TMD FFs from SIDIS

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Fragmentation Functions 2018

Stresa, Feb. 21st 2018

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Outline

1) TMD FFs

2) extraction from SIDIS data

3) what's next

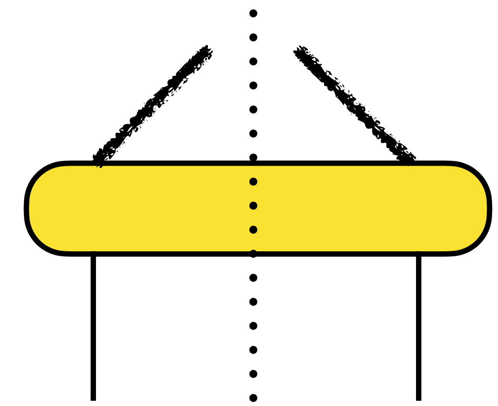
TMD FFs

Definition

[Parton Fragmentation Functions (Metz-Vossen) - DOI: 10.1016/j.ppnp.2016.08.003](https://doi.org/10.1016/j.ppnp.2016.08.003)

$$\Delta_{ij}(z, P_{h\perp}) = \frac{1}{2z} \sum_{\mathcal{X}} \int \frac{d\xi^+ \xi_T^2}{(2\pi^3)}\; e^{ik\xi}\; \langle 0|\psi_i(\xi)|h\mathcal{X}\rangle\langle\mathcal{X}h|\overline{\psi}_j(0)|0\rangle$$

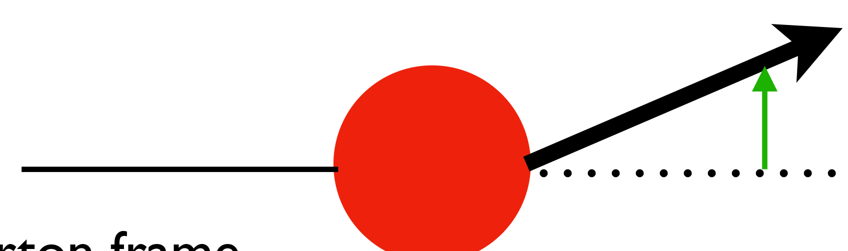
Hadronic variables
(probabilistic interpretation)

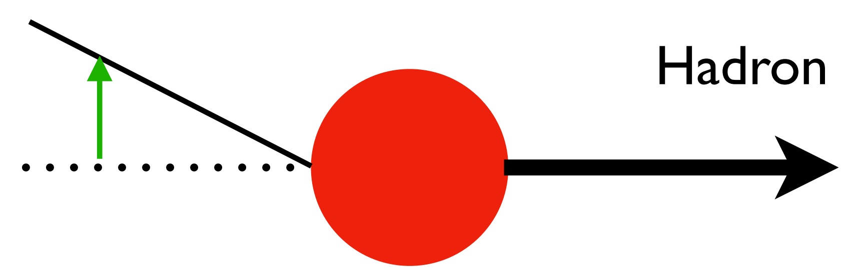


Single hadron FF

J. Collins - Foundations of pQCD (2011)

Although the parton frame is a natural one for defining fragmentation functions as number densities, it is inconvenient for derivations of factorization. The problem is that, in a physical process, there is an integral over parton momentum, and so the parton-frame axes are not fixed. Neither parton momenta nor the resulting parton-frame axes can be determined from experimentally measured quantities. Therefore we will express the definitions of fragmentation functions in hadron-frame coordinates. In the derivation of factorization, we will use a hadron frame defined in terms of measured quantities.





Different frames for different purposes

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TMD FFs

quark pol.

hadron pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_1, $H_{1T}^\perp$

Twist-2 table

Diagonal: also collinear
Red: T-odd (but universal, unlike TMD PDFs)
Blue: T-even

A similar table exists for gluon TMD FFs

Correlator for spin 1/2 hadron:

Dirac matrix parametrized by
quark TMD FFs

$$D_1^{a\to h}(z, P_\perp^2; \mu, \zeta)$$

Evolution equations with respect to two scales:
- UV renormalization
- rapidity renormalization

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TMDs and their evolution

FT of TMDs :

$$\tilde{F}_i(x, b_T; Q, Q^2) = \tilde{F}_i(x, b_T, \mu_{\hat{b}}, \mu_{\hat{b}}^2) \times$$

$$\exp\left\{ \int_{\mu_{\hat{b}}}^{Q} \frac{d\mu}{\mu} \gamma_F[\alpha_s(\mu), Q^2/\mu^2] \right\} \left(\frac{Q^2}{\mu_{\hat{b}}^2} \right)^{-K(\hat{b}_T;\mu_{\hat{b}})-g_K(\overline{b}_T;\{\lambda\})}$$

Sudakov form factor : perturbative and **nonperturbative** contributions

(input) TMD distribution : Wilson coefficients and **intrinsic part**

Collinear distribution!

$$\tilde{F}_i(x, b_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) = \sum_{j=q,\bar{q},g} C_{i/j}(x, \hat{b}_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) \otimes f_j(x; \mu_{\hat{b}}) \tilde{F}_{i,NP}(x, \overline{b}_T; \{\lambda\})$$

Nonperturbative parts : **power corrections to perturbative calculations**

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TMDs and their evolution

Distribution for intrinsic transverse momentum (and its FT):

$$\tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\})$$

Which form ?

Soft gluon emission

$$g_K(\bar{b}_T; \{\lambda\})$$

TMDs and their evolution

Distribution for intrinsic transverse momentum (and its FT):

$$\tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\})$$

Which form ?

Soft gluon emission

$$g_K(\bar{b}_T; \{\lambda\})$$

Separation of **b_T regions**

$$\hat{b}_T(b_T; b_{\min}, b_{\max}) \sim \begin{cases} b_{\max} \ , & b_T \to +\infty \\ b_T \ , & b_{\min} \ll b_T \ll b_{\max} \\ b_{\min} \ , & b_T \to 0 \end{cases}$$

High b_T limit : avoid Landau pole

Low b_T limit : recover fixed order expression

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Extraction from SIDIS

What do we know ?

(only a selection of results!)

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	✘	✘	✔	✔	309

[courtesy A. Bacchetta]

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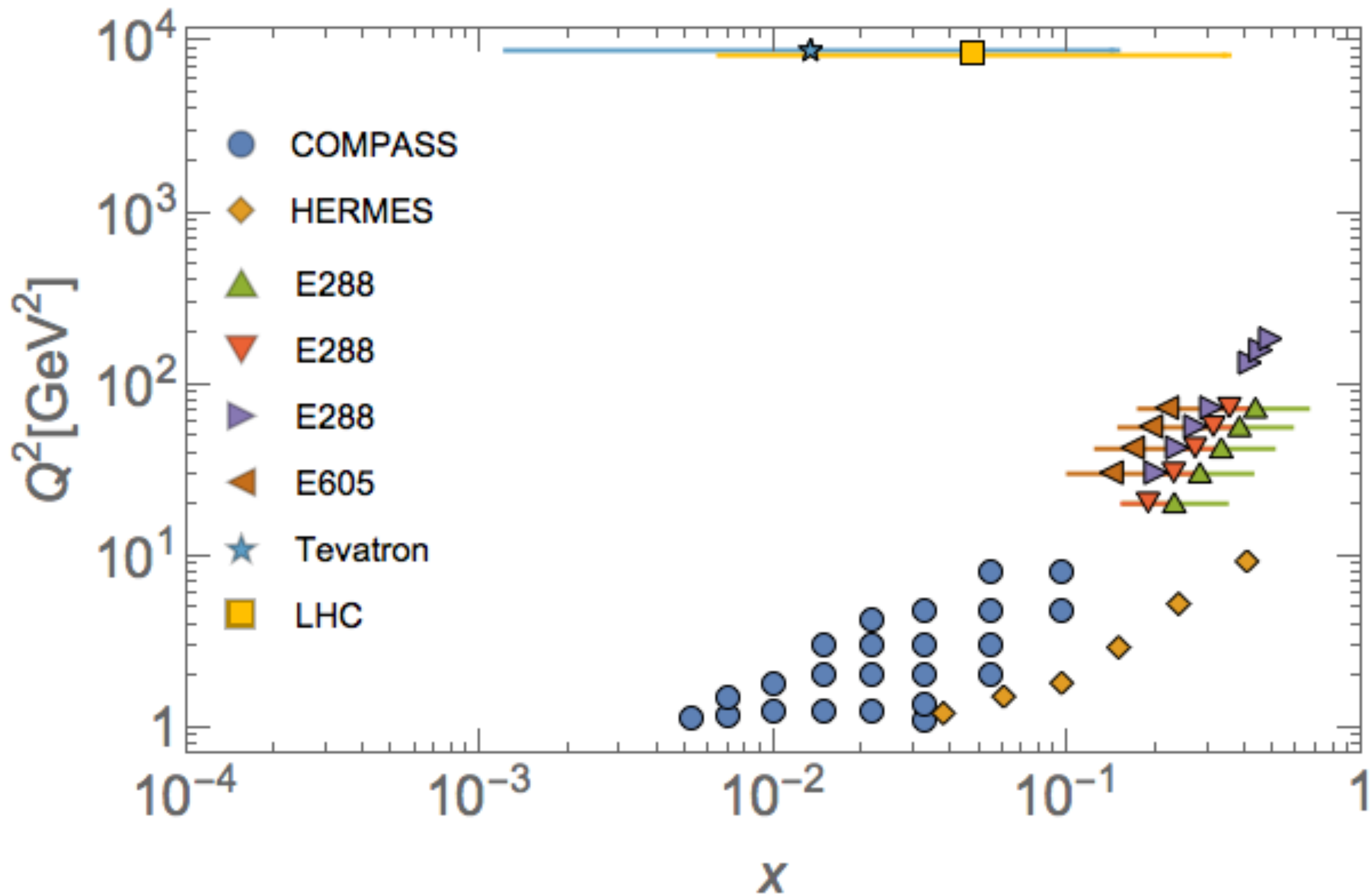
Data sets and kinematic coverage



Electron-positron annihilation data are still **missing** (only some azimuthal asymmetries are available)

crucial for analyses of TMD FFs !!

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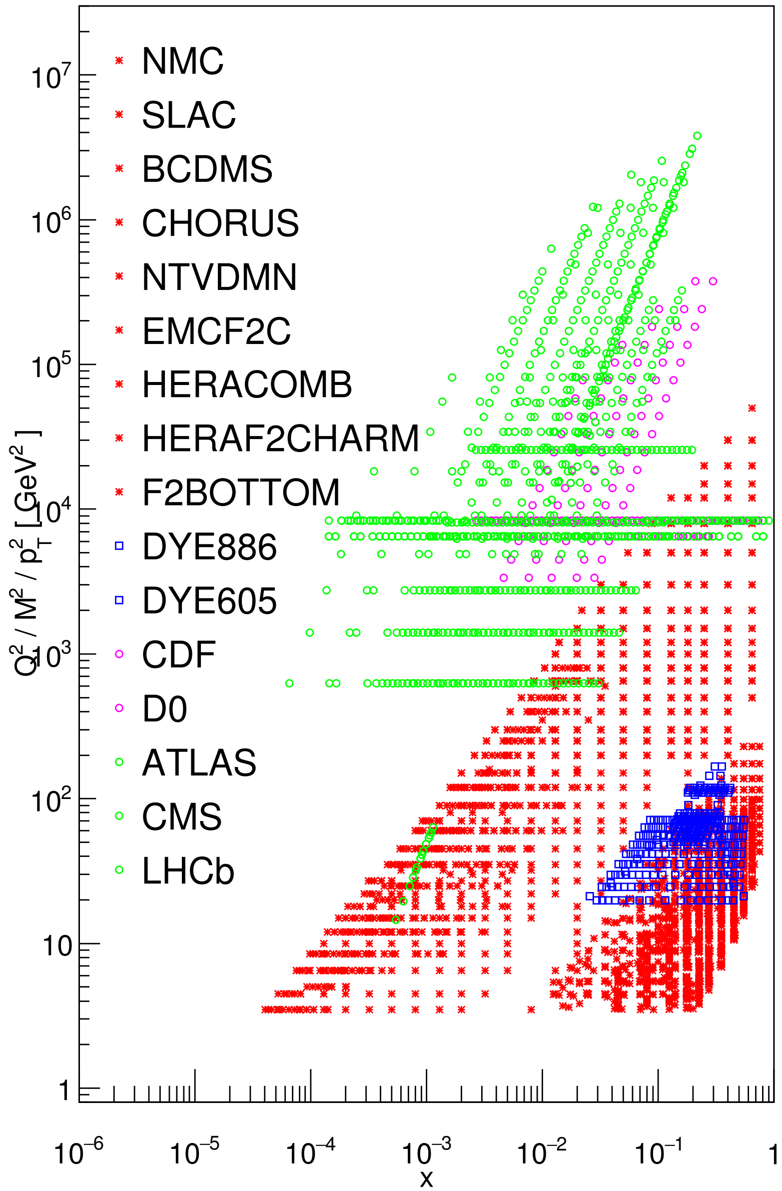
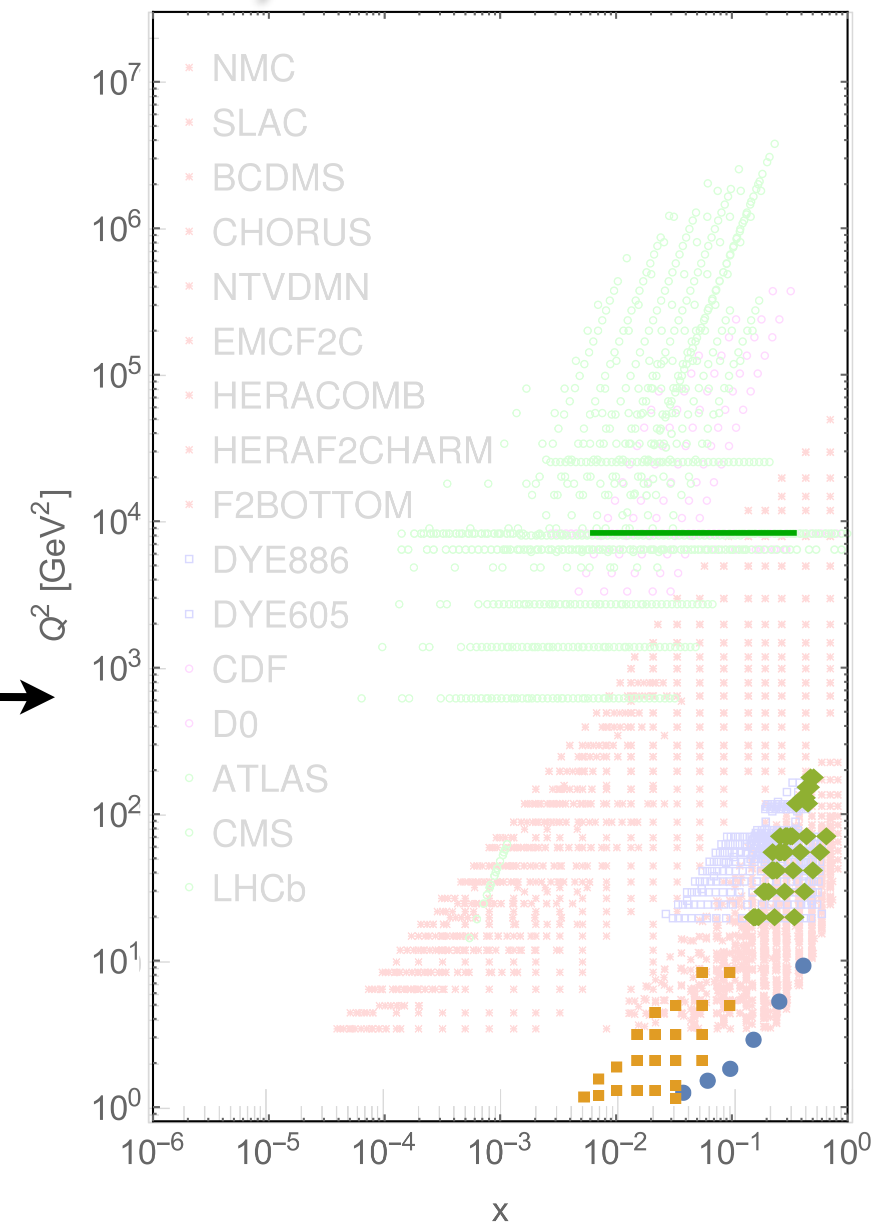
Comparison with collinear PDF fits

see talk by E. Nocera at POETIC2016



data sets available:

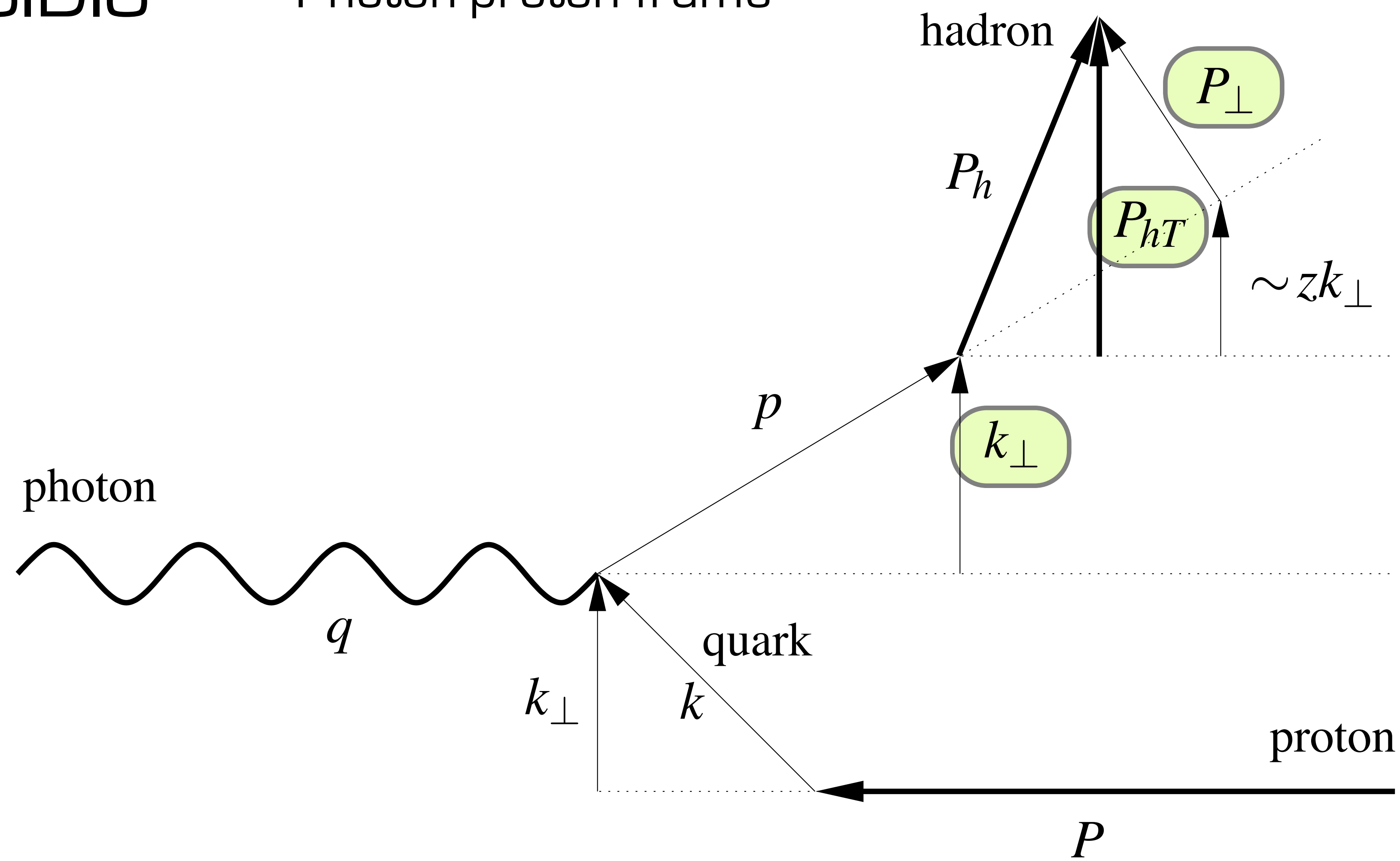
collinear PDFs
vs
TMD PDFs



Transverse momenta in SIDIS

SIDIS

Photon-proton frame



TMD FF

TMD PDF

Longitudinal scaling variable

$$z = \frac{P \cdot P_h}{P \cdot q}$$

Observed transverse momentum

$$P_{hT} \approx zk_\perp + P_\perp$$

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Data sets and selections - SIDIS

