational readout we could competely cancel the radiation pressure noise at one frequency
To cancel the radiation pressure noise at all frequencies, we would need a frequency dependent angle of readout quadrature



Injecting a squeezed vacuum

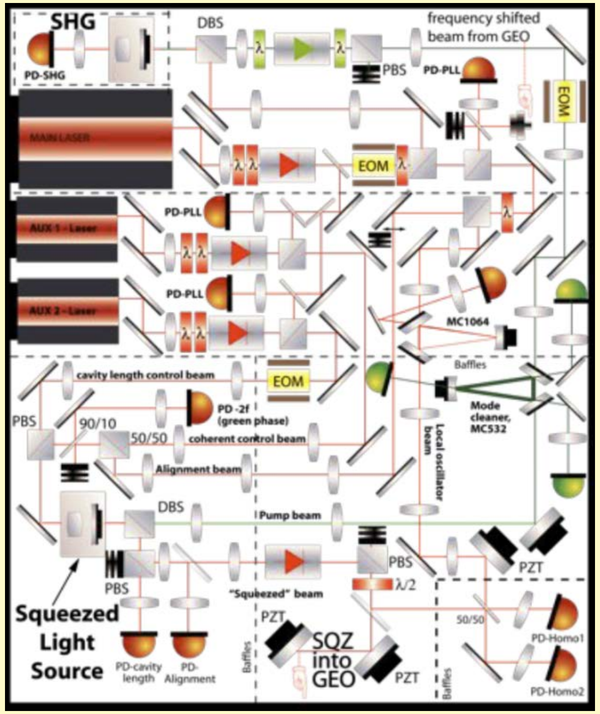
- Quantum fluctuations enter the interferometer through the dark port
- We can inject an electromagnetic mode in a squeezed vacuum state
- If the squeezing angle is appropriately choosen SQL can be evaded
- Once again, optimal squeezing angle is frequency dependent

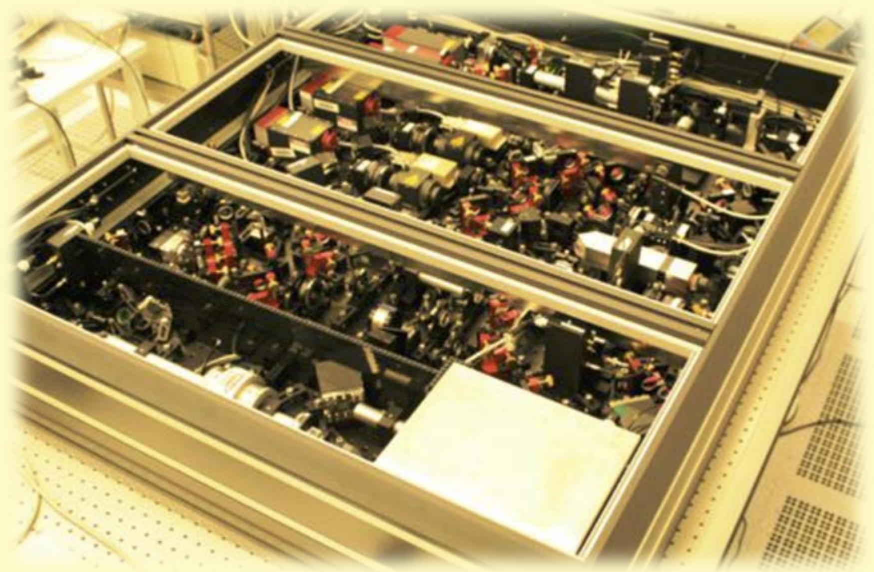




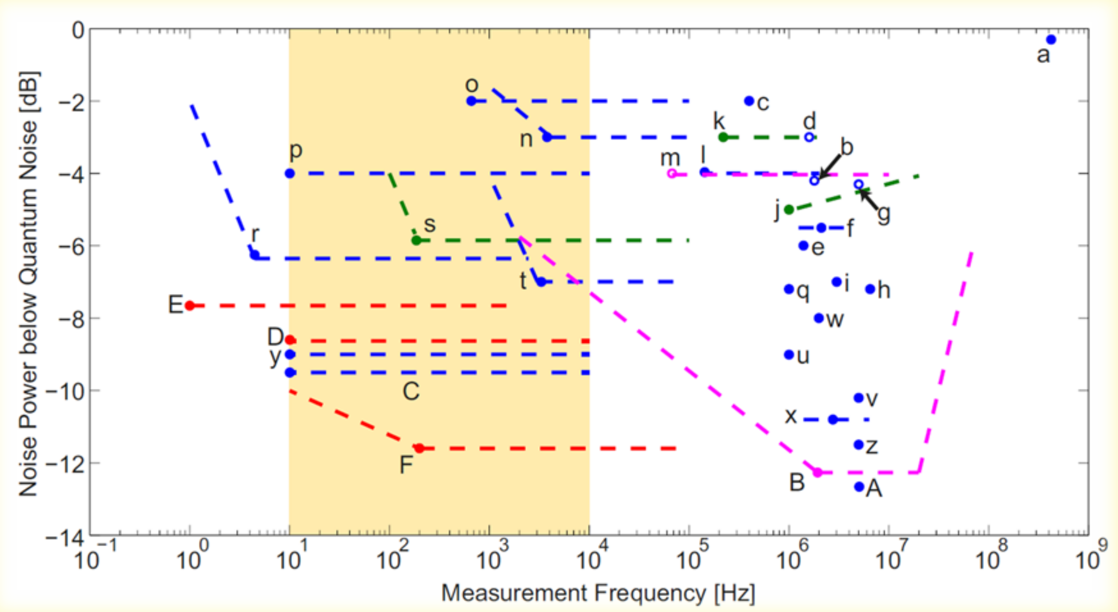
Kimble H J, Levin Y, Matsko A B, Thorne K S and Vyatchanin S P 2002 Conversion of conventional gravitational-wave interferometers into quantum nondemolition interferometers by modifying their input and/or output optics *Phys. Rev.* D **65** 02202

Squeezing vacuum generation





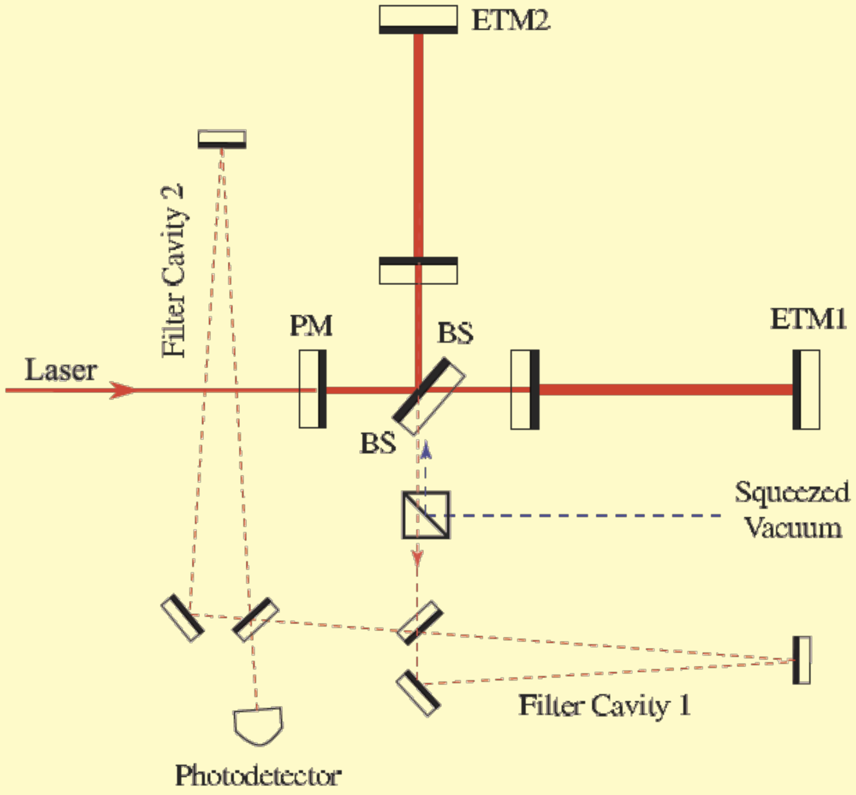
- Standard way: nonlinear crystals and optical parametric amplification
- Over past decade, squeezing made incredible progresses
- Squeezing at low frequencies (as low as 1Hz)
- Squeezing factor 10dB (QN reduction by a factor 3)

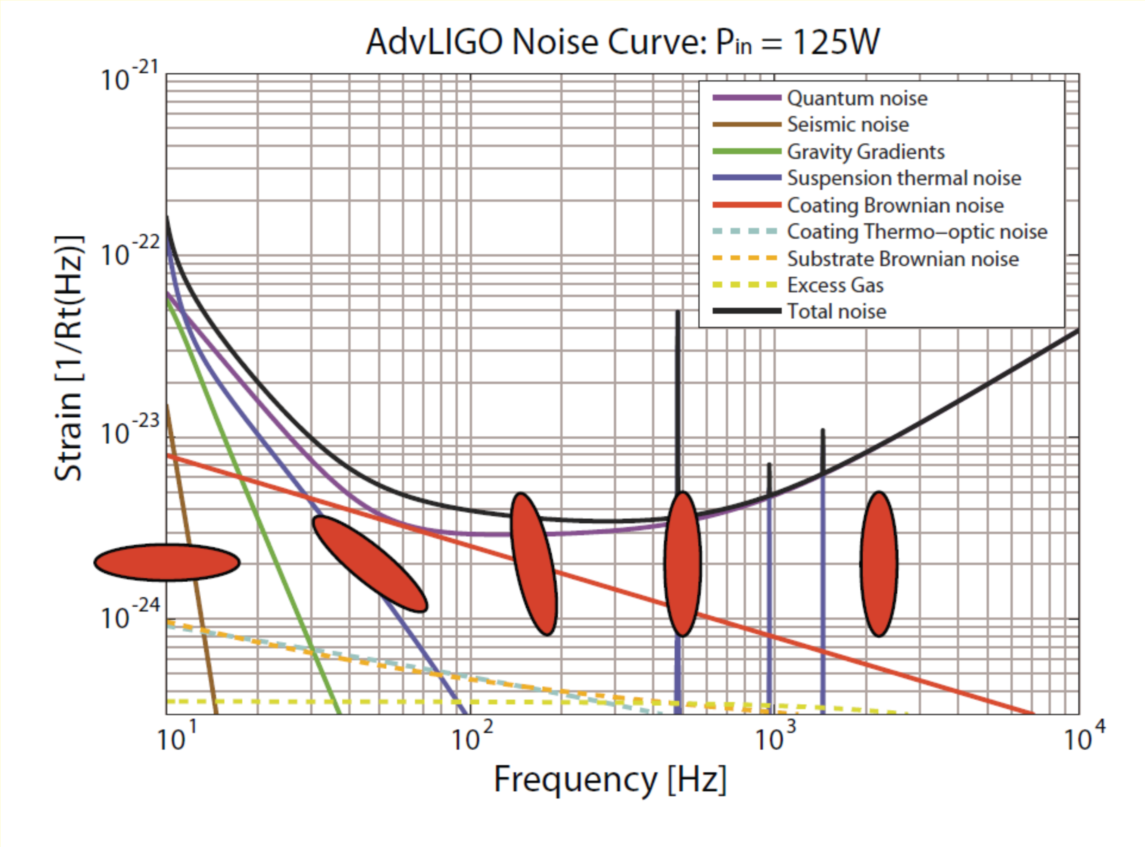


Courtesy: S.Y.Chua, Ph.D. Thesis (2013)

Frequency dependent squeezing angle

- Amplitude fluctuations should be reduced at low frequency
- Phase fluctuations should be reduced at higher frequencies
- The transition bandwidth should be of the order of 100 Hz
- It is possible to use optical cavities as filters

