Accidental Peccei-Quinn symmetry in a model of flavour

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INFN invisibles neutrinos, dark matter & dark energy physics

Weak decays violate *CP*

- CKM phase: $\delta^q \approx 69^\circ$
- PMNS phase: $\delta^\ell \sim -90^\circ$ (?)

Strong *CP* problem

- *CP*-violating term allowed by Standard Model symmetries

$$\mathcal{L} \supset \bar{\theta}\frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- but no *CP* violation is observed in strong interactions!
- $\bar{\theta} \lesssim 10^{-10}$ from neutron EDM [Pendlebury et al '15]

Solution: Peccei-Quinn mechanism [Peccei, Quinn '77, Weinberg '78, Wilczek '78]

PQ mechanism introduces an **axion**

Ingredients in a PQ solution

- Global $U(1)_{PQ}$ symmetry with chiral anomaly
- Complex scalar field $\varphi \to \langle\varphi\rangle$ which breaks $U(1)_{PQ}$

$U(1)_{PQ}$ does not need to be put in by hand!
$\rightarrow$ *accidental* PQ symmetry

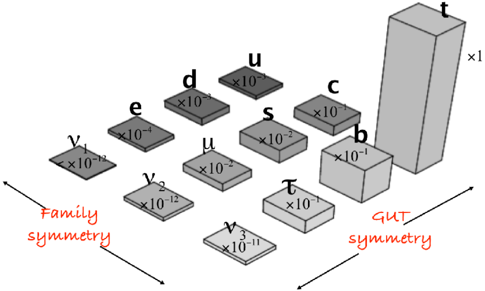
In recent paper [FB, Chun, King '17] we connected an accidental $U(1)_{PQ}$ to the flavons that control Yukawa structures
$\rightarrow$ the axion is *flavoured*

See also

- Flaxion [Ema, Hamaguchi, Moroi, Nakayama '16]
- Axiflavon [Calibbi, Goertz, Redigolo, Ziegler, Zupan '16]

A **unified** model of flavour

- Pati-Salam gauge group $SU(4)_C \times SU(2)_L \times SU(2)_R$
  Vertical unification
- A_4 family symmetry
  Horizontal unification (of LH fermions)



[King, 1301.1340]

Can accommodate all quarks/leptons, masses + mixings

Features

- LH fermions unified in triplets of A_4: e.g. $F_1 = (u, c, t)$
- They couple to scalar fields ϕ that are
  - triplets under A_4
  - singlets under Pati-Salam
- Yukawa couplings become *dynamical*: $y_{ij} F F^c H \to \frac{\langle \phi_i \rangle}{M} F_i F_j^c H$
- Lagrangian has accidental $U(1)_{PQ}$ symmetry!
- Axion lives inside (multiple) ϕ
- PQ scale f_a fixed by same mechanics that fix flavour scale

$f_a \gtrsim 10^{12}$ GeV is close to cosmological upper bound

Dark matter axion? Mass: $m_a \sim 1 - 10\mu$eV

Axion couplings to matter predicted by Yukawa structures

Flavour violation via axion interactions!

Flavoured axions contribute to many flavour-violating decays (suppressed by v_{PQ})

Example: $K^+ \to \pi^+ a$ (e.g. NA62 experiment)

$$\mathrm{Br}(K^+ \to \pi^+ a) = \frac{1}{\Gamma(K^+)} \frac{\left|V^d_{21}\right|^2}{16\pi} \frac{m_K^3}{v_{PQ}^2} \left(1 - \frac{m_\pi^2}{m_K^2}\right)^3 \left|f_+(0)\right|^2$$

with $f_+(0) \approx 1$.

- Experiments [E787, E949] contrain the ratio

$$\frac{v_{PQ}}{|V^d_{21}|} \gtrsim 7 \times 10^{11}\ \mathrm{GeV}$$

- NA62 experiment predicts order of magnitude improvement in limit – approaching predicted range from model!