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Reference	[61]			
Cuts	$Q^2 > 1.4$ GeV^2 $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5$ GeV			
Points	190	190	189	187
Max. Q^2	9.2 GeV^2			
x range	$0.06 < x < 0.4$			

TMD factorization
($P_{hT}^2/z^2 \ll Q^2$)

avoid target fragmentation [?]
(low z)
and exclusive contributions [?]
(high z)

Problem with normalization
in the previous release

	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Reference	[74]				[75]	
Cuts	$Q^2 > 1.4$ GeV^2 $0.20 < z < 0.74$ $P_{hT} < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5$ GeV					
Points	190	190	189	189	3125	3127
Max. Q^2	9.2 GeV^2				10 GeV^2	
x range	$0.04 < x < 0.4$				$0.005 < x < 0.12$	
Notes					Observable: $m_{\text{norm}}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$, eq. (3.1)	

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Features

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 arXiv:1703.10157	LO-NLL	✔	✔	✔	✔	8059

PROs

almost a **global fit** of quark unpolarized TMDs

includes **TMD evolution**

replica (bootstrap) fitting methodology

kinematic dependence in intrinsic part of TMDs

intrinsic momentum: **beyond the Gaussian** assumption

CONs

no "pure" info on TMD FFs

accuracy of TMD evolution : not the state of the art

only "low" transverse momentum (no fixed order and Y-term)

flavor separation in the transverse plane : problematic

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Intrinsic transverse momentum

$$f^a_{1NP}(x,k_\perp^2) = \frac{1}{\pi}\frac{(1+\lambda k_\perp^2)}{g_{1a}+\lambda g_{1a}^2}\; e^{-\frac{k_\perp^2}{g_{1a}}}$$

$$\hat{x} = 0.1$$
$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

weighted sum of two Gaussian distributions:

same widths for TMD PDFs

different widths for TMD FFs

$$\hat{z} = 0.5$$
$$g_{3,4}(z) = N_{3,4} \frac{(z^\beta+\delta)(1-z)^\gamma}{(\hat{z}^\beta+\delta)(1-\hat{z})^\gamma}$$

$$D^{a\to h}_{1NP}(z,P_\perp^2) = \frac{1}{\pi}\frac{1}{g_{3a\to h}+(\lambda_F/z^2)g_{4a\to h}^2}\left(e^{-\frac{P_\perp^2}{g_{3a\to h}}} + \lambda_F \frac{P_\perp^2}{z^2}\; e^{-\frac{P_\perp^2}{g_{4a\to h}}}\right)$$

Inspired by **model calculations**:

Matevosyan et al.
Phys. Rev. D85, 014021 (2012), 1111.1740
Bacchetta et al.
Phys. Lett. B659, 234 (2008), 0707.3372
Bacchetta at al.
Phys. Rev. D65, 094021 (2002), hep-ph/0201091

There are **11 free parameters** in a flavor independent scenario (one for evolution)

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Models - evolution and b_T regions

$$g_K(b_T; g_2) = -g_2 \frac{b_T^2}{2}$$

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)$$

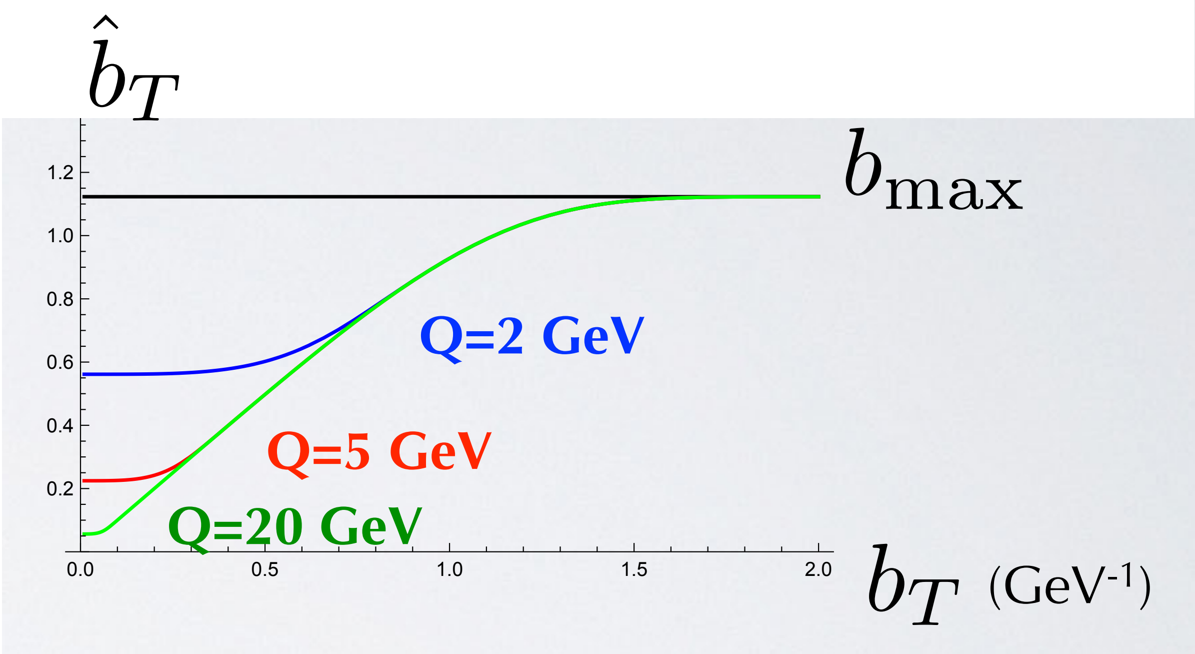
$\nearrow\ b_{\max}\ , \quad b_T \to +\infty$

$\searrow\ b_{\min}\ , \quad b_T \to 0$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$

These choices guarantee that for Q=1 GeV the TMD coincides with the NP model



Models - evolution and b_T regions

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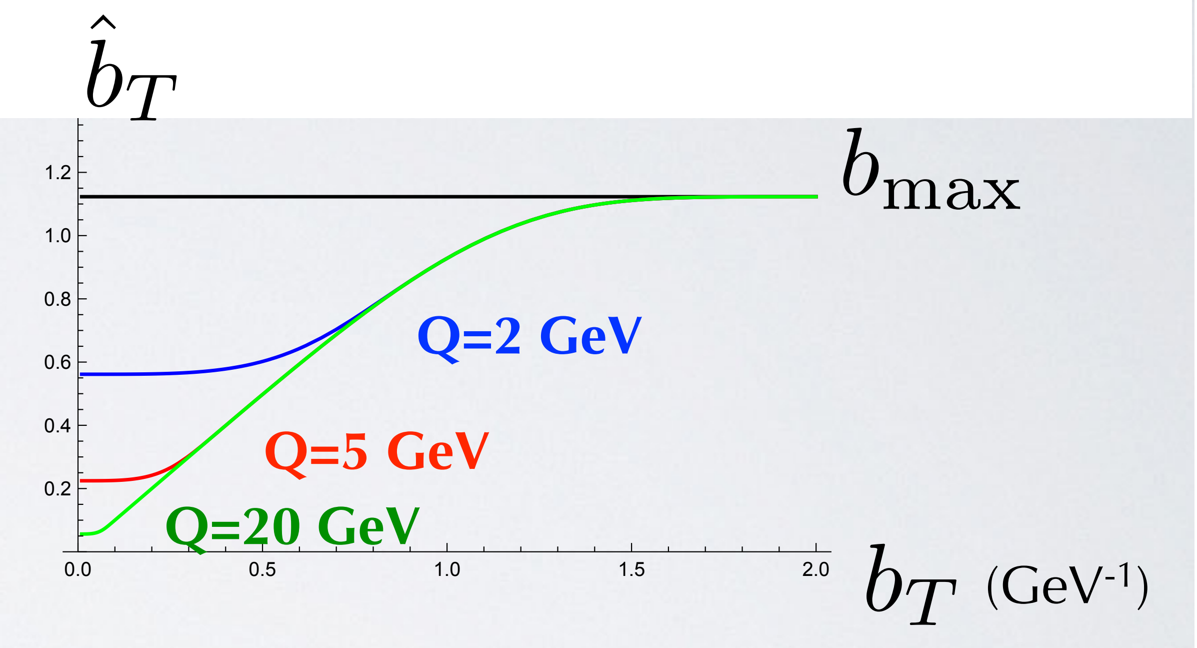
$\nearrow \; b_{\max} \,, \quad b_T \to +\infty$

$\searrow \; b_{\min} \,, \quad b_T \to 0$

$b_{\min} \sim 1/Q \,, \quad \mu_{\hat{b}} < Q$

The phenomenological importance of b_{min} is a signal that -especially in SIDIS data at **low Q**- we are exiting the TMD region, entering the collinear factorization region



Agreement data-theory

Flavor independent scenario

Flavor independent configuration | 11 parameters

Points	Parameters	χ^2	χ^2/d.o.f.
8059	11	12629 ± 363	1.55 ± 0.05

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Points	190	190	189	187
χ^2/points	4.83	2.47	0.91	0.82

	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Points	190	190	189	189	3125	3127
χ^2/points	3.46	2.00	1.31	2.54	1.11	1.61

	E288 [200]	E288 [300]	E288 [400]	E605
Points	45	45	78	35
χ^2/points	0.99	0.84	0.32	1.12

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Points	31	14	37	8
χ^2/points	1.36	1.11	2.00	1.73

Hermes P/D into π+: problems at low z

Hermes kaons better than pions: larger uncertainties from FFs

Compass : better agreement due to #points and normalization

Let's see what happens with the new data

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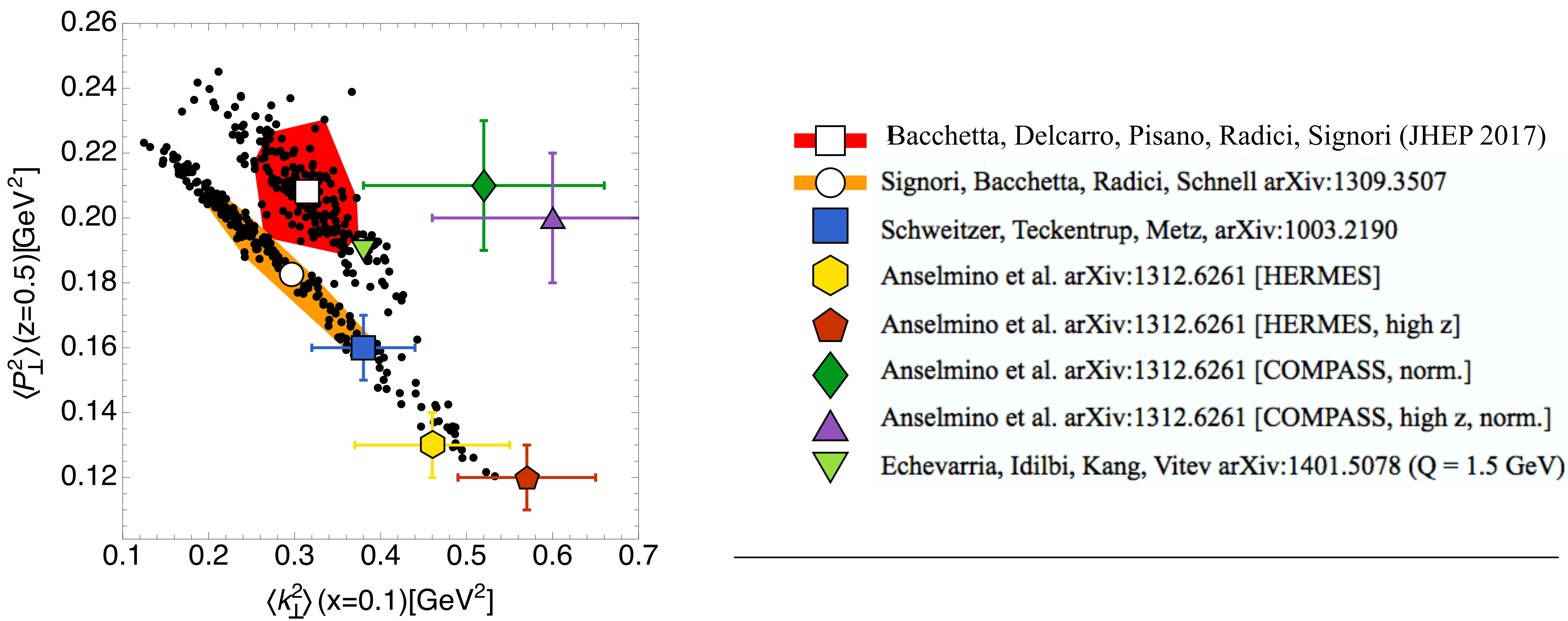
Average transverse momenta

Flavor ind. scenario



Red/orange regions : **68% CL** from replica method

Inclusion of **DY/Z diminishes the correlation**

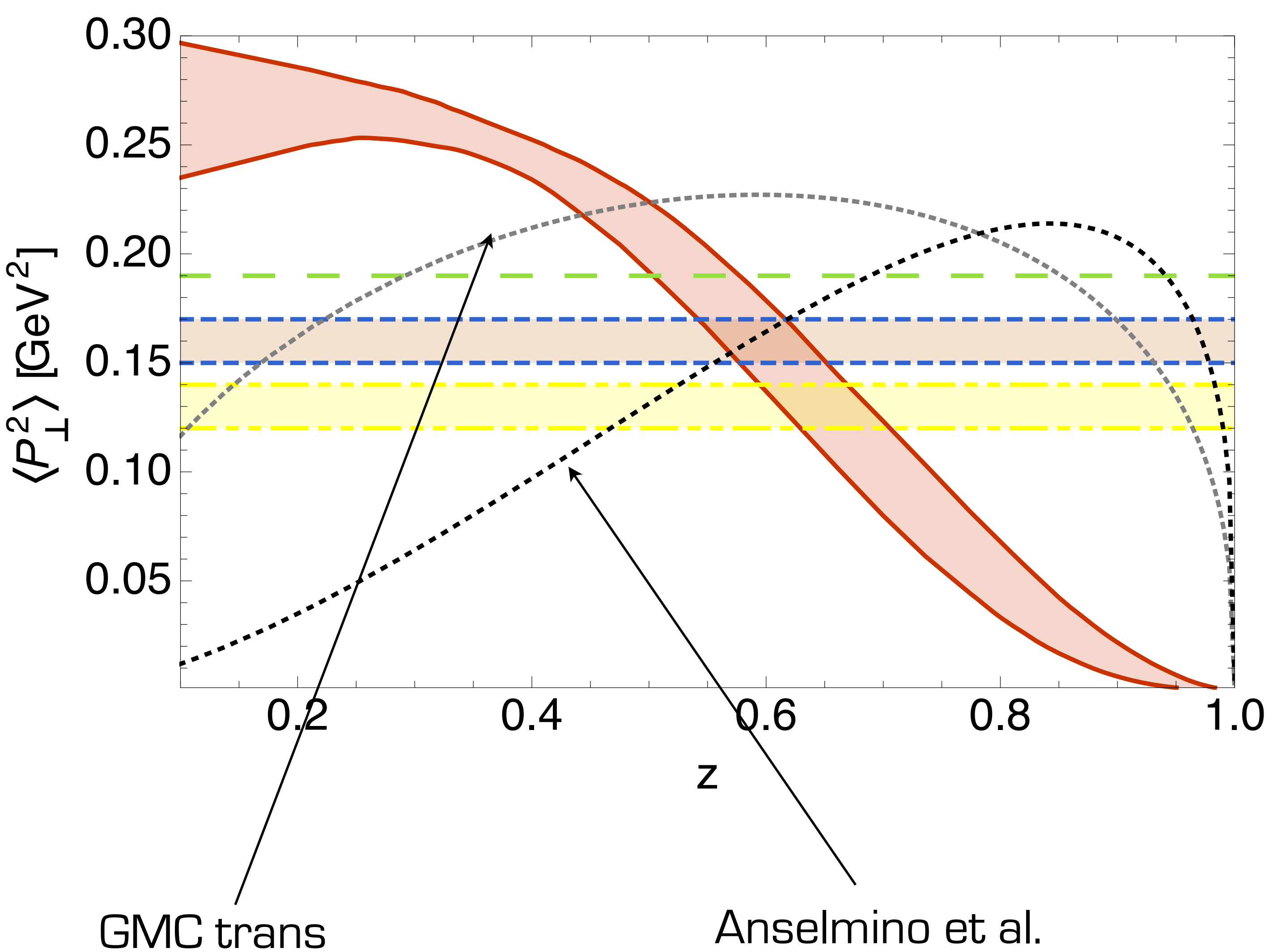
Inclusion of **Compass** increases the $\langle P_\perp^2 \rangle$ and reduces its spread

e+e- data would further reduce the correlation

Kinematic dependence

$$\langle P_\perp^2 \rangle(z) = \frac{\int d^2P_\perp \; P_\perp^2 \; D_1^{a\to h}(z, P_\perp^2, Q = 1 \text{ GeV})}{\int d^2P_\perp \; D_1^{a\to h}(z, P_\perp^2, Q = 1 \text{ GeV})}$$

Average square transverse momentum in TMD FF



Color code : same as previous slide

Flavor-independent scenario: no differences in quark/hadron flavor

z-dependence : important to fit the data

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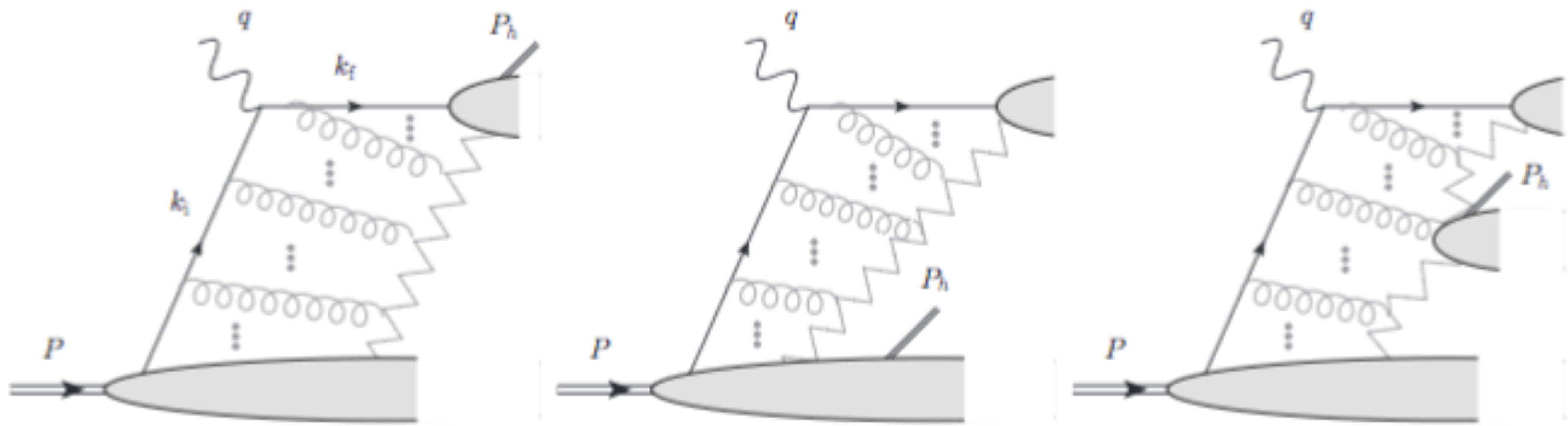
What's next

