Di-hadron Fragmentation Functions (DiFF)

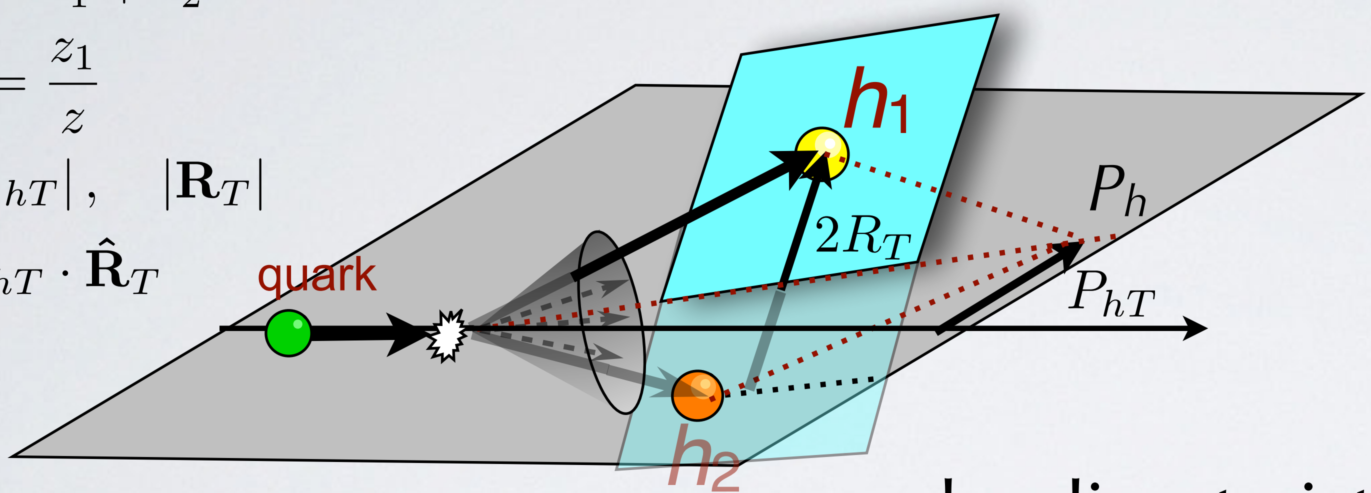
*Bianconi et al., P.R. D**62** (00) 034008*

$z = z_1 + z_2$

$\xi = \frac{z_1}{z}$

$|\mathbf{P}_{hT}|, \quad |\mathbf{R}_T|$

$\hat{\mathbf{P}}_{hT} \cdot \hat{\mathbf{R}}_T$



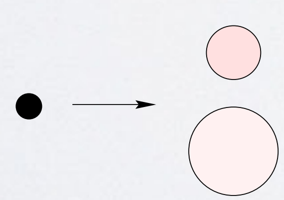
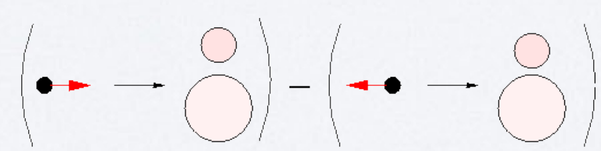
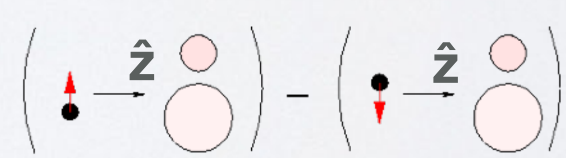
$P_h^\mu = P_1^\mu + P_2^\mu$

$R^\mu = (P_1^\mu - P_2^\mu) / 2$

$$R_T^2 = \frac{z_1 z_2}{z^2} M_h^2 - \frac{z_2}{z} M_1^2 - \frac{z_1}{z} M_2^2$$

*Bacchetta & Radici, P.R.D**67** (03) 094002*

leading twist, $R_T^2 \ll Q^2$

		Quark polarization		
		Unpolarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Hadron polarization	**U**	D_1 	$G_1^\perp$ $\mathbf{s}_L \cdot \mathbf{P}_{hT} \times \mathbf{R}_T$ 	$H_1^{\sphericalangle}$ $\hat{\mathbf{z}} \cdot \mathbf{s}_T \times \mathbf{R}_T$ $H_1^\perp$ $\hat{\mathbf{z}} \cdot \mathbf{s}_T \times \mathbf{P}_{hT}$ 

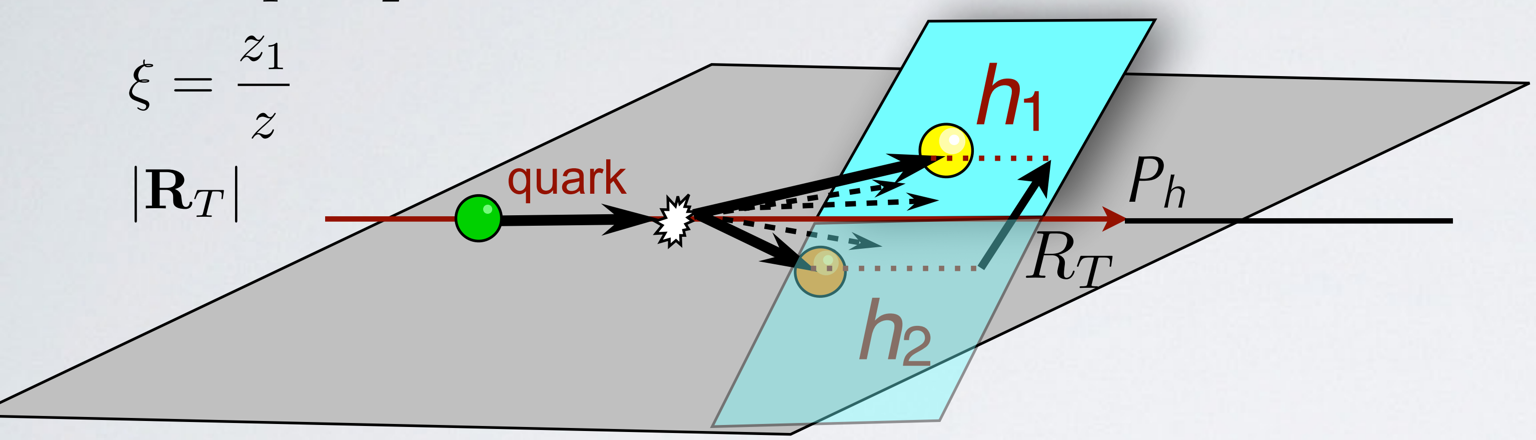
Di-hadron Fragmentation Functions (DiFF)

Bianconi et al.,
P.R. D62 (00) 034008

$z = z_1 + z_2$

$\xi = \dfrac{z_1}{z}$

$|\mathbf{R}_T|$



$P_h^\mu = P_1^\mu + P_2^\mu$

$R^\mu = (P_1^\mu - P_2^\mu) / 2$

$\int d\mathbf{P}_{hT}$ collinear framework

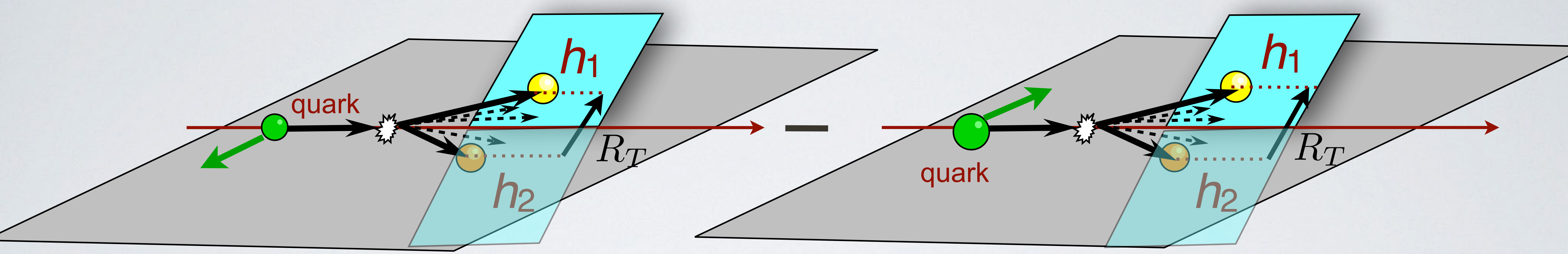
leading twist, $R_T^2 \ll Q^2$

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Hadron polarization	U	D_1	$G_1^\perp$ $\mathbf{s}_L \cdot \mathbf{P}_{hT} \times \mathbf{R}_T$	$H_1^{\sphericalangle}$ $\hat{\mathbf{z}} \cdot \mathbf{s}_T \times \mathbf{R}_T$ $H_1^\perp$ $\hat{\mathbf{z}} \cdot \mathbf{s}_T \times \mathbf{P}_{hT}$

chiral-odd partner of transversity

Access to transversity via DiFF

Collins, Heppelman, Ladinsky, N.P. ***B420*** *(94)*



correlation between

quark polarization and $\boldsymbol{R}_T = (z_2 \boldsymbol{P}_{1T} - z_1 \boldsymbol{P}_{2T}) / z$

or, equivalently, azimuthal orientation of (h_1, h_2) plane

(only if $h_1 \neq h_2$)

effect encoded in $h_1(x)\ H_1^{\sphericalangle}(z, M_h^2)$

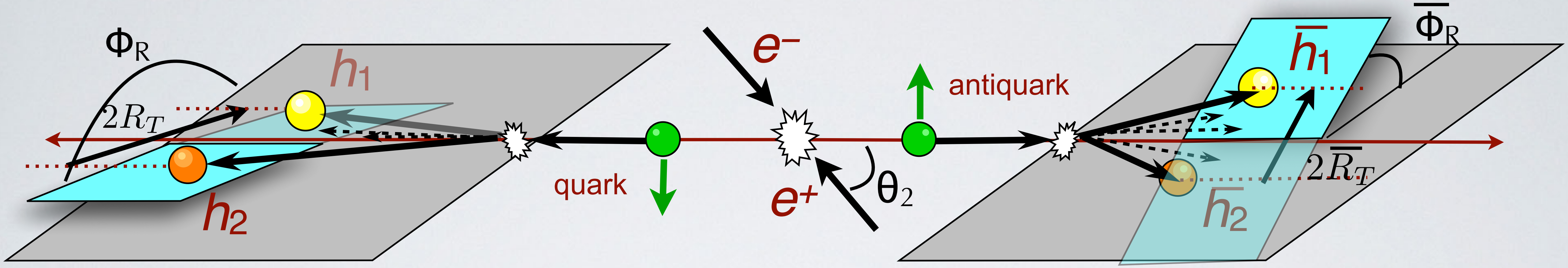
$z = z_1 + z_2$

$P_h^2 = M_h^2$ **<=>** $\boldsymbol{R}_T^2$

alternative to Collins effect

*Radici, Jakob, Bianconi, P.R.D****65*** *(02) 074031*
*Bacchetta & Radici, P.R.D****67*** *(03) 094002*
Bacchetta et al., JHEP ***1303*** *(13) 119*
Radici et al., JHEP ***1505*** *(15) 123*