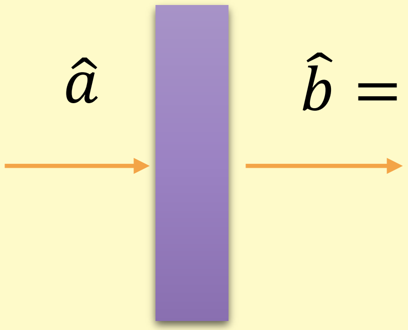




... but they should be large

- noisy
- expensive

The role of losses

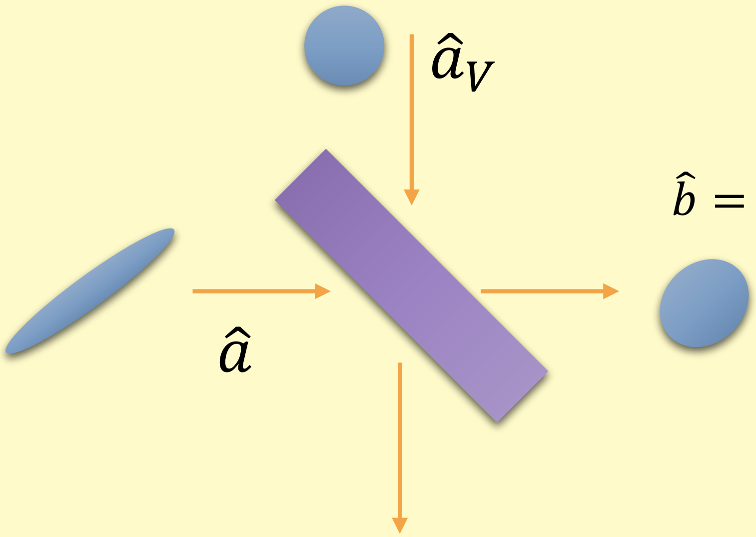


$\hat{b} = \sqrt{\eta}\hat{a}$

Naïve model: not consistent with unitarity: commutation rules are not preserved.

$$[\hat{a}, \hat{a}^{+}] = [\hat{b}, \hat{b}^{+}] = 1$$

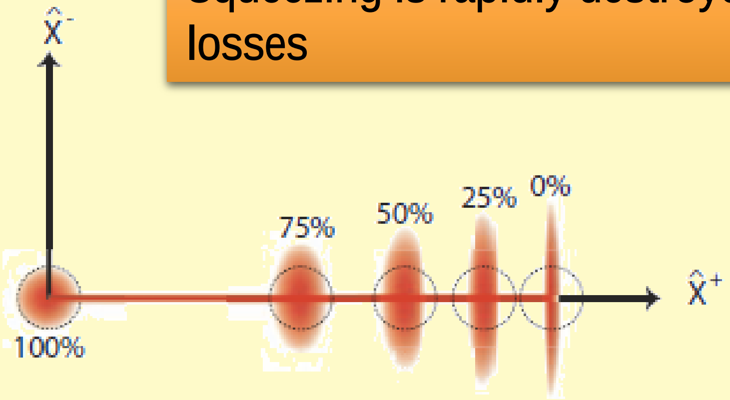
A consistent model:



$\hat{b} = \sqrt{\eta}\hat{a} + \sqrt{1-\eta}\,\hat{a}_V$

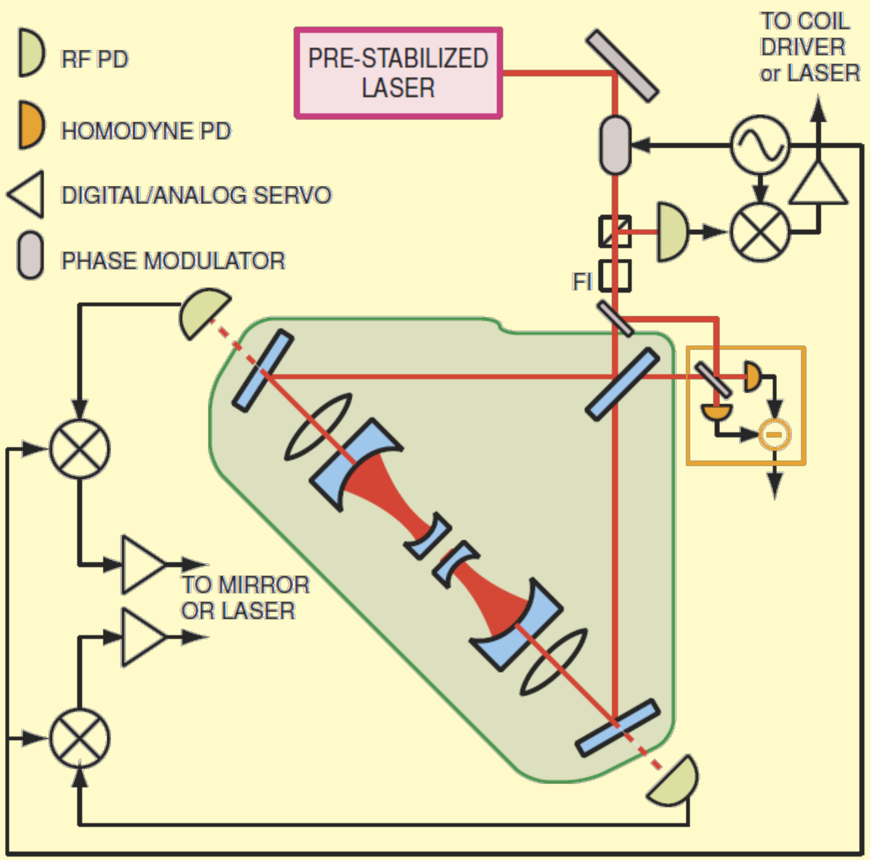
$$\Delta q^2 = e_s^{-2r} \rightarrow \eta e^{-2r} + (1-\eta)$$

Squeezing is rapidly destroyed by losses



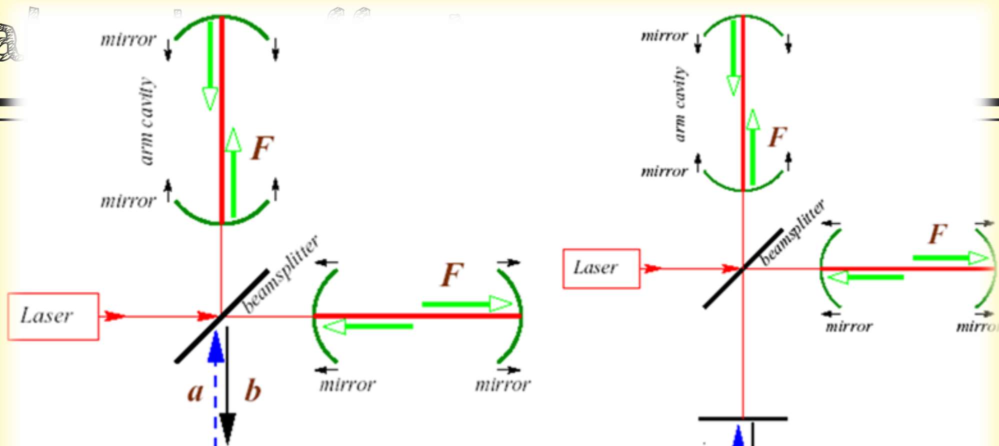
Ponderomotive squeezing generation

- It is possible to test directly ponderomotive squeezing?
  - Losses
  - Thermal noise
  - Light mirrors
- Several attempts in progress
- Two cavities scheme to cancel out laser noise
- Work in progress…..

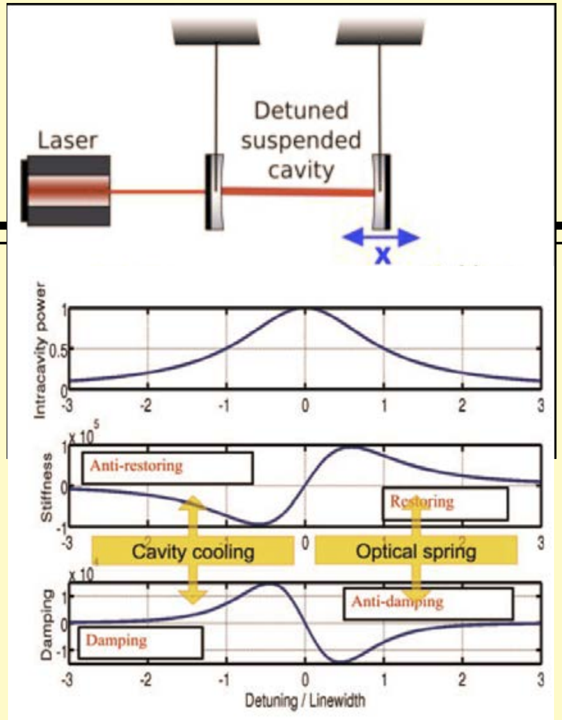


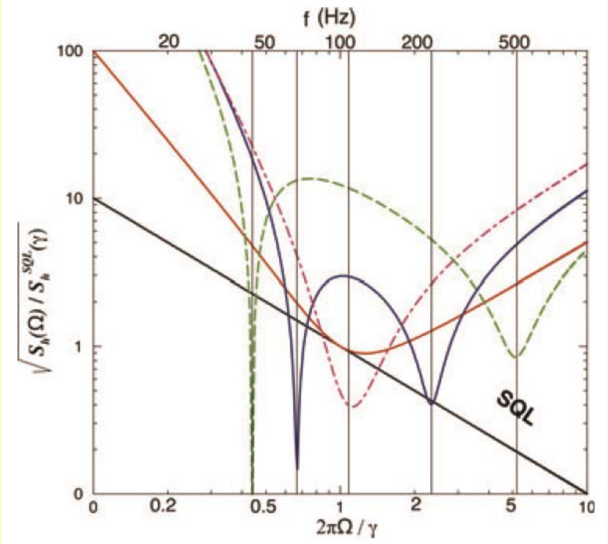
Squeezed-state source using radiation-pressure-induced rigidity
Phys. Rev. A 73, 023801 – Thomas Corbitt, Yanbei Chen, Farid Khalili, David Ottaway, Sergey Vyatchanin, Stan Whitcomb, and Nergis Mavalvala

Optica



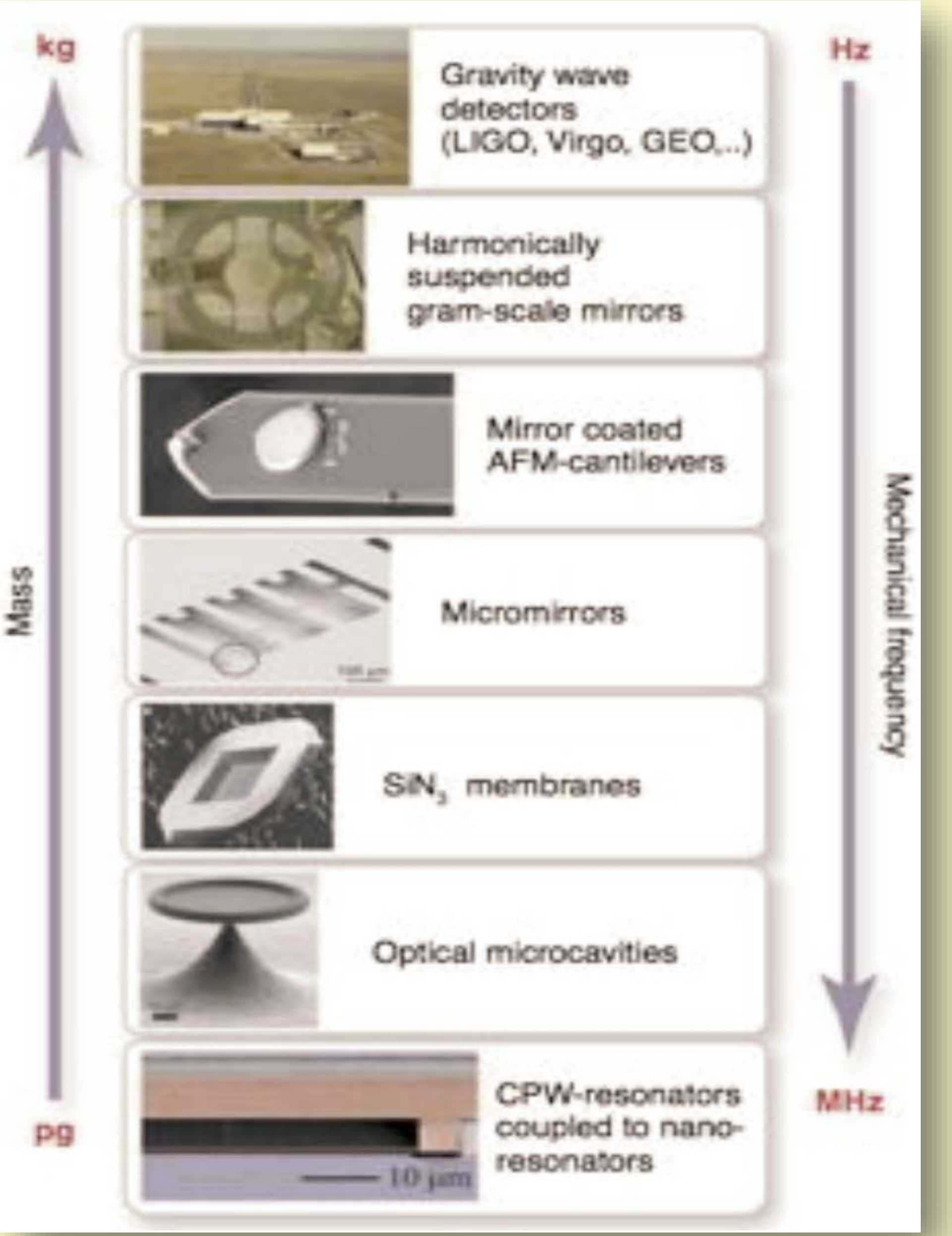
Buonanno and Chen, Phys. Rev. D ***65*** *(2002)*