Just a selection of topics to feed the discussion

Target vs current vs central regions



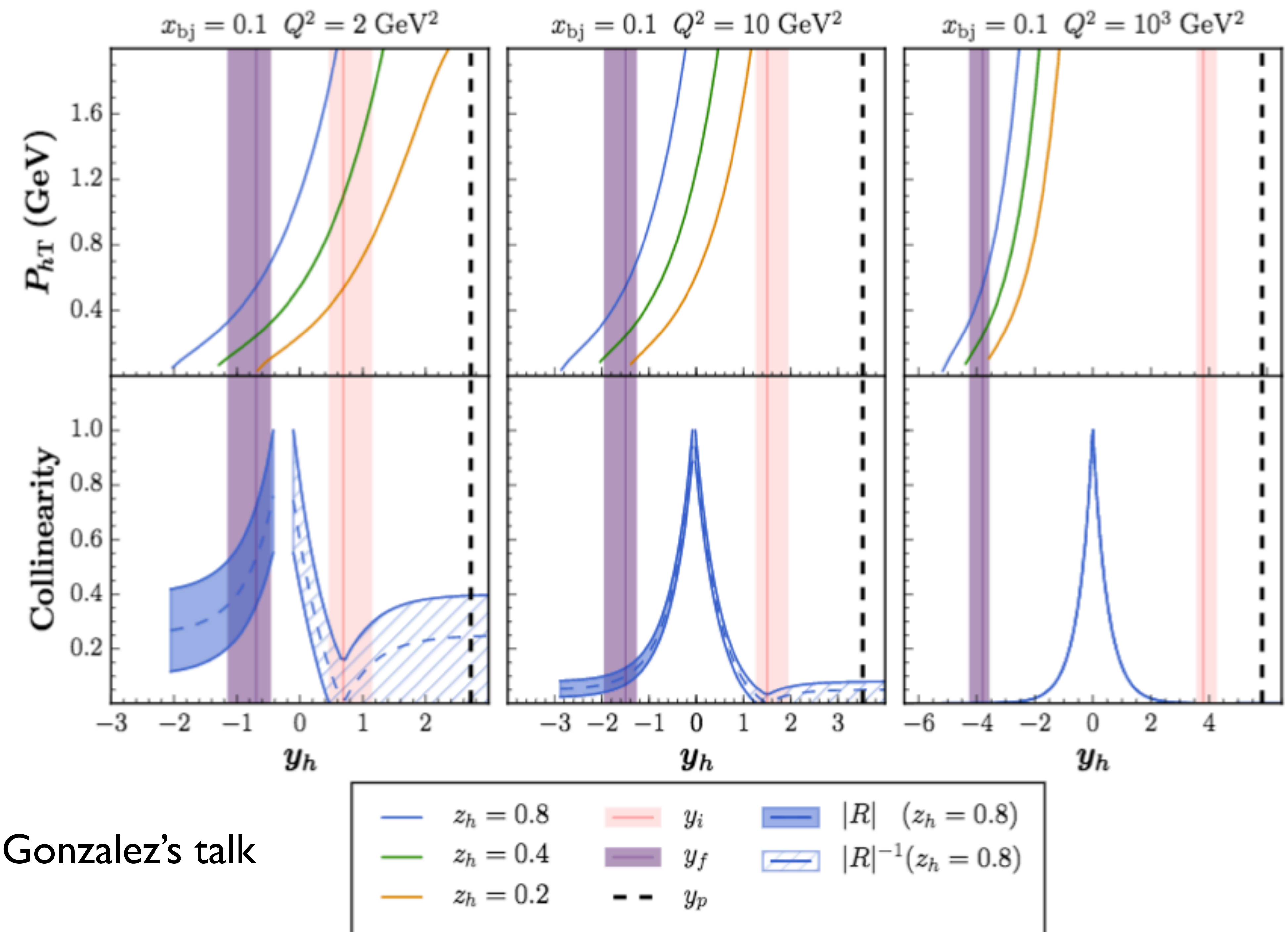
(a) (b) (c)

current fragmentation

target fragmentation

“soft” fragmentation

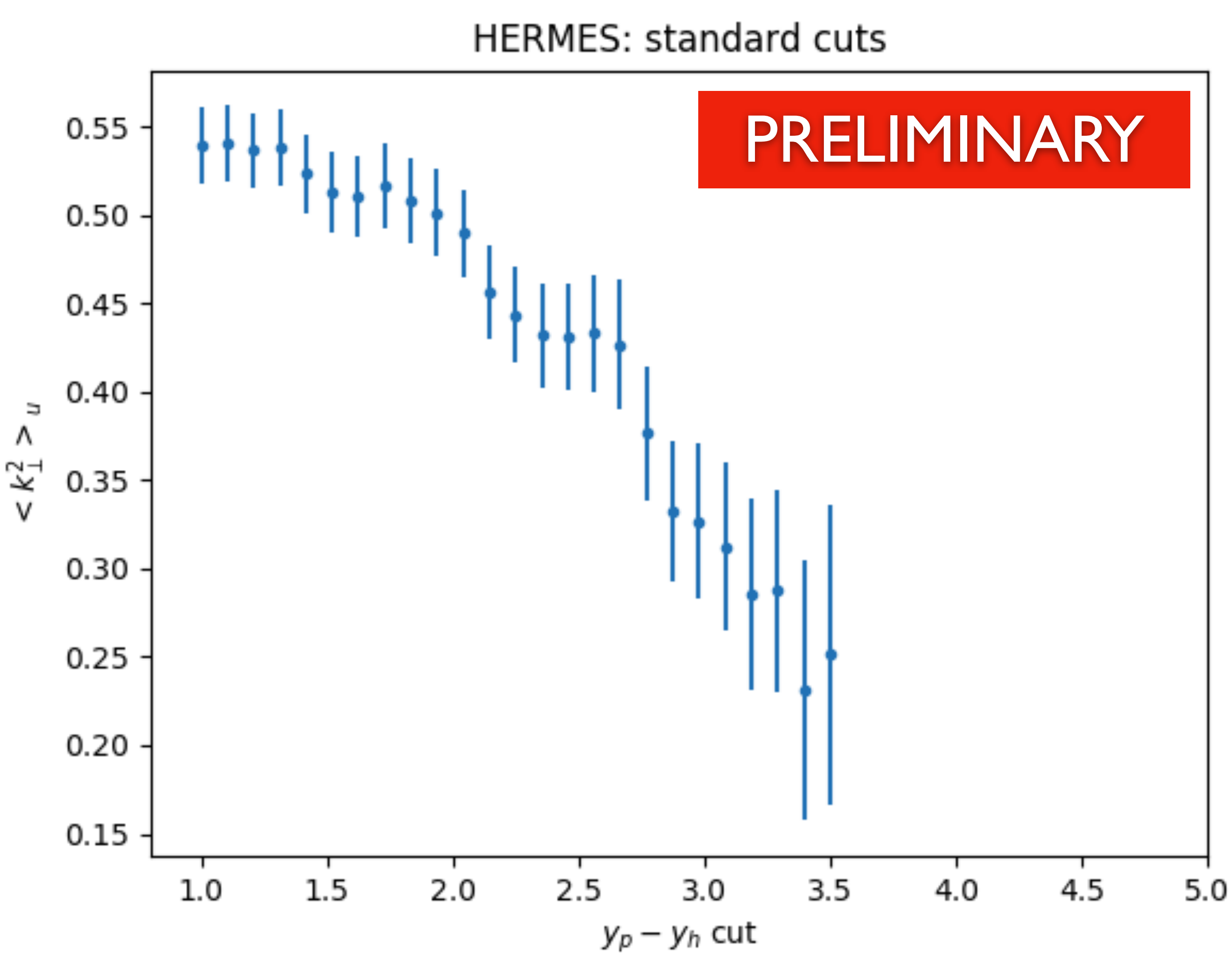
Target vs current vs central regions

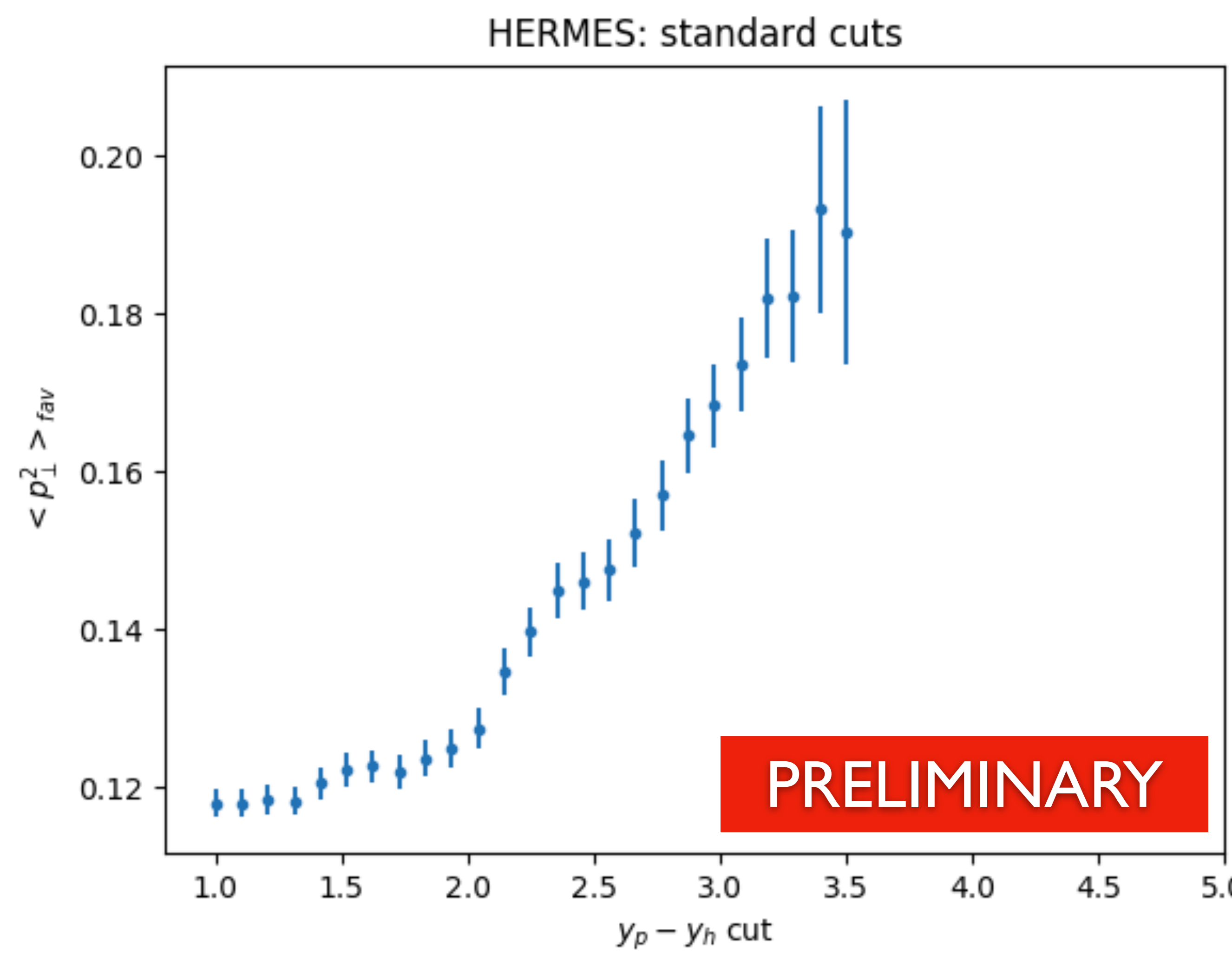


See O. Gonzalez's talk

"collinearity" criterion

Target vs current vs central regions





Description of Hermes data within the quark parton model

"rapidity difference" criterion

Widths described as a function of the rapidity difference between the incoming proton and outgoing hadron

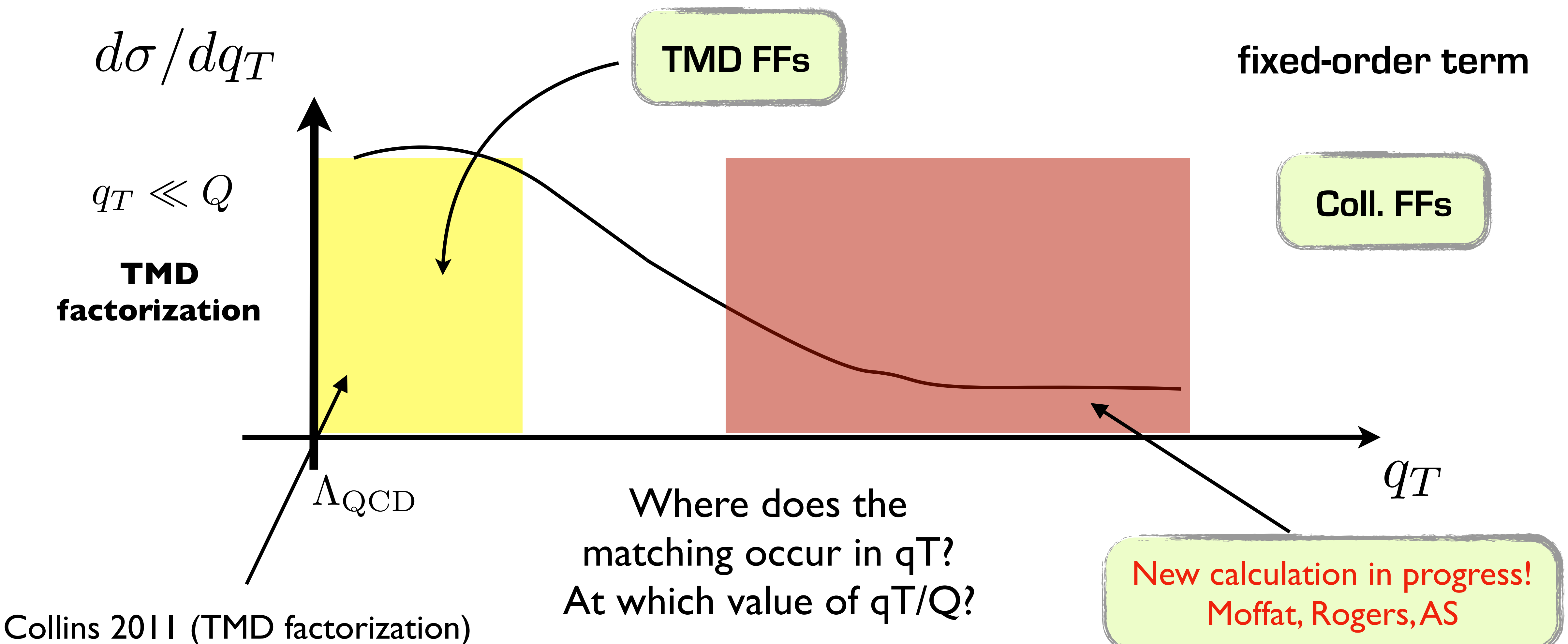
M. Albright et al. 2018

TMD FFs from e+e-

$e^+e^- \to h_1\ h_2\ X$

Completing the formalism to study TMD FFs

fixed Q, variable q_T

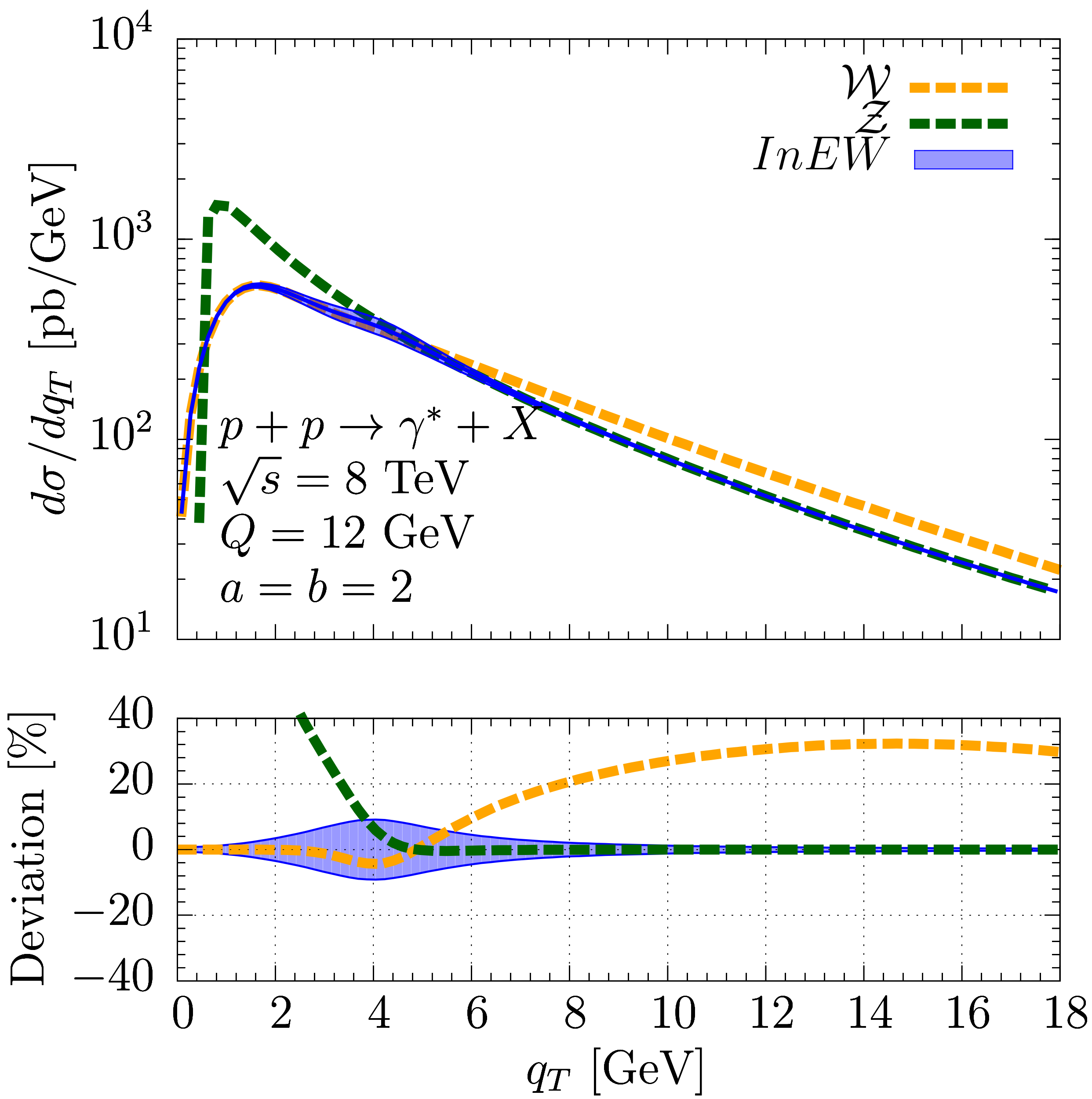


Several phenomenological works, e.g.:
* Bacchetta, Echevarria, Mulders, Radici, AS - JHEP 2015