extraction of DiFF from e⁺e⁻



back-to-back hadron pairs → $\cos(\Phi_R+\bar{\Phi}_R)$ modulation

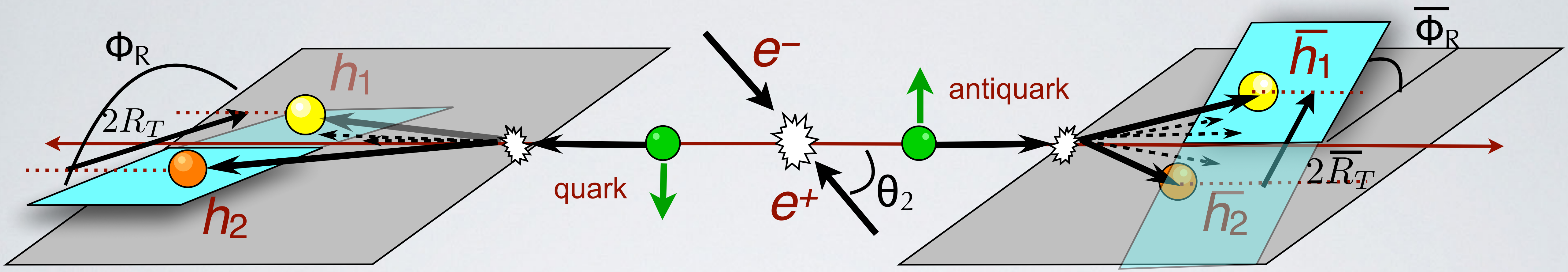
*Artru & Collins, Z.Ph. C**69** (96) 277*

Boer, Jakob, Radici,
*P.R.D**67** (03) 094003*

$$A^{\cos(\phi_R+\bar{\phi}_R)} = \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,\frac{|\boldsymbol{R}_T|}{M_h}\,\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\,\frac{\sum_q e_q^2\, H_1^{\sphericalangle\, q}(z,M_h^2)\ \bar{H}_1^{\sphericalangle\, \bar{q}}(\bar{z},\bar{M}_h^2)}{\sum_q e_q^2\, D_1^q(z,M_h^2)\ \bar{D}_1^{\bar{q}}(\bar{z},\bar{M}_h^2)}$$

same as
in SIDIS

extraction of DiFF from e^+e^-



back-to-back hadron pairs → $\cos(\Phi_R+\bar{\Phi}_R)$ modulation

*Artru & Collins, Z.Ph. C**69** (96) 277*

Boer, Jakob, Radici,
*P.R.D**67** (03) 094003*

$$A^{\cos(\phi_R+\bar{\phi}_R)} = \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,\frac{|\boldsymbol{R}_T|}{M_h}\,\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\,\frac{\sum_q e_q^2\,H_1^{\sphericalangle\, q}(z,M_h^2)\;\bar{H}_1^{\sphericalangle\,\bar{q}}(\bar{z},\bar{M}_h^2)}{\sum_q e_q^2\,D_1^q(z,M_h^2)\;\bar{D}_1^{\bar{q}}(\bar{z},\bar{M}_h^2)}$$

same as
in SIDIS

for D_1

Belle data for
$A^{\cos(\Phi_R+\Phi_R)}$

*Vossen et al., P.R.L. **107** (11) 072004*

first extraction of DiFF, but using PYTHIA

*Courtoy et al., P.R.D**85** (12) 114023*

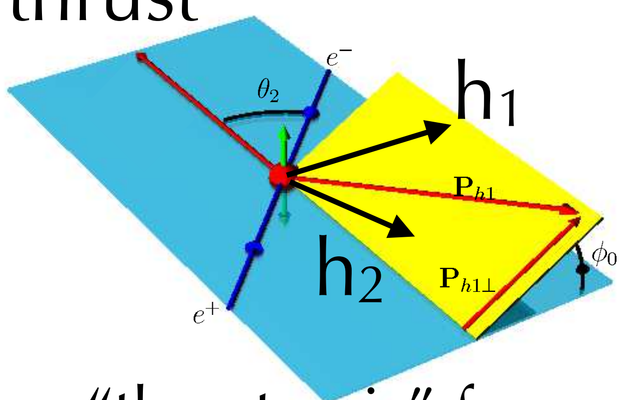
*Radici et al., JHEP **1505** (15) 123*

new Belle data for unpolarized cross section

e^+e^- cross section for $(\pi\pi)$ in same hemisphere

$e^+ e^- \rightarrow (h_1 h_2)\, X$

thrust



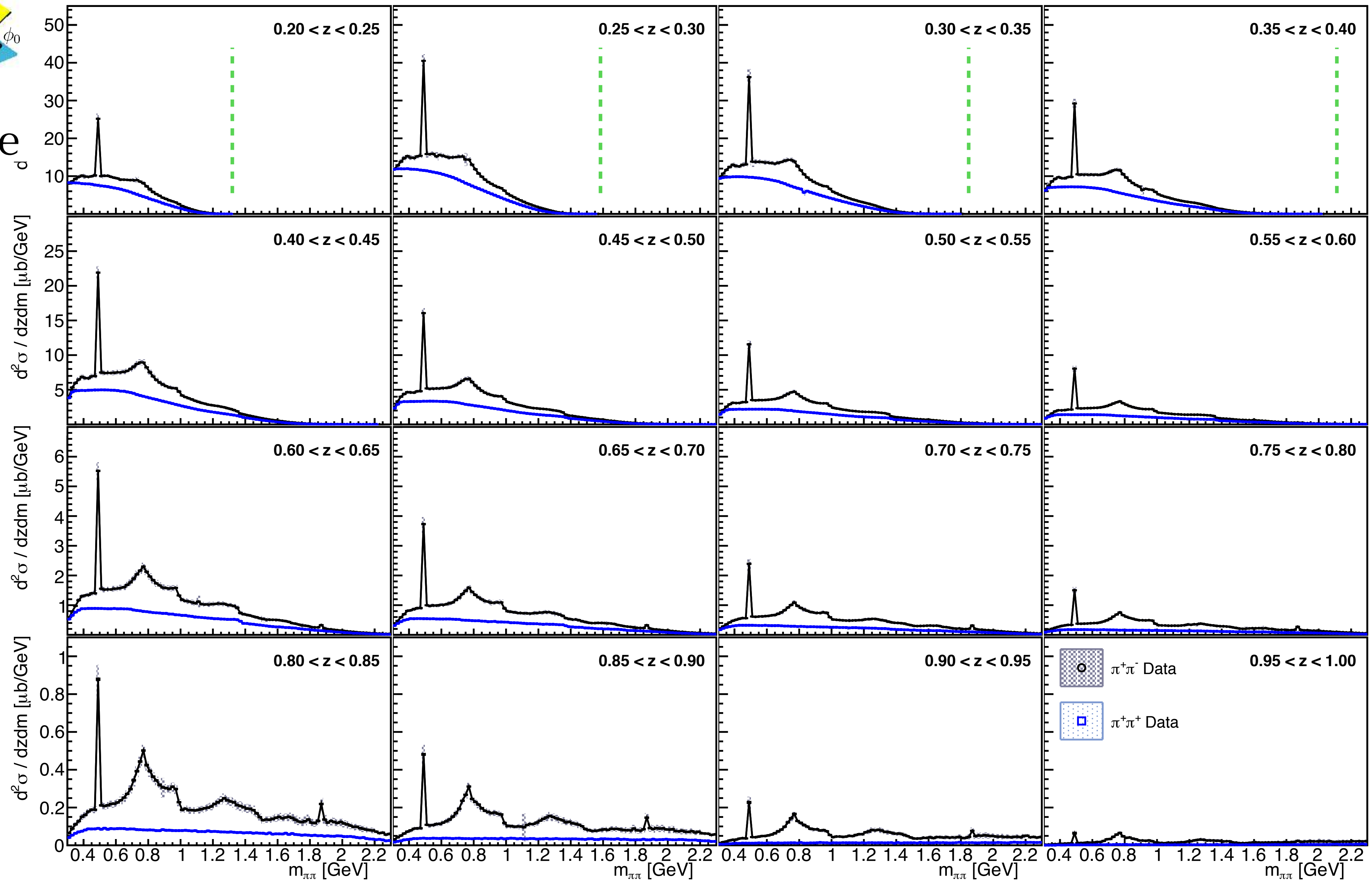
"thrust-axis" frame

$$\frac{d\sigma^0}{dz\, dM_h}$$

● $\pi^+\pi^-$

● $\pi^+\pi^+$

R. Seidl et al. (BELLE)
*P.R. D**96** (17) 032005*

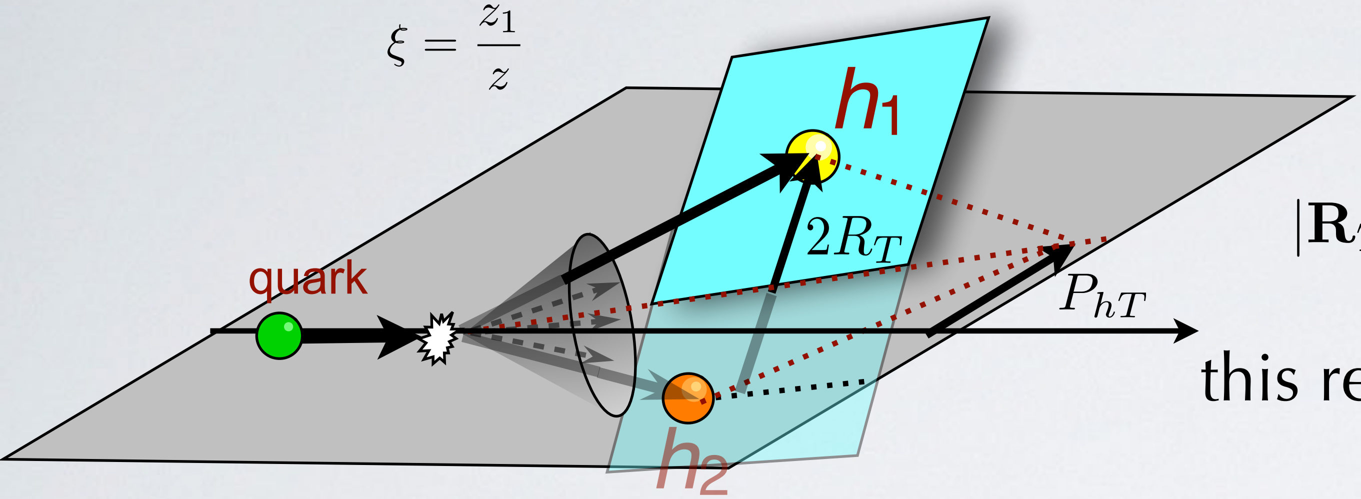


universality of $H_1^{\sphericalangle}$

$$z = z_1 + z_2$$

$$\xi = \frac{z_1}{z}$$

$$\mathrm{DiFF}(z, \xi, |\mathbf{k}_T|, |\mathbf{R}_T|, \hat{\mathbf{k}}_T \cdot \hat{\mathbf{R}}_T) = \frac{1}{\pi} \sum_n \frac{\cos n(\phi_k - \phi_R)}{1 + \delta_{n0}} \, \mathrm{DiFF}^{[n]}(z, \xi, |\mathbf{k}_T|, |\mathbf{R}_T|)$$