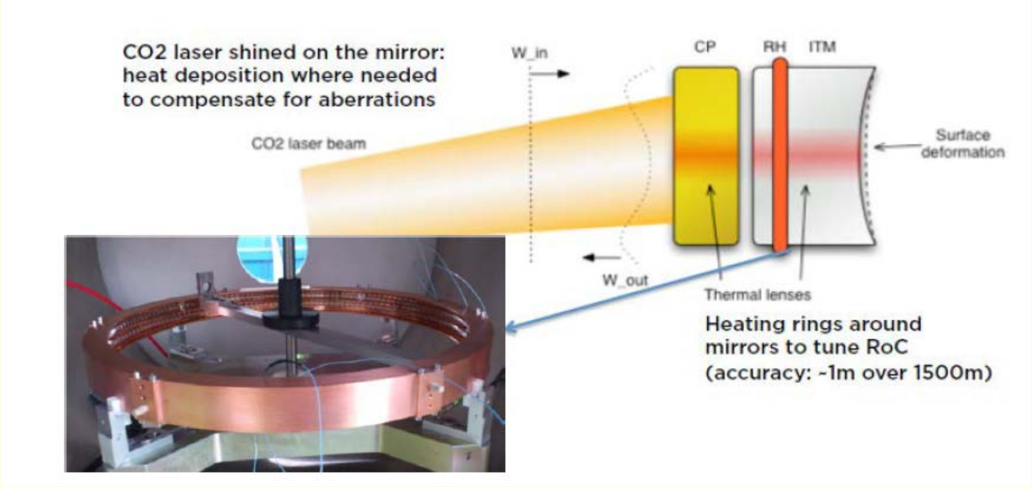
- With Signal Recycling back action is no more limited to a single quadrature:
- A linear restoring force is generated
- Optical and mechanical modes are coupled.
- The system is unstable. A control feedback must be introduced.

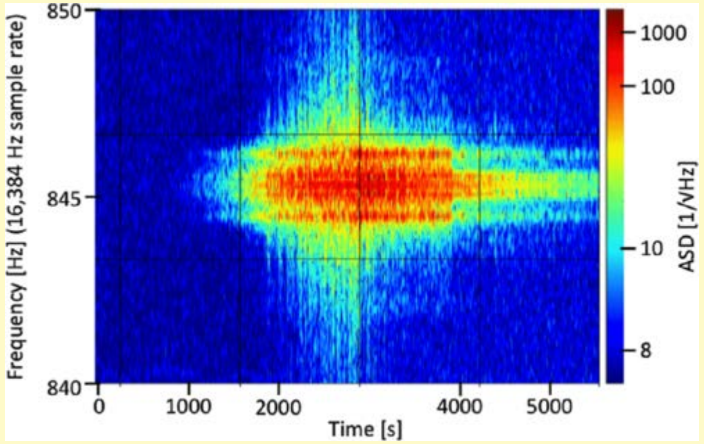
- **Measurement of weak forces require quantum-limited sensors**, i.e., working at the sensitivity limits imposed by Heisenberg uncertainty principle
- SQL is not a fundamental limit for GW detection
- Several proposals to evade SQL, starting to be tested (this is very far from a complete review)
- Exploring the boundary between micro and macro world
- Very rich phenomenology (again, touched very marginally here)

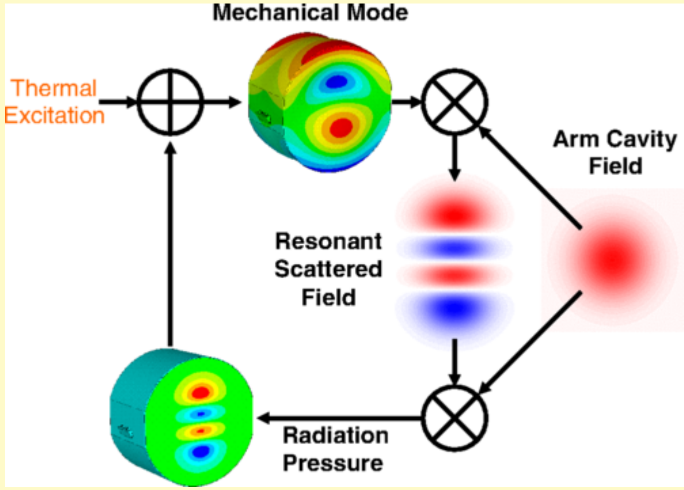


High power lasers

- Brute force approach to reduce shot noise
- With squeezing, in principle an handle to reduce optical noise at will
- Hower, there is not a free lunch:
  - Thermal lensing effects
  - Thermo-optic noise
  - Parametric instabilities



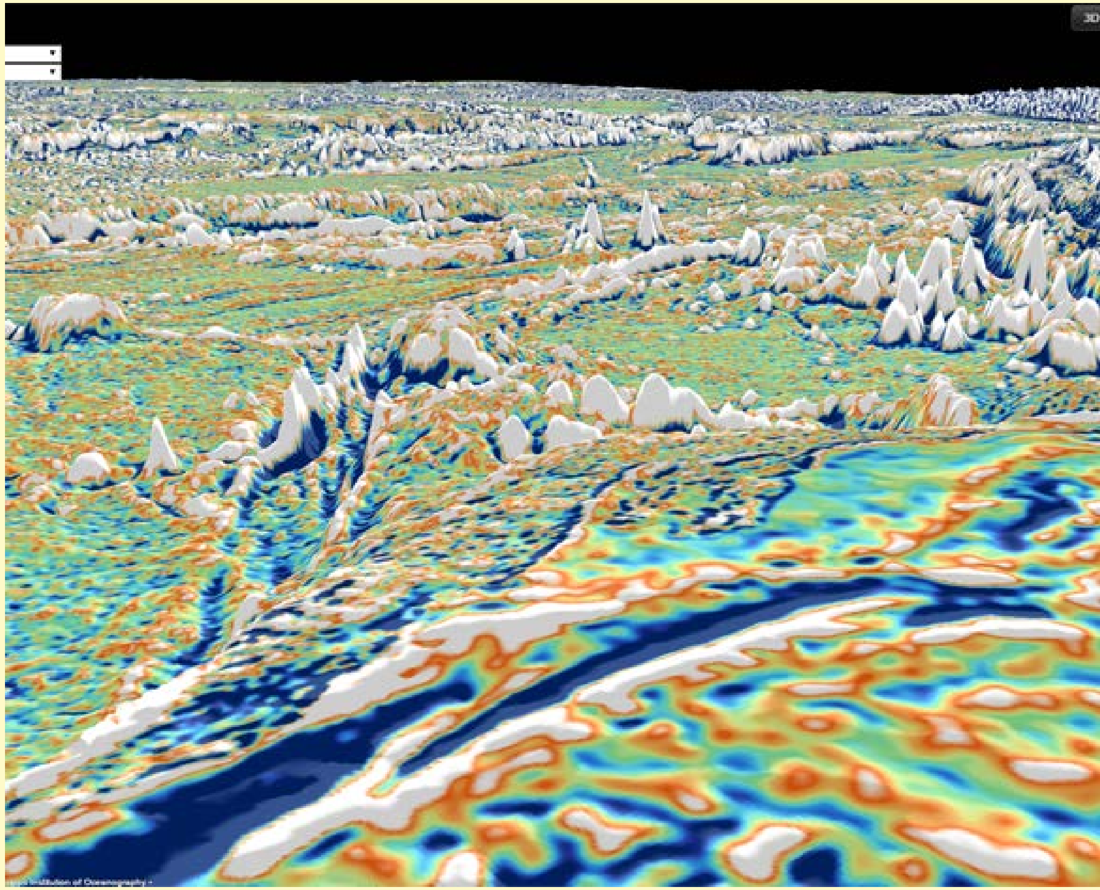




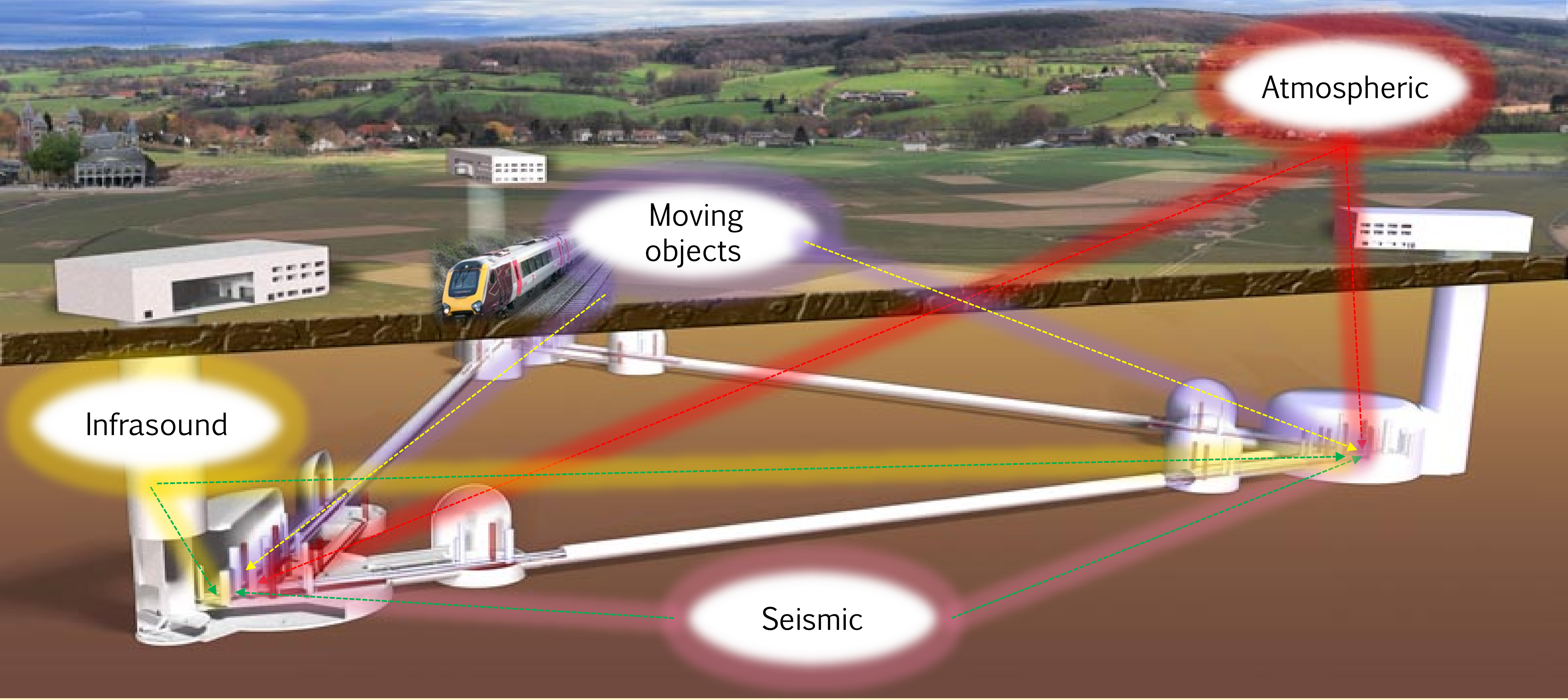
Evans et al., PRL 114, 161102 (2015)

