Fragmentation Functions 2018

Regina Palace – Stresa (Italy)

Transverse parton momenta in single inclusive hadron production in e^+e^- annihilation processes

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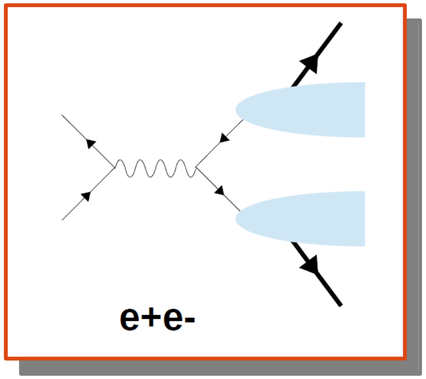
In collaboration with J.O. Gonzalez Hernandez, R. Taghavi

e^+e^- scattering processes



- Recent data on Collins azimuthal asymmetries from BELLE, BaBar and BES III
- No modern data available (yet) on unpolarized cross sections

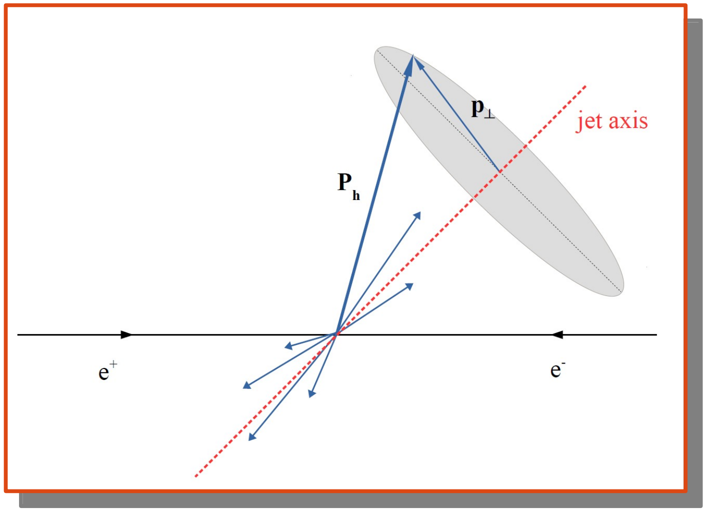
TASSO, MARKII data available for unpolarized $e^+e^- \rightarrow h\,X$

- p_T distributions
- Different energies

TMD evolution

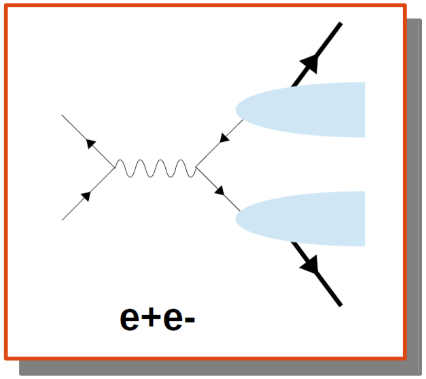
- Integrated over z
- No charge separation

Big limitation



- Direct observation of the hadron momentum component, transverse to the fragmenting parent parton (jet axis)

e^+e^- scattering processes



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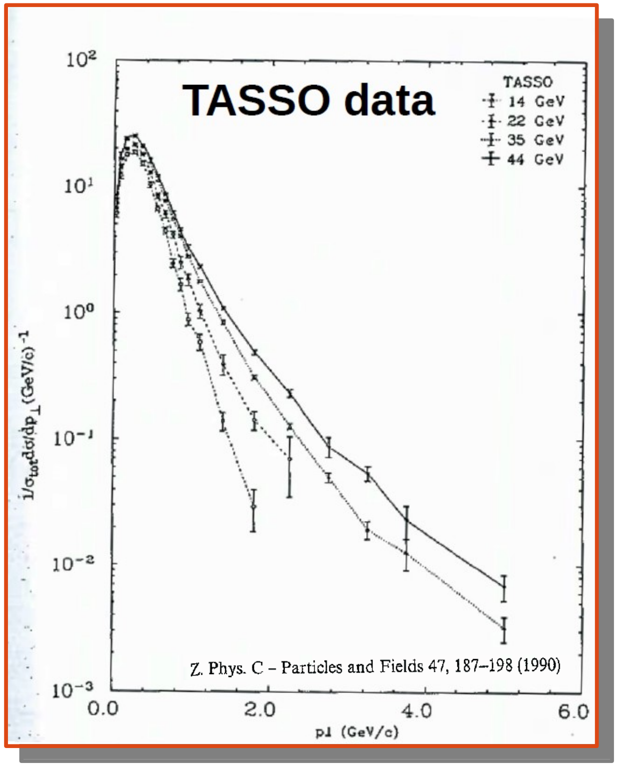


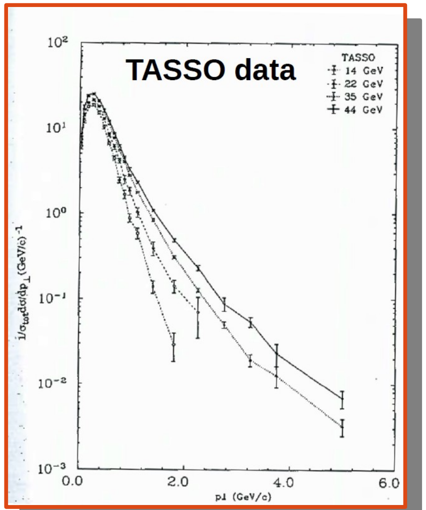
- Direct observation of the hadron momentum component, transverse to the fragmenting parent parton (jet axis)