Other interesting decays

- $\mu^+ \to e^+ a$ (e.g. MEG experiment)
- $\mu^+ \to e^+ a\gamma$
- $K_L^0 \to \pi^0 a$ (e.g. KOTO, KLEVER)
- $B^\pm \to K^\pm(\pi^\pm)a$ (e.g. Belle-II, LHCb)

Other searches

- Haloscopes (e.g. ADMX): searching for axion DM
  (ADMX sensitive to predicted mass range ($m_a \sim$ a few μeV)
- Helioscopes (e.g. CAST, IAXO)
  probing $g_{a\gamma}$ and g_{ae}

1. The great advantage of the unified approach is that predictions are *correlated* and *fixed*.

2. Once flavour observables are determined, all axion couplings are immediately known.

3. Potentially rich phenomenology, but heavy suppression by f_a.

4. No new scalar field is needed to accommodate axion – already present in the theory!

Backup slides

Leptons

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ_{12}^{ℓ} /°	33.57	$32.81 \to 34.32$	32.88	$32.72 \to 34.23$
θ_{13}^{ℓ} /°	8.460	$8.310 \to 8.610$	8.611	$8.326 \to 8.882$
θ_{23}^{ℓ} /°	41.75	$40.40 \to 43.10$	39.27	$37.35 \to 40.11$
δ^{ℓ} /°	261.0	$202.0 \to 312.0$	242.6	$231.4 \to 249.9$
y_e $/10^{-5}$	1.004	$0.998 \to 1.010$	1.006	$0.911 \to 1.015$
y_μ $/10^{-3}$	2.119	$2.106 \to 2.132$	2.116	$2.093 \to 2.144$
y_τ $/10^{-2}$	3.606	$3.588 \to 3.625$	3.607	$3.569 \to 3.643$
Δm_{21}^2 $/10^{-5}\,\mathrm{eV}^2$	7.510	$7.330 \to 7.690$	7.413	$7.049 \to 7.762$
Δm_{31}^2 $/10^{-3}\,\mathrm{eV}^2$	2.524	$2.484 \to 2.564$	2.540	$2.459 \to 2.616$
m_1 /meV			0.187	$0.022 \to 0.234$
m_2 /meV			8.612	$8.400 \to 8.815$
m_3 /meV			50.40	$49.59 \to 51.14$
$\sum m_i$ /meV		< 230	59.20	$58.82 \to 60.19$
α_{21}			10.4	$-38.0 \to 70.1$
α_{31}			272.1	$218.2 \to 334.0$
$m_{\beta\beta}$ /meV			1.940	$1.892 \to 1.998$

We set $\tan\beta = 5$, $M_{\mathrm{SUSY}} = 1$ TeV and $\bar{\eta}_b = -0.24$

Quarks

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ^q_{12} /°	13.03	12.99 → 13.07	13.04	12.94 → 13.11
θ^q_{13} /°	0.1471	0.1418 → 0.1524	0.1463	0.1368 → 0.1577
θ^q_{23} /°	1.700	1.673 → 1.727	1.689	1.645 → 1.753
δ^q /°	69.22	66.12 → 72.31	68.85	63.00 → 75.24
y_u /10^{-6}	2.982	2.057 → 3.906	3.038	1.098 → 4.957
y_c /10^{-3}	1.459	1.408 → 1.510	1.432	1.354 → 1.560
y_t	0.544	0.537 → 0.551	0.545	0.530 → 0.558
y_d /10^{-5}	2.453	2.183 → 2.722	2.296	2.181 → 2.966
y_s /10^{-4}	4.856	4.594 → 5.118	4.733	4.273 → 5.379
y_b	3.616	3.500 → 3.731	3.607	3.569 → 3.643

We set $\tan\beta = 5$, $M_{\mathrm{SUSY}} = 1$ TeV and $\bar{\eta}_b = -0.24$

Input parameters

Parameter	Value
$a\,/10^{-5}$	$1.246\,e^{4.047i}$
$b\,/10^{-3}$	$3.438\,e^{2.080i}$
c	-0.545
$y_d^0\,/10^{-5}$	$3.053\,e^{4.816i}$
$y_s^0\,/10^{-4}$	$3.560\,e^{2.097i}$
$y_b^0\,/10^{-2}$	3.607
$\epsilon_{13}\,/10^{-3}$	$6.215\,e^{2.434i}$
$\epsilon_{23}\,/10^{-2}$	$2.888\,e^{3.867i}$
B	$10.20\,e^{2.777i}$
x	5.880

Parameter	Value
m_a /meV	3.646
m_b /meV	1.935
m_c /meV	1.151
η	2.592
ξ	2.039

Pati-Salam $[SU(4)_C \times SU(2)_L \times SU(2)_R]$

- Left-handed fermions in

$$F_i \sim (4, 2, 1)_i = \begin{pmatrix} u_r & u_g & u_b & \nu \\ d_r & d_g & d_b & e \end{pmatrix}_i$$

- Right-handed fermions in

$$F_i^c \sim (\bar{4}, 1, 2)_i = \begin{pmatrix} u_r^c & u_g^c & u_b^c & N^c \\ d_r^c & d_g^c & d_b^c & e^c \end{pmatrix}_i$$

