Fragmentation Functions 2018
Feb. 19-22, 2018 - Stresa, IT

recent results and future plans

BELLE

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“skip introduction!” [organizer's mail]

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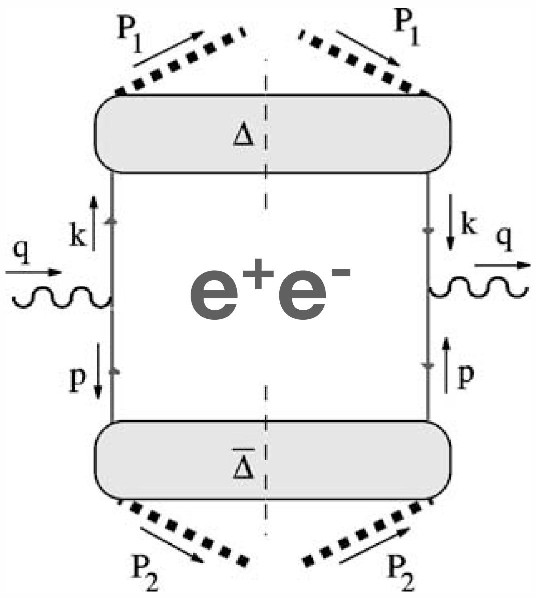
instead add some personal spice
(to trigger discussion)

fragmentation in e^+e^- annihilation

- single-inclusive hadron production,
  $e^+e^- \rightarrow hX$
  - D_1 fragmentation fctn.
  - $D_{1T}^{\perp}$ spontaneous transv. pol.



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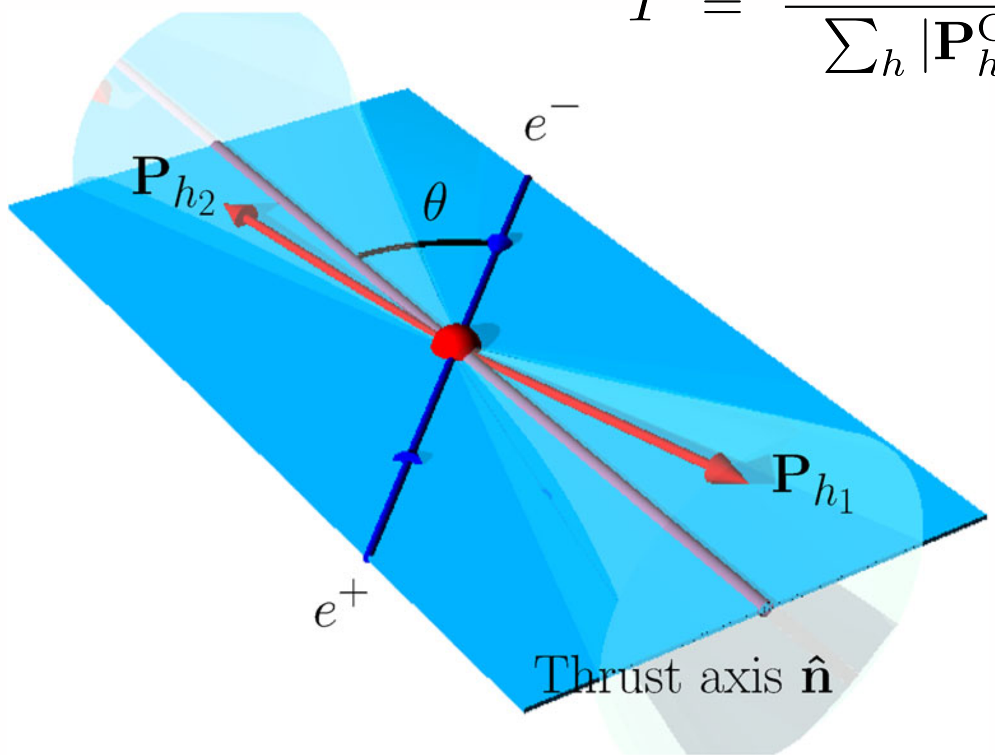
  - D_1 fragmentation fctn.
  - $D_{1T}^{\perp}$ spontaneous transv. pol.
- inclusive "back-to-back" hadron pairs, **$e^+e^- \to h_1h_2X$**
  - product of FFs
  - flavor, transverse-momentum, and/or polarization tagging

Thrust (axis):

$$T \stackrel{\max}{=} \frac{\sum_h |\mathbf{P}_h^{\rm CMS} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\rm CMS}|}$$



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- inclusive "back-to-back" hadron pairs, **$e^+e^- \rightarrow h_1h_2X$**
  - product of FFs
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- inclusive same-hemisphere hadron pairs, **$e^+e^- \rightarrow h_1h_2X$**
  - dihadron fragmentation

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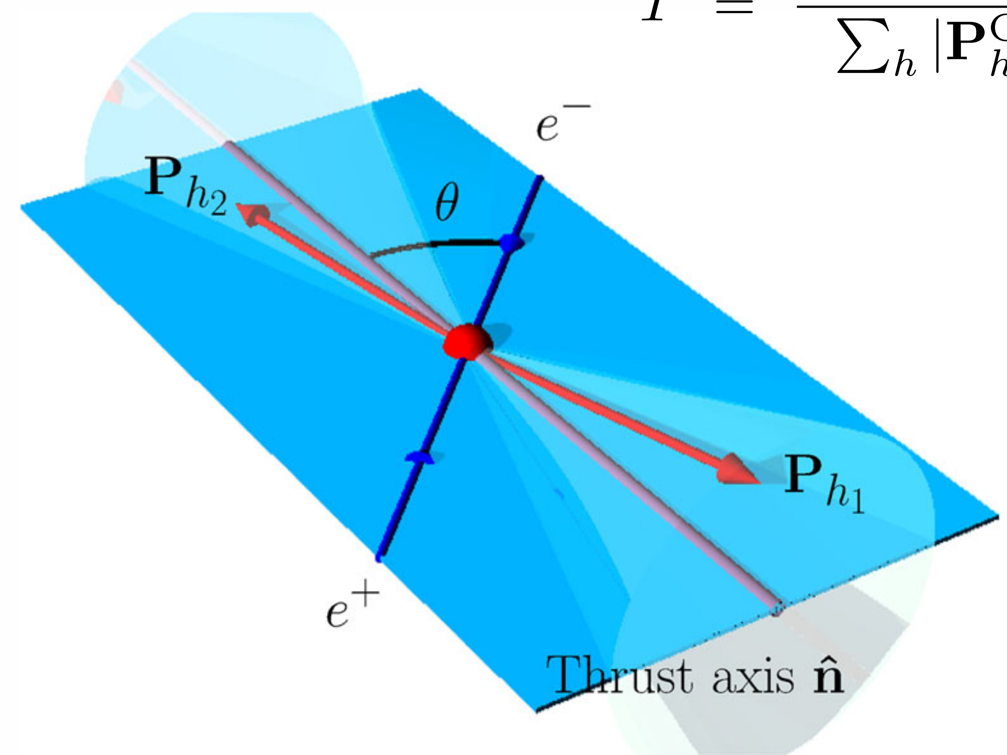
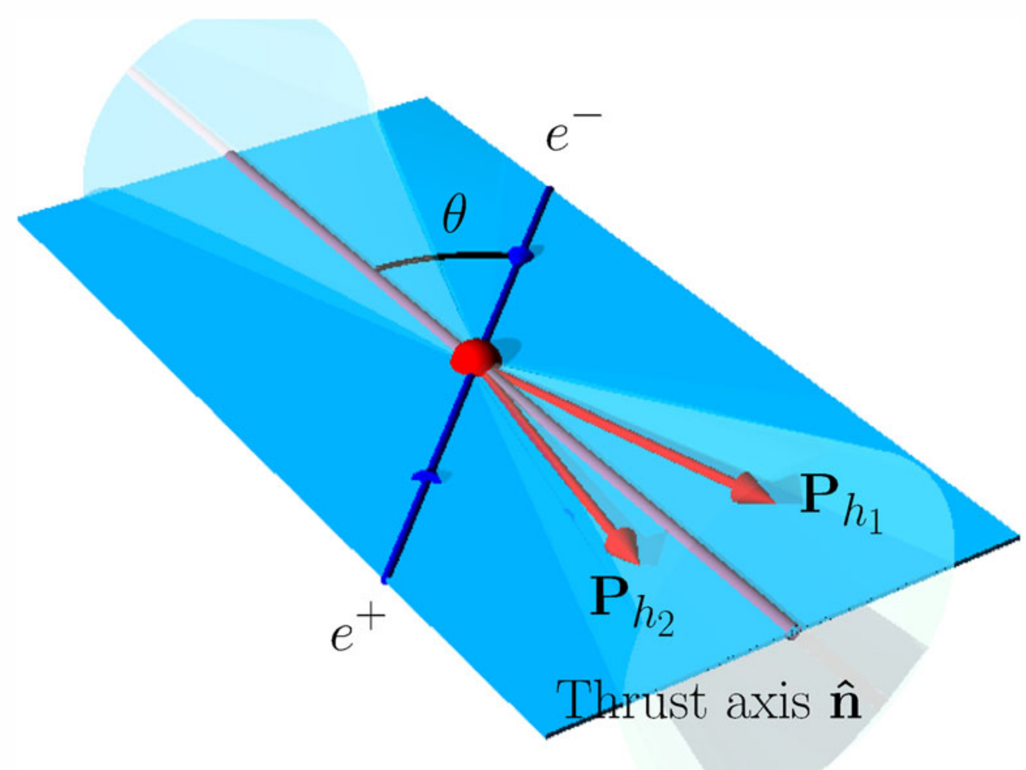
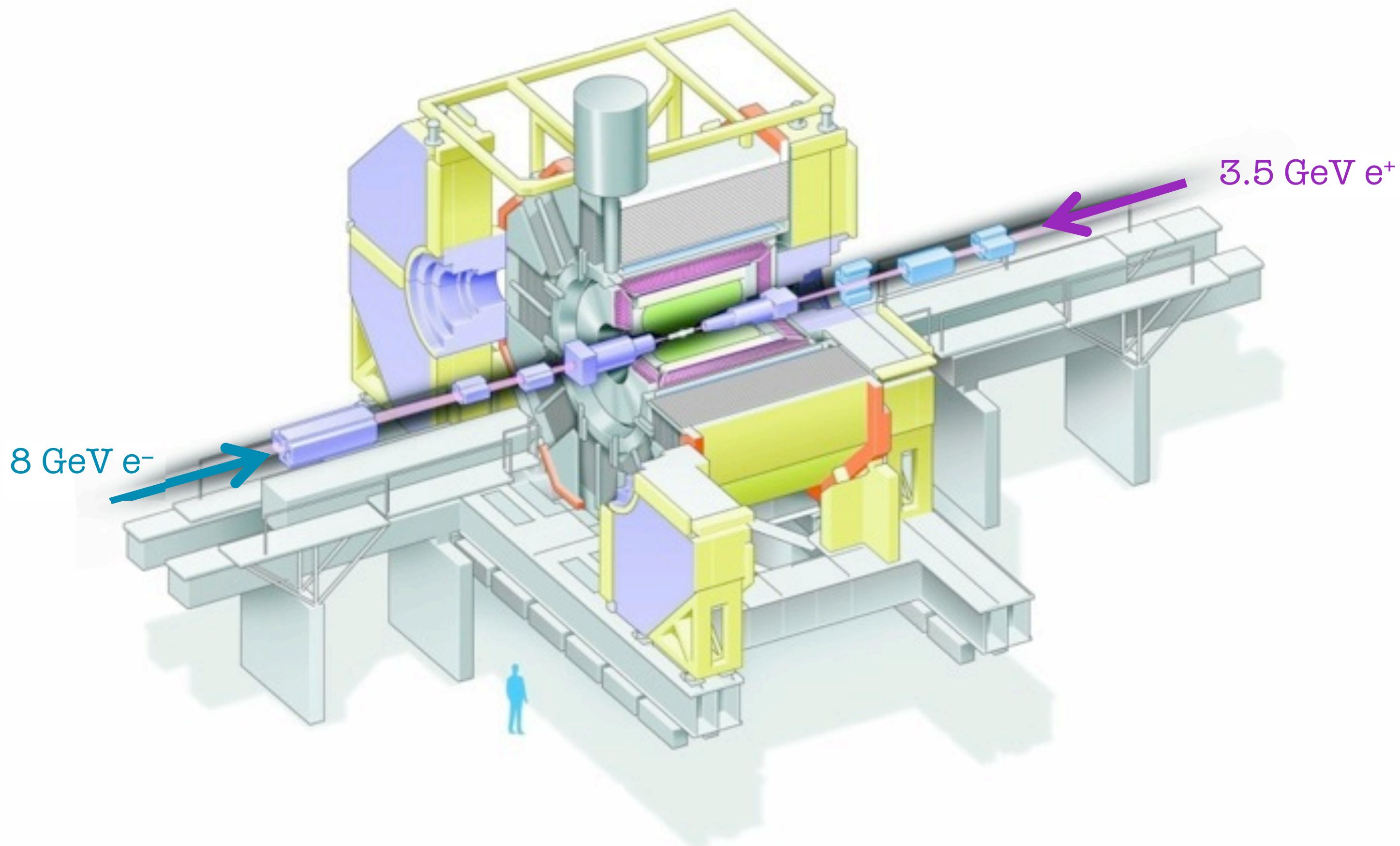
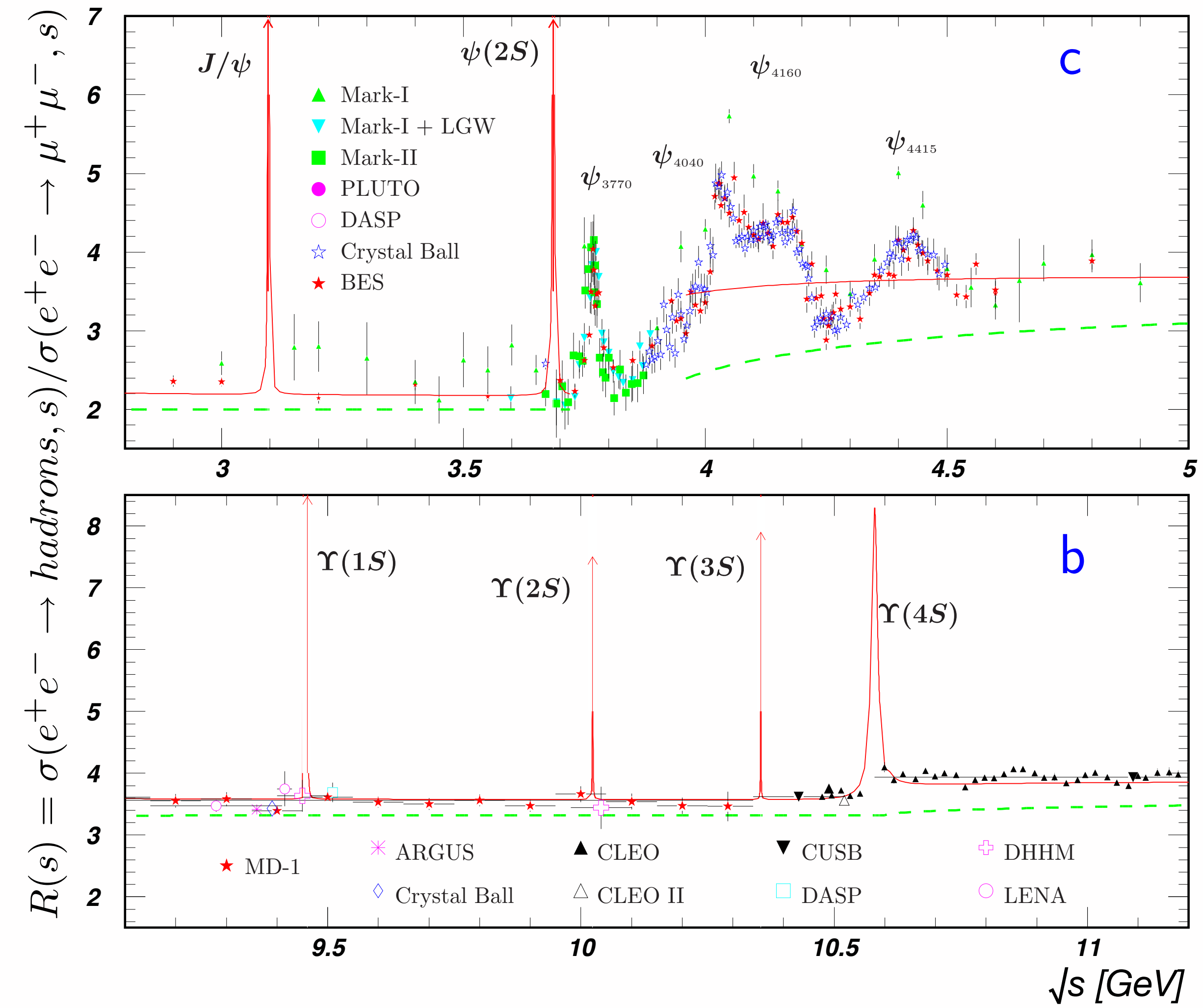




e^+e^- annihilation at Belle





- KEKB: asymmetric beam-energy e^+e^- collider near/at Υ(4S) resonance (10.58 GeV)
- large azimuthally symmetric geometric acceptance
- particle ID -> π, K, p results

Υ(4S) on resonance	Υ(4S) off resonance "udsc continuum"
655 fb^{-1}	68 fb^{-1}

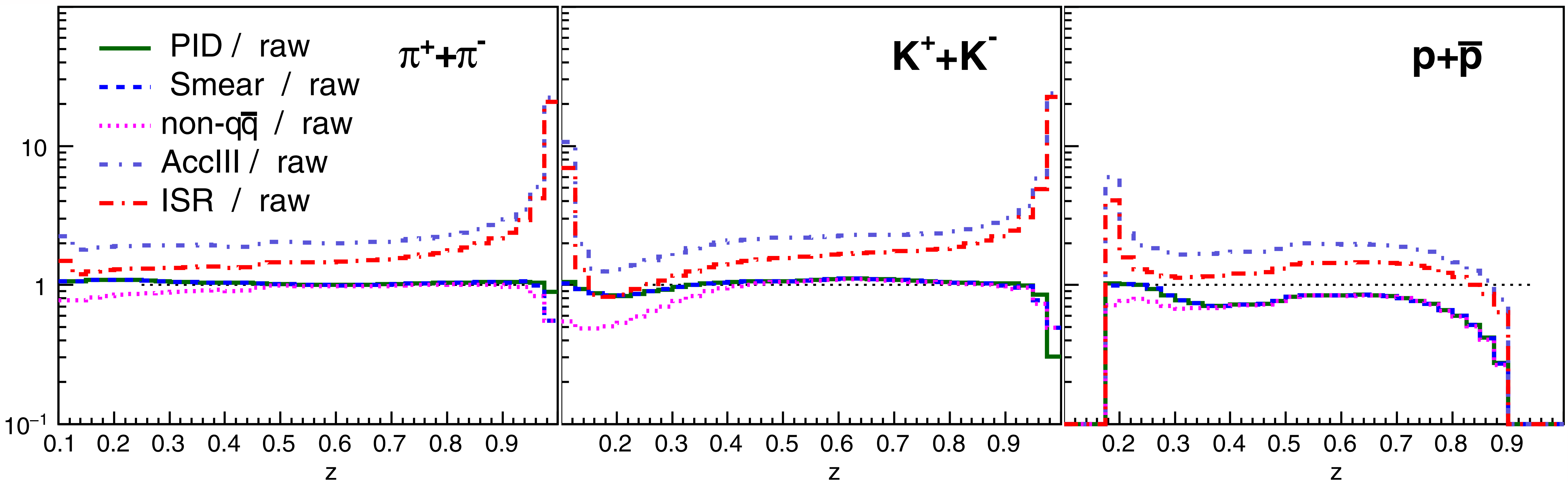
[published results shown here]

from hadron yields to cross sections

- hadron yields observed undergo series of corrections
  - particle (mis)identification [e.g., not every identified pion was a pion]
  - smearing unfolding [e.g., measured and true momentum might differ]
  - non-$q\bar{q}$ processes [e.g., two-photon processes, Υ -> BB, ...]
  - "4π" correction [selection criteria and limited geometric acceptance]
  - QED radiation [initial-state radiation (ISR)]
  - optional: weak-decay removal (e.g., "prompt fragmentation")

from hadron yields to cross sections

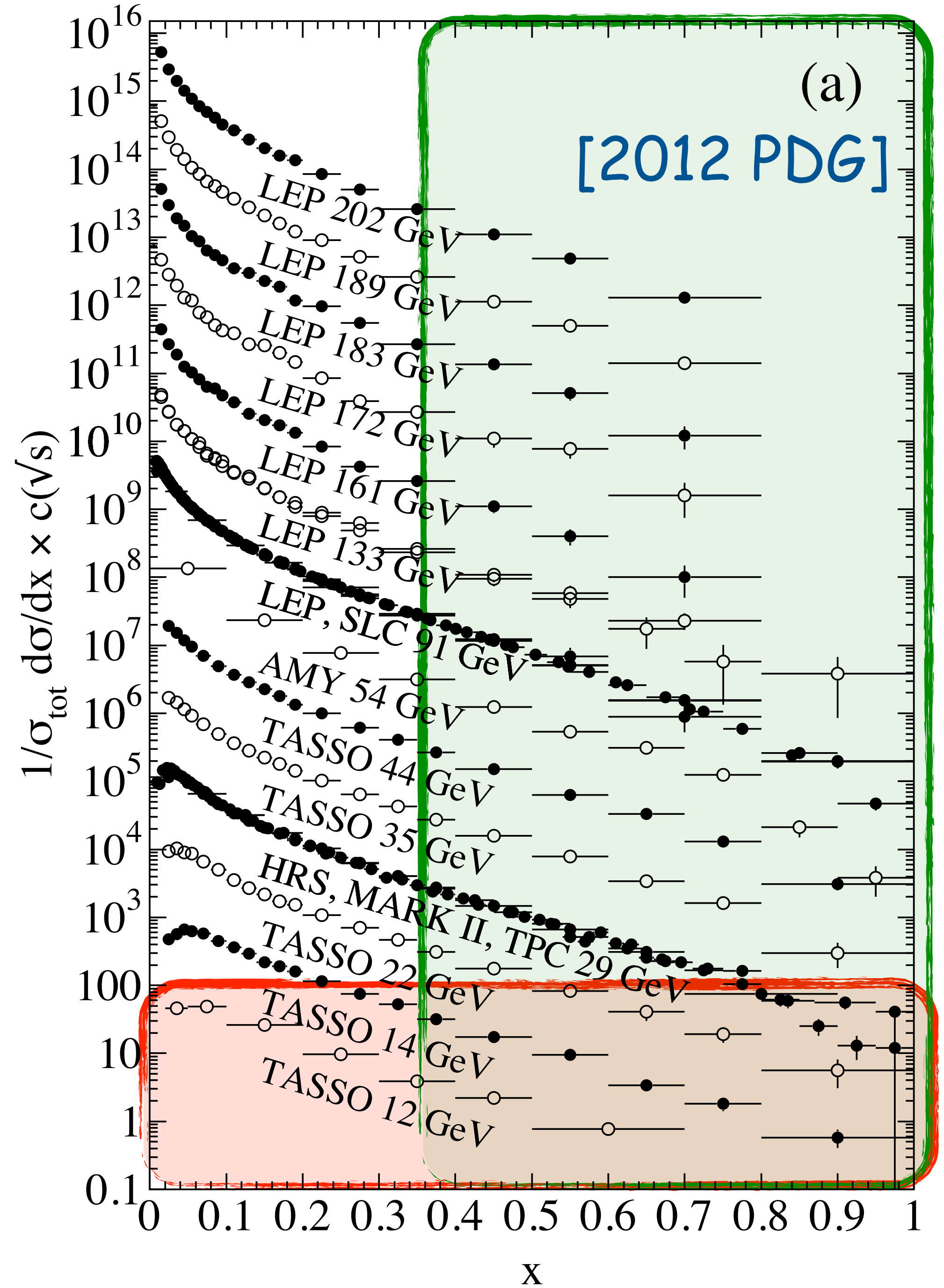
- **example**: single-hadron cross sections
  - cumulative effect of correction steps



  - largest effect for mesons from acceptance and ISR correction
  - larger PID correction for protons than for mesons

single-hadron production

- before 2013: lack of precision data at (moderately) high z and at low √s
  - limits analysis of evolution and gluon fragmentation
  - limited information in kinematic region often used in semi-inclusive DIS



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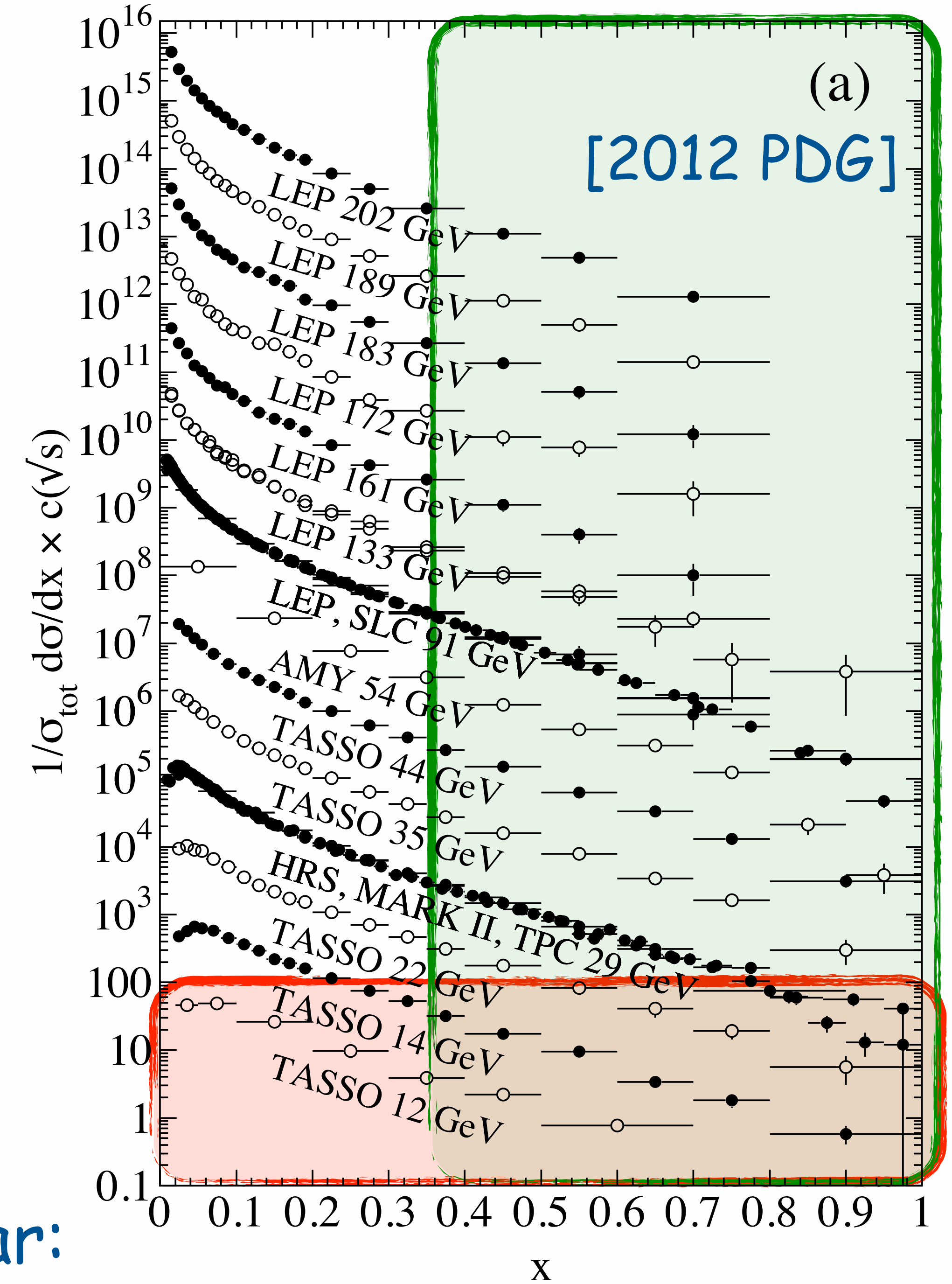
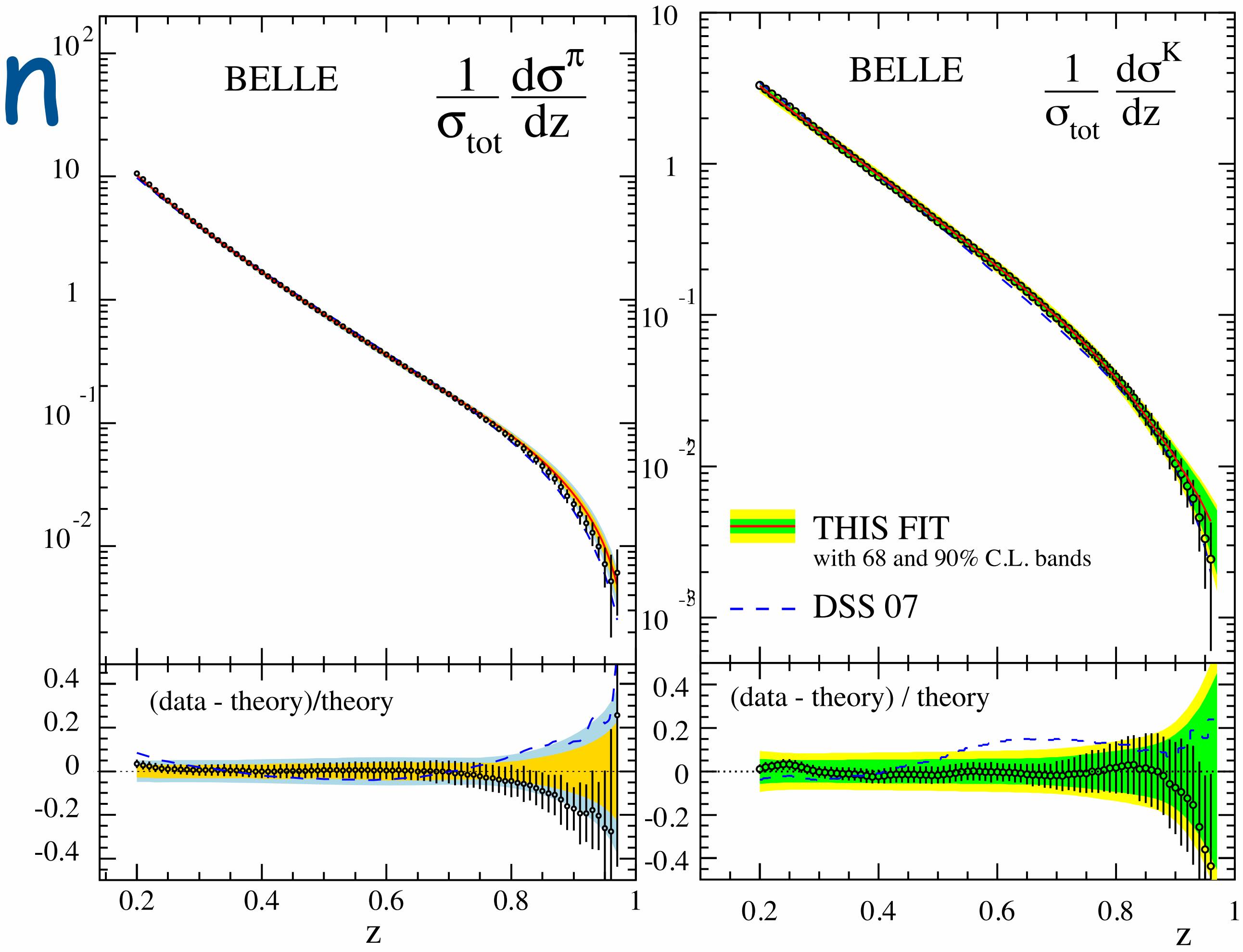