Unpolarized cross section

$$\frac{d\sigma^h}{dz\,d^2\boldsymbol{p}_\perp} = L_{\mu\nu}W^{\mu\nu} = \frac{4\pi\alpha^2}{3s}\,z\,F_1^h(z,p_\perp;Q^2)$$

$$W^{\mu\nu} = W^{\mu\nu}_{TMD} + W^{\mu\nu}_{coll}$$

$$W^{\mu\nu}_{TMD} \propto \sum_f |\mathcal{H}_f(Q;\mu)|^{\mu\nu} D_{h/f}(z,\boldsymbol{p}_\perp;\mu,\zeta_D)$$



$$D_{h/f}(z,z\boldsymbol{k}_\perp;\mu,\zeta_D) \equiv \frac{1}{(2\pi)^2}\int d^2\boldsymbol{b}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{b}_\perp}\tilde{D}_{h/f}(z,\boldsymbol{b}_\perp;\mu,\zeta_D)\,,$$

$$\tilde{D}_{h/f}(z,\boldsymbol{b}_\perp;\mu,\zeta_D) \equiv$$

$$\sum_j \left[\tilde{C}_{j/f}\otimes d_{h/j}(z;\mu_b)/z^2\right]$$

$$\times\exp\left\{\int_{\mu_b}^{\mu}\frac{d\tilde{\mu}}{\tilde{\mu}}\left[\gamma_D(\alpha_s(\tilde{\mu});1)-\gamma_K(\alpha_s(\tilde{\mu}))\log\left(\frac{\sqrt{\zeta_D}}{\tilde{\mu}}\right)\right]\right\}$$

$$\times\exp\left\{\tilde{K}(b_*;\mu_b)\log\left(\frac{\sqrt{\zeta_D}}{\mu_b}\right)\right\}$$

Calculable within perturbative QCD

$$\times\exp\left\{g_{h/j}(z,b_\perp)+g_K(b_\perp)\log\left(\sqrt{\frac{\zeta_D}{\zeta_D^{(0)}}}\right)\right\}.$$

Non-perturbative information

Modeling the cross section

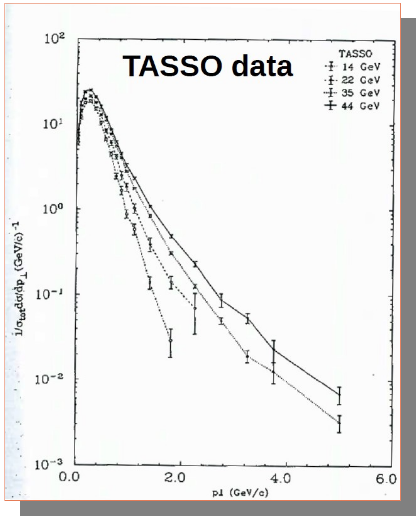
Boglione, Gonzalez-Hernandez, Taghavi, Phys. Lett. B772 (2017) 78-86

Assuming factorization ...

$$\left.\frac{d\sigma^h}{dz\,d^2\boldsymbol{p}_\perp}\right|_{model} = \frac{4\pi\alpha^2}{3s}\sum_q e_q{}^2\, D_q^h(z, p_\perp; Q^2)$$

$$= \frac{4\pi\alpha^2}{3s}\sum_q e_q{}^2\, D_q^h(z, Q^2)\, h(p_\perp)$$

To leading order



Use this ...

To get information about that ...

Issues to investigate:

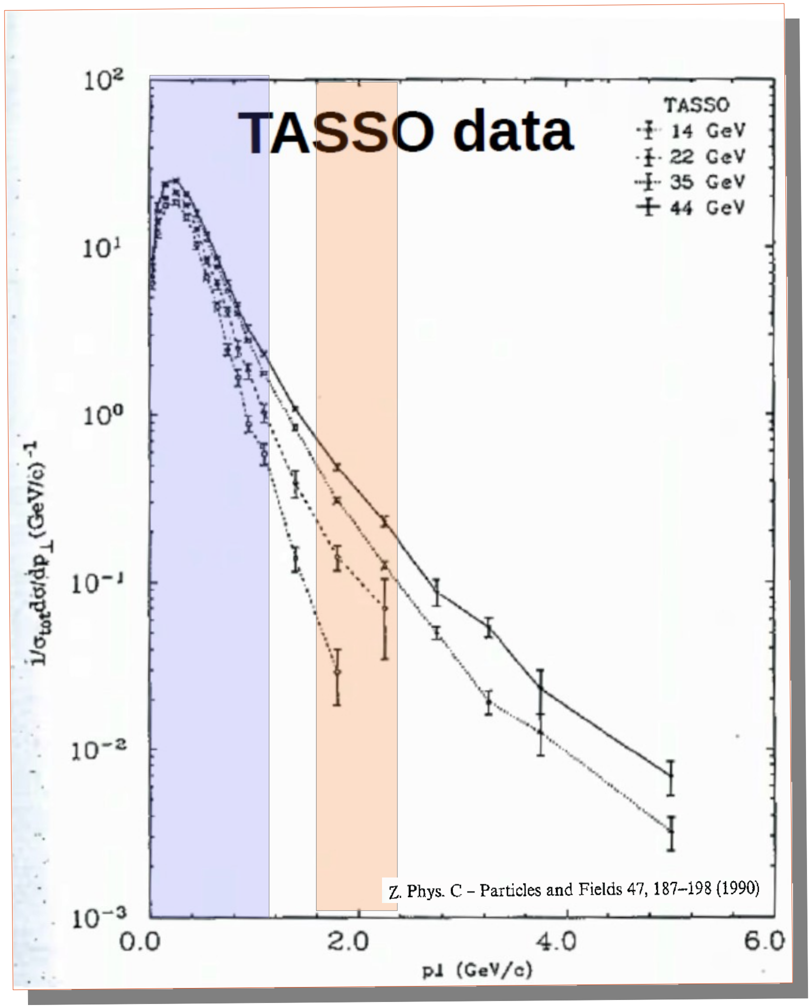
- Appropriate functional form for $g_{j/p}$
- Scale evolution induced by g_k

$$\tilde{D}_{h/q}(z, \boldsymbol{b}_\perp; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2}\right) e^{\Gamma_D(Q)}\right] \exp\left\{g_{j/P}(x, b_\perp) + g_K(b_\perp)\log\left(\frac{Q}{Q_0}\right)\right\}$$

Modeling the cross section

Boglione, Gonzalez-Hernandez, Taghavi, Phys. Lett. B772 (2017) 78-86

The lack of information about **z** hinders a full TMD extraction of the FF.