FUNDAMENTAL NOISES

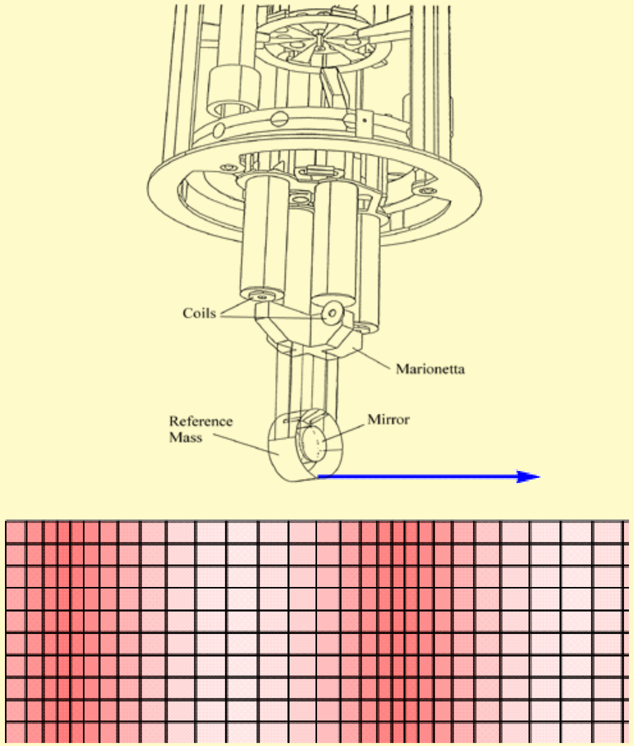
Gravity fluctuations



GGN: direct coupling between environmental fluctuation of mass density and test masses



Seismic Gravity Gradient Noise



$$\rho_0(x)\vec{\nabla}\cdot\vec{u}(x,t) \neq 0$$

Compressional effects

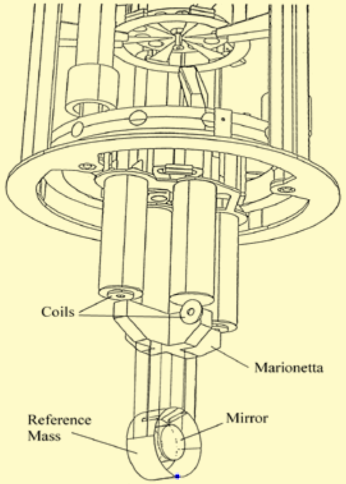
- Mass density fluctuates.....

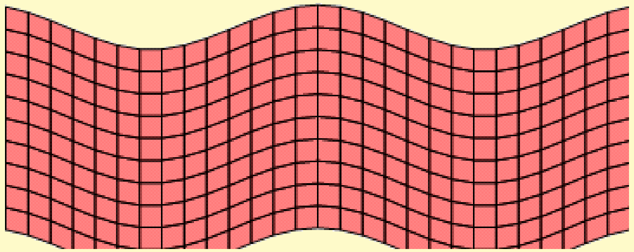
$$\rho(x,t) = \rho_0(x) + \delta\rho(x,t)$$

- ..... and generates fluctuations of gravitational field

$$\delta\Phi(x,t) = G\int \frac{\vec{\nabla}\cdot[\rho_0(x')\vec{u}(x',t)]}{|x-x'|}d^3x'$$

Seismic Gravity Gradient Noise





$$\vec{u}(x,t)\cdot\vec{\nabla}\rho_0(x)\neq 0$$

Dragging effects

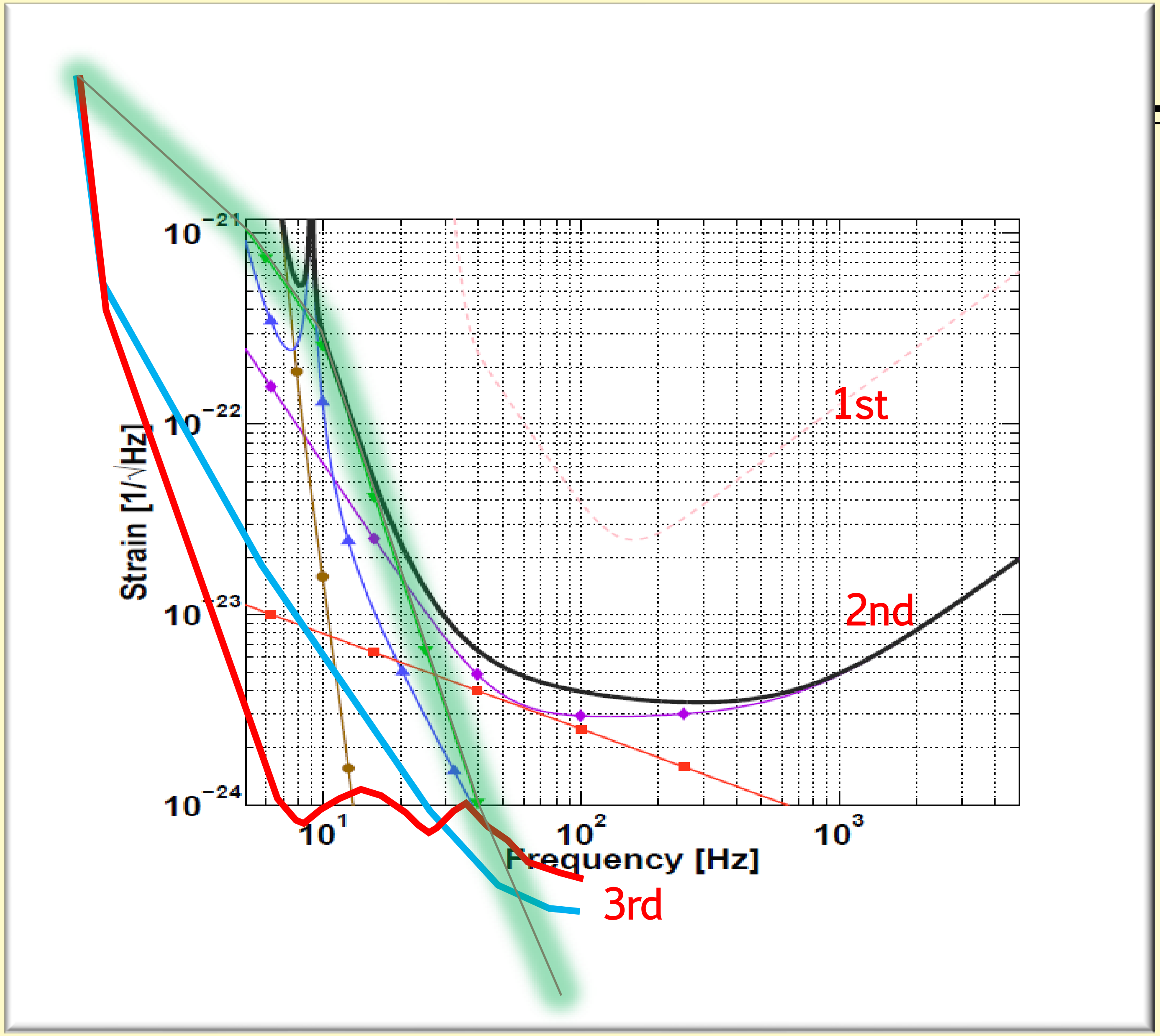
- Mass density fluctuates.....

$$\rho(x,t)=\rho_0(x)+\delta\rho(x,t)$$

- ..... and generates fluctuations of gravitational field

$$\delta\Phi(x,t)=G\int\frac{\vec{\nabla}\cdot[\rho_0(x')\vec{u}(x',t)]}{|x-x'|}d^3x'$$

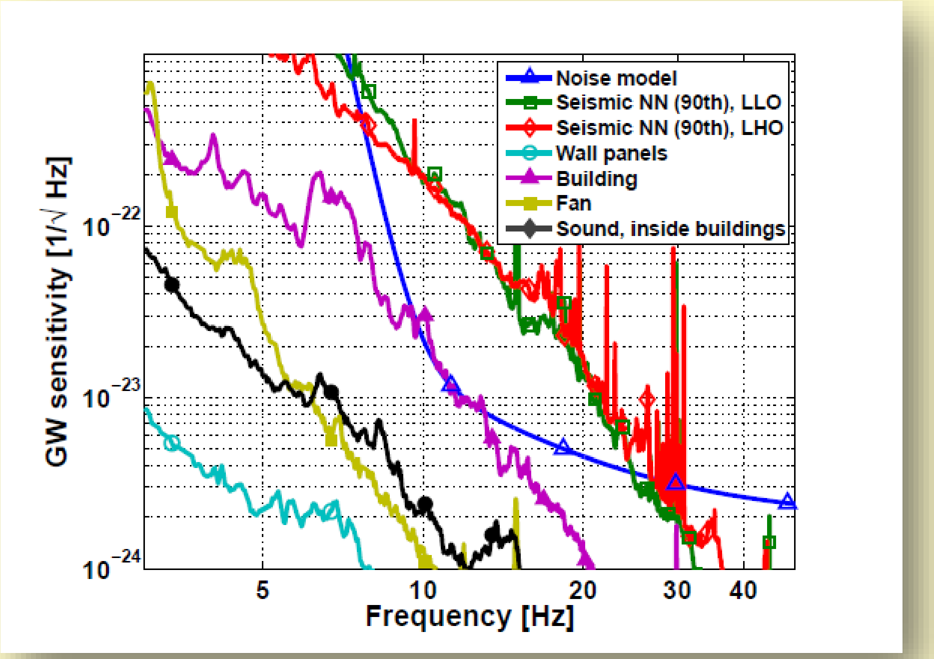
Relevance



GGN equivalent strain

ET-B sensitivity

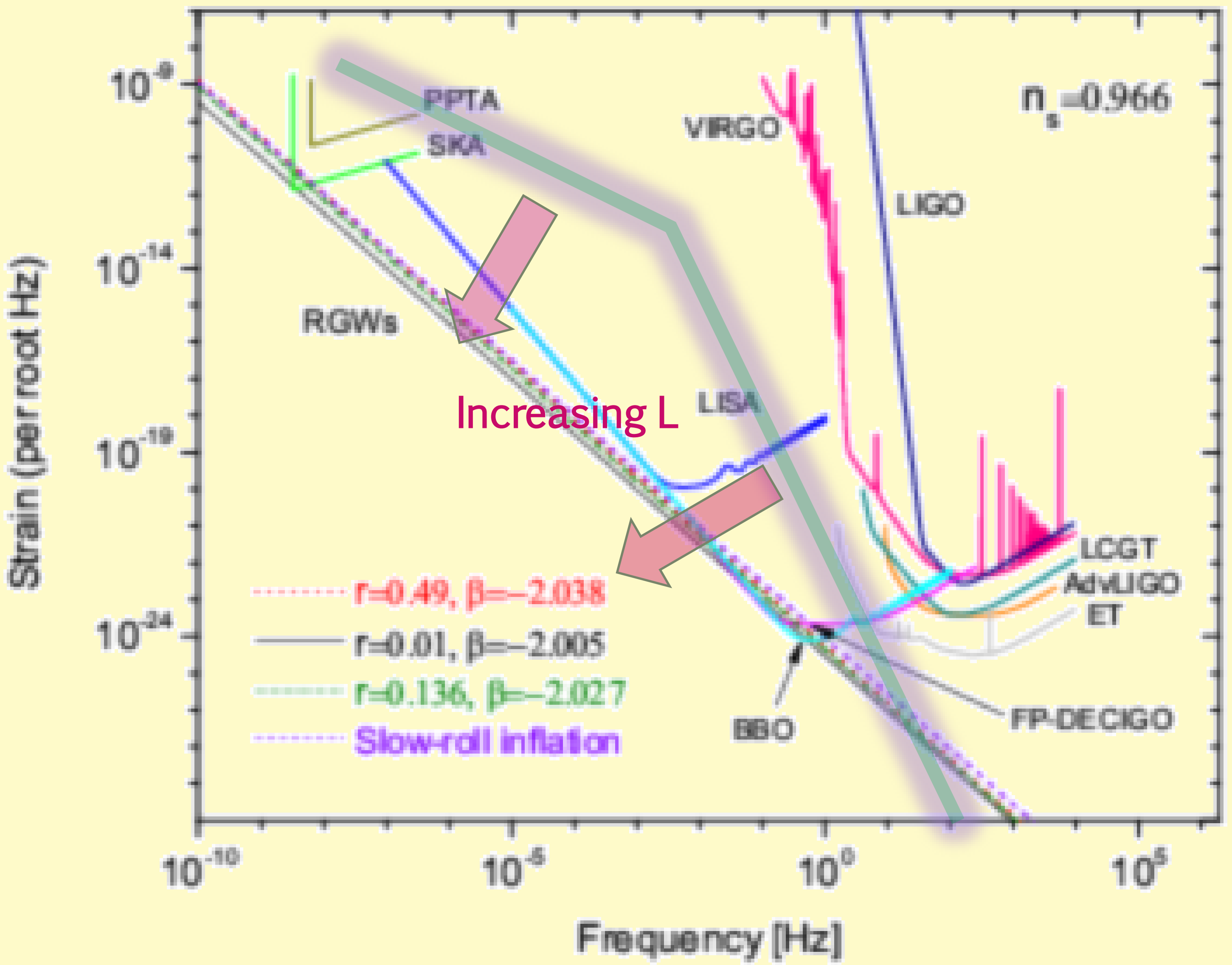
ET-C sensitivity



From: Subtraction of Newtonian noise using optimized sensor arrays Jennifer C. Driggers, Jan Harms, and Rana X. Adhikari Phys. Rev. D 86, 102001