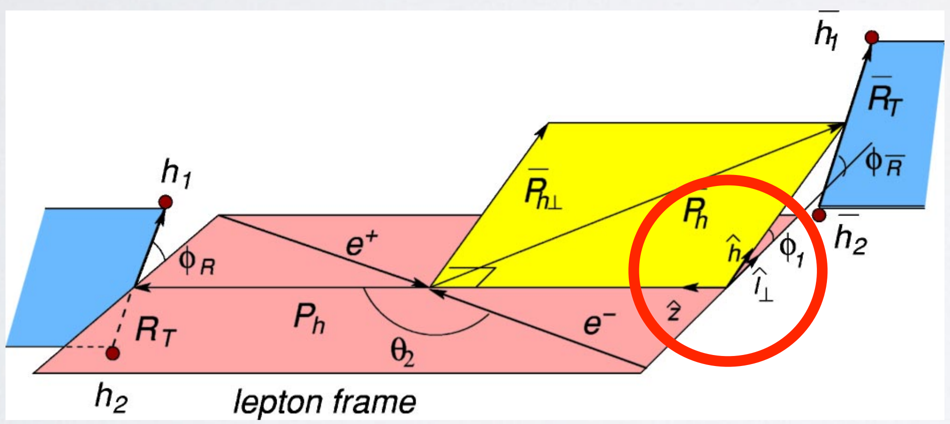


$$|\mathbf{R}_T| \, H_1^{\sphericalangle}(z, M_h^2) = z^2 \int d\mathbf{k}_T \int d\xi \, \left[H_1^{\sphericalangle\,[0]} + |\mathbf{k}_T| \, H_1^{\perp\,[1]} \right]$$

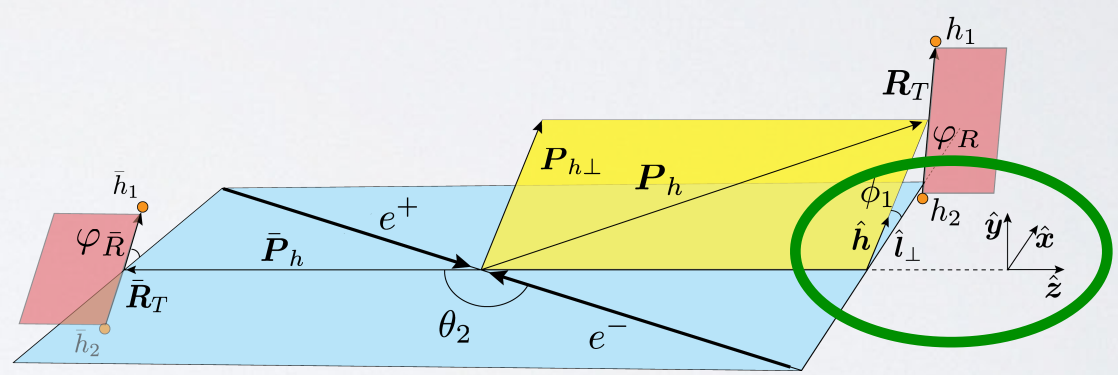
this result in SIDIS cannot be reproduced from e+e−

*Bacchetta & Radici, P.R.D**69** (04) 074026*

*Boer, Jakob, Radici, P.R.D**67** (03) 094003*



*Boer, Jakob, Radici, P.R.D**67** (03) 094003*



Matevosyan, Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas, arXiv:1802.01578

in the cross section, all terms $\perp \hat{\mathbf{h}}$ change sign → $H_1^{\sphericalangle}$ in $A^{\cos(\phi_R + \bar{\phi}_{\bar{R}})}$ same as in SIDIS A_{UT}

compatibility restored !

helicity DiFF $G_1^\perp$

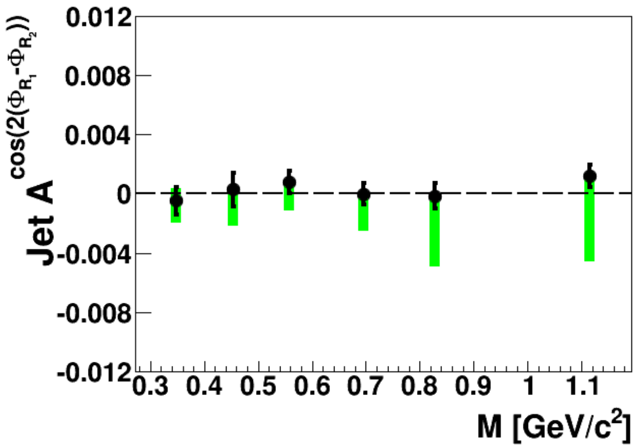
Matevosyan, Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas, arXiv:1802.01578

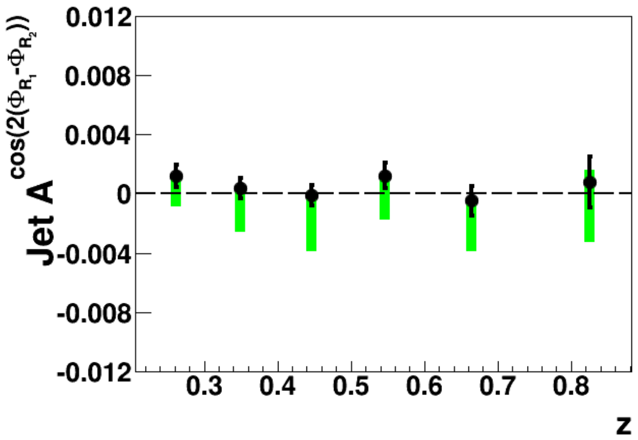
$$\langle T\rangle = \int d\xi \int d\bar{\xi} \int d\phi_R \int d\bar{\phi}_{\bar{R}} \int d\mathbf{q}_T \, T \, d\sigma$$

proposed "jet handedness" asymmetry to extract $G_1^\perp$ vanishes $\langle \cos 2(\phi_R - \bar{\phi}_{\bar{R}})\rangle = 0$

confirms BELLE findings

Abdesselam et al. (BELLE) arXiv:1505.08020





which asymmetry to get $G_1^\perp$?

$$\langle \mathbf{q}_T^2 (2\cos(\phi_R - \bar{\phi}_{\bar{R}}) - \cos(2\phi_1 - \phi_R - \bar{\phi}_{\bar{R}})\rangle$$
$$\propto \sum_q G_1^{\perp q}(z, M_h^2)\, \bar{G}_1^{\perp \bar{q}}(\bar{z}, \bar{M}_h^2)$$

Matevosyan et al., arXiv:1712.06384

$$G_1^\perp(z, M_h^2) \equiv G_1^{\perp,[0],(1)}(z, M_h^2) - G_1^{\perp,[2],(1)}(z, M_h^2)$$

$$G_1^{\perp,[n],(p)}(z, M_h^2) = z^2 \int d\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M_h^2}\right)^p \int d\xi \, \frac{|\mathbf{R}_T|}{M_h} \, G_1^{\perp,[n]}(z, \xi, |\mathbf{k}_T|, |\mathbf{R}_T|)$$

N.B. $\langle \cos(\phi_R - \bar{\phi}_{\bar{R}})\rangle = 0$ but $\langle \cos(\phi_R - \bar{\phi}_{\bar{R}})\rangle(\mathbf{q}_T^2) \neq 0$ → there must be a node

The power of DiFFs

collinear framework → factorization theorems

$e^+e^- \to (\pi^+\pi^-)(\pi^+\pi^-)X$ BELLE

$$A_{e^+e^-} \sim \frac{H_1^{\sphericalangle}\ \overline{H_1^{\sphericalangle}}}{D_1\ \overline{D_1}}$$

prediction
*Boer, Jakob, Radici, P.R. D**67** (03) 094003*
extraction
*Courtoy et al., P.R. D**85** (12) 114023*

universality of DiFF and usual DGLAP evolution
*Ceccopieri, Radici, Bacchetta, P.L.**B650** (07) 81*

STAR **$p\,p^\uparrow \to (\pi^+\pi^-)X$**

$$A_{pp} \sim \frac{f_1 \otimes h_1 \otimes H_1^{\sphericalangle}}{f_1 \otimes f_1 \otimes D_1}$$

prediction
*Bacchetta and Radici, P.R. D**70** (04) 094032*
test universality
*Radici et al., P.R.D**94** (16) 034012*

SIDIS: $e\,p^\uparrow \to e'\,(\pi^+\pi^-)X$ COMPASS hermes

$$A_{\text{SIDIS}} \sim \frac{h_1\ H_1^{\sphericalangle}}{f_1\ D_1}$$

prediction
*Radici, Jakob, Bianconi, P.R.D**65** (02) 074031*
*Bacchetta & Radici, P.R. D**67** (03) 094002*
extraction
*Bacchetta, Courtoy, Radici, P.R.L. **107** (11) 012001*
*Bacchetta, Courtoy, Radici, JHEP **1303** (13) 119*
*Radici et al., JHEP **1505** (15) 123*