- by now, results available from Belle and BaBar:
  - **Belle Collaboration, Phys. Rev. Lett. 111 (2013) 062002: $\pi^\pm$, $K^\pm$**
  - **Belle Collaboration, Phys. Rev. D92 (2015) 092007: $\pi^\pm$, $K^\pm$, $p+\bar{p}$**
  - BaBar Collaboration, Phys. Rev. D88 (2013) 032011: $\pi^\pm$, $K^\pm$, $p+\bar{p}$

single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z (z<0.98)
- included in recent DEHSS fits [e.g. PRD 91, 014035 (2015)]



single-hadron production

- very precise data for charged pions and kaons
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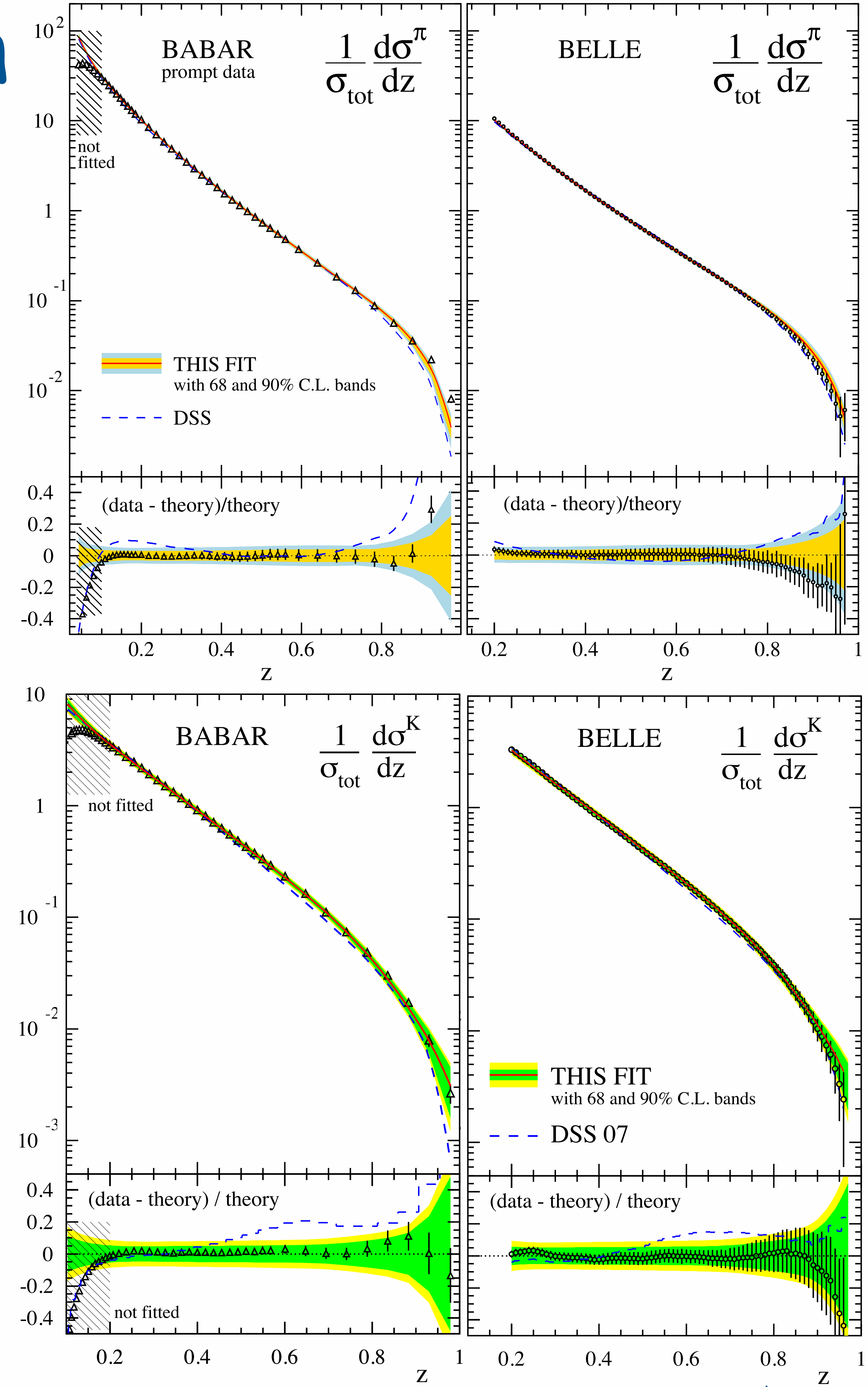
- included in recent DEHSS fits
  - slight tension at low-z for BaBar and high-z for Belle



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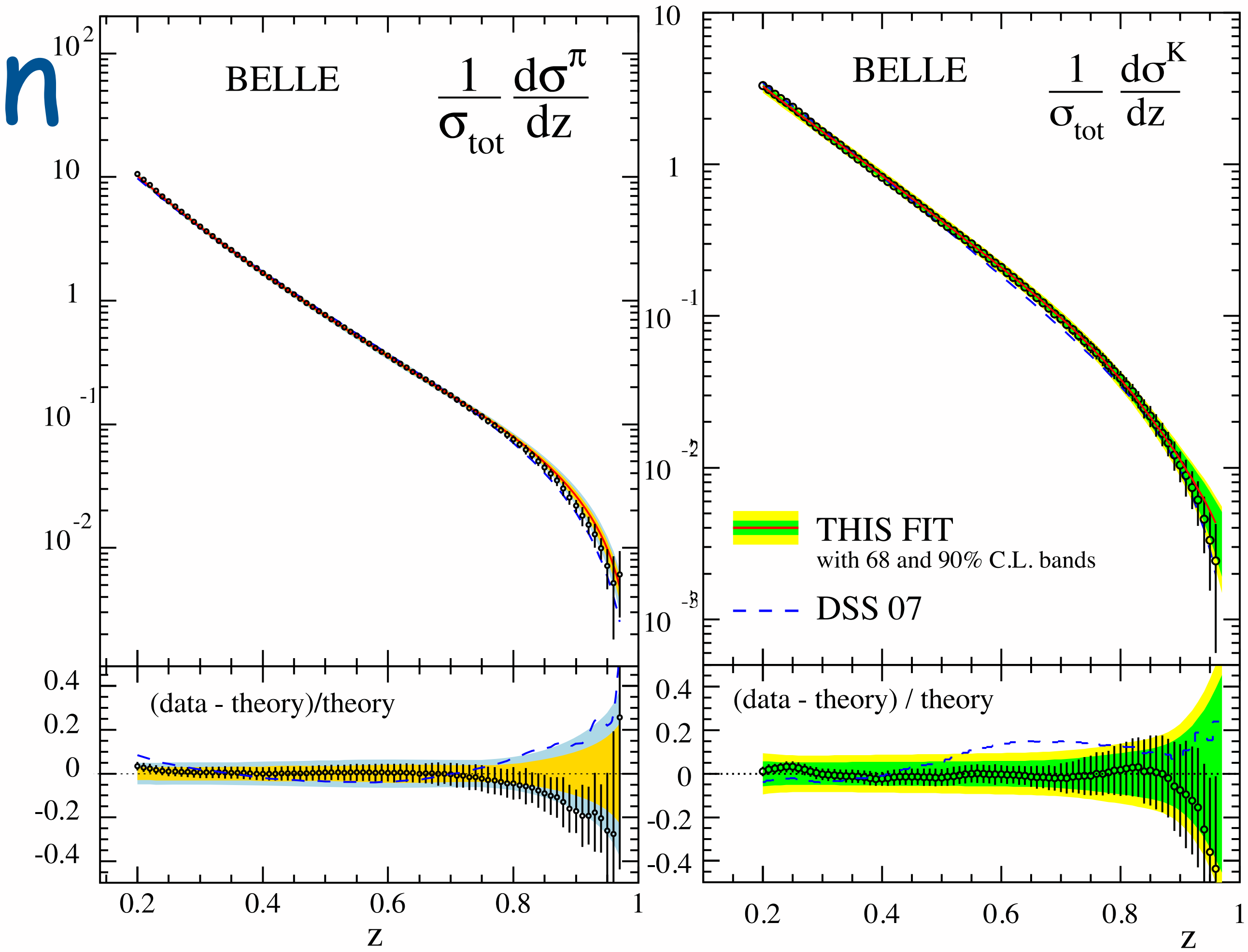
- radiative corrections **undone** in FF fits

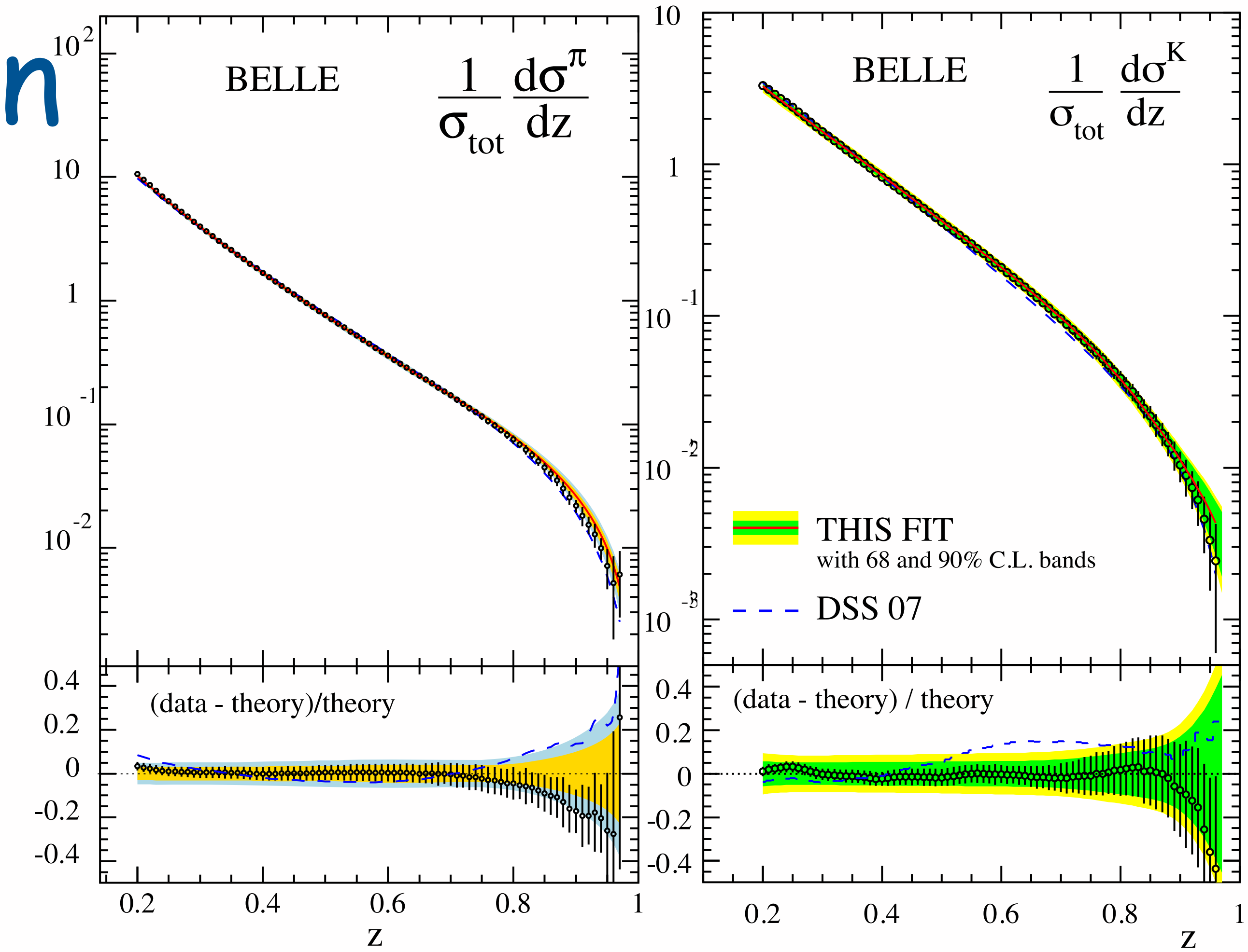


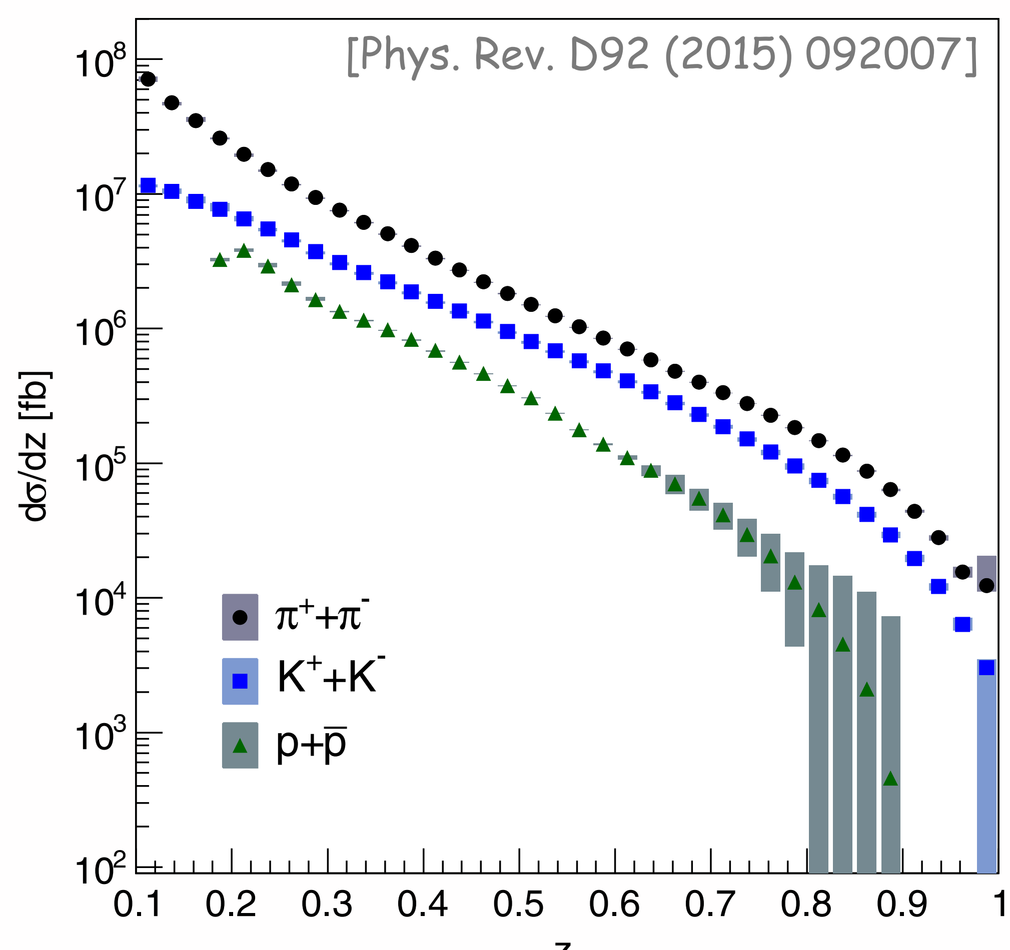
[EPJC 77 (2017) 516, NNFF1.0]

In the case of the BELLE experiment we multiply all data points by a factor $1/c$, with $c = 0.65$ for charged pions and kaons [69] and with c a function of z for protons/antiprotons [53]. This correction is required in order to treat the BELLE data consistently with all the other SIA measurements included in NNFF1.0. The reason is that a kinematic cut on radiative photon events was applied to the BELLE data sample in the original analysis instead of unfolding the radiative QED effects. Specifically, the energy scales

single-hadron production

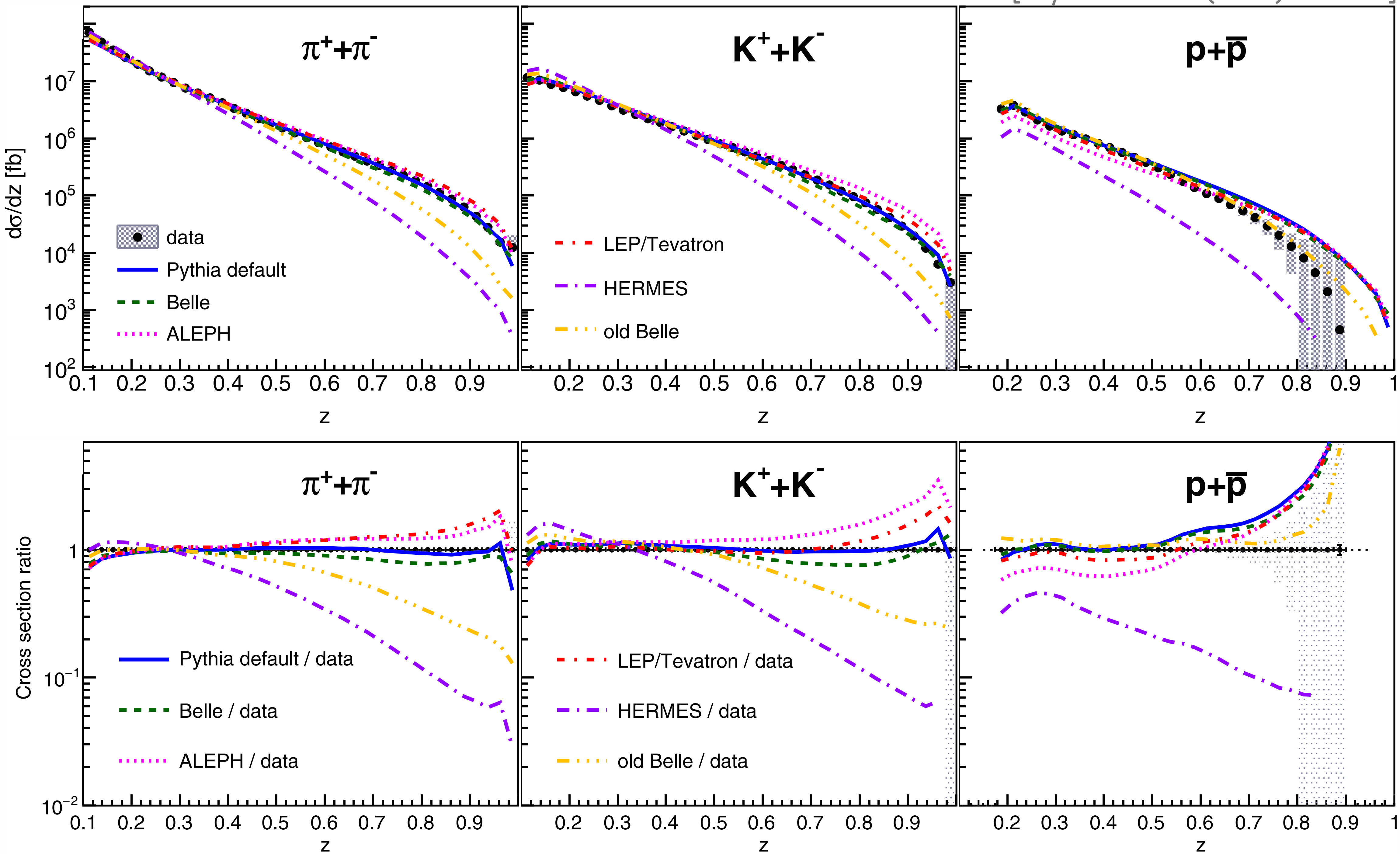
- very precise data for charged pions and kaons
- Belle data available up to very large z (z<0.98)
- included in recent DEHSS fits [e.g. PRD 91, 014035 (2015)]
- radiative corrections **undone** in FF fits
- **new**: data for protons and anti-protons
  - not (yet) included in DEHSS, but in NNFF 1.0
  - similar z dependence as pions
  - about ~⅕ of pion cross sections





single-hadron production - MC comparison

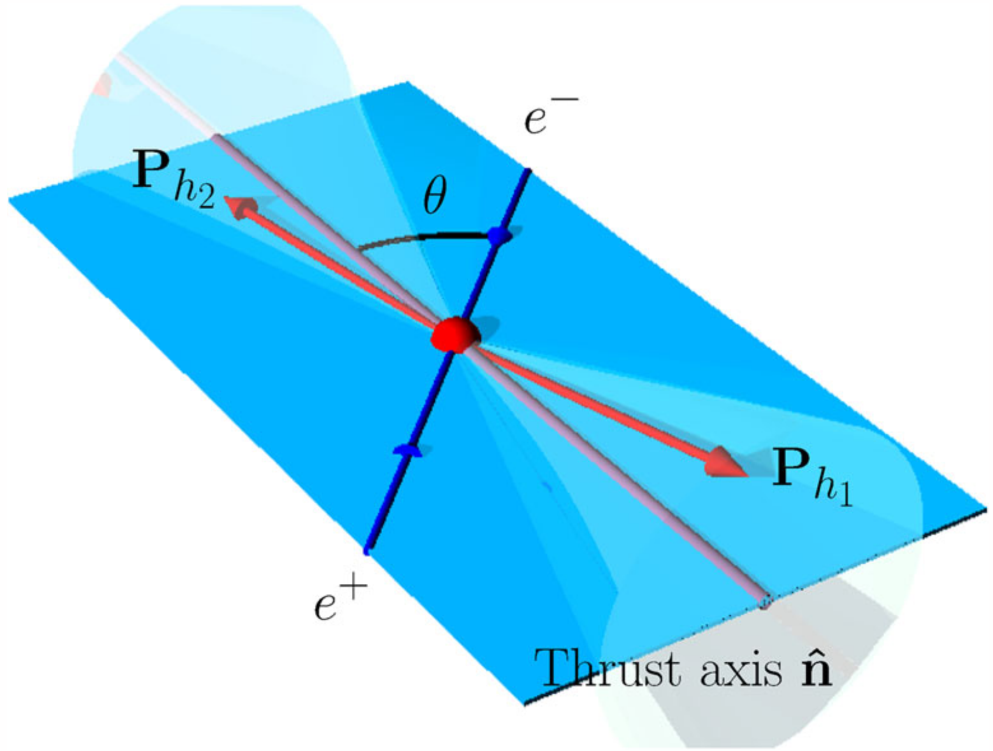
[Phys. Rev. D92 (2015) 092007]

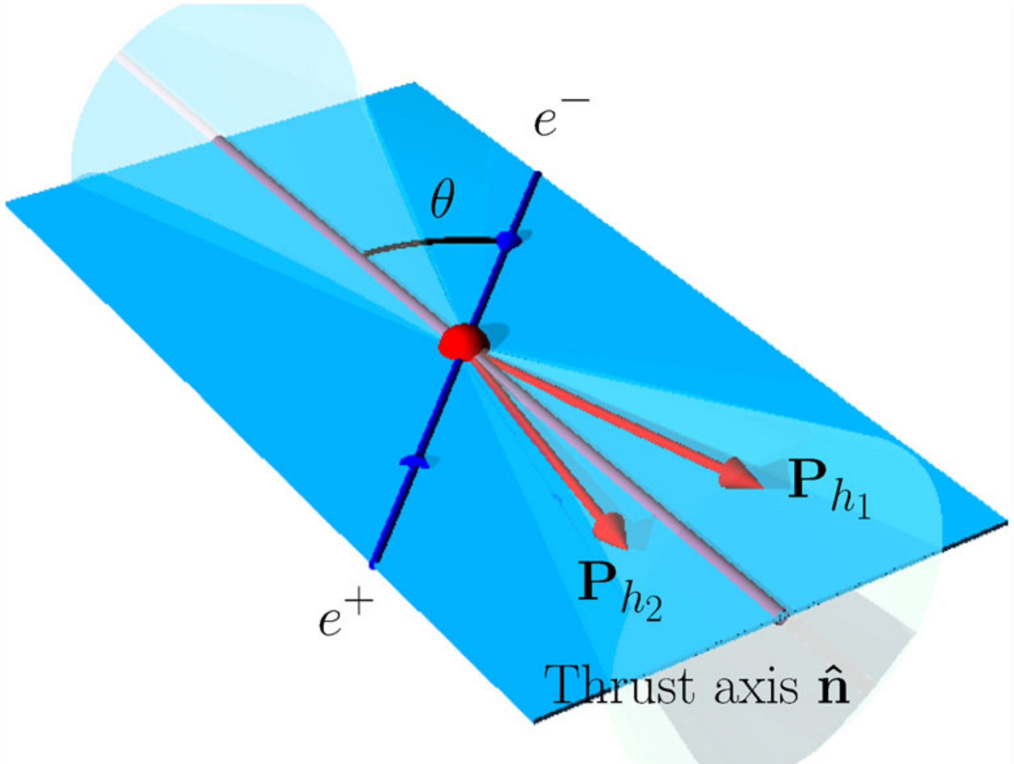


- pion and kaon data reasonably well described by Jetset
- protons difficult to reproduce, especially at large z: MC overshoots data

hadron-pair production

- single-hadron production has low discriminating power for parton flavor
- can use 2nd hadron in opposite hemisphere to "tag" flavor
  - mainly sensitive to product of single-hadron FFs
- if hadrons in same hemisphere: **dihadron fragmentation**
  - a la de Florian & Vanni [Phys. Lett. B 578 (2004) 139]
  - a la Collins, Heppelmann & Ladinsky [Nucl. Phys. B 420 (1994) 565]; Boer, Jacobs & Radici [Phys. Rev. D 67 (2003) 094003]

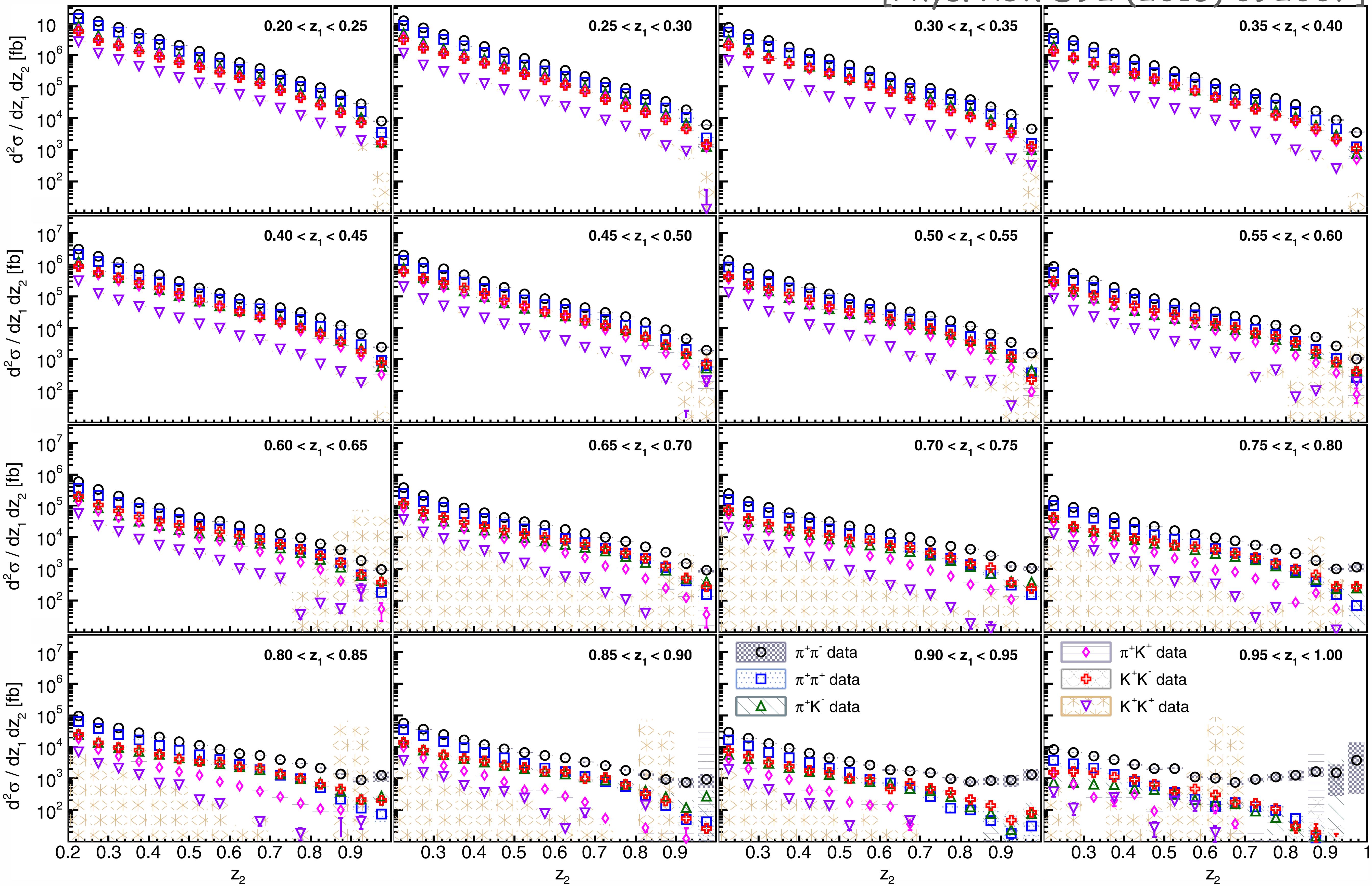




hadron-pair production

no hemisphere selection

[Phys. Rev. D92 (2015) 092007]