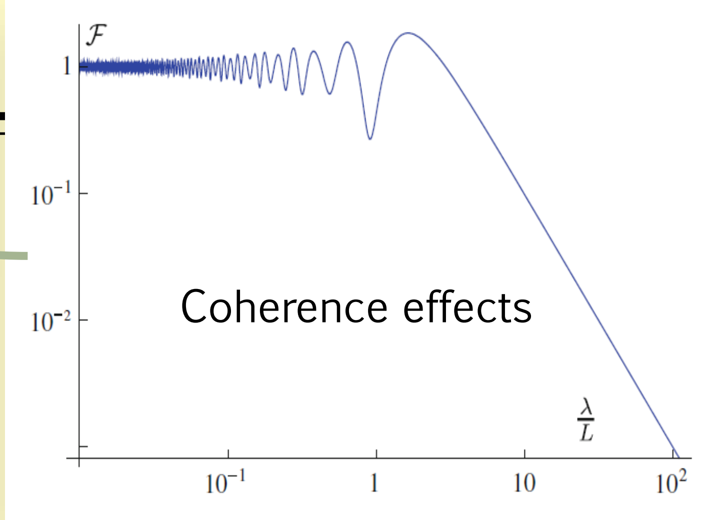
Extending the lower frequency limit

- GGN is a low frequency fundamental limit for the sensitivity of earth bound detectors
- When non mitigated by coherence effects $S^{1/2}_{\mathrm{h},GGN} \propto f^{-4}$
- GGN strain equivalent noise decreases by increasing L
- Coherence effects are relevant for $f < v_s/L$.
- The detailed shape of the seismic spectrum should be taken into account



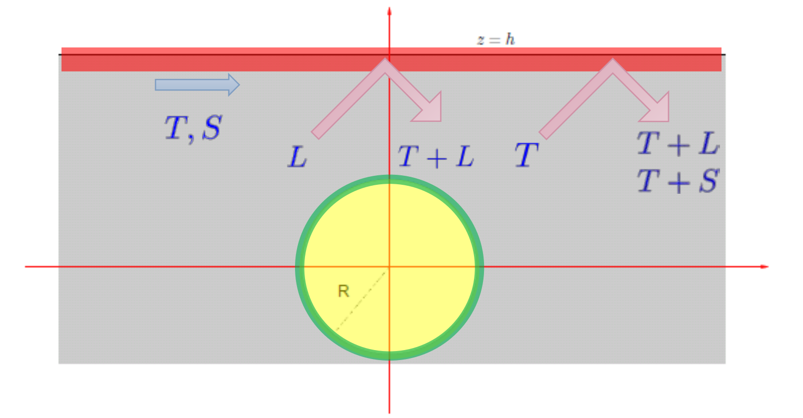
Gravity Gradient Noise: modelization and estimation

$$S_h^{NN}(\omega) = \frac{4\pi G^2 \rho_0^2}{L^2\omega^4} \int \mathcal{N}^2 S_K^g(\omega) \left[2e^{\beta_L K z} - \frac{1+2\beta_L+\beta_T^2}{\beta_T^2+1} e^{Kz}\right]^2 \mathcal{F}(KL) K dK$$



$$K S_K^g(\omega) = \frac{2\pi}{\mathcal{N}^2 \beta_L^2} \left(\frac{\beta_T^2+1}{\beta_T^2-1}\right)^2 \int C_{vv}^{seism}(r;\omega) J_0(Kr)\, r dr$$

Example: underground cavity with $\lambda \gg R$

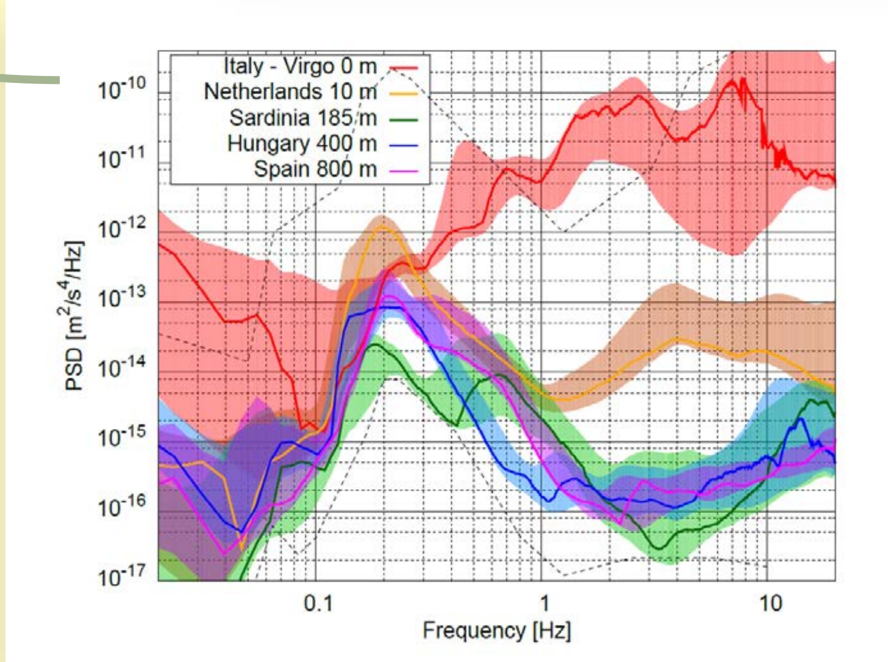


Bulk contribution

- Cavity contribution

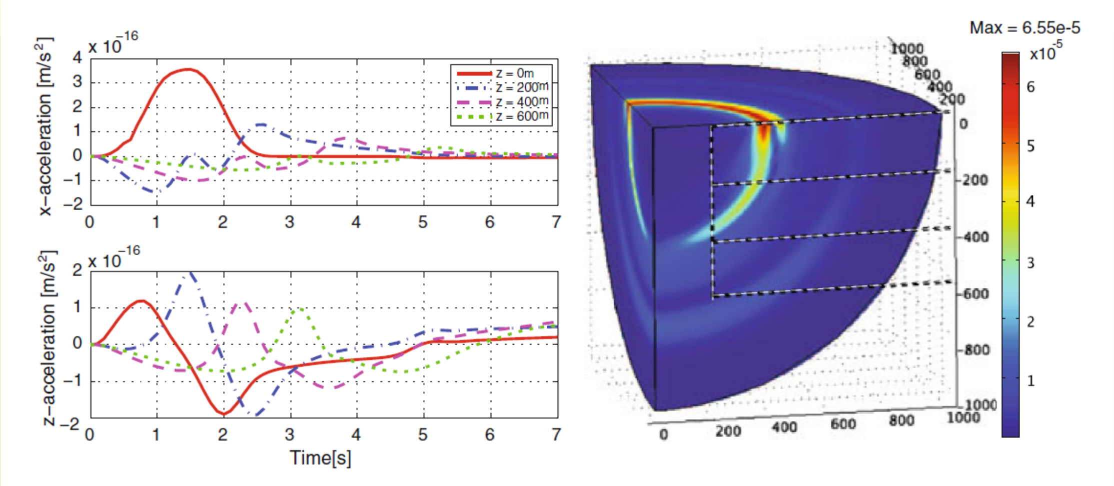
+ Surface contribution

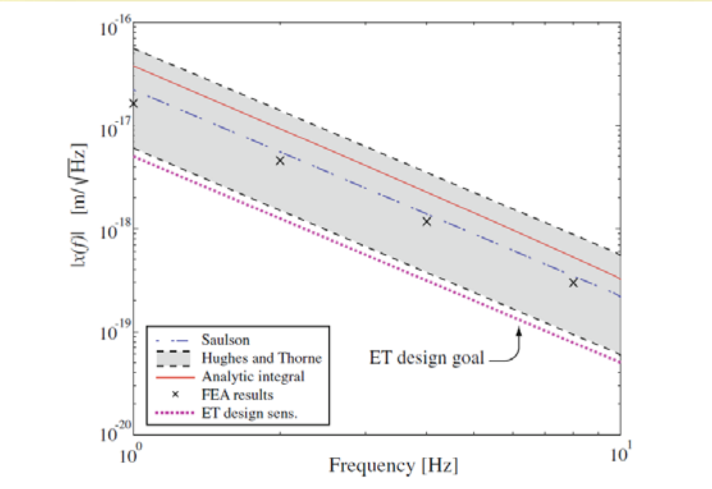
+ Cav. surface contr.

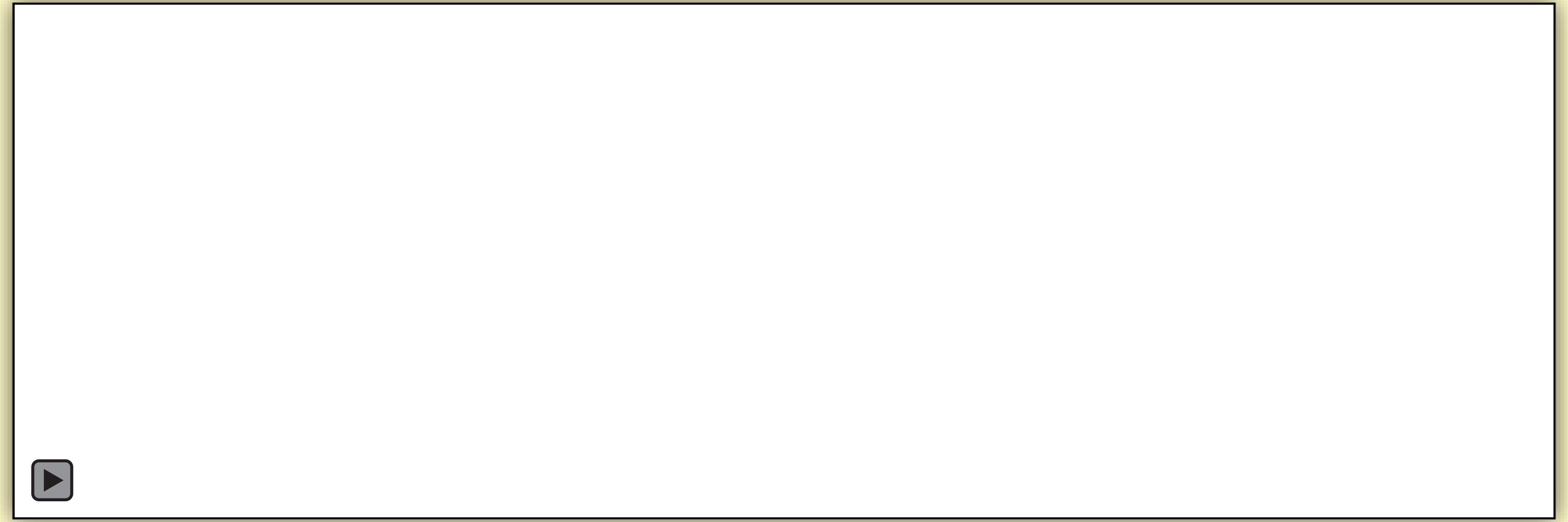


Seismic Gravity Gradient Noise: FE modelization

- Useful to deal with complex situations and/or complex excitations
  - Non trivial geometry/morphology (for example, anisotropy)
  - Infrastructure effects
  - Effect of localized sources (for example, once again, anisotropy)
  - Time domain (short scale non stationarity)
- Can require a large computational power, in particular if the dynamics must be simulated by them
- Need validation: comparison with analytical estimates is important (when an overlap is possible)

