

FF 2018

Workshop on Fragmentation Functions

19-22 Febr. 2018, Stresa (Italy)



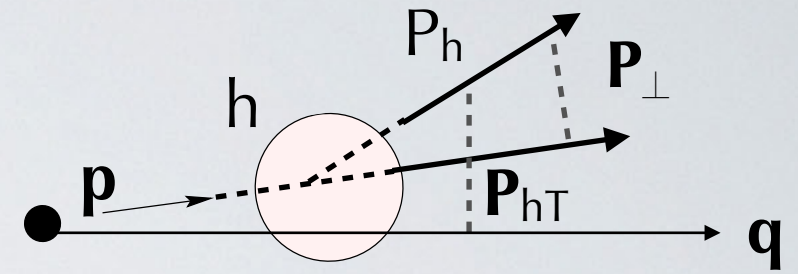
Marco Radici
INFN - Pavia


















Theory open issues

TMD FF map

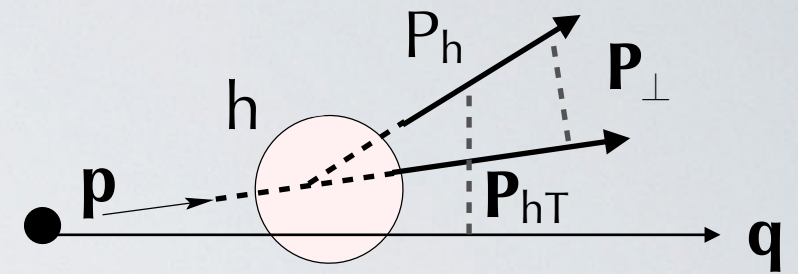
leading twist
 $S_h \leq 1/2$



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarization	U	D_1  <i>Unpolarized</i>		H_1^\perp  -  <i>Collins</i>
	L		G_{1L}  - 	H_{1L}^\perp  - 
Hadron	T	D_{1T}^\perp  - 	G_{1T}  - 	H_1  -  H_{1T}^\perp  - 

TMD FF map

leading twist
 $S_h \leq 1/2$



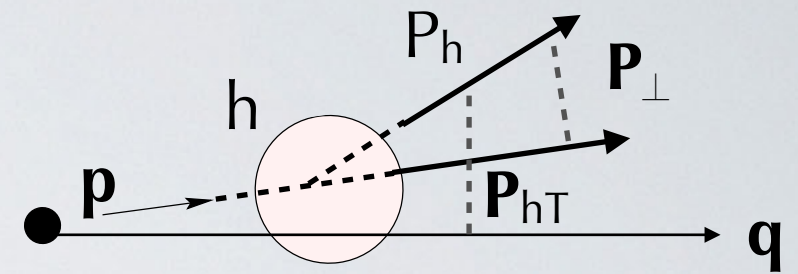
$$S_h = 0$$

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Hadron Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp <i>Collins</i>
	L		G_{1L}	H_{1L}^\perp
Hadron	T	D_{1T}^\perp	G_{1T}	H_1 H_{1T}^\perp

most of the time, detection of final unpolarized mesons (π , K ..) \Rightarrow use only first row of table

TMD FF map

leading twist
 $S_h \leq 1/2$



$S_h = 1/2$

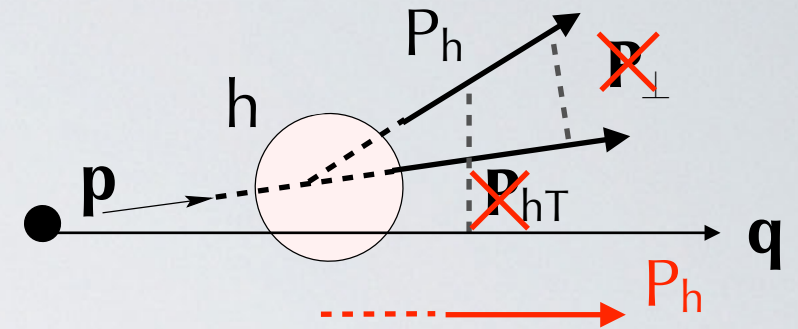
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
Hadron	T	D_{1T}^\perp -	G_{1T} -	H_1 -
				H_{1T}^\perp -

data on Λ^\uparrow production from BELLE / COMPASS (and CERN- NA48/OPAL/ATLAS
 HERA-B
 old FermiLab)











collinear FF map

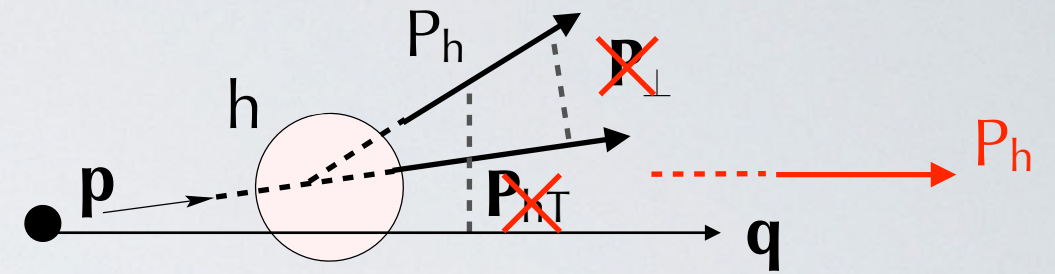
leading twist
 $S_h \leq 1/2$



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
Hadron	T	D_{1T}^\perp -	G_{1T} -	H_1 - H_{1T}^\perp -

collinear FF map

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Hadron	U	D_1  <i>Unpolarized</i>		H_1^\perp  <i>Collins</i>
	L		G_{1L} 	H_{1L}^\perp 
	T	D_{1T}^\perp 	G_{1T} 	H_1  H_{1T}^\perp 











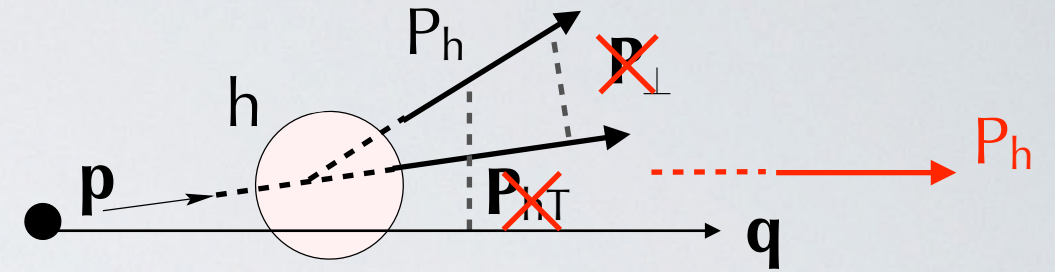
“standard”

NLO

analysis of e^+e^- data for π^\pm, K^\pm $0.1 \lesssim z \lesssim 0.9$ Ex: JAMFF

collinear FF map

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Hadron	U	D_1  <i>Unpolarized</i>		H_1^\perp  <i>Collins</i>
	L		G_{1L} 	H_{1L}^\perp 
	T	D_{1T}^\perp 	G_{1T} 	H_1  H_{1T}^\perp 



“standard”
















NLO

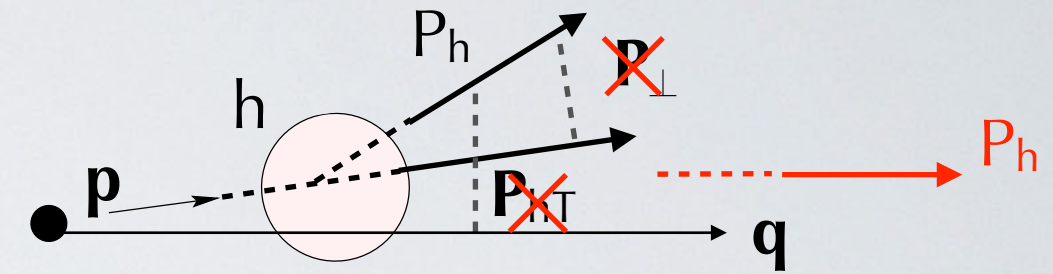
analysis of e^+e^- data for π^\pm, K^\pm $0.1 \lesssim z \lesssim 0.9$ Ex: JAMFF

NNLO

1st: ASR15 (only π^\pm , no error)
then NNFF1.0 (“standard” data + $\overset{(-)}{p}$)

collinear FF map

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Hadron	U	D_1  <i>Unpolarized</i>		H_1^\perp  -  <i>Collins</i>
	L		G_{1L}  - 	H_{1L}^\perp  - 
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“standard”

NLO

analysis of e^+e^- data for π^\pm, K^\pm $0.1 \lesssim z \lesssim 0.9$ Ex: JAMFF
















NNLO

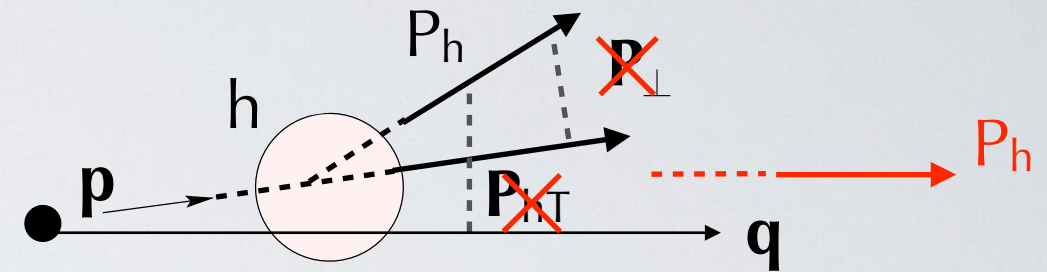
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then NNFF1.0 (“standard” data + $\overset{(-)}{p}$)

NNLO+NNLL

AKSR17 (only π^\pm , no error, small z)

collinear FF map

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Hadron	U	D_1  <i>Unpolarized</i>		H_1^\perp  -  <i>Collins</i>
	L		G_{1L}  - 	H_{1L}^\perp  - 
	T	D_{1T}^\perp  - 	G_{1T}  - 	H_1  -  H_{1T}^\perp  - 



“standard”

NLO

analysis of e^+e^- data for π^\pm, K^\pm $0.1 \lesssim z \lesssim 0.9$ Ex: JAMFF

NNLO

1st: ASR15 (only π^\pm , no error)
then NNFF1.0 (“standard” data + $\overset{(-)}{p}$)

NNLO+NNLL

AKSR17 (only π^\pm , no error, small z)

global fit

DSS 2015 (π^\pm) + DSS 2017 (K^\pm) : e^+e^- , SIDIS, p-p data

collinear π FF at NLO: DSS 2015

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L}	H_{1L}^\perp
	T	D_{1T}^\perp	G_{1T}	H_1 - H_{1T}^\perp

$$D_1(z) \quad \bullet \longrightarrow \bigcirc \quad h = \pi$$

De Florian, Sassot, Epele, Hernandez-Pinto, Stratmann, P.R. D91 (15) 014035

DSS 2007 \rightarrow major update DSS 2015 (only for $q \rightarrow \pi$)

first time
use of LHC data \rightarrow

- more/better data for e^+e^- (BELLE, BaBar)
- SIDIS (Hermes, COMPASS)
- p-p data (STAR), also on π^- / π^+
- LHC (ALICE)
- new error analysis



Iterative Hessian (IH) + $\|N\| \chi^2$ -penalty

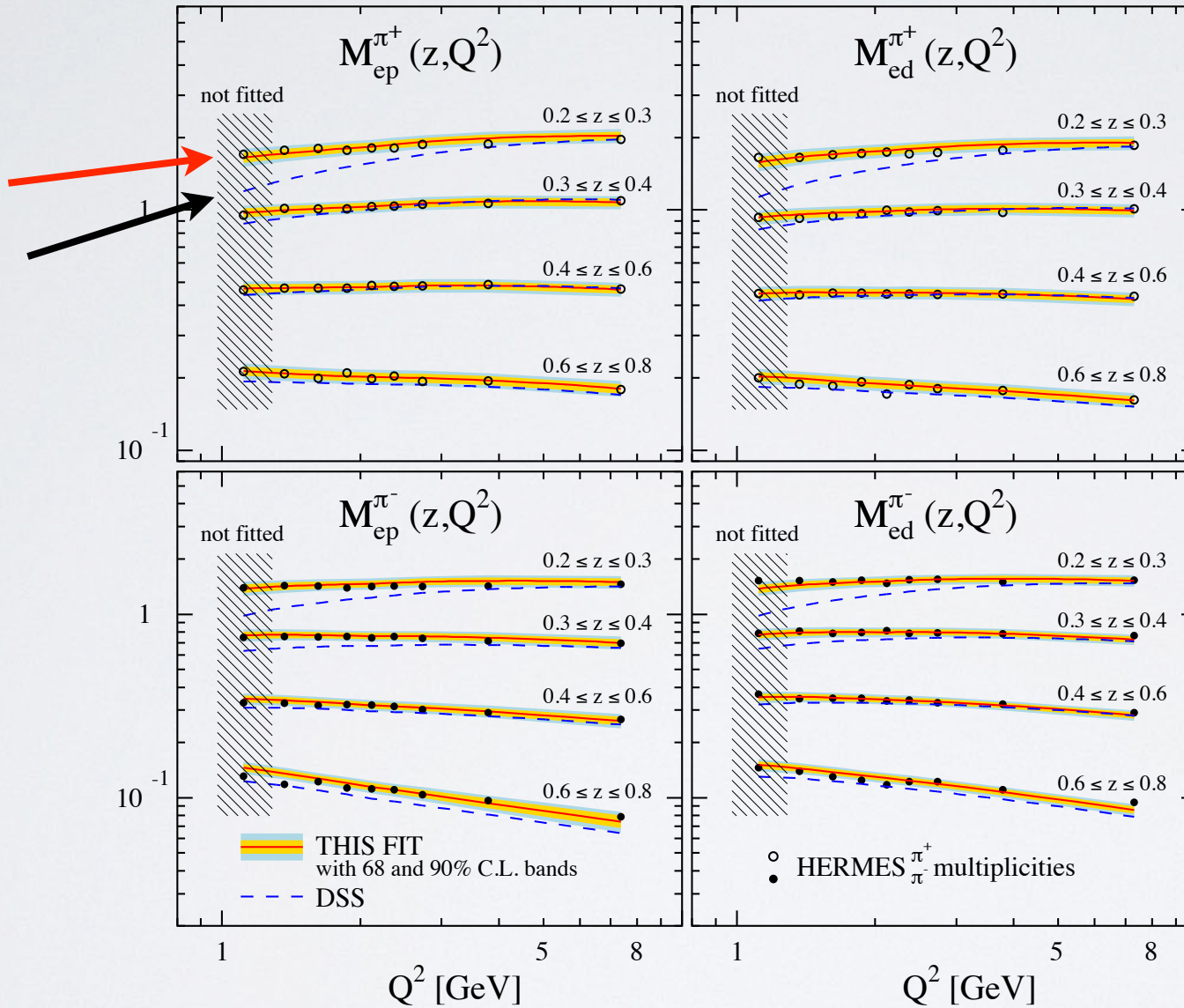
- 973 data points, 28 parameters [$0.05 \leq z$]
- **global $\chi^2/\text{dof} \sim 2.2 \rightarrow 1.2$**

collinear π FF at NLO: DSS 2015



Hermes multiplicities

DSS 2015
DSS 2007



(MSTW +)
DSS 2007 \rightarrow **DSS 2015**

HERMES data χ^2
2.86 \rightarrow **1.37**

\rightarrow talk Kunne

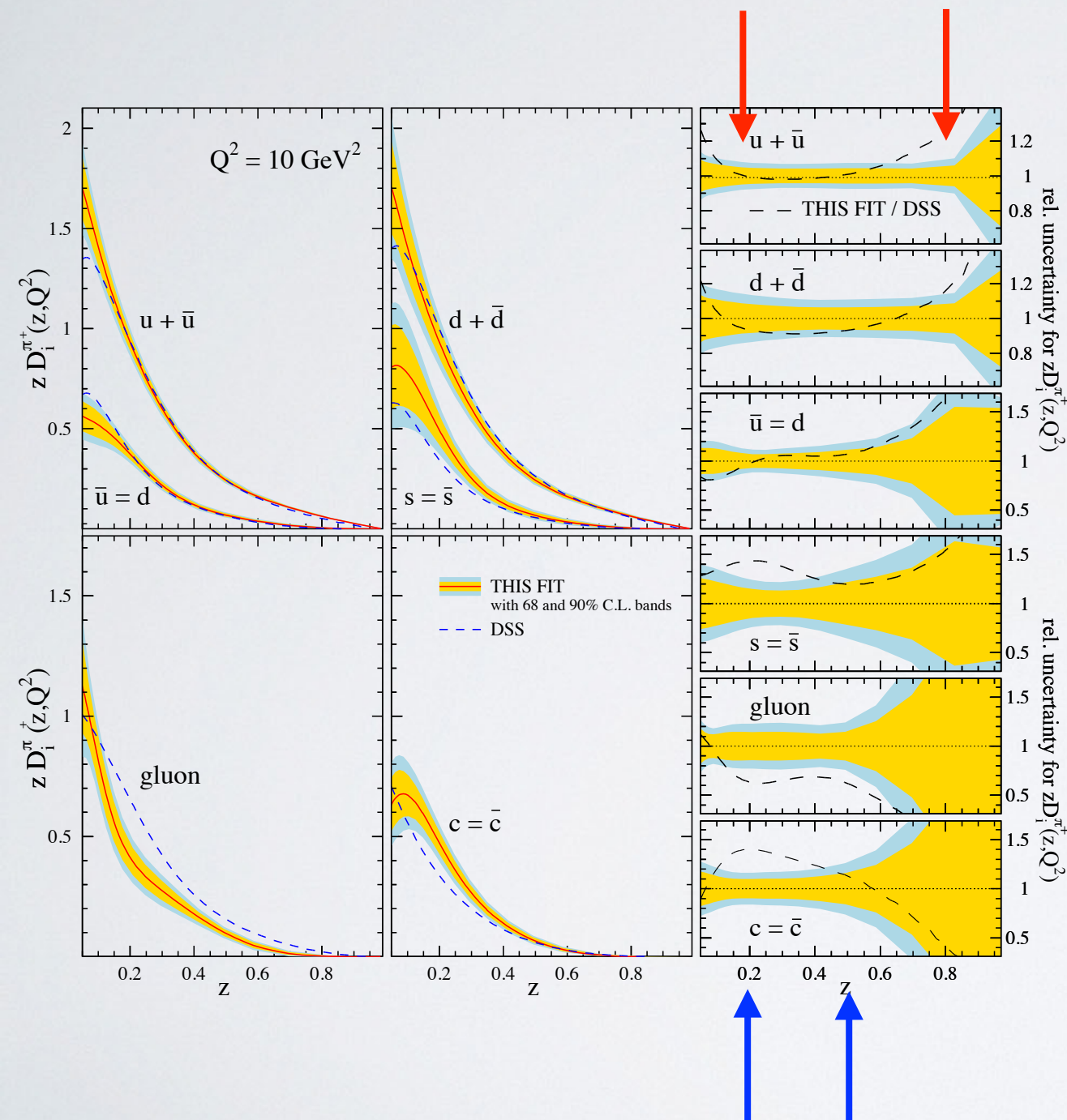


De Florian, Sassot, Epele, Hernandez-Pinto, Stratmann, P.R. D91 (15) 014035

collinear π FF at NLO: DSS 2015

De Florian, Sassot, Epele, Hernandez-Pinto, Stratmann, P.R. D91 (15) 014035

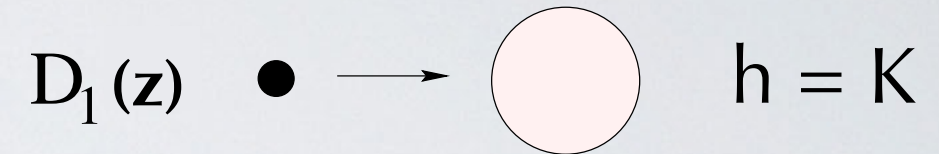
caveat



- major improvement only for total up & down channels: rel. uncertainty $\lesssim 10\%$ for $0.2 < z < 0.8$
- Compass data for SIDIS multiplicities for deuteron target only
- for other channels, improvement upon DSS 2007 only for $0.2 < z < 0.5$

collinear K FF at NLO: DSS 2017

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
	T	D_{1T}^\perp -	G_{1T} -	H_1 - H_{1T}^\perp -



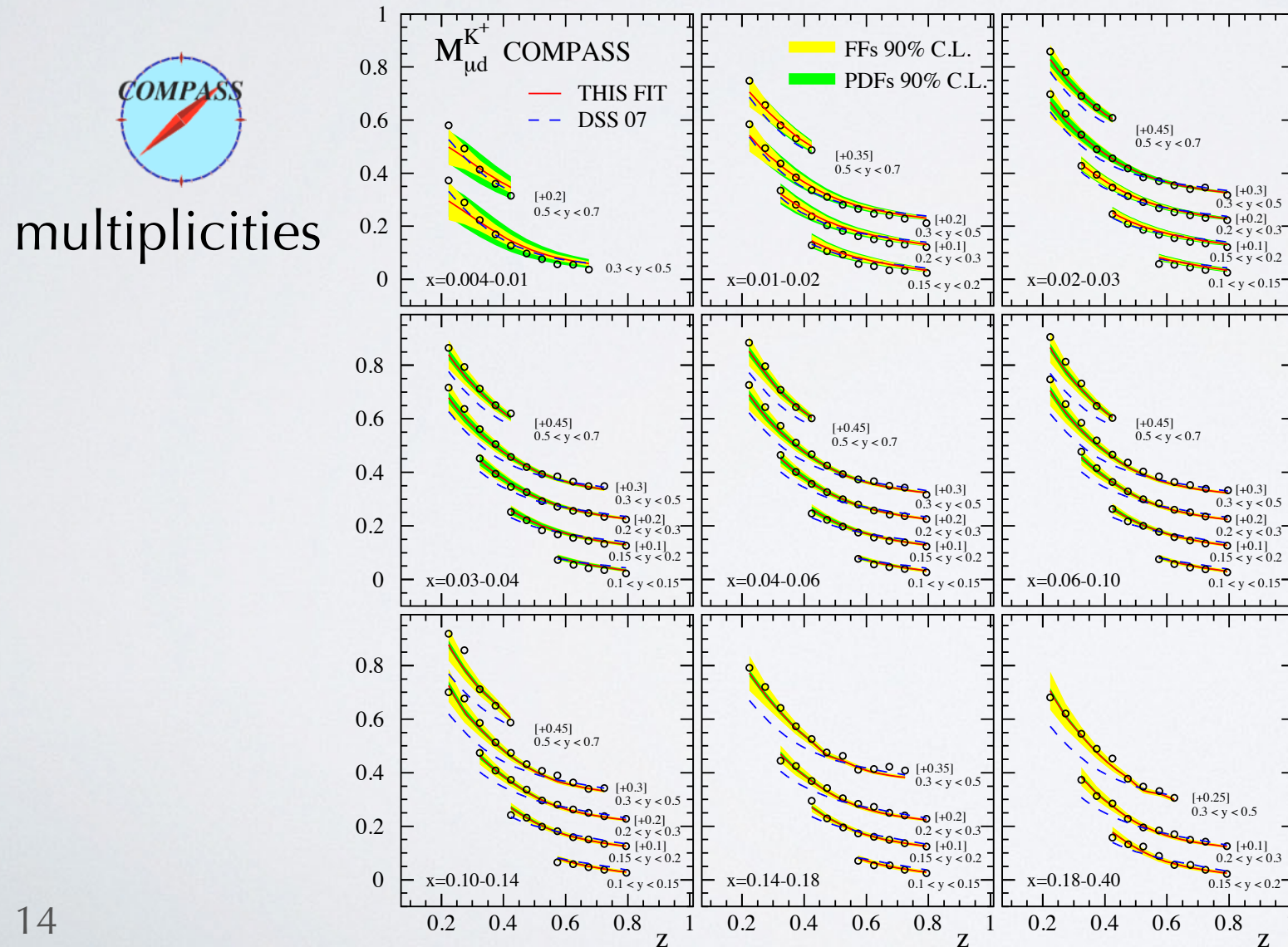
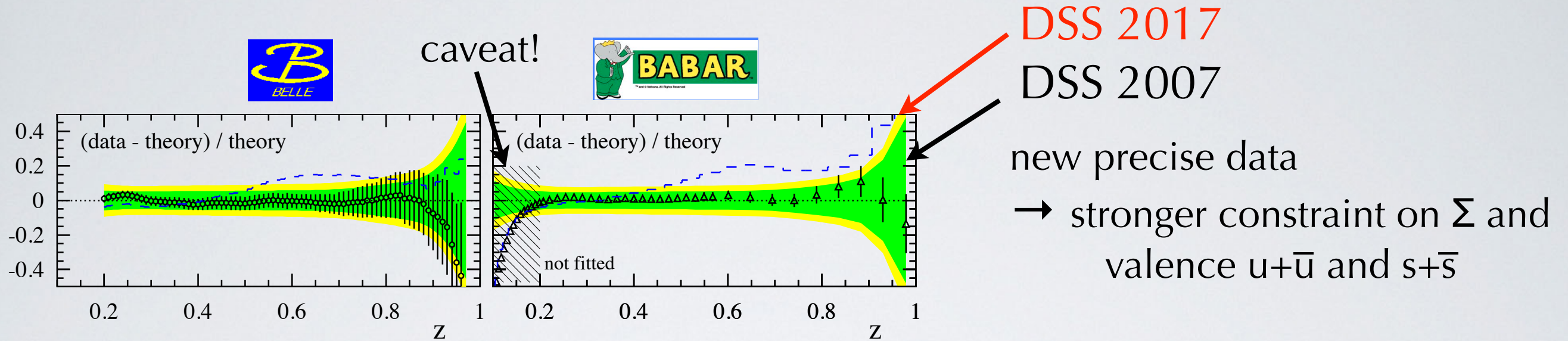
De Florian, Epele, Hernandez-Pinto, Sassot, Stratmann, P.R. D95 (17) 094019

DSS 2007 \rightarrow major update DSS 2017 for $q \rightarrow K$

- talks \rightarrow Schnell
Stolarski
Drachenberg
- newer precise data for e^+e^- (BELLE, BaBar)
 - final (Hermes) and new (COMPASS) SIDIS data
 - new p-p data (STAR), also on K^+ / K^-
 - LHC data (ALICE) on K / π
 - same error analysis as for π FF
 - 1194 data points, 20 parameters [$0.1 \lesssim z$]
 - **global $\chi^2/\text{dof} \sim 1.83 \rightarrow 1.08$**



collinear K FF at NLO: DSS 2017



DSS 2007 → DSS 2017
 (+ MMHT14)

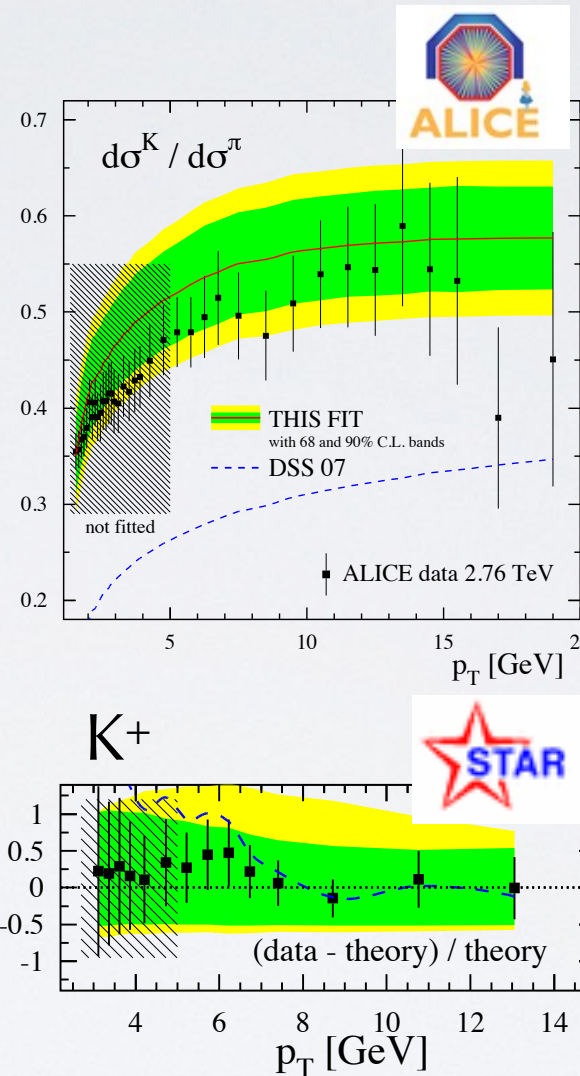
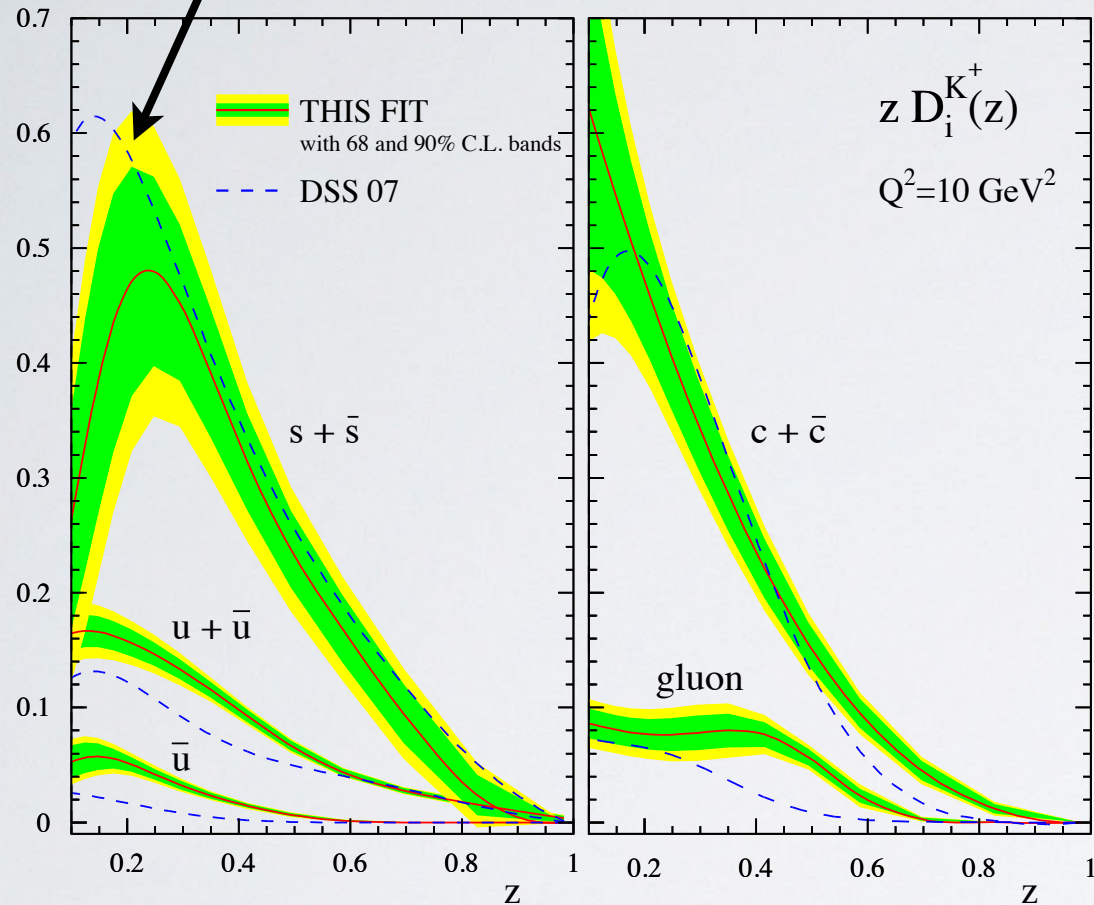
global χ^2
 1.83 → 1.08

(z, Q^2) and (z, x) projections
 → better flavor separation

collinear K FF at NLO: DSS 2017

De Florian, Epele, Hernandez-Pinto, Sassot, Stratmann, P.R. D95 (17) 094019

important for $s(x)$ extraction



caveat

- good global fit including p-p data only if $p_T > 5 \text{ GeV}$

- DSS 2007 within errors
 → is gluon constrained from p-p data?

- best fit with flavor symmetric unfavored channels; SIDIS data not enough sensitive

- problem with $z < 0.2$ in BaBar data

collinear FF at NLO: JAMFF

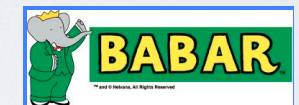
		Quark polarization		
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Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L}	H_{1L}^\perp
	T	D_{1T}^\perp	G_{1T}	H_1 - H_{1T}^\perp

$$D_1(z) \bullet \longrightarrow \bigcirc \quad h = \pi, K$$

Sato, Ethier, Melnitchouk, Hirai, Kumano, Accardi, P.R. D94 (16) 114004

new fit from JAM collaboration: JAMFF (for $q \rightarrow h = \pi, K$)

- only e^+e^- data from SLAC + LEP + KEK + DESY
- 459 data for π , 391 for K [$0.05 \lesssim z \lesssim 0.95$]
- 18 parameters for π , 24 for K
- Iterative Monte Carlo methodology
- **global $\chi^2/\text{dof} \sim 1.31$ (π), 1.01 (K)**

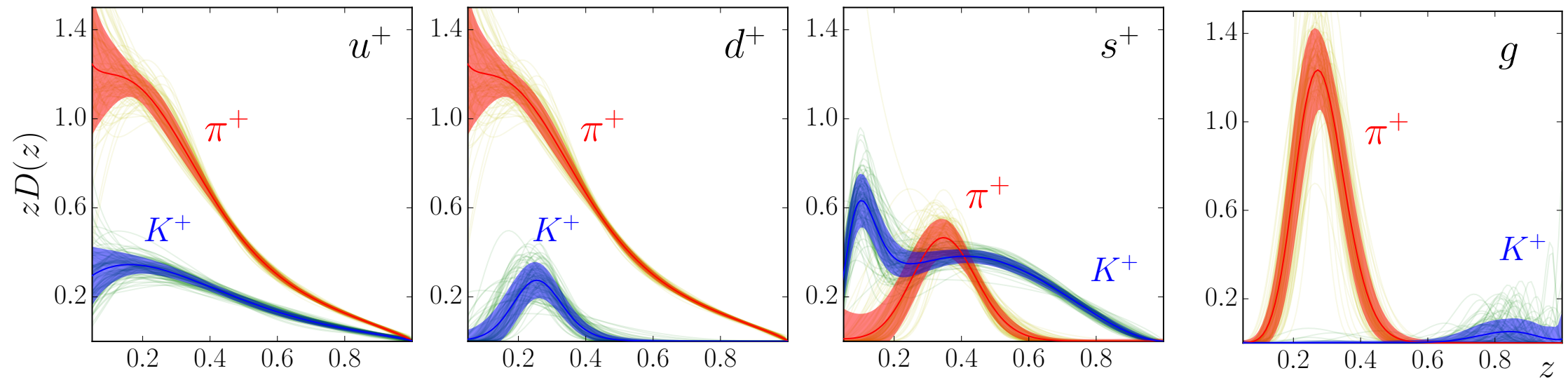


new method

collinear FF at NLO: JAMFF
















$$q^+ = q + \bar{q}$$

$$Q^2 = 1 \text{ GeV}^2$$



Sato, Ethier, Melnitchouk, Hirai, Kumano, Accardi, P.R. D94 (16) 114004

collinear π FF at NNLO: ASR15

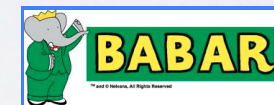
		Quark polarization		
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Nucleon Polarization	U	D_1  <i>Unpolarized</i>		H_1^\perp  -  <i>Collins</i>
	L		G_{1L}  - 	H_{1L}^\perp  - 
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$$D_1(z) \bullet \longrightarrow \bigcirc \quad h = \pi$$

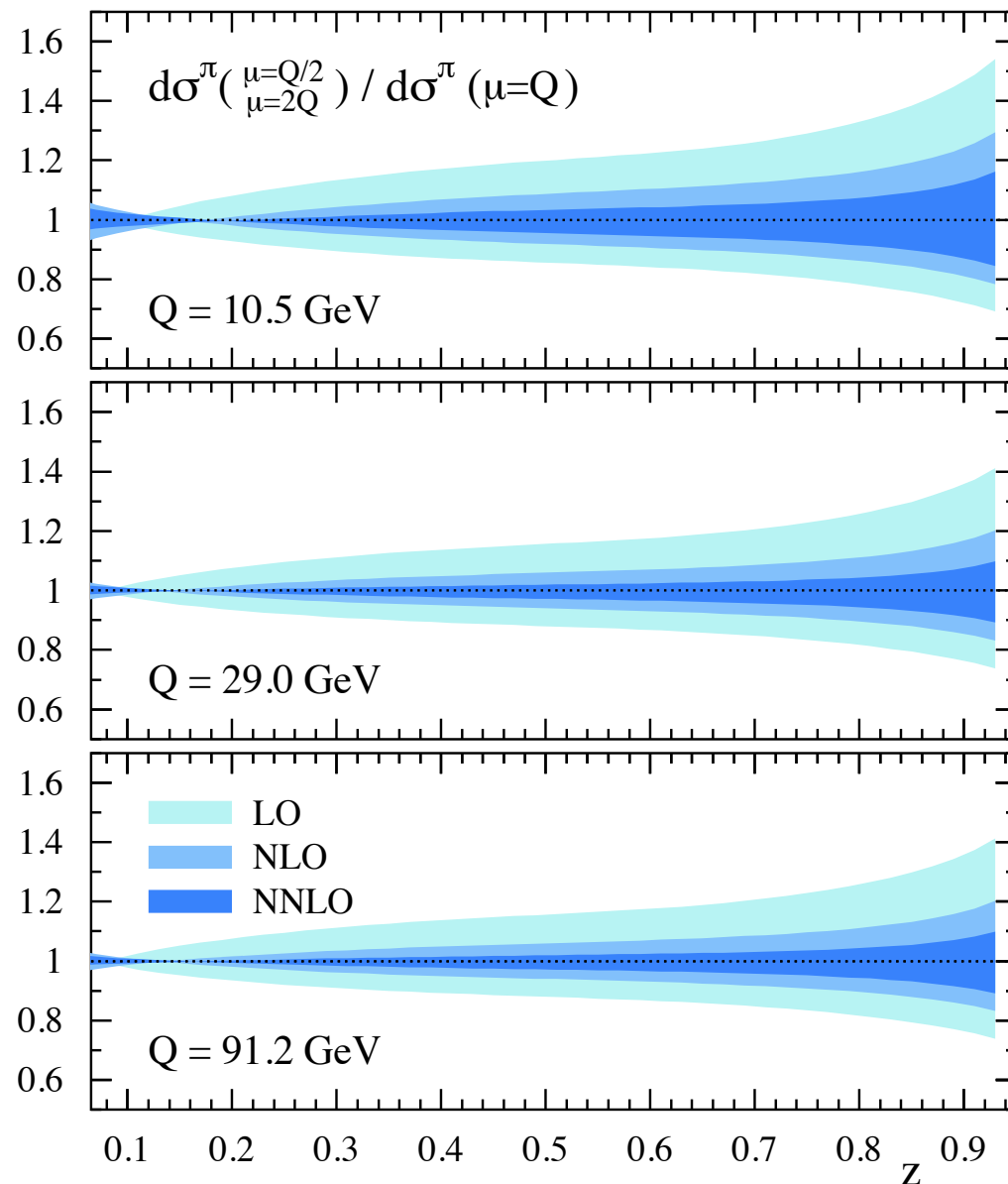
Anderle, Stratmann, Ringer, P.R. D92 (15) 114017

first extraction at NNLO: ASR15 ($q \rightarrow h=\pi$ only)

- only $e^+ e^-$ data from reduced set of SLAC + LEP + KEK + DESY
- 288 data points [$0.075 \lesssim z \lesssim 0.95$]
- 16 parameters
- **global χ^2/dof :**
 - LO=0.89**
 - NLO=0.70**
 - NNLO=0.64**



collinear π FF at NNLO: ASR15



scale uncertainty

$$\frac{\{ d\sigma[Q/2], d\sigma[Q], d\sigma[2Q] \}}{d\sigma[Q]}$$



- benefits of NNLO
- smaller scale uncertainty
 - more flat K factor \rightarrow 1

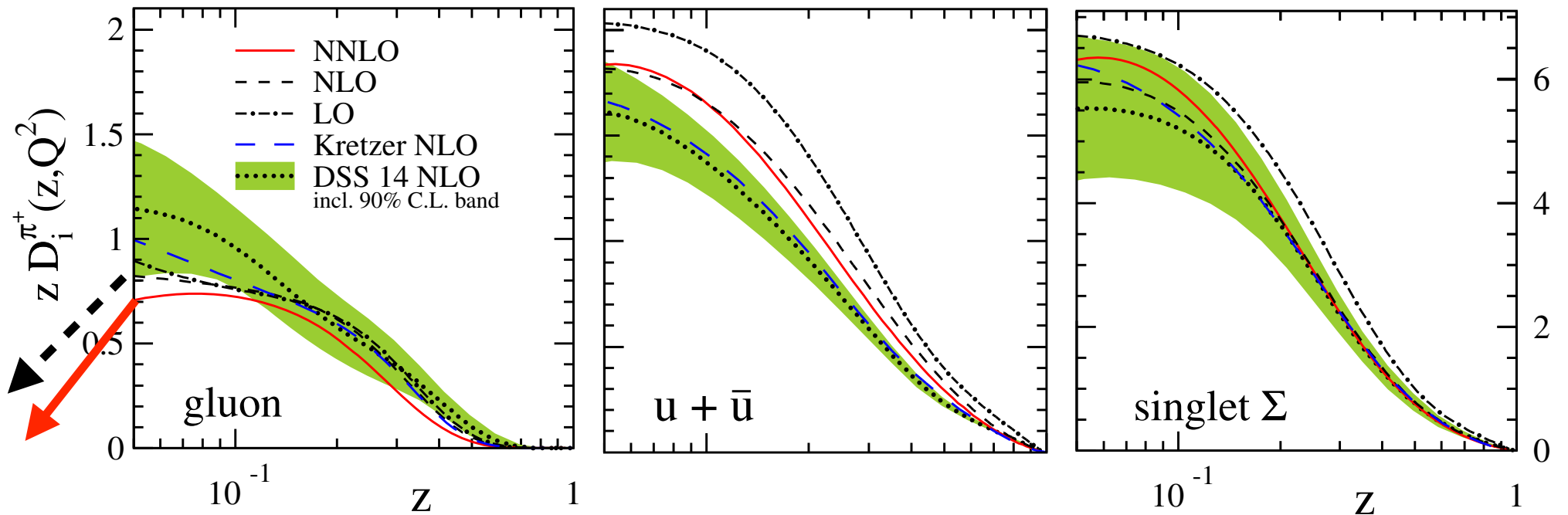
collinear π FF at NNLO: ASR15

Anderle, Ringer, Stratmann,
P.R. D92 (15) 114017

$Q^2 = 10 \text{ GeV}^2$

DSS 2015
NLO

ARS15
NLO
NNLO



caveat

- smaller data set w.r.t. "standard"
- no statistical error analysis
- NLO for some channels is incompatible with previous DSS 2015

collinear FF at NNLO: NNFF1.0

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L}	H_{1L}^\perp
	T	D_{1T}^\perp	G_{1T}	H_1 - H_{1T}^\perp

$$D_1(z) \bullet \longrightarrow \bigcirc \quad h = \pi, K, \bar{p}$$

Bertone, Carrazza, Hartland, Nocera, Rojo, E.P.J. C77 (17) 516

new fit from NNPDF collaboration: NNFF1.0 (for $q \rightarrow h = \pi, K$)

- only e^+e^- data from SLAC + LEP + KEK + DESY
- 1173 data points [$0.02 / 0.075 \approx z \approx 0.9$]
- same NN method as for NNPDFx.x
- 185 parameters for a 4-layers NN with 2-5-3-1 nodes

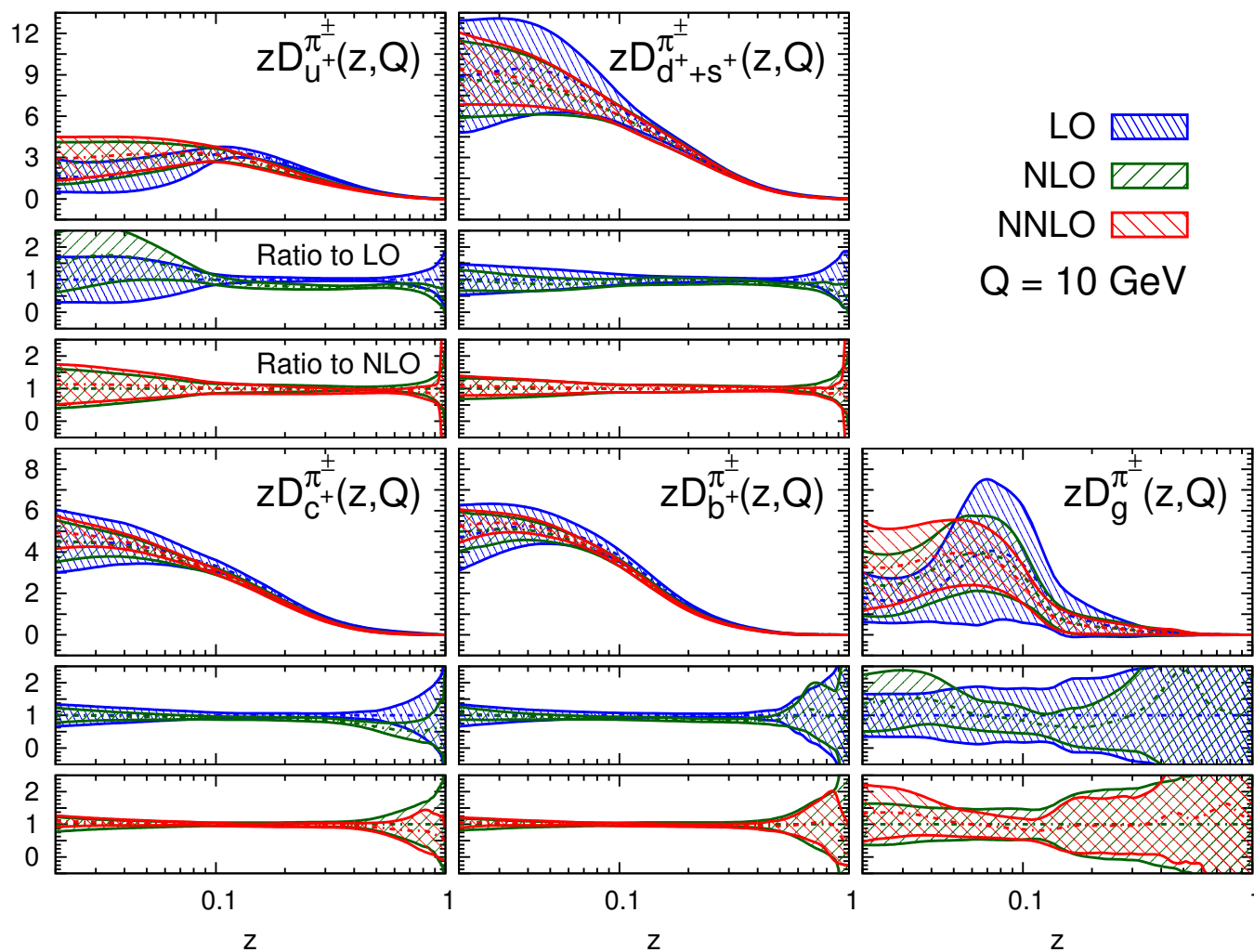


more details \longrightarrow talk Bertone



collinear FF at NNLO: NNFF1.0

tension between
BaBar & BELLE (large z)



Exp.	$\chi^2/N_{\text{dat}} (h = \pi^\pm)$			$\chi^2/N_{\text{dat}} (h = K^\pm)$			$\chi^2/N_{\text{dat}} (h = p/\bar{p})$		
	LO	NLO	NNLO	LO	NLO	NNLO	LO	NLO	NNLO
BELLE	0.60	0.11	0.09	0.21	0.32	0.33	0.10	0.31	0.50
BABAR	1.91	1.77	0.78	2.86	1.11	0.95	4.74	3.75	3.25
TASSO12	0.70	0.85	0.87	1.10	1.03	1.02	0.69	0.70	0.72
TASSO14	1.55	1.67	1.70	2.17	2.13	2.07	1.32	1.25	1.22
TASSO22	1.64	1.91	1.91	2.14	2.77	2.62	0.98	0.92	0.93
TPC (incl.)	0.46	0.65	0.85	0.94	1.09	1.01	1.04	1.10	1.08
TPC (<i>uds</i> tag)	0.78	0.55	0.49	—	—	—	—	—	—
TPC (<i>c</i> tag)	0.55	0.53	0.52	—	—	—	—	—	—
TPC (<i>b</i> tag)	1.44	1.43	1.43	—	—	—	—	—	—
TASSO30	—	—	—	—	—	—	0.25	0.19	0.18
TASSO34	1.16	0.98	1.00	0.27	0.44	0.36	0.82	0.81	0.78
TASSO44	2.01	2.24	2.34	—	—	—	—	—	—
TOPAZ	1.04	0.82	0.80	0.61	1.19	0.99	0.79	1.21	1.19
ALEPH	1.68	0.90	0.78	0.47	0.55	0.56	1.36	1.43	1.28
DELPHI (incl.)	1.44	1.79	1.86	0.28	0.33	0.34	0.48	0.49	0.49
DELPHI (<i>uds</i> tag)	1.30	1.48	1.54	1.38	1.49	1.32	0.47	0.46	0.45
DELPHI (<i>b</i> tag)	1.21	0.99	0.95	0.58	0.49	0.52	0.89	0.89	0.91
OPAL	2.29	1.88	1.84	1.67	1.57	1.66	—	—	—
SLD (incl.)	2.33	1.14	0.83	0.86	0.62	0.57	0.66	0.65	0.64
SLD (<i>uds</i> tag)	0.95	0.65	0.52	1.31	1.02	0.93	0.77	0.76	0.78
SLD (<i>c</i> tag)	3.33	1.33	1.06	0.92	0.47	0.38	1.22	1.22	1.21
SLD (<i>b</i> tag)	0.45	0.38	0.36	0.59	0.67	0.62	1.12	1.29	1.33
Total dataset	1.44	1.02	0.87	1.02	0.78	0.73	1.31	1.23	1.17

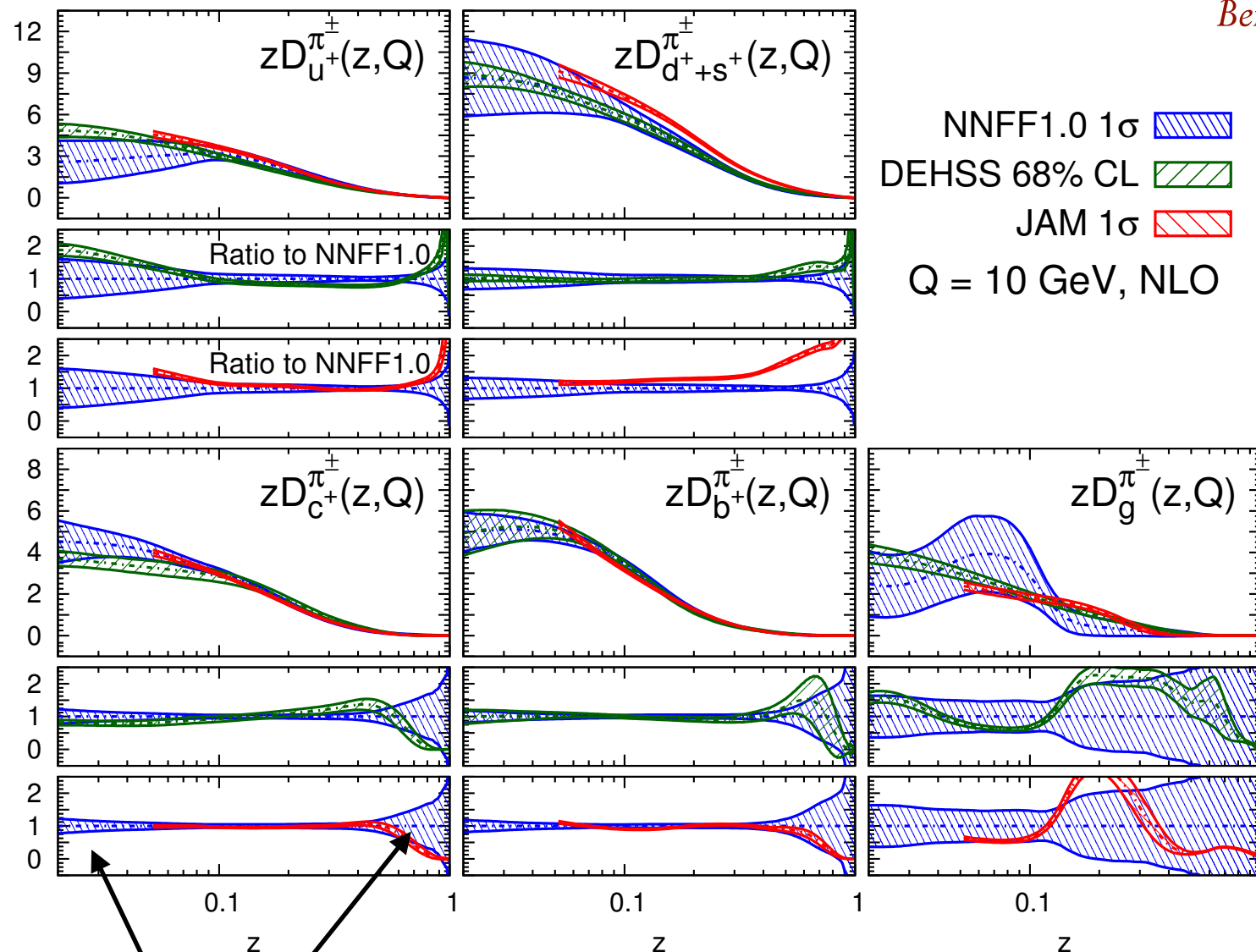
very good overall χ^2

NNPDF

Bertone, Carrazza, Hartland, Nocera, Rojo, E.P.J. C77 (17) 516

comparison at NLO: NNFF1.0 - JAMFF - DSS

Bertone, Carrazza, Hartland, Nocera, Rojo, E.P.J. C77 (17) 516



differences due to different z cuts
(target mass corrections at low z!)

in fact, similar cuts give similar results
(particularly for gluon)
















larger NN uncertainty where less or no data

expect larger uncertainty on D_g (NLO only) than D_q

- visible in NNFF1.0
- not visible in DSS15 (bound from p-p data?)
- not visible in JAMFF (rigid functional form?)

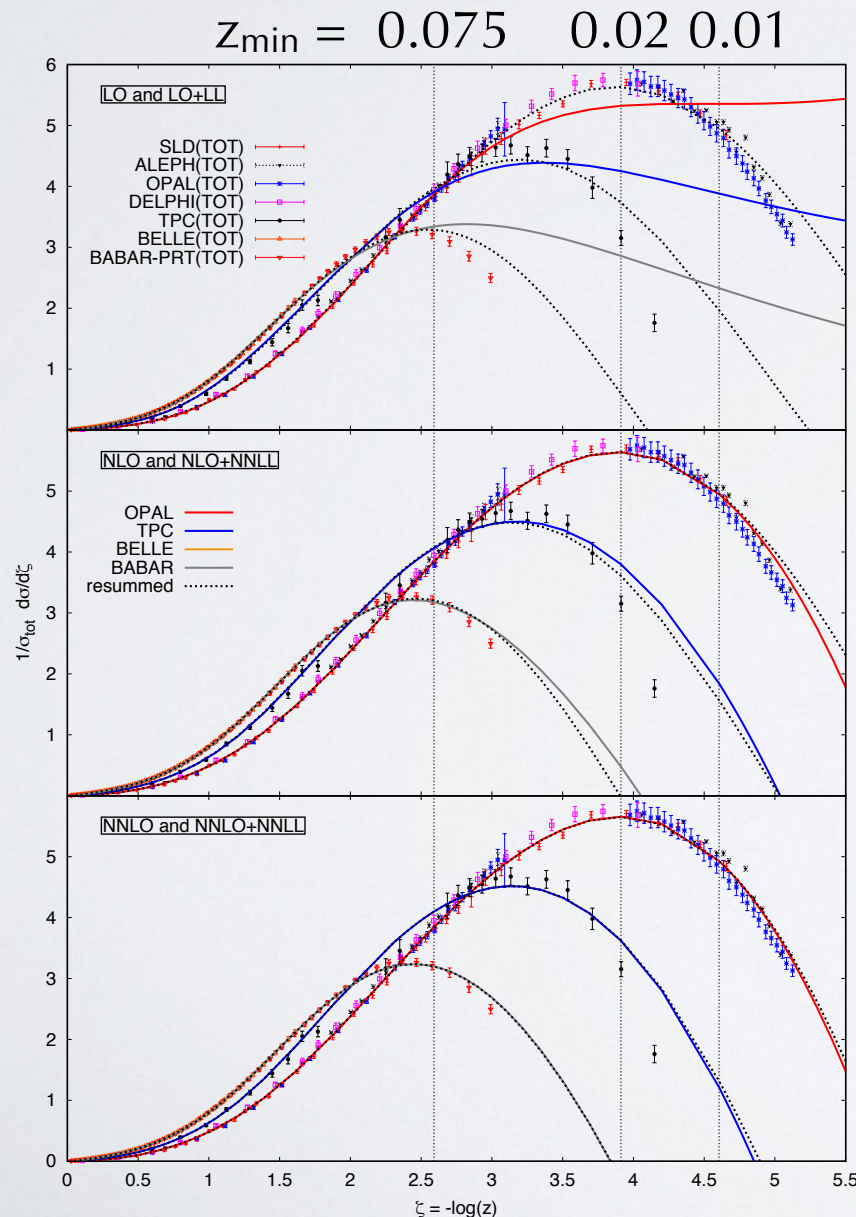
➔ talk Bertone

collinear π FF at NNLO+NNLL: AKSR17

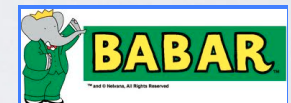
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Nucleon Polarization	U	D_1  <i>Unpolarized</i>		H_1^\perp  -  <i>Collins</i>
	L		G_{1L}  - 	H_{1L}^\perp  - 
	T	D_{1T}^\perp  - 	G_{1T}  - 	H_1  -  H_{1T}^\perp  - 

$$D_1(z) \quad \bullet \longrightarrow \bigcirc \quad h = \pi$$

first extraction with resummation
at NNLO + NNLL : AKSR17
($q \rightarrow h=\pi$ and small z only)



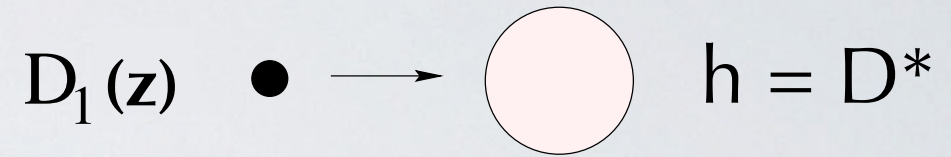
- only e^+e^- data from SLAC + LEP + KEK + DESY
- 436 data points, 19 parameters



accuracy	χ^2	norm shift	χ^2/dof
LO	1260.78	29.02	2.89
NLO	354.10	10.93	0.81
NNLO	330.08	8.87	0.76
LO+LL	405.54	9.83	0.93
NLO+NNLL	352.28	11.27	0.81
NNLO+NNLL	329.96	8.77	0.76

collinear D^* FF at NLO

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
	T	D_{1T}^\perp -	G_{1T} -	H_1^\perp - H_{1T}^\perp -

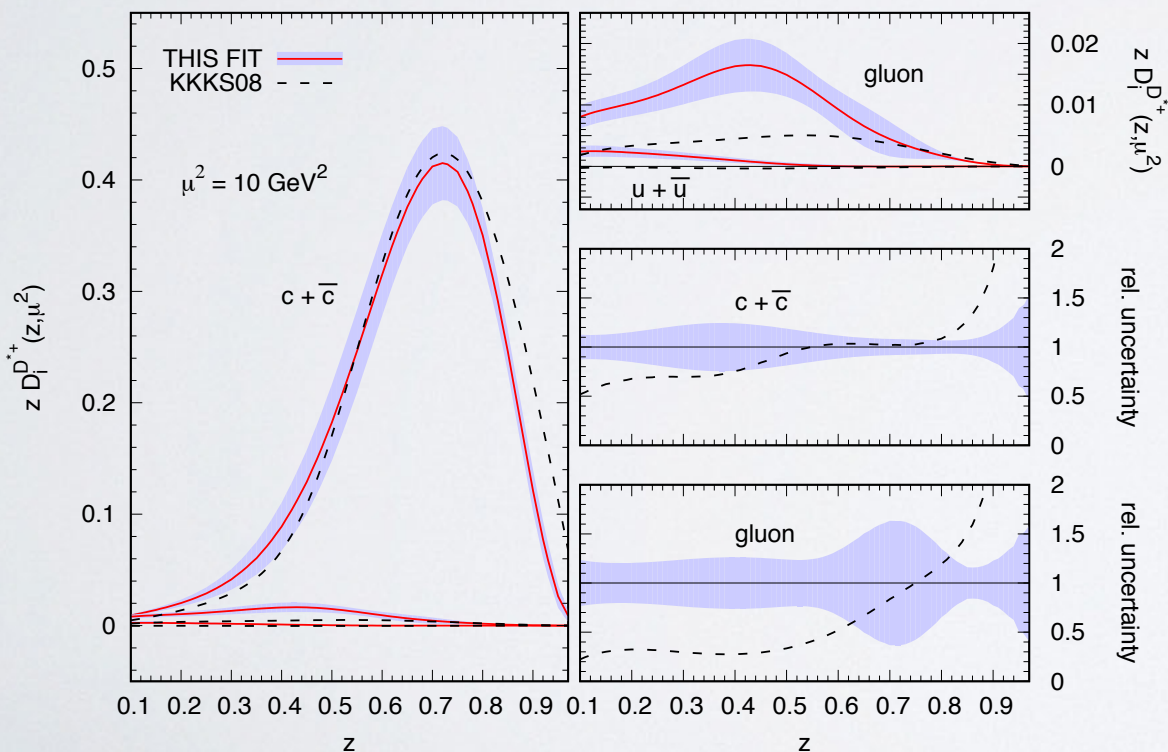


NLO analysis ($q \rightarrow h=D^*$ only)

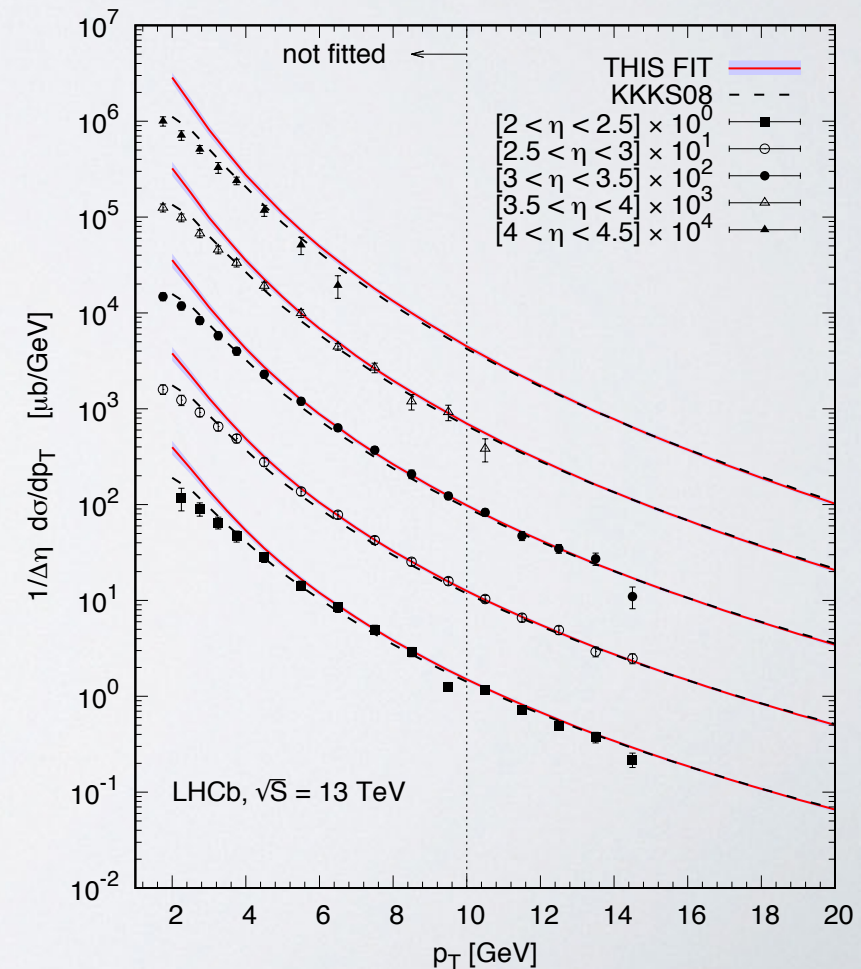
- LEP e^+e^- data
- ATLAS, ALICE, CDF, LHCb $p-p \rightarrow D^* X$
- ATLAS $p-p \rightarrow (\text{jet } D^*) X$
- 96 data points, 9 parameters
- **global $\chi^2/\text{dof} \sim 1.18$**



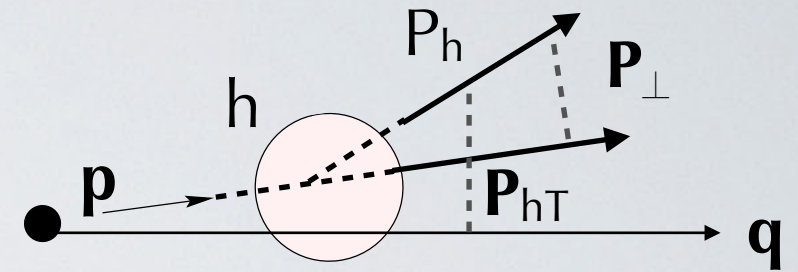
talk Scimemi


















Z dependence
 p_T



unpolarized TMD FF


















$$S_h = 0$$

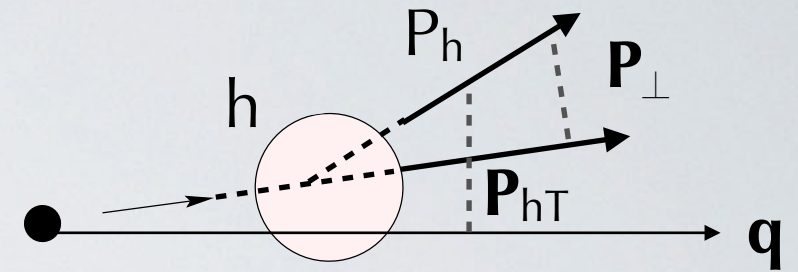
		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Hadron Polarization	U	D_1  <i>Unpolarized</i>		H_1^\perp  -  <i>Collins</i>
	L		G_{1L}  - 	H_{1L}^\perp  - 
Hadron	T	D_{1T}^\perp  - 	G_{1T}  - 	H_1  -  H_{1T}^\perp  - 

What do we know about the \mathbf{P}_{hT} dependence of D_1 ?

unpolarized TMD FF

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1  <i>Unpolarized</i>		H_1^\perp  -  <i>Collins</i>
	L		G_{1L}  - 	H_{1L}^\perp  - 
	T	D_{1T}^\perp  - 	G_{1T}  - 	H_1  -  H_{1T}^\perp  - 

$$D_1(z, \mathbf{P}_{hT})$$

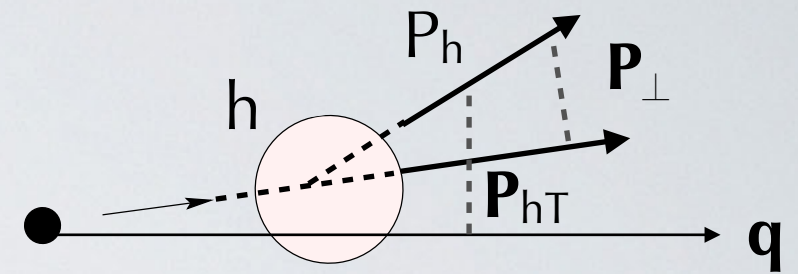


1. Does the \mathbf{P}_{hT} dependence change with **flavor ?**
2. Does the \mathbf{P}_{hT} dependence change with **z ?**
3. Does the \mathbf{P}_{hT} dependence change with **energy \sqrt{s} ?**
4. Does the \mathbf{P}_{hT} dependence change with **scale Q^2 ?**

unpolarized TMD FF

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
	T	D_{1T}^\perp -	G_{1T} -	H_1 - H_{1T}^\perp -

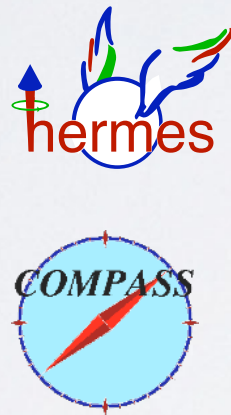
$$D_1(z, \mathbf{P}_{hT})$$



data

fits

SIDIS



multiplicities

PV2013, TO2014
(EIKV14), PV2016

e^+e^-

TASSO (DESY)
MARKII (SLAC)

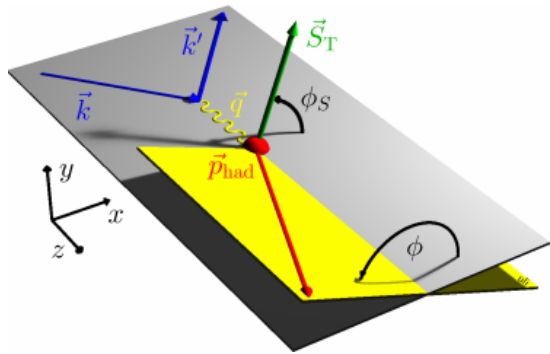
cross section

TO2017

PLUTO (DESY)

$\langle P_{hT}^2 \rangle$

D₁ from unintegrated SIDIS multiplicities

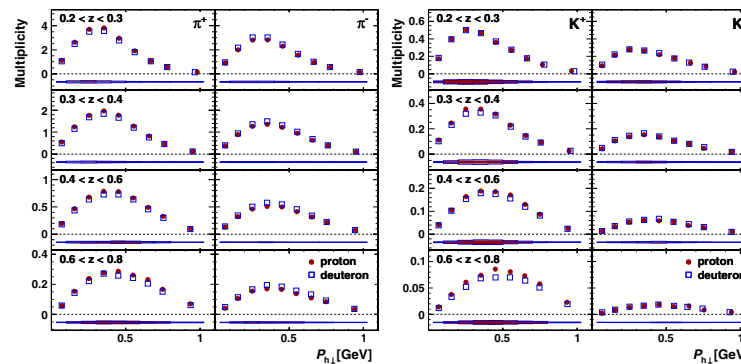


$$M_N^h = \frac{d\sigma_N^h/dx dz dP_{hT}^2 dQ^2}{d\sigma_{\text{DIS}}/dx dQ^2} \approx \frac{\sum_q e_q^2 [f_1^q \otimes D_1^q](x, z, P_{hT}^2; Q^2)}{\sum_q e_q^2 f_1^q(x; Q^2)}$$

$$P_{hT}^2/z \ll Q^2$$



- target: proton, deuteron
- final state: π^+ , π^- , K^+ , K^-
- 2688 points

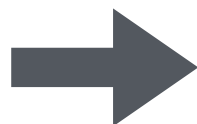
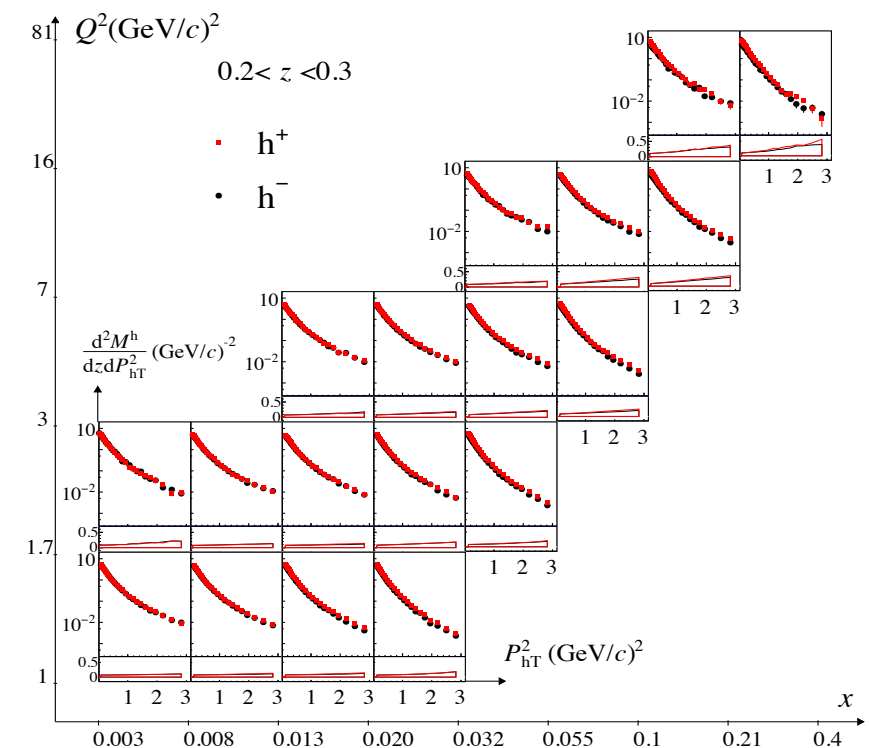


*Airapetian et al.,
P.R. D87 (13) 074029*



- about 5000 data points
- target: deuteron
- final state:
 - h^+ , h^- (run 2004)
 - π^+ , π^- , K^+ , K^- (run 2006)

*Aghasyan et al.,
P.R. D97 (18) 032006*



talk Bressan

D₁ from unintegrated SIDIS multiplicities

available fits

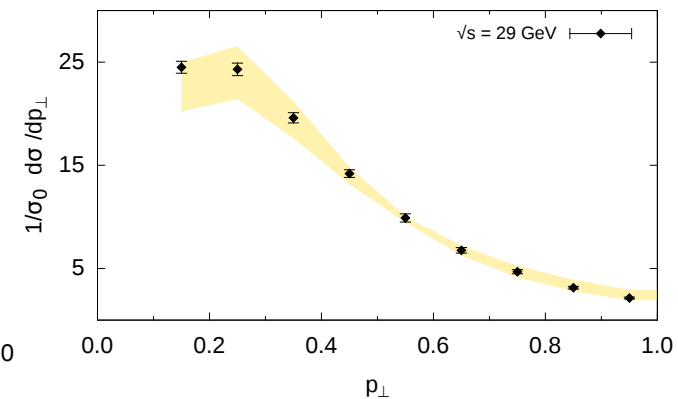
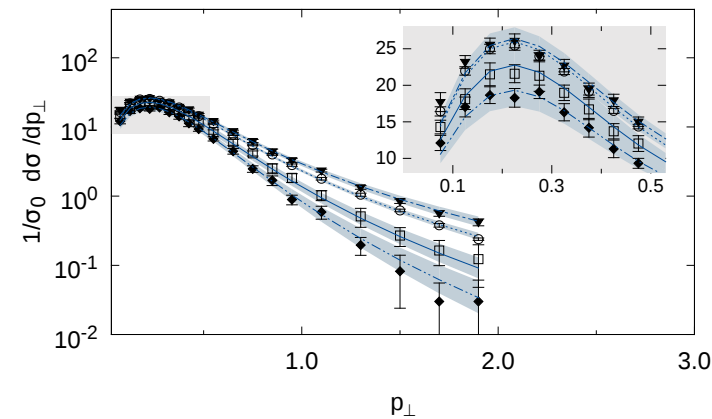
	Framework	Hermes	Compass	# points
Pavia 2013 <i>Bacchetta et al.,</i> <i>JHEP 1311 (13) 194</i>	Gaussian $\langle \mathbf{p}_T^2 \rangle_q(z)$ 7 parameters no evolution	✓	✗	1538
Torino 2014 <i>Anselmino et al.,</i> <i>JHEP 1404 (14) 005</i>	Gaussian $\langle \mathbf{p}_T^2 \rangle$ (1 parameter) only collinear DGLAP evolution $N_y = A + B y$ ($y = Q^2/xs$) (C)	✓ separately	✓ separately	576 (H) 6284 (C)
↓ Framework of TMD evolution ↓				
EIKV 2014 <i>Echevarria et al.,</i> <i>P.R.D89 (14) 074013</i>	TMD framework, NLL level not a real fit	1 bin	(x, Q^2)	(?)
Pavia 2016 Bacchetta et al., <i>JHEP 1706 (17) 081</i>	TMD framework, NLL level first global fit (includes DY and Z⁰)	✓	✓	8156

➔ talk Signori

D₁ from old e⁺e⁻ data

available fits	Framework	# points	TASSO	MARKII PLUTO
	↓ Framework of TMD evolution ↓			
Torino 2017 <i>Boglione et al., P.L. B772 (17) 78</i>	LO + LL analysis 1) Gaussian with $\langle p_T^2 \rangle = a + b z^2 \log(Q/Q_0)$ 2) Power law with $\alpha = \alpha_0 + \alpha_1 \log(Q/Q_0)$ peak & normalization fitted	76	✓	✗

- fit of TASSO, cross-check on MARKII (+ PLUTO)
- hints of p_T-broadening with Q
- power law good, Gaussian bad



- z integrated, $\langle z \rangle \approx 0.1$ p_T/ $\langle z \rangle \sim Q$ [14-44 GeV] TMD factor. broken?
- need to extend to p_T ≈ 3 GeV to reproduce $\langle p_T^2 \rangle$
- equivalent fit with H_p ≠ TMD factorization

caveat

What do we know about $D_1(z, P_{hT})$?

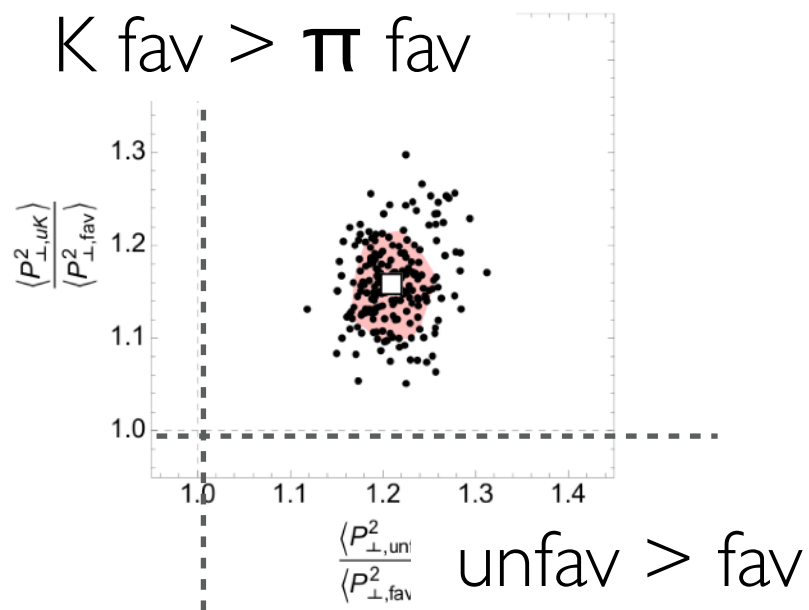
I. does P_{hT} dependence change with flavor ?

What do we know about $D_1(z, P_{hT})$?

I. does P_{hT} dependence change with flavor ?

Pavia 2013

(Hermes)



$\chi^2/\text{dof} = 1.63$

Torino 2014

(Hermes)

flavor indep. $\rightarrow \chi^2/\text{dof} = 1.69$

unfav > fav $\rightarrow \chi^2/\text{dof} = 1.60$

Pavia 2016

(global)

flavor indep.

global $\chi^2/\text{dof} \sim 1.55$

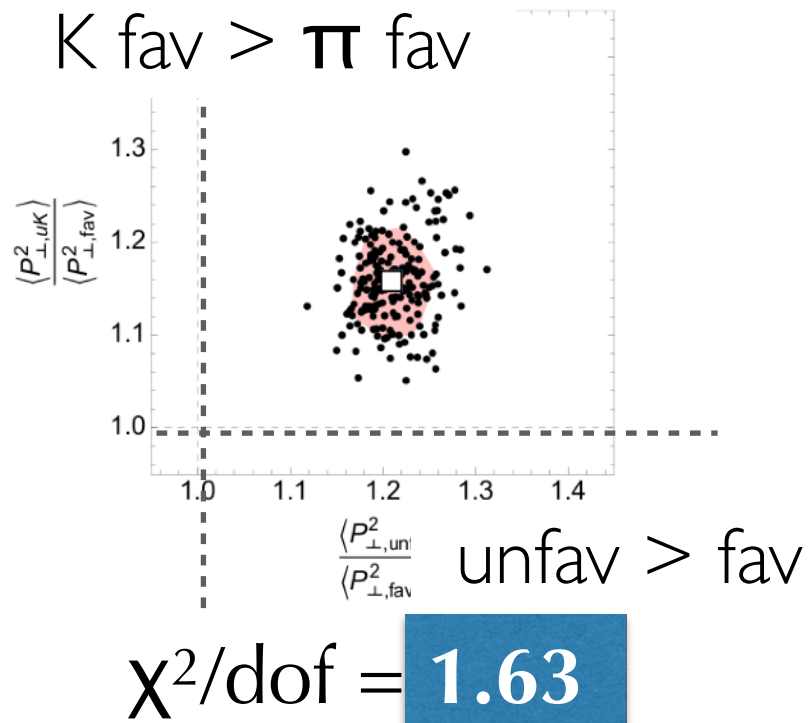
(flavor dep. in progress)

What do we know about $D_1(z, P_{hT})$?

I. does P_{hT} dependence change with flavor ?

Pavia 2013

(Hermes)



Torino 2014

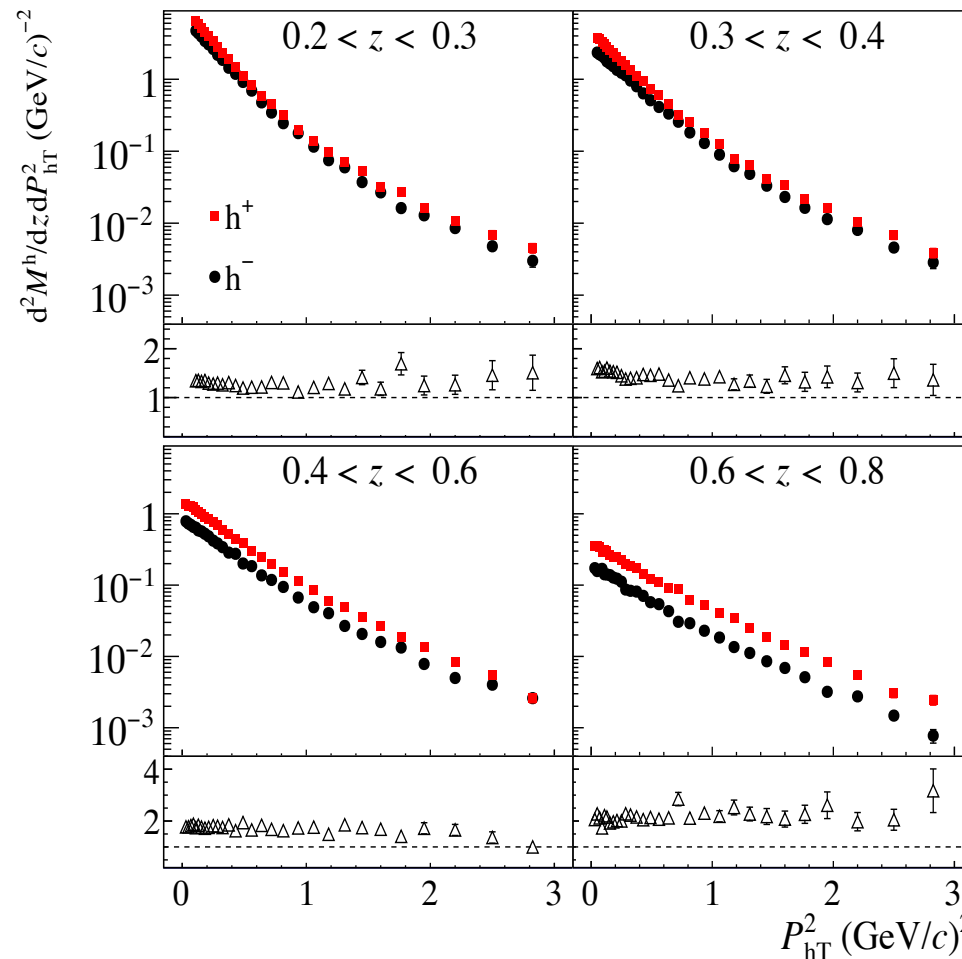
(Hermes)

flavor indep. $\rightarrow \chi^2/\text{dof} = 1.69$
 unfav > fav $\rightarrow \chi^2/\text{dof} = 1.60$

Pavia 2016

(global)

flavor indep.
 global $\chi^2/\text{dof} \sim 1.55$
 (flavor dep. in progress)



Answer :
 visible at large z



Aghasyan et al.,
 P.R. D97 (18) 032006

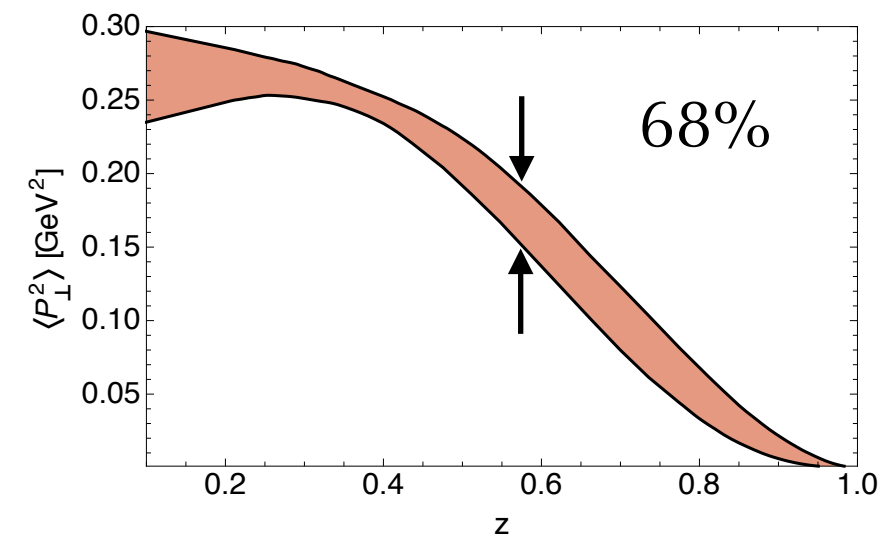
What do we know about $D_1(z, P_{hT})$?

2. does P_{hT} dependence change with z ?

What do we know about $D_1(z, P_{hT})$?

2. does P_{hT} dependence change with z ?

Bacchetta et al., JHEP 1706 (17) 081



Pavia 2016

$$\langle P_{hT}^2 \rangle(z) = \langle \widehat{P}_{hT}^2 \rangle \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

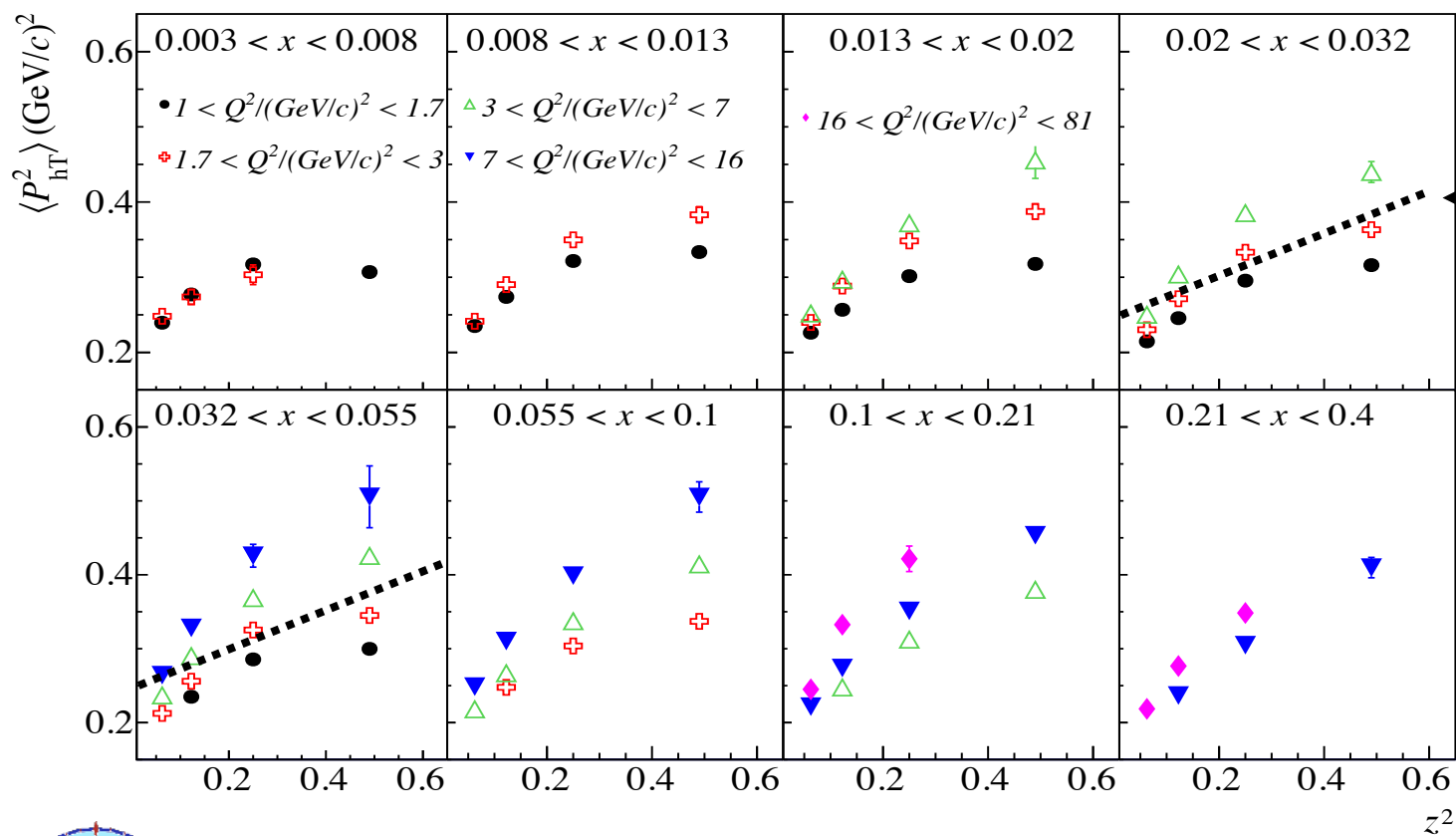
$$\beta = 2.68 \pm 0.08$$

$$\delta = 3.36 \pm 0.12 \quad \hat{z} = 0.5$$

$$\gamma = 0.04 \pm 0.004$$

What do we know about $D_1(z, P_{hT})$?

2. does P_{hT} dependence change with z ?

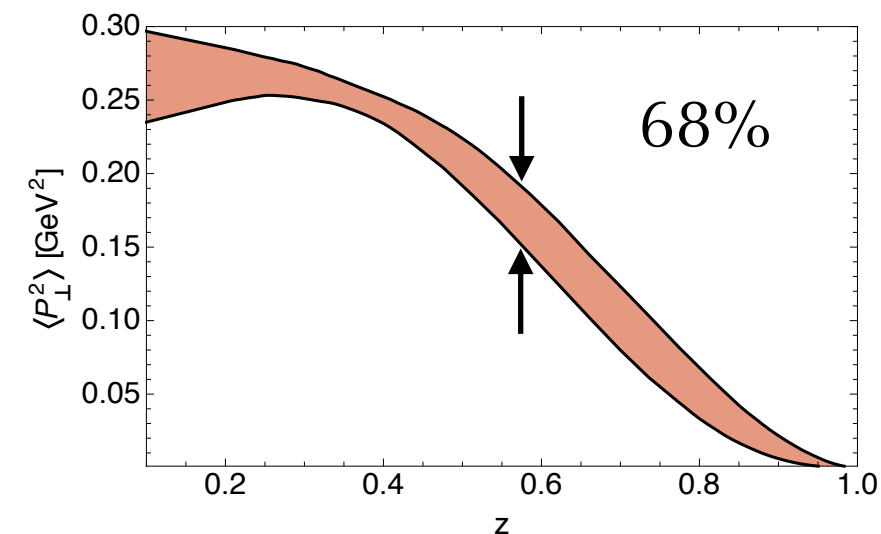


detected hadron fragm. parton initial parton

$$\langle P_{hT}^2 \rangle = \langle P_{\perp}^2 \rangle + z^2 \langle k_{\perp}^2 \rangle$$

Gaussian 0.25 0.28

Bacchetta et al., JHEP 1706 (17) 081



Aghasyan et al., P.R. D97 (18) 032006

Answer:
nonlinear z^2 dependence
at large z (small x, Q^2)

Pavia 2016

$$\langle P_{hT}^2 \rangle(z) = \langle \widehat{P}_{hT}^2 \rangle \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

$$\beta = 2.68 \pm 0.08$$

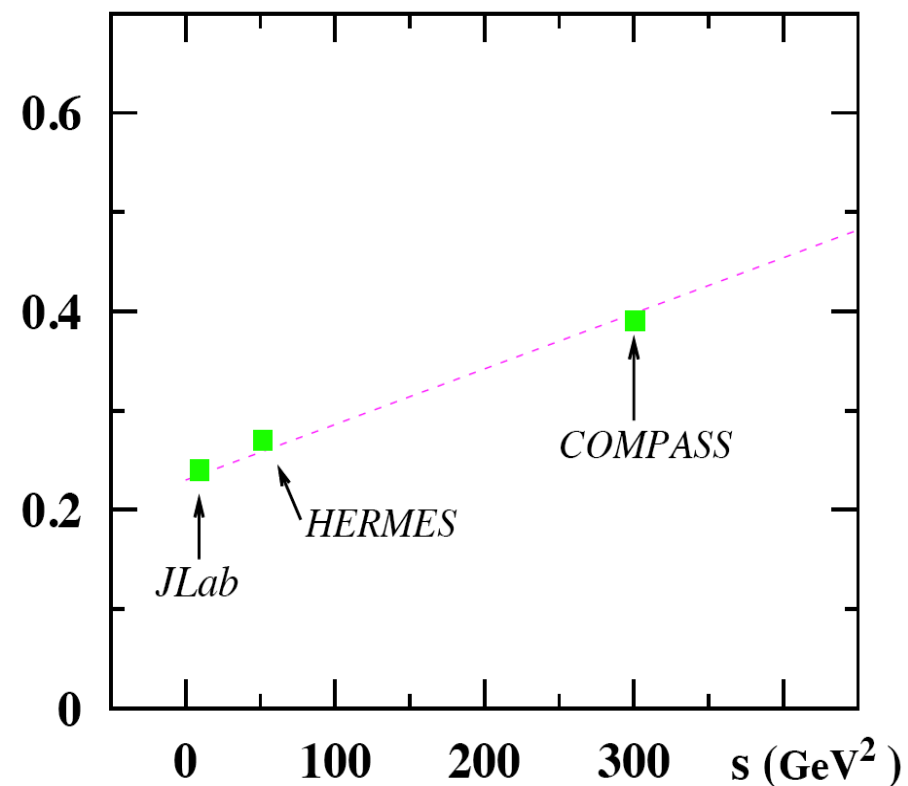
$$\delta = 3.36 \pm 0.12 \quad \hat{z} = 0.5$$

$$\gamma = 0.04 \pm 0.004$$

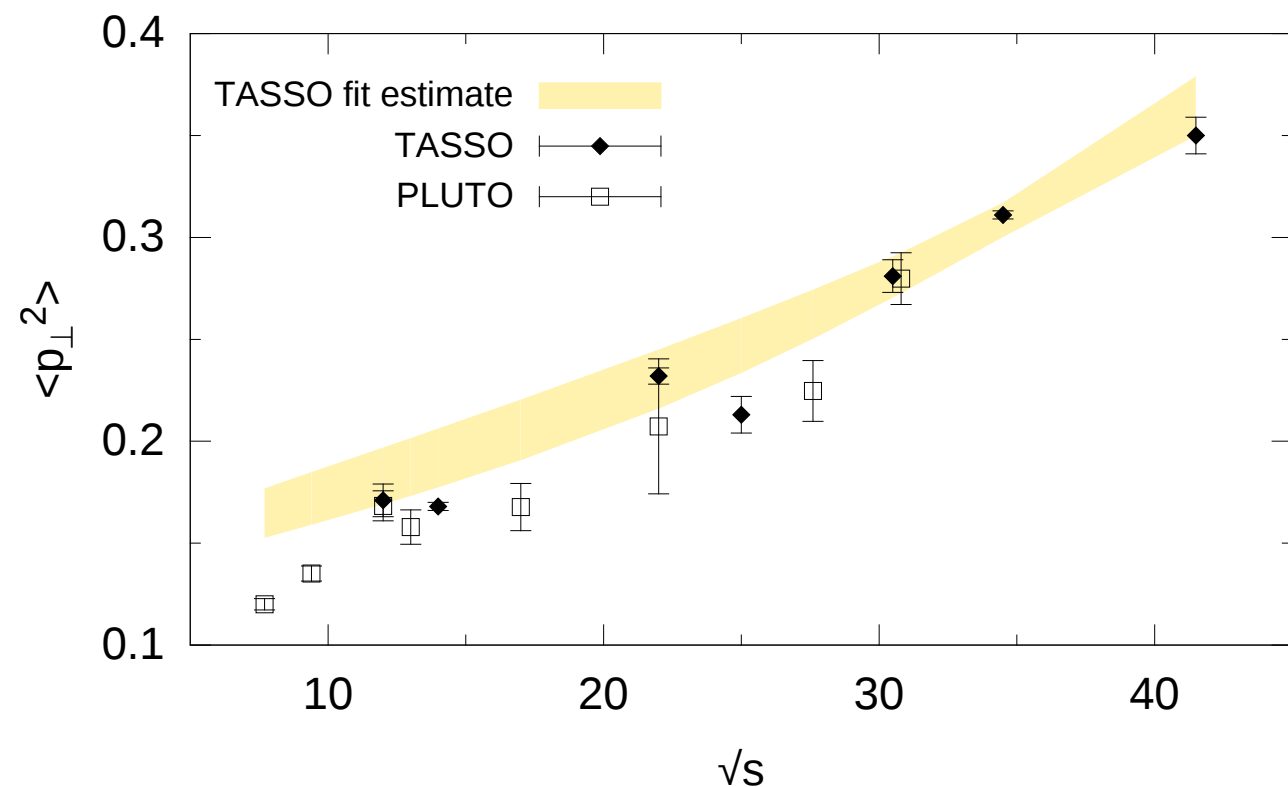
What do we know about $D_1(z, P_{hT})$?

3. does P_{hT} dependence change with energy \sqrt{s} ?

$\langle p_T^2 \rangle$ of Gaussian



Schweitzer, Teckentrup, Metz, P.R. D81 (10) 094019



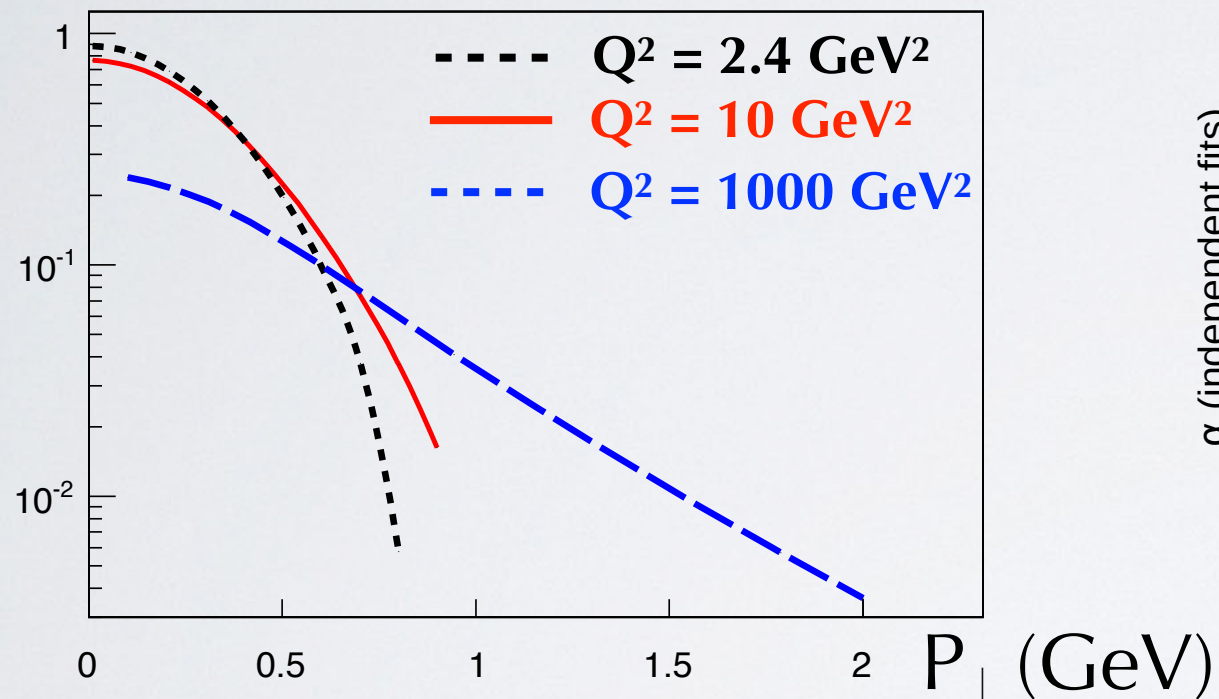
Boglione, Gonzalez, Taghavi, P.L. B772 (17) 78

Answer: it is likely,
but need better e^+e^- data

What do we know about $D_1(z, P_{hT})$?

4. does P_{hT} dependence change with scale Q^2 ?

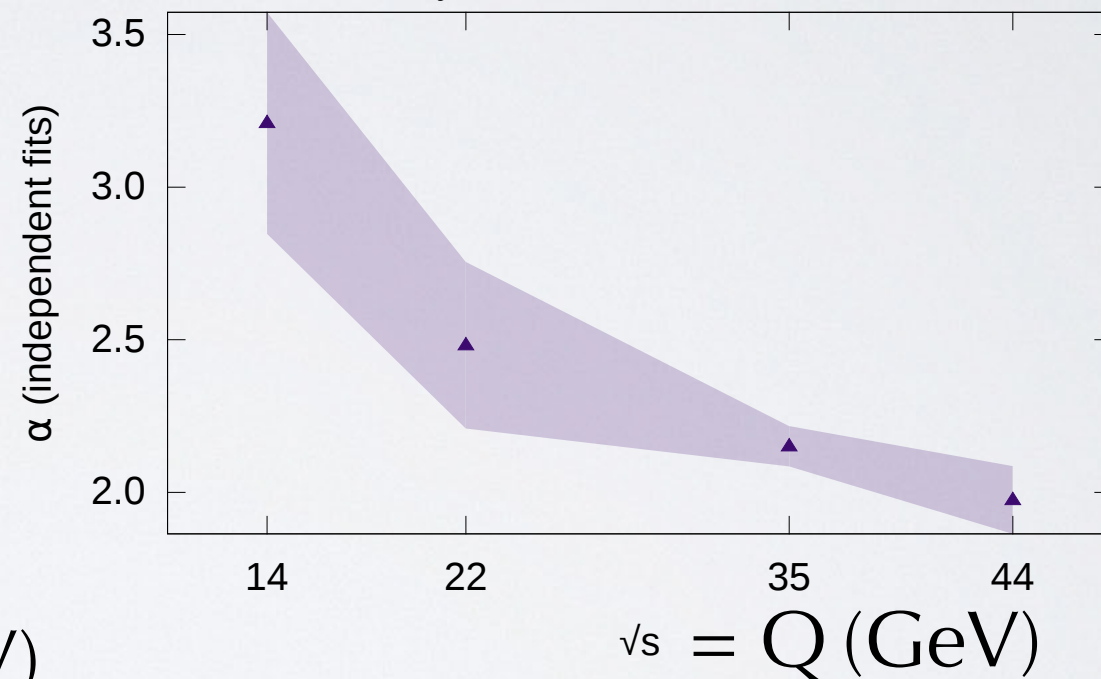
$D_1^{u \rightarrow \pi^+}$



Kang, Prokudin, Sun, Yuan, P.R. D93 (16) 014009

strong dependence
predicted

$$\alpha = \alpha_0 + \alpha_1 \log \frac{Q}{Q_0} \quad \text{power law} \sim \frac{1}{(p_T^2 + p_{T0}^2)^\alpha}$$

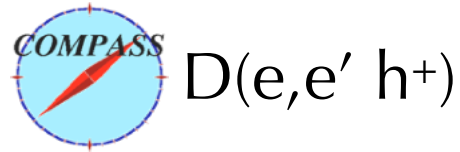


Boglione, Gonzalez, Taghavi, P.L. B772 (17) 78

log-like behavior
observed (so far..)

What do we know about $D_1(z, P_{hT})$?

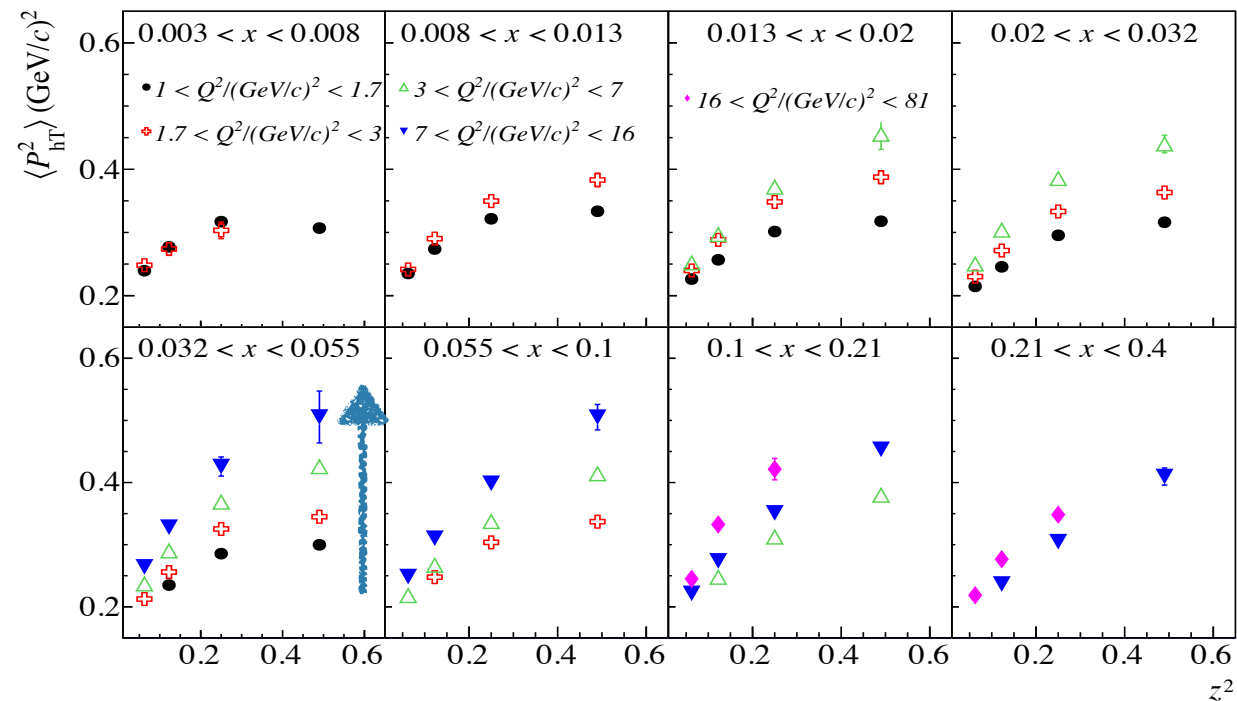
4. does P_{hT} dependence change with scale Q^2 ?



Bacchetta et al.,
JHEP 1706 (17) 081

Pavia 2016

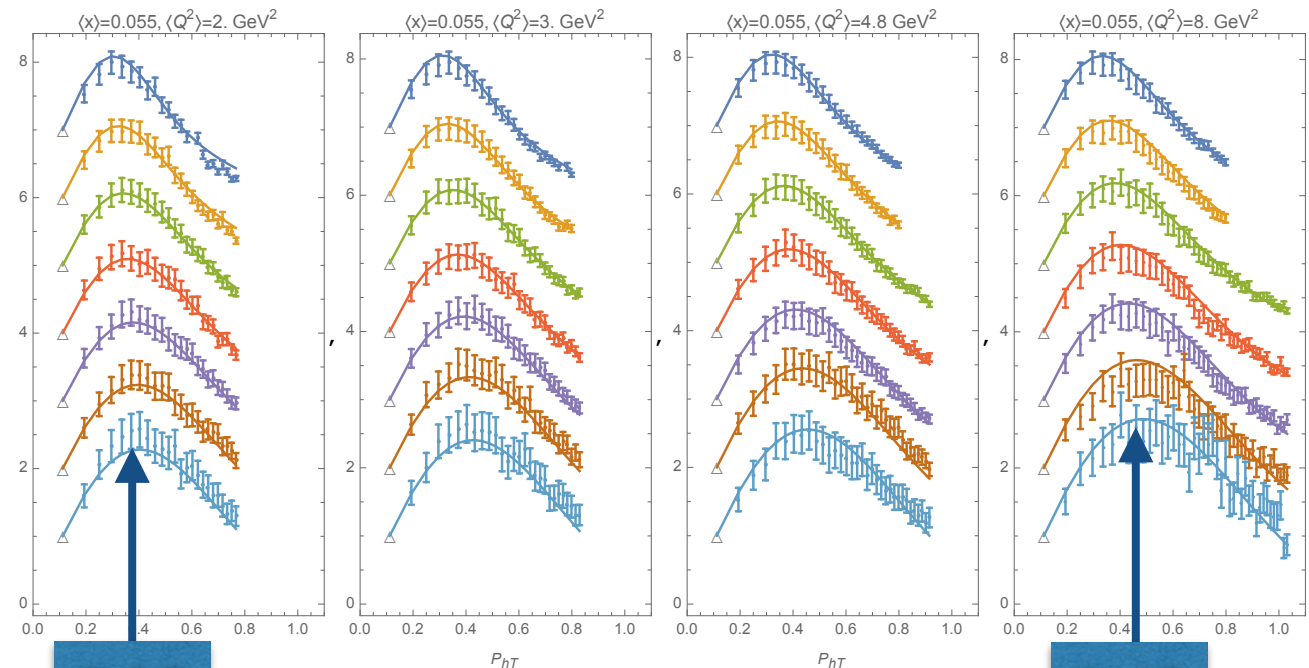
Aghasyan et al., P.R. D97 (18) 032006



$\langle Q^2 \rangle = 2 \rightarrow$

...

$\rightarrow 8 \text{ GeV}^2$



0.35

$\langle x \rangle = 0.055$
 $\chi^2/\text{dof} = 1.49$

0.45

$P_{hT} \text{ (GeV)}$

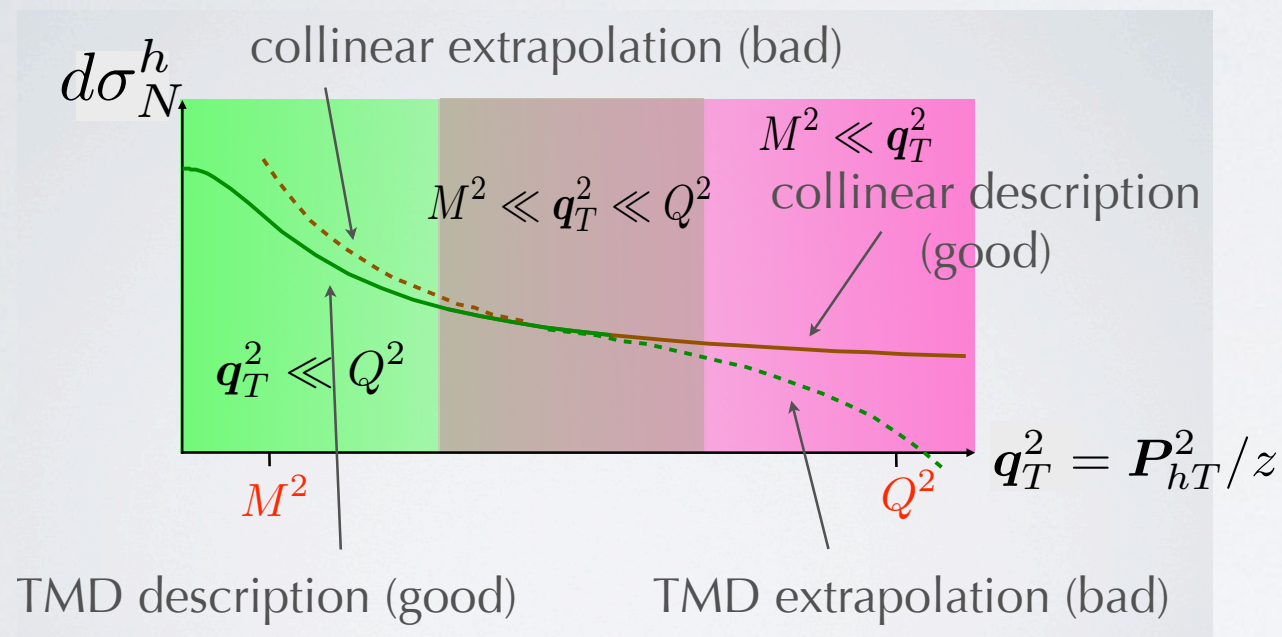
at SIDIS scales, very moderately

The matching problem in SIDIS

SIDIS unpolarized cross section

$$\frac{d\sigma_N^h}{dx dz d\mathbf{P}_{hT}^2 dQ^2} \approx \sum_q e_q^2 [f_1^q \otimes D_1^q] \mathcal{H}_q(Q^2) + Y(Q^2, \mathbf{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

$$\mathbf{q}_T^2 = \mathbf{P}_{hT}^2/z$$



need to match collinear
(fixed-order) description such that

$$\int_0^\infty d\mathbf{P}_{hT}^2 \frac{d\sigma_N^h}{dx dz d\mathbf{P}_{hT}^2 dQ^2} = \frac{d\sigma_N^h}{dx dz dQ^2}$$

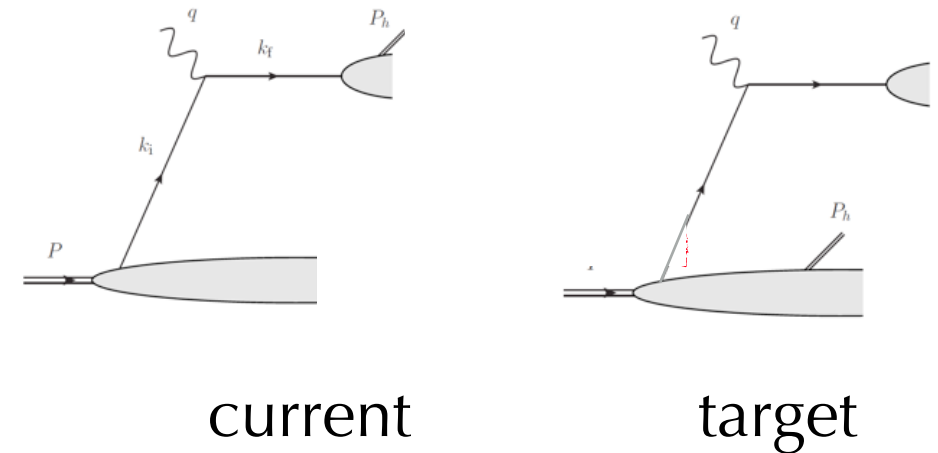
various prescriptions

*Collins, Gamberg, Prokudin, Rogers, Sato, Wang,
P.R. D94 (16) 034014*

*Echevarria, Kasemets, Lansberg, Pisano, Signori,
arXiv:1801.01480*

The factorization problem in SIDIS

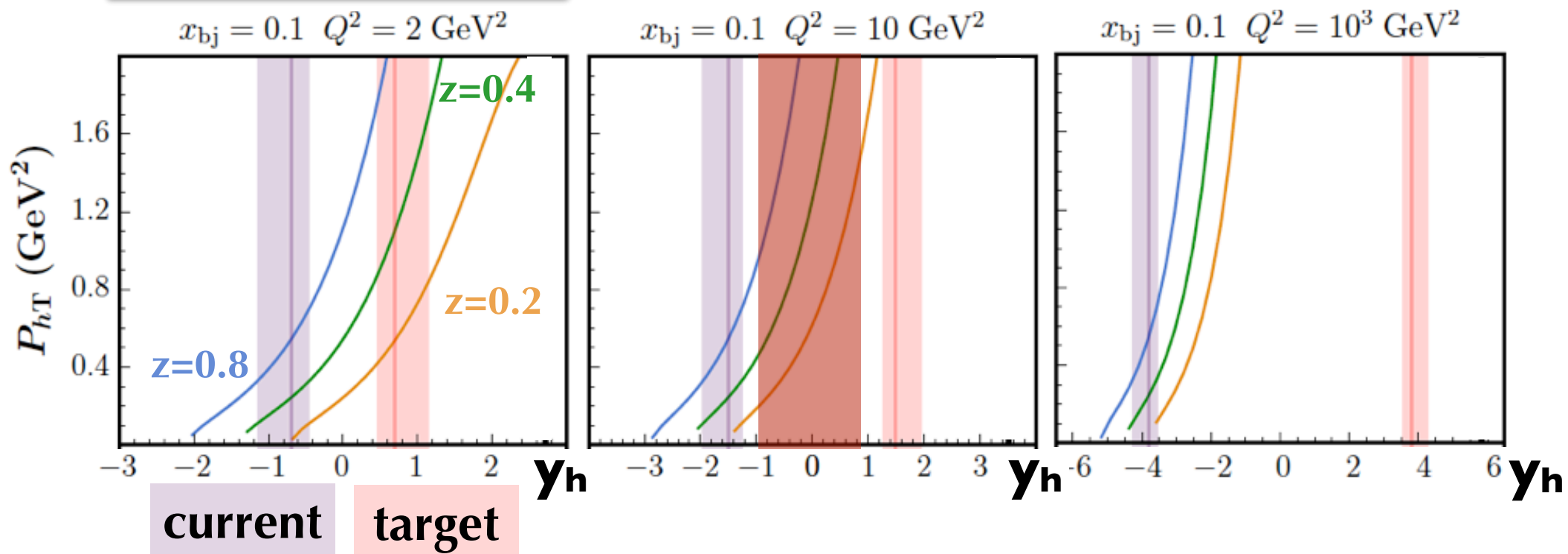
factorization th.'s for (current) fragmentation and (target) fracture functions assume that current and target regions are well separated in rapidity

$$y_h = \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$


at   it's true for few P_{hT}

at  still Y term is relevant

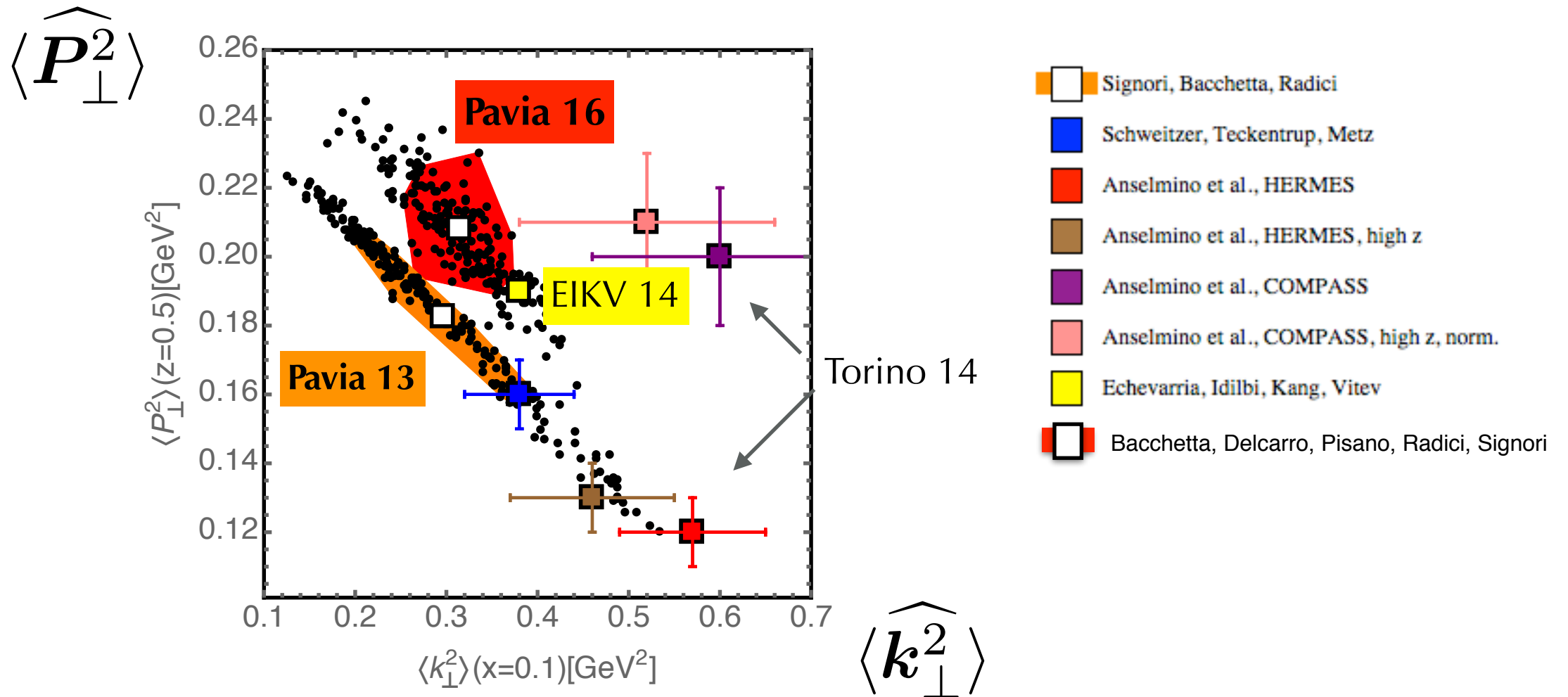
at EIC it's ok



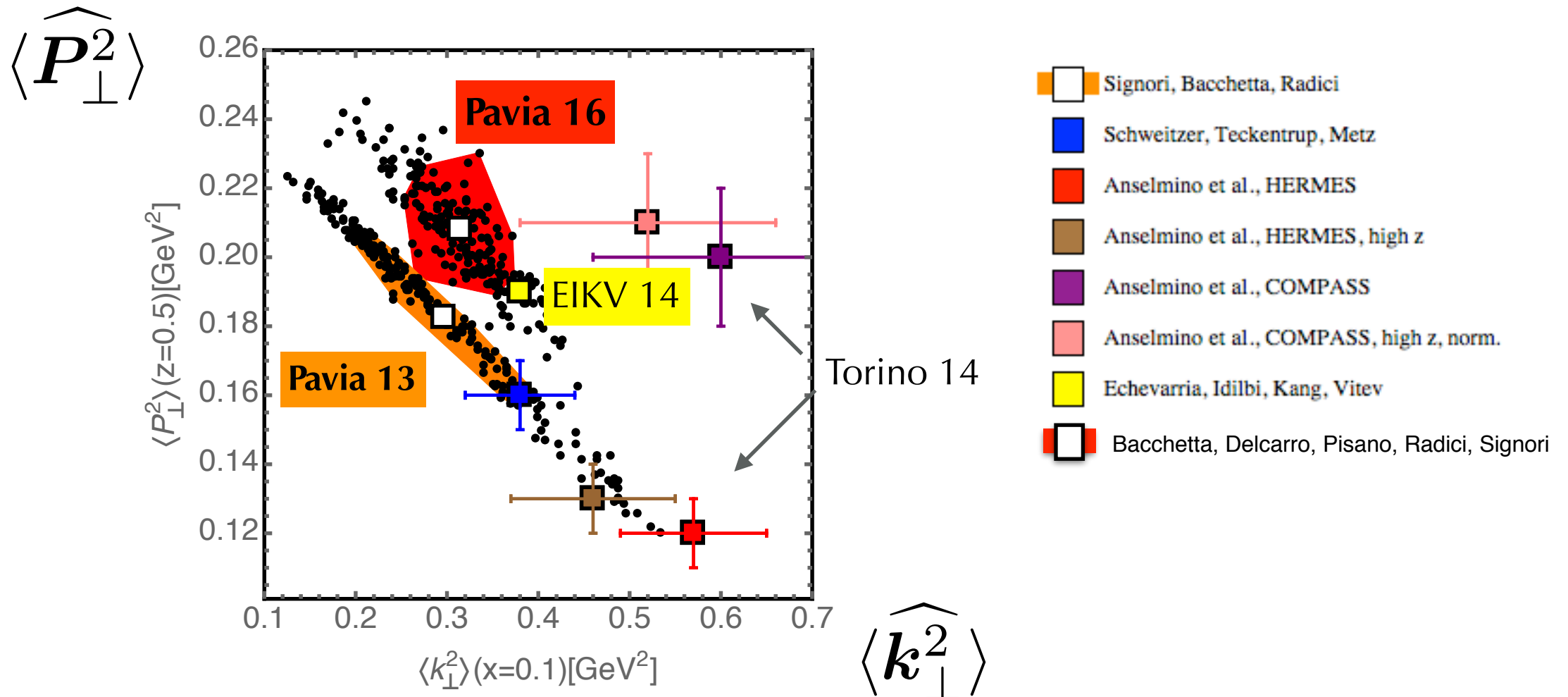
Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato, P.L. B766 (17) 245

 talk Gonzalez

The anticorrelation problem in SIDIS



The anticorrelation problem in SIDIS

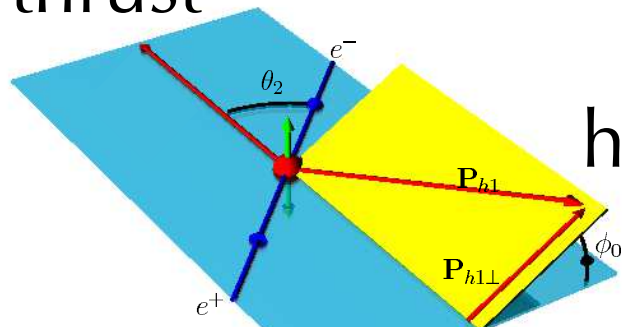


need independent determination of $\langle \mathbf{P}_{hT}^2 \rangle$
 → extract $D_I(z, P_{hT})$ from large set of e^+e^- data

e^+e^- cross section

$$e^+ e^- \rightarrow h X$$

thrust

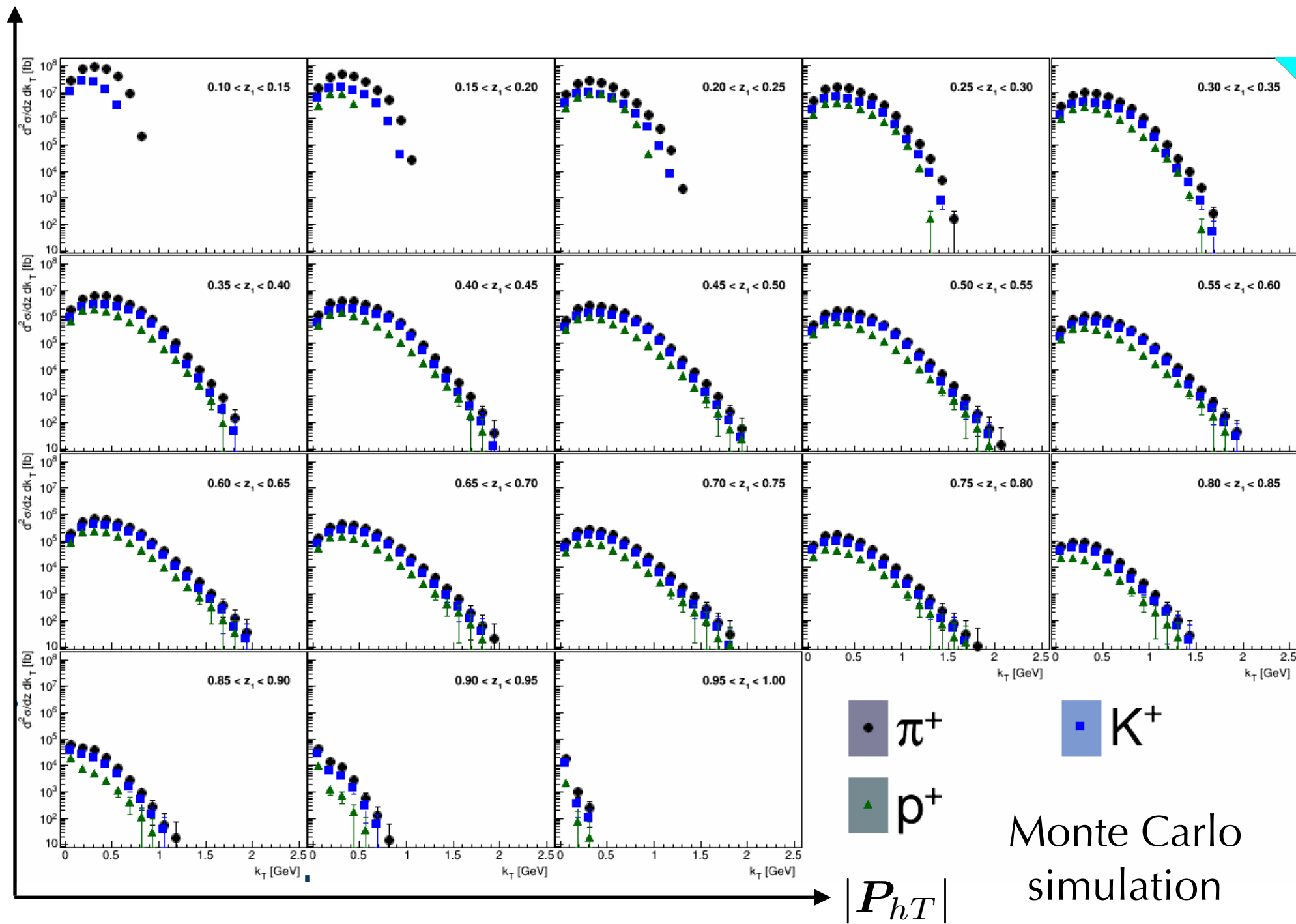


"thrust-axis" frame

$$\frac{d\sigma}{dz dP_{hT}}$$



R. Seidl, talk at SPIN2016

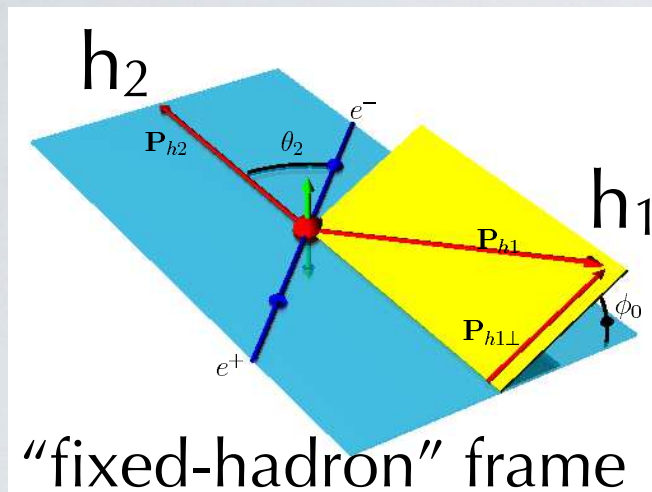


preliminary Belle data

➔ talk Seidl

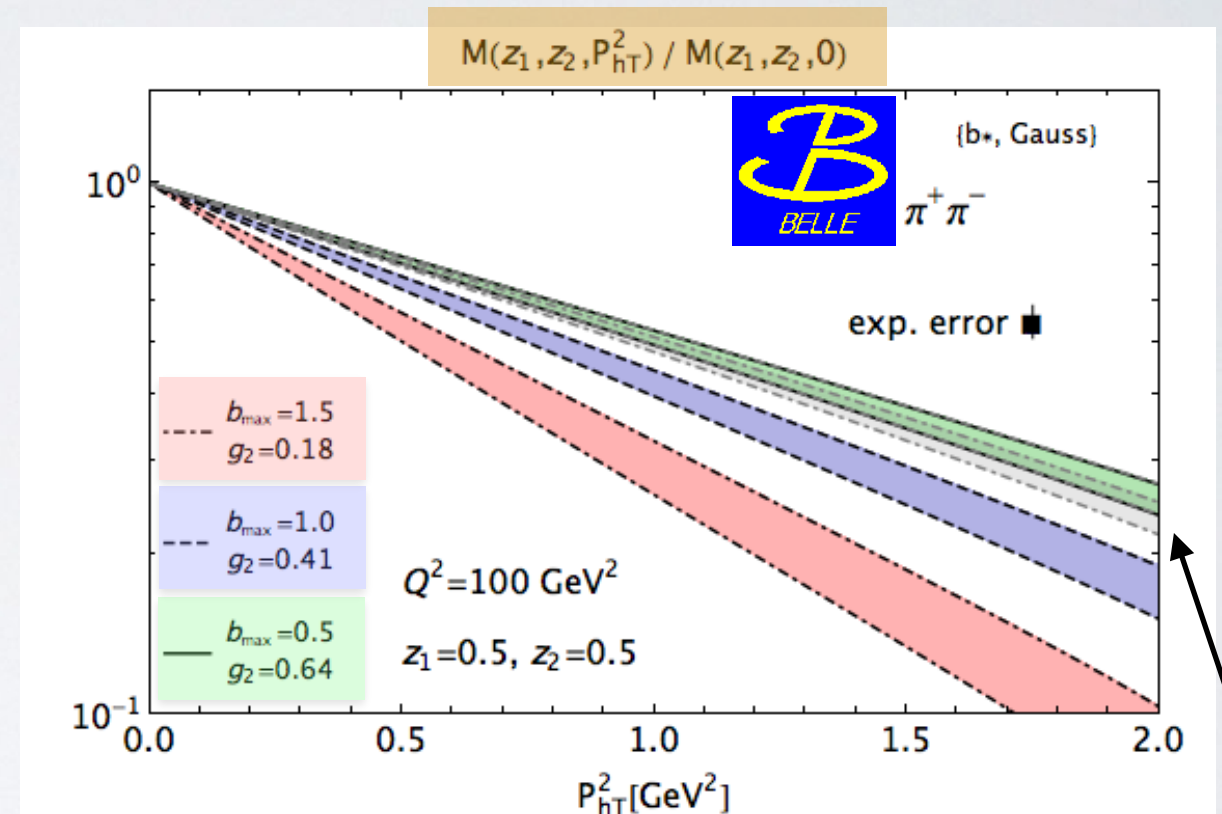
e^+e^- unintegrated multiplicity

$$e^+ e^- \rightarrow h_1 h_2 X$$



$$M = \frac{d\sigma^{h_1 h_2} / dz_1 dz_2 d\mathbf{P}_{h_1 T}^2 dy}{d\sigma_{\text{incl}} / dz_2 dy} \approx \frac{\sum_q e_q^2 [D_1^q \otimes D_1^{\bar{q}}](z_1, z_2, \mathbf{P}_{h_1 T}^2; Q^2)}{\sum_q e_q^2 D_1^q(z_2; Q^2)}$$

$$\frac{\mathbf{P}_{h_1 T}^2}{z_1} \ll Q^2$$



$$D_1^q(z, \mathbf{b}_T; Q^2) = R(Q^2, \mu_b^2(b_T, b_{\text{max}})) e^{-\frac{1}{2} g_2 b_T^2 \log \frac{Q}{Q_0}} D_1^q(z, \mathbf{b}_T; \mu_b)$$

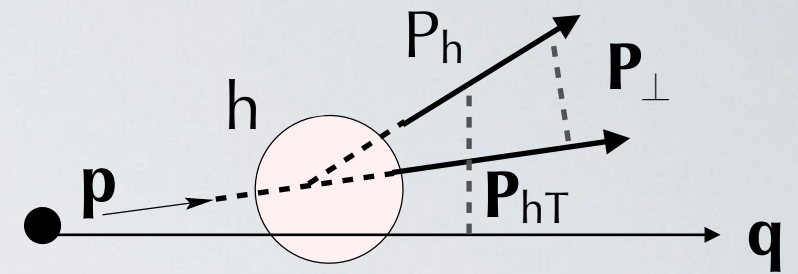
parameters of nonperturbative evolution

Bacchetta et al., JHEP 1511 (15) 076

a 7% error at BELLE scale can constrain nonperturbative parameters $\{b_{\text{max}}, g_2\}$

sensitivity to C_1 in $\mu_b = C_1 / b_T$: $C_1/2 < C_1 < 2 C_1$

TMD FF map



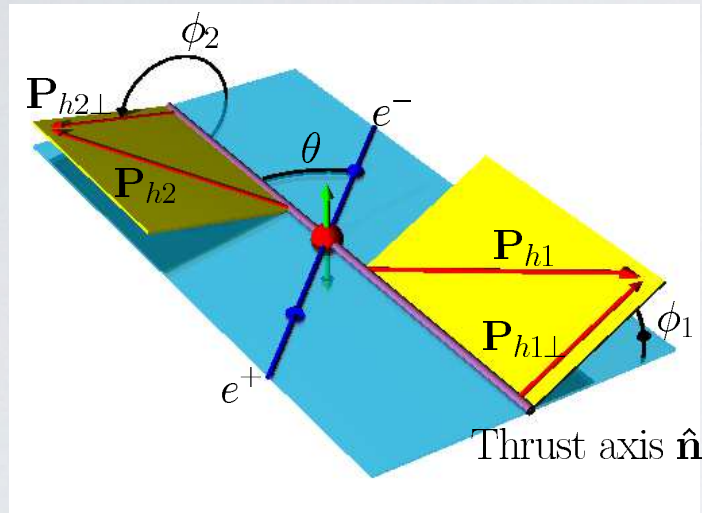
$$S_h = 0$$

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
Hadron	T	D_{1T}^\perp -	G_{1T} -	H_1 - H_{1T}^\perp -

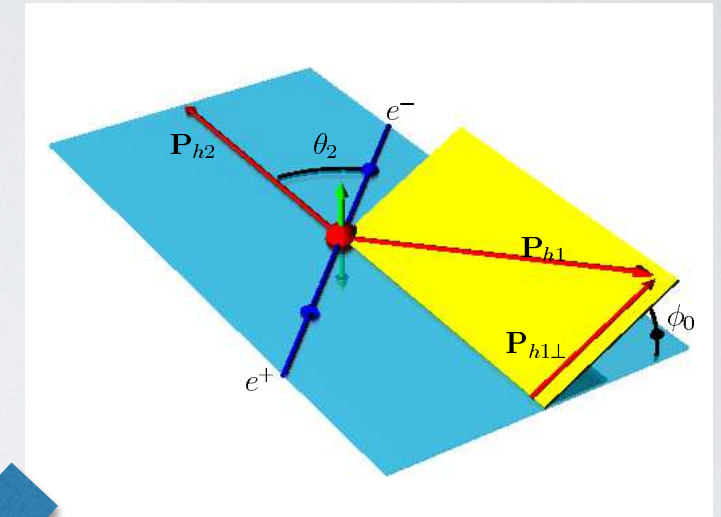
What do we know about the Collins function ?

e^+e^- Collins effect

“thrust axis” frame : A_{12}



“fixed hadron” frame : A_0



not obvious QCD generalization of TMD factorization formula because of thrust axis definition

$$A_0^{e^+e^-} \sim \frac{\sin^2 \theta}{1 + \cos^2 \theta} \cos 2\phi_0 \frac{\sum_q e_q^2 \left[H_{1,q}^\perp(z_1, \mathbf{p}_{1T}^2) \otimes H_{1,\bar{q}}^\perp(z_2, \mathbf{p}_{2T}^2) \right] (z_1, z_2, \mathbf{P}_{1\perp}^2)}{\sum_q e_q^2 \left[D_1^q(z_1, \mathbf{p}_{1T}^2) \otimes D_1^{\bar{q}}(z_2, \mathbf{p}_{2T}^2) \right] (z_1, z_2, \mathbf{P}_{1\perp}^2)}$$

$$\dots \otimes \dots \rightarrow \int d\mathbf{p}_{1T} d\mathbf{p}_{2T} \delta \left(\mathbf{p}_{1T} + \mathbf{p}_{2T} + \frac{\mathbf{P}_{1\perp}}{z_1} \right) \dots$$

$$R_{\text{exp}} \equiv \frac{d\sigma}{d\sigma_0}$$

Unlike-sign
Like-sign

$$\frac{R_{\text{exp}}^U}{R_{\text{exp}}^L} \approx 1 + A_0^{e^+e^-} \left(\frac{\pi^+\pi^- + \pi^-\pi^+}{\pi^+\pi^- + \pi^-\pi^+} \right) - A_0^{e^+e^-} \left(\frac{\pi^+\pi^+ + \pi^-\pi^-}{\pi^+\pi^+ + \pi^-\pi^-} \right)$$

Unlike

Like

Unlike-sign
Charged

$$\frac{R_{\text{exp}}^U}{R_{\text{exp}}^C} \approx 1 + A_0^{e^+e^-} \left(\frac{\pi^+\pi^- + \pi^-\pi^+}{\pi^+\pi^- + \pi^-\pi^+} \right) - A_0^{e^+e^-} \left(\frac{\text{all } \pi\pi}{\text{all } \pi\pi} \right)$$

to kill false asymmetries

Data for e^+e^- Collins effect



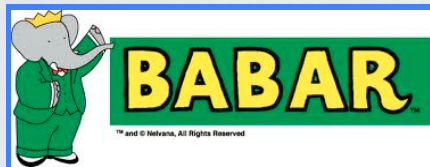
Abe et al., P.R.L. 96 (06) 232002
Seidl et al., P.R. D78 (08) 032011
D86 (12) 039905(E)

$$A_{12}^{U/L/C}(z_1, z_2)$$

$$A_0^{U/L/C}(z_1, z_2)$$

$$s = Q^2 = 112 \text{ GeV}^2$$

→ talk Seidl



Lees et al., P.R. D90 (14) 052003
Lees et al., P.R. D92 (15) 111101

$$A_{12}^{U/L/C}(z_1, z_2, P_{1T}, P_{2T})$$

$$A_0^{U/L/C}(z_1, z_2, P_{1T})$$

$$s = Q^2 = 112 \text{ GeV}^2$$

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$$A_0^{U/L/C}(z_1, z_2)$$

KK and $K\pi$ pairs



Ablikim et al., P.R.L. 116 (16) 042001

$$A_0^{U/L/C}(z_1, z_2, P_{1T})$$

$$s = Q^2 = 13 \text{ GeV}^2$$

→ talk Anulli

Data for e^+e^- Collins effect



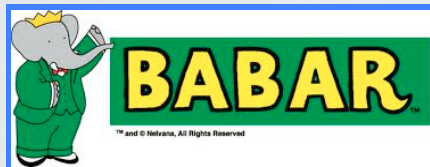
Abe et al., P.R.L. 96 (06) 232002
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KK and $K\pi$ pairs



Ablikim et al., P.R.L. 116 (16) 042001

$$A_0^{U/L/C}(z_1, z_2, P_{1T})$$

$$s = Q^2 = 13 \text{ GeV}^2$$

phase transition :
direct access to transverse dynamics
and to its chiral-odd QCD evolution

→ talk Anulli

e⁺e⁻ Collins effect

available fits

both perform global fits (SIDIS + e⁺e⁻)
with χ^2/dof in [0.85 - 1.2]

	Framework	Belle	BaBar A ₀ (z ₁ ,z ₂ ,P _{IT}) U / L / C	# points	BaBar A ₁₂ U / L / C	BESIII A ₀ (z ₁ ,z ₂ ,P _{IT}) U / L / C
Torino 2015 <i>Anselmino et al., P.R.D92 (15) 114023</i>	Gaussian, fixed width various params. for fav (z) unfav (z) = N _{unf} D ₁ (z) only chiral-odd collinear DGLAP evolution 5 parameters	✓	✓	122	predicted	predicted
KPSY 2015 <i>Kang et al., P.R.D93 (16) 014009</i>	TMD evolution in CSS scheme at NLO + NLL level * $\hat{H}^{(3)}(z) \propto D_1(z)$ fav (z) \neq unfav (z) only homogeneous evo eqs. 7 parameters	✓	✓	122	✗	predicted

$$* H_1^{\perp q}(z, \mathbf{b}_T; Q^2) = \sum_i \left(\delta C_{q/i} \otimes \hat{H}^{(3)i} \right) (z, b_*; \mu_b) e^{S(b_*, \mu_b; Q)} e^{S_{\text{NP}}(b_T; Q)} H_{\text{NP}}^q(z, \mathbf{b}_T; Q_0^2)$$

Moments of Collins function : z dep.

favored

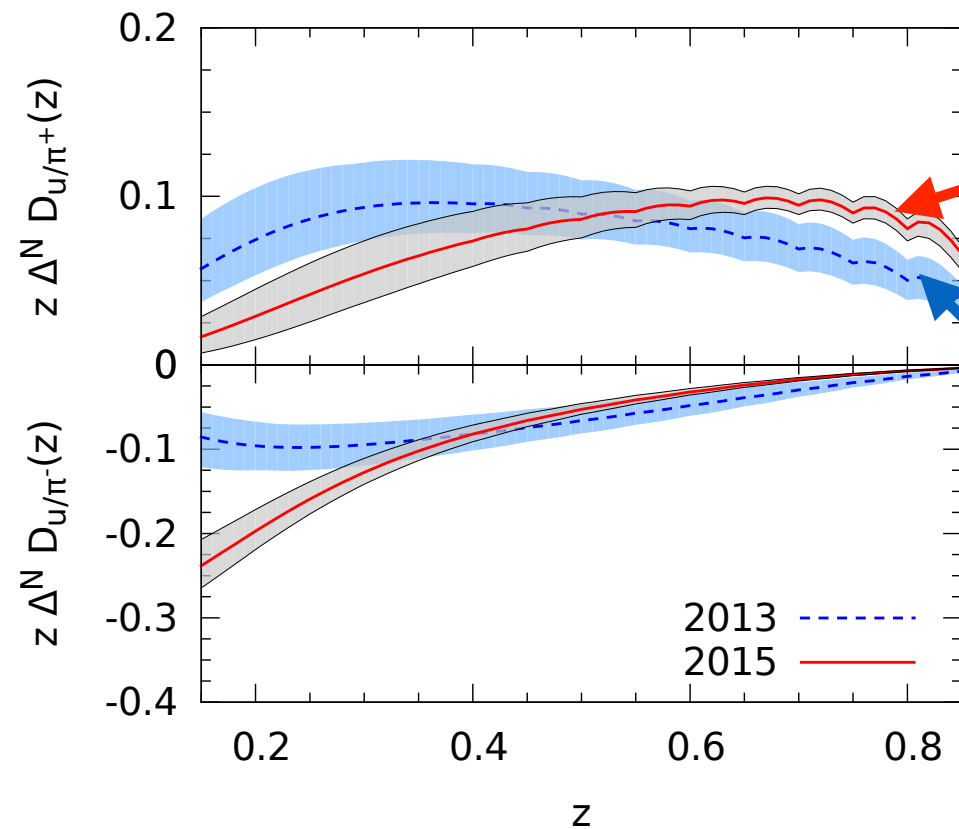
$$\Delta^N D_{h/q}(z) = 4 H_1^{\perp(1/2)q \rightarrow h}(z)$$

$$H_1^{\perp(n)}(z) = \int d\mathbf{P}_\perp \frac{1}{2} \left(\frac{\mathbf{P}_\perp^2}{z^2 m_h^2} \right)^n H_1^\perp(z, \mathbf{P}_\perp^2)$$

unfavored

$$\hat{H}^{(3)}(z) = -2z M_h H_1^{\perp(1)}(z)$$

- similar results
- both very good χ^2/dof
- R^U/R^L or R^U/R^C
- not very sensitive to TMD evolution

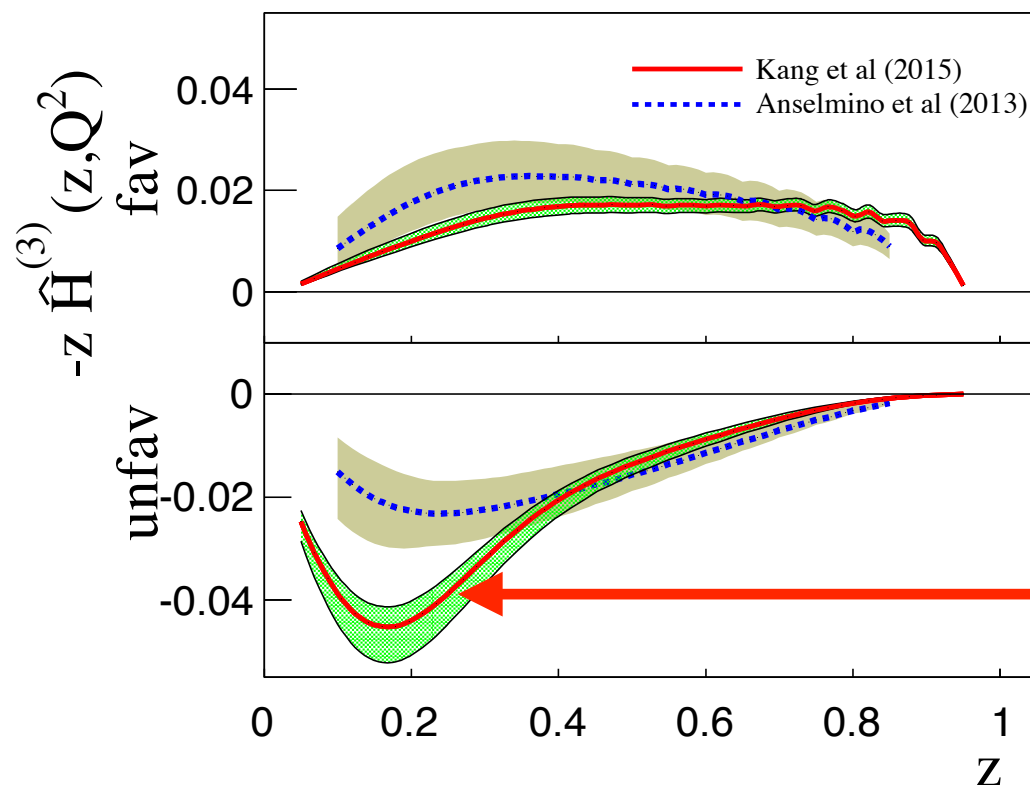


Anselmino et al.,
P.R. D92 (15) 114023

Anselmino et al.,
P.R. D87 (13) 094019

Torino 2013

Torino 2015



Torino 2013

KPSY 2015

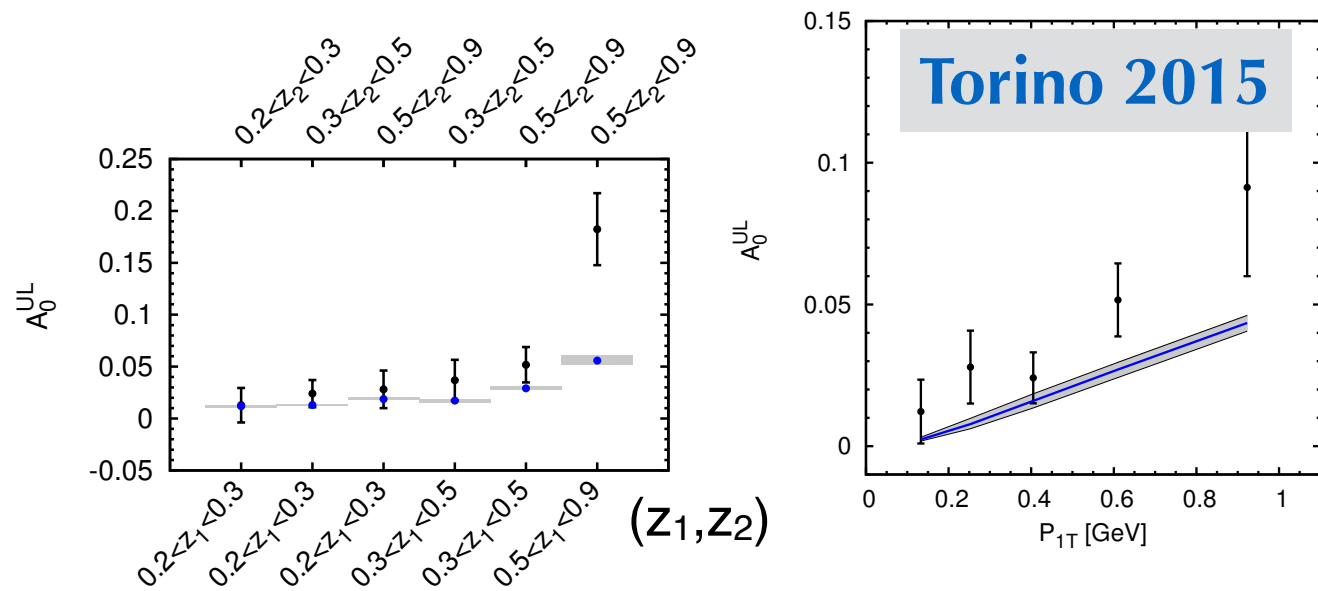
Kang et al.,
P.R. D93 (16) 014009

Predicting the **BESIII** asymmetry



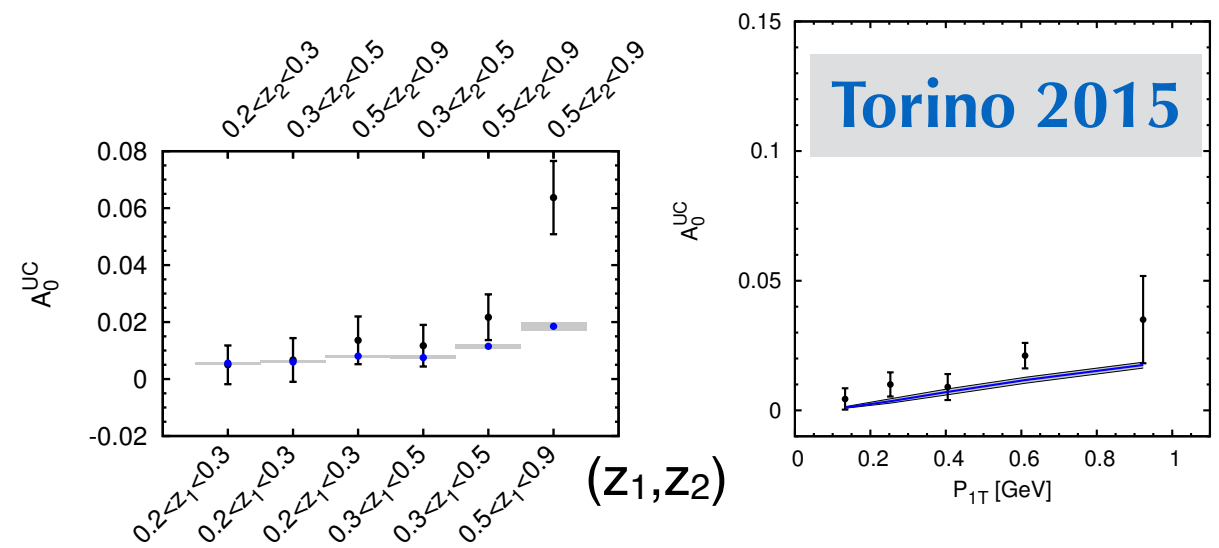
A_0^{UL}

$$A_0^{e^+e^-} \left(\frac{\pi^+\pi^- + \pi^-\pi^+}{\pi^+\pi^- + \pi^-\pi^+} \right) - A_0^{e^+e^-} \left(\frac{\pi^+\pi^+ + \pi^-\pi^-}{\pi^+\pi^+ + \pi^-\pi^-} \right)$$

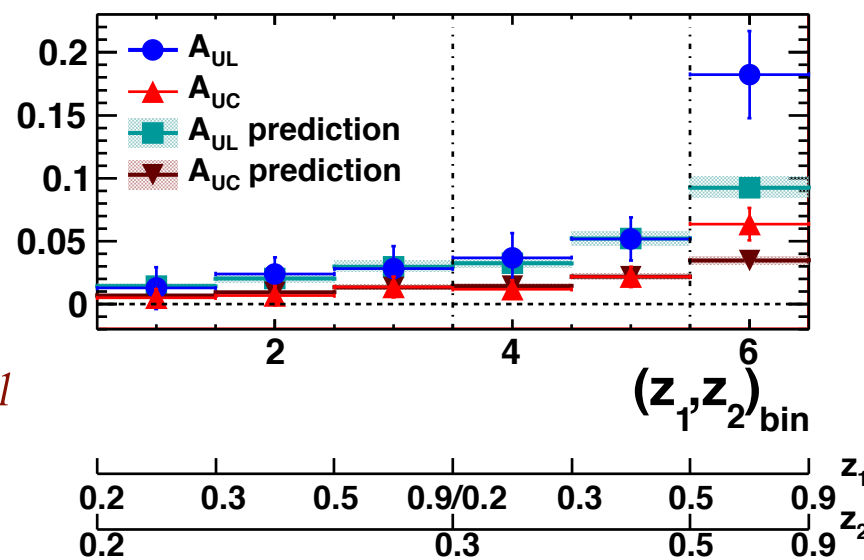


A_0^{UC}

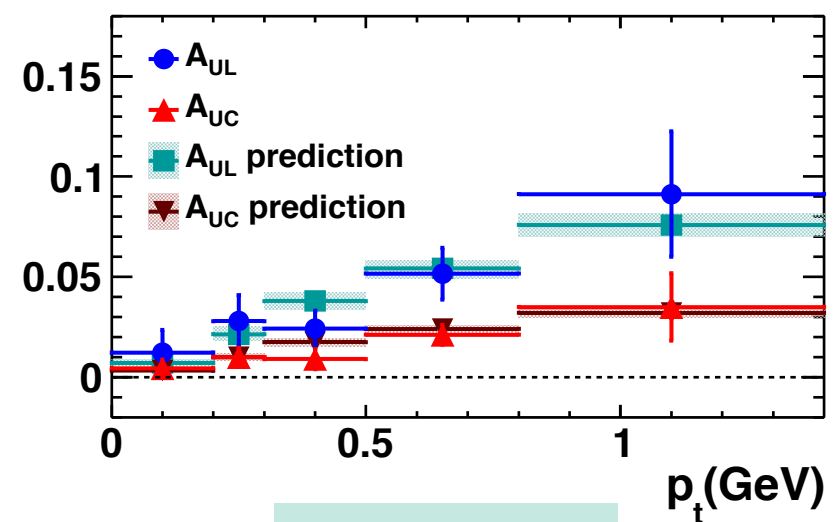
$$A_0^{e^+e^-} \left(\frac{\pi^+\pi^- + \pi^-\pi^+}{\pi^+\pi^- + \pi^-\pi^+} \right) - A_0^{e^+e^-} \left(\frac{\text{all } \pi\pi}{\text{all } \pi\pi} \right)$$



Anselmino et al., P.R. D92 (15) 114023



Ablikim et al., P.R.L. 116 (16) 042001



predictions =

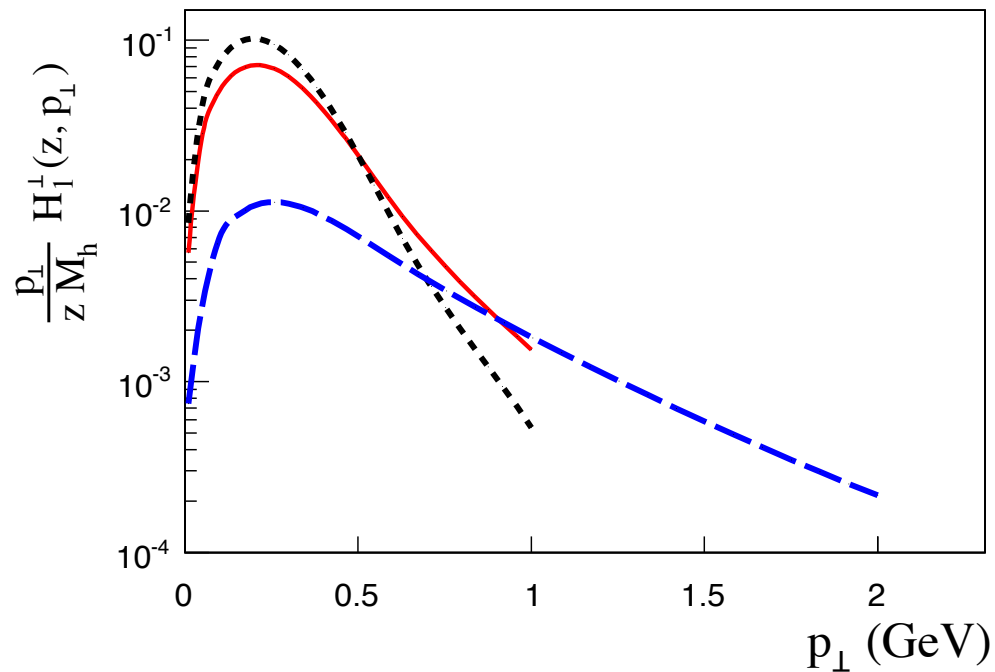
A_0^{UL} **KPSY 2015**

A_0^{UC} **KPSY 2015**

TMD evolution on Collins funct.



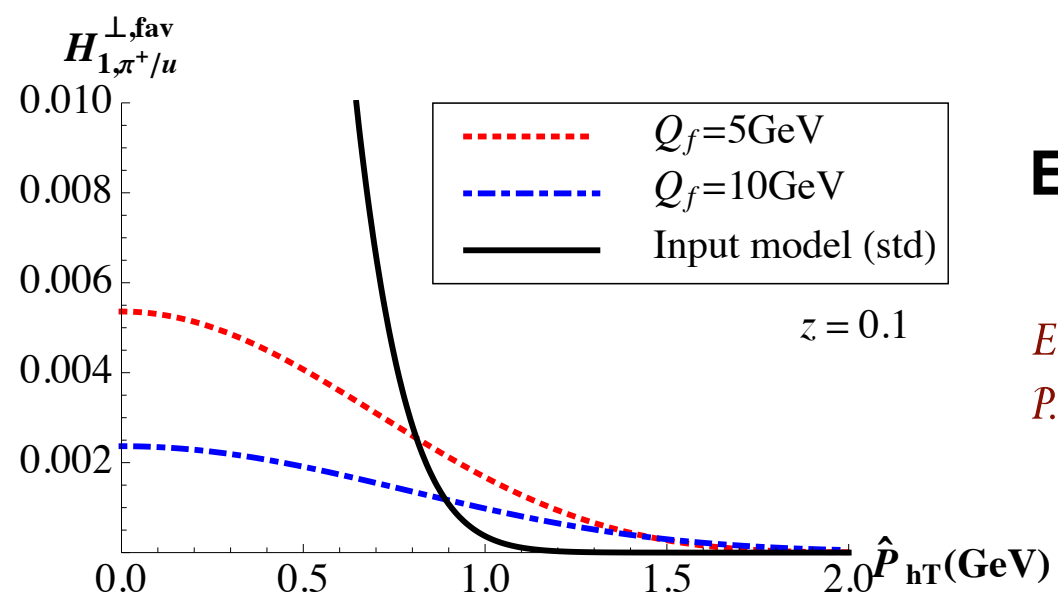
different broadening
in the early range
 $Q_0^2 \text{ — } 10 \div 20 \text{ GeV}^2$



..... $Q^2 = 2.4 \text{ GeV}^2$
 — $Q^2 = 10 \text{ GeV}^2$
 - - - $Q^2 = 1000 \text{ GeV}^2$

KPSY 2015

*Kang et al.,
P.R. D93 (16) 014009*



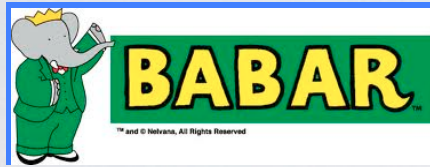
..... $Q_f = 5 \text{ GeV}$
 - - - $Q_f = 10 \text{ GeV}$
 — Input model (std)

$z = 0.1$

EIS 2014

*Echevarria, Idilbi, Scimemi,
P.R. D90 (14) 014003*

Collins funct. for Kaons



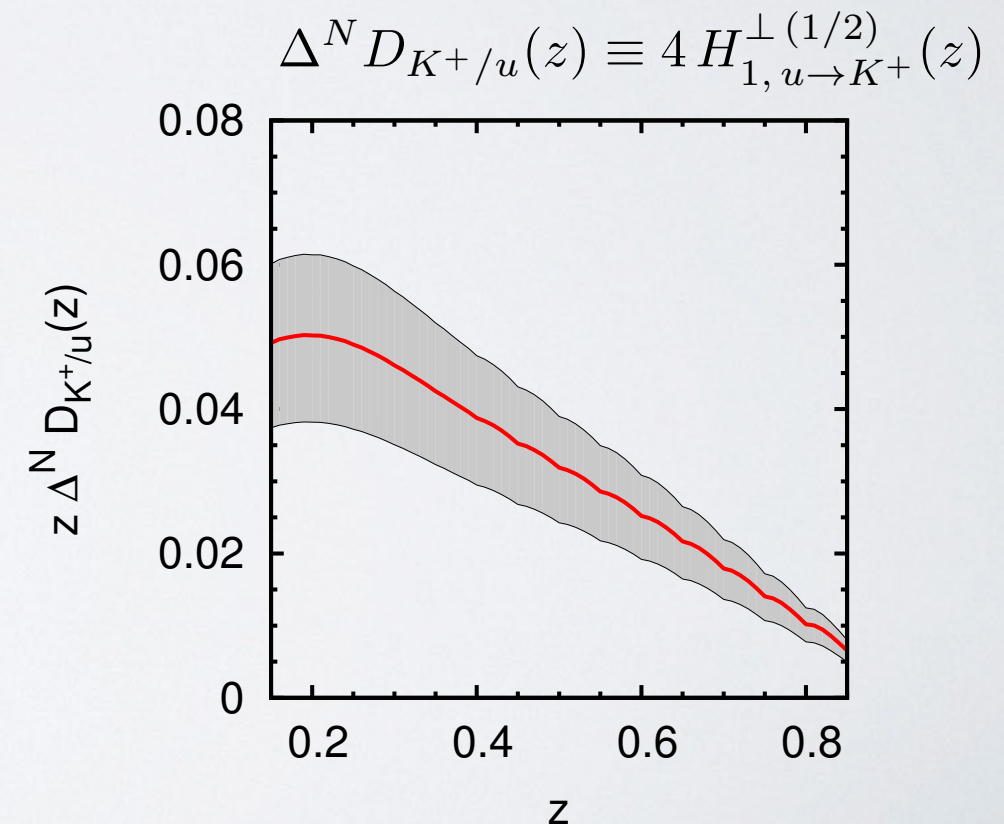
Lees et al., P.R. D92 (15) 111101

64 data: $A_0^{U/L/C}$ for πK and KK pairs

Torino 2016

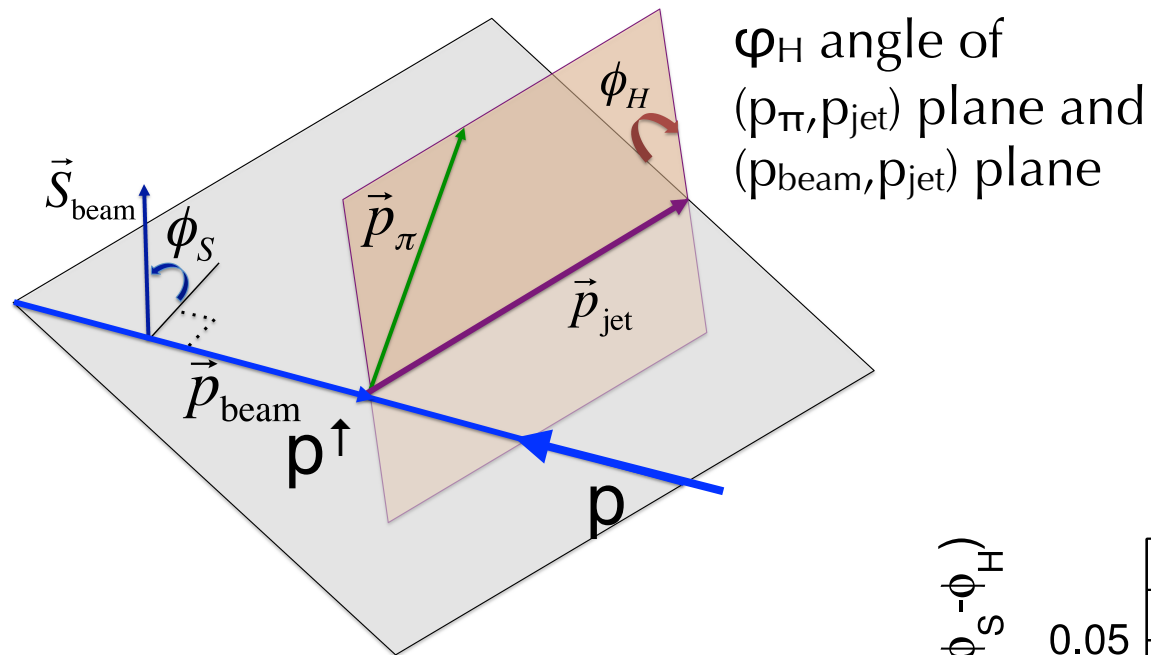
- same p_T dependence as for π (Gaussian with fixed parameter)
- z dependence as $D_1^{q \rightarrow K}(z)$
- only non-chiral-odd DGLAP evolution of z -dependence
- 2 parameters: normalization for favored u and unfavored
- global $\chi^2/\text{dof} = 0.89$

- favored $u \rightarrow K^+$ determined and positive
- favored $s \rightarrow K^-$ undetermined (also in sign)
- unfavored undetermined
because of too few data



Anselmino et al., P.R.D93 (16) 034025

Collins effect for hadron-in-jet



- collinear collision, TMD only in fragmentation
- assuming factorization at large P_{jetT}

$$A_{UT}^{\sin(\phi_S - \phi_H)} \propto \frac{h_1^q \otimes f_1^{\bar{q}} \otimes H_1^{\perp q}}{f_1^q \otimes f_1^{\bar{q}} \otimes D_1^q}$$

PDF & TMDFF from SIDIS + e^+e^- analysis

--- DMP+2013 (no evolution)

Anselmino et al., P.R. D87 (13) 094019

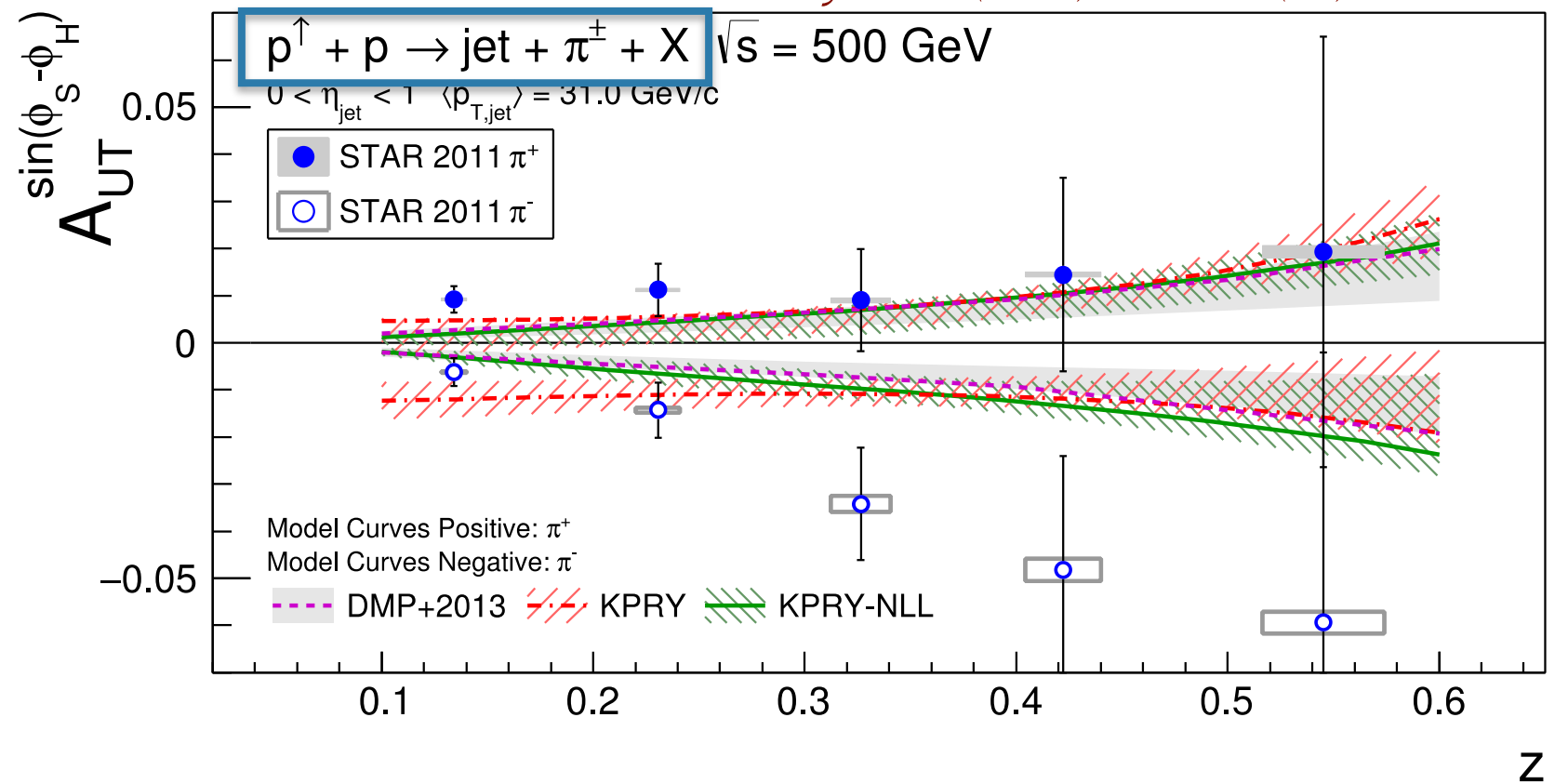
D'Alesio et al., P.L. B773 (17) 300

--- KPRY (no evolution)

--- KPRY-NLL (TMD evolution)

Kang et al., P.L. B774 (17) 635

Adamczyk et al. (STAR), P.R. D97 (18) 032004



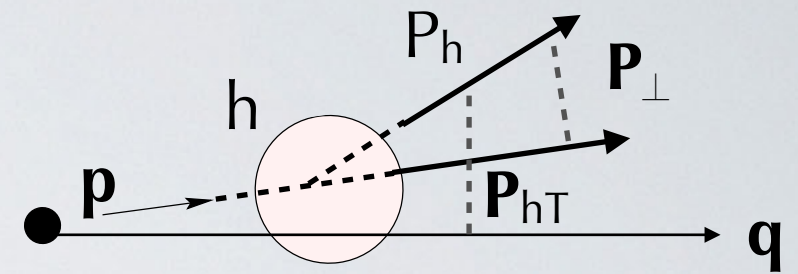
is Collins function for quark universal ?

gluon signature: $A_{UT}^{\sin(\phi_S - 2\phi_H)}$

➔ talks Pisano, Scimemi Drachenberg

TMD FF map

leading twist
 $S_h \leq 1/2$



$S_h = 1/2$

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
Hadron	T	D_{1T}^\perp -	G_{1T} -	H_1 -
				H_{1T}^\perp -

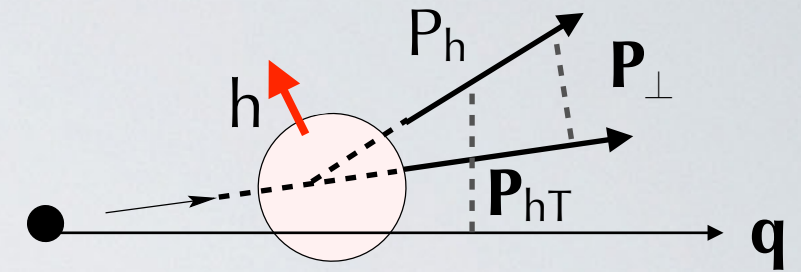
data on Λ^\uparrow production from BELLE / COMPASS (and CERN- NA48/OPAL/ATLAS
 HERA-B
 old FermiLab)



Access to $D_{1T\perp}$

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L}	H_{1L}^\perp
	T	D_{1T}^\perp	G_{1T}	H_1 - H_{1T}^\perp

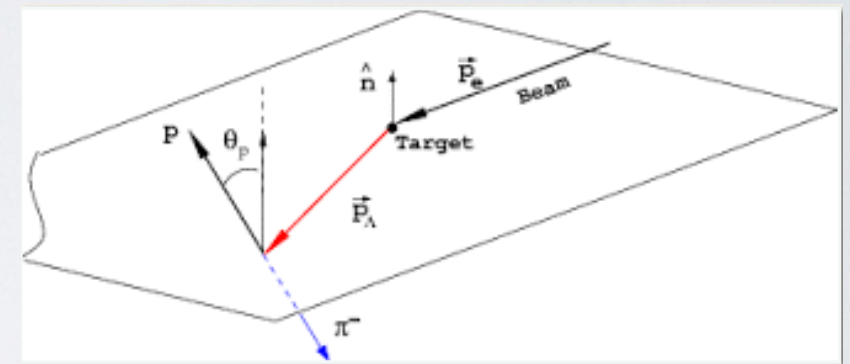
$$D_{1T\perp}(z, \mathbf{P}_{hT})$$



encodes “spontaneous” polarization of h

assuming TMD factorization,
p-Be and p-p data from FermiLab
can be interpreted as

$$\frac{1}{N} \frac{dN}{d \cos \theta_p} = 1 + \alpha P \cos \theta_p$$



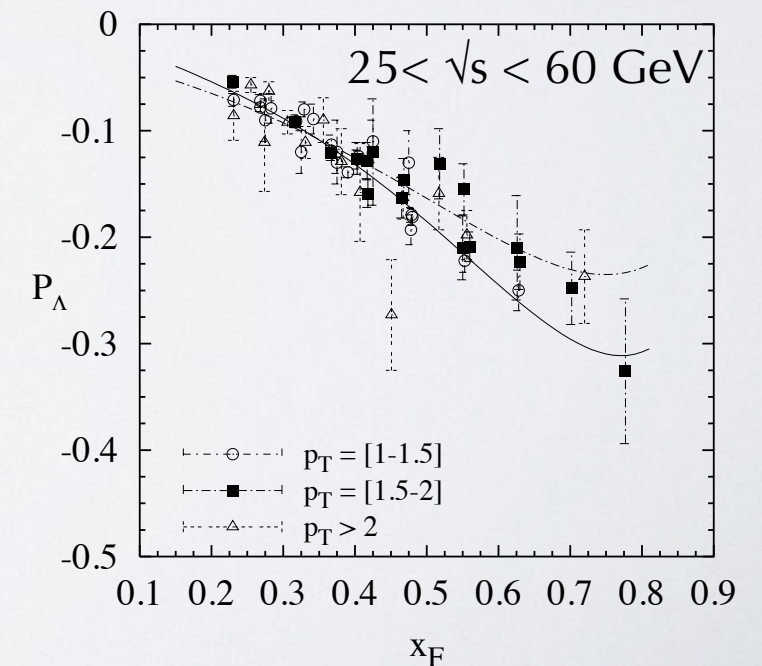
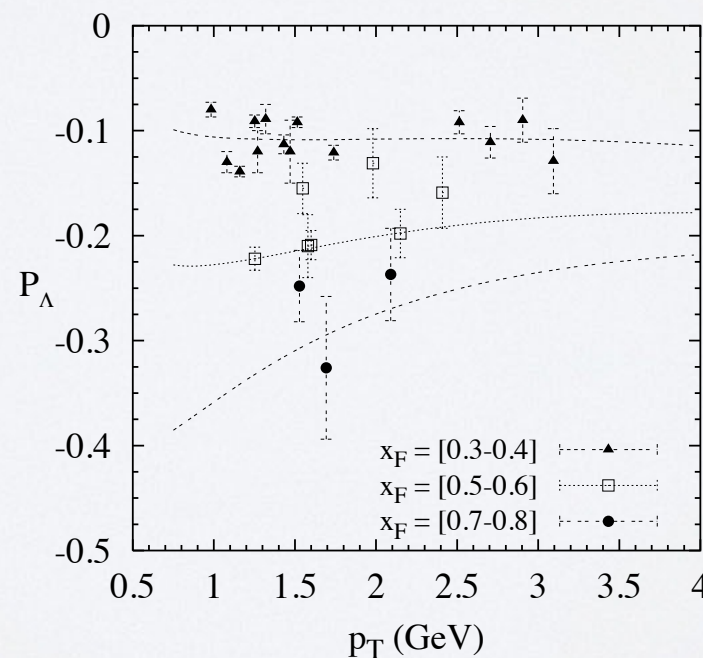
$$P_\Lambda \propto \frac{f_1^q \otimes f_1^{\bar{q}} \otimes D_{1T}^{\perp q}}{f_1^q \otimes f_1^{\bar{q}} \otimes D_1^q}$$

Anselmino, Boer, D'Alesio, Murgia, P.R. D63 (01) 054029

caveat

$d\sigma_0$ turns out too small w.r.t. data
factorization broken?

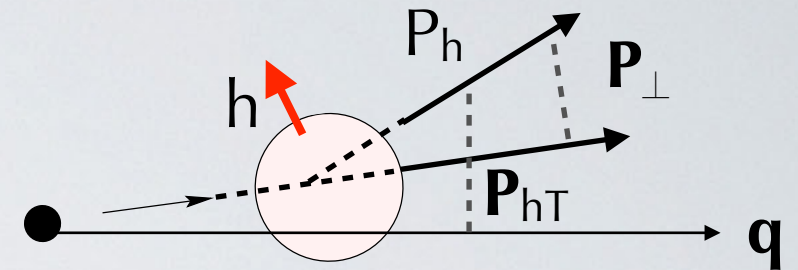
➔ talk Boer



Λ polarization data

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
	T	D_{1T}^\perp -	G_{1T} -	H_1 - H_{1T}^\perp -

$$D_{1T^\perp}(z, \mathbf{P}_{hT})$$



encodes “spontaneous” polarization of h

extraction of D_{1T^\perp} never attempted from e^+e^- so far

new data from Belle on $e^+e^- \rightarrow \Lambda^\uparrow(\bar{\Lambda}^\uparrow) + X$
 $e^+e^- \rightarrow \Lambda^\uparrow(\bar{\Lambda}^\uparrow) + \pi(K) + X$
 with full $(z_\Lambda, P_{\Lambda T})$ dependence



Abdesselam et al. (BELLE),
 arXiv:1611.06448

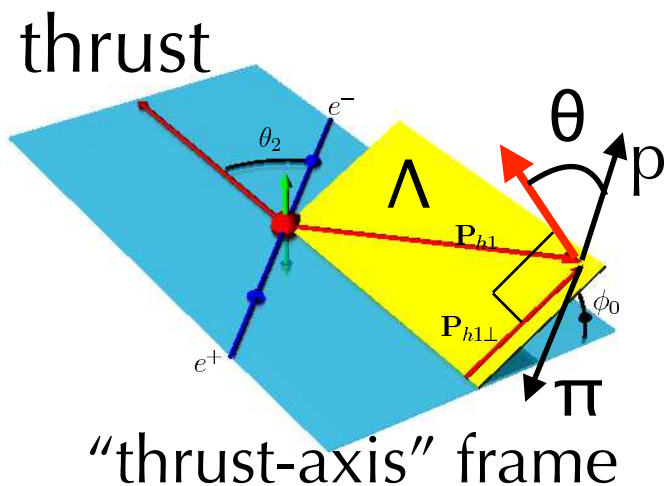
ongoing attempt to interpret data in
 collinear factorization up to twist-3 and NLO:

$$P_\Lambda \leftrightarrow D_T(z) \text{ “intrinsic twist-3”}$$

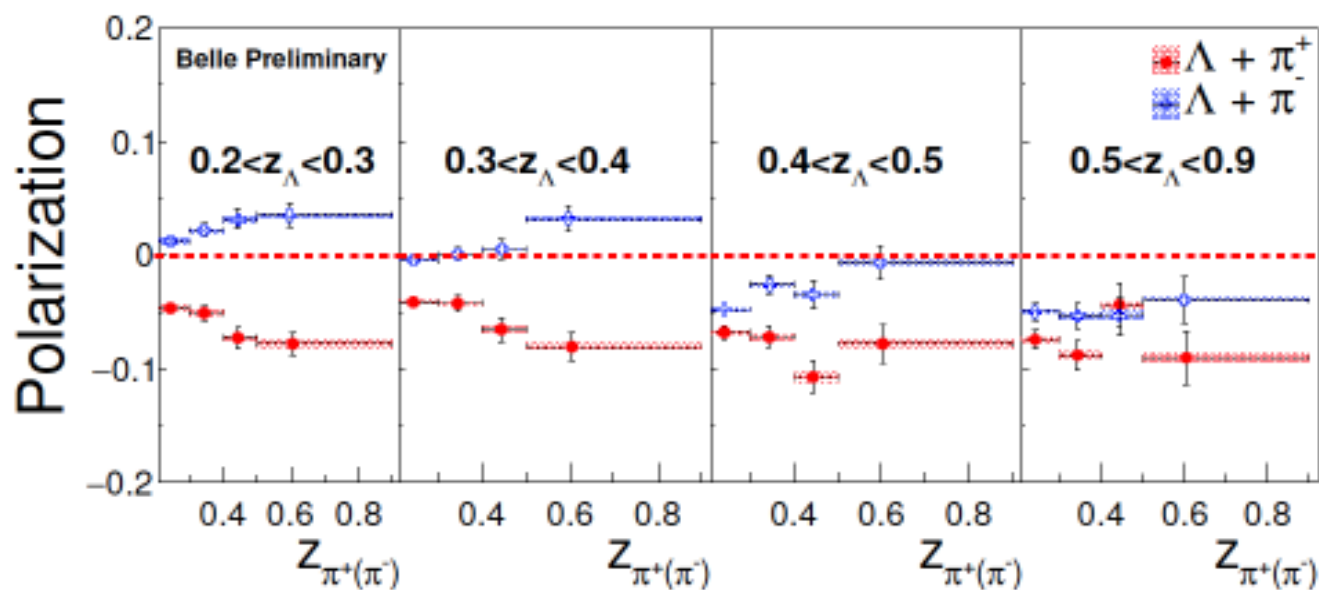