Identify the region where TMD effects are dominant

For fully differential cross sections, matching region is Expected to be at

$$p_\perp \sim zQ$$

Use experimental **<z>** to make an estimate

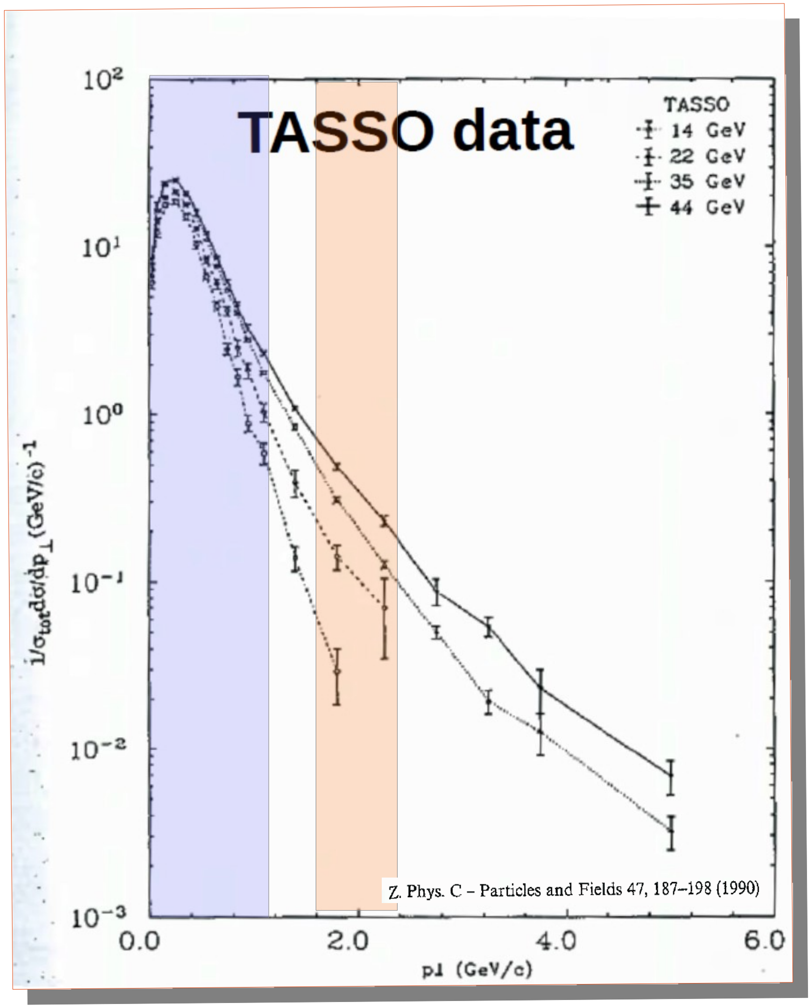
$$p_\perp \sim 2\,\mathrm{GeV}$$

We start by concentrating on a restricted range:

$$p_\perp < 1\ \mathrm{GeV}$$

Modeling the $p_\perp$ dependence

Boglione, Gonzalez-Hernandez, Taghavi, Phys. Lett. B772 (2017) 78-86



$$D_q^h(z, p_\perp) = D_q^h(z)\, h(p_\perp)$$

The lack of information about **z** hinders a full TMD extraction of the FF.

- Gaussian $p_\perp$ dependence

$$h(p_\perp) = \frac{e^{-p_\perp{}^2/\langle p_\perp{}^2\rangle}}{\pi \langle p_\perp{}^2\rangle}$$

- Power law $p_\perp$ dependence

$$h(p_\perp) = 2(\alpha - 1) M^{2\,(\alpha-1)} \frac{1}{\left(p_\perp^2 + M^2\right)^\alpha}$$

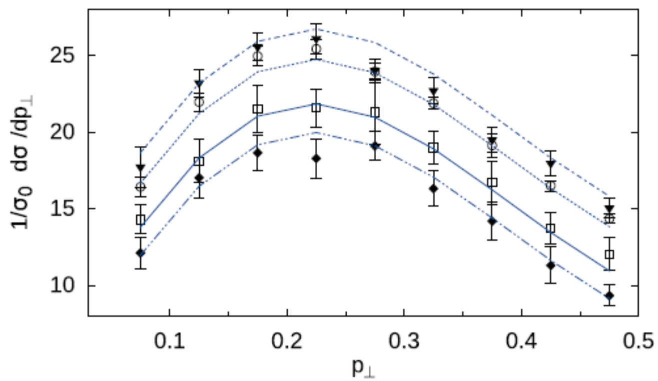
$$M^2 = (2\alpha - 1)\, p_{0\perp}{}^2$$
$$p_{0\perp} = 0.212\ \text{GeV}\,.$$