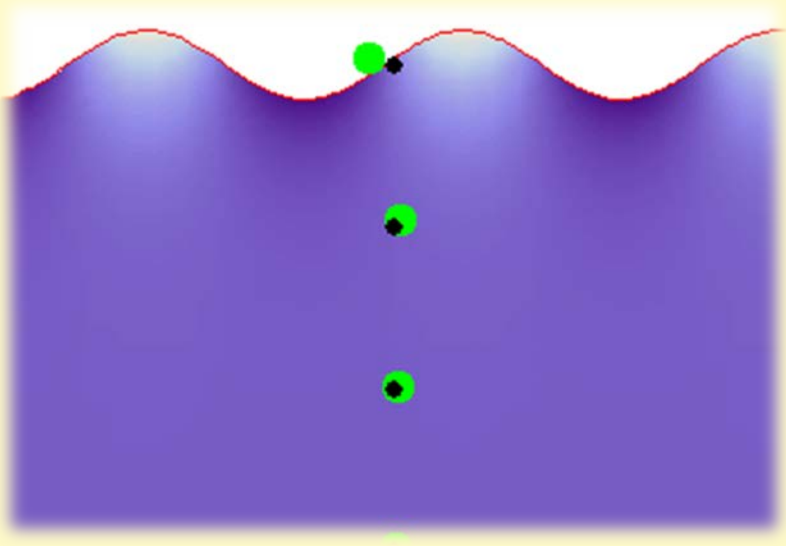


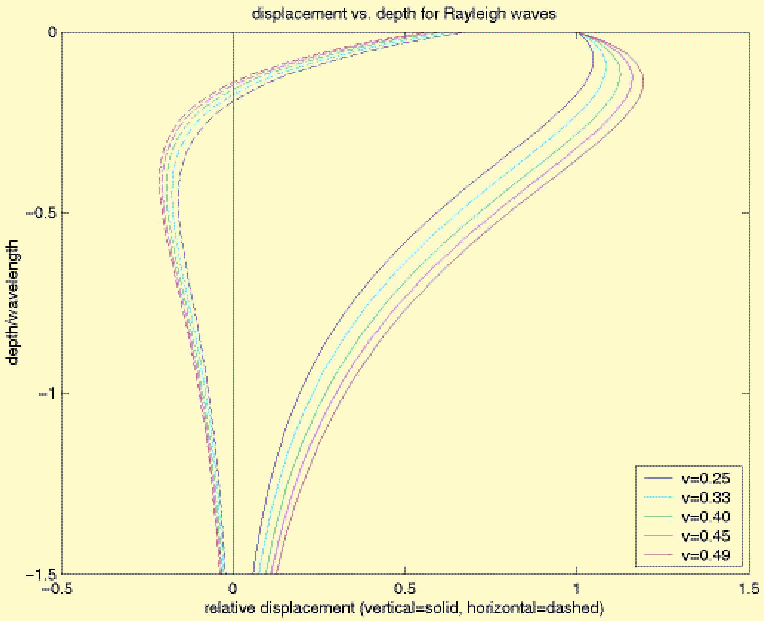


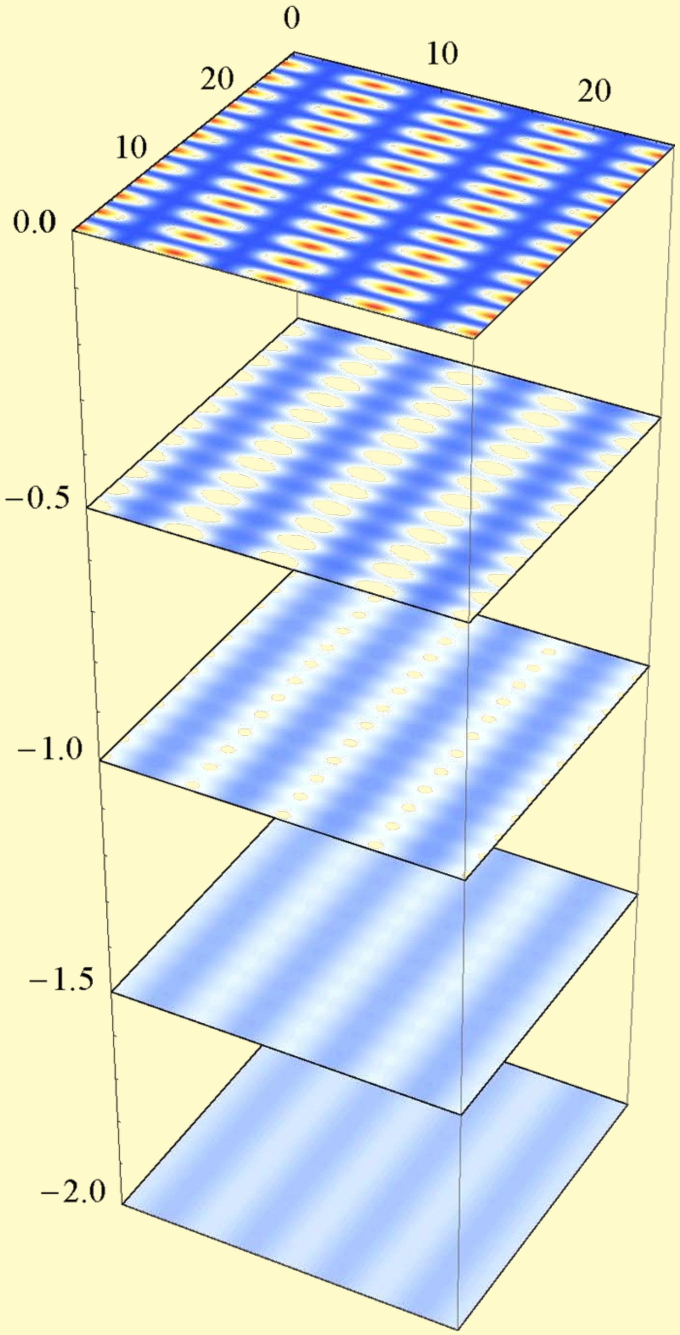


Mitigation: going underground

- GGN is (exponentially) averaged to zero on a scale $\frac{\lambda}{2\pi}$
  - Surface contributions are damped
  - Volume contributions come from a $O(\lambda)$ layer around the test mass
- Surface waves should be dominant





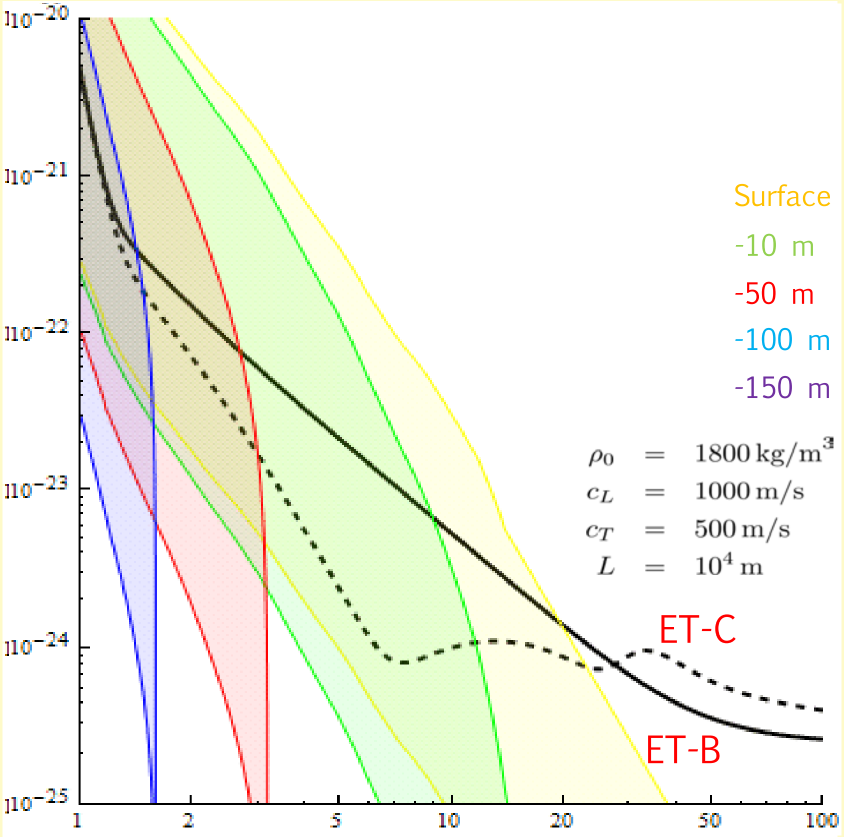


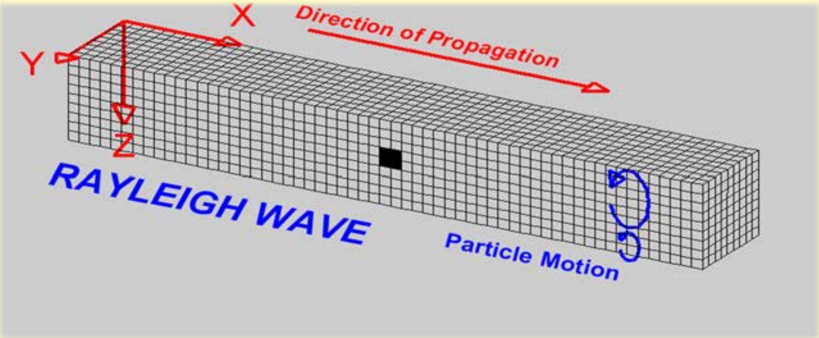
Reduction with the depth

S. Hild et al., "Pushing towards the ET sensitivity using 'conventional' technology" http://arxiv.org/abs/0810.0604

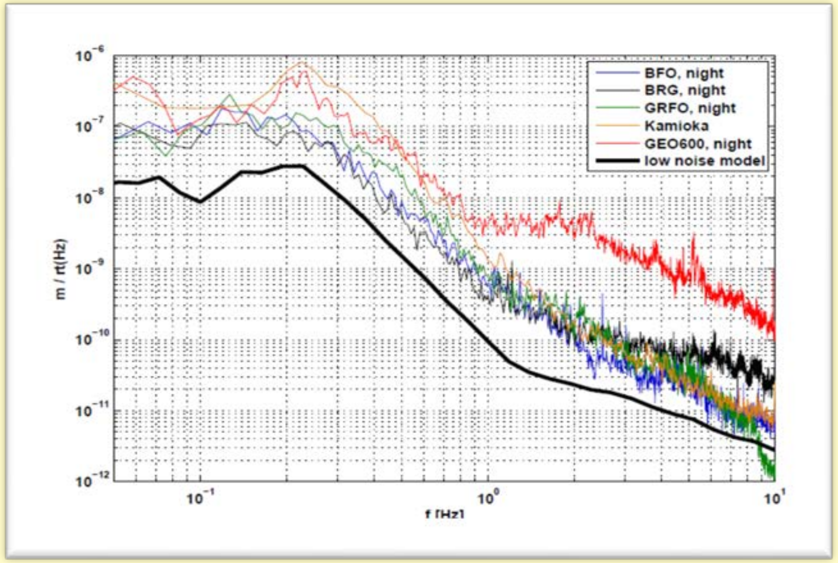
S.Hild et al., "A Xylophone Configuration for a third Generation Gravitational Wave Detector" http://arxiv.org/abs/0906.2655

M.G. Beker et al., "Mitigating noise in future GW observatories in the 1-10 Hz band", GRG 43:623-656 (2011)





- Simple model
  - Homogeneous medium
  - Surface waves only
  - Stationarity
  - No strong local sources
- Validation
  - Prediction about seismic correlations



Mitigation: subtraction

Stationary, gaussian case:

- Measure some set of auxiliary quantities (correlated with the noise to subtract)

- Construct the "optimally subtracted" signal as

$$h_s(t) = h(t) - \sum_i \int \Lambda_i\left(t - t'\right) s\left(t'\right) dt'$$

- Define the efficiency of the subtraction procedure as

$$\eta = 1 - \frac{S_s(\omega)}{S(\omega)}$$

with

$$\langle h_s^*(\omega) h_s(\omega') \rangle = 2\pi\delta\left(\omega - \omega'\right) S_s(\omega)$$

$$\langle h^*(\omega) h(\omega') \rangle = 2\pi\delta\left(\omega - \omega'\right) S(\omega)$$

We need:

- Spectral correlation between two sensors

$$\langle s_i^*(\omega) s_j(\omega') \rangle = 2\pi\delta\left(\omega - \omega'\right) C_{ij}(\omega)$$

  - It is a function of the positions of the sensors i and j
  - Contributions from seismic noise and measurement noise
  - In principle, contributions from self gravitation effects and gravitational waves and (be careful at very low frequency)