A_4

- Left-handed fermions in triplet

$$F \sim 3 = (F_1, F_2, F_3)$$

- Right-handed fermions in singlets

$$F_1^c,\ F_2^c,\ F_3^c \sim 1$$

Constrained sequential dominance (CSD) [King '99, '00, '02]

- SD originally devised for neutrinos:
  1) $N_{\rm atm} \rightarrow$ atmospheric mass m_{ν_3} and mixing $\theta_{23} \sim 45^\circ$
  2) $N_{\rm sol} \rightarrow$ solar mass m_{ν_2} and solar+reactor mixing θ_{12}, θ_{13}
  3) $N_{\rm dec}$, if present, nearly decoupled from theory $\rightarrow m_{\nu_1} \ll m_{\nu_{2,3}}$

CSD(n) with two neutrinos:

$$Y^\nu = \begin{pmatrix} 0 & b & * \\ a & nb & * \\ a & (n-2)b & * \end{pmatrix}, \qquad M_R \sim \mathrm{diag}(M_{\rm atm}, M_{\rm sol}, M_{\rm dec})$$

$$\begin{aligned} m^\nu &= v^2 Y^\nu M_R^{-1} (Y^\nu)^T \\ &= m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix} + m_c \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \end{aligned}$$

- In unified scenario, CSD is extended to the quarks!
- Consider $n = 4$ [King '13]. With Y^d diagonal,

$$Y^u = Y^\nu = \begin{pmatrix} 0 & b & * \\ a & 4b & * \\ a & 2b & * \end{pmatrix}$$

- To first approximation, Cabibbo angle

$$\theta^q_{12} \approx \frac{Y^u_{12}}{Y^u_{22}} \approx \frac{1}{4}$$

- This is compellingly close to the true value $\theta^q_{12} \approx 0.227$.

- CSD(4) achieved by A_4 triplet flavons ϕ
- Flavons acquire VEVs with particular alignments:

$$\langle\phi_1^u\rangle = v_{\phi_1^u}(0,1,1), \qquad \langle\phi_1^d\rangle = v_{\phi_1^d}(1,0,0)$$
$$\langle\phi_2^u\rangle = v_{\phi_2^u}(1,4,2), \qquad \langle\phi_2^d\rangle = v_{\phi_2^d}(0,1,0)$$

- Example: first-generation up-type quarks

$$W \supset \frac{(F\cdot\phi_1^u)h_u F_1^c}{M} \to v_u \frac{v_{\phi_1^u}}{M}\left(F_1 F_2 F_3\right)\begin{pmatrix}0\\1\\1\end{pmatrix} F_1^c$$

- Alignments can be fixed by A_4 and orthogonality arguments, implemented by a superpotential

Full Yukawa/mass superpotential

$$W_F^{\text{eff}} = (F \cdot h_3)F_3^c + \frac{(F \cdot \phi_1^u)h_u F_1^c}{\langle \Sigma_u \rangle} + \frac{(F \cdot \phi_2^u)h_u F_2^c}{\langle \Sigma_u \rangle} + \frac{(F \cdot \phi_1^d)h_d F_1^c}{\langle \Sigma_{15}^d \rangle} + \frac{(F \cdot \phi_2^d)h_{15}^d F_2^c}{\langle \Sigma_d \rangle} + \frac{(F \cdot \phi_1^u)h_d F_1^c}{\langle \Sigma_d \rangle}$$

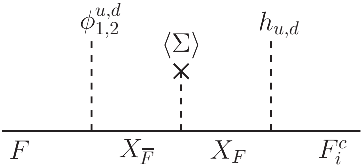
$$W_{\text{Maj}}^{\text{eff}} = \frac{\overline{H^c}\,\overline{H^c}}{\Lambda}\left(\frac{\xi^2}{\Lambda^2}F_1^c F_1^c + \frac{\xi}{\Lambda}F_2^c F_2^c + F_3^c F_3^c + \frac{\xi}{\Lambda}F_1^c F_3^c\right)$$

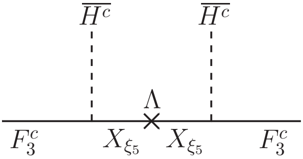
Notes

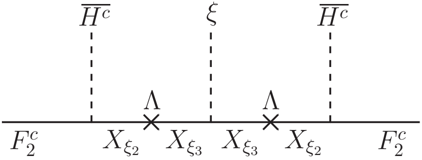
- $\overline{H^c} \sim (4,1,2)$ breaks $SU(4)_C \to SU(3)_C$, generates RHν masses
- $\Sigma \sim (1/15,1,1) \to \langle \Sigma \rangle \lesssim M_{\text{GUT}}$
- $\xi \sim (1,1,1) \to \langle \xi \rangle /\Lambda \sim 10^{-5}$