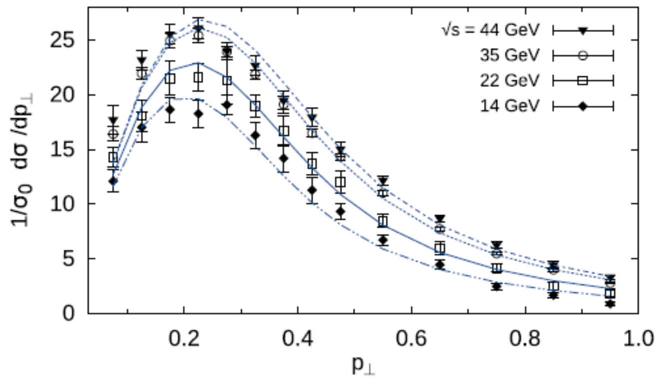
Modeling the $p_\perp$ dependence

Fit of TASSO data,
using **gaussian** $p_\perp$ dependence

1) $$\frac{1}{\sigma_0}\frac{d\sigma^h}{dp_\perp}\bigg|_{model} \longrightarrow \pi\, p_\perp N \left[\int dz\, \frac{\sum_q e_q^2 D_q^h(z;Q^2)}{\sum_q e_q^2}\right] h(p_\perp) + \delta\, Q$$

Normalization problematic !





The lack of information about **z** hinders a full TMD extraction of the FF.

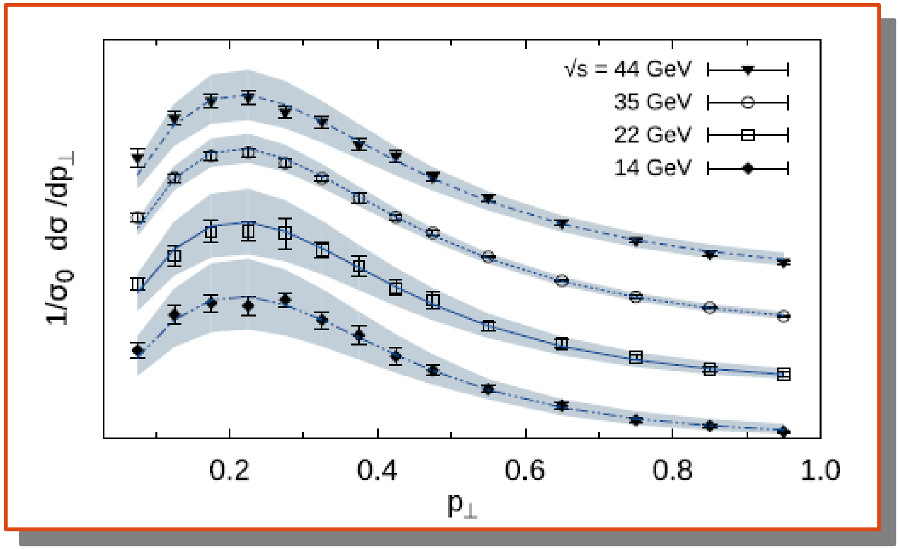
2) $$\langle p_\perp^2 \rangle = 2\, g_1 + 2\, g_2\, z^2 \log\left(\frac{Q}{2\, Q_0}\right)$$

Tension between the shape of the peak and that of the tale

Fit of TASSO data,
using **power law** $p_\perp$ dependence

$$h(p_\perp) = 2(\alpha - 1) \mathrm{M}^{2\,(\alpha-1)} \frac{1}{(p_\perp^2 + \mathrm{M}^2)^\alpha}$$

1) α extracted separately for each √s value



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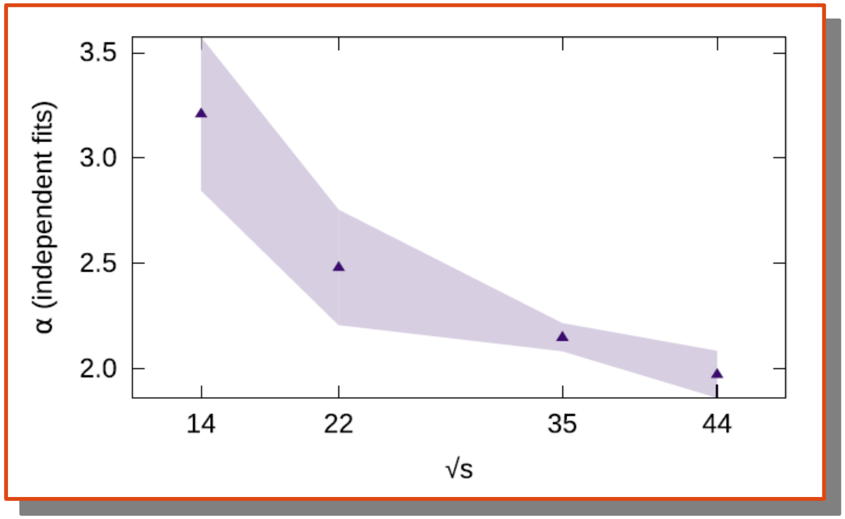
Interpreting our results ...