extraction of transversity from first global fit

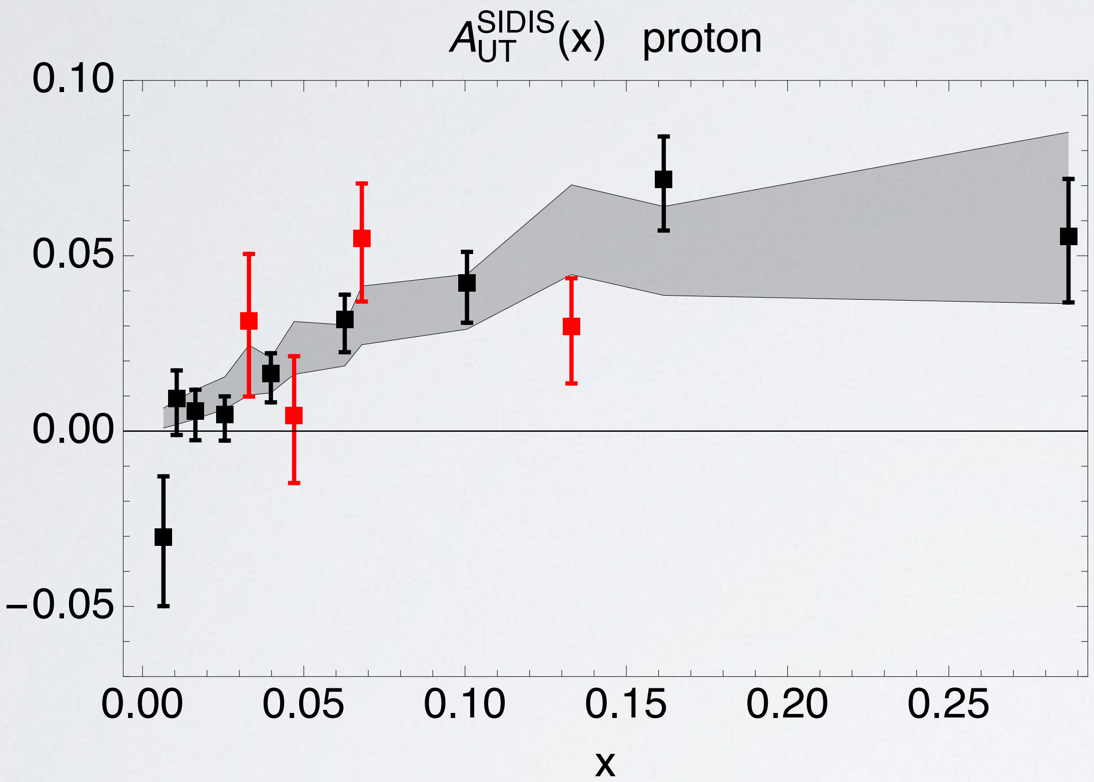
Radici & Bacchetta, arXiv:1802.05212

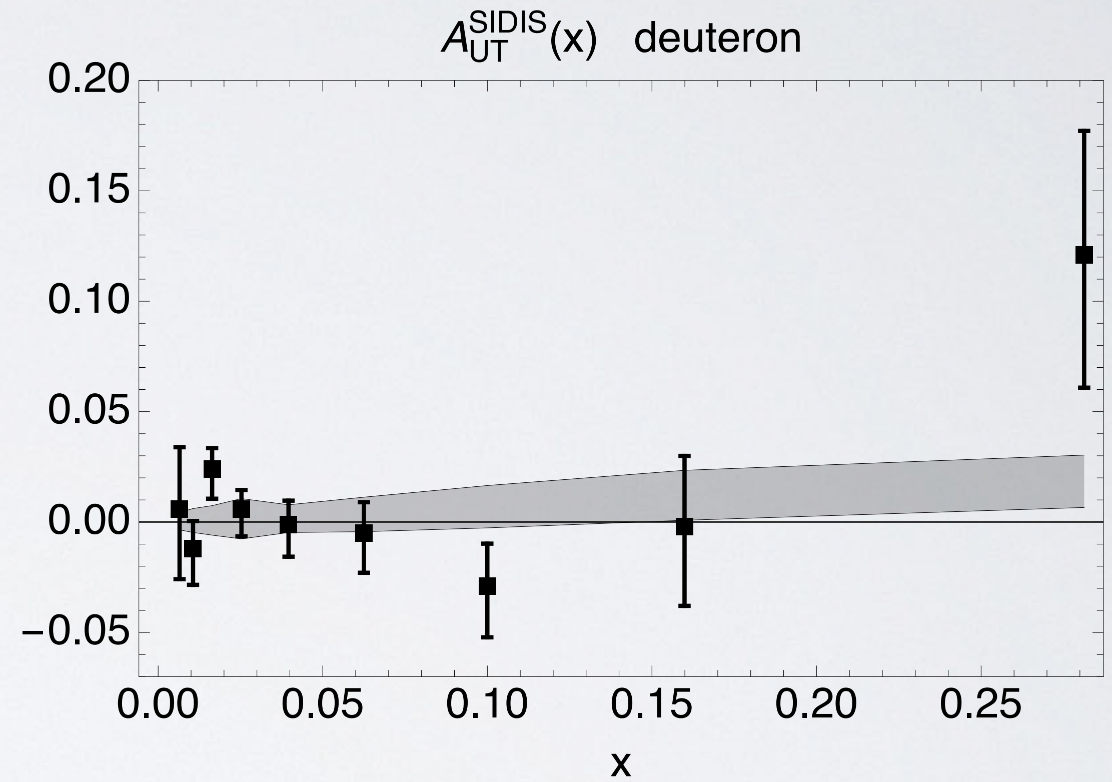
global fit : the SIDIS data

COMPASS — *Braun et al., E.P.J. Web Conf.* ***85*** *(15) 02018*

hermes — *Airapetian et al., JHEP* ***0806*** *(08) 017*

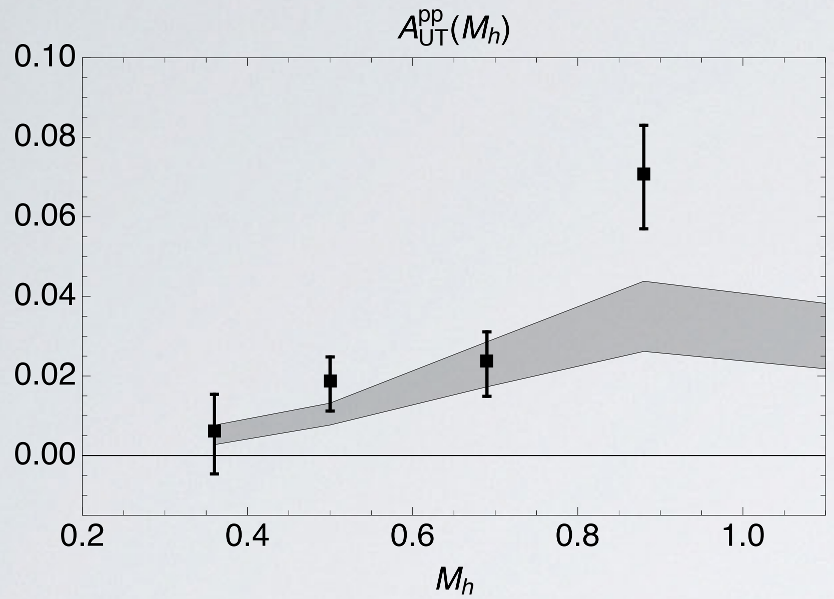
COMPASS — *Adolph et al., P.L.* ***B713*** *(12)*



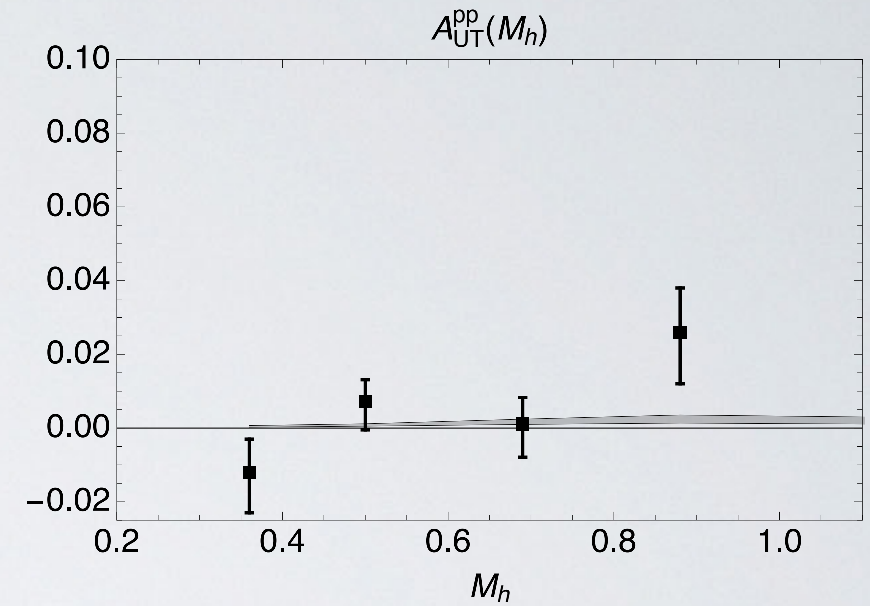


90% replicas

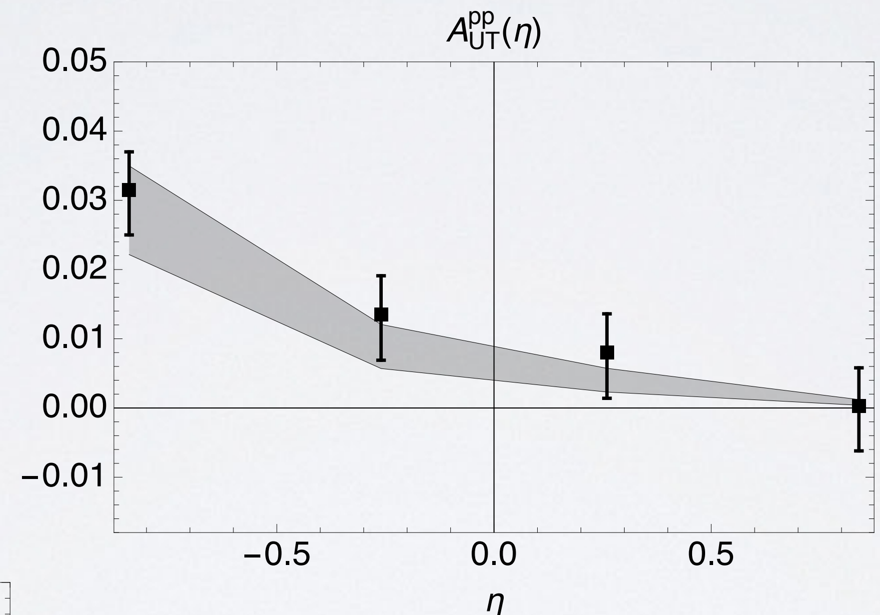
global fit : the STAR data



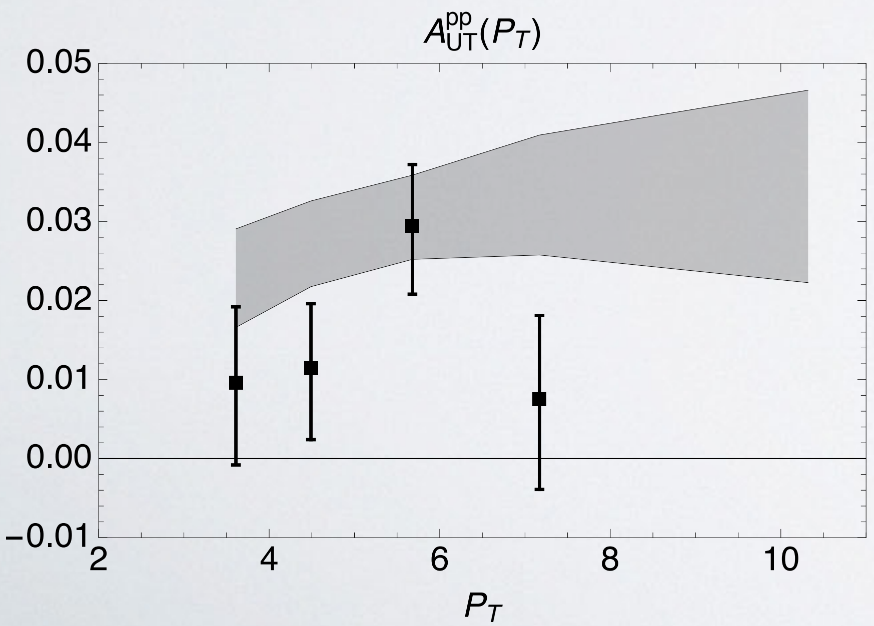
Adamczyk et al. (STAR),
P.R.L. ***115*** *(2015) 242501*