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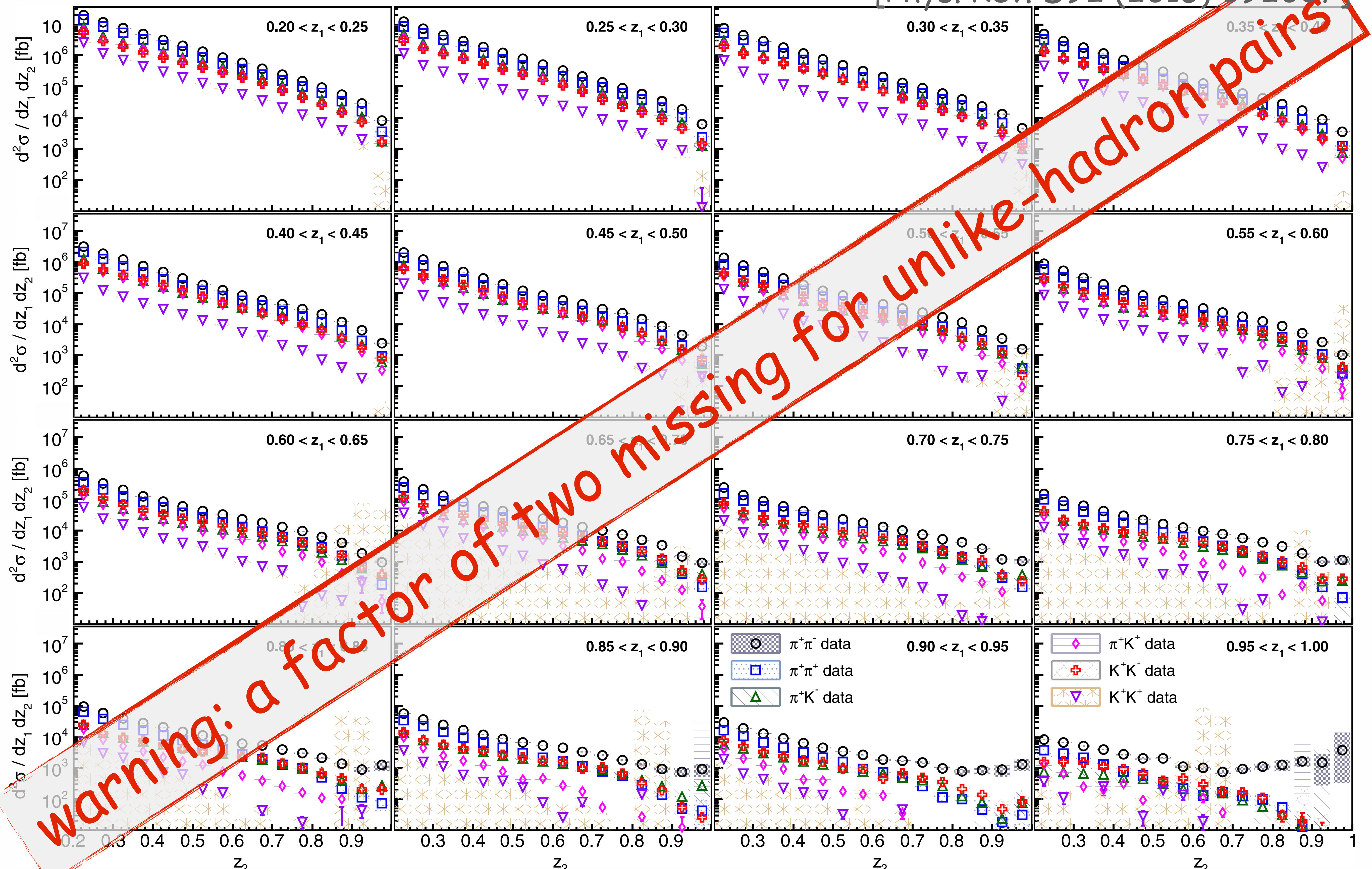

hadron-pair production

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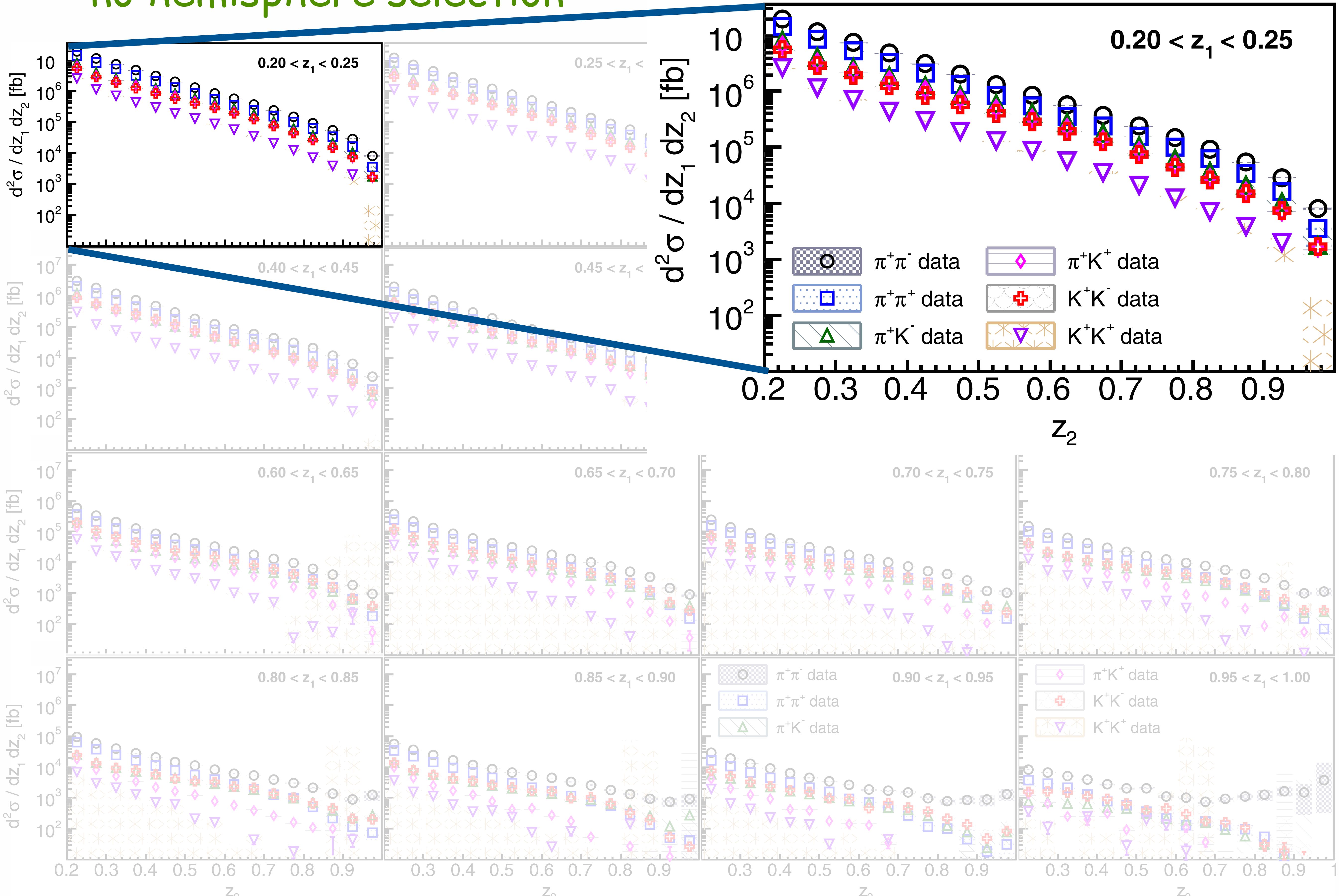

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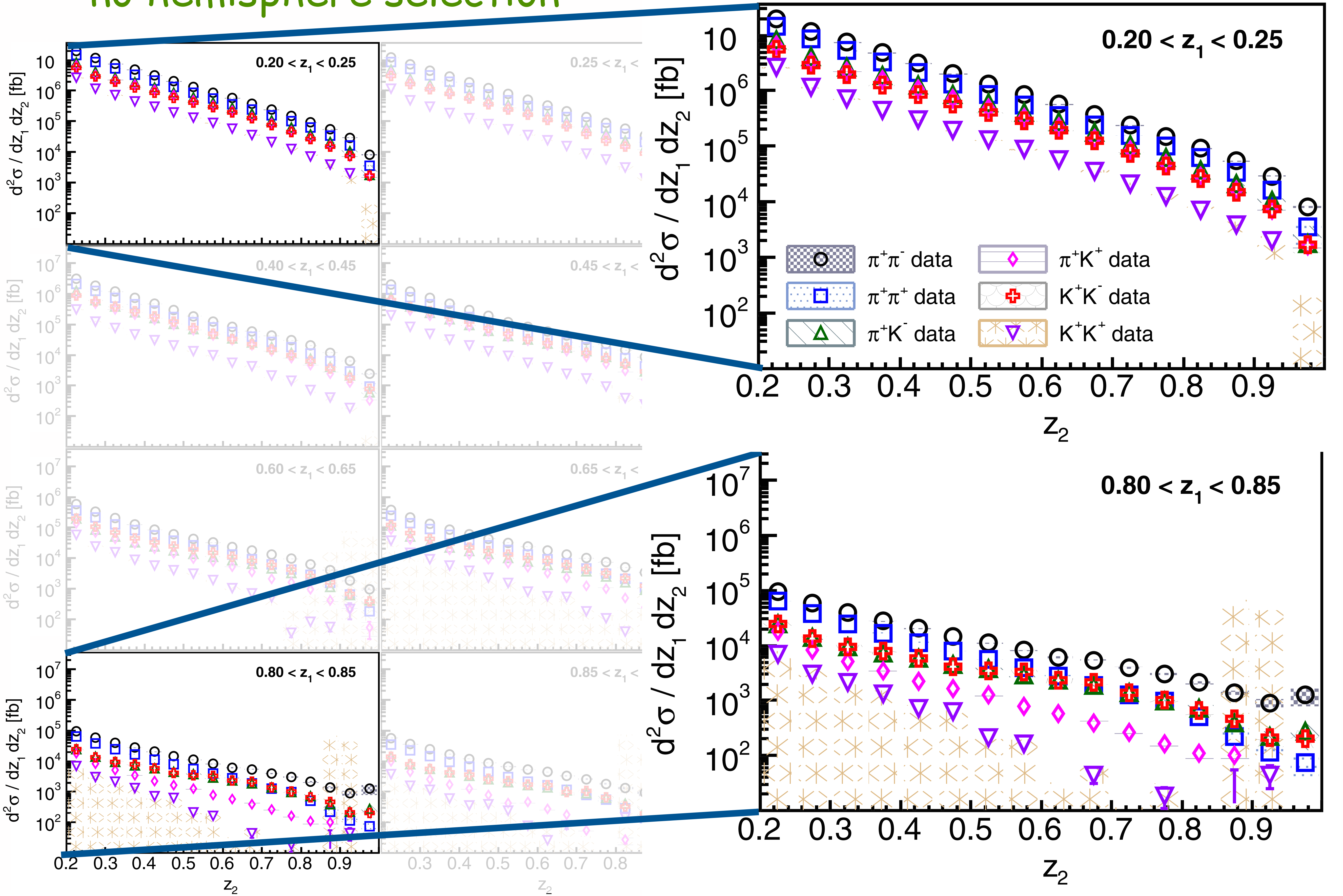
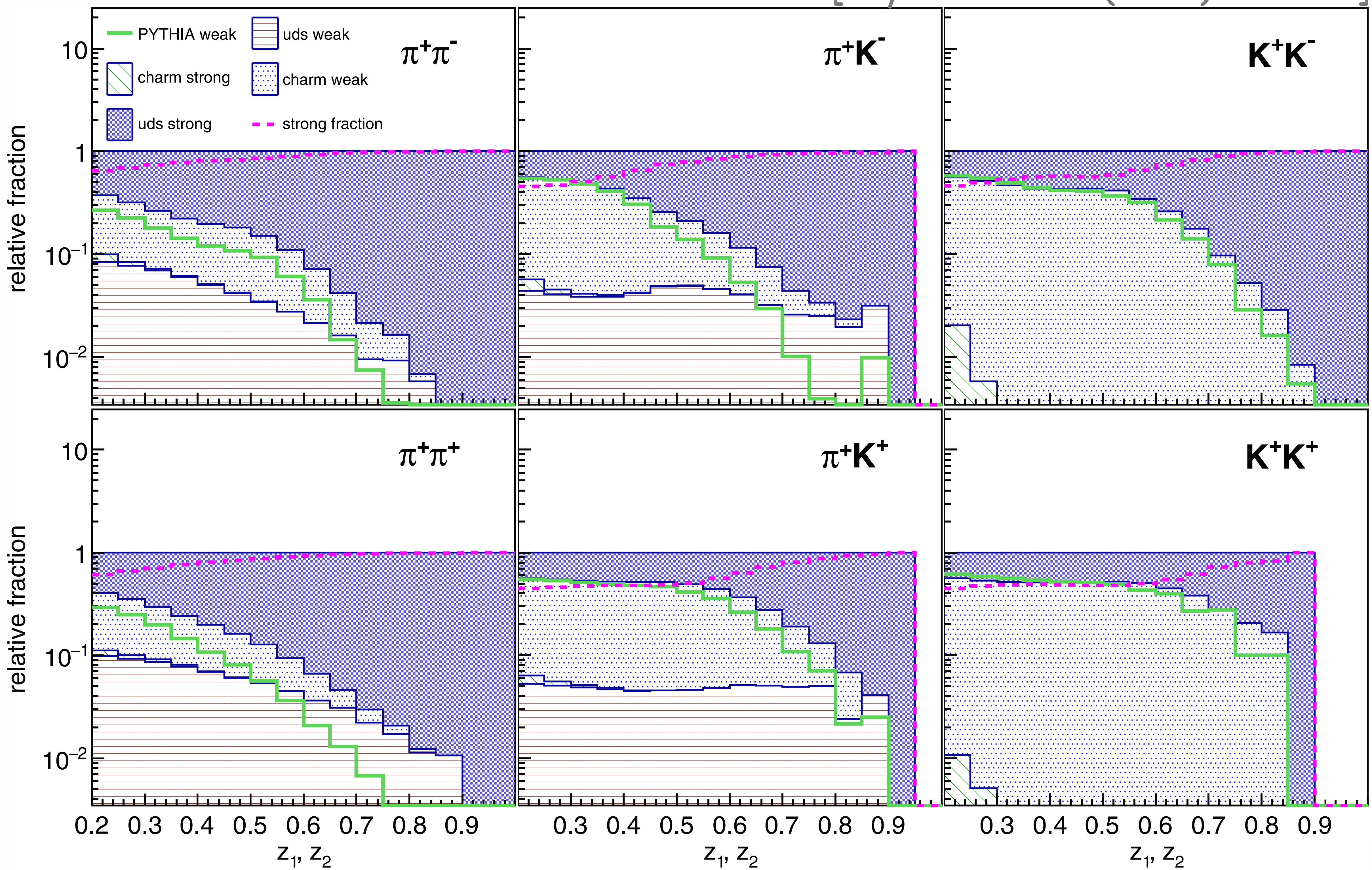


hadron-pairs: weak-decay contributions

- not all hadrons originate from uds quarks but, e.g., from D decay
  - here: only $z_1=z_2$ diagonal bins

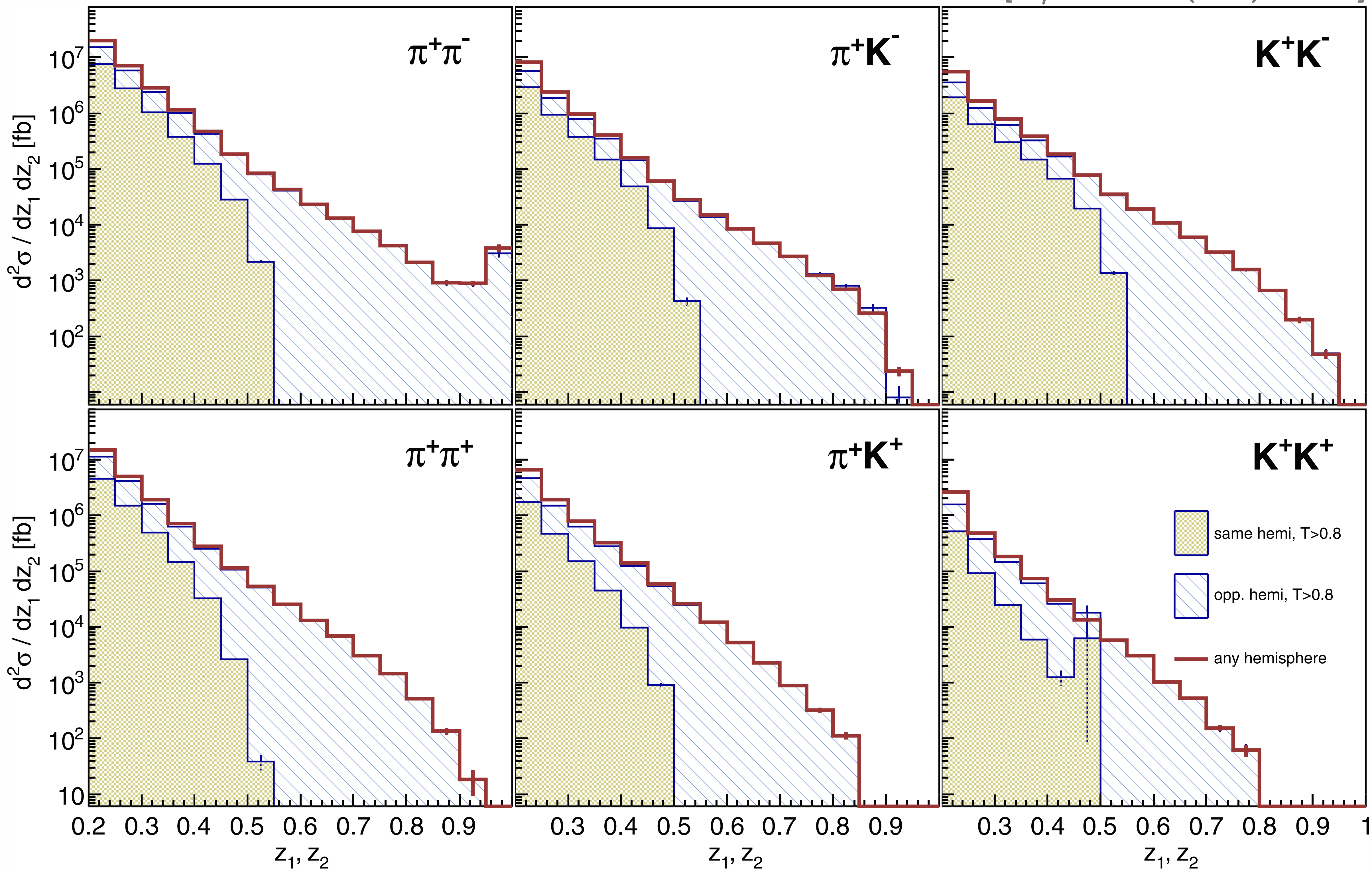
[Phys. Rev. D92 (2015) 092007]



hadron-pairs: topology comparison

- any hemisphere vs. opposite- & same-hemisphere pairs
  - same-hemisphere pairs with kinematic limit at $z_1=z_2=0.5$

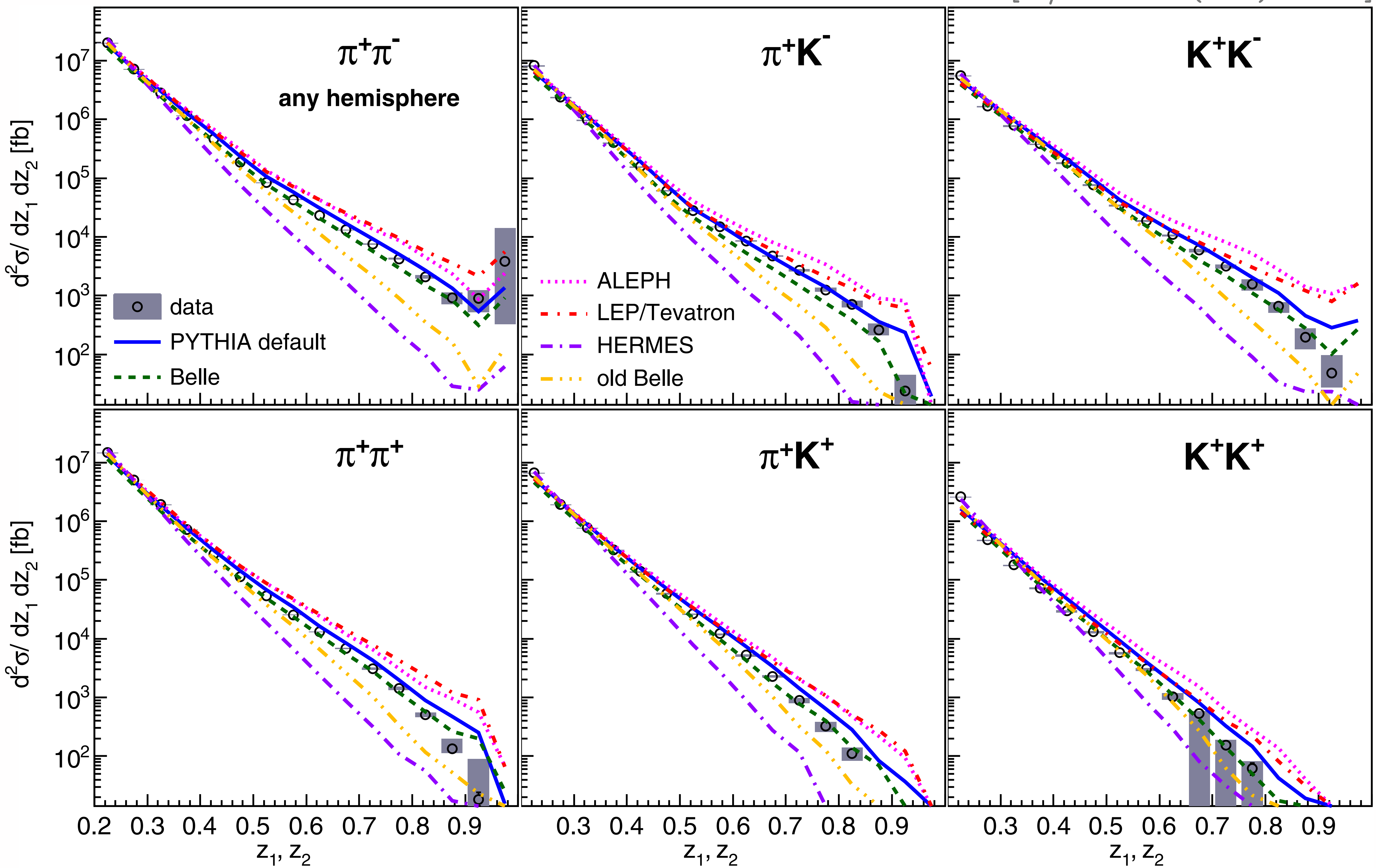
[Phys. Rev. D92 (2015) 092007]



hadron-pairs: comparison with PYTHIA

- generally good agreement at low z
  - at large z only present Belle and PYTHIA default tunes satisfactory

[Phys. Rev. D92 (2015) 092007]



single-hadron FFs from hadron pairs

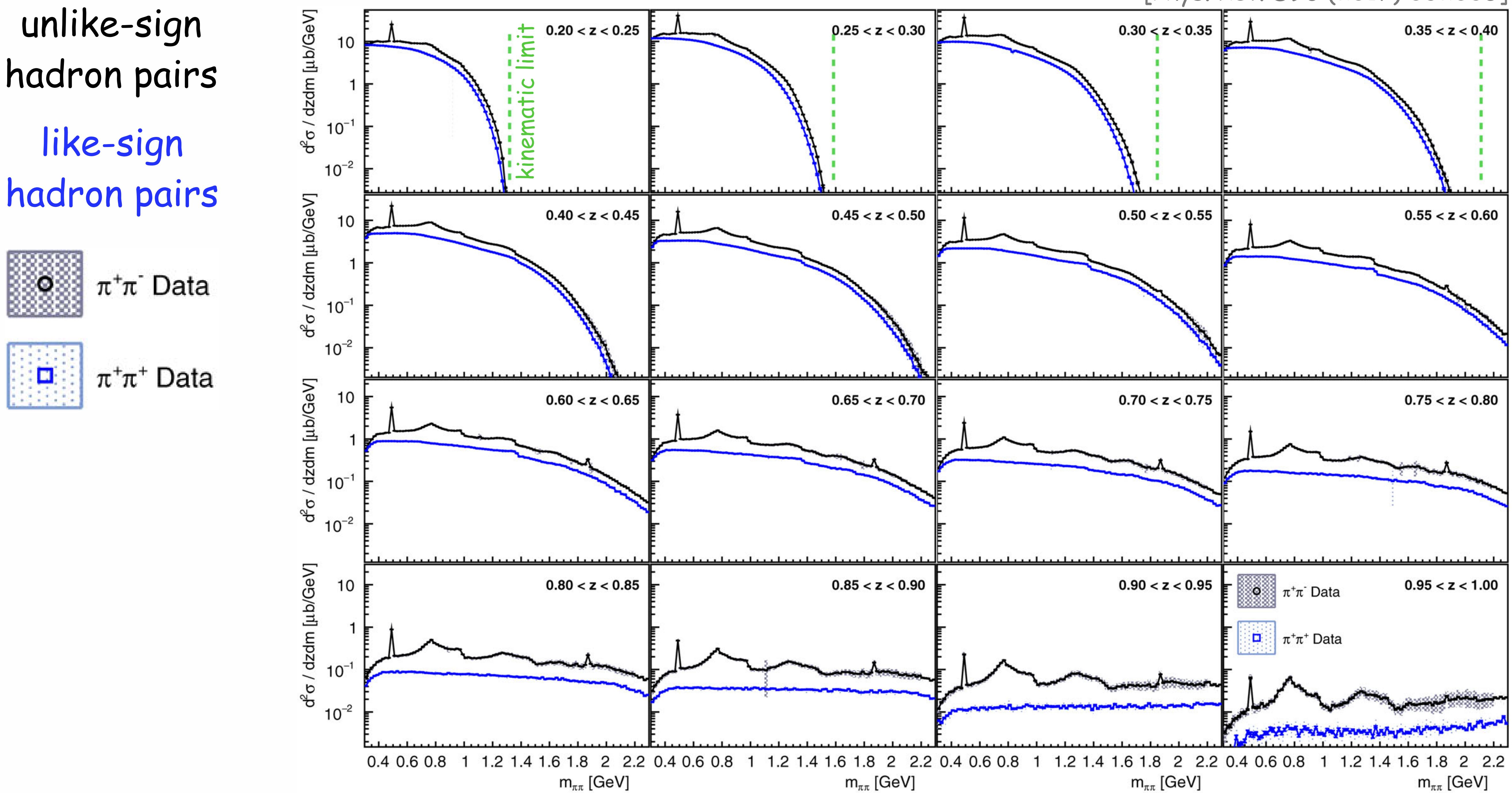
- binning in z_1 and z_2 alone does not discriminate single-hadron from dihadron fragmentation
  - indeed, at low z substantial contribution from hadrons that originate from same quark
  - already at NLO, interpretation in terms of [product of] single-hadron FFs clouded
- Altarelli et al. advocated different choice of variable set [NPB 160 (1979) 301]: $z = 2P_1 q / Q^2$ ($=2E_h/\sqrt{s}$) & $u = P_1 P_2 / (P_1 q)$
- is it necessary to redo analysis? Or live with data available:
  - large-z region kinematically suppresses same-quark fragmentation
  - thrust axis can be used to define hemispheres ... but then cross sections dependent on thrust value, hence not fully inclusive

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign
hadron pairs

like-sign
hadron pairs

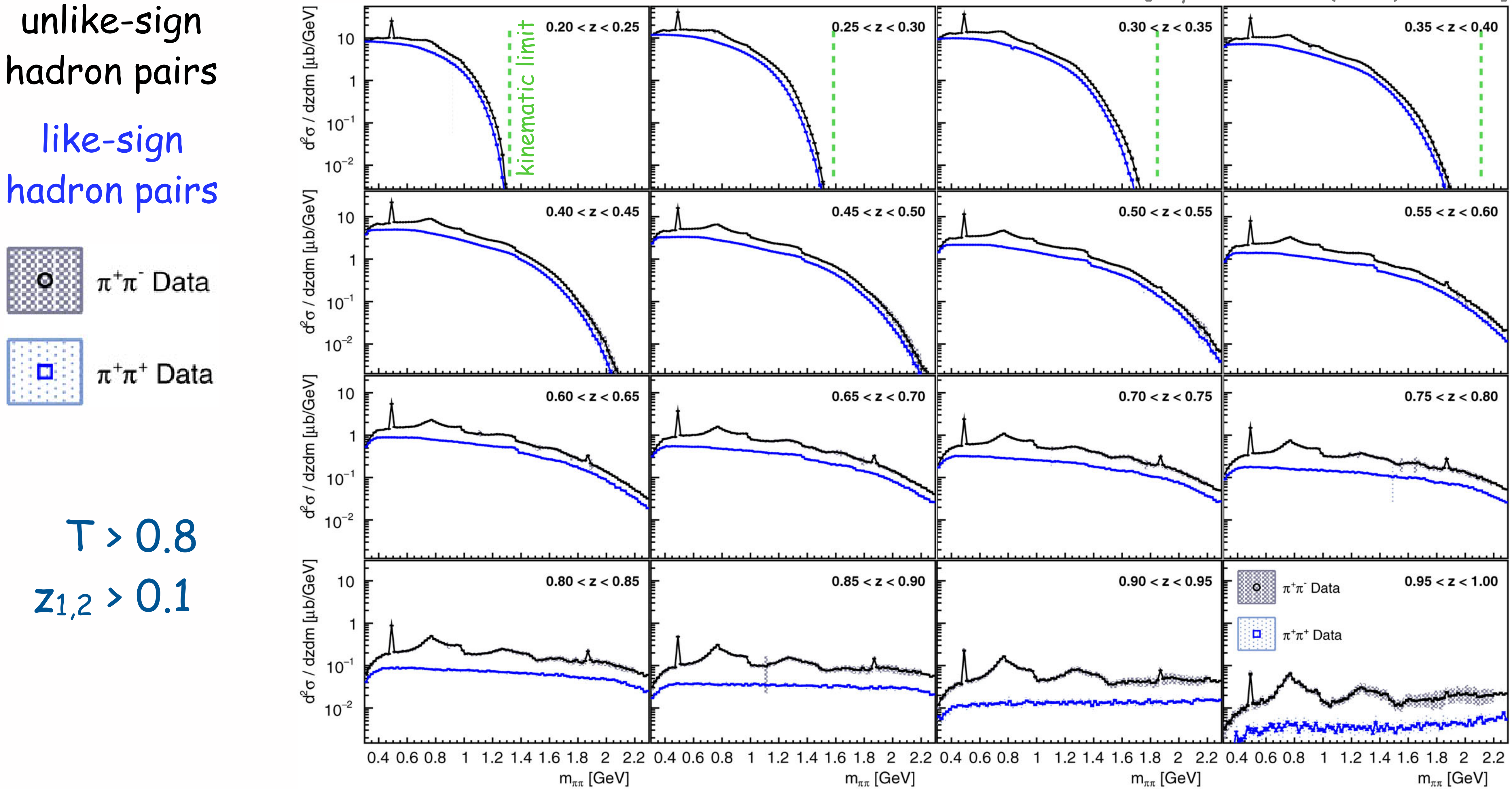


same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign hadron pairs

like-sign hadron pairs



T > 0.8

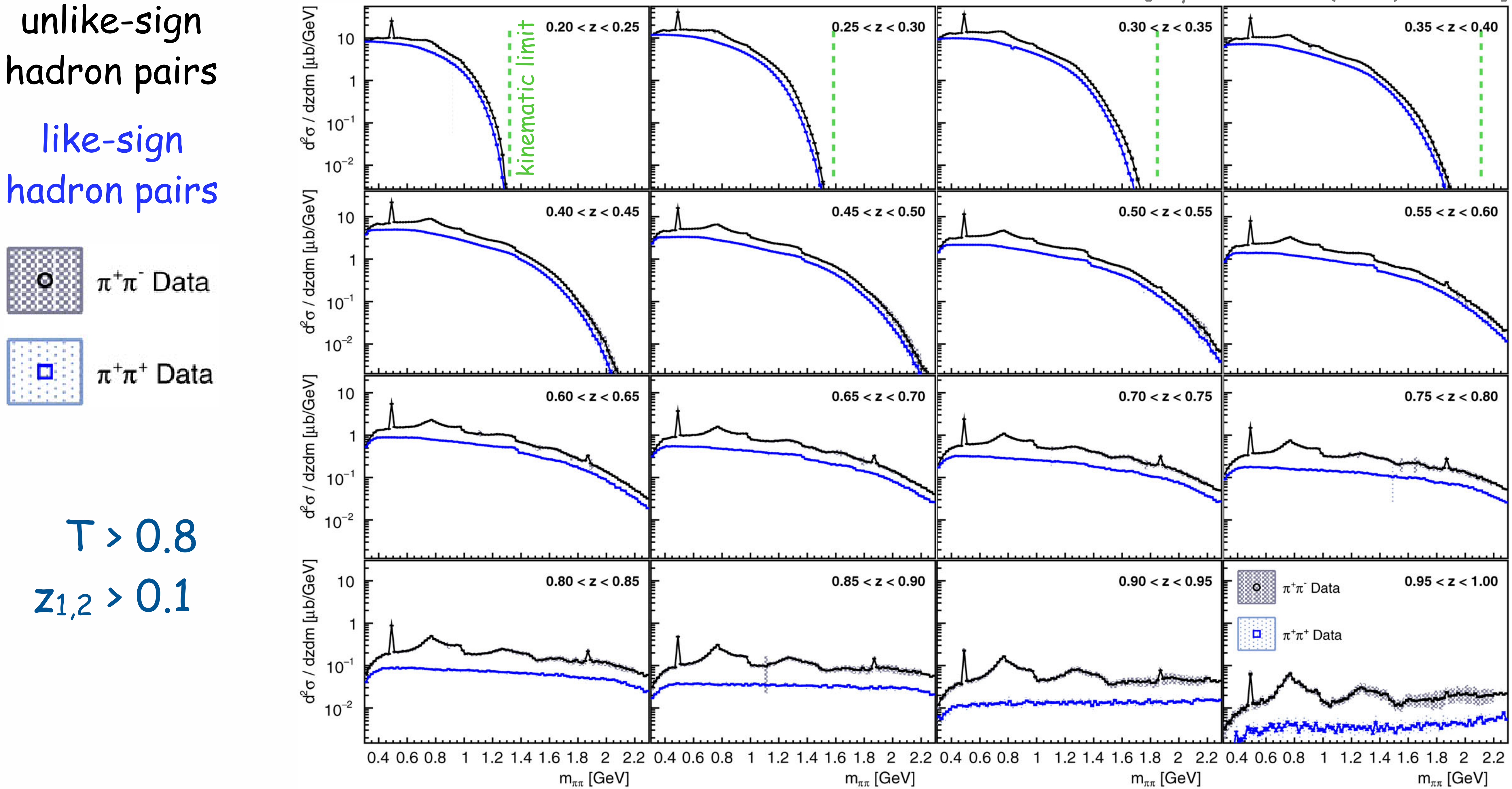
$z_{1,2} > 0.1$

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign hadron pairs

like-sign hadron pairs



$T > 0.8$

$z_{1,2} > 0.1$

- unlike-sign pairs with clear decay and resonance structure: K_s, ρ^0 ...
- like-sign pairs with much smoother and smaller cross sections