MODEL

$$h(p_\perp) = 2(\alpha - 1)\mathrm{M}^{2\,(\alpha-1)} \frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$



TMD scheme

(under the assumption that integration over z does not alter the structural form of the non perturbative exponential)

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\tilde{g}(b_\perp)\log\left(\frac{Q}{Q_0}\right)\right\}$$

$$b_\perp^{\alpha_0}\exp\left\{\tilde{g}(b_\perp)\log\left(\frac{Q}{Q_0}\right)\right\} \propto b_\perp^{\alpha}$$

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha}\log\left(\frac{Q}{Q_0}\right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha}\log(\nu\, b_\perp)$$

Modeling the $p_\perp$ dependence

Fit of TASSO data,
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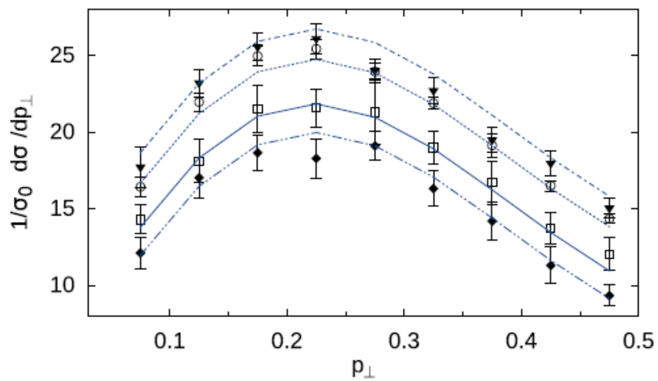
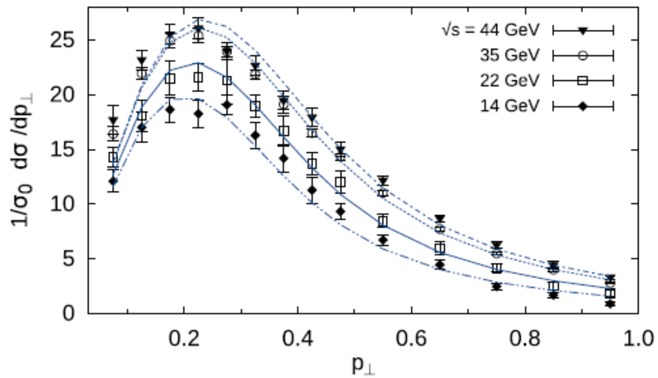
1) $$\frac{1}{\sigma_0}\frac{d\sigma^h}{dp_\perp}\bigg|_{model} \longrightarrow \pi\, p_\perp N\left[\int dz\, \frac{\sum_q e_q^2\, D_q^h(z;Q^2)}{\sum_q e_q^2}\right] h(p_\perp) + \delta\, Q$$

Normalization problematic !





2) $$\langle p_\perp^2 \rangle = 2\, g_1 + 2\, g_2\, z^2 \log\left(\frac{Q}{2\, Q_0}\right)$$

Tension between the shape of the peak and that of the tale

The lack of information about **z** hinders a full TMD extraction of the FF.

Fit of TASSO data,
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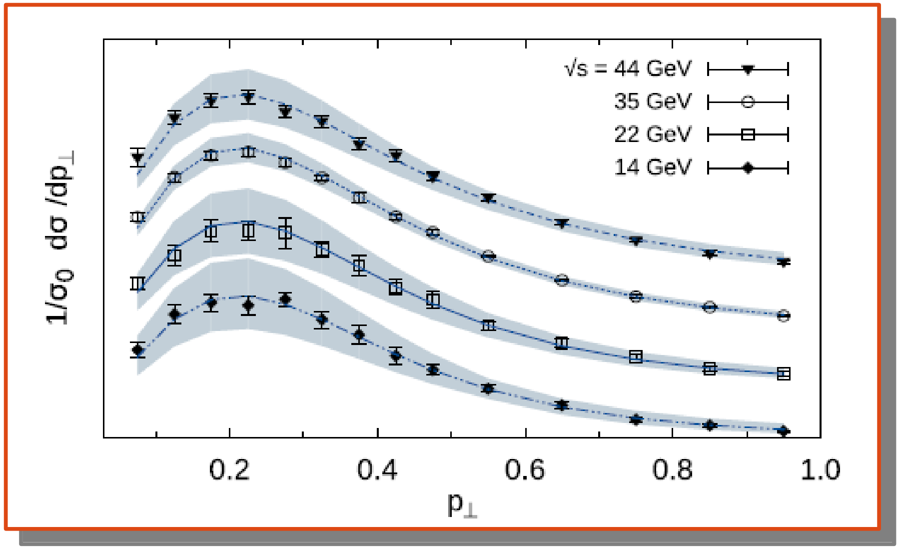

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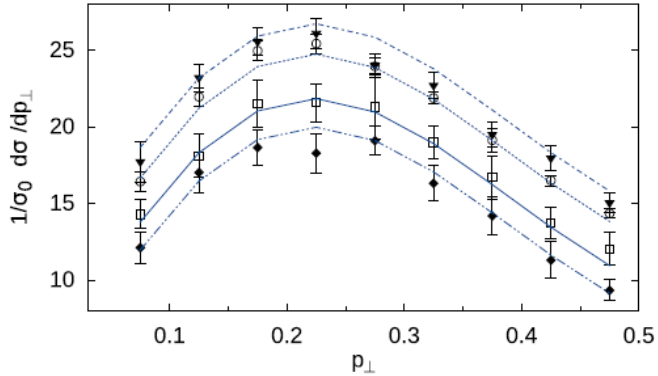