Matching

Echevarria, Kasemets, Lansberg, AS, Pisano
1801.01480

development of a new scheme (**InEW** - **inverse error weighting**)
and comparison to improved CSS subtraction



$$d\sigma(q_T,Q)=\omega_1\ \mathcal{W}(q_T,Q)+\omega_2\ \mathcal{Z}(q_T,Q)$$

$$\omega_1\sim\Delta\mathcal{W}^{-2}\ ,\ \Delta\mathcal{W}\sim\mathcal{O}\left(\frac{q_T}{Q}\right)^a+\mathcal{O}\left(\frac{m}{Q}\right)^{a'}$$

$$\omega_2\sim\Delta\mathcal{Z}^{-2}\ ,\ \Delta\mathcal{Z}\sim\mathcal{O}\left(\frac{m}{q_T}\right)^b$$

Drell-Yan at LHC
(Q=12 GeV)
(data available)

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Matching

Echevarria, Kasemets, Lansberg, AS, Pisano
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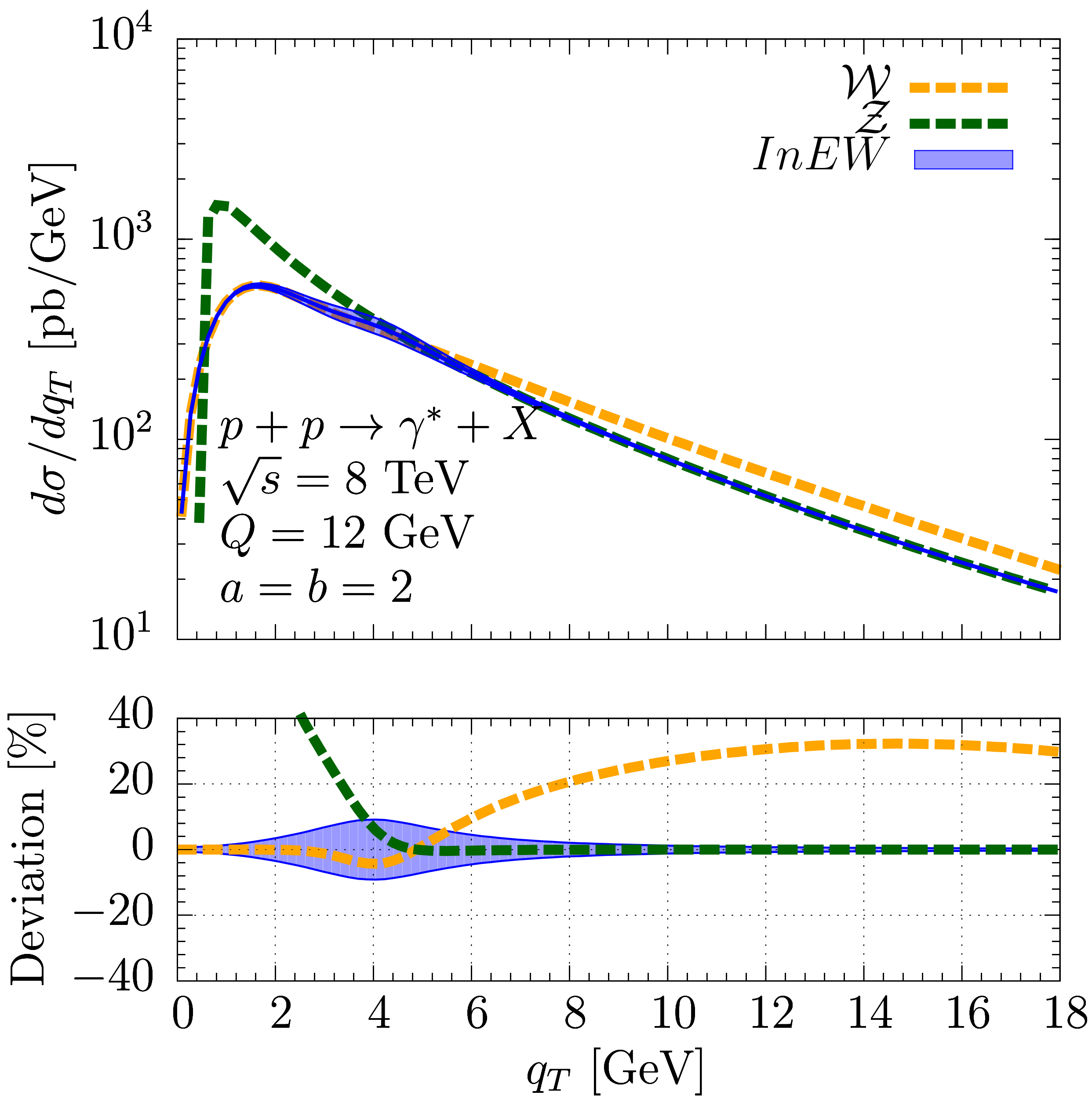

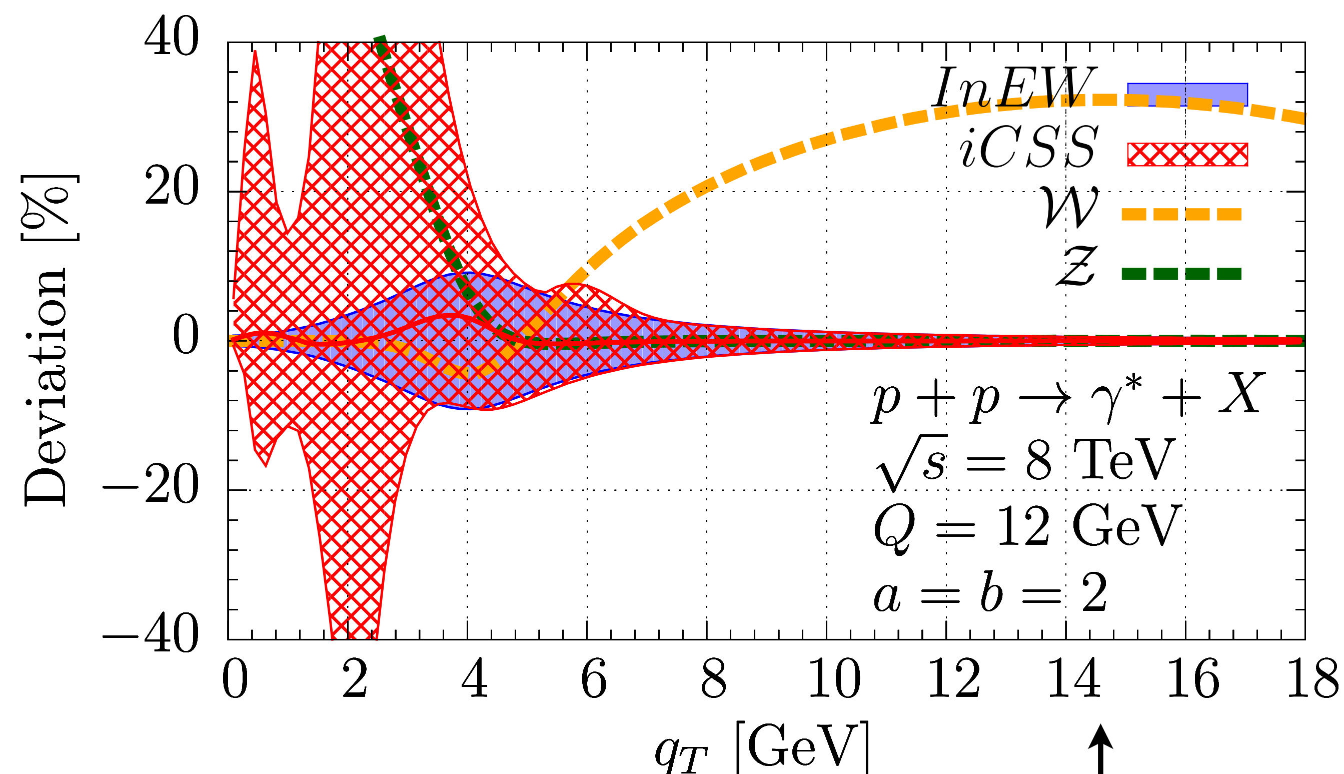

Drell-Yan at LHC
(Q=12 GeV)
(data available)

InEW vs iCSS

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Inputs for discussion

FORMALISM:
* definition of the fragmentation regions
* which variables shall we use to describe the momentum fractions? (see Gunar's talk)
* matching schemes: how to estimate uncertainties associated to the matching prescription
* jet fragmentation functions

PERTURBATIVE ASPECTS:
* implementation of evolution (transverse momentum, threshold resummation, zeta prescription, …?)
* fixed-order calculations in SIDIS : is it sufficient to describe data at higher qT ? Or do we need power corrections/higher twist, contributions from soft region, etc ?

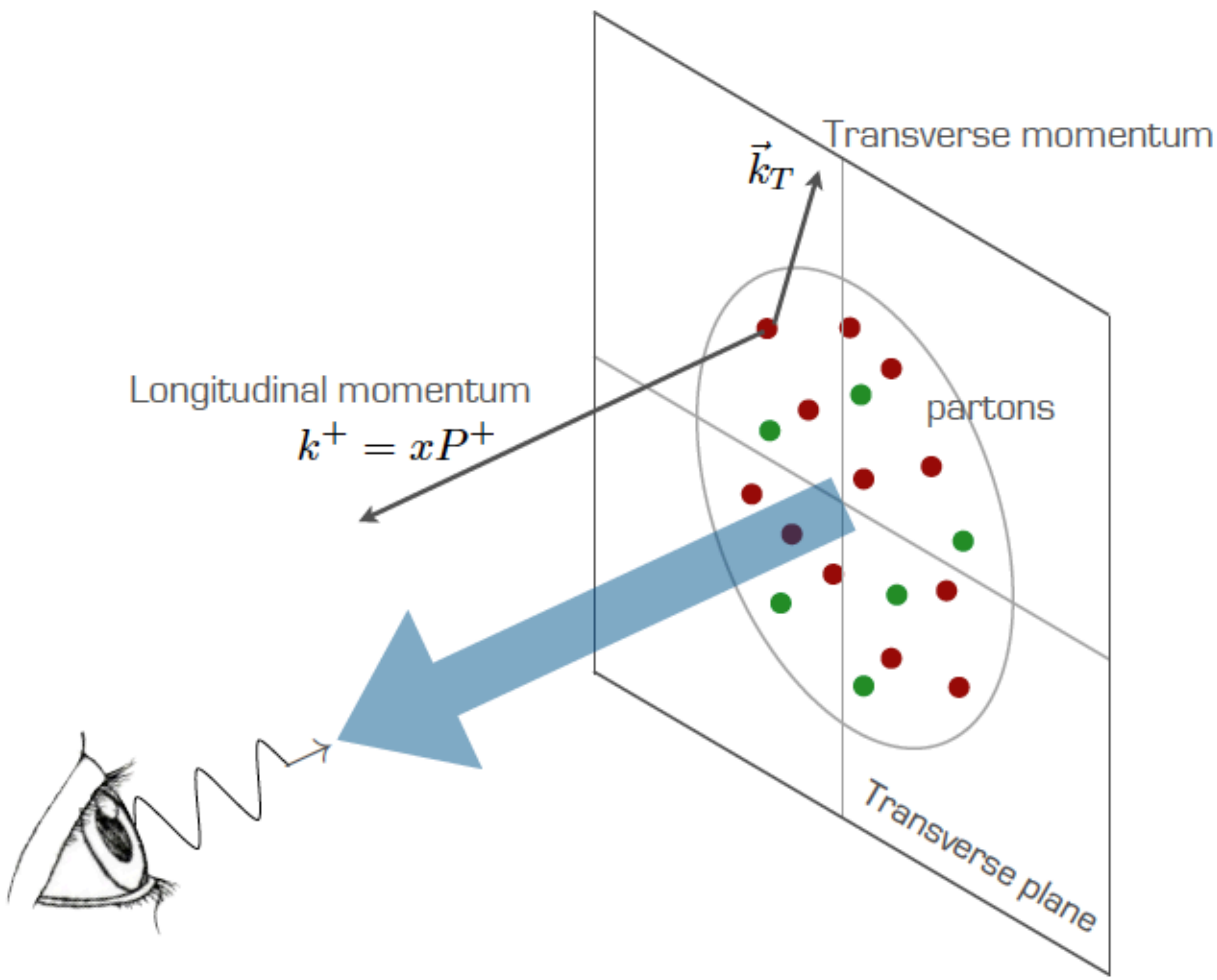
NONPERTURBATIVE ASPECTS:
* functional form at low transverse momentum
* its kinematic dependence
* its flavor dependence
* nonperturbative contribution to TMD evolution

DATA :
* impact of the new release of Compass data
* A Fixed Target Experiment at the LHC ?
* what can be done with the forthcoming e+e- data concerning TMD FFs (also including matching to high qT)
* how well does the fixed order describes data at large transverse momentum
* …

Backup

quark TMD PDFs

$$\Phi_{ij}(k, P; S) \sim \text{F.T.}\ \langle PS \,|\, \bar{\psi}_j(0)\ U_{[0,\xi]}\ \psi_i(\xi)\ |PS\rangle|_{LF}$$



extraction of a **quark**
not collinear with the proton

Status of TMD phenomenology

Theory, data, fits : we are in a position to start validating the formalism

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Only first attempts

Limited data, theory, fits

see, e.g, Bacchetta, Radici, arXiv:1107.5755
Anselmino, Boglione, Melis, PRD86 (12)
Echevarria, Idilbi, Kang, Vitev, PRD 89 (14)
Anselmino, Boglione, D'Alesio, Murgia, Prokudin, arXiv:1612.06413
Anselmino et al., PRD87 (13)
Kang et al. arXiv:1505.05589

Lu, Ma, Schmidt, arXiv:0912.2031
Lefky, Prokudin arXiv:1411.0580
Barone, Boglione, Gonzalez, Melis, arXiv:1502.04214

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Beware of different notations...

Amsterdam	Seattle (arXiv:1108.1713)	
p	k	momentum of parton in distribution function
$\boldsymbol{p}_T$	$\boldsymbol{k}_\perp$	parton transverse momentum in distribution function
k	p	momentum of fragmenting parton
$\boldsymbol{k}_T$	$\boldsymbol{p}_\perp$	trans. momentum of fragmenting parton w.r.t. final hadron
$\boldsymbol{K}_T$	$\boldsymbol{P}_\perp$	trans. momentum of final hadron w.r.t. fragmenting parton
$\boldsymbol{P}_{h\perp}$	$\boldsymbol{P}_{hT}$	transverse momentum of final hadron w.r.t. virtual photon

Let's agree on the notation!

Collinear and TMD factorization

Let's consider a process with
three separate scales:

(SIDIS, Drell-Yan, e+e- to hadrons,
pp to quarkonium, …)

hadronic mass scale $\Lambda_{\mathrm{QCD}} \ll q_T \ll Q$ **hard scale**

(related to the)
transverse momentum of the observed particle

The ratios Λ_{QCD}/Q $\Lambda_{\mathrm{QCD}}/q_T$ q_T/Q

select the factorization theorem that we rely on.

According to their **values** we can access **different "projections"** of hadron structure

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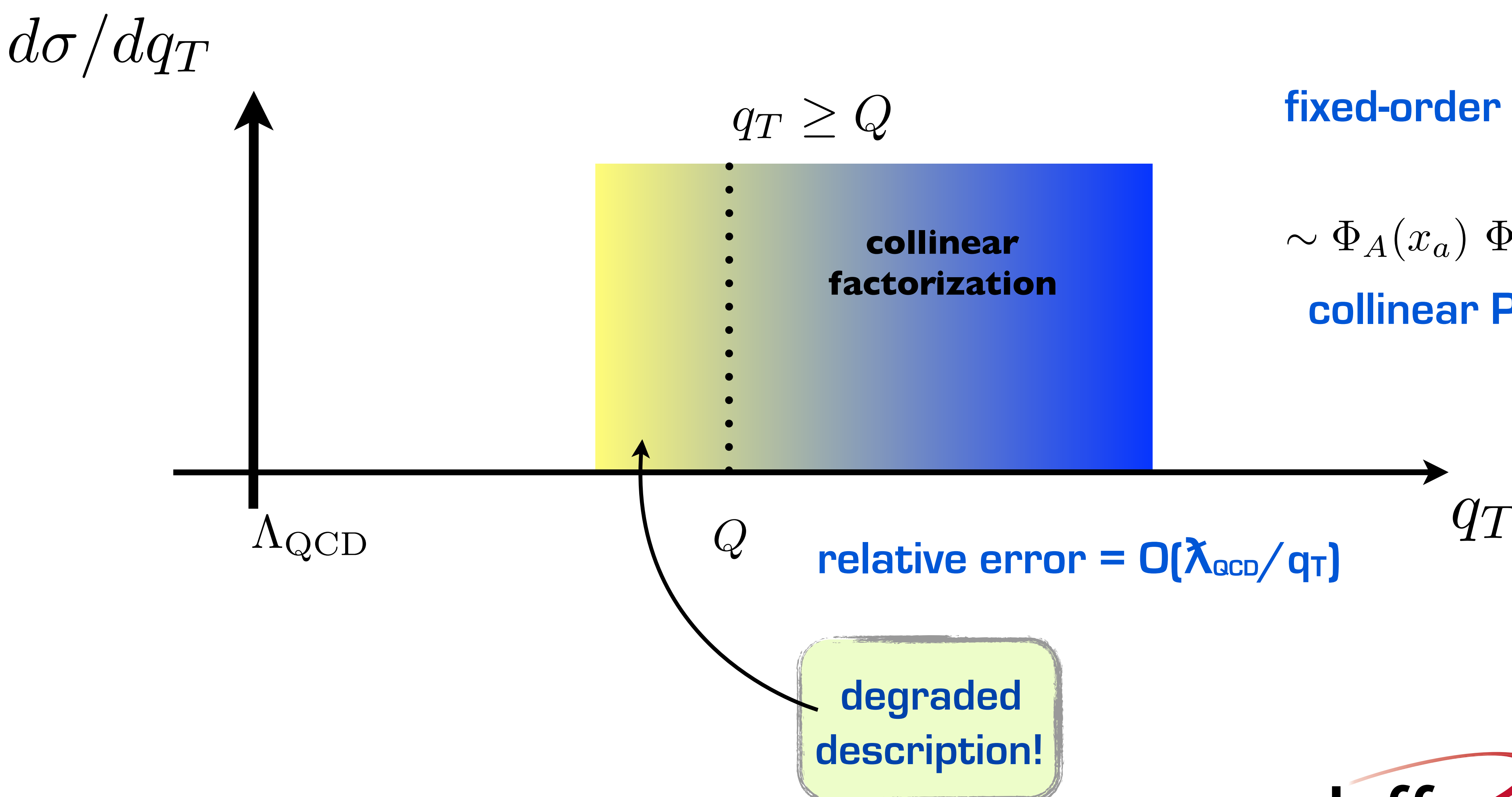
Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T