● 3rd family ● Up quarks ● Down quarks/charged leptons

Sample diagrams







Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}'_5$	R	$U(1)_{PQ}$
F	$(4,2,1)$	3	1	1	1	1	0
$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	$\alpha, \alpha^3, 1$	$\beta, \beta^2, 1$	$\gamma^3, \gamma^4, 1$	1	$-2, -1, 0$
$\overline{H^c}$	$(4,1,2)$	1	1	1	1	0	0
H^c	$(\bar{4},1,2)$	1	1	1	1	0	0
$\phi^u_{1,2}$	$(1,1,1)$	3	α^4, α^2	β^2, β	γ^2, γ	0	$2, 1$
$\phi^d_{1,2}$	$(1,1,1)$	3	α^3, α	β^2, β	γ^2, γ	0	$2, 1$
h_3	$(1,2,2)$	3	1	1	1	0	0
h_u	$(1,2,2)$	$1''$	α	1	1	0	0
h^u_{15}	$(15,2,2)$	1	α	1	1	0	0
h_d	$(1,2,2)$	$1'$	α^3	1	1	0	0
h^d_{15}	$(15,2,2)$	$1'$	α^4	1	1	0	0
Σ_u	$(1,1,1)$	$1''$	α	1	1	0	0
Σ_d	$(1,1,1)$	$1'$	α^3	1	1	0	0
Σ^d_{15}	$(15,1,1)$	$1'$	α^2	1	1	0	0
ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2

Discrete $\mathbb{Z}_N$ symmetries

- $\mathbb{Z}_5$
  - Shaping symmetry of original A to Z model
  - Ensures CSD(4)
- $\mathbb{Z}_3$
  - Ensures PQ symmetry at renormalisable level
  - Forbids most off-diagonal terms in $Y^{d,e}$ (new!)
- $\mathbb{Z}_5'$
  - Protects PQ symmetry to sufficient order

Yukawa and mass matrices

$$Y^u = Y^\nu = \begin{pmatrix} 0 & b & \epsilon_{13} c \\ a & 4b & \epsilon_{23} c \\ a & 2b & c \end{pmatrix} \qquad Y^d = \begin{pmatrix} y_d^0 & 0 & 0 \\ B y_d^0 & y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix}$$

$$Y^e = \begin{pmatrix} -(y_d^0/3) & 0 & 0 \\ B y_d^0 & x y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & M_{13} \\ 0 & M_2 & 0 \\ M_{13} & 0 & M_3 \end{pmatrix}$$

Neutrino matrix after seesaw,

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{i\xi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

PQ charges

$$W_F^{\text{eff}} \sim \underset{0\ \ 0}{(F \cdot h_3)} \underset{0}{F_3^c} + \underset{0\ \ 2}{(F \cdot \phi_1^u)} \underset{0}{h_u} \underset{-2}{F_1^c} + \underset{0\ \ 1}{(F \cdot \phi_2^u)} \underset{0}{h_u} \underset{-1}{F_2^c}$$

$$+ \underset{0\ \ 2}{(F \cdot \phi_1^d)} \underset{0}{h_d} \underset{-2}{F_1^c} + \underset{0\ \ 1}{(F \cdot \phi_2^d)} \underset{0}{h_{15}^d} \underset{-1}{F_2^c} + \underset{0\ \ 2}{(F \cdot \phi_1^u)} \underset{0}{h_d} \underset{-2}{F_1^c}$$

$$W_{\text{Maj}}^{\text{eff}} \sim \underset{0}{\overline{H^c}} \underset{0}{\overline{H^c}} (\underset{2}{\xi} \underset{2}{\xi} \underset{-2}{F_1^c} \underset{-2}{F_1^c} + \underset{2}{\xi} \underset{-1}{F_2^c} \underset{-1}{F_2^c} + \underset{0}{F_3^c} \underset{0}{F_3^c} + \underset{2}{\xi} \underset{-2}{F_1^c} \underset{0}{F_3^c})$$

Notes

- PQ symmetry realised also at renormalisable level
- Higgs sector completely neutral $\rightarrow$ no GUT-scale PQ breaking
- $U(1)_{PQ}$ assignments unique
- Third family is neutral