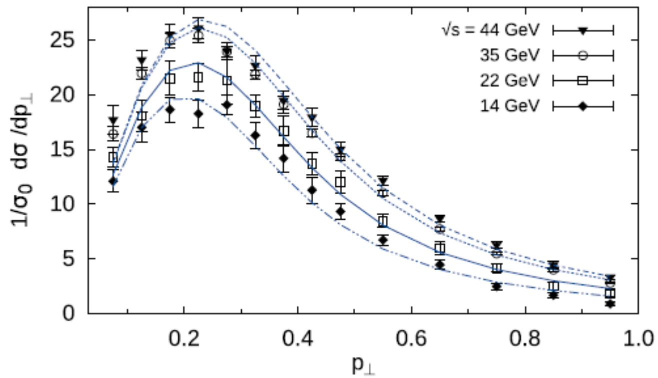

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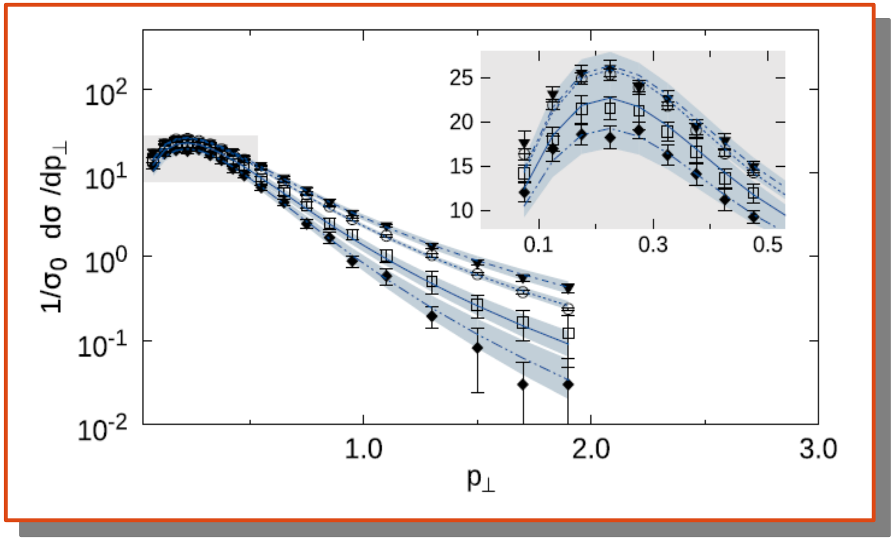
Power-law fits can be
Extended to much larger $p_\perp$



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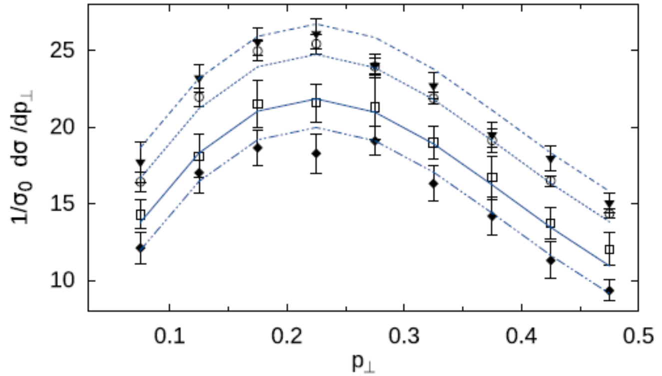

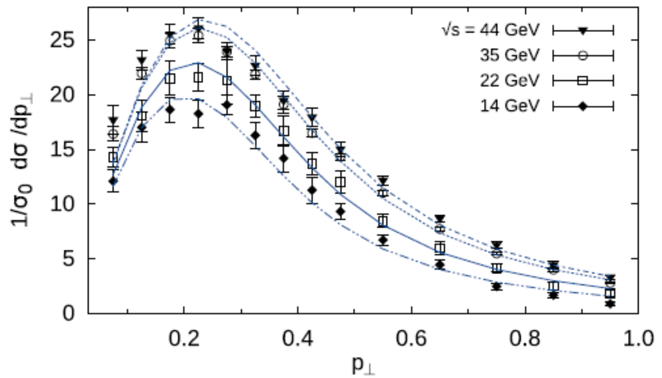

$$\langle p_\perp^2\rangle = 2\,g_1 + 2\,g_2\, z^2 \log\left(\frac{Q}{2\,Q_0}\right)$$

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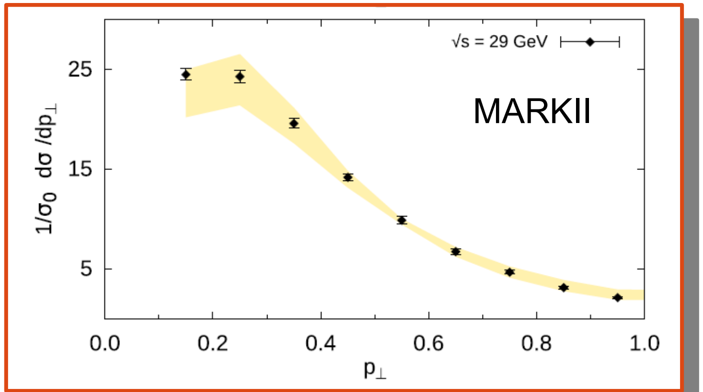

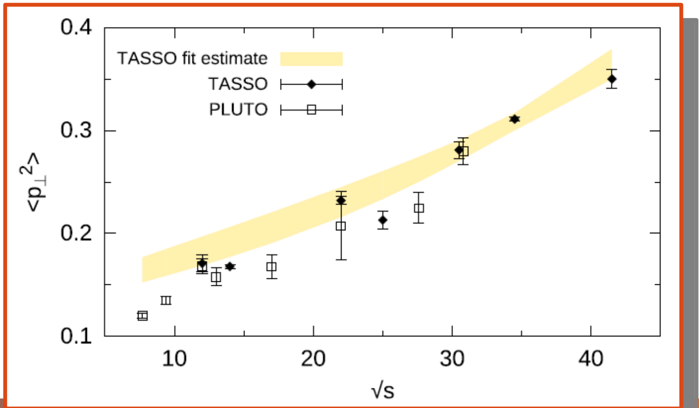

Interpreting our results ...