fixed-order term

$\sim \Phi_A(x_a)\ \Phi_B(x_b)$

collinear PDFs

relative error = $O(\Lambda_{QCD}/q_T)$

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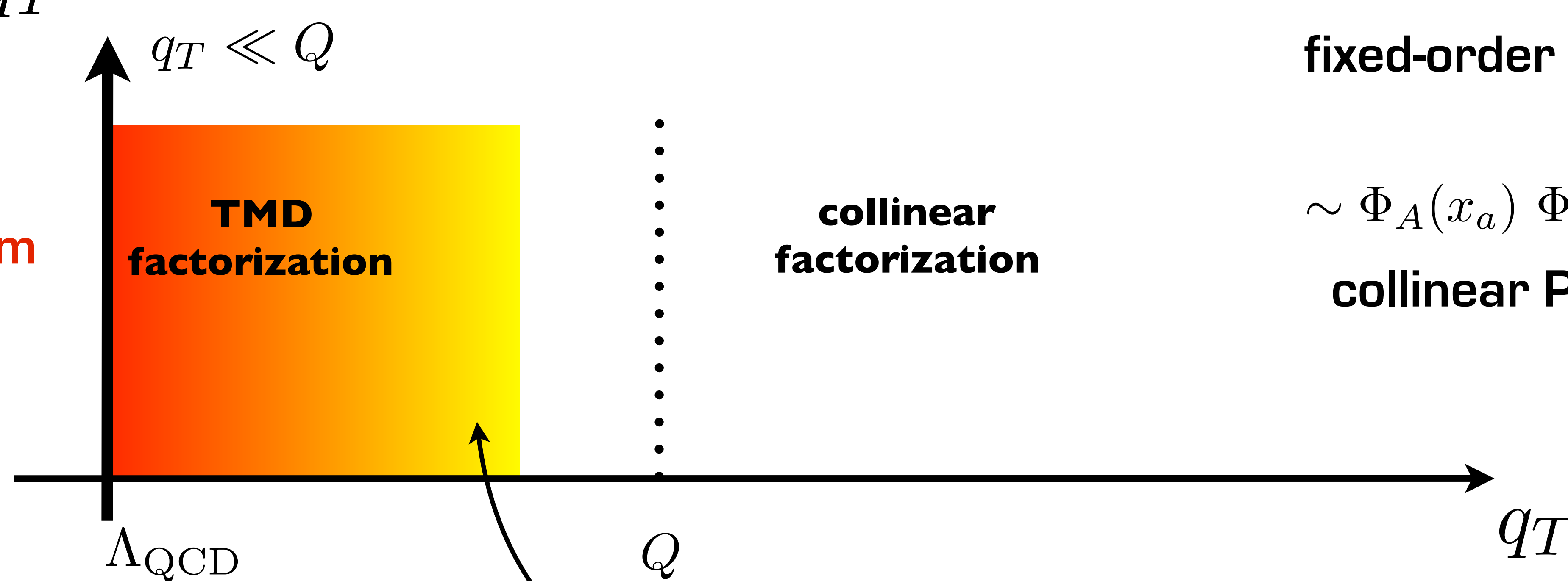
Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T

$d\sigma / dq_T$



resummed term (W)

fixed-order term

$\sim \Phi_A(x_a)\ \Phi_B(x_b)$

collinear PDFs

$\Phi_A(x_a, \mathbf{k}_{Ta})\ \Phi_B(x_b, \mathbf{k}_{Tb})\ \sim$ **TMD PDFs**

relative error = O(q_T/Q)

degraded description!

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Collinear and TMD factorization

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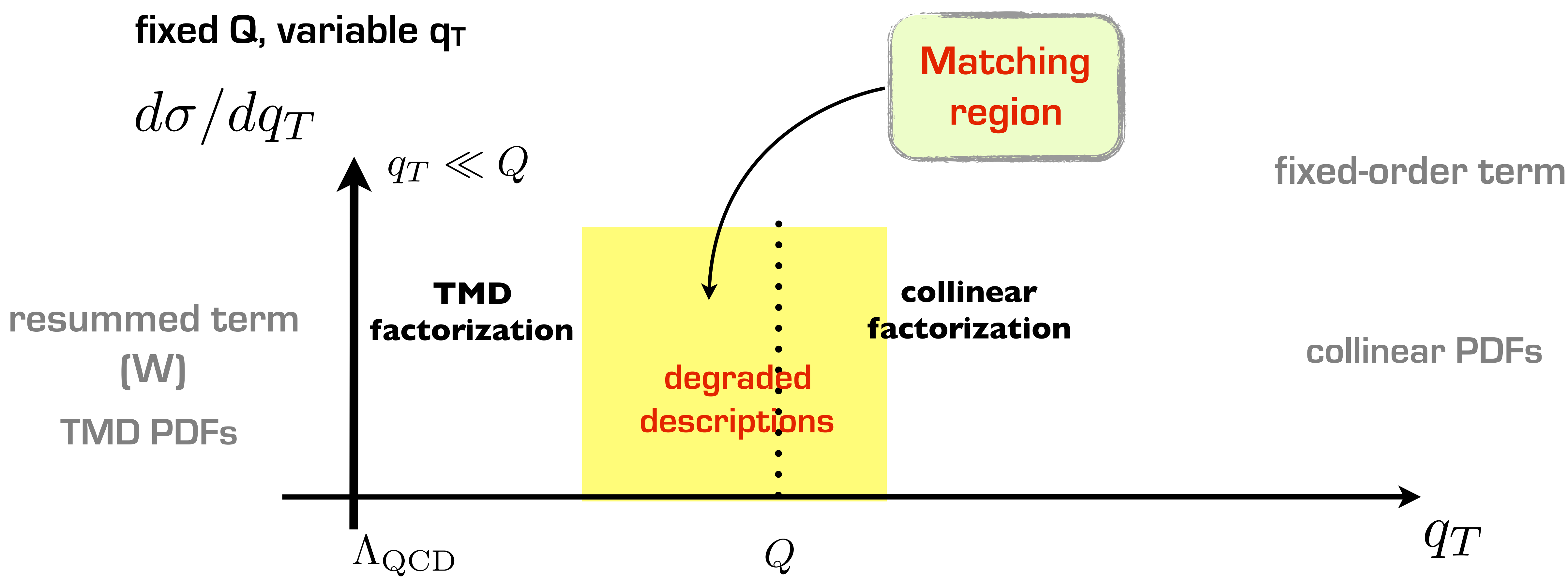


W, relative error = $O(q_T/Q)$

F.O., relative error = $O(\Lambda_{QCD}/q_T)$

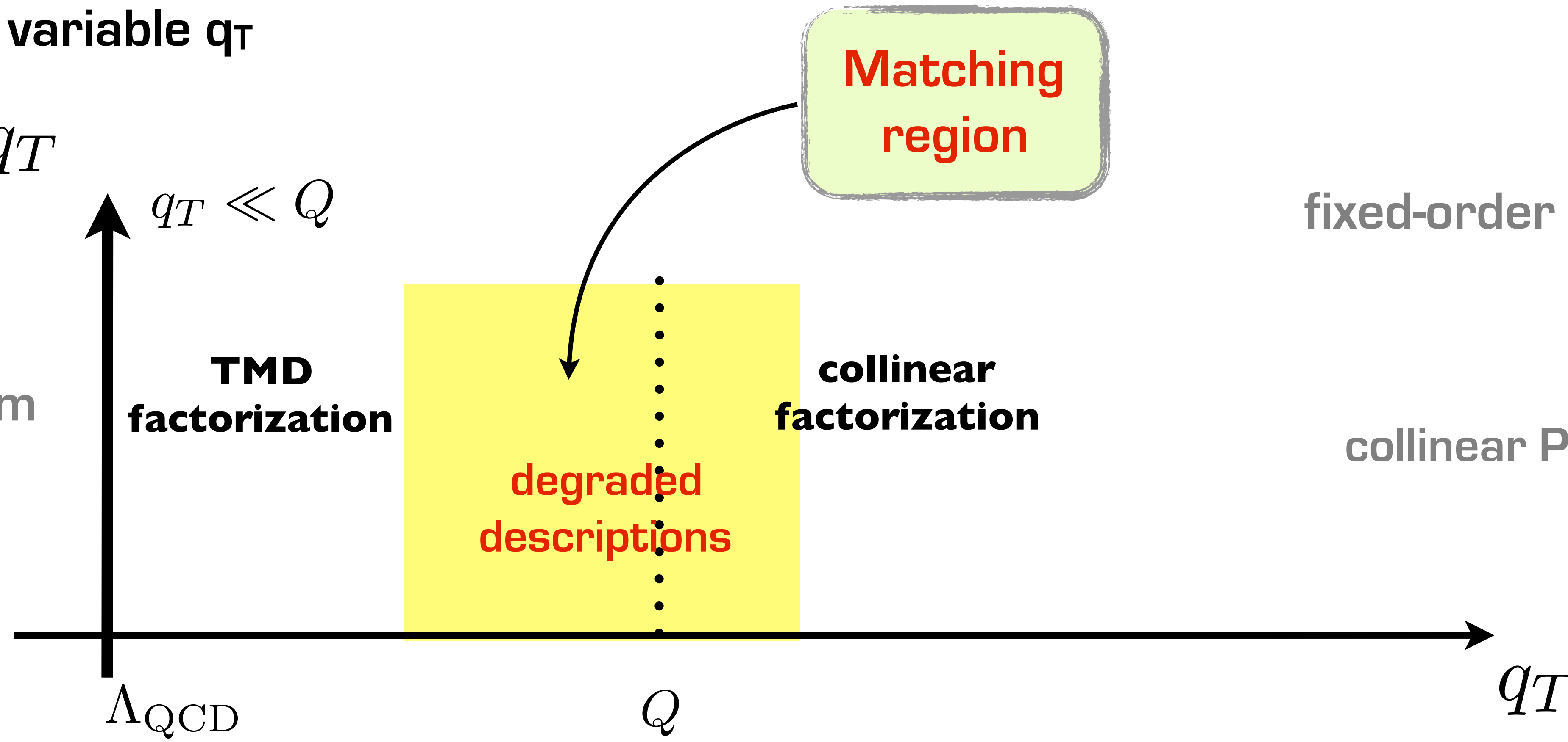
We need a prescription to deal with the region where both descriptions are not good

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T

$d\sigma/dq_T$



fixed-order term

collinear PDFs

resummed term (W)

TMD PDFs

The extraction of the **nonperturbative part of TMDs** is affected by the **description of the whole q_T range**

Crucial, especially at **low Q** (e.g. JLab kinematics), where the **regions shrink**

polarization ?

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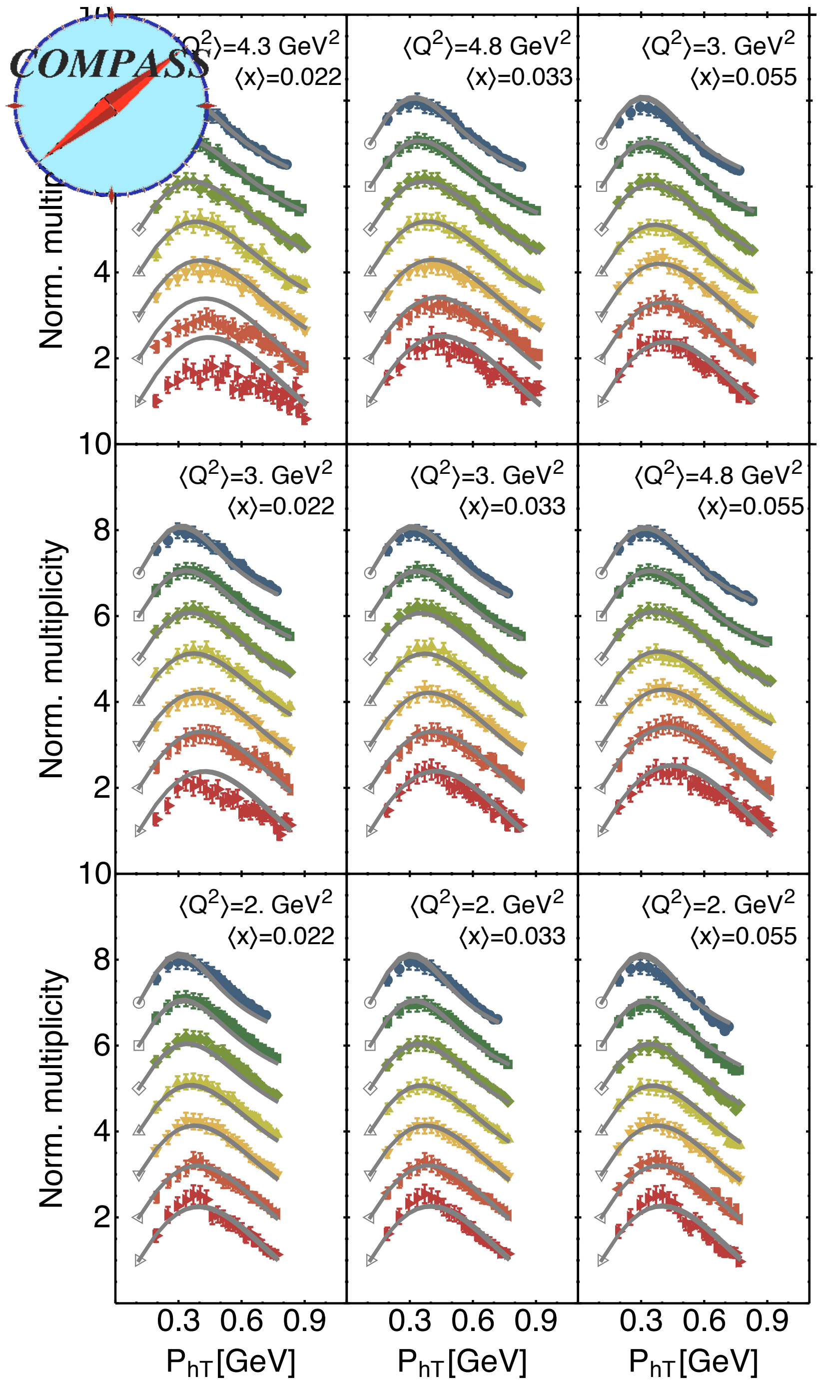
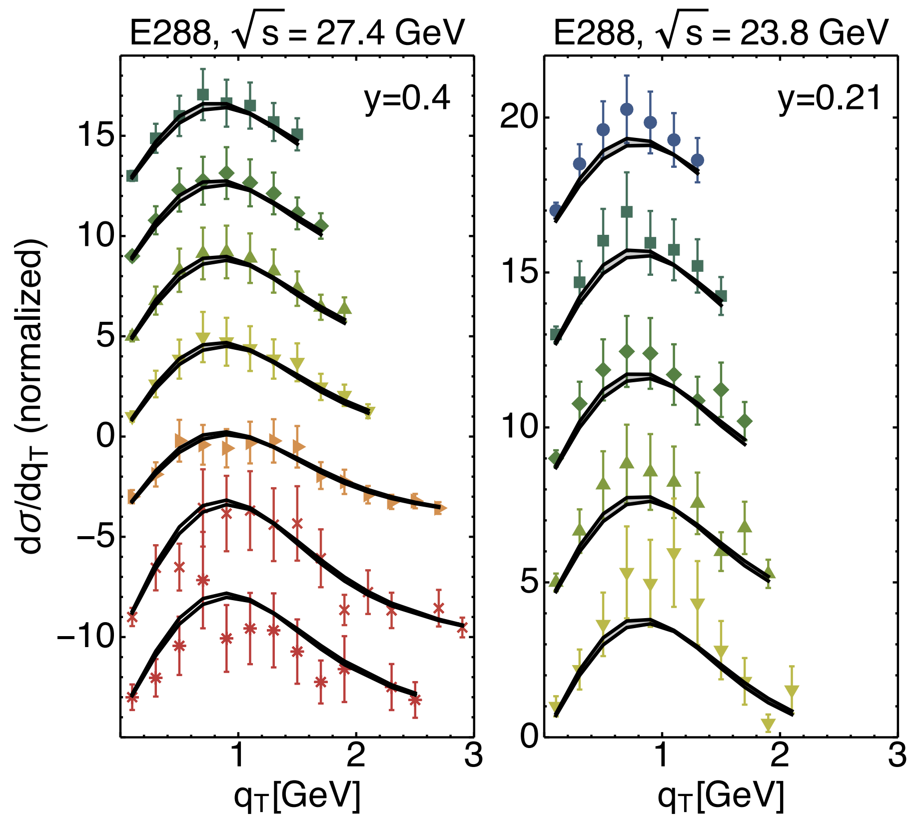
Global fit