same-hemisphere data: M_{h1h2} dependence

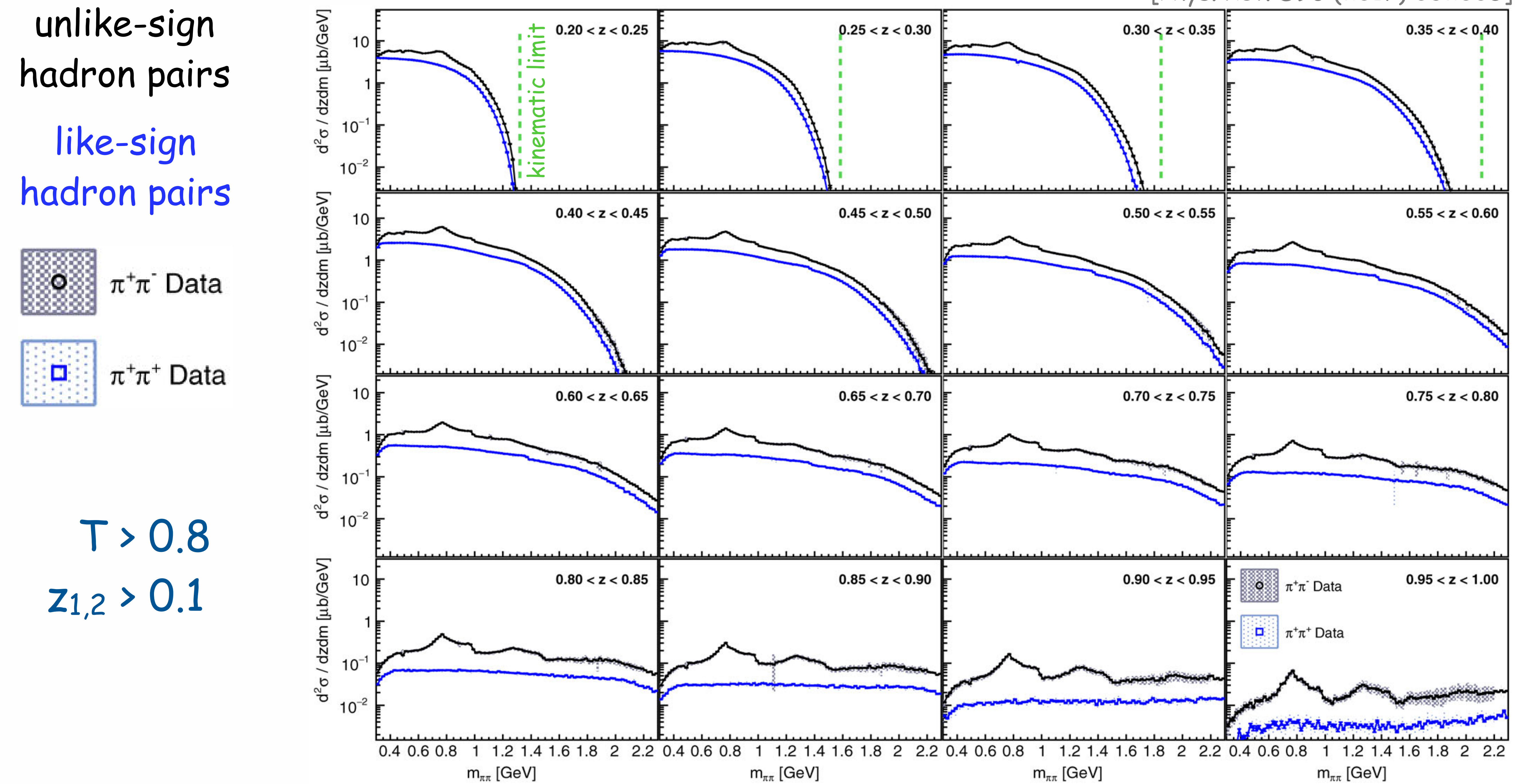
unlike-sign hadron pairs

like-sign hadron pairs

$T > 0.8$

$z_{1,2} > 0.1$

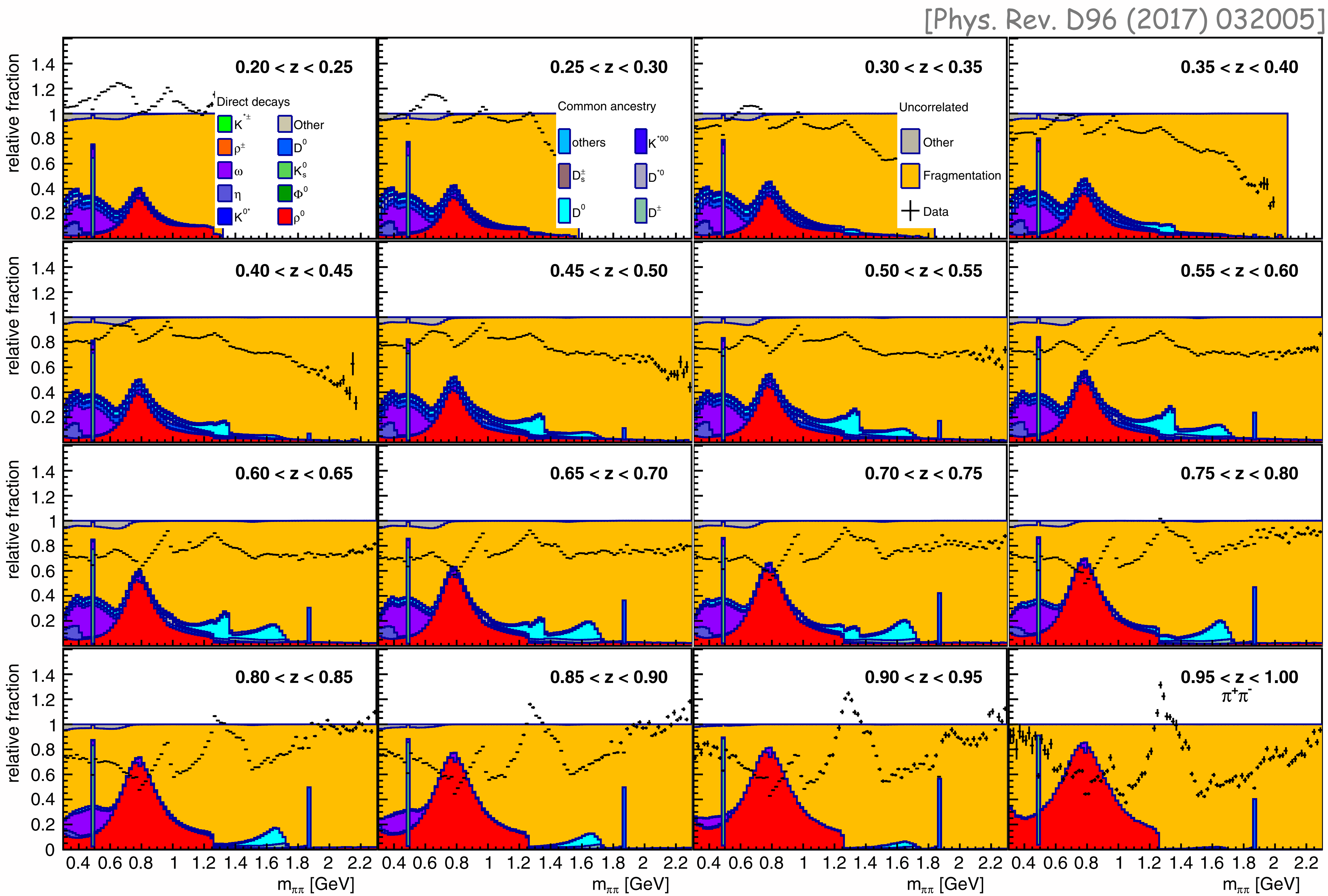
[Phys. Rev. D96 (2017) 032005]



- cross sections after (MC-based) **removal of weak-decay contributions**
  - relies on good description of those channels in Pythia

same-hemisphere data: M_{h1h2} dependence

unlike-sign pion pairs



$T > 0.8$

$z_{1,2} > 0.1$

- decomposition based on PYTHIA simulation
- clear differences in invariant-mass dependence between MC and data

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

like-sign pioin pairs

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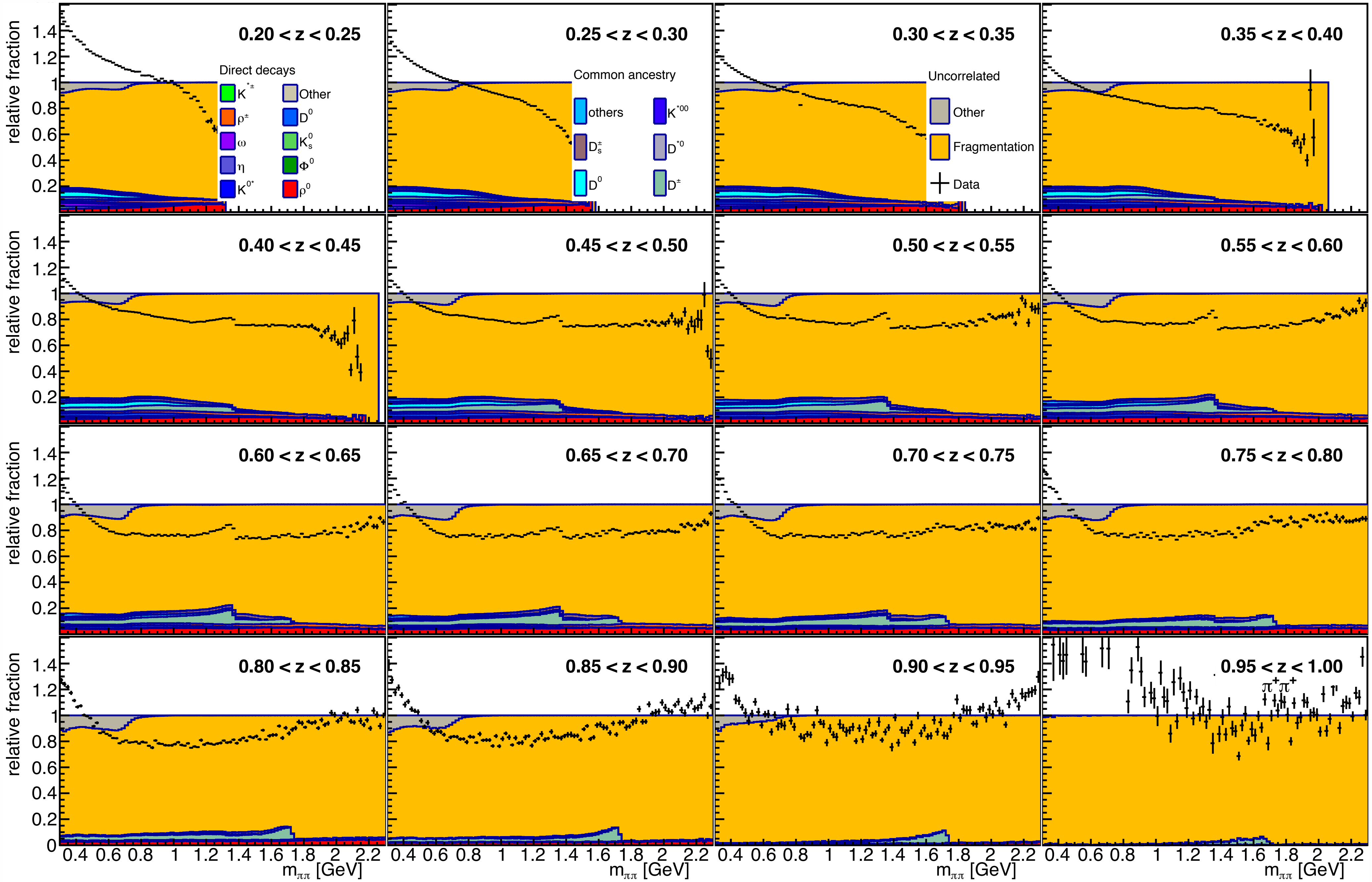


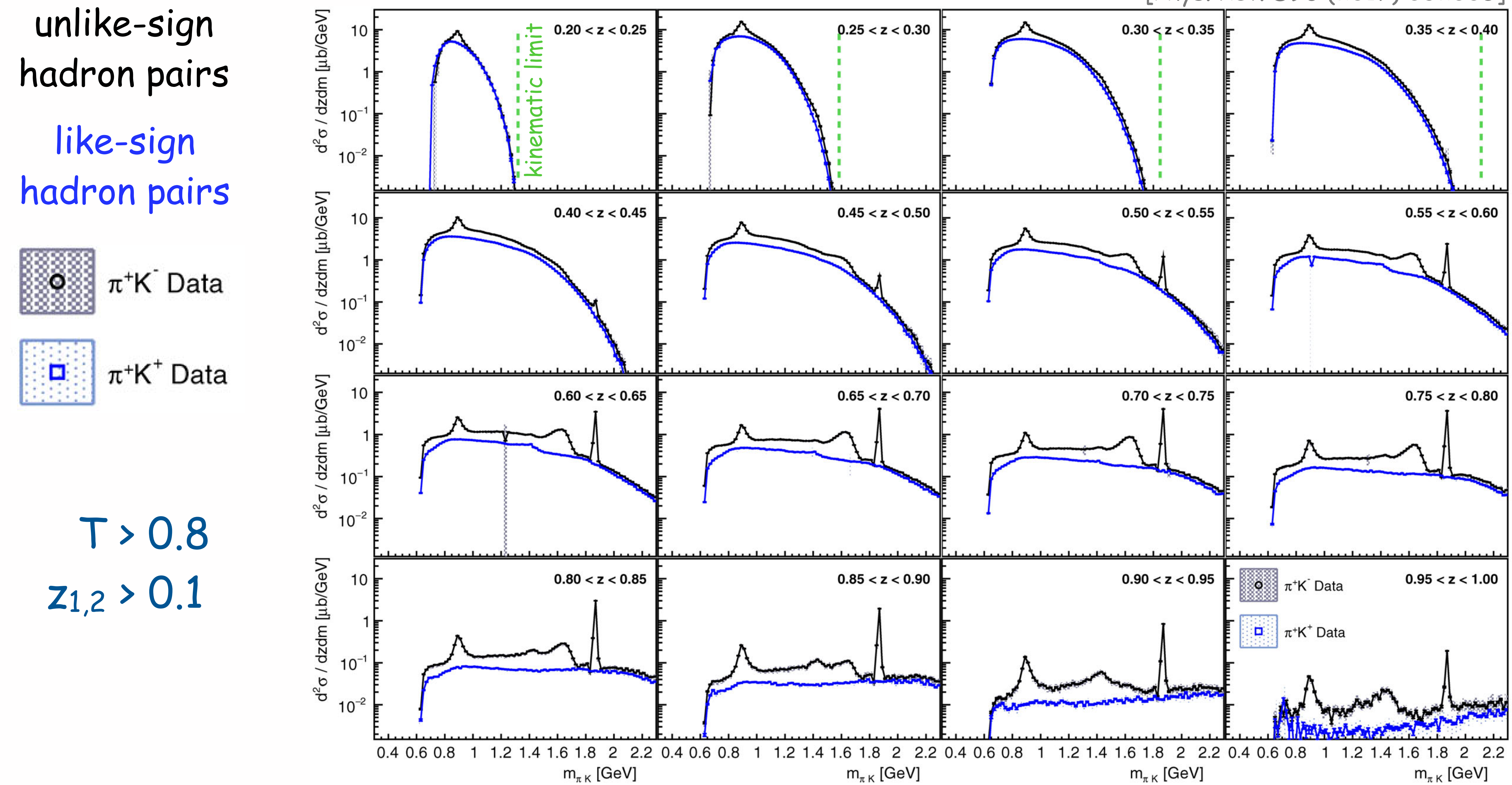
- decomposition based on PYTHIA simulation
- though no strong resonance structure still clear MC/data discrepancy

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign hadron pairs

like-sign hadron pairs



T > 0.8

$z_{1,2}$ > 0.1

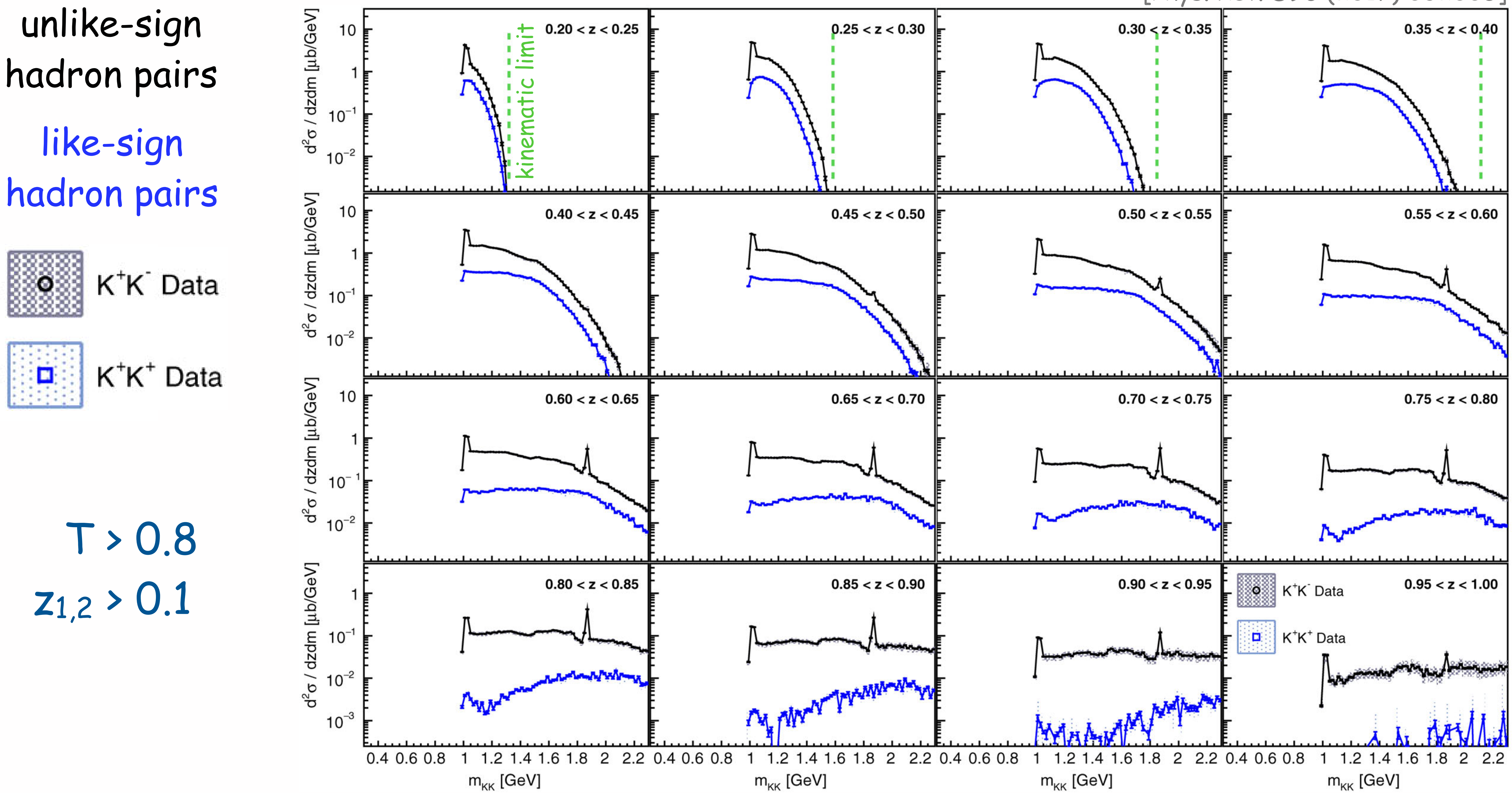
- unlike-sign πK pairs with clear K* and increased D-decay contributions

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign hadron pairs

like-sign hadron pairs



$T > 0.8$

$z_{1,2} > 0.1$

- unlike-sign kaon pairs with (again) a decay structure (e.g. ϕ and D)
- like-sign kaon pairs strongly suppressed at larger z

some more details

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign hadron pairs

like-sign hadron pairs

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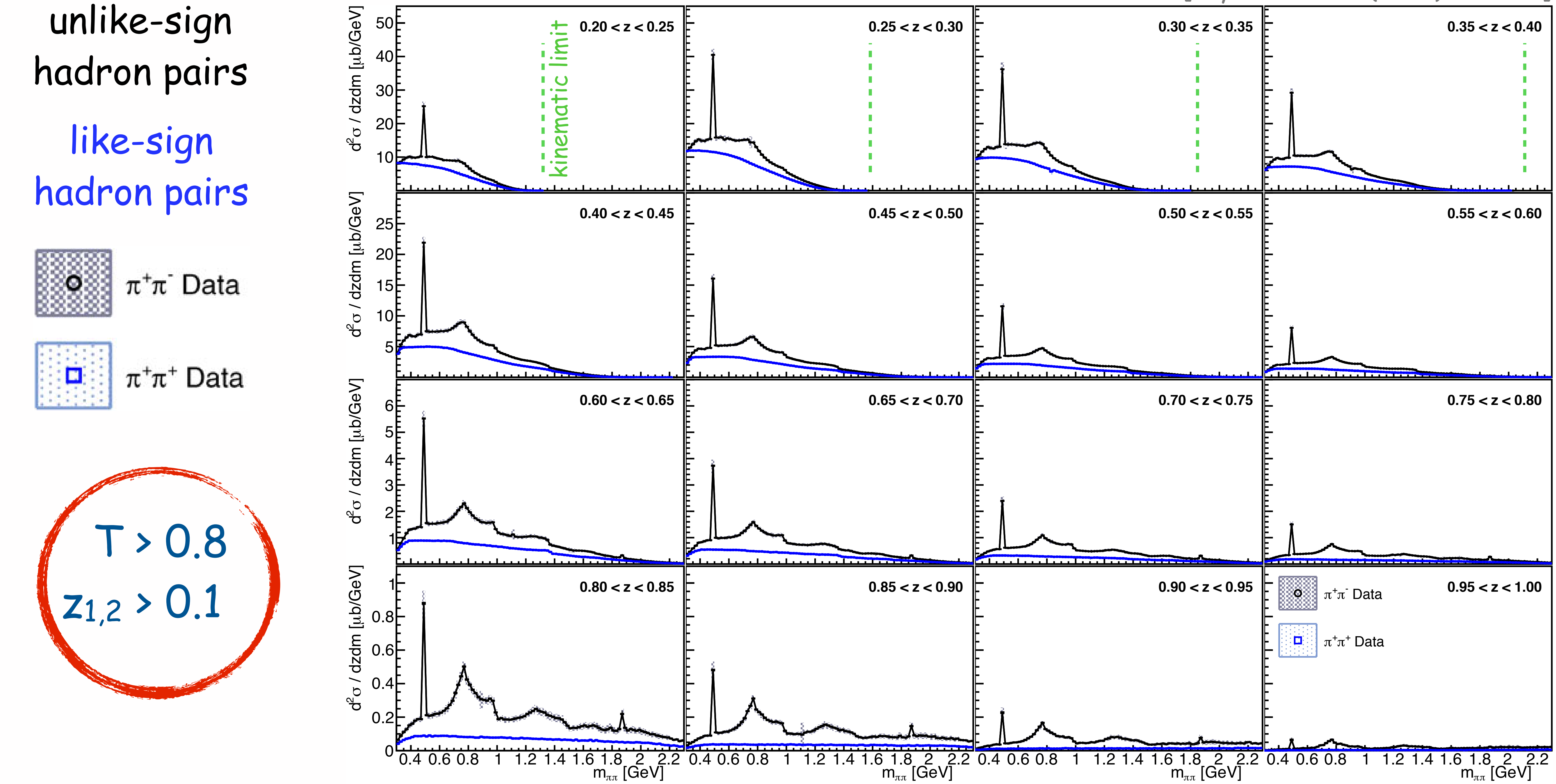

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign hadron pairs

like-sign hadron pairs

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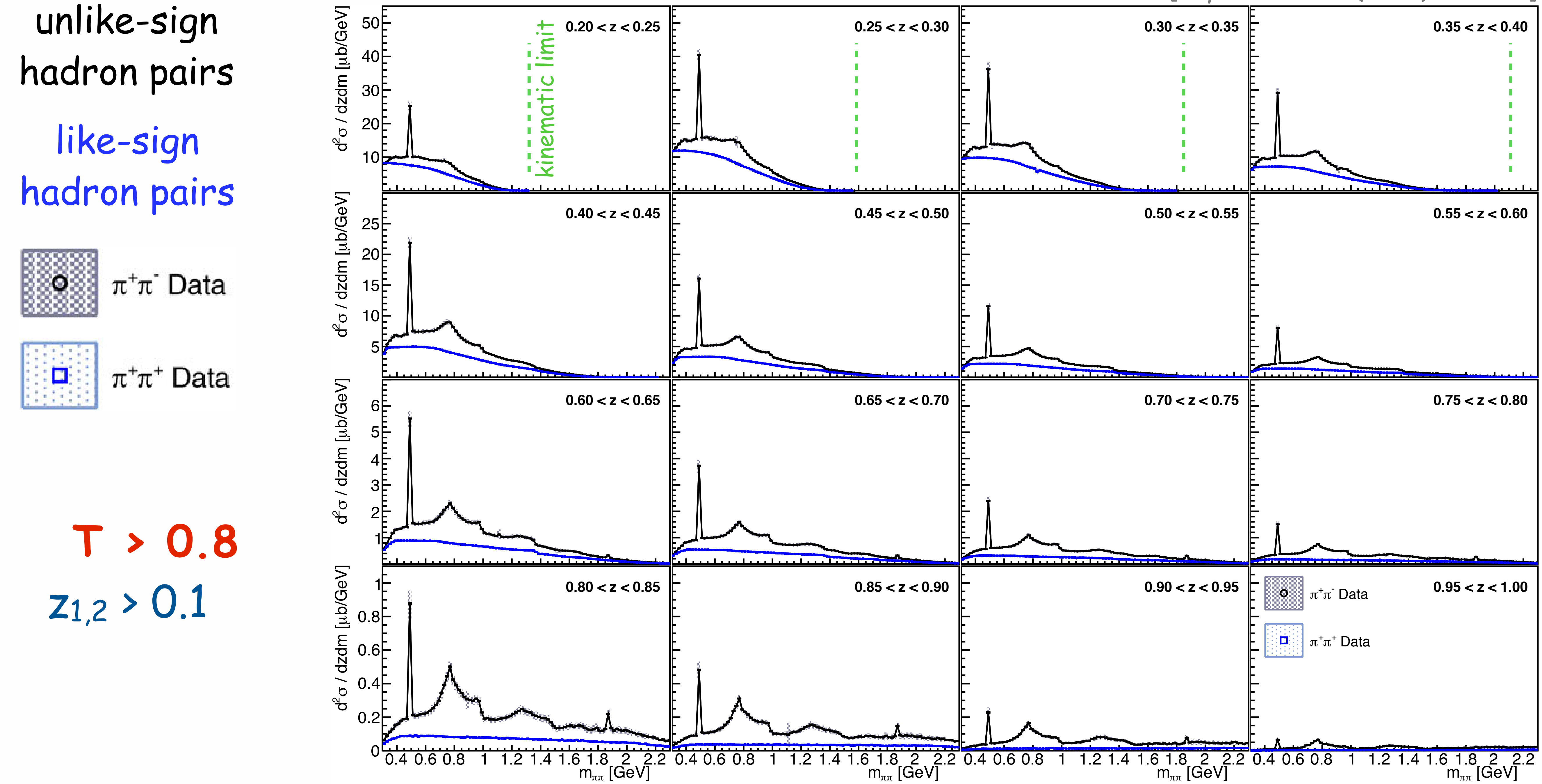


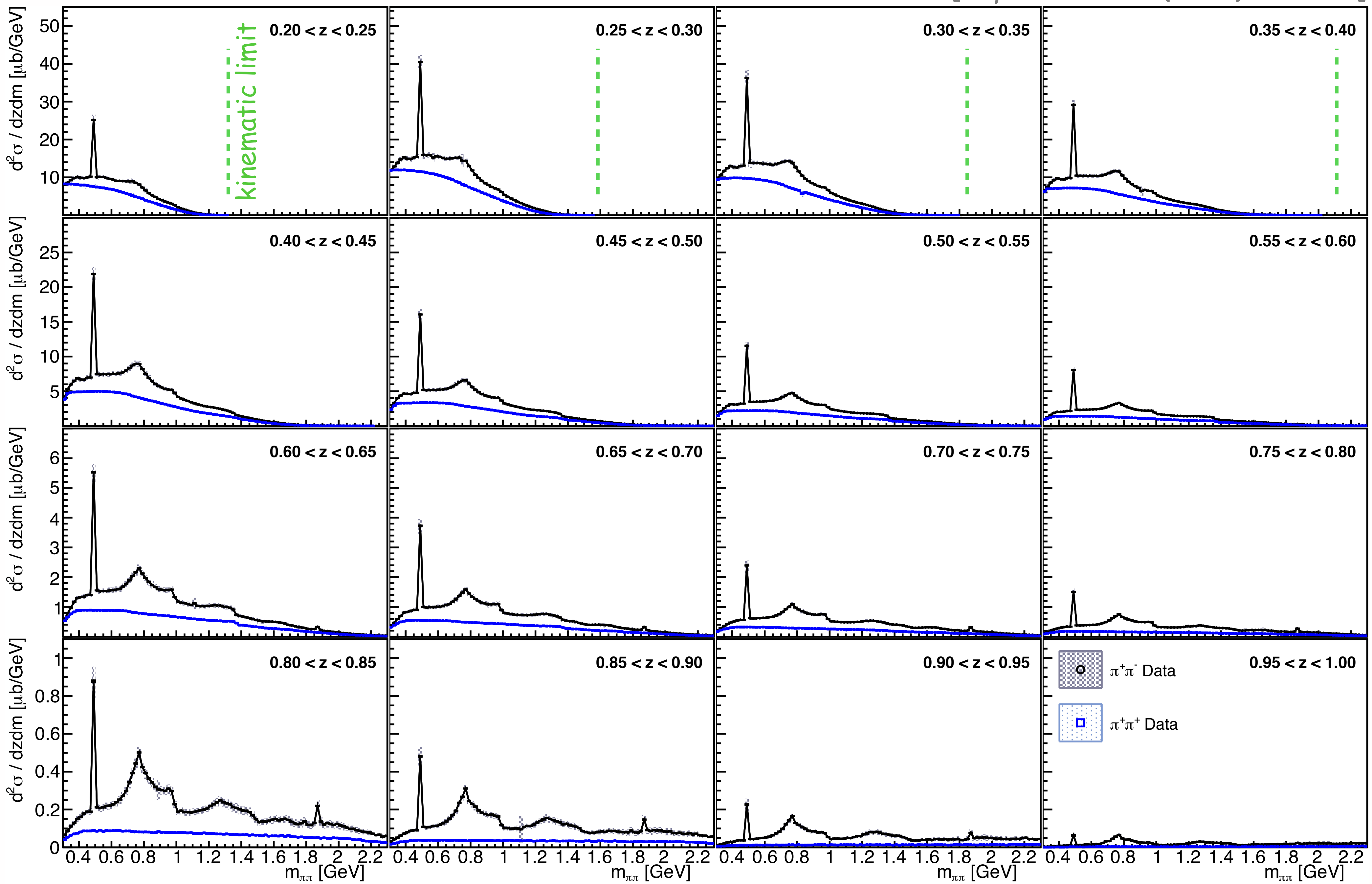
- **thrust** very useful experimentally to suppress BG and to define hemispheres
- potentially difficult to incorporate in phenomenology (unlike thrust axis?)