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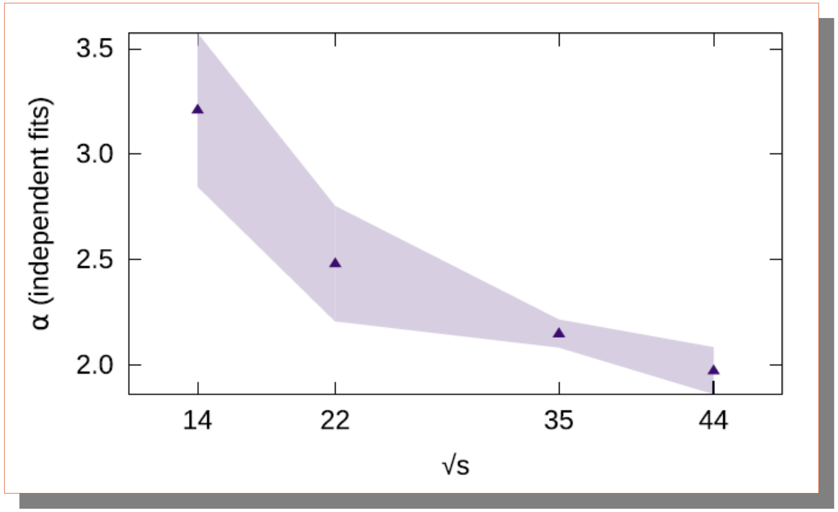

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Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

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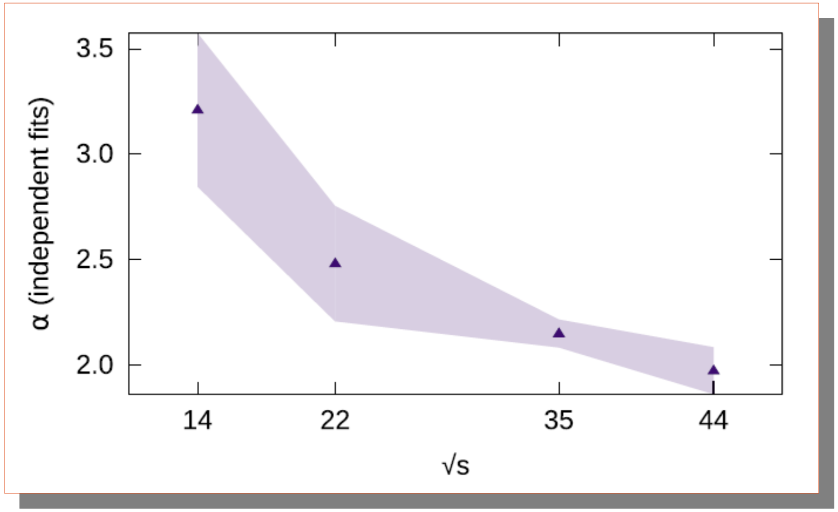

TMD scheme

(under the assumption that integration over z does not alter the structural form of the non perturbative exponential)

There are caveats on this interpretation:

- it is consistent with theoretical expectations but it is not unique.
- Lack of information on z-dependence of the TMD FF in the TASSO and MARK II measurements (and possible correlations between Q and z of different origin) hinders a more solid conclusion about TMD evolution effects in these data sets.

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

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$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha}\log(\nu\, b_\perp)$$

Conclusions

- Phenomenological studies of TMD factorization and evolution have come a long way.
- Some issues remain open and need further investigation, especially as far as phenomenology is concerned:
  - Difficult to work in b_T space where we loose phenomenological intuition
- P_T distributions of SIDIS cross sections over the full P_T range will have to be further investigated (matching and all that).
- Similarly, we need to study e+e- cross sections over the full $p_\perp$ range
- Simultaneous fits of SIDIS, Drell-Yan and e^+e^- annihilation data are recommended, but they should be performed within a consistent and solid framework where they can be implemented.
- Data selection is crucial in global fitting:
  - not too many
    (only data within the ranges where the TMD evolution schemes work should be considered)
  - not too few
    (too strict a selection can bias the fit results and neglect important information from experimental data)

Back-up slides

Table 1
Upper and central panels: center of mass energies and corresponding z_h mean values for the TASSO and MARK II cross sections. Lower panel: center of mass energies corresponding to PLUTO measurements of $\langle p_\perp^2 \rangle$.

Experiment	c.m. energy	$\langle z_h \rangle$
TASSO	14 GeV	0.13
	22 GeV	0.11
	35 GeV	0.09
	44 GeV	0.08
MARK II	29 GeV	0.09
PLUTO	7.7 GeV	–
	9.4 GeV	–
	12.0 GeV	–
	13.0 GeV	–
	17.0 GeV	–
	22.0 GeV	–
	27.6 GeV	–

Table 2
Fits of the TASSO four sets of cross sections, corresponding to the Gaussian parameterization of the $p_\perp$ distributions. Parametrization I refers to the usual choice of Eq. (14), Parametrization II refers to the Gaussian corrected by a Q-dependent shift, see Eq. (15), while Parametrization III corresponds to a Gaussian distribution with a Q dependent width, as in Eq. (16).

Parametrization	Normalization	Gaussian width	χ^2_{pt}
Gaussian – I $p_\perp \in [0.03 - 0.50]$ GeV 36 data points	$N = \{N_{14}, N_{22}, N_{35}, N_{44}\}$ $N_{14} = 2.3 \pm 0.2$, $N_{22} = 2.7 \pm 0.2$ $N_{35} = 3.1 \pm 0.1$, $N_{44} = 3.2 \pm 0.1$	$\langle p_\perp^2 \rangle$ = constant $\langle p_\perp^2 \rangle = 0.118 \pm 0.004$ GeV^2	5.9
Gaussian – II $p_\perp \in [0.03 - 0.50]$ GeV 36 data points	N, $\delta\, Q$ $N = 1.8 \pm 0.2$ $\delta = 0.22 \pm 0.03$ GeV^{-2}	$\langle p_\perp^2 \rangle$ = constant $\langle p_\perp^2 \rangle = 0.098 \pm 0.005$ GeV^2	0.74
Gaussian – III $p_\perp \in [0.03 - 1.00]$ GeV 56 data points	$N = \{N_{14}, N_{22}, N_{35}, N_{44}\}$ $N_{14} = 2.7 \pm 0.2$, $N_{22} = 3.3 \pm 0.3$ $N_{35} = 4.0 \pm 0.1$, $N_{44} = 4.3 \pm 0.2$	$\langle p_\perp^2 \rangle = 2g_1 + 2g_2 z^2 \log \frac{Q}{3.2}$ $g_1 = 0.013 \pm 0.004$ GeV^2 $g_2 = 2.6 \pm 0.3$ GeV^2	2.7