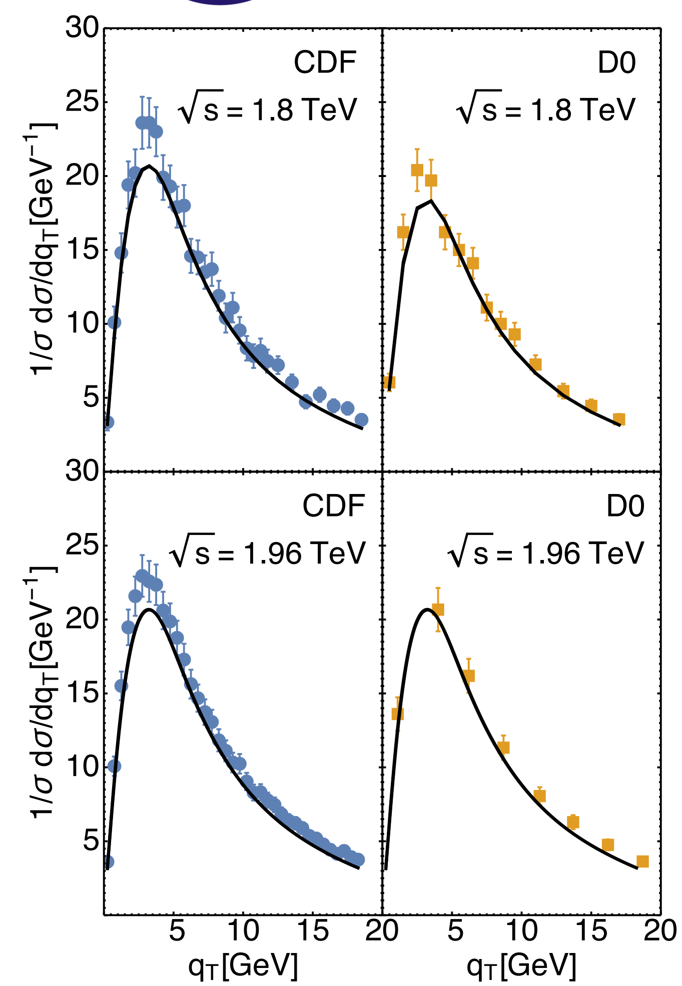
SIDIS



Drell-Yan

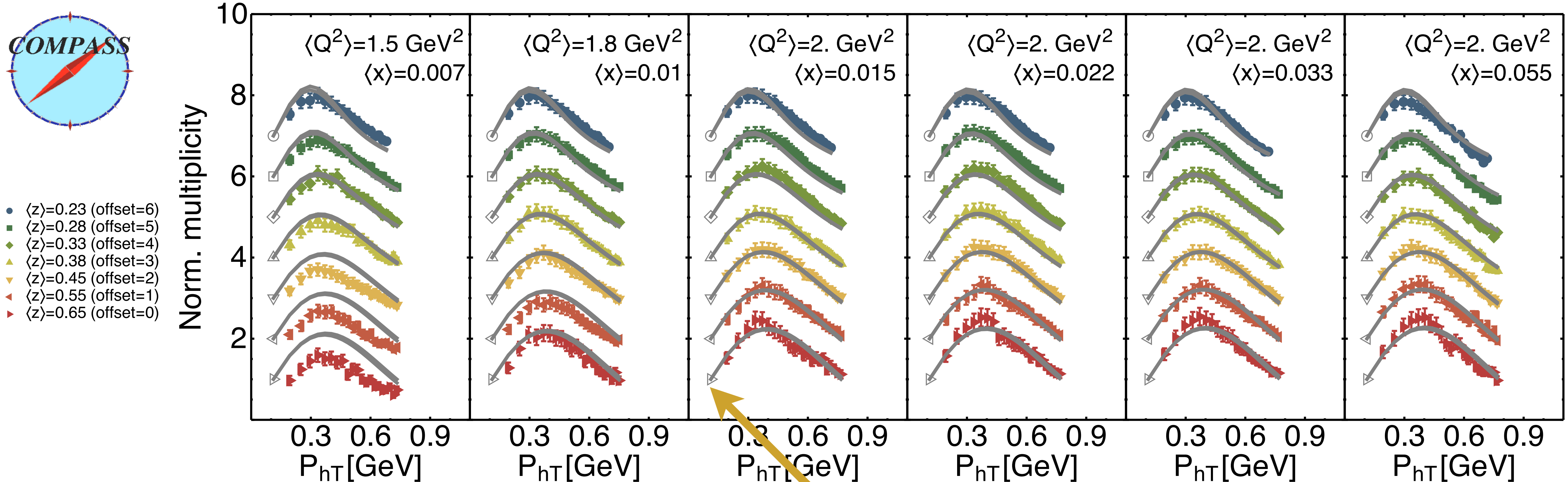


Z production



Bacchetta et al. JHEP 1706 (2017) 081

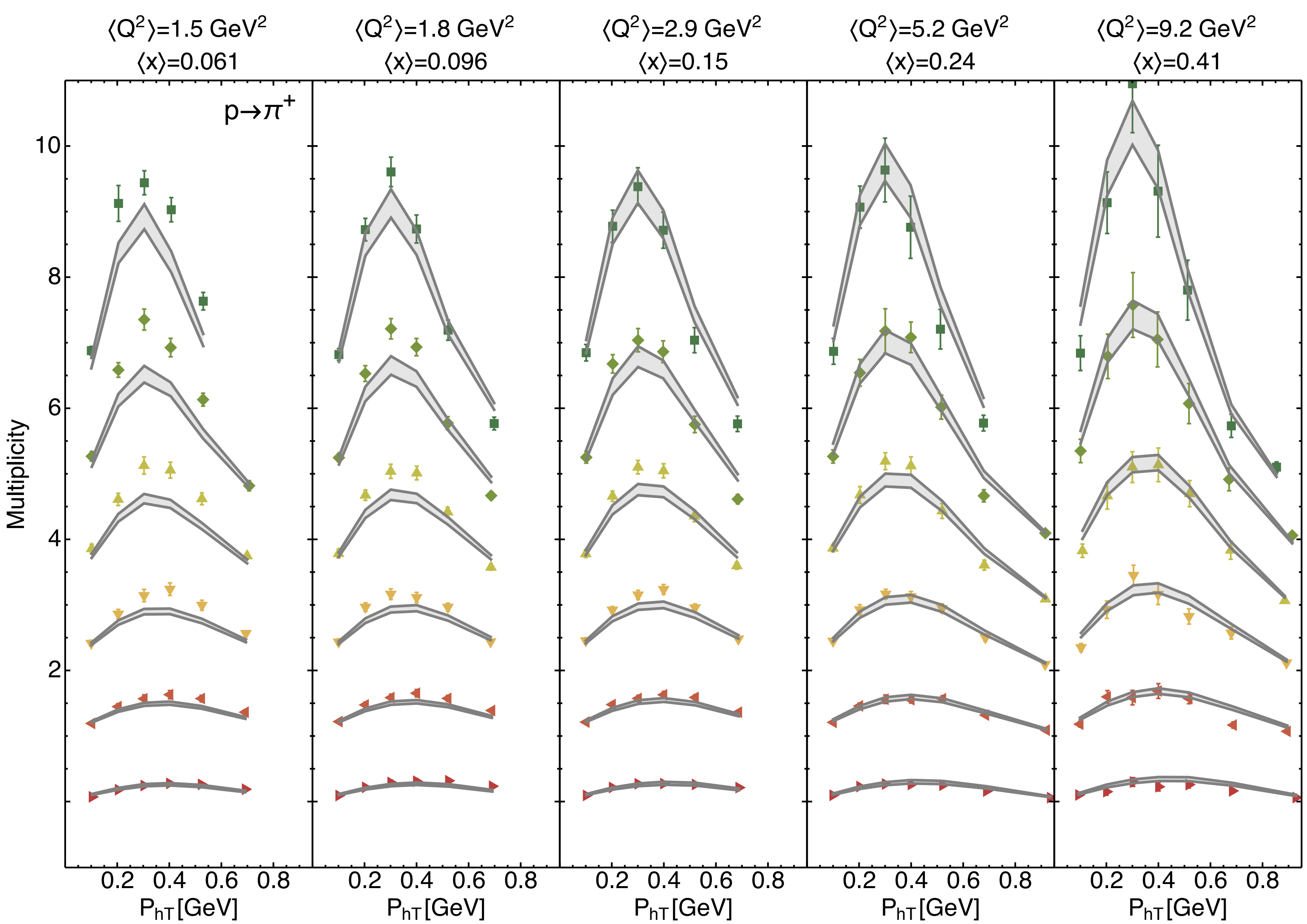
COMPASS, selected bins



First points are not fitted, but used as normalization

Deuteron h^- $\chi^2/\text{dof} = 1.58$

HERMES, selected bins



$\chi^2/\mathrm{dof} = 4.80$

The worst of all channels...

However **normalizing** the theory curves to the first bin, without changing the parameters of the fit, χ^2/dof becomes good

Contributions to chi2 mainly from **normalization**, not shape (also in Z-boson production)

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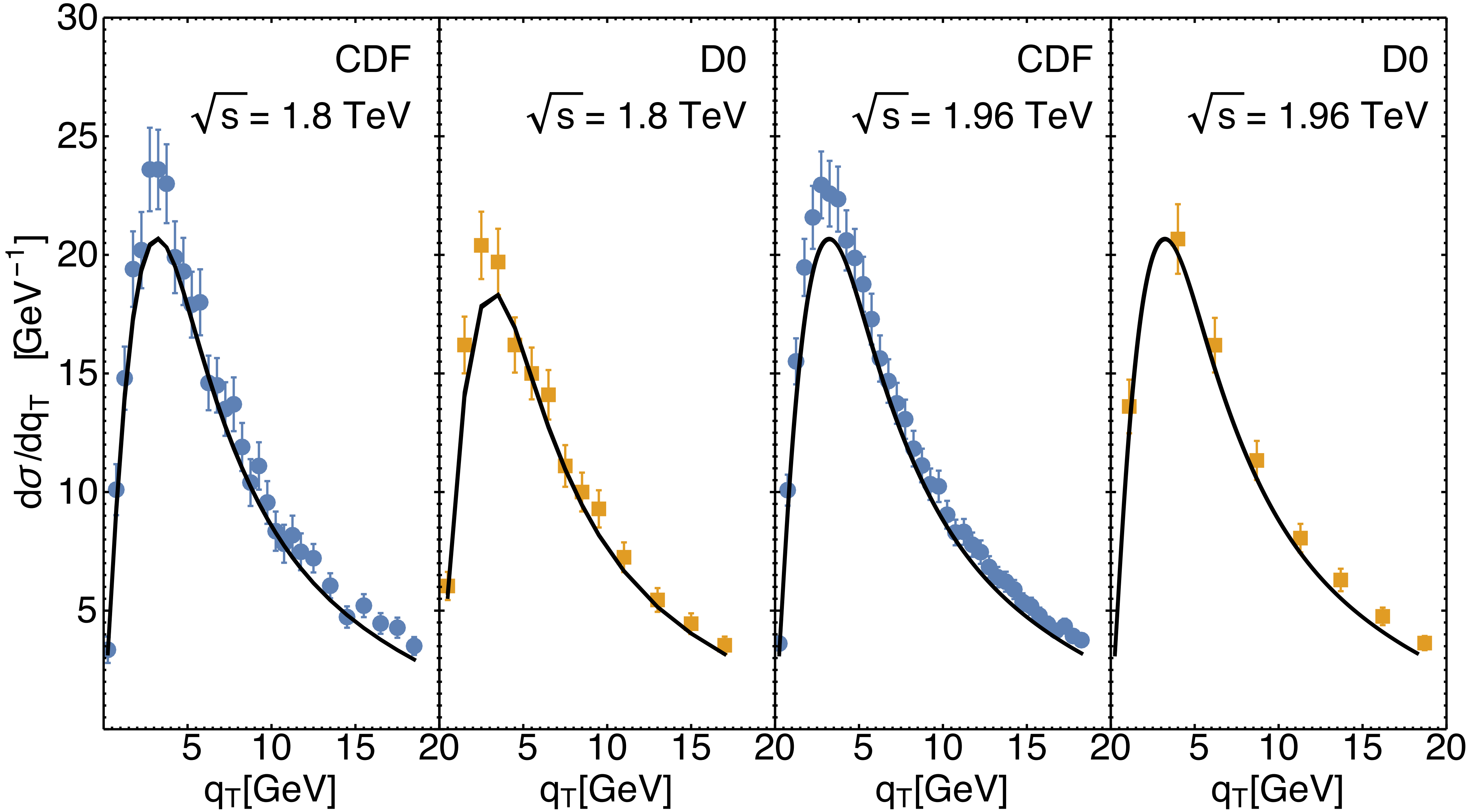
Z-boson @ Fermilab

Narrow bands, driven mainly by **g_2 values** (reduced sensitivity to intrinsic k_T)

Contributions to chi2 mainly from **normalization**, not shape



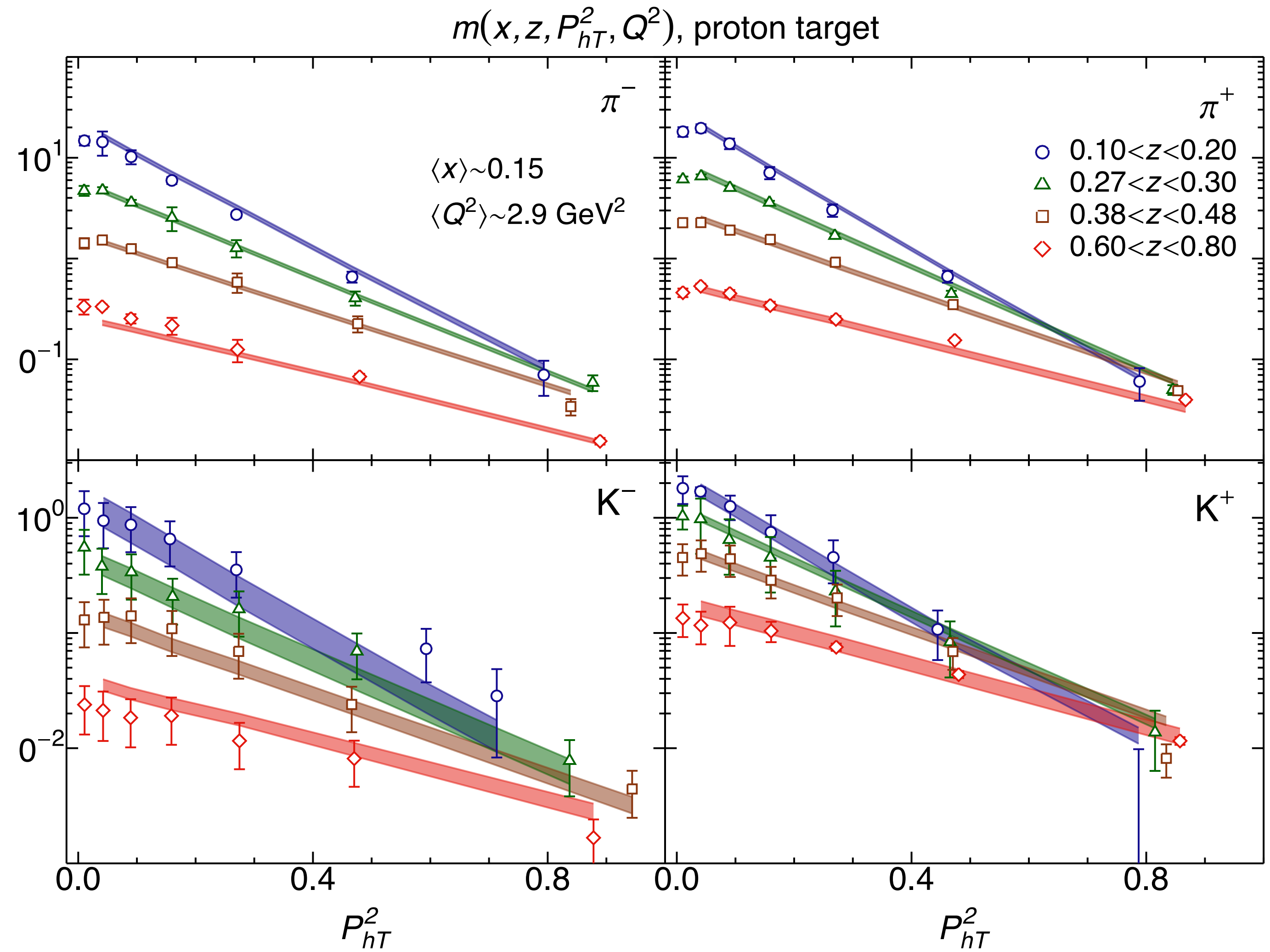
Pavia / Amsterdam / Bilbao 2013

proton target global χ^2 / d.o.f. = 1.63 ± 0.12

no flavor dep. 1.72 ± 0.11

π^-
1.80 ± 0.27
1.83 ± 0.25

K^-
0.78 ± 0.15
0.87 ± 0.16



π^+
2.64 ± 0.21
2.89 ± 0.23

K^+
0.46 ± 0.07
0.43 ± 0.07

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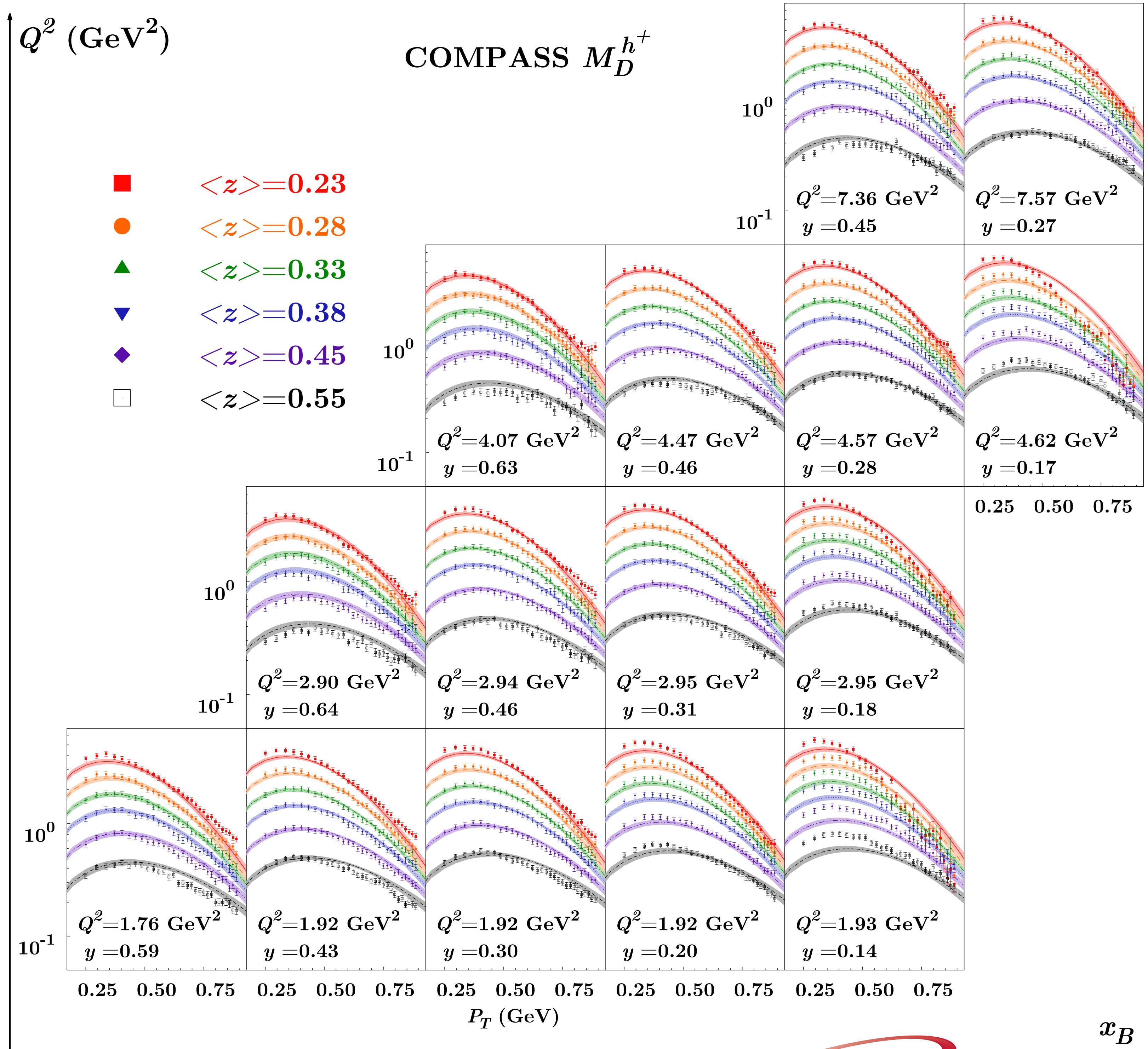
Torino / JLab 2014