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign hadron pairs

like-sign hadron pairs



$T > 0.8$

$z_{1,2} > 0.1$

- experimental constraints on individual z restricts phase space of hadron pairs, however, not easy to avoid (detection requirements!)
- among others leads to mixing of partial-wave contributions [GS, QCDE'17]

same-hemisphere data: M_{h1h2} dependence

[Phys. Rev. D96 (2017) 032005]

unlike-sign
π pairs

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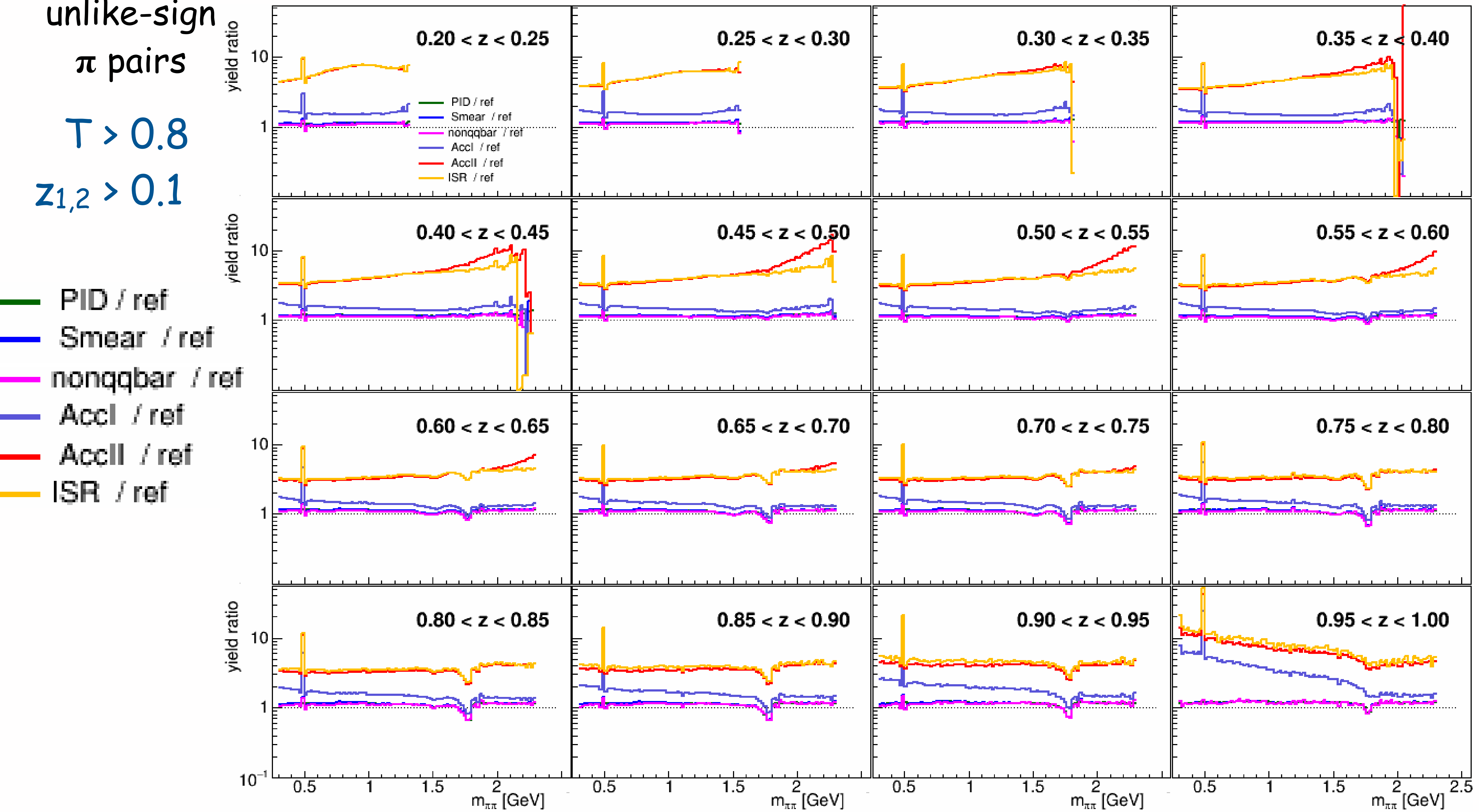
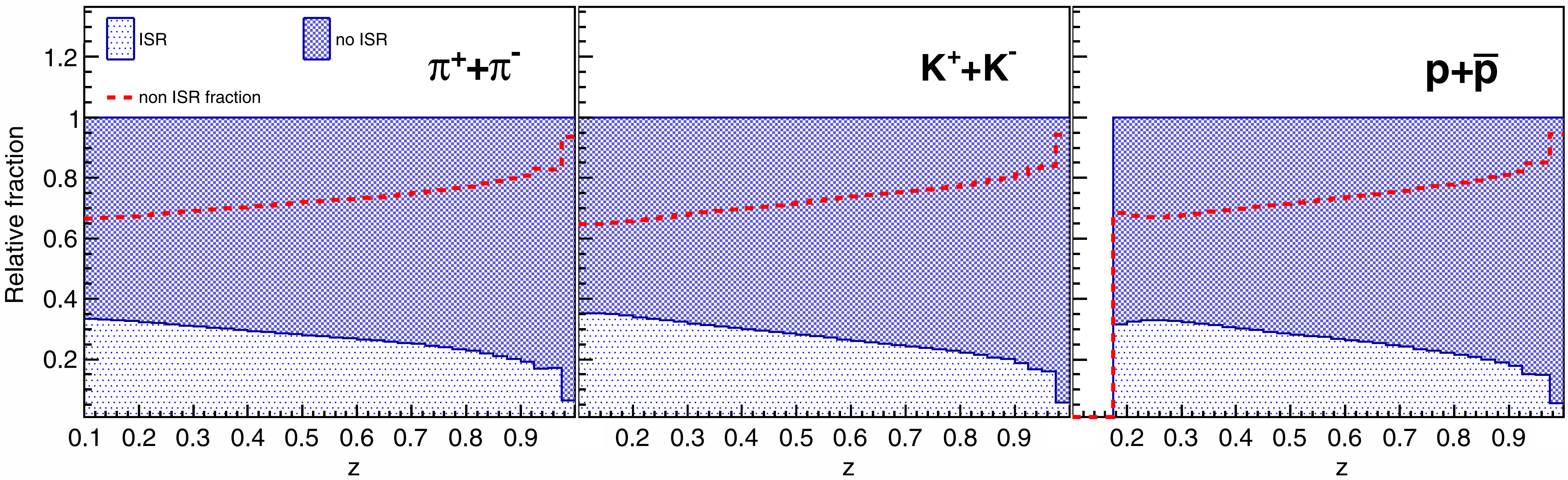


- large correction to 4π (red) ➠ large systematic uncertainties (>stat.unc.)

➡ necessary to report in 4π or sufficient in accepted range in θ_{thrust} ?

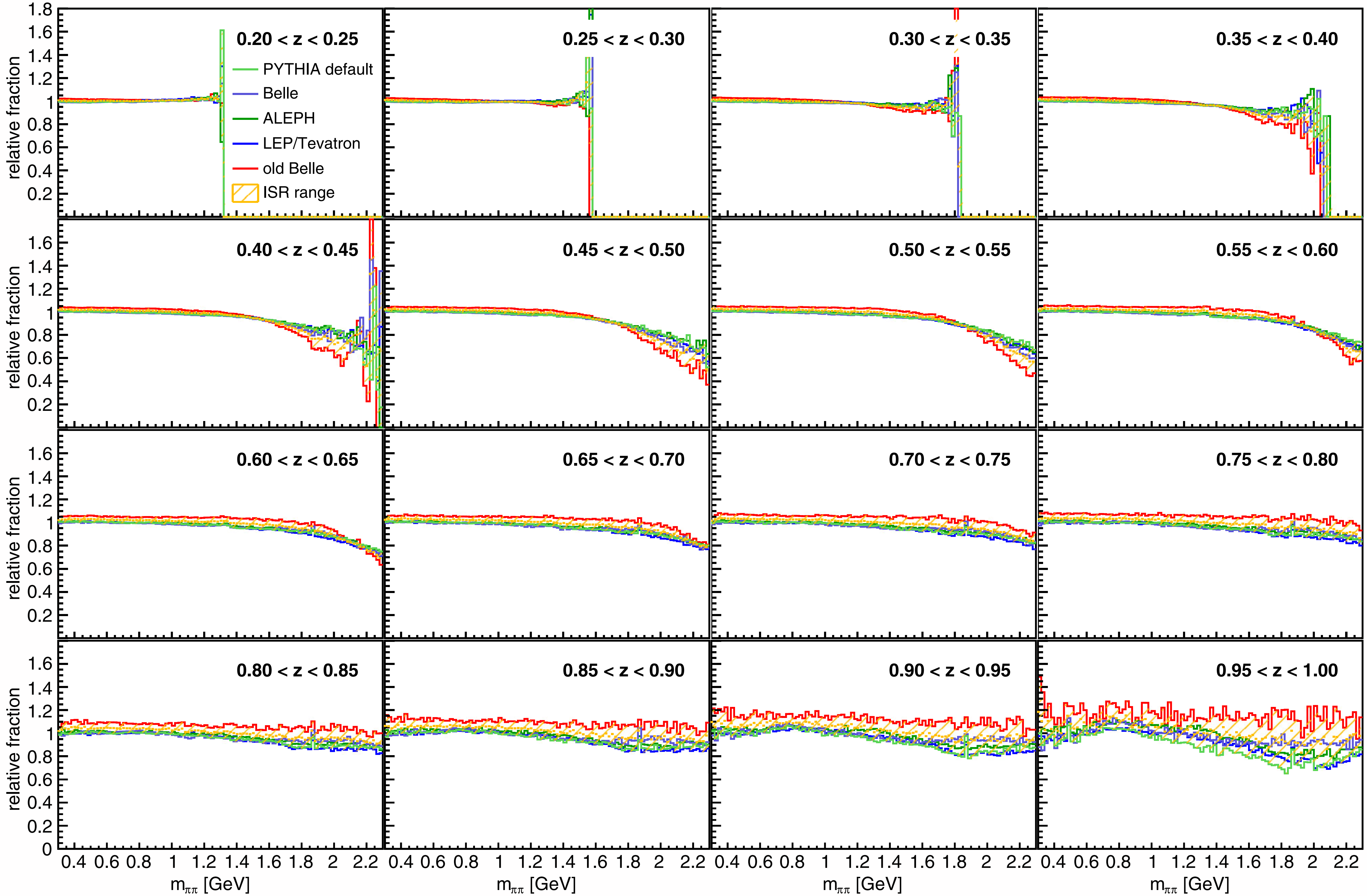
ISR corrections - PRD92 (2015) 092007



- relative fractions of hadrons as a function of z originating from ISR or non-ISR events (≡ energy loss less than 0.5%)
- large non-ISR fraction at large z, as otherwise not kinematically reachable (remember $z = E_h / 0.5\sqrt{s_{nominal}}$)

☛ Ralf

ISR corrections - PRD96 (2017) 032005

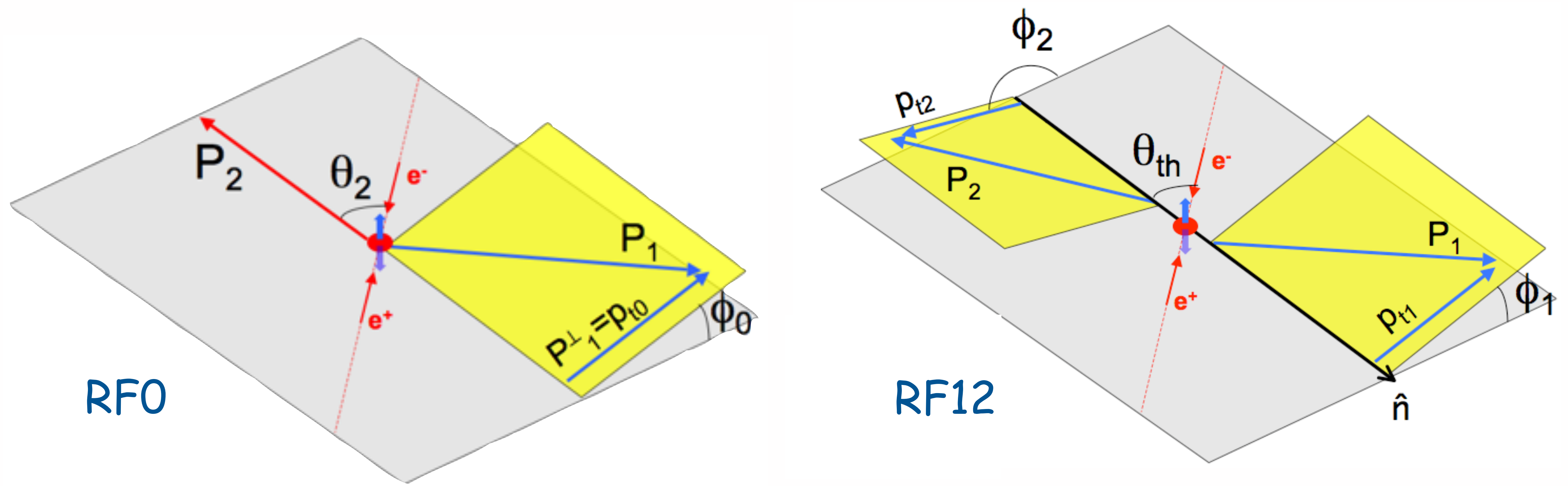


- non-ISR / ISR fractions based on PYTHIA switch MSTP(11)
  - several PYTHIA tunes used for estimate of systematic uncertainty

polarization

hadron-pairs: angular correlations

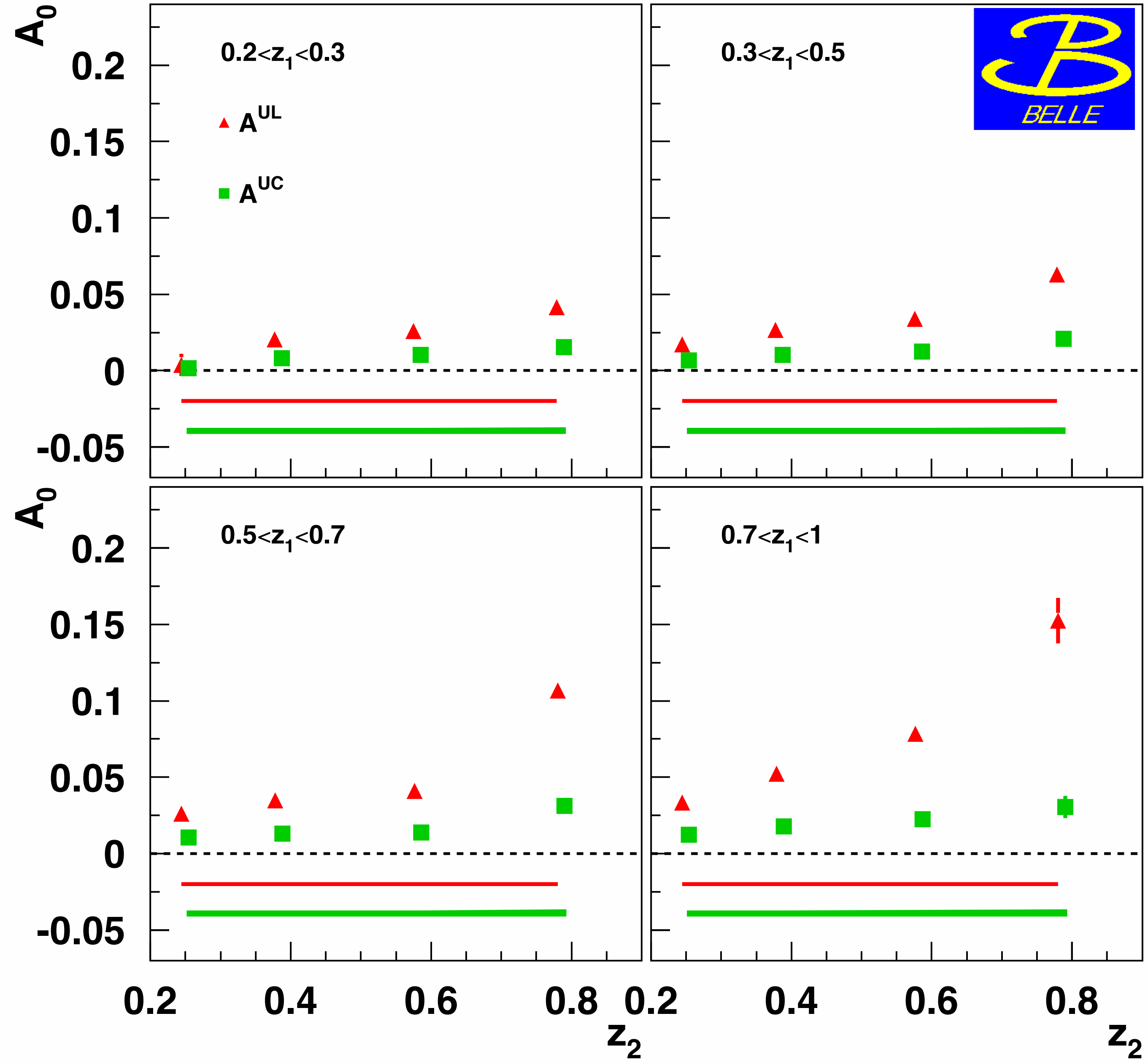
- angular correlations between nearly back-to-back hadrons used to tag transverse quark polarization -> Collins fragmentation functions
  - RF0: one hadron as reference axis -> $\cos(2\phi_0)$ modulation
  - RF12: thrust (or similar) axis -> $\cos(\phi_1+\phi_2)$ modulation



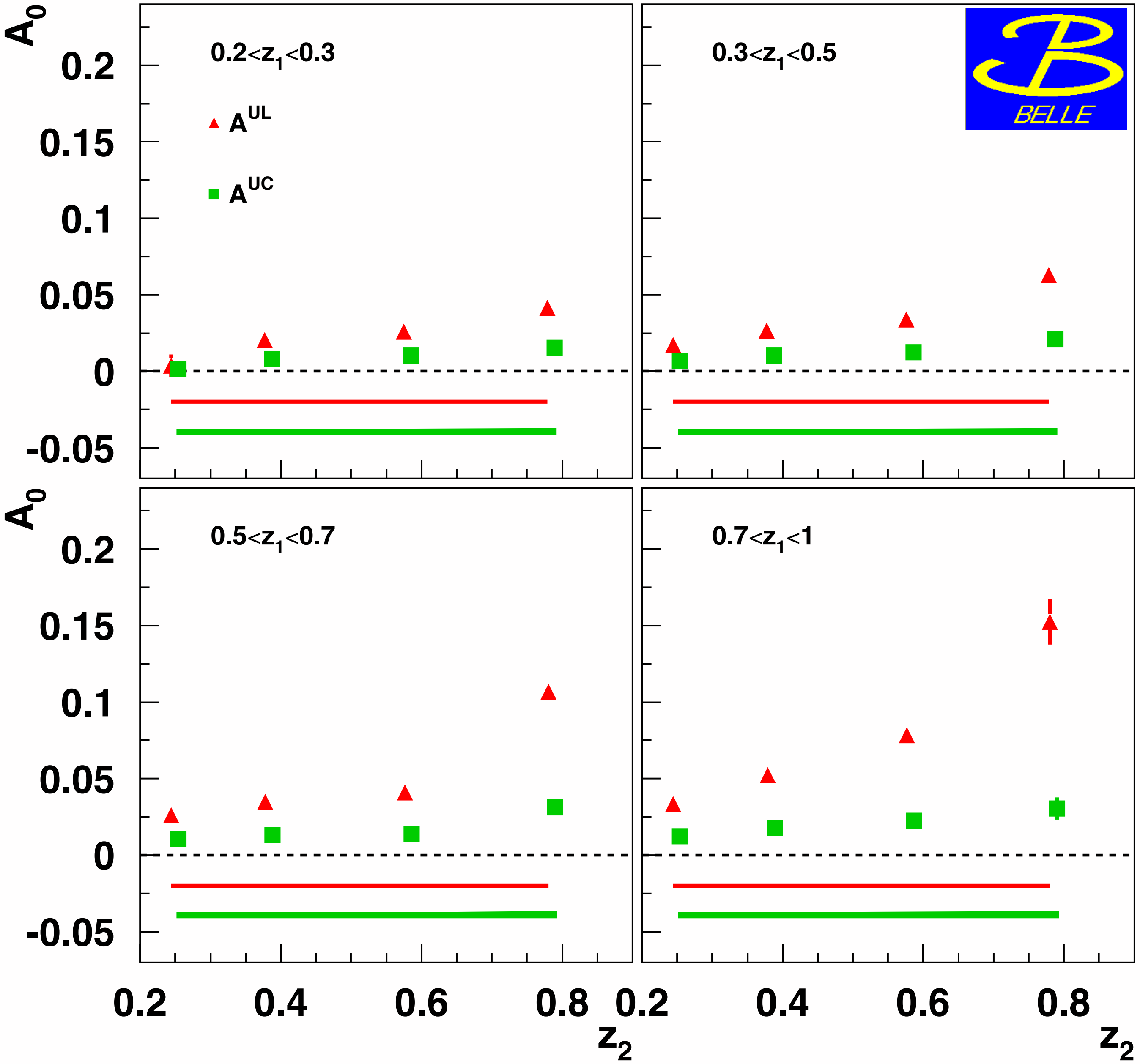
- RF0 and RF12: different convolutions over transverse momenta
- debatable: MC used to "correct" thrust axis to $q\bar{q}$ axis
- form double ratios to cancel flavor-independent sources of asymmetries

Collins asymmetries

- pioneering measurement of Collins asymmetries by Belle for charged pions
  [PRL 96 (2006) 232002, PRD 78 (2008) 032011, PRD 86 (2012) 039905(E)]

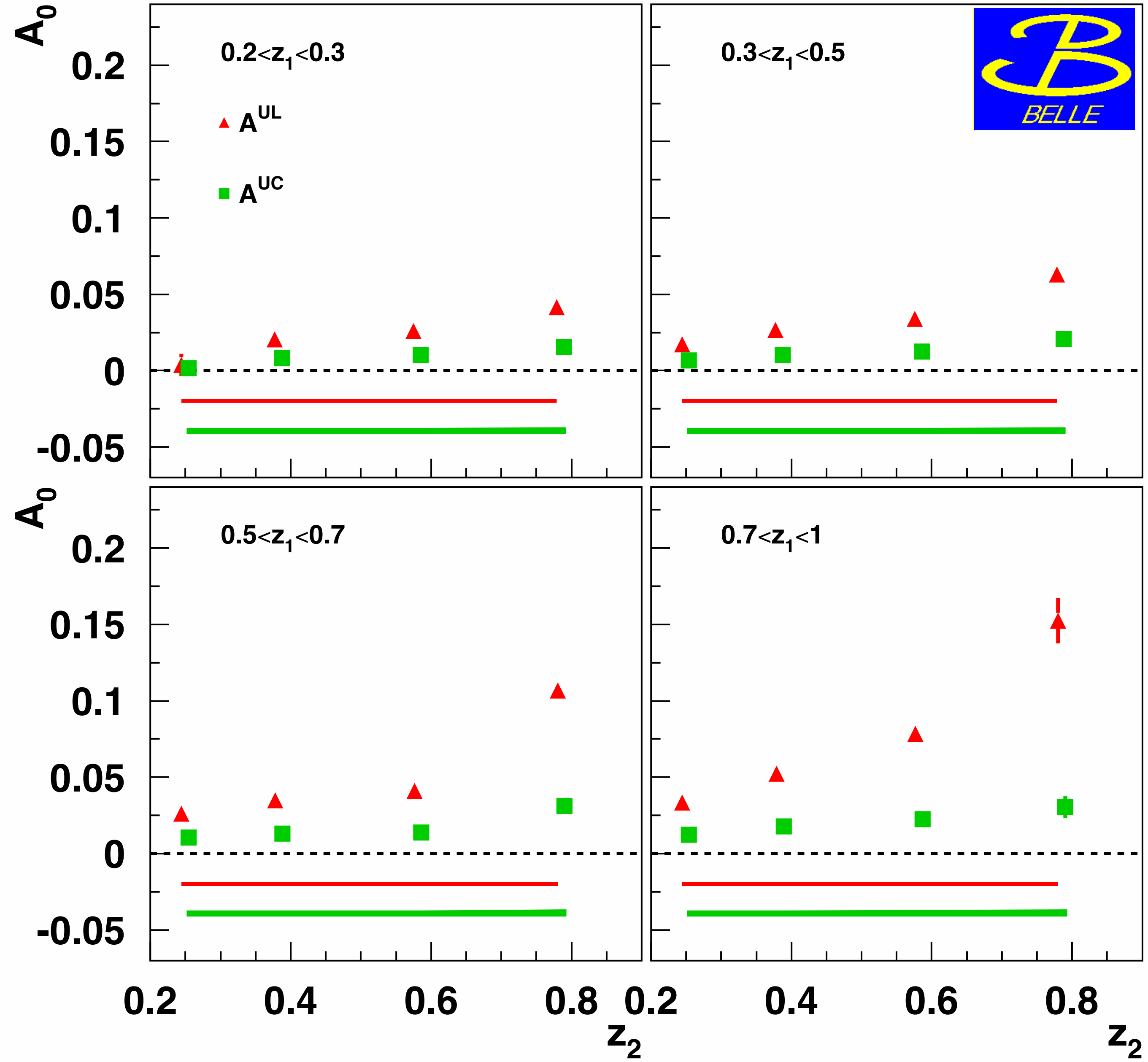


Collins asymmetries



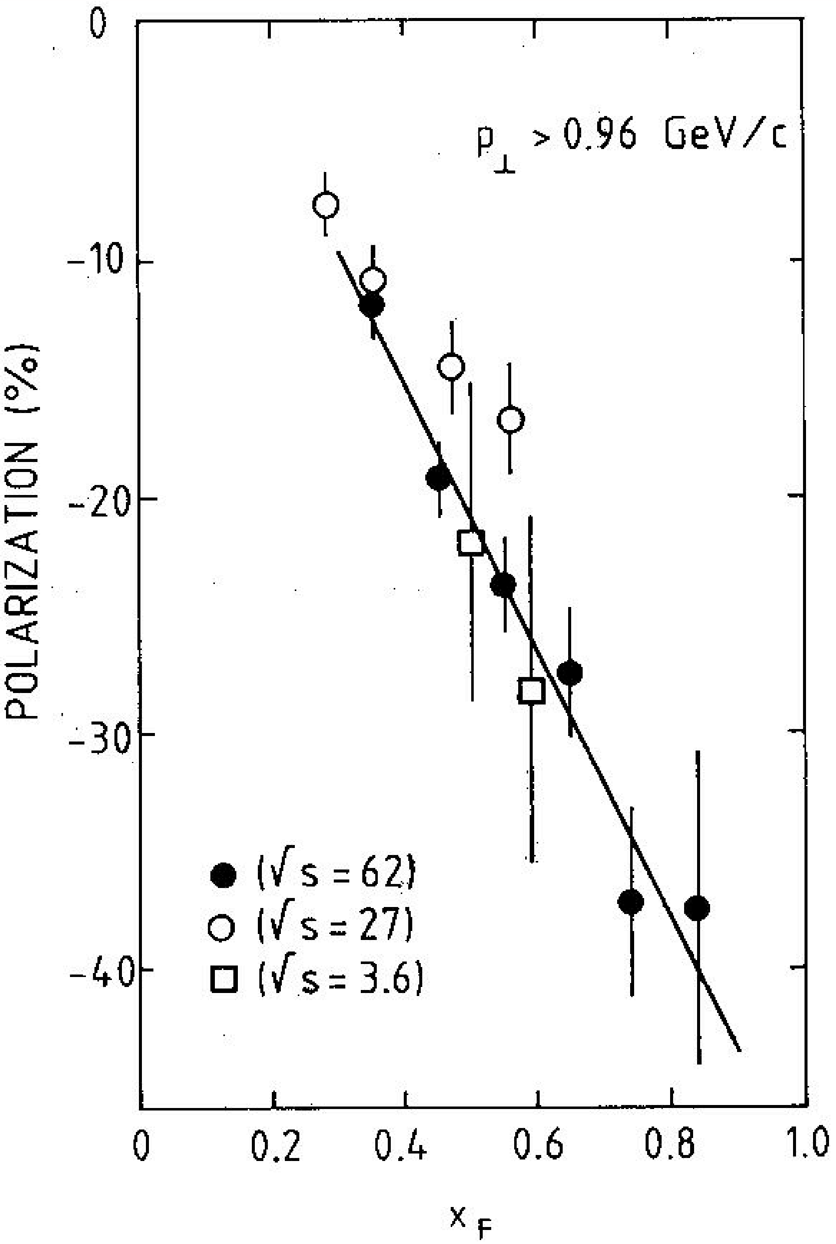
- pioneering measurement of Collins asymmetries by Belle for charged pions
  [PRL 96 (2006) 232002, PRD 78 (2008) 032011, PRD 86 (2012) 039905(E)]
- binned in (z_1, z_2)
  - significant asymmetries rising with z
  - used for first transversity and Collins FF extractions

Collins asymmetries

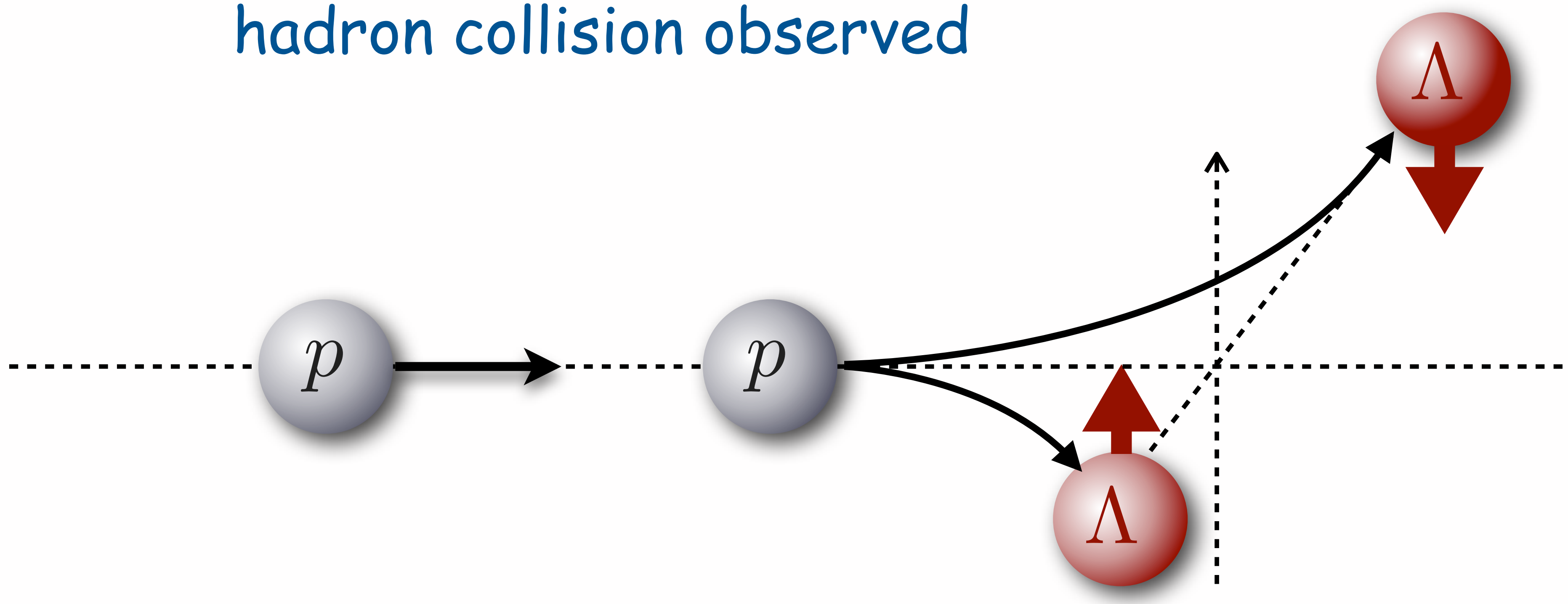


- pioneering measurement of Collins asymmetries by Belle for charged pions
  [PRL 96 (2006) 232002, PRD 78 (2008) 032011, PRD 86 (2012) 039905(E)]
- binned in (z_1, z_2)
  - significant asymmetries rising with z
  - used for first transversity and Collins FF extractions
- (very) close to release: π^0 and η asymmetries
  - RF12 asymmetries only, corrected to thrust axis
  - 2d binning (z_1, z_2) but also in transverse momentum (e.g., z-p_t bins)