Table 3
Fits of the TASSO four cross sections, corresponding to the power-law parameterization of the $p_\perp$ distributions. Parametrization I refers to 4 independent fits (one for each data set corresponding to a different c.m. energy) using the functional form of Eq. (17), with constant α and M^2 parameters. Parametrization II refers to the simultaneous fit of the four data sets, using the functional form of Eq. (17), with constant α and M^2 parameters. Parametrization III refers to the simultaneous fit of the four data sets, using the functional form of Eq. (17), with Q-dependent α and M^2 parameters. Parametrization IV refers to the same case as III, but now the fit is performed on the extended range $p_\perp < 2$ GeV, for which we free the parameter $p_{0\perp}$.

Parametrization	Normalization $N = \{N_{14}, N_{22}, N_{35}, N_{44}\}$	Parameters	χ^2_{pt}
Power-law – I	$N_{14} = 2.6 \pm 0.1$	$\alpha = \{\alpha_{14}, \alpha_{22}, \alpha_{35}, \alpha_{44}\}$	$\chi^2_{14} = 0.35$
$p_\perp \in [0.03 - 1.00]$ GeV	$N_{22} = 3.2 \pm 0.2$		$\chi^2_{22} = 0.30$
14×4 data point	$N_{35} = 4.0 \pm 0.1$	$\alpha_{14} = 3.3 \pm 0.4$, $\alpha_{22} = 2.5 \pm 0.3$	$\chi^2_{35} = 0.88$
	$N_{44} = 4.4 \pm 0.2$	$\alpha_{35} = 2.2 \pm 0.1$, $\alpha_{44} = 2.0 \pm 0.1$	$\chi^2_{44} = 0.84$
Power-law – II	$N_{14} = 2.6 \pm 0.2$	α = constant	
$p_\perp \in [0.03 - 1.00]$ GeV	$N_{22} = 3.3 \pm 0.2$		
56 data points	$N_{35} = 4.0 \pm 0.1$	$\alpha = 2.2 \pm 0.1$	2.87
	$N_{44} = 4.2 \pm 0.2$		
Power-law – III	$N_{14} = 2.6 \pm 0.2$	$\alpha = \alpha_0 + \tilde{\alpha} \log(Q/Q_0)$	
$p_\perp \in [0.03 - 1.00]$ GeV	$N_{22} = 3.3 \pm 0.2$	$Q_0 = 14$ GeV	
56 data points	$N_{35} = 4.0 \pm 0.1$		0.66
	$N_{44} = 4.4 \pm 0.2$	$\alpha_0 = 3.1 \pm 0.4$, $\tilde{\alpha} = -1.0 \pm 0.4$	
Power-law – IV	$N_{14} = 2.6 \pm 0.2$	$\alpha = \alpha_0 + \tilde{\alpha} \log(Q/Q_0)$	
$p_\perp \in [0.03 - 2.00]$	$N_{22} = 3.2 \pm 0.3$	$Q_0 = 14$ GeV	
76 data points	$N_{35} = 4.0 \pm 0.1$	$\alpha_0 = 3.5 \pm 0.3$, $\tilde{\alpha} = -1.1 \pm 0.3$	0.95
	$N_{44} = 4.3 \pm 0.2$	$p_{0\perp} = 0.219 \pm 0.005$ GeV	

Counter-example

- Start from a simple picture in which:

$$\frac{d\sigma}{dz\, d^2 \boldsymbol{p}_\perp} \propto \left(\frac{1}{p_\perp^2 + z\, \tilde{M}^2} \right)^{\beta_1 + \beta_2\, z}$$

- Account for z integration by some average value <z>:

$$\frac{d\sigma}{d^2 \boldsymbol{p}_\perp} \propto \left(\frac{1}{p_\perp^2 + \langle z \rangle\, \tilde{M}^2} \right)^{\beta_1 + \beta_2\, \langle z \rangle}$$

- Obtain a good description of data by using the measured <z> values, which exhibit a seemingly logarithmic behavior.

- In this way correlations between z and Q are not related to TMD factorization, but rather of a different origin.

Interpreting our results ...

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\, d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\left(\lambda_\Gamma(b_*) + g_K(b_\perp)\right)\log\left(\frac{Q}{Q_0}\right)\right\}\Bigg|_{b_\perp \to z\, b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

MODEL $$h(p_\perp) = 2(\alpha-1)\mathrm{M}^{2\,(\alpha-1)}\frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}\right\} \xrightarrow{\text{large}\, b_\perp} \frac{1}{2^\alpha\,\pi\,\Gamma(\alpha)}\left(\frac{b_\perp}{\mathrm{M}}\right)^{\alpha-1}\sqrt{\frac{\pi}{2}}\,\frac{e^{-b_\perp \mathrm{M}}}{\sqrt{b_\perp \mathrm{M}}}\left[1 + \mathcal{O}\left(\frac{1}{b_\perp \mathrm{M}}\right)\right]$$