COMPASS $M_D^{h^+}$

simple Gaussian ansatz

$\chi^2/\text{dof} = 3.79$
with ad-hoc normalization

see Compass coll. Erratum

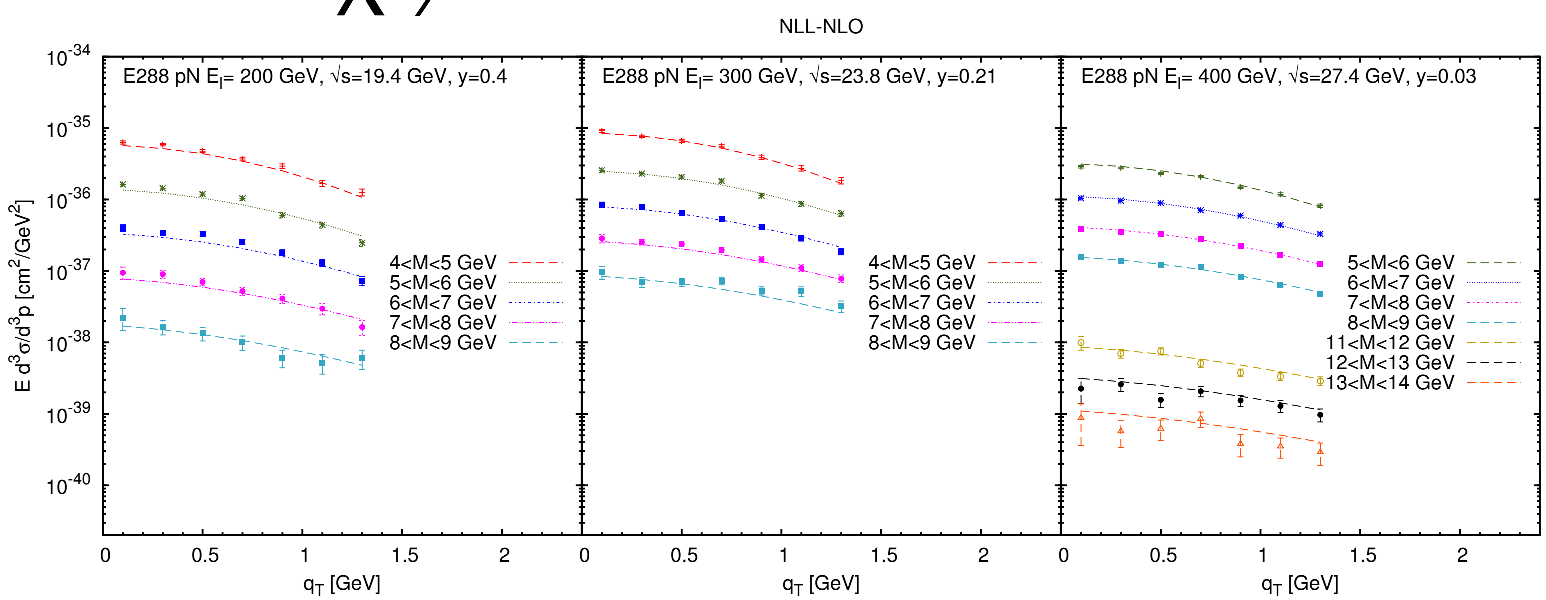


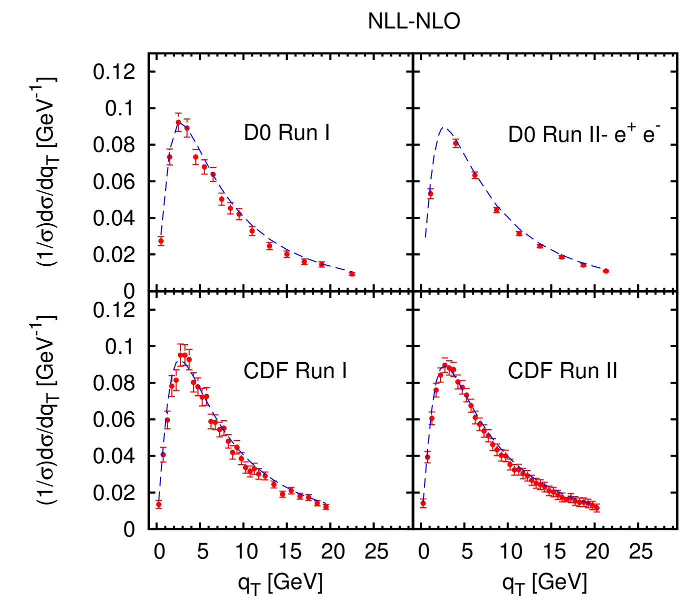
Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (14)

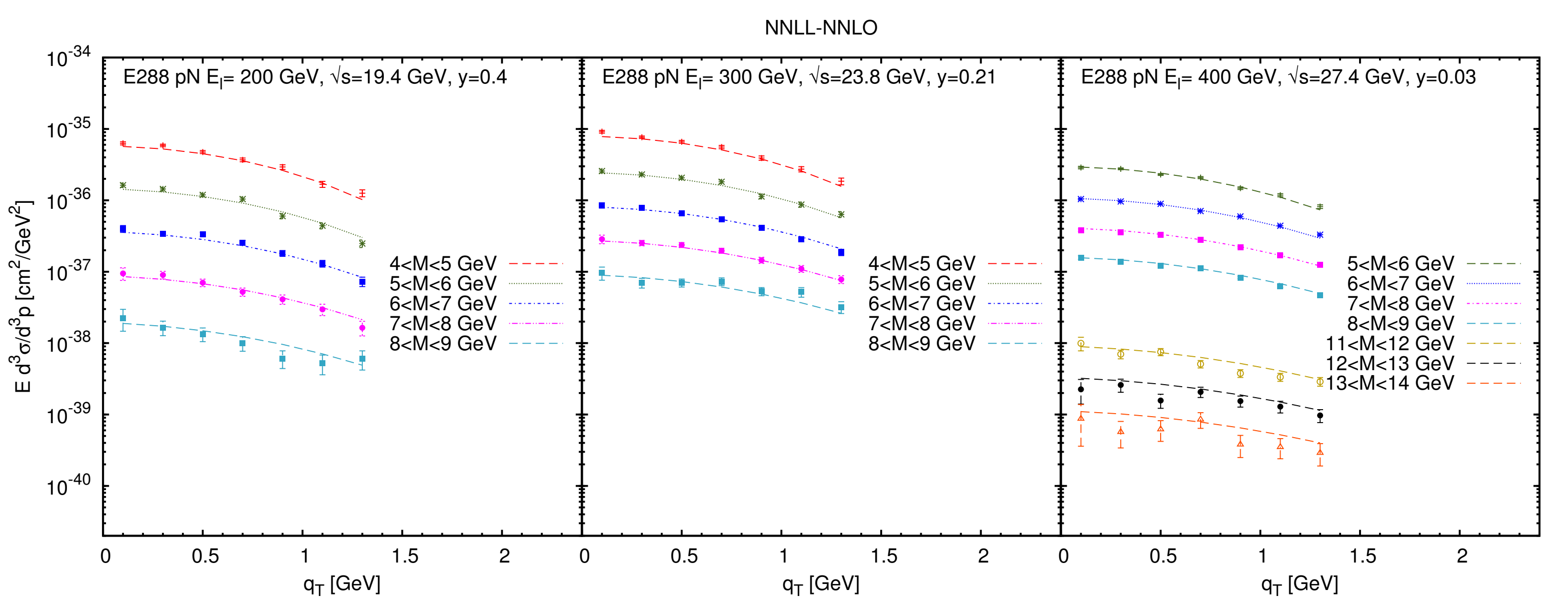
DEMS 2014

D'Alesio, Echevarria, Melis, Scimemi, JHEP 1411 (14)

$\chi^2/\text{dof} = 0.81$



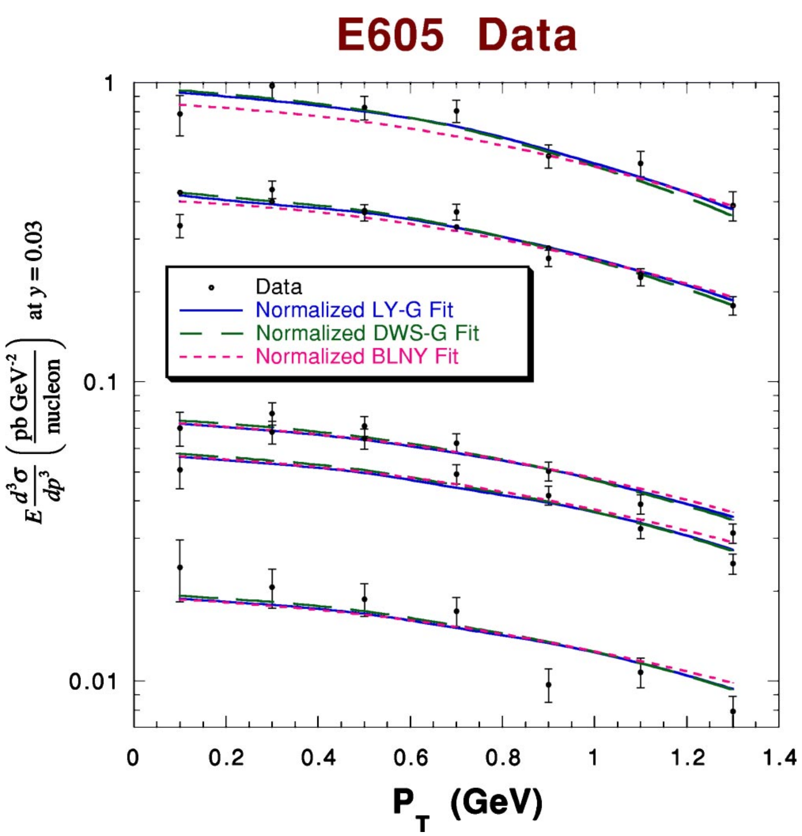


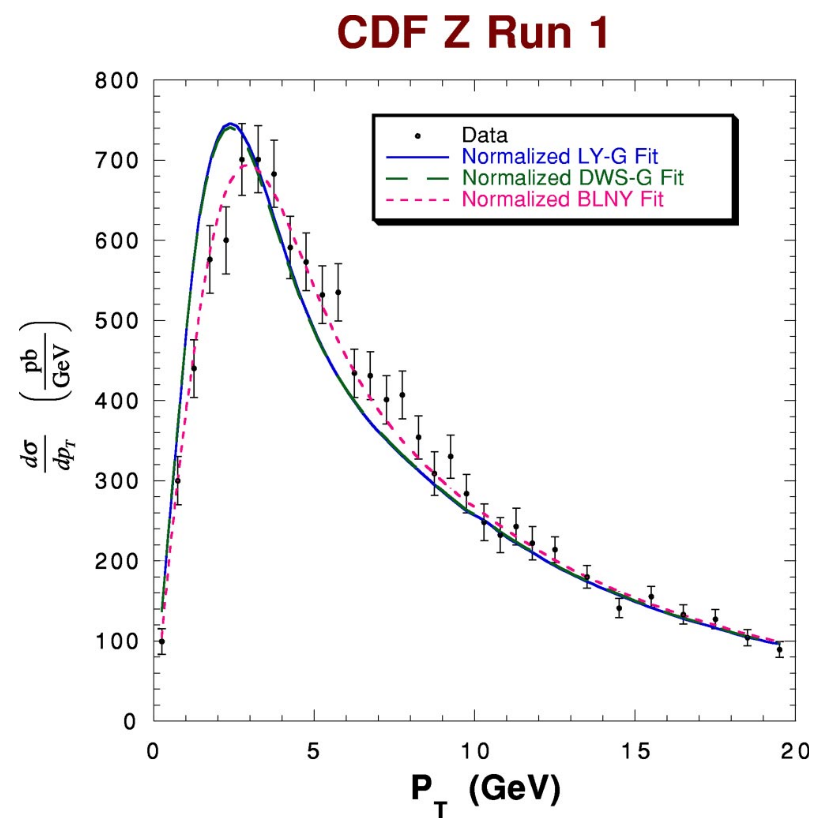


NLO-NNLL analysis with evaluation of theoretical uncertainties

very good

KN 2006





≈100 data points

$Q^2 > 4$ GeV

$$Q_0 = 3.2 \text{ GeV}$$

$$\frac{1}{\langle b_T^2 \rangle} = \frac{1}{2}\left(0.21 + 0.68 \log\left(\frac{Q}{2Q_0}\right) - 0.25 \log(10x)\right)$$

$$b_{\max} = 0.5 \text{ GeV}^{-1}$$

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

$$\frac{1}{\langle b_T^2 \rangle} = \frac{1}{2}\left(0.20 + 0.184 \log\left(\frac{Q}{2Q_0}\right) - 0.026 \log(10x)\right)$$

$$b_{\max} = 1.5 \text{ GeV}^{-1}$$

KN 2006

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EIKV 2014

Parametrizations for intrinsic momenta
and soft gluon emission :

$$F_{NP}(b_T, Q)^{\mathrm{pdf}} = \exp\left[-b_T^2\left(g_1^{\mathrm{pdf}} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

$$F_{NP}(b_T, Q)^{\mathrm{ff}} = \exp\left[-b_T^2\left(g_1^{\mathrm{ff}} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

Pros and Cons :

1) a global analysis of SIDIS and DY/Z/W data

2) TMD evolution at LO-NLL

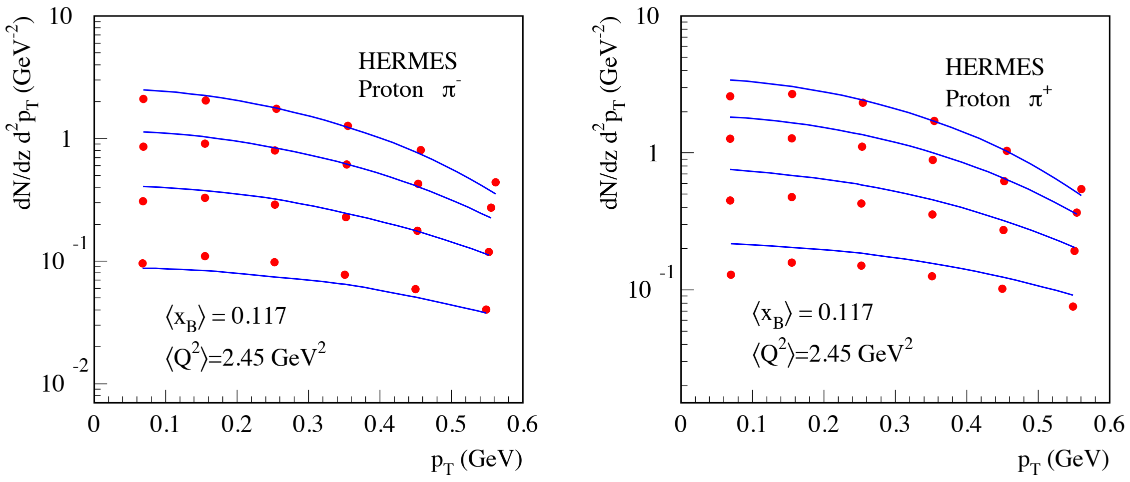
3) multidimensionality not exploited

4) chi-square not provided

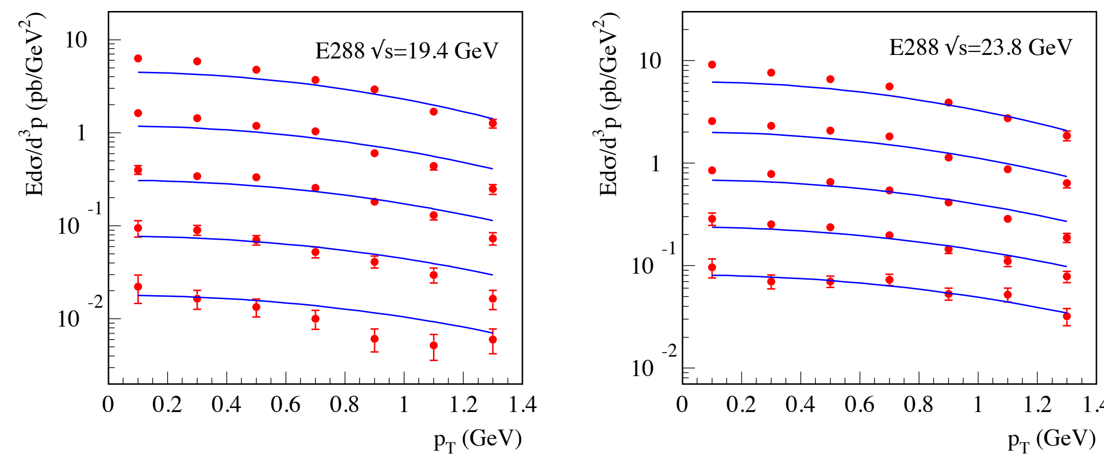
5) can’t be considered as a “complete” fit

EIKV 2014

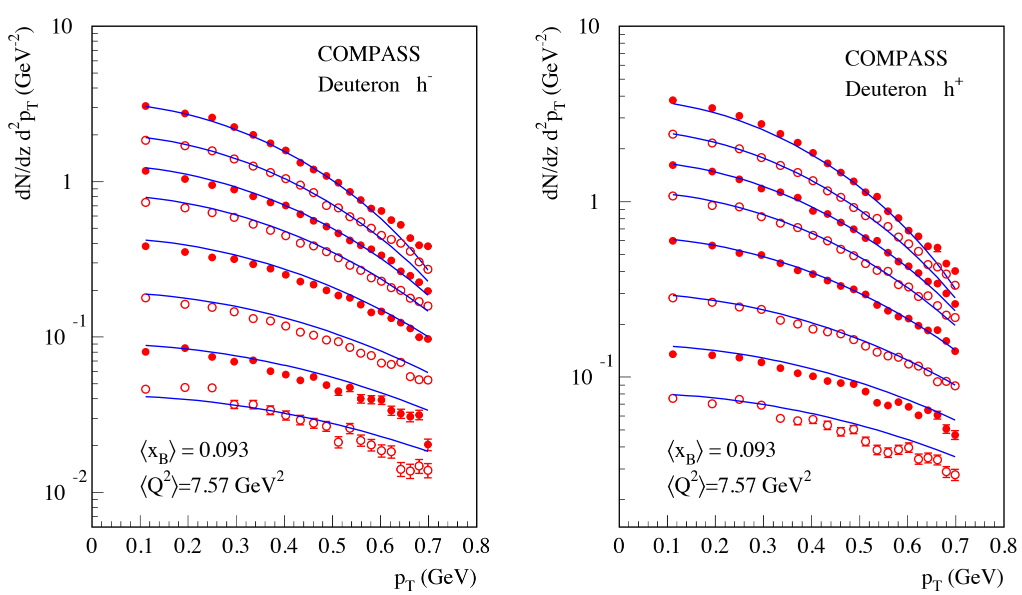
SIDIS



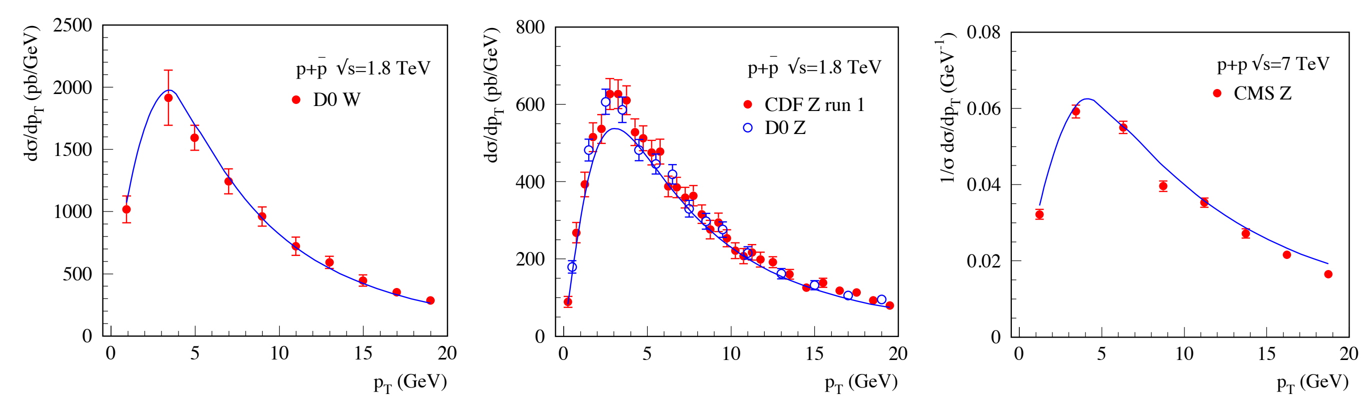
DRELL-YAN



SIDIS



W AND Z PRODUCTION



$$b_{\max} = 1.5\ \text{GeV}^{-1}$$

$$g_2 = 0.16$$

Echevarria et al. arXiv:1401.5078

Other studies

CSS formalism on DY/Z/W data:

1) Davies-Webber-Stirling (DOI: 10.1016/0550-3213(85)90402-X)

2) Ladinsky-Yuan (DOI: 10.1103/PhysRevD.50.R4239)

3) BLNY (DOI: 10.1103/PhysRevD.63.013004)

4) Hirai, Kawamura, Tanaka (DOI: 10.3204/DESY-PROC-2012-02/136) - complex-b prescription

...