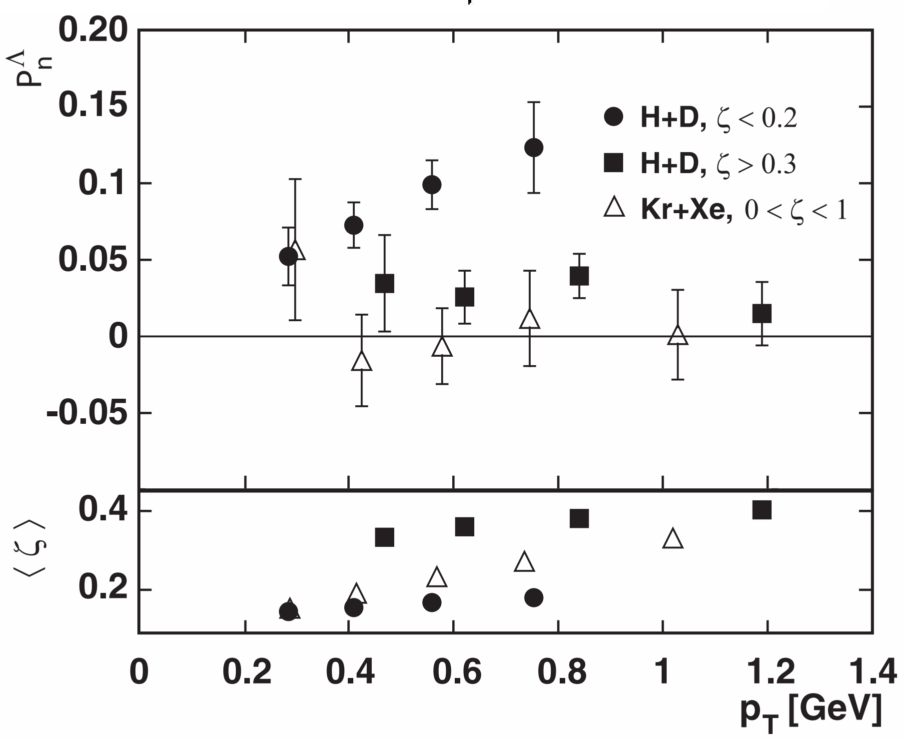
polarizing fragmentation



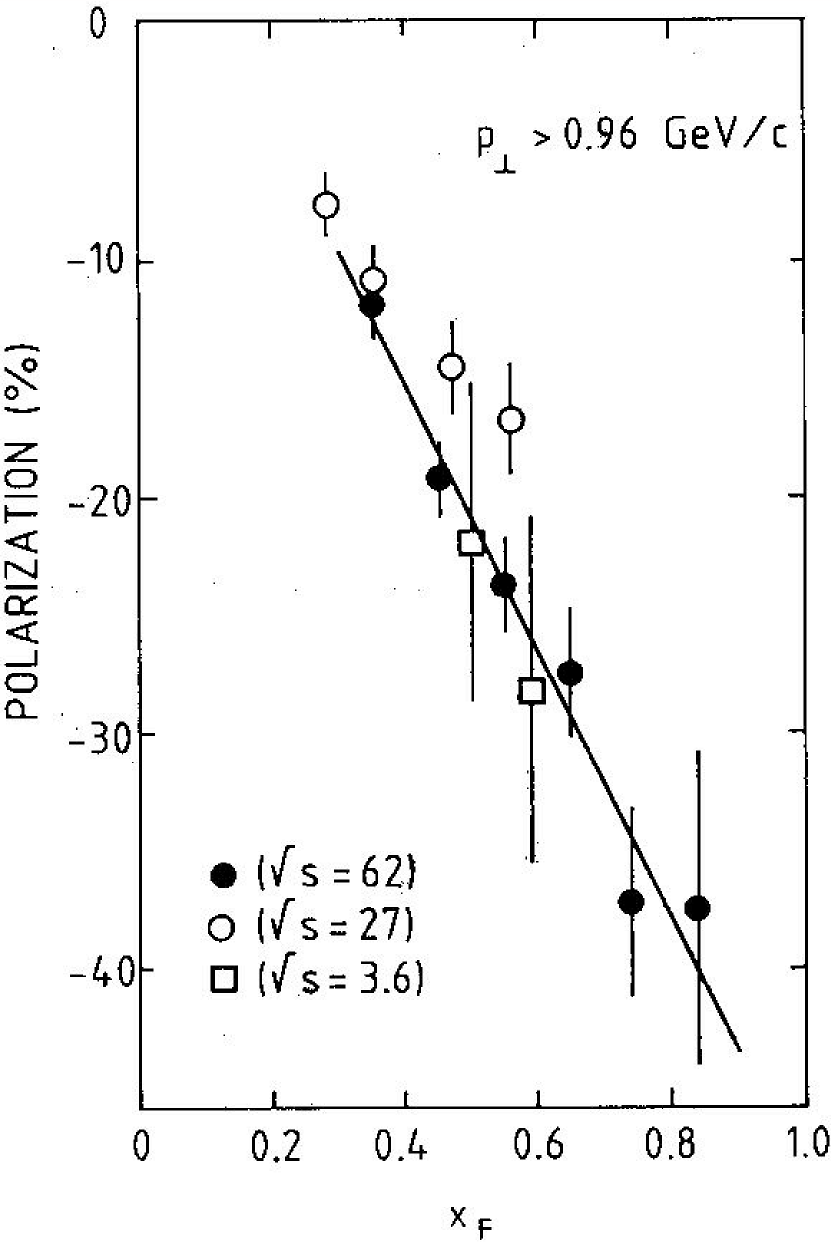
- large hyperon polarization in unpolarized hadron collision observed



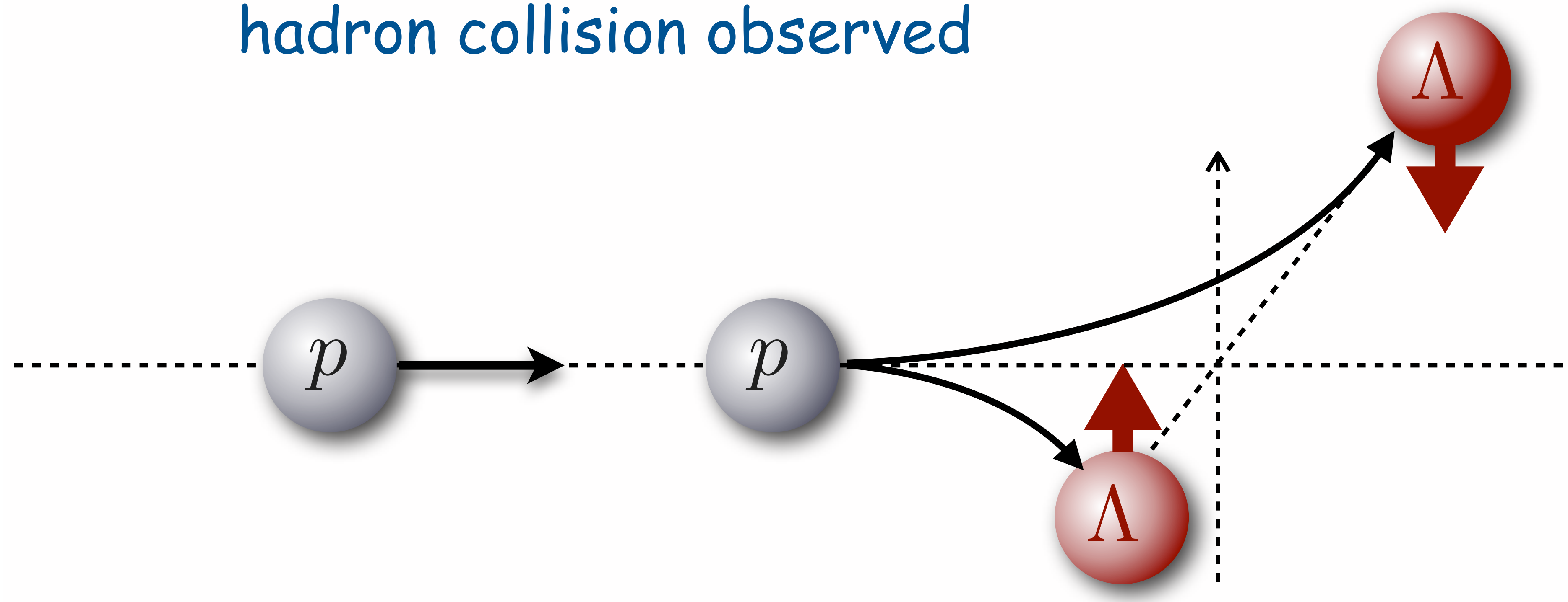
- … as well as in inclusive lepto-production

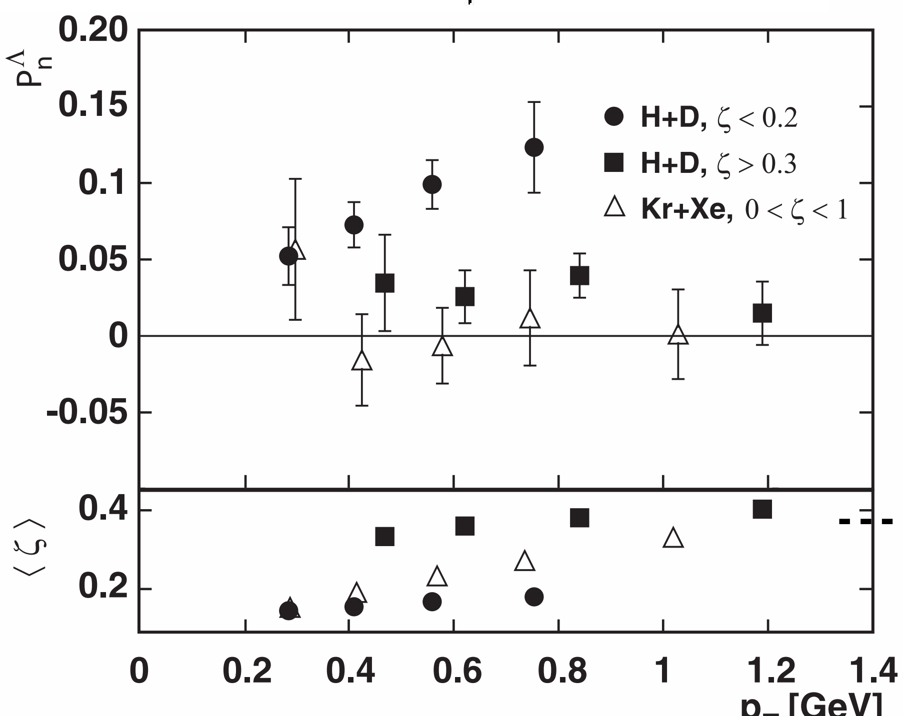


polarizing fragmentation

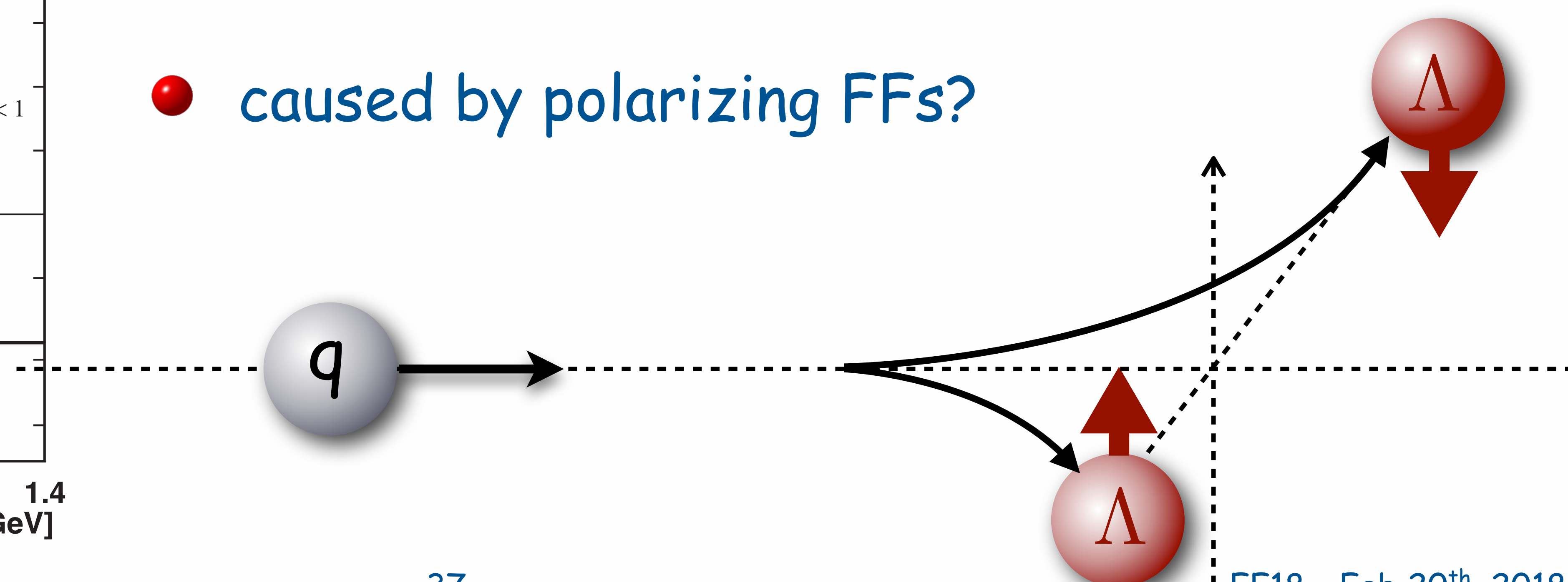


- large hyperon polarization in unpolarized hadron collision observed



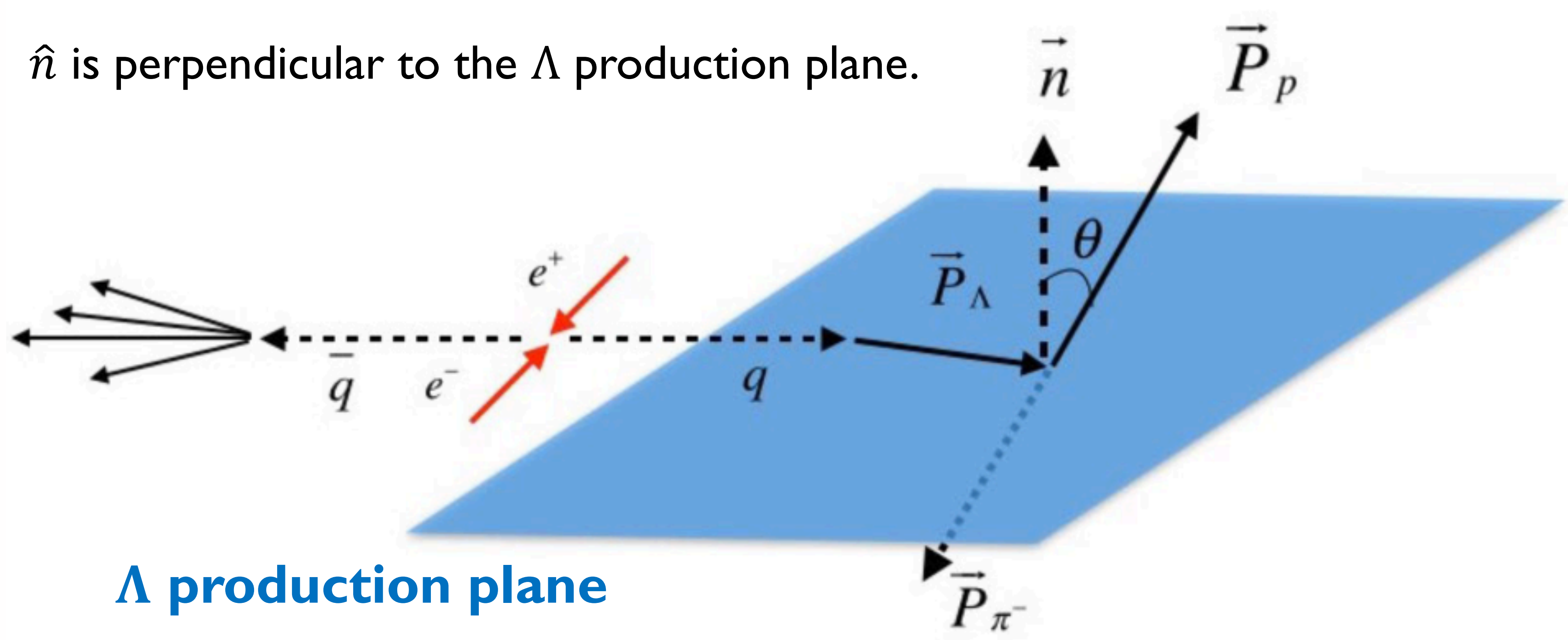


- … as well as in inclusive lepto-production
- caused by polarizing FFs?



polarizing fragmentation function

- polarization measured normal to production plane, i.e $\propto$(“q” × P_Λ)
  (note that in figure sign is reversed)

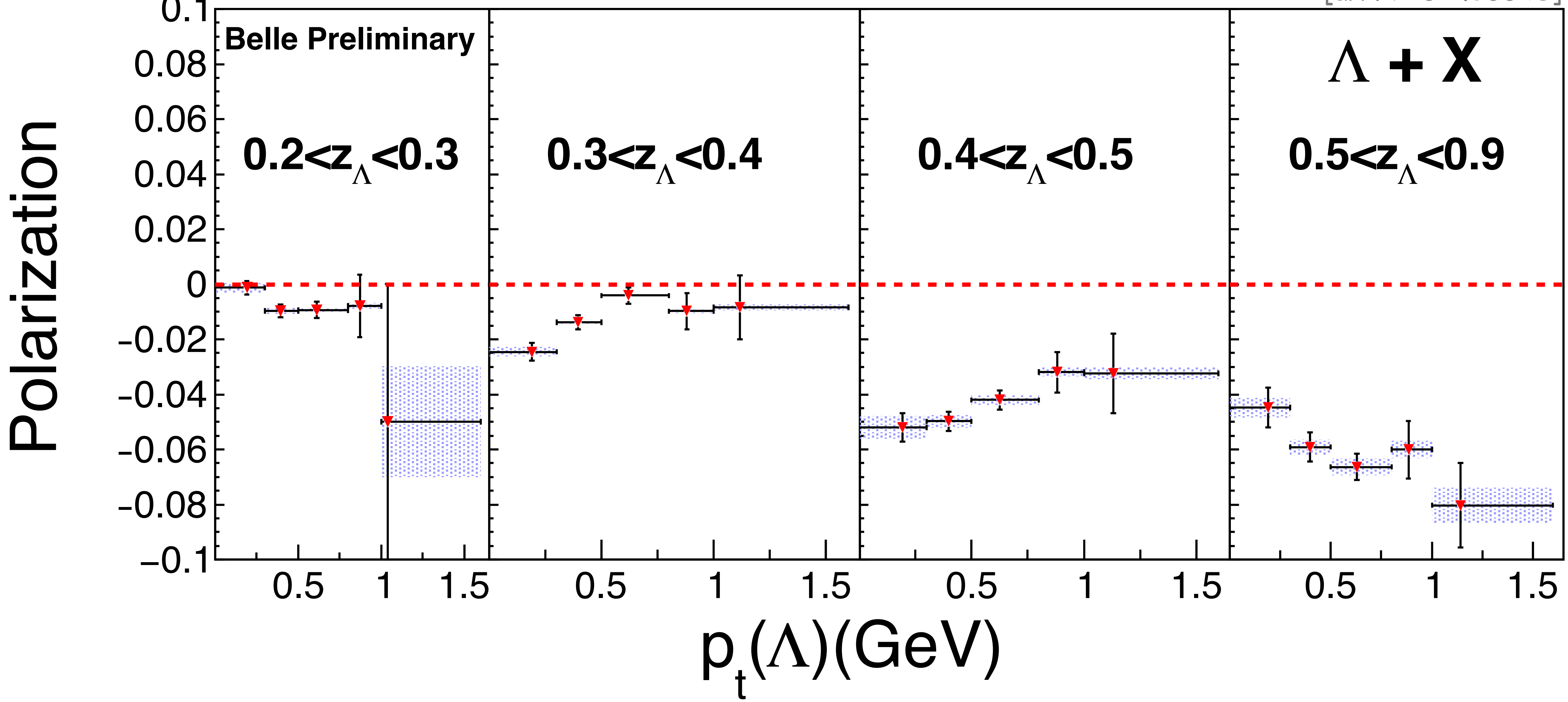


- need reference axis to define transverse momentum
  - “thrust frame” - use thrust axis
  - “hadron frame” - use momentum direction of “back-to-back” hadron
- use self-analyzing weak decay of Λ to determine polarization

polarizing fragmentation function

- polarization measured as function of z and p_t

[arXiv:1611.06648]

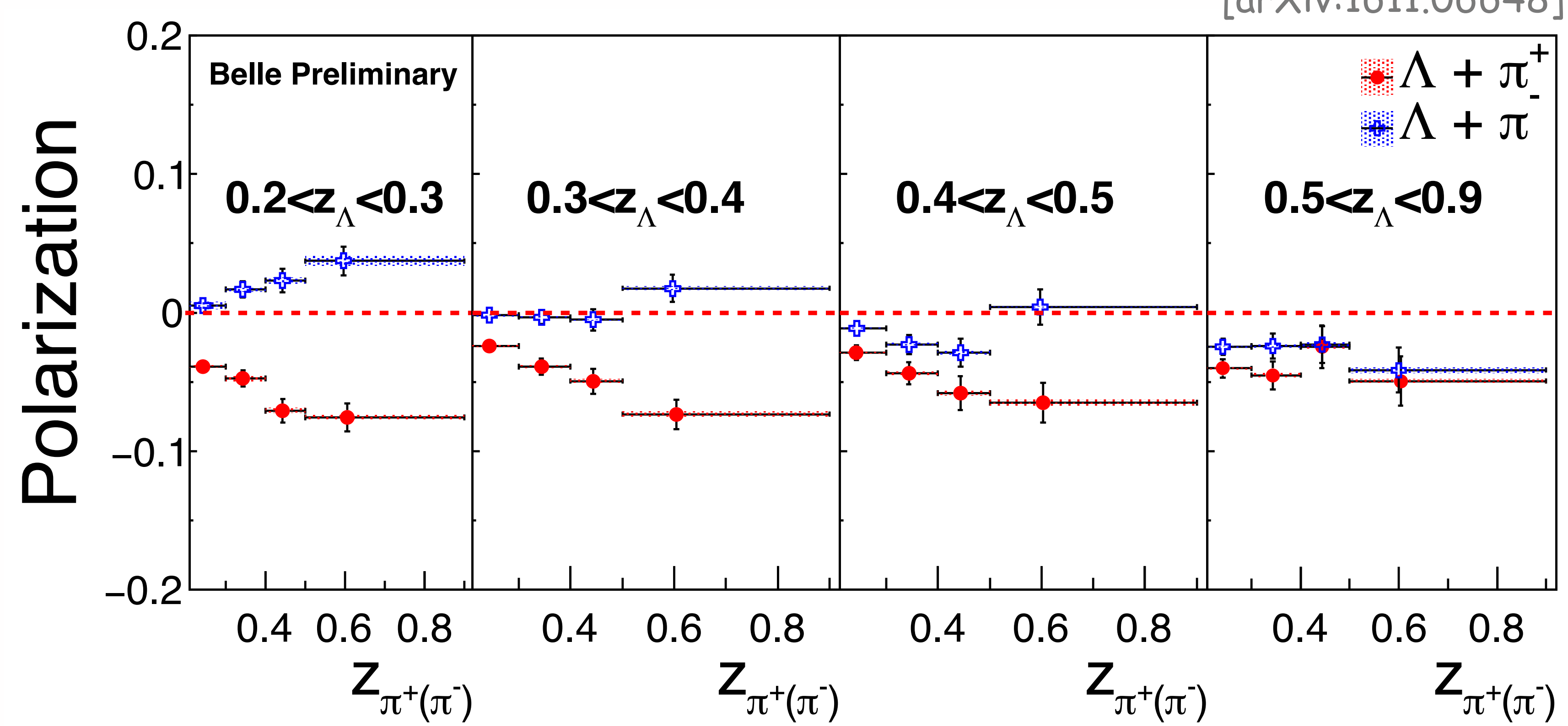


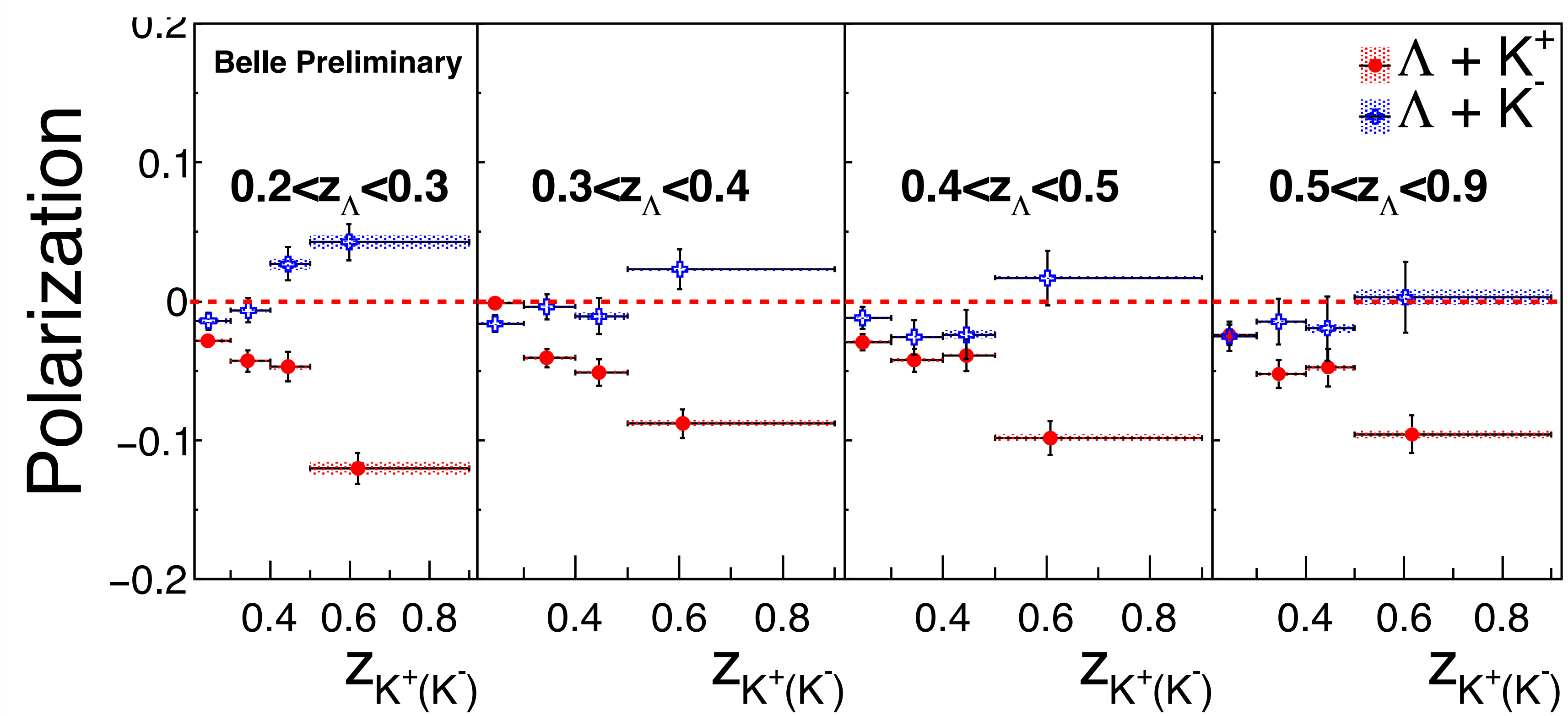
- strong dependence on both kinematics
  - unexpected/surprising behavior for p_t -> 0

polarizing fragmentation function

- flavor-tagging through hadrons in opposite hemisphere:

[arXiv:1611.06648]





- large-z_h hadrons tag quark flavor more efficiently

➡ enlarges differences between oppositely charged hadrons

- MC-based quark-flavor decomposition in backup

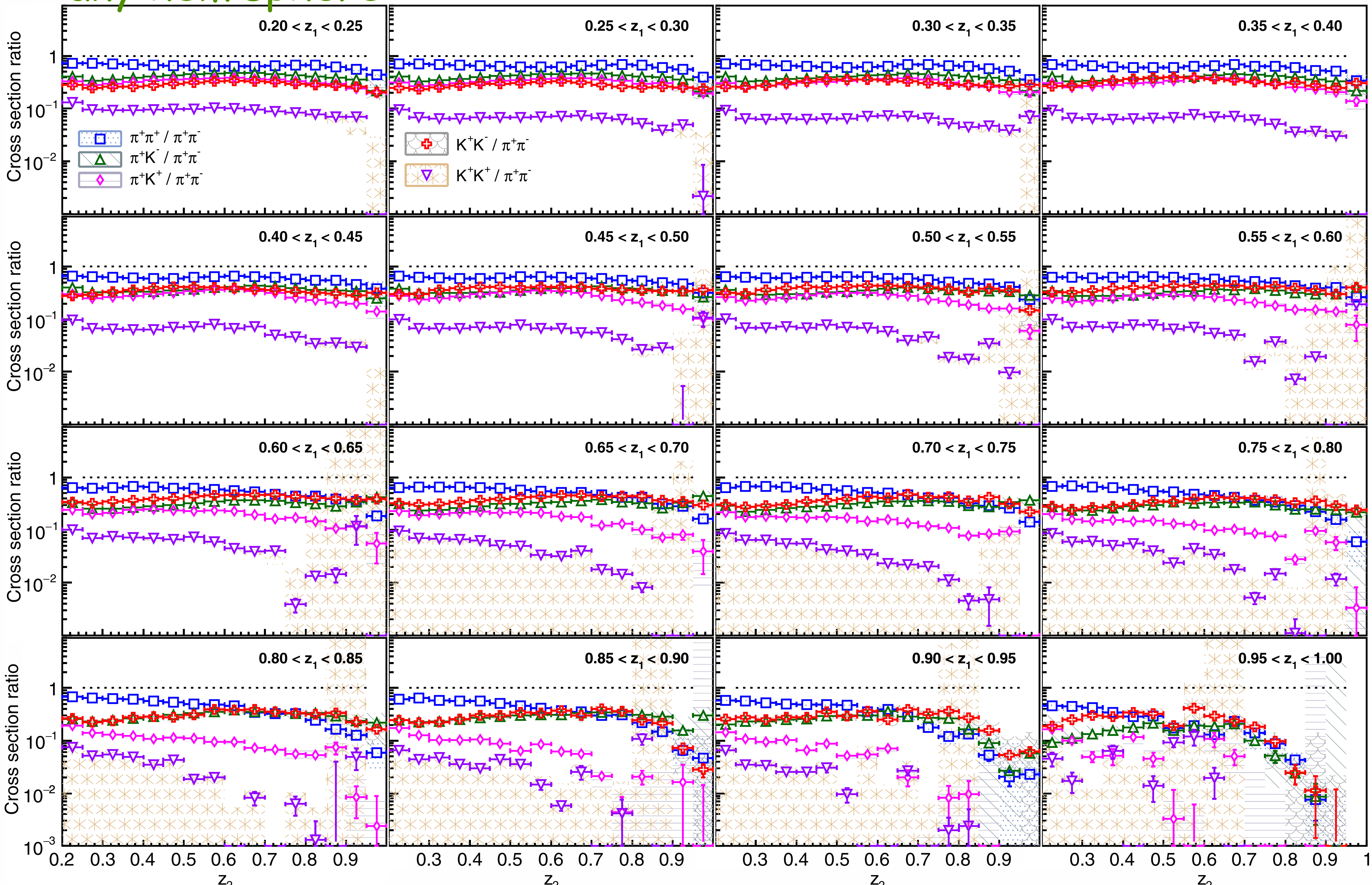
summary of future Belle(2) results on FFs

- production cross sections of hyperons and charmed baryons (submitted to Phys. Rev. D, arXiv:1706.06791)
- transverse polarization of inclusively produced Λ^0 hyperons (arXiv:1611.06648)
- π^0 and η Collins asymmetries
- k_T-dependent D_1 FFs ☛ Ralf
  - hadron-to-thrust
  - nearly back-to-back hadrons
- hadron-pair cross sections and asymmetries revisited: fully differential and/or differential in other variables?
- helicity-dependent dihadron fragmentation: $G_1^\perp$ ("jet handedness")
- $\sqrt{s}$ scan using ISR ☛ Ralf

backup slides

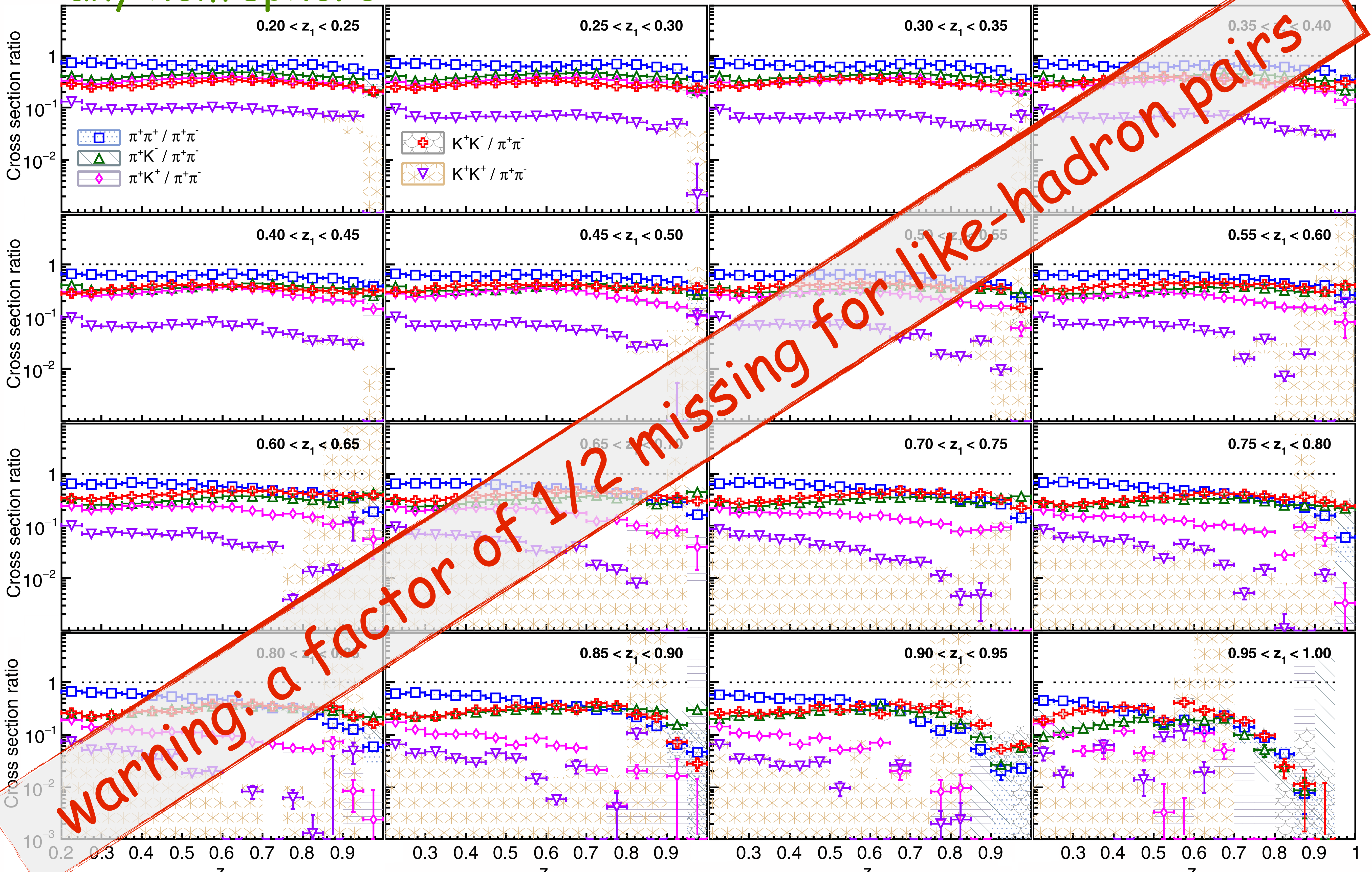
hadron-pair cross sections relative to pi+pi-

any hemisphere

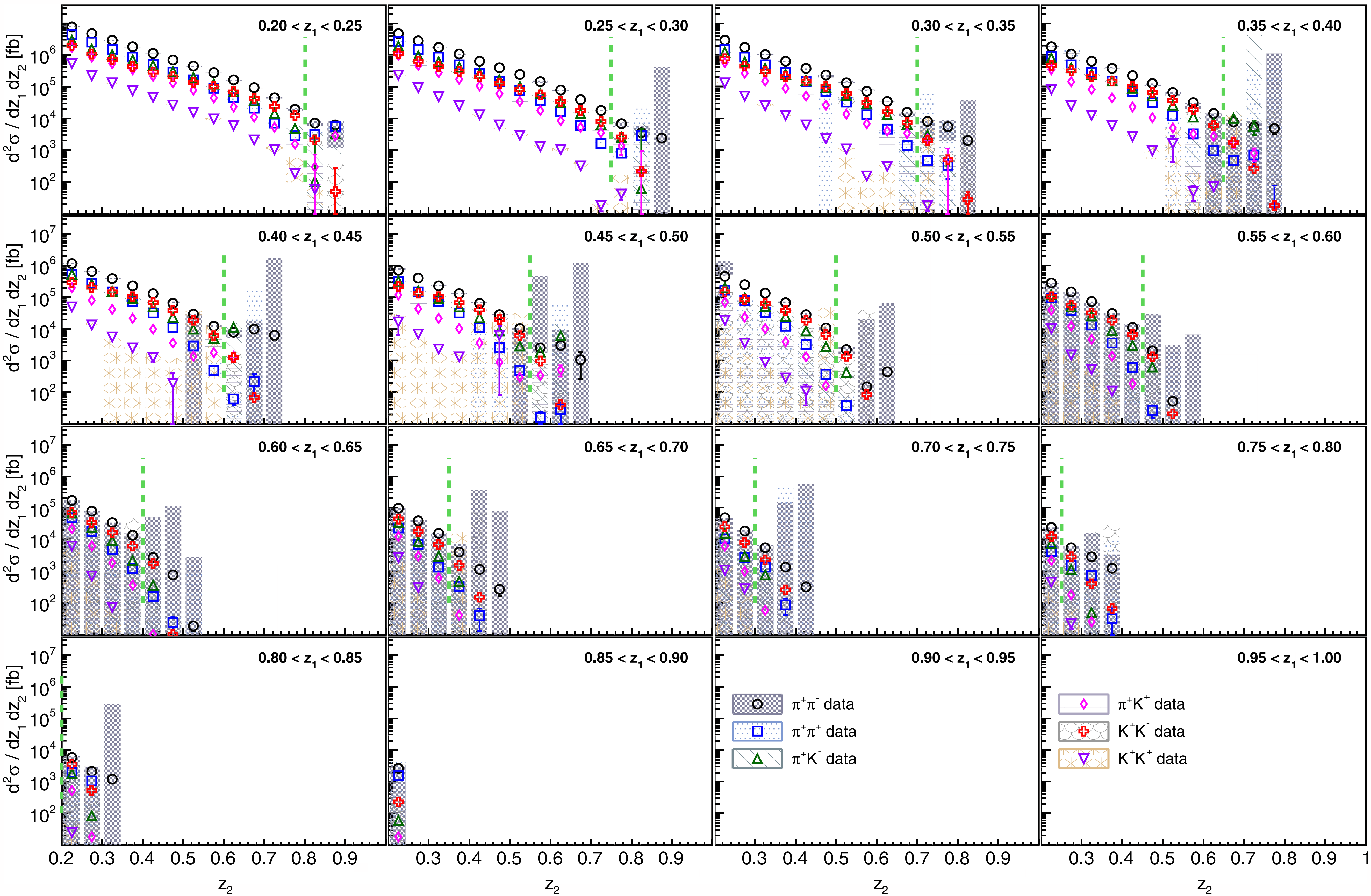


hadron-pair cross sections relative to pi+pi-

any hemisphere

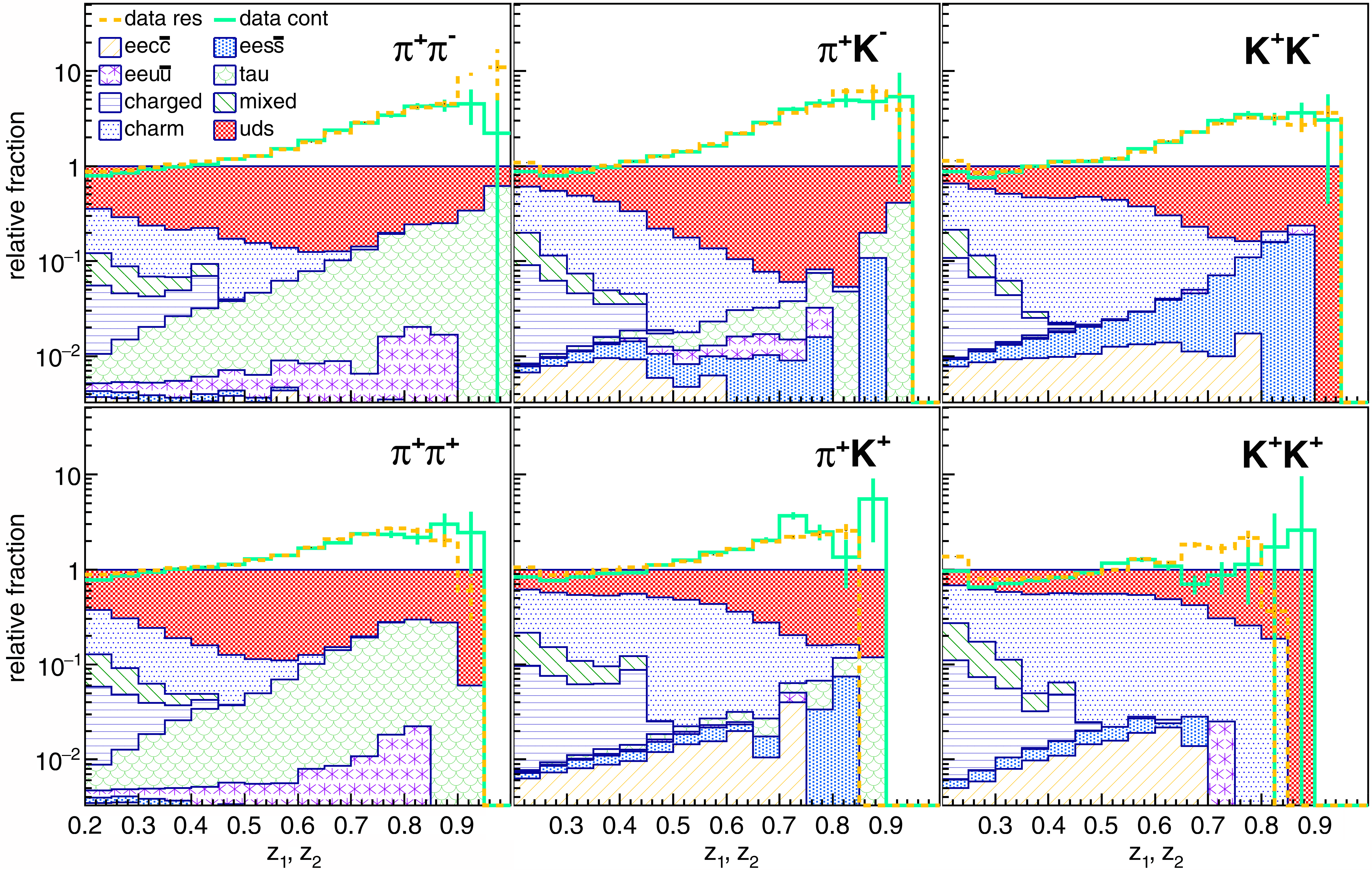


same-hemisphere hadron pairs

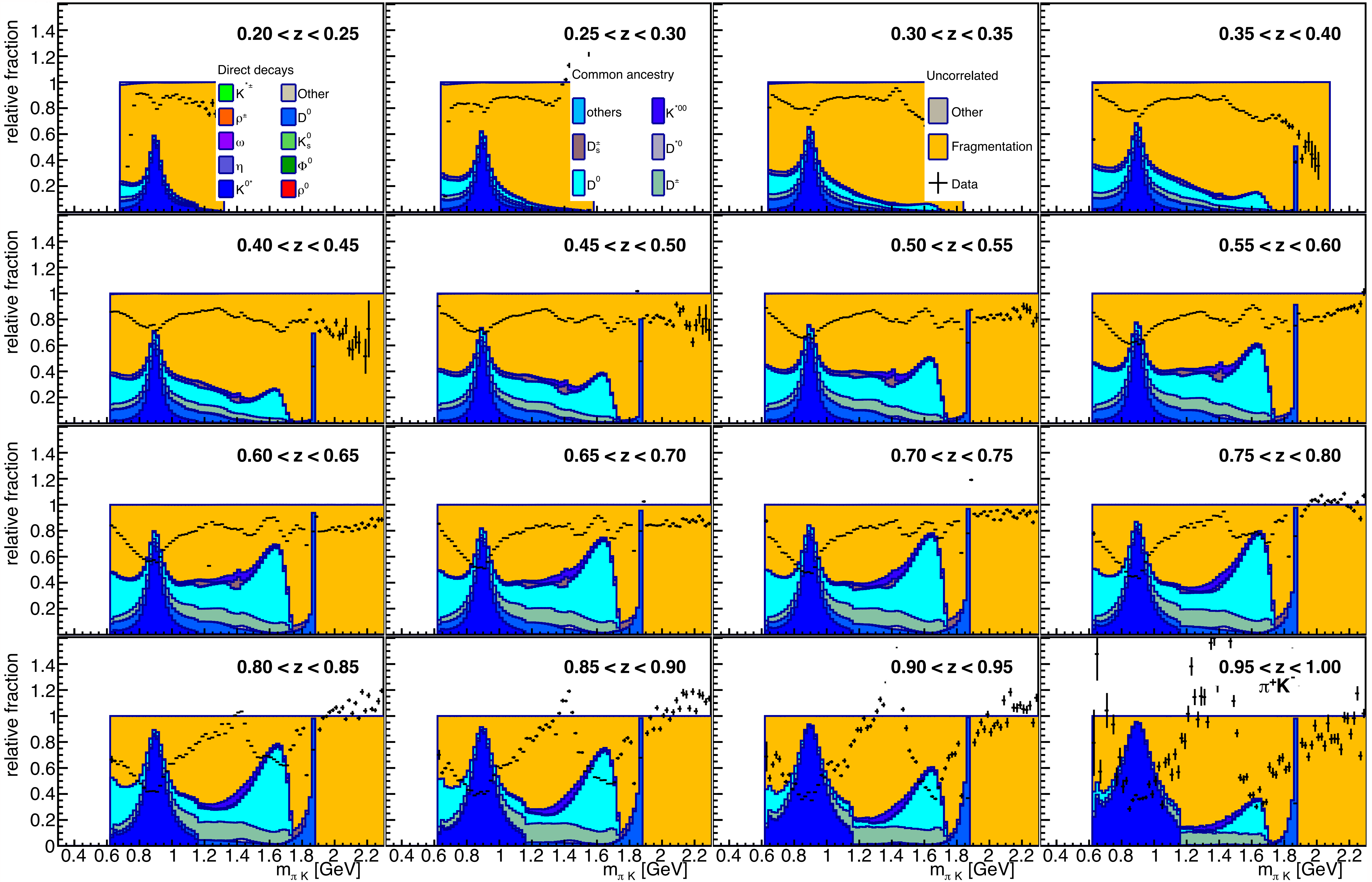


hadron-pairs: subprocess contributions

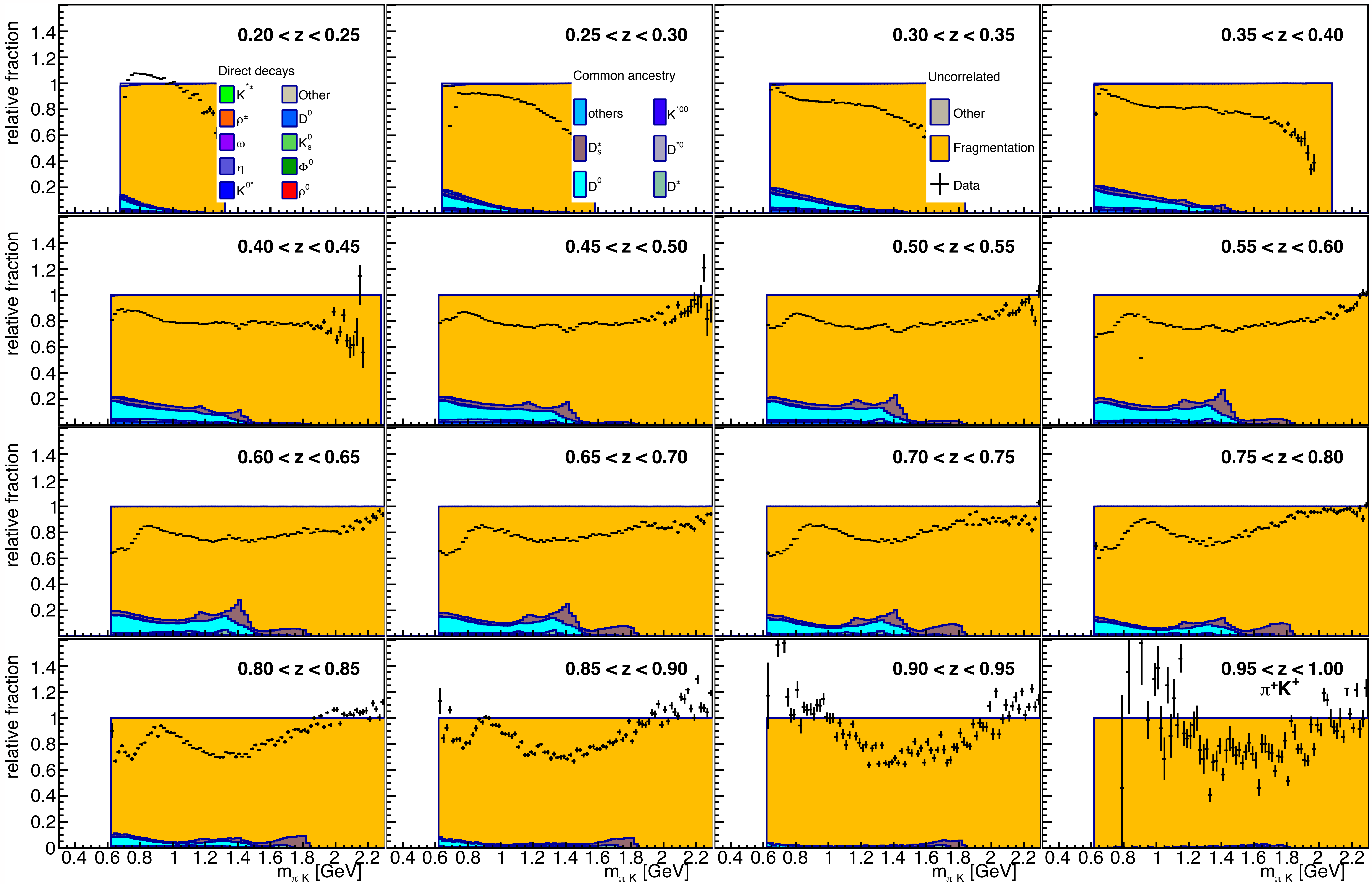
[Phys. Rev. D92 (2015) 092007]



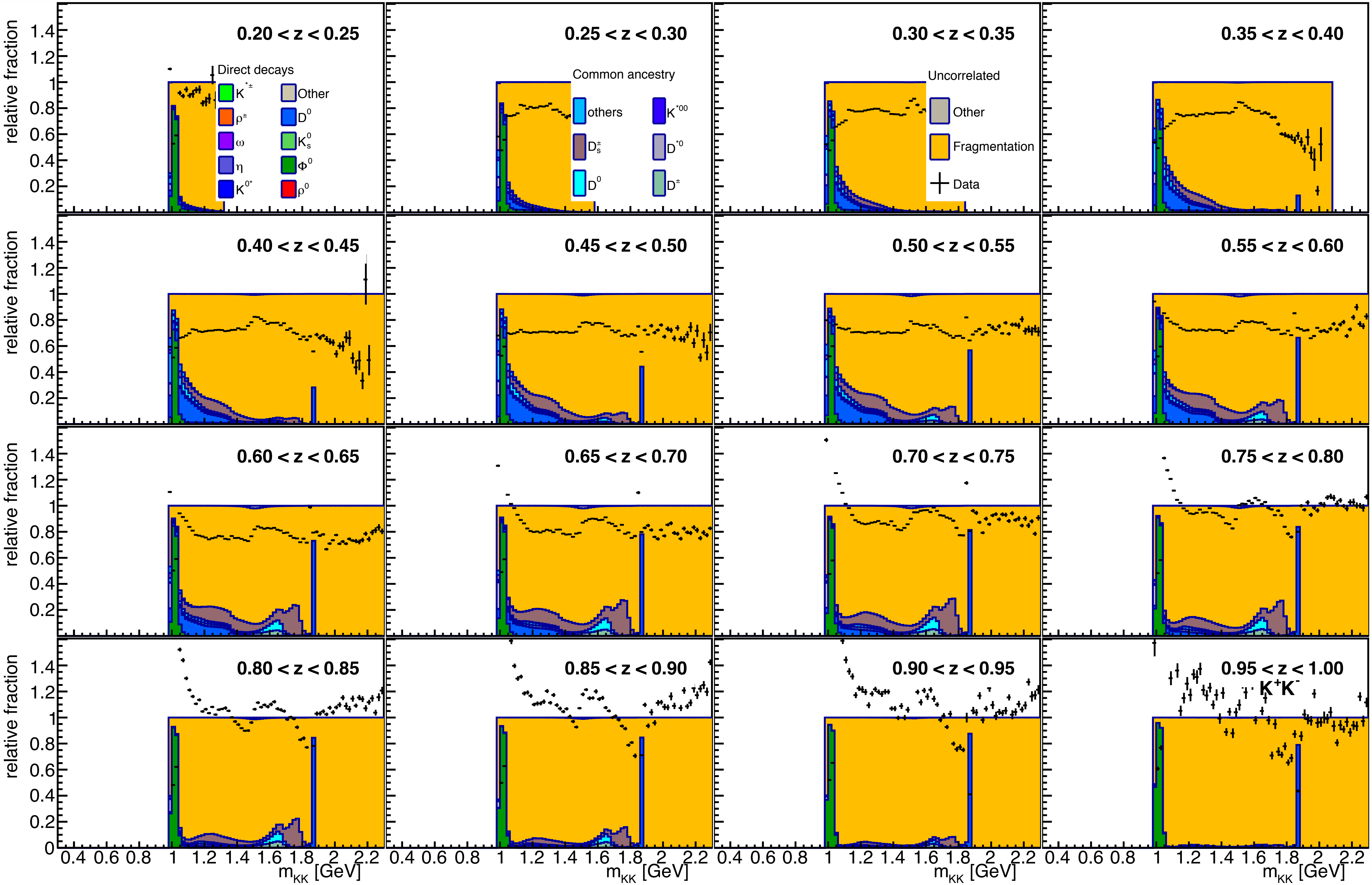
unlike-sign pi-K cross sections



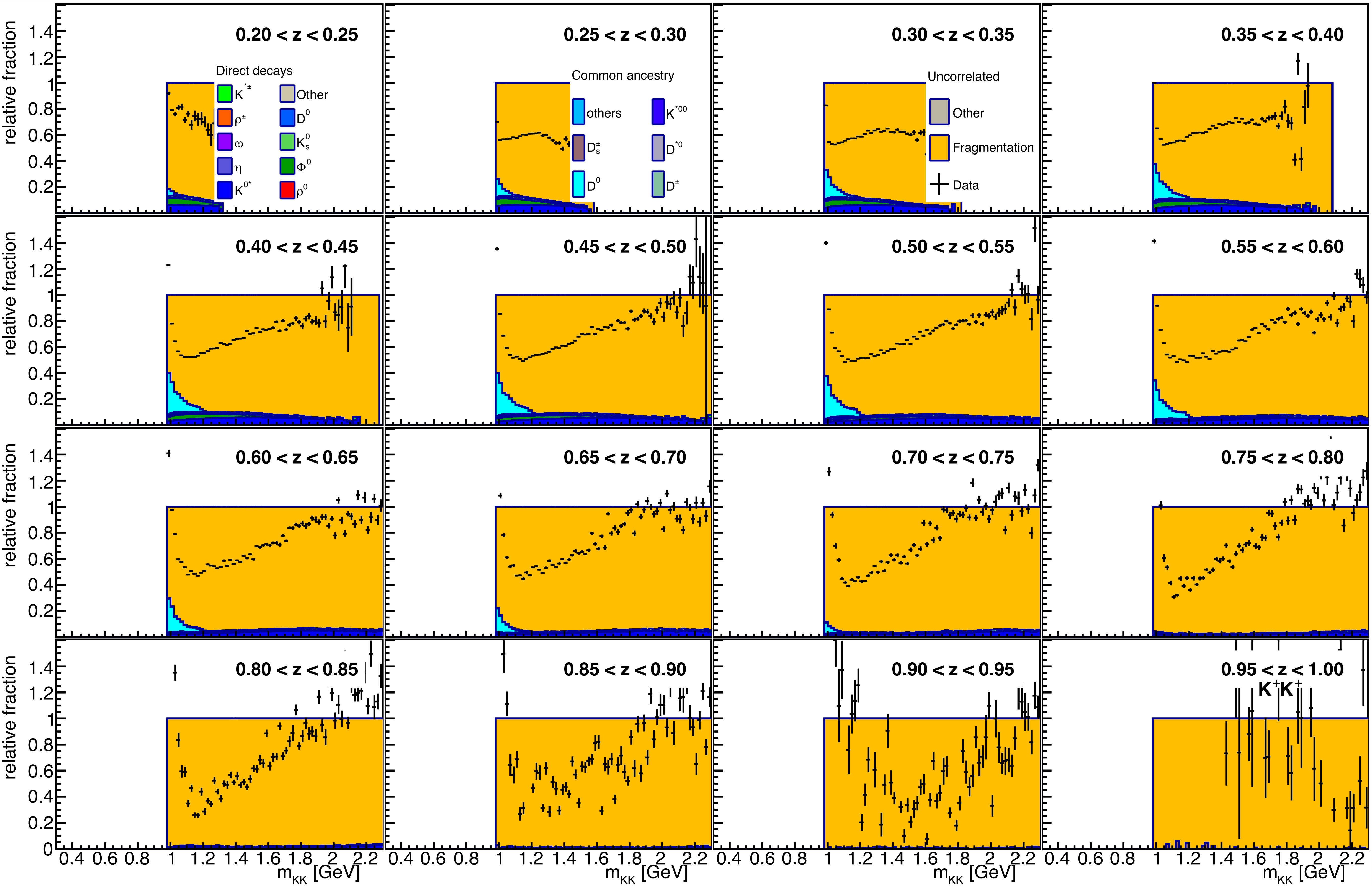
like-sign pi-K cross sections



unlike-sign di-kaon cross sections

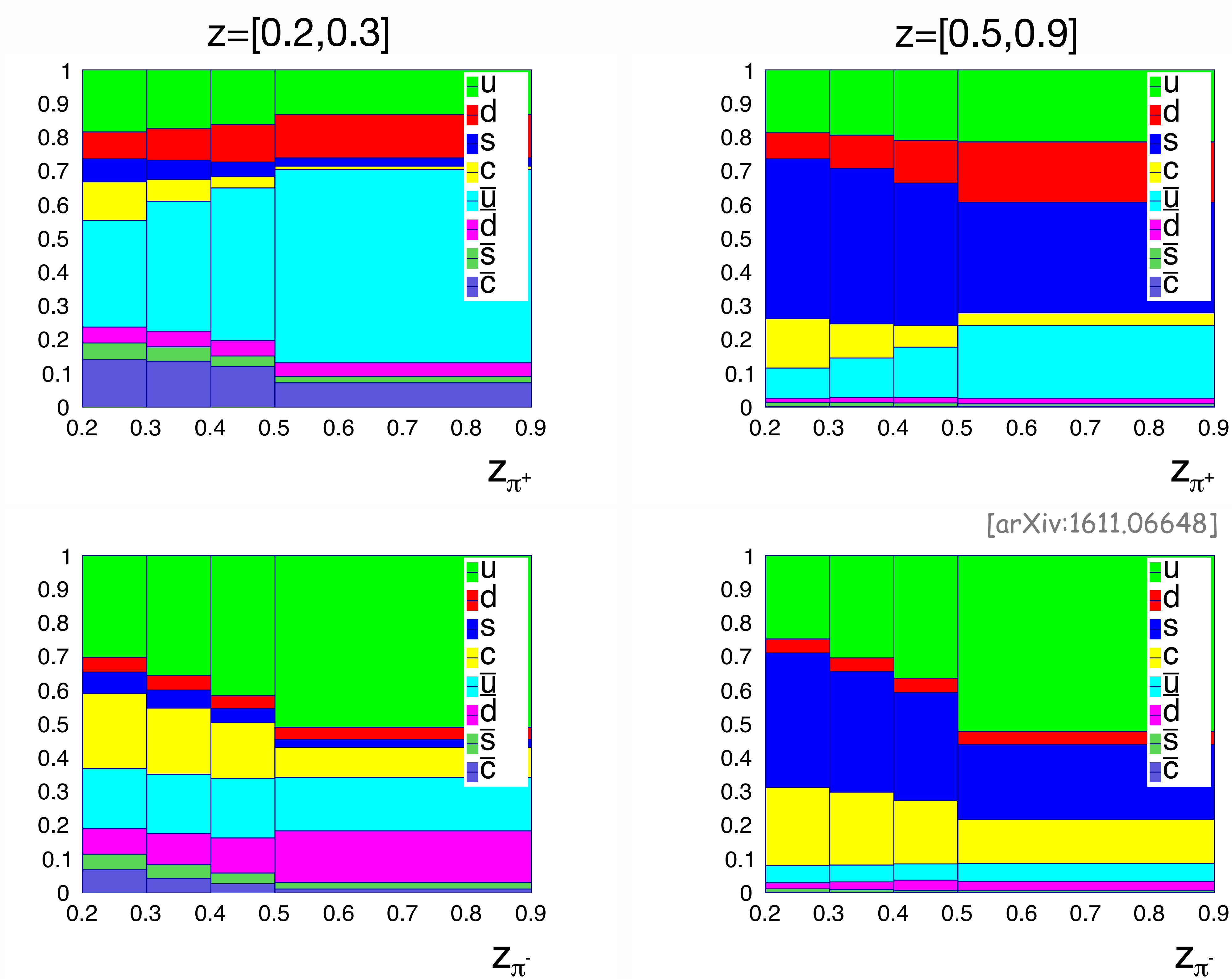


like-sign di-kaon cross sections



quark-flavor contributions to Lambda prod.

- flavor tagging through opposite-hemisphere hadrons



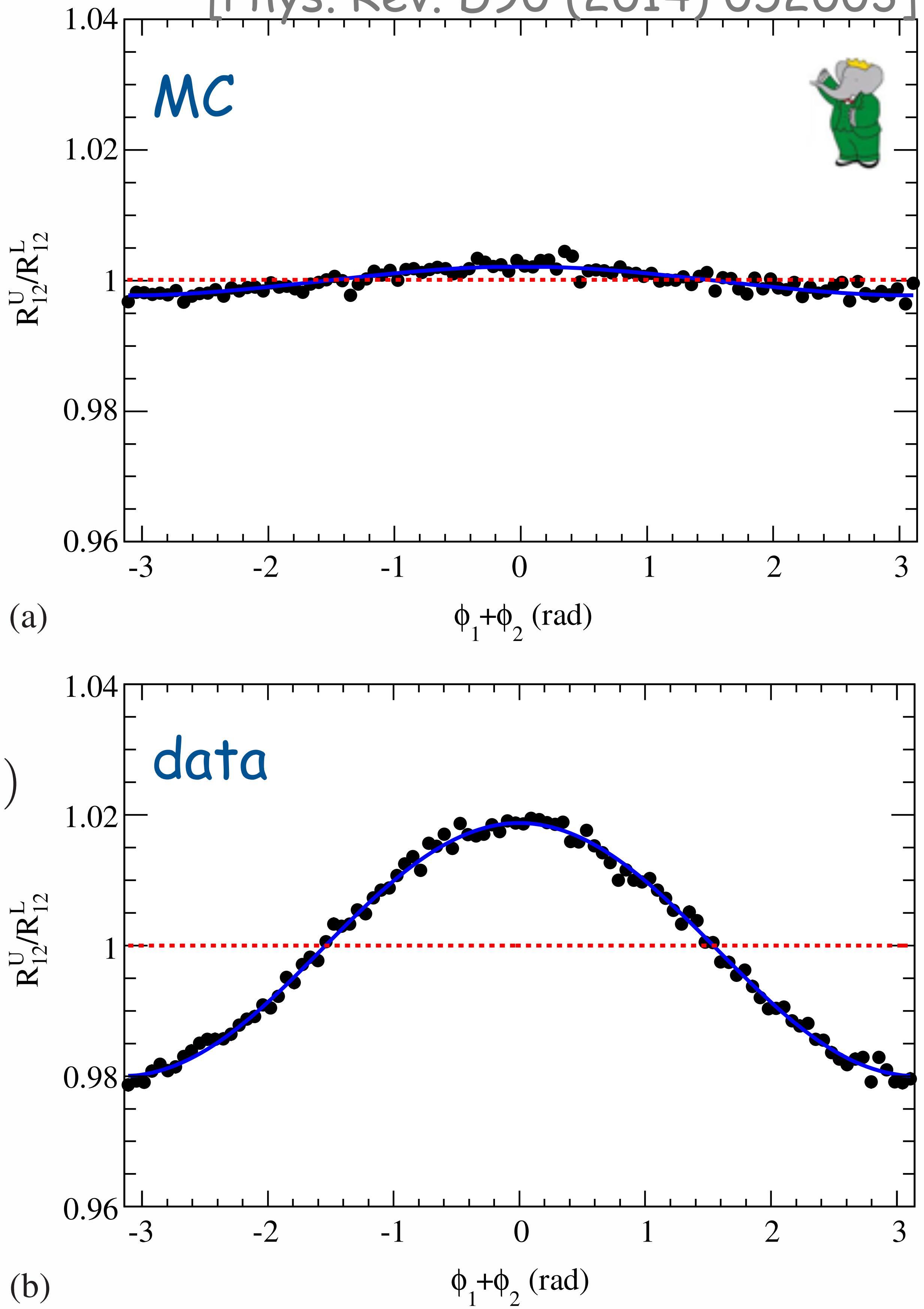
hadron-pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., MC)
- construct double ratio of normalized-yield distributions R_{12}, e.g. unlike-/like-sign:

$$\frac{R_{12}^U}{R_{12}^L} \simeq \frac{1 + \langle \frac{\sin^2\theta_{\rm th}}{1+\cos^2\theta_{\rm th}} \rangle G^U \cos(\phi_1 + \phi_2)}{1 + \langle \frac{\sin^2\theta_{\rm th}}{1+\cos^2\theta_{\rm th}} \rangle G^L \cos(\phi_1 + \phi_2)}$$

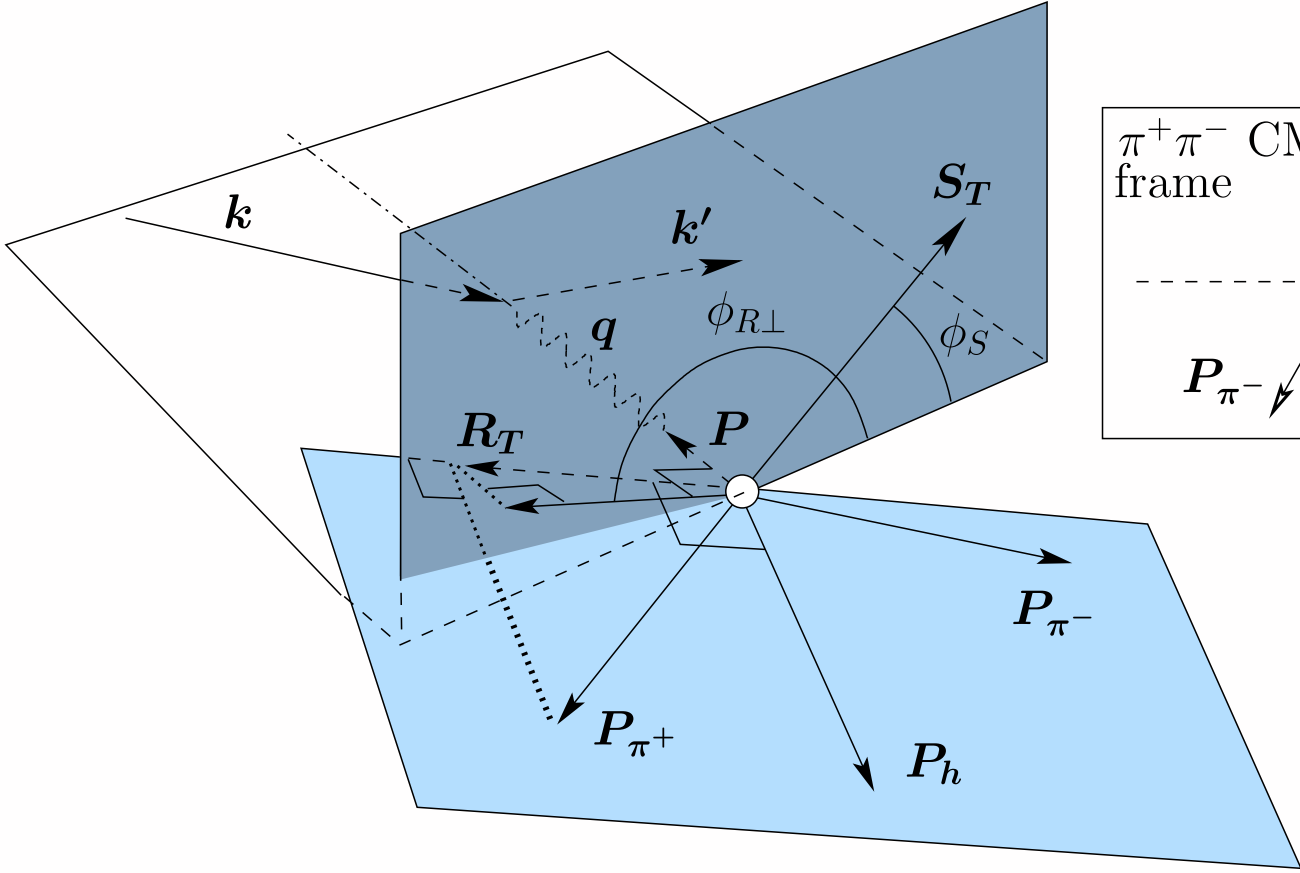
$$\simeq 1 + \left\langle \frac{\sin^2\theta_{\rm th}}{1+\cos^2\theta_{\rm th}} \right\rangle \{G^U - G^L\} \cos(\phi_1 + \phi_2)$$

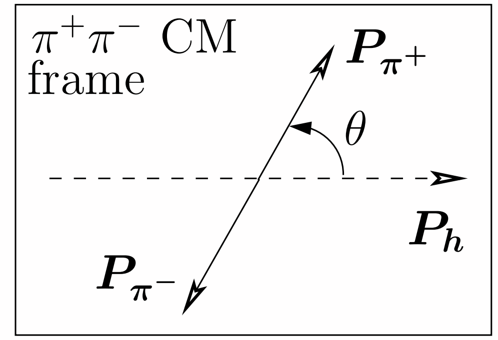
[Phys. Rev. D90 (2014) 052003]



- suppresses flavor-independent sources of modulations
- A^{UL} specific combinations of FFs
- remaining MC asym.'s: systematics

“pitfalls” in dihadron fragmentation

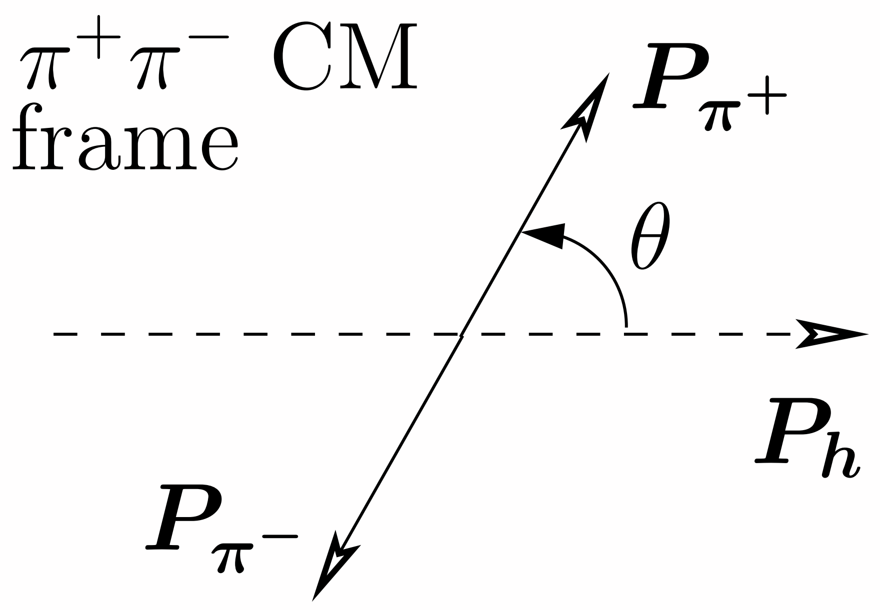




"appetizer"

- dihadron FFs: alternative path to extract (collinear) transversity
  - exploit orientation of hadron's relative momentum, correlate with target polarization
- complication: SIDIS cross section now differential in 9(!) variables
- integration over polar angle eliminates, in theory, a number of contributing FFs (partial waves)
- experimental constraints limit acceptance in polar angle, most prominently the minimum-momentum requirements

simple case study



basic assumptions:

- dihadron pair with equal-mass hadrons; here: pions
- e^+e^- annihilation, thus energy fractions z translates directly to energy/momentum of particles/system as primary energy is "fixed" (-> simplifies Lorentz boost)
- without loss of generality, focus on B factory and use primary quark energy $E_0 = 5.79$GeV
- minimum energy of each pion in lab frame: 0.1 E_0 (i.e., $z_{min} = 0.1$)

application of Lorentz boost

- can easily apply Lorentz boost using the invariant mass of the dihadron M and its energy zE_0 to arrive at condition on θ, e.g., polar angle of pions in center-of-mass frame:

$$\cos\theta \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M^2)(M^2 - 4m_\pi^2)]}} E_0 M$$

- as both pions have to fulfill the constraint on the minimum energy:

$$\cos(\pi - \theta) = -\cos\theta \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M^2)(M^2 - 4m_\pi^2)]}} E_0 M$$

thus:

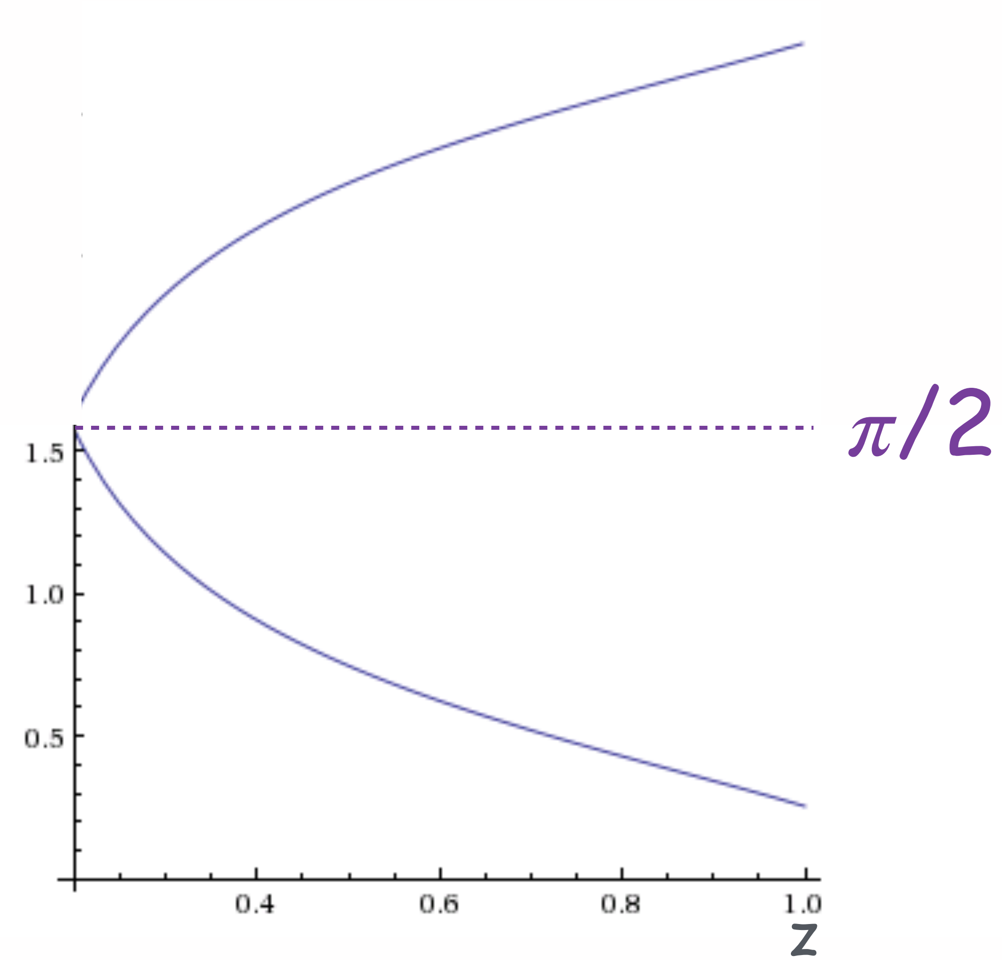
$$|\cos\theta| \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M^2)(M^2 - 4m_\pi^2)]}} E_0 M$$

- translates to a symmetric range around $\pi/2$

  (can be easily understood because at $\pi/2$ the pions will have both the same energy in the lab and easily pass the z_{min} requirement, while in the case of one pion going backward in the CMS, that pion will have less energy in the lab frame ... and maybe too little)

impact of z_{min}=0.1 on accepted polar range

- (again without loss of generality) let's assume M=0.5 GeV (and take already the $\cos^{-1}$):



- all theta between the purple lines (and the mirror range above the dashed line) are accepted
- clearly limited, especially at low z

partial-wave expansion of dihadron FF

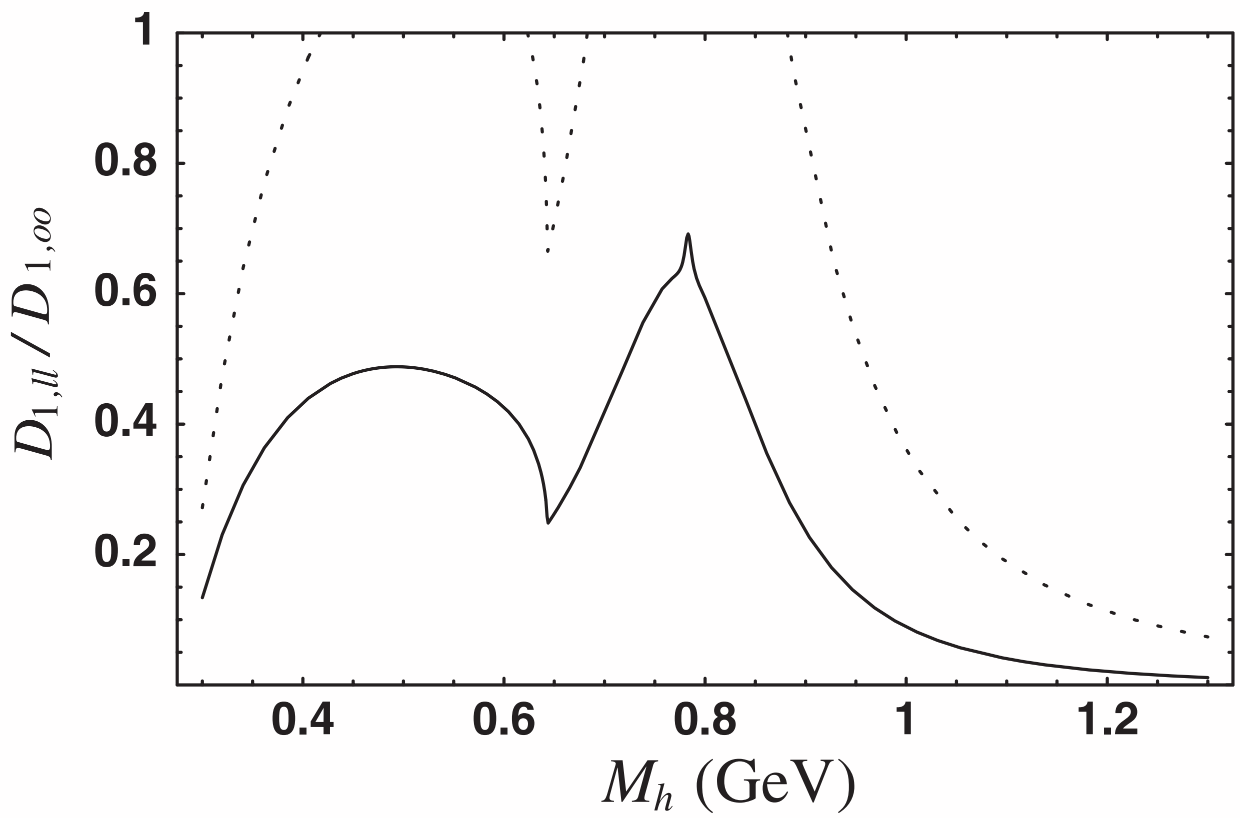
- partial-wave expansion worked out in Phys. Rev. D67 (2003) 094002
- for the particular case here, use Phys. Rev. D74 (2006) 114007, in particular Eq. (12), and (later on) Figure 5:

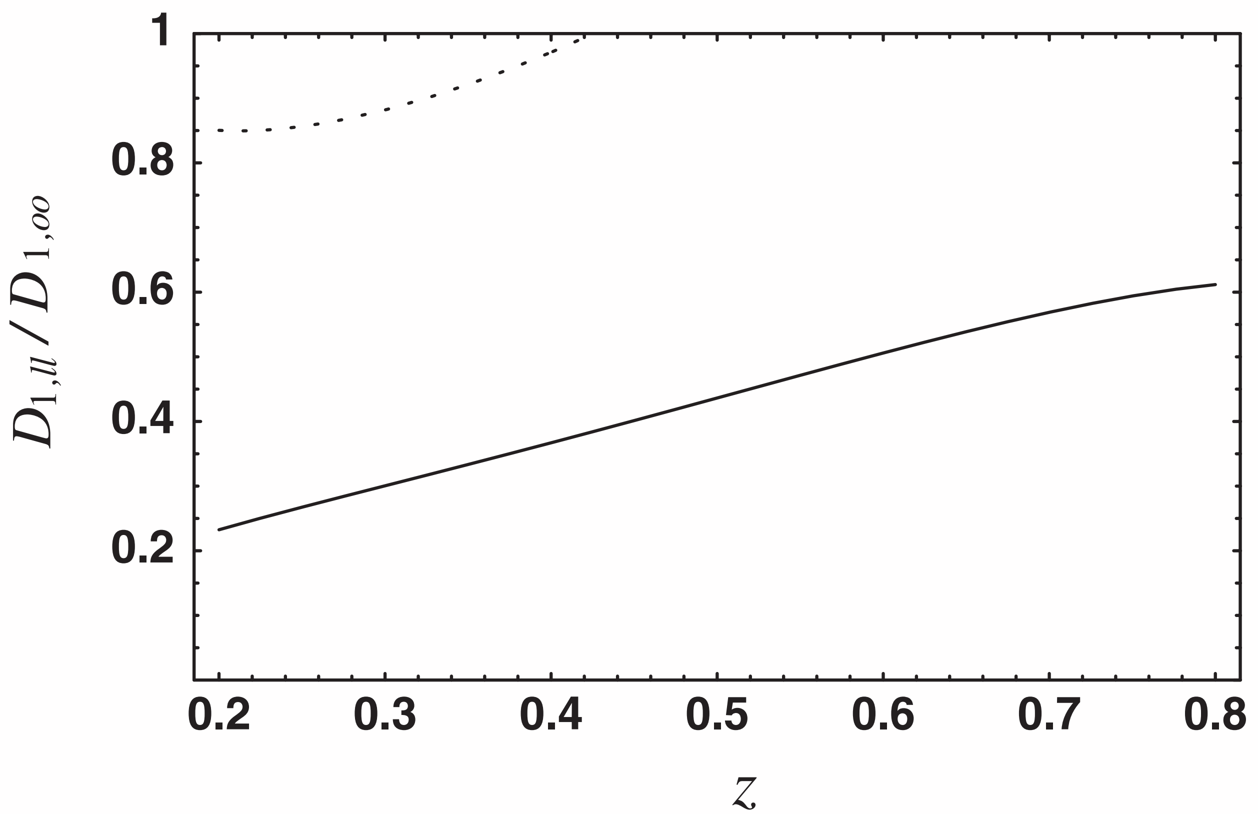
$$D_1^q(z, \cos\theta, M_h^2) \approx D_{1,oo}^q(z, M_h^2) + D_{1,ol}^q(z, M_h^2)\cos\theta + D_{1,ll}^q(z, M_h^2)\tfrac{1}{4}(3\cos^2\theta - 1), \qquad (12)$$

- it is the first contribution ($D_{1,oo}$) that is used in "collinear extraction" of transversity (and subject of a current Belle analysis)
  - it is also the only one surviving the integration over θ
- the $D_{1,ol}$ contribution vanishes upon integration over θ as long as the theta range is symmetric around $\pi/2$ (as it is the case here)
- the $D_{1,ll}$ term, however, will in general contribute in case of only partial integration over θ — the question is how much?

$D_{1,ll}$ contribution to dihadron fragmentation

- $D_{1,ll}$ is unknown and can't be calculated using first principles
- it can not be extracted from cross sections integrated over θ
- upon (partial) integration there is no way to disentangle the two contributions
- in PRD74 (2006) 114007, a model for dihadron fragmentation was tuned to PYTHIA and used to estimate the various partial-wave contributions
- its Figure 5 gives an indication about the relative size of $D_{1,ll}$ vs. $D_{1,oo}$:





effect of partial integration

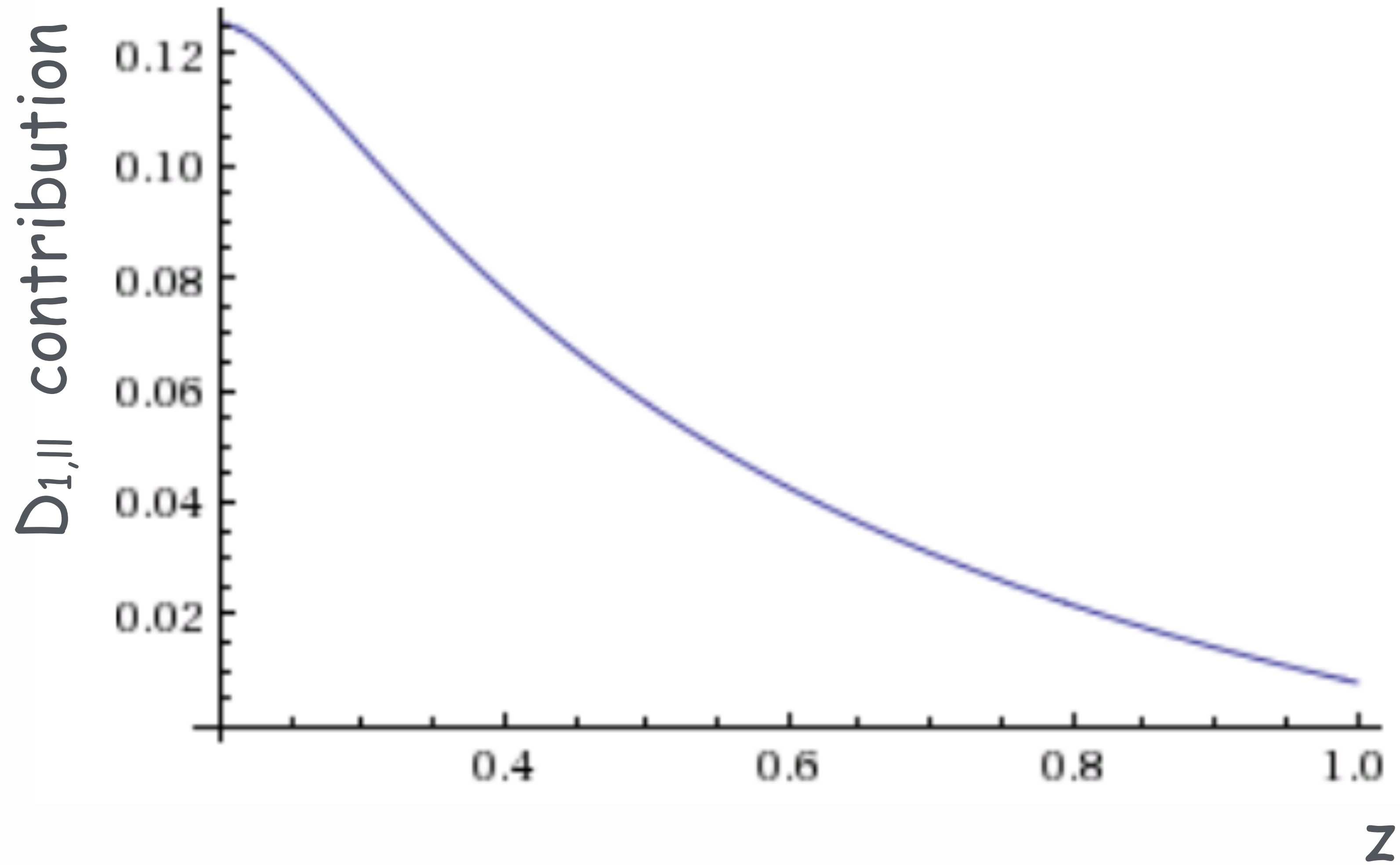
- as both contributions — $D_{1,ll}$ and $D_{1,oo}$ — will be affected by the partial integration, look at relative size of the $D_{1,ll}$ to $D_{1,oo}$ modulations when subjected to integration:

$$\frac{D_{1,ll}}{D_{1,oo}} \frac{\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta \, \frac{1}{4}(3\cos^2\theta - 1)}{\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta} = -\frac{1}{4}(1-\cos^2\theta_0)\,\frac{D_{1,ll}}{D_{1,oo}}$$

- without limit in the polar-angular range ($\theta_0 = 0$) -> no contribution from $D_{1,ll}$ (sanity check!)
- the relative size of the partial integrals reaches a maximum of 25% for z=0.2 (i.e., pions at 90 degrees in center-of-mass system)
- in order to estimate the $D_{1,ll}$ contribution, one "just" needs the relative size of $D_{1,ll}$ vs. $D_{1,oo}$, e.g., Figure 5 of PRD74 (2006) 114007
  - let's take for that size 0.5 (rough value for M=0.5 GeV)

effect of partial integration

- … $D_{1,ll} / D_{1,oo}$ ~0.5 results in an up to O(10%) effect on the measured cross section:



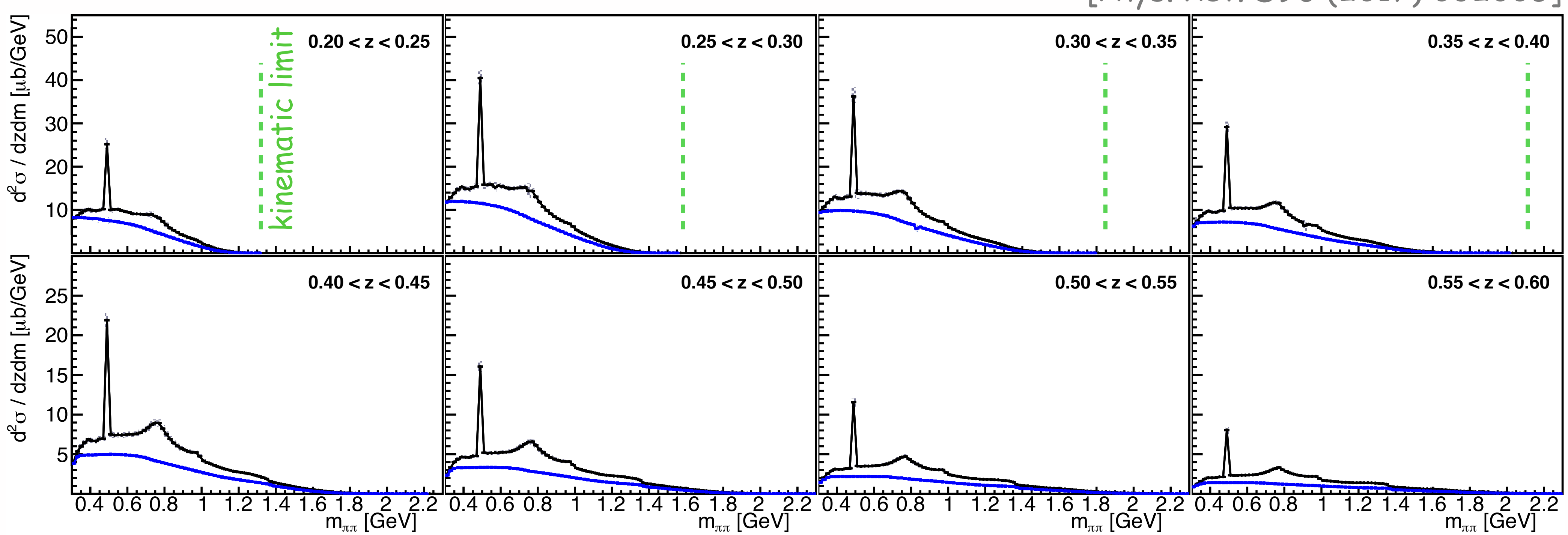
- depending on the sign of $D_{1,ll}$, the partial integration thus leads to a systematic **underestimation** (positive $D_{1,ll}$) or **overestimation** (negative $D_{1,ll}$) **of the "integrated" dihadron cross section**
    - **leads to overestimate/underestimate of extracted transversity**

cross-section reduction

$$\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta D_1^q(z,\cos\theta,M_h) \simeq \int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta \left[D_{1,oo}^q(z,M_h) + D_{1,ll}^q(z,M_h)\frac{1}{4}(3\cos^2\theta-1)\right]$$

$$\propto \cos\theta_0 \left[1-\frac{1}{4}\left(1-\cos^2\theta\right)\frac{D_{1,ll}^q(z,M_h)}{D_{1,oo}^q(z,M_h)}\right] D_{1,oo}^q(z,M_h)$$

[Phys. Rev. D96 (2017) 032005]



- effect very strong [and even present if no p-wave contribution]