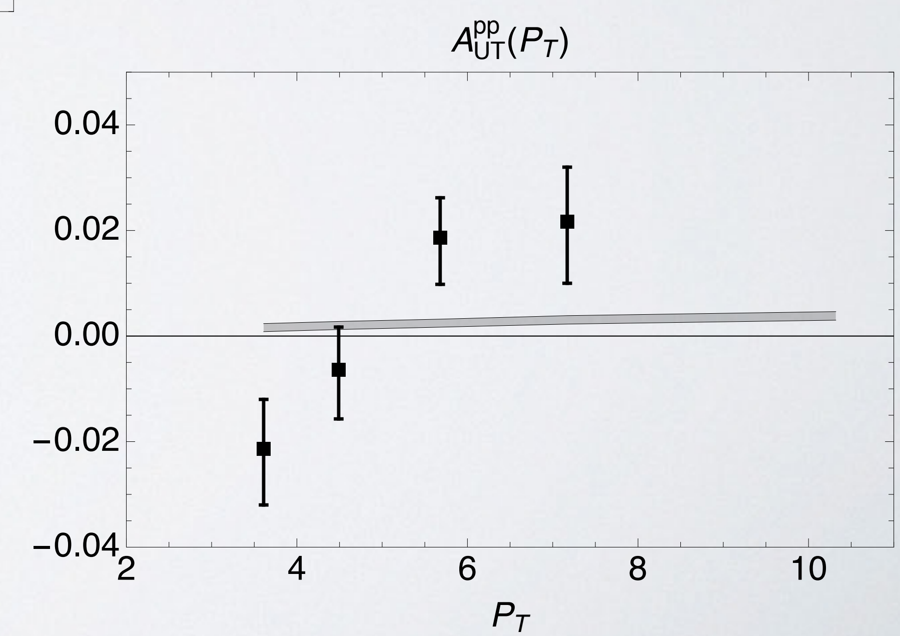
η<0



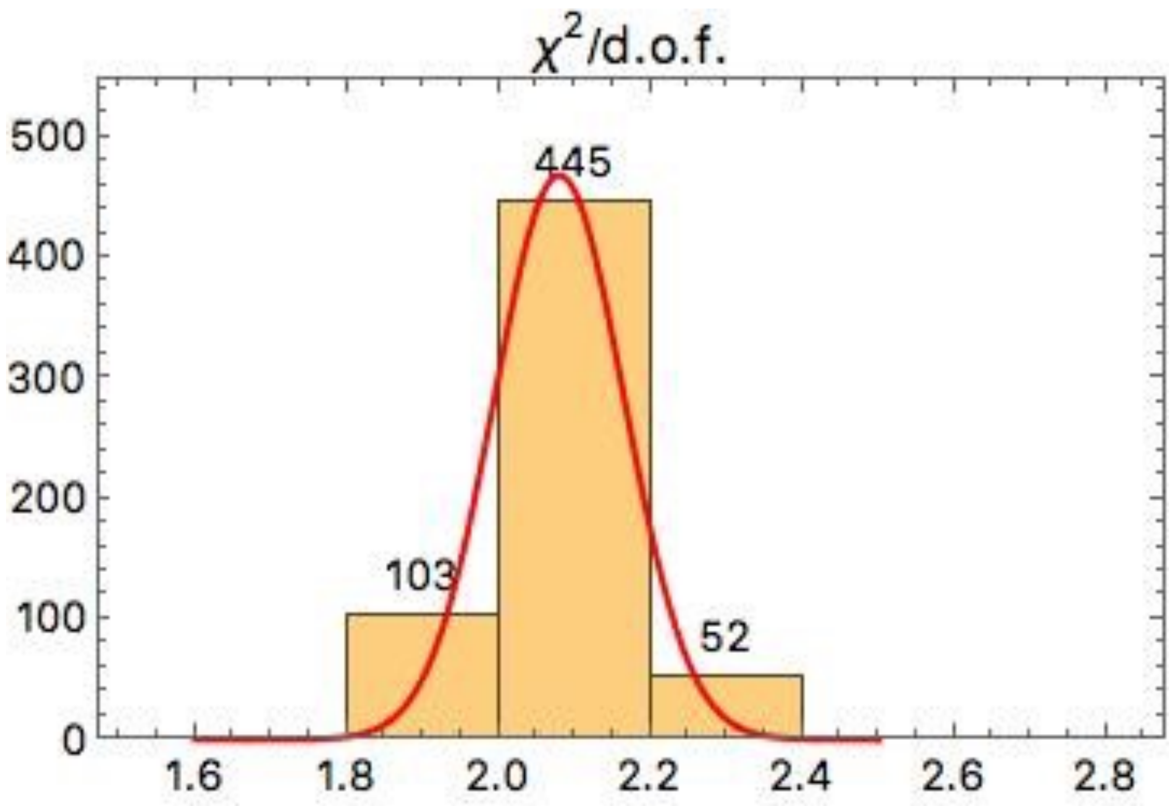
η>0



90% uncertainty band



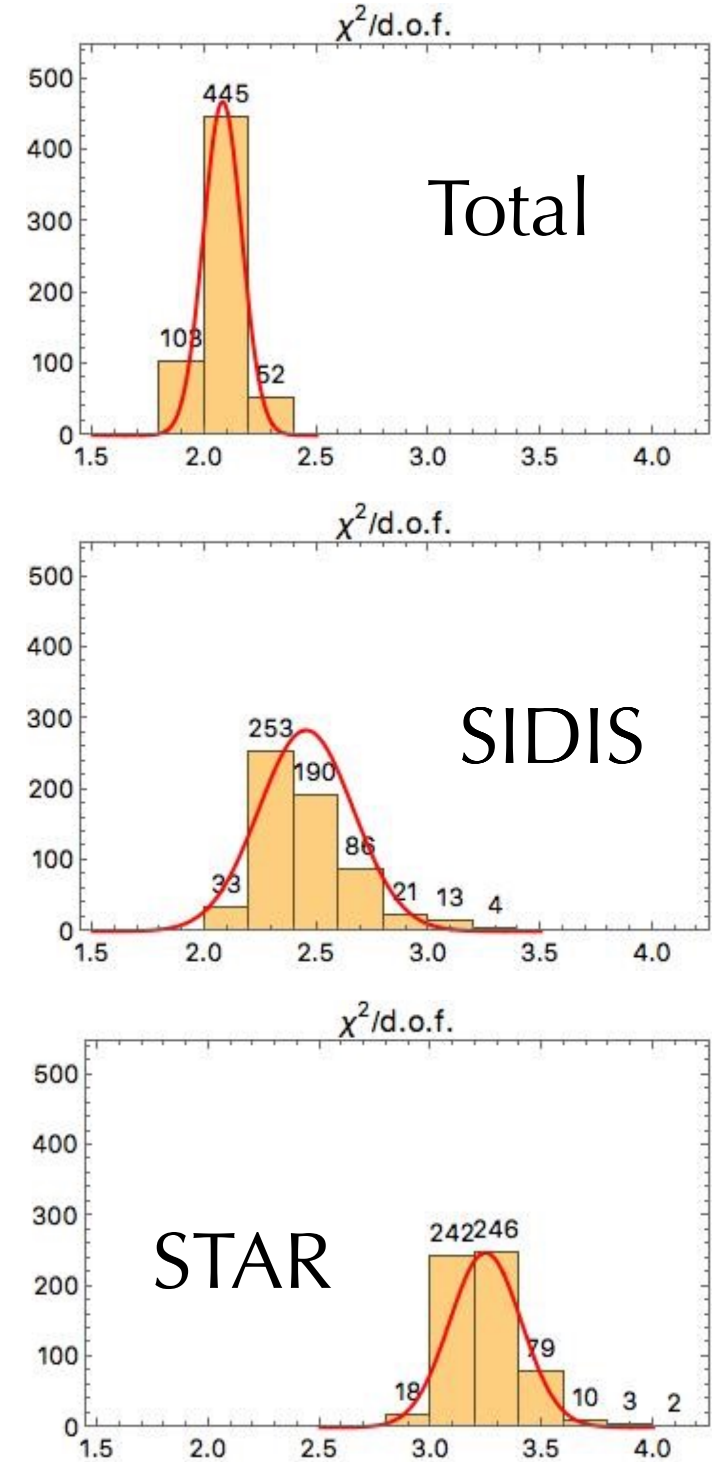
χ^2 of the fit



$\chi^2/\mathrm{dof} = 2.08 \pm 0.09$

38%

62%



~75% (of 38%) from COMPASS (higher precision)
but ~40% (of 75%) from deuteron bins

theoretical uncertainties

Single-Spin Asymmetry
in p-p↑ collisions

$$A_{UT}(\eta, M_h, P_T) = \frac{d\sigma_{UT}}{\boxed{d\sigma_0}}$$

typical cross section for a+b→c+d process

$$d\sigma_0 \propto \sum_{a,b,c,d} \int \frac{dx_a\, dx_b}{8\pi^2 \bar{z}}\, f_1^a(x_a)\, f_1^b(x_b)\, \frac{d\hat{\sigma}_{ab\to cd}}{d\hat{t}}\, D_1^c(\bar{z}, M_h)$$

quark D_1^q is well constrained by e^+e^- (Montecarlo) but

we don't know anything about the gluon D_1^g (e^+e^- doesn't help..)