- Correlation between a sensor and the detector output

$$\langle s_i^*(\omega) h(\omega') \rangle = 2\pi\delta\left(\omega - \omega'\right) N_i(\omega)$$

  - It is a function of the position of the sensor relative to the test masses

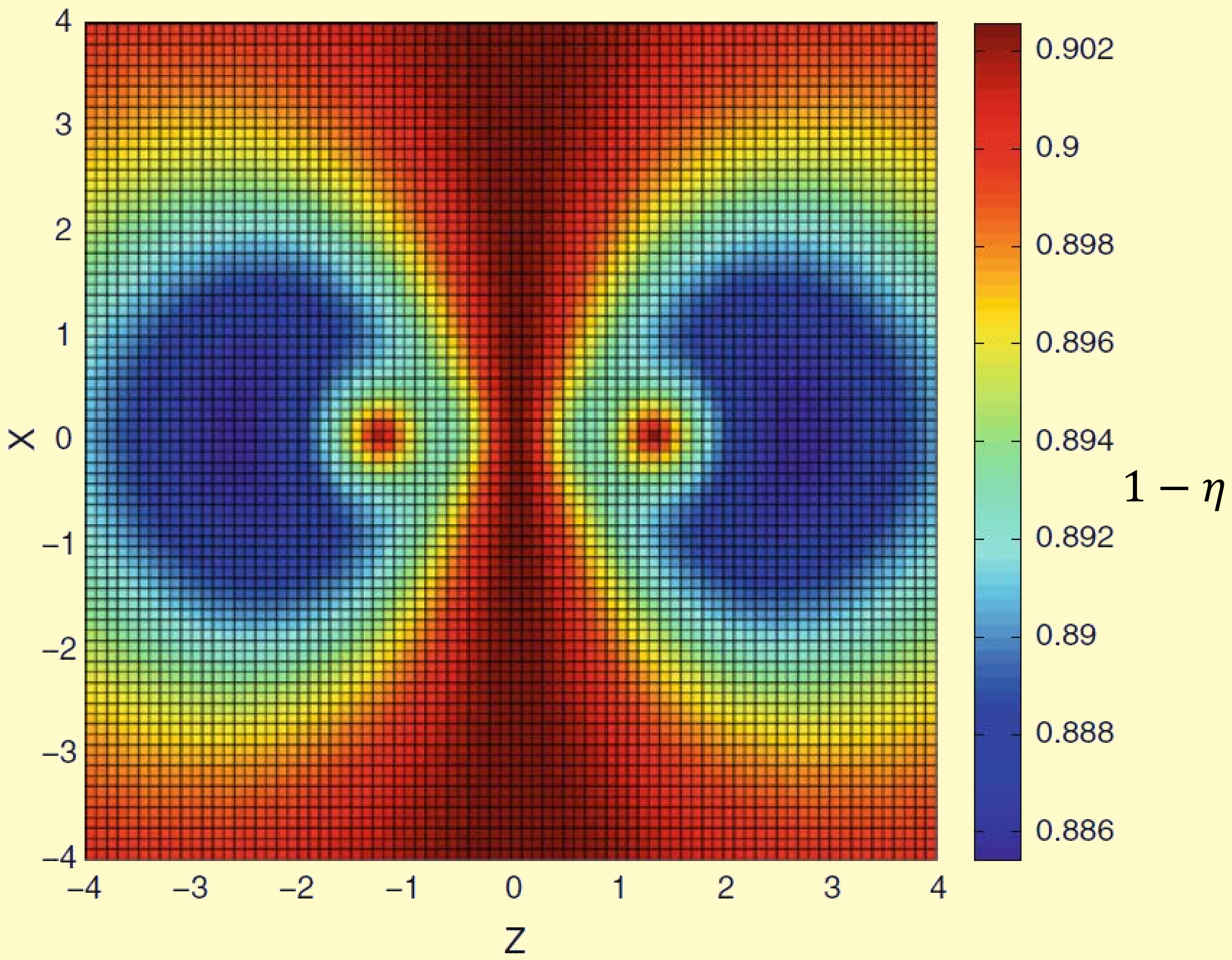
Mitigation: subtraction

- Results for the efficiency:

$$\eta = \frac{N_i^\star \left[C^{-1}\right]_{ij} N_j}{S(\omega)}$$

  - Assumption: sensors dominated by seismic noise

- η is a nonlinear function of the sensor positions and orientation
  - Large N_i is good: strong correlation between auxiliary sensor and GGN
  - Small C_{ij} is good, if measurement error is small: auxiliary meaurements uncorrelated, larger information available
  - If measurement error is not negligible: some correlation between sensors can be tolerated

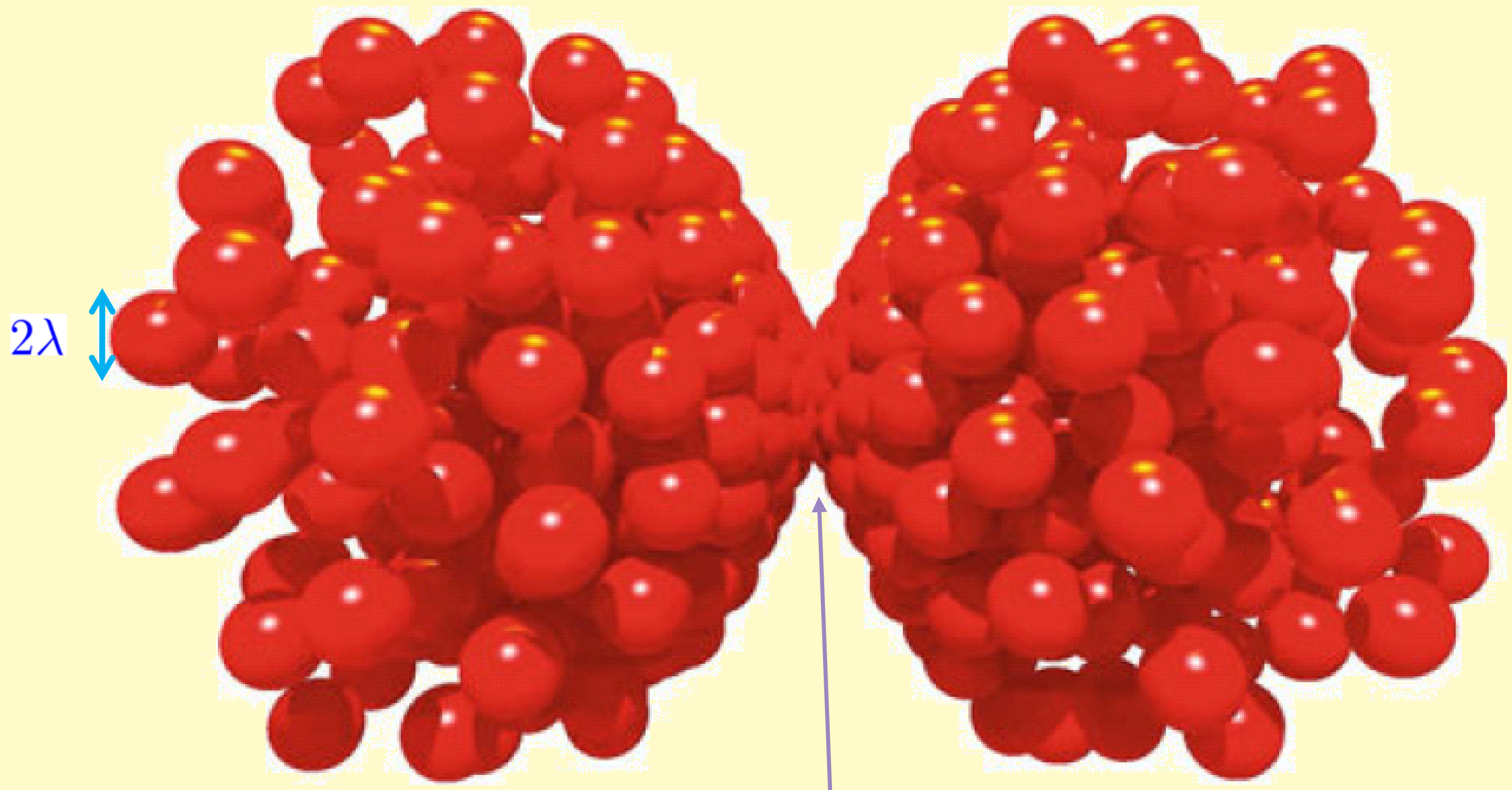


An example: given two sensors located in the optimal way (on the surface), where is convenient to put the third, to optimize the subtraction at some frequency?

- Test mass in the origin
- Coordinates normalized to λ

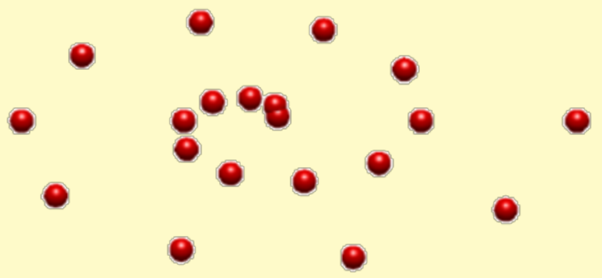
Subtraction: full optimization of sensor positions