Breaking $U(1)_{PQ}$

- $\phi_i^f \to \langle \phi_i^f \rangle \sim v_{\phi_1^f}$ breaks all discrete symmetries *and* $U(1)_{PQ}$
- PQ-breaking scale

$$v_{PQ}^2 = (N_a f_a)^2 = \sum_{\phi} x_{\phi}^2 v_{\phi}^2$$

- Dominated by largest VEV: $\langle \phi_2^u \rangle$ (related to charm mass)

Axion

$$a = \frac{1}{v_{PQ}} \sum_{\varphi} x_{\varphi} v_{\varphi} a_{\varphi}$$

Domain wall number

$$N_a \equiv \left| 6x_F + 2\sum_i x_{F_i^c} \right| = \left| 6(0) + 2(-2 + -1 + 0) \right| = 6$$

Protecting the PQ symmetry

Consider terms like

$$\frac{\{\phi\}^n}{M_P^n} W$$

These generate a PQ-breaking axion mass

$$m_*^2 \sim m_{3/2}^2 \frac{v_{PQ}^{n-2}}{M_P^{n-2}}$$

[Holman et al '92]

[Kamionkowski, March-Russell '92]

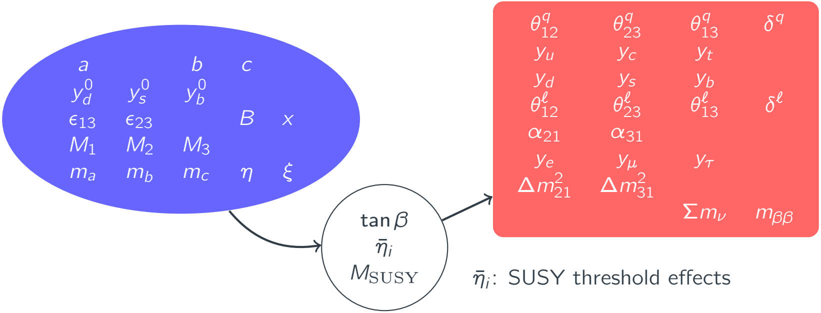
[Barr, Seckel '92]

We require $m_*^2/m_a^2 < 10^{-10}$, where

$$m_a^2 \approx m_\pi^2 \frac{f_\pi^2}{f_a^2}$$

To protect our solution, we forbid all PQ-violating terms like $\{\phi\}^n$ up to $n = 7$ (or $dim = 10$)!

Fitting to quark and lepton mixing data



$\bar{\eta}_i$: SUSY threshold effects

Simple MCMC

- Minimise χ^2 to find best fit

$$\chi^2 = \sum_i \left(\frac{P(x_i) - \mu_i}{\sigma_i} \right)^2$$

- Calculate 95% credible intervals (hpd)

Measured values run up to M_{GUT} (assuming MSSM) [Antusch, Maurer '13]

Driving axion VEVs

$$W_{\text{driving}} = P^{u,d}_{1,2}\left(\bar{\phi}^{u,d}_{1,2}\phi^{u,d}_{1,2} - M^2\right) + P_\xi\left(\bar{\xi}\xi - M^2\right),$$

Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}'_5$	R	$U(1)_{PQ}$
$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	β^2,β	γ^2,γ	0	2, 1
$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	β^2,β	γ^2,γ	0	2, 1
ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2
$\bar{\phi}^u_{1,2}$	$(1,1,1)$	3	α,α^3	β,β^2	γ^3,γ^4	0	$-2,-1$
$\bar{\phi}^d_{1,2}$	$(1,1,1)$	3	α^2,α^4	β,β^2	γ^3,γ^4	0	$-2,-1$
$\bar{\xi}$	$(1,1,1)$	1	α	β	γ^3	0	-2

Yukawa matrices can be diagonalised by bi-unitary matrices $V^{u,d}_{L,R}$, $U^{e}_{L,R}$

$$
\begin{aligned}
Y^{u,\mathrm{diag}} &= V^u_L Y^u (V^u_R)^\dagger, \\
Y^{d,\mathrm{diag}} &= V^d_L Y^d (V^d_R)^\dagger, \\
Y^{e,\mathrm{diag}} &= U^e_L Y^e (U^e_R)^\dagger.
\end{aligned}
$$

We transform the fields by

$$
\begin{aligned}
Q &\to (V^u_L)^\dagger Q, \\
d^c &\to (V^d_R)^\dagger d^c, \\
u^c &\to (V^u_R)^\dagger u^c.
\end{aligned}
$$

Then $Y^u \to Y^{u,\mathrm{diag}}$, $Y^d \to V_{\mathrm{CKM}} Y^{d,\mathrm{diag}}$, where $V_{\mathrm{CKM}} = V^u_L (V^d_L)^\dagger$.