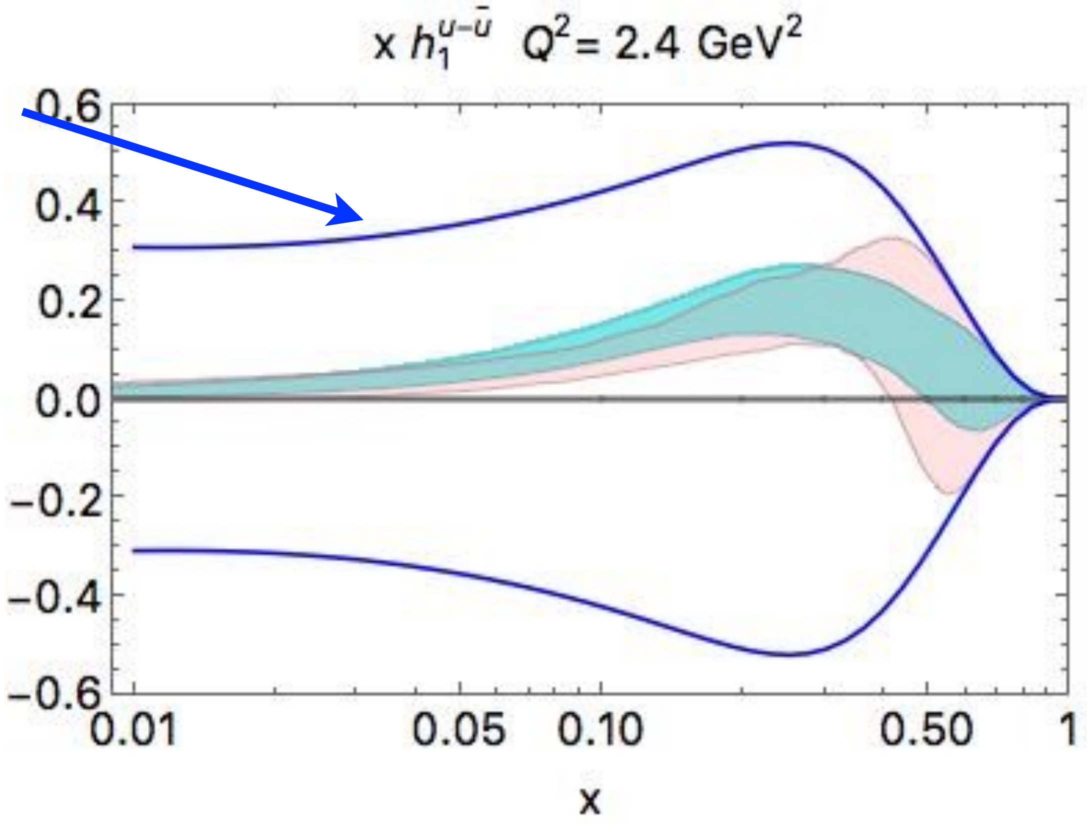
our choice: compute $d\sigma_0$ with $D_1^g(Q_0) = \begin{cases} 0 \\ D_1^u(Q_0)/4 \\ D_1^u(Q_0) \end{cases}$

deteriorates our e^+e^- fit as $\chi^2/\text{dof} = \begin{cases} 1.69 & 1.28 \\ 1.81 & 1.37 \\ 2.96 & 2.01 \end{cases}$