- Using a simple model for the correlations
- 512 sensors
- At a fixed frequency

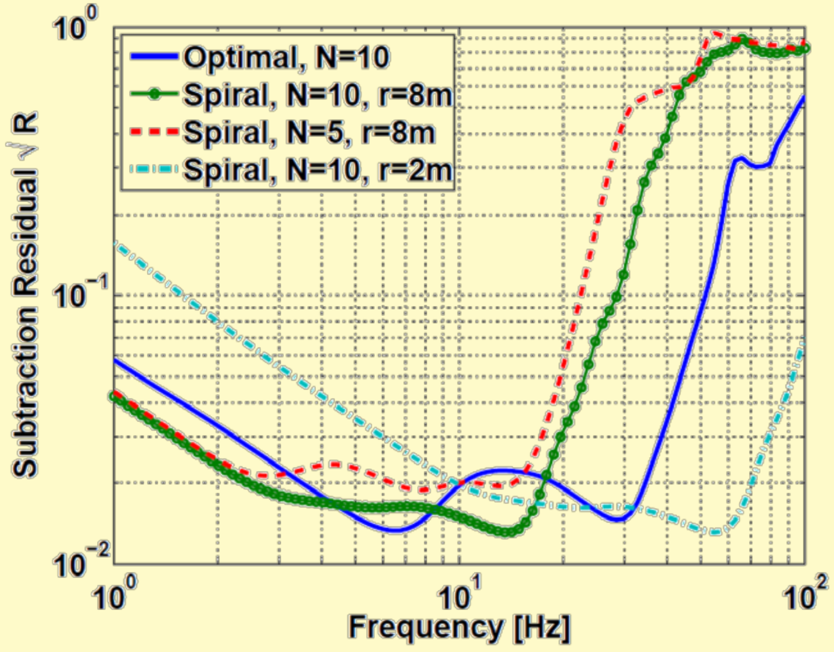


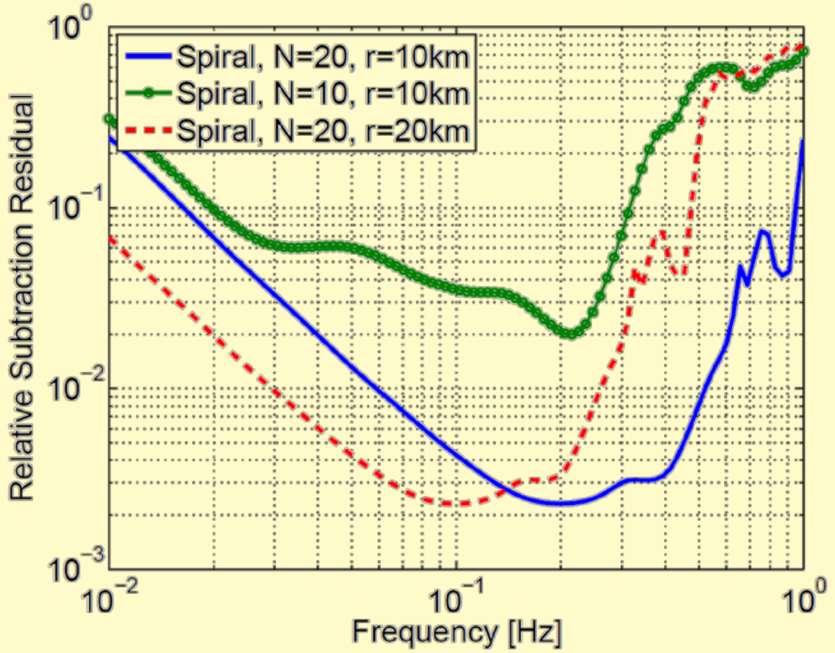
Test mass here

Subtraction efficiency

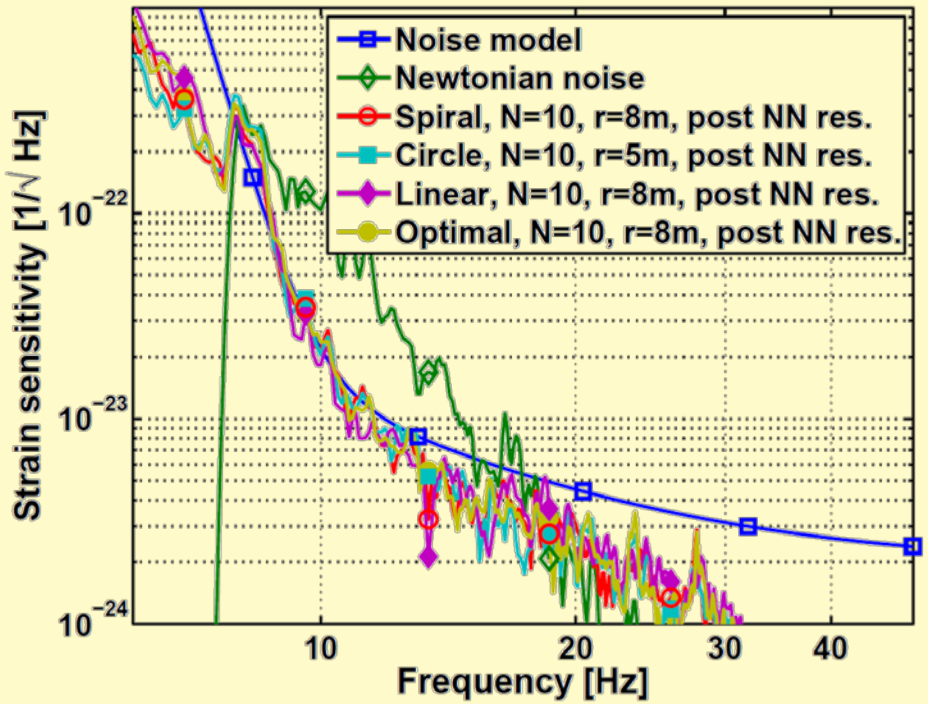


From: Subtraction of Newtonian noise using optimized sensor arrays
Jennifer C. Driggers, Jan Harms, and Rana X. Adhikari Phys. Rev. D 86, 102001





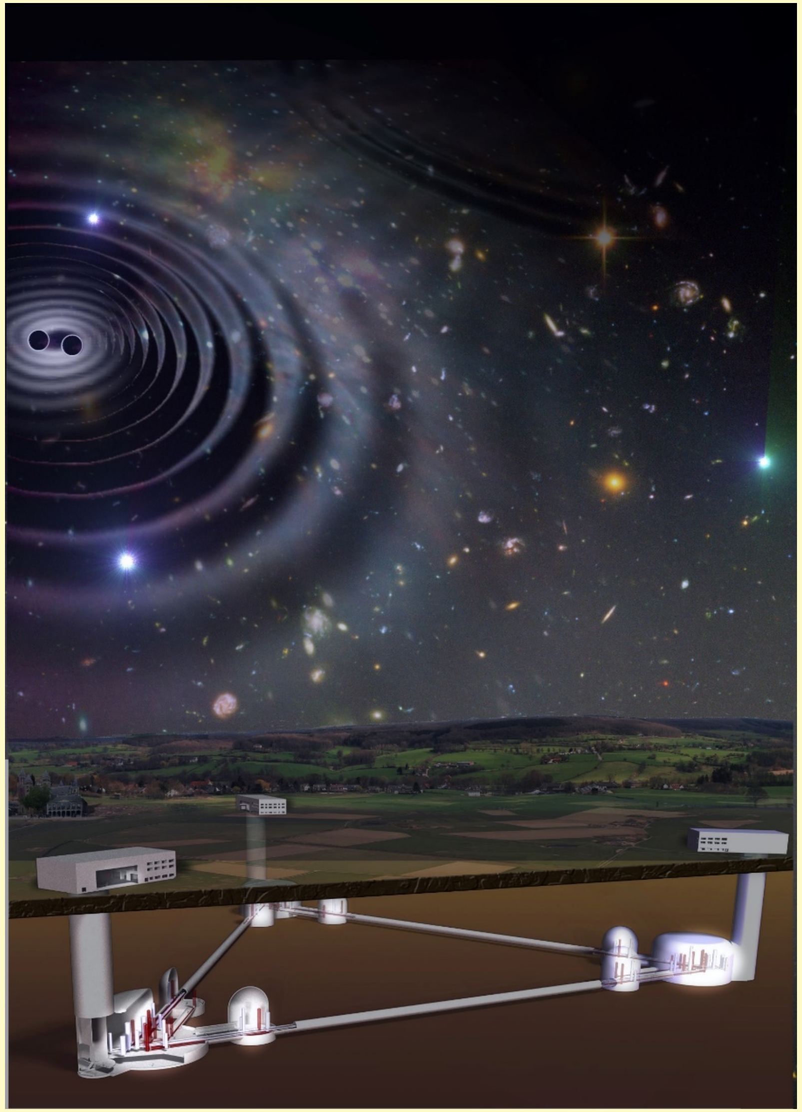
- Specific sensor placement is not critical
- Detailed model needed:
  - Volume waves
  - Scattering effects
- Enough improvement for a third generation detector
- Good in the low frequency region

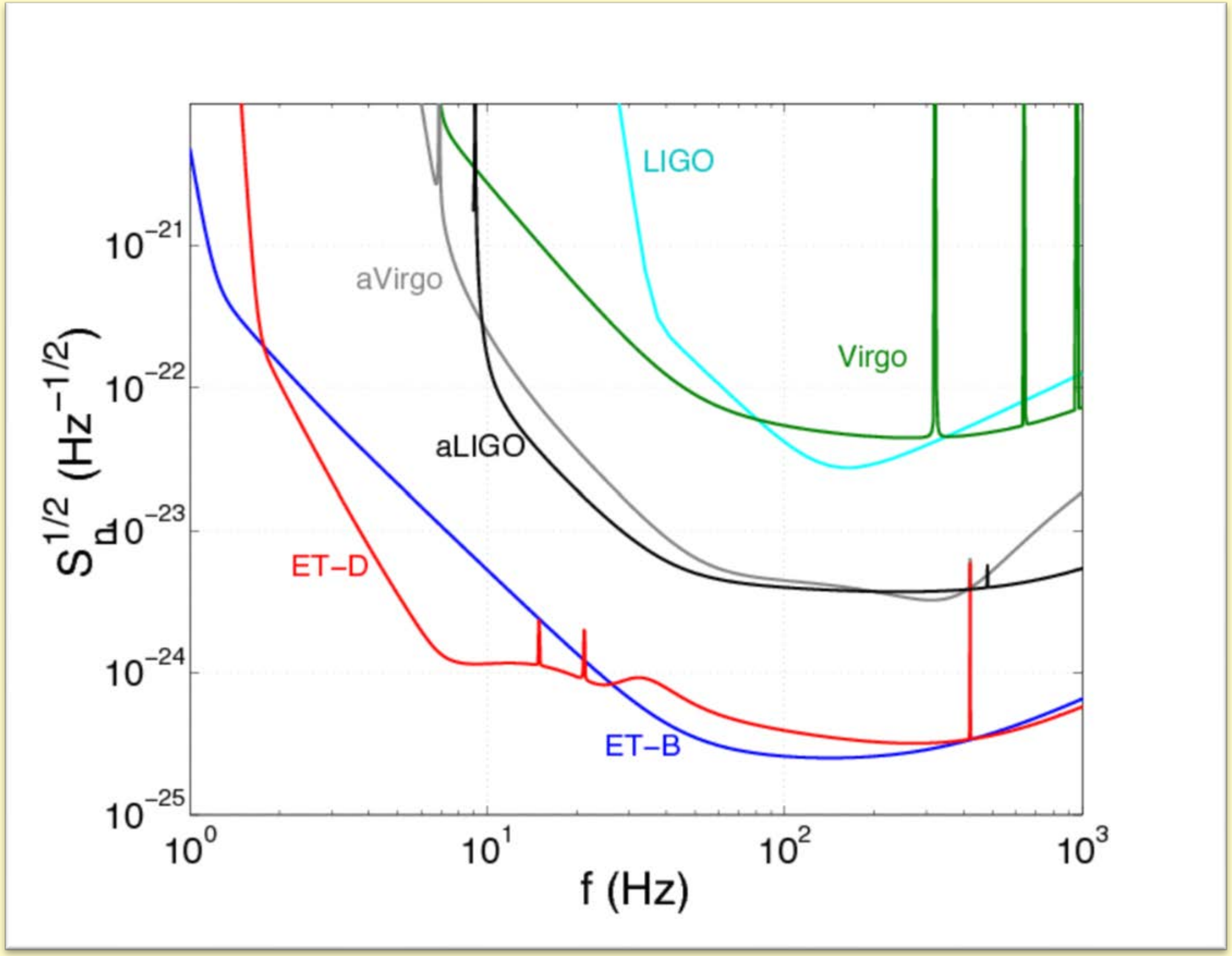


WHAT NEXT?

Third generation detectors

What next? The third generation detectors

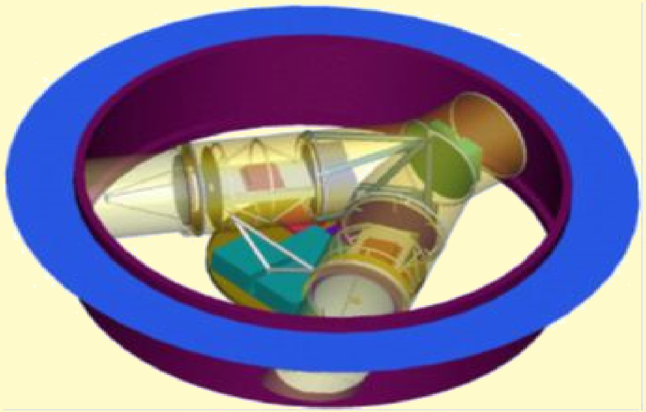


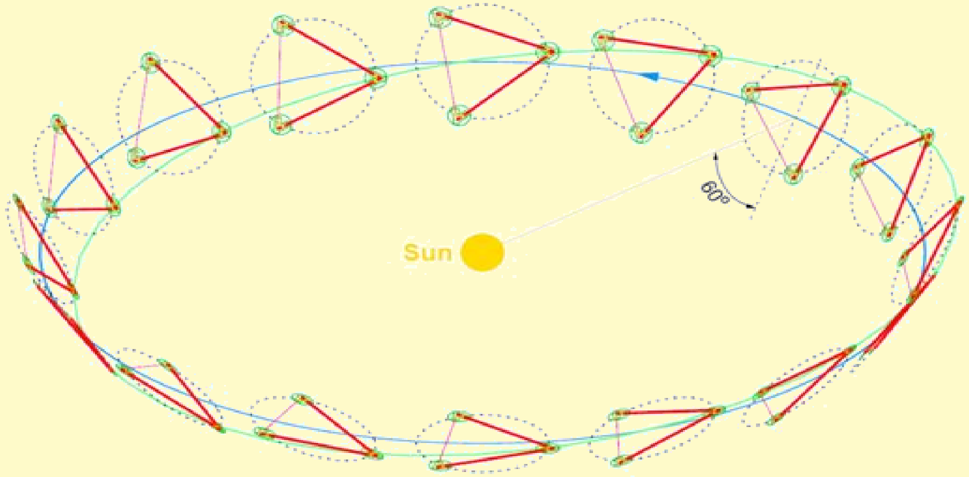


What next? Space detectors

eLISA Space Based GW Detector

- Laser Interferometer in Space Antenna, LISA, provides unique capabilities
  - Immune to seismic noise
  - Long baseline provides 0.001 - 1Hz GW spectrum sensitivity needed for observing massive black hole mergers
- Multiple identical or similar detectors to improve detection confidence





LISA: a mission to detect and observe gravitational waves, O Jennrich, in Gravitational Wave and Particle Astrophysics, Proc SPIE v5500