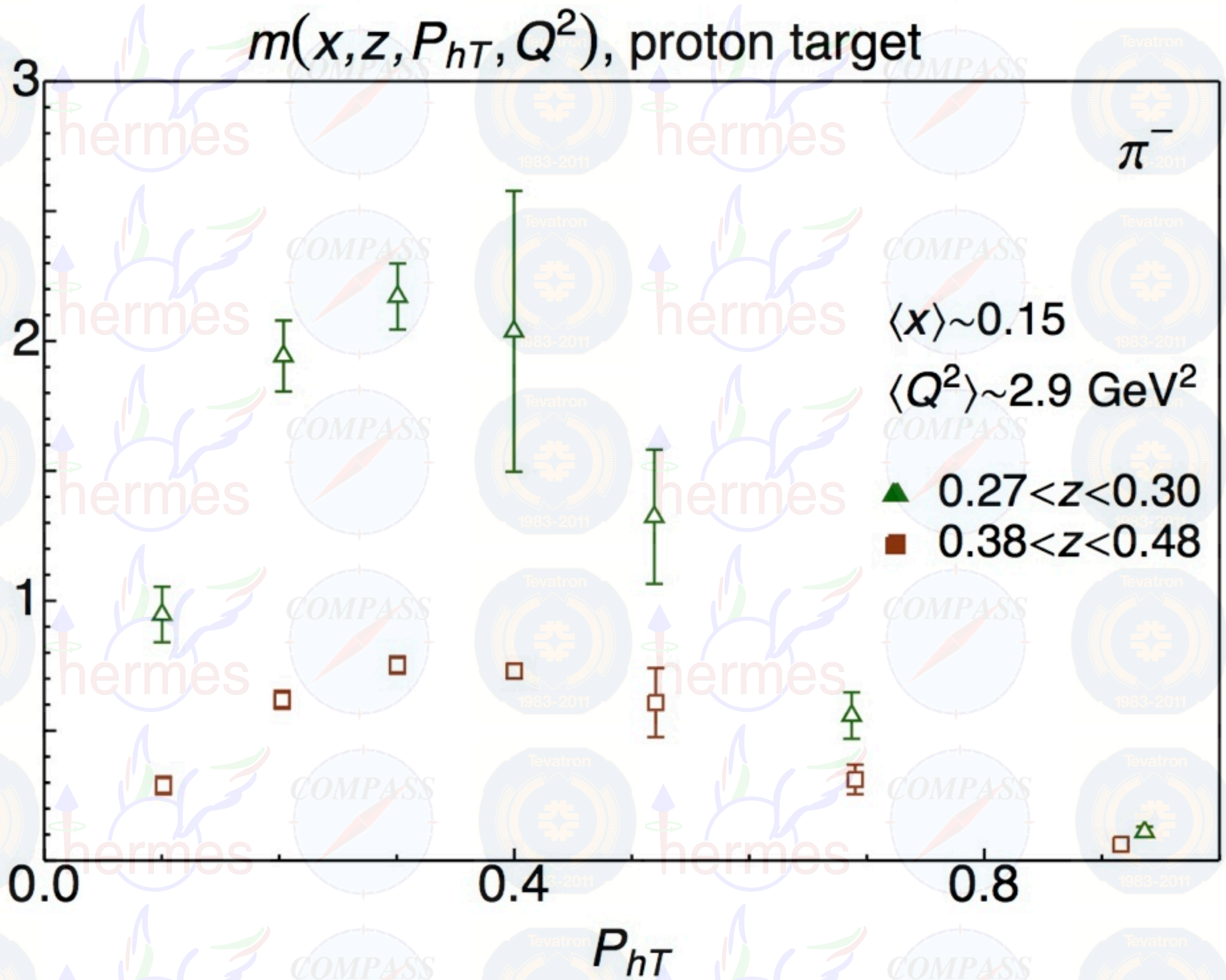
combined SIDIS/DY/W/Z :

5) Sun, Yuan (arXiv:1308.5003)

6) Isaacson, Sun, Yuan, Yuan (arXiv:1406.3073)

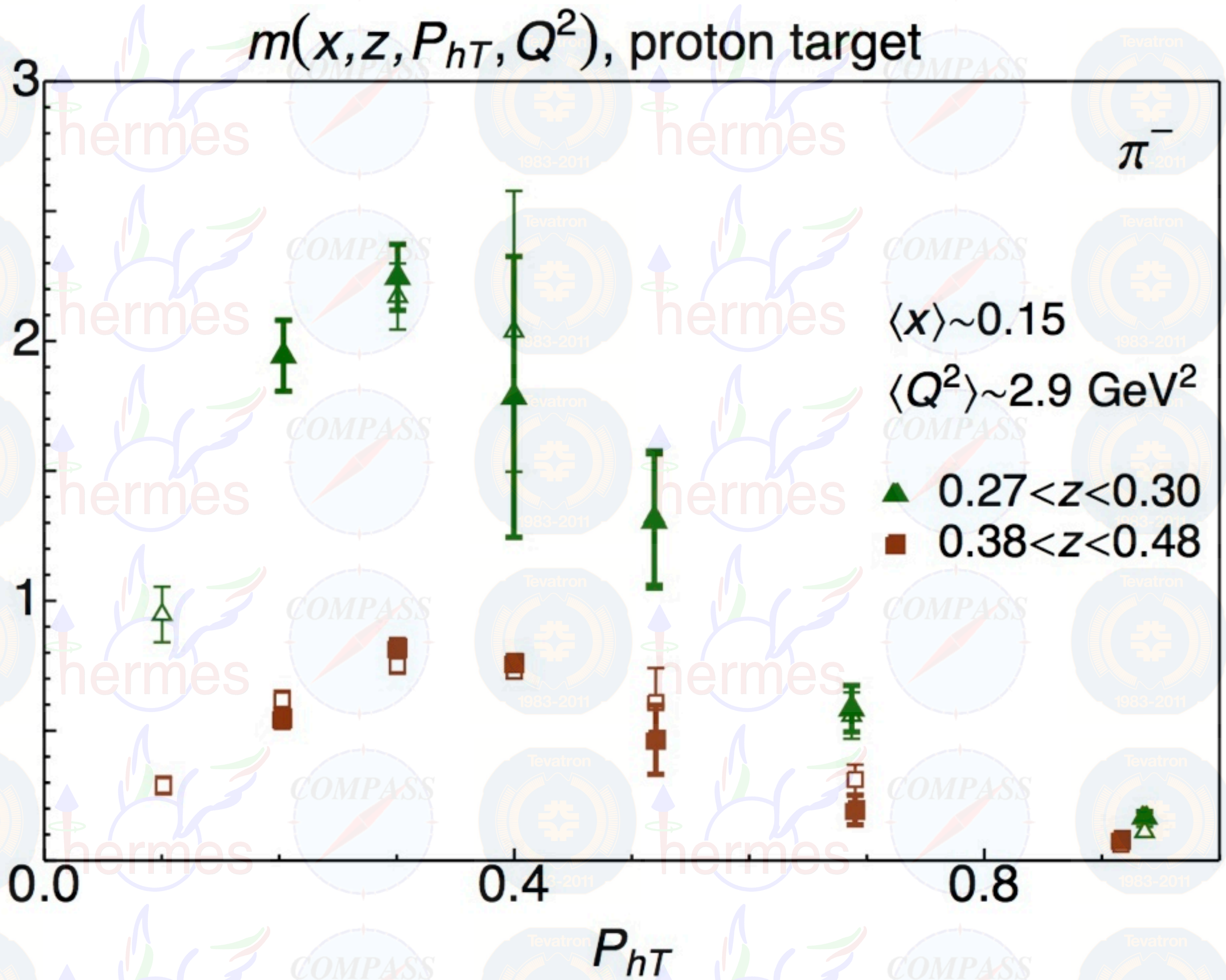
...

The replica method



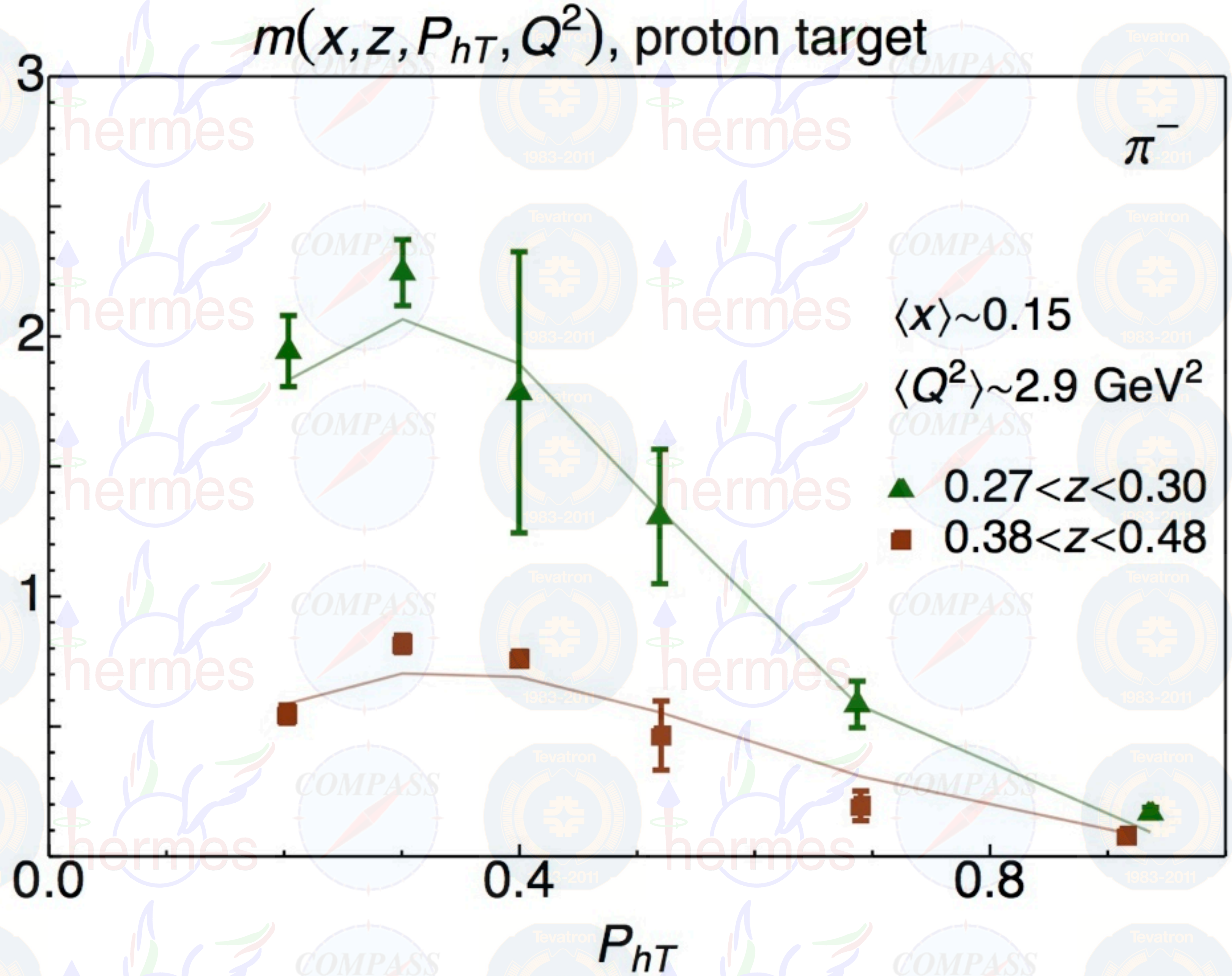
Sample of original data

The replica method



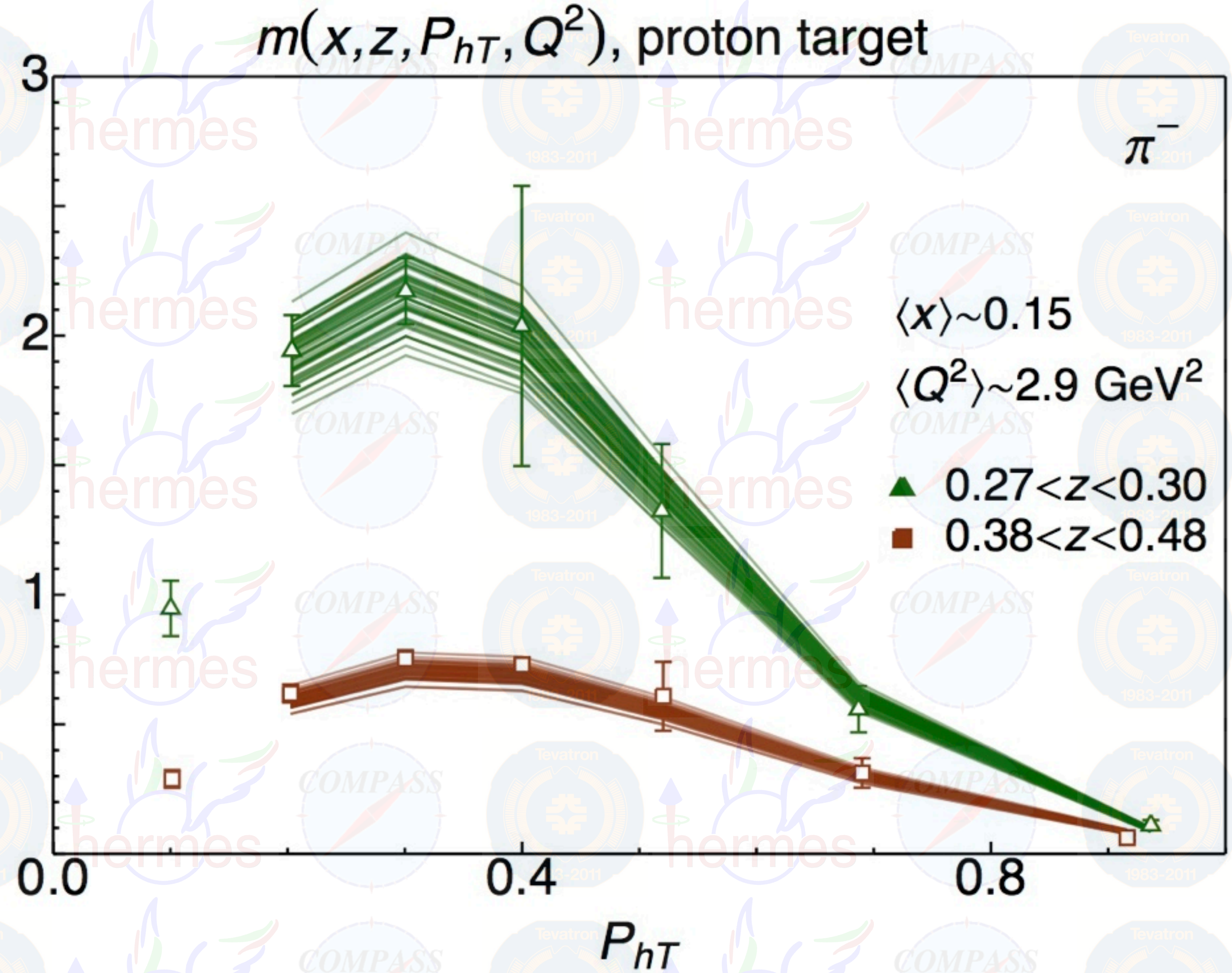
Replica of the original data with Gaussian noise

The replica method



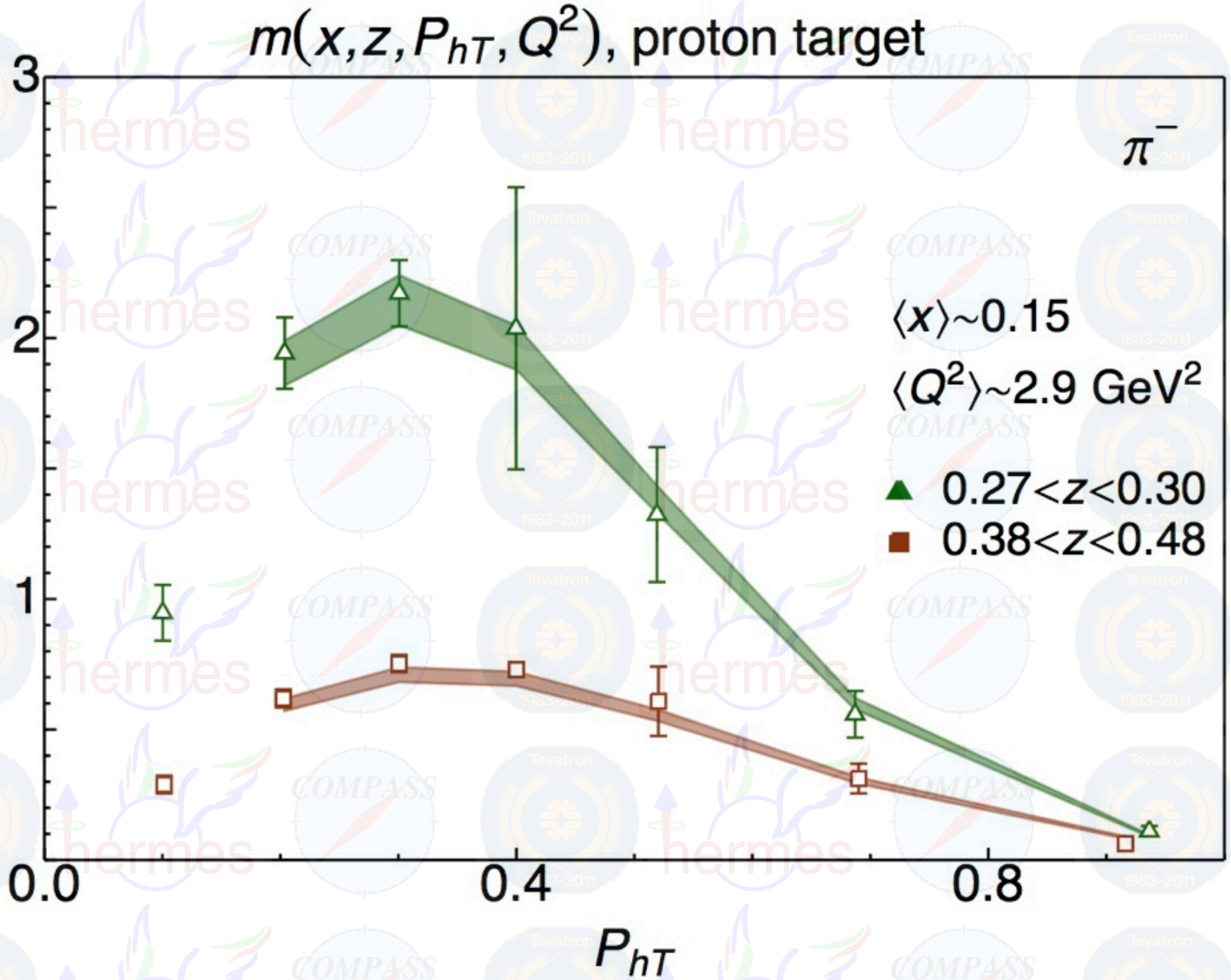
Fit of the replicated data

The replica method



Repeat the generation and the fit N times

The replica method



Obtain **distributions of best values** -
calculate **68% CL bands**

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