background ρ channels

comparison with previous fit

Soffer bound



global fit

old fit

*Radici et al., JHEP **1505** (15) 123*

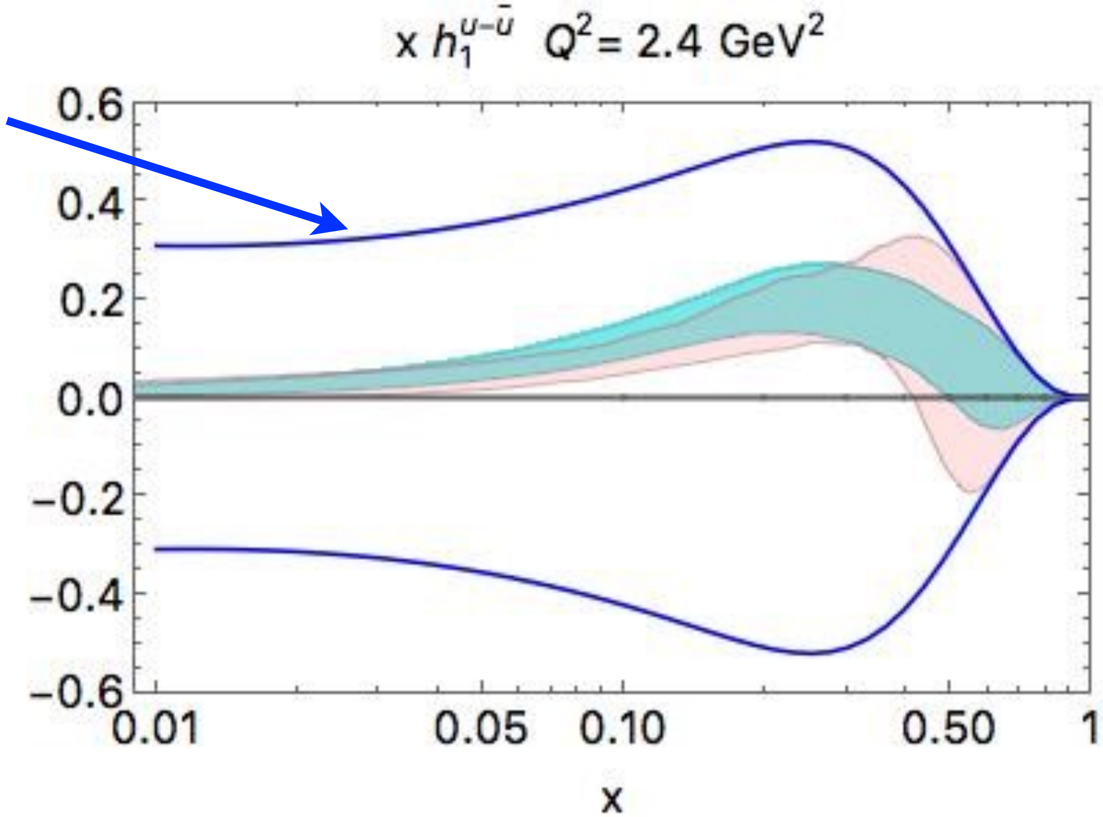
global fit

up

higher precision

comparison with previous fit

Soffer bound



global fit

old fit

Radici et al.,
*JHEP **1505** (15) 123*

global fit

up

higher precision

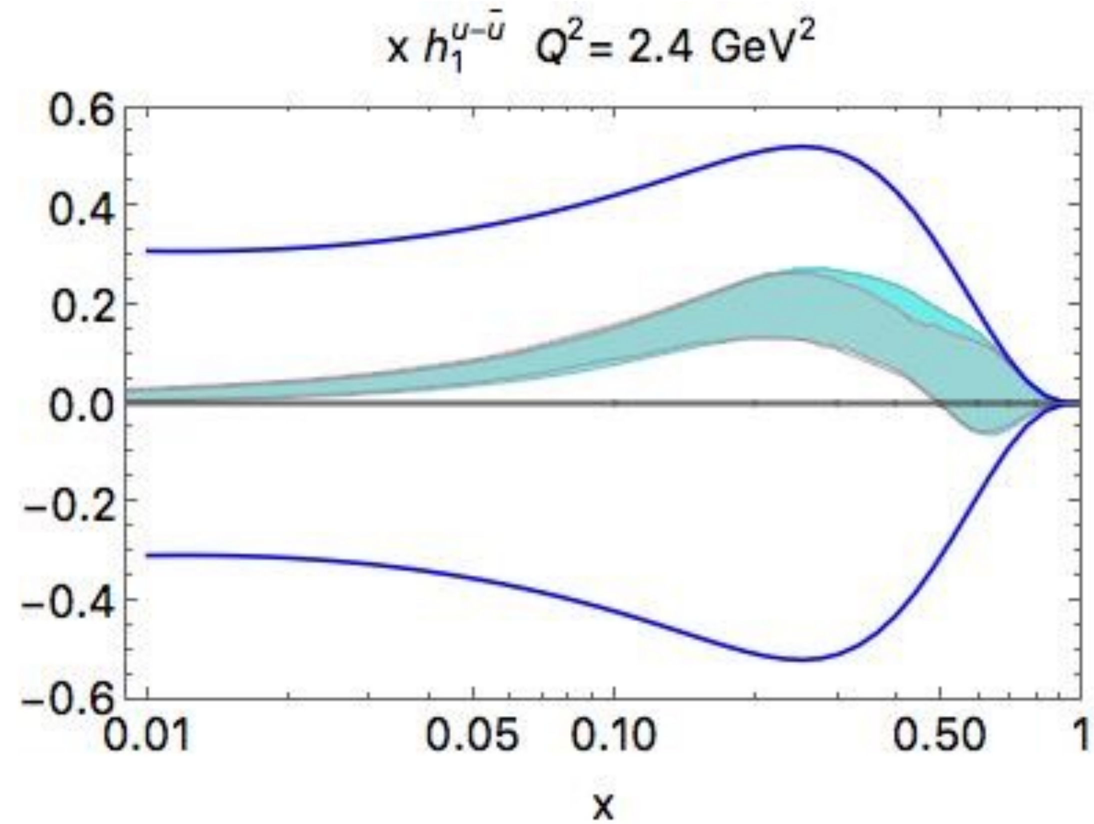
global fit

up

insensitive to uncertainty on gluon D_1

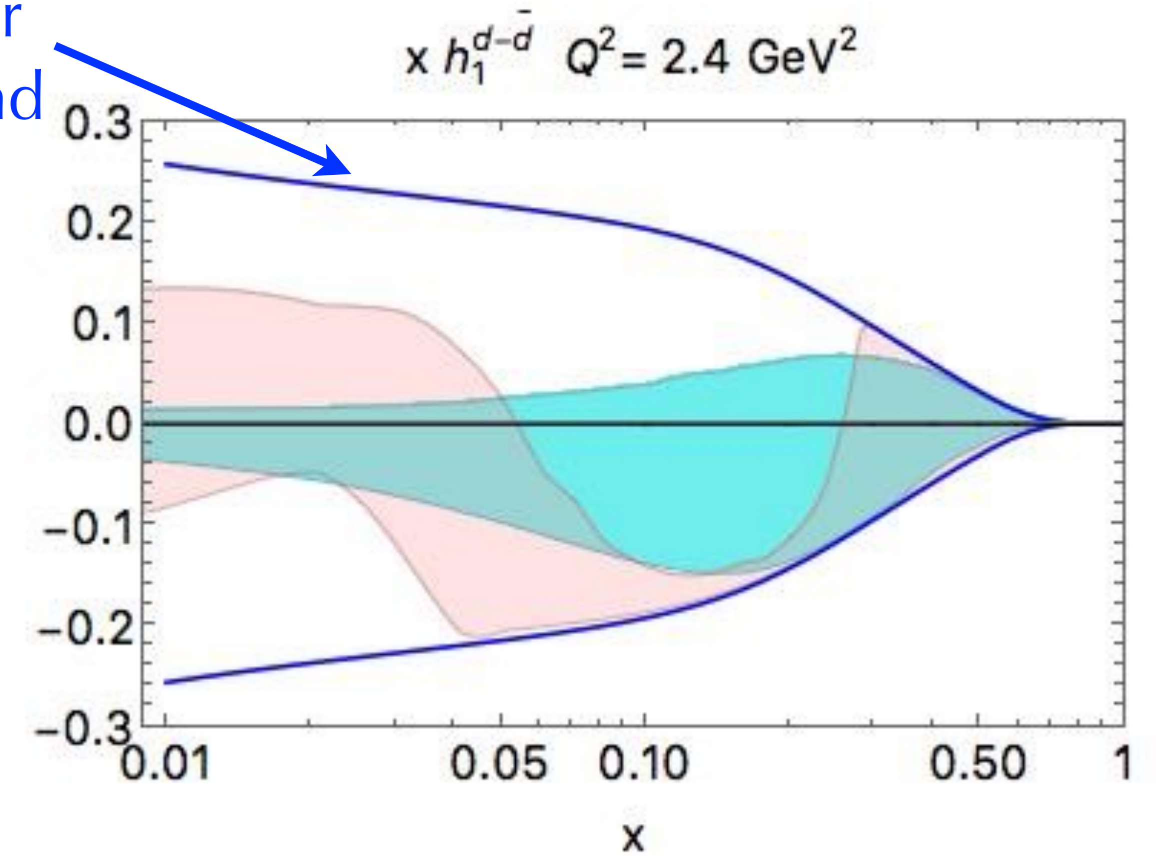
$D_1^g(Q_0) = 0$

$$D_1^g(Q_0) = \begin{cases} 0 \\ D_1^u/4 \\ D_1^u \end{cases}$$



comparison with previous fit

Soffer bound



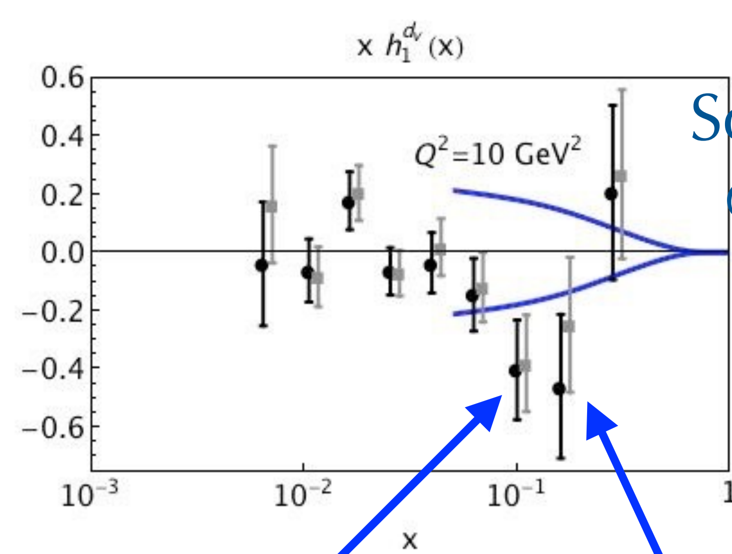
down

global fit

old fit

effect of STAR data : saturation of Soffer bound practically disappeared

*Radici et al., JHEP **1505** (15) 123*



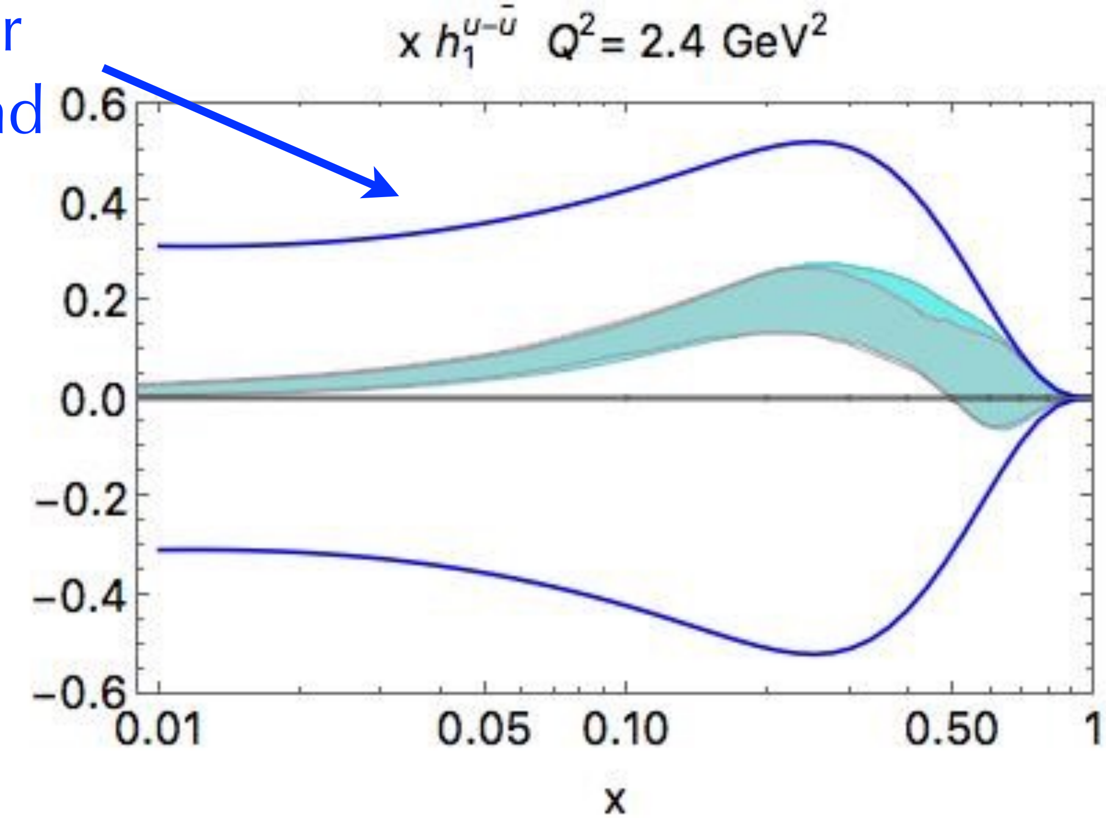
Soffer bound @10 GeV^2

Q^2 ~ 9

Q^2 ~15 GeV^2

sensitivity to gluon $D_1{}^g$

Soffer bound



global fit

up

insensitive to uncertainty on gluon D_1

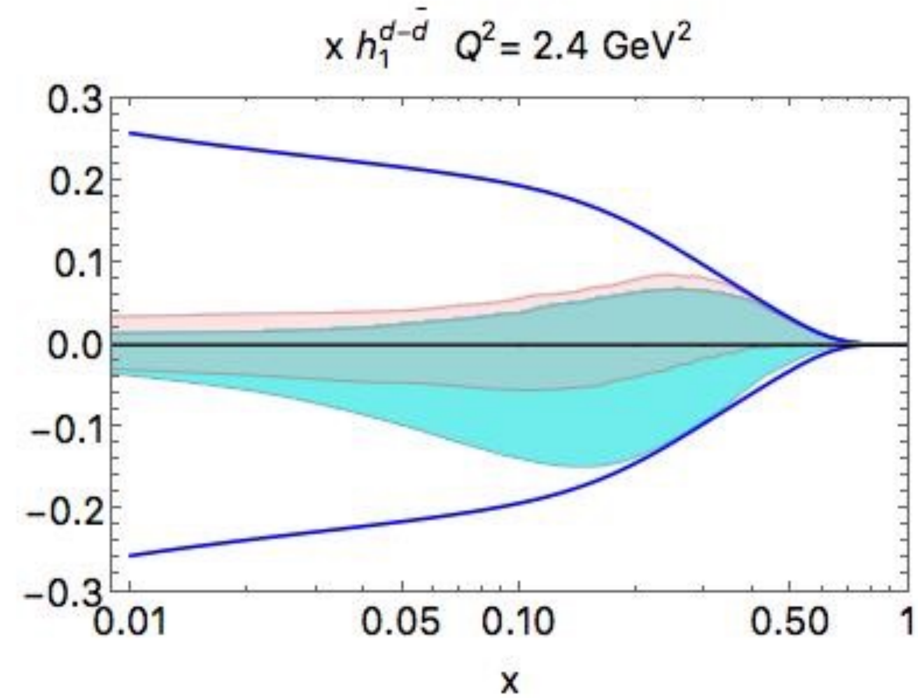
global fit

down

sensitive to uncertainty on gluon D_1

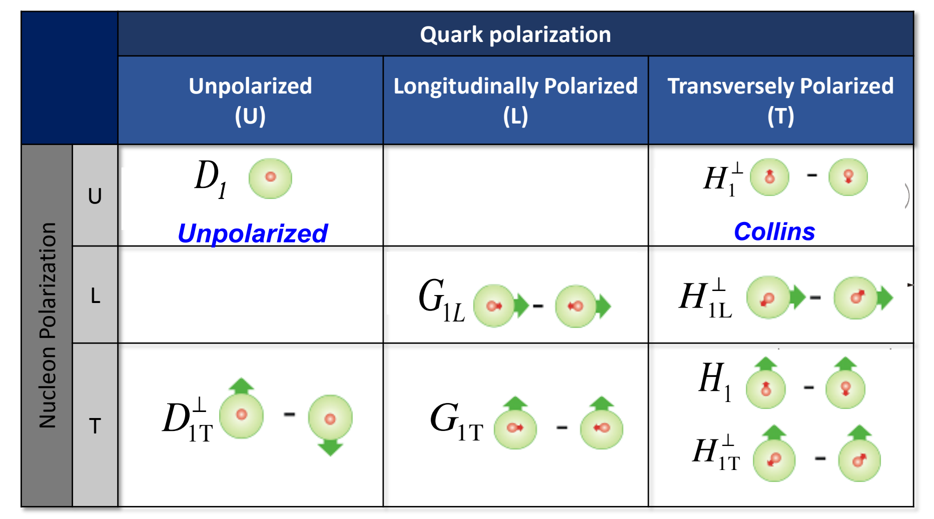
$D_1{}^g (Q_0) = 0$

$$D_1{}^g (Q_0) = \begin{cases} 0 \\ D_1{}^u/4 \\ D_1{}^u \end{cases}$$



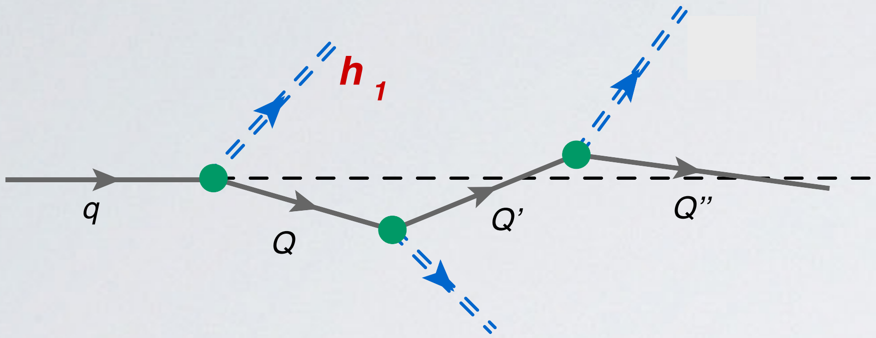
need dihadron multiplicities from RHIC and better deuteron data from COMPASS

MC-based models of FF



- extended NJL quark-jet model
- string fragmentation model

extended NJL quark-jet model



quark hadronization chain

Nambu-Jona Lasinio model at each vertex

$D_1^{q \to h}(z, p_T^2)$ = MC statistical average of proper multiplicities

results for q=*u,d,s* and h= π, K, ρ, K*, φ, N

*Matevosyan et al., P.R. D**83** (11) 114010 D**86** (12) 059904(E)*

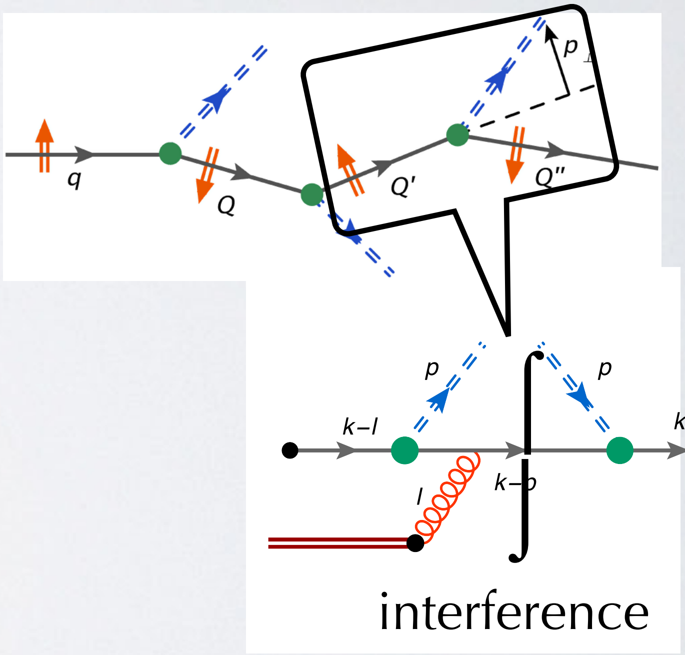
- p_T-dependence deviates from Gaussian
- $\langle p_T^2 \rangle$: flavor- and nonlinear z-dependent

*Matevosyan et al., P.R. D**85** (12) 014021*

- includes transverse momentum and spin correlations

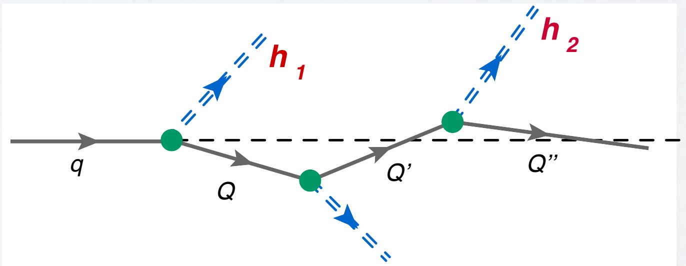
*Bentz et al., P.R. D**94** (16) 034004*

*Matevosyan et al., P.R. D**95** (17) 014021*



- extended to DiFF

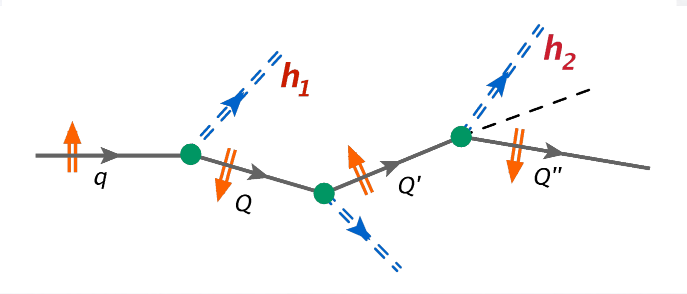
*Matevosyan et al., P.R. D**88** (13) 094022*



- and polarized DiFF

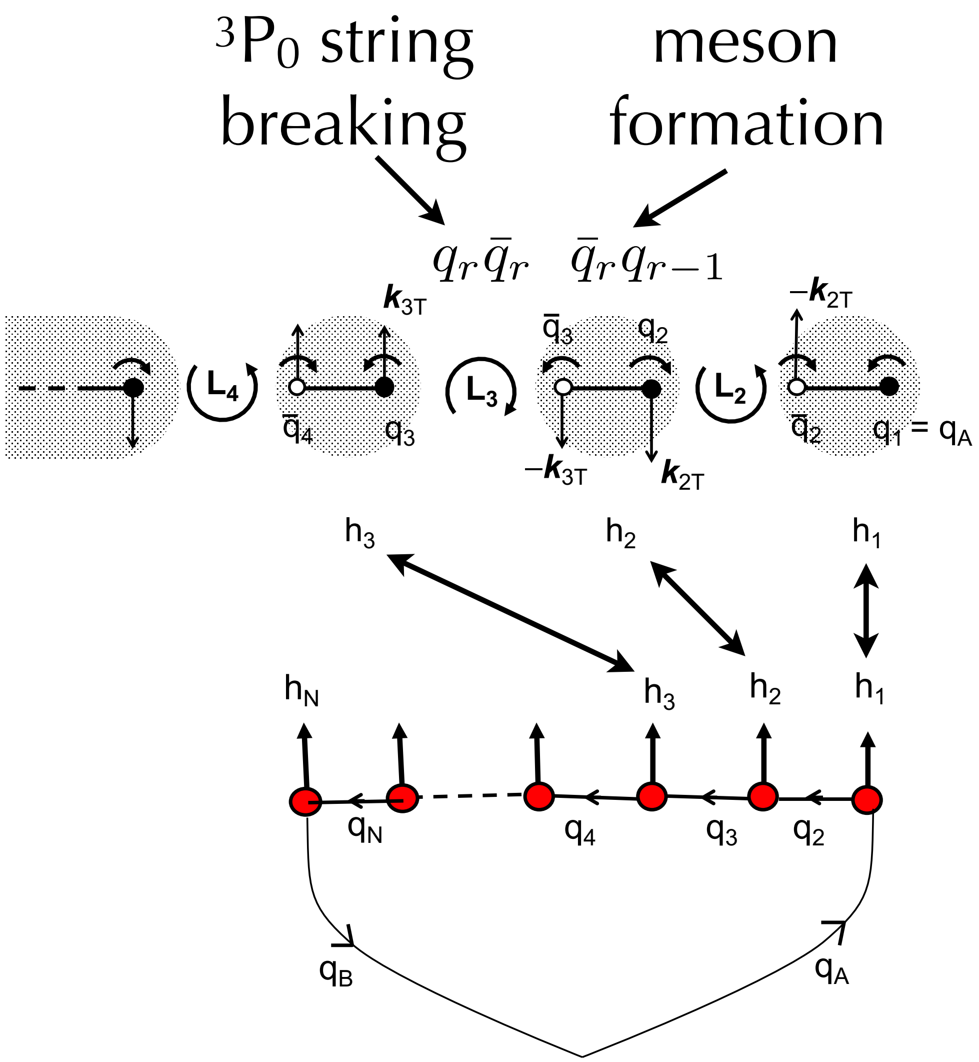
*Matevosyan et al., P.R. D**96** (17) 074010*

*Matevosyan et al., P.R. D**97** (18) 014019*



$H_1^{\sphericalangle}$ as a recursive Collins effect

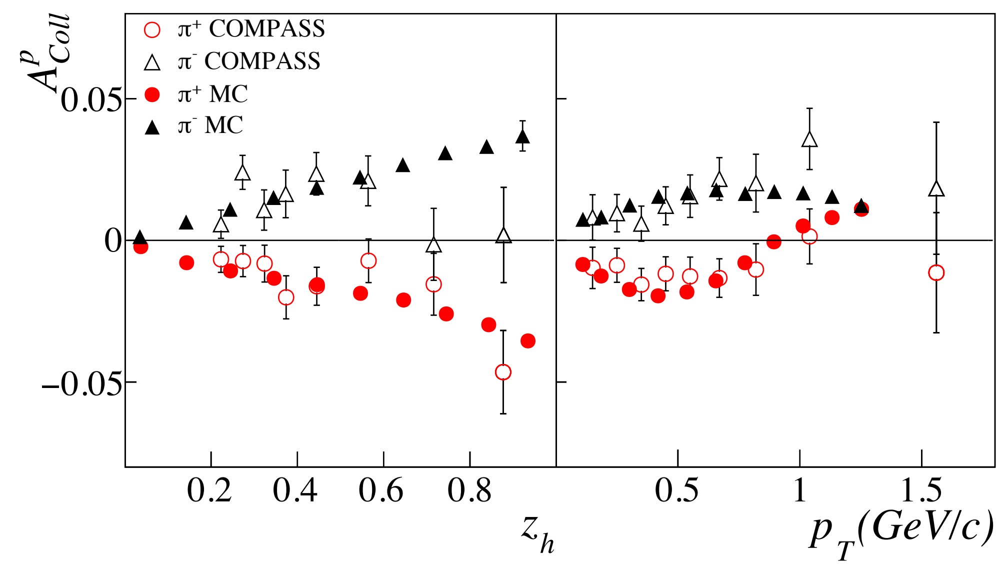
string fragmentation model



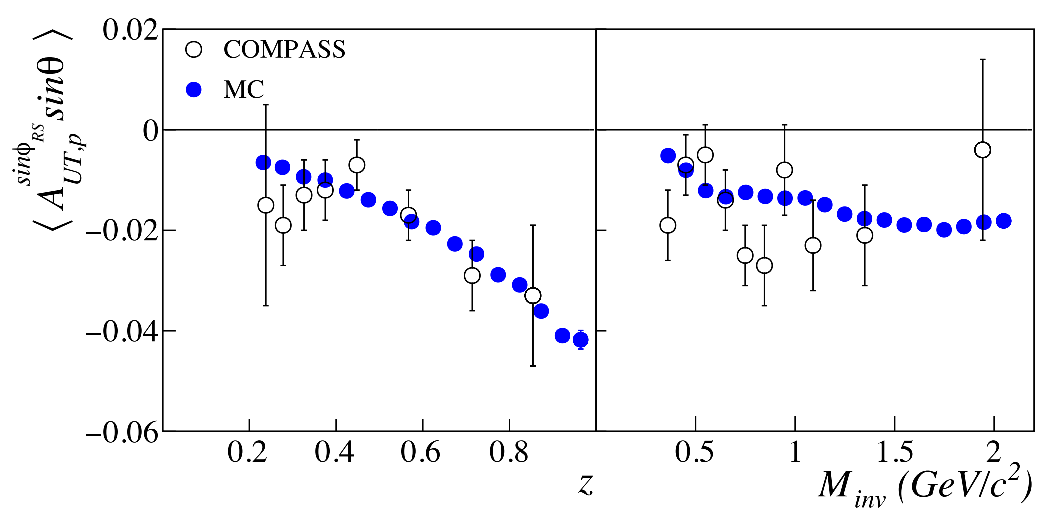
$$q_A \bar{q}_B \rightarrow h_1 + h_2 + \ldots + h_N$$

Kerbizi, Artru, Belghobsi, Bradamante, Martin, arXiv:1802.00962

SIDIS Collins effect



SIDIS di-hadron effect



no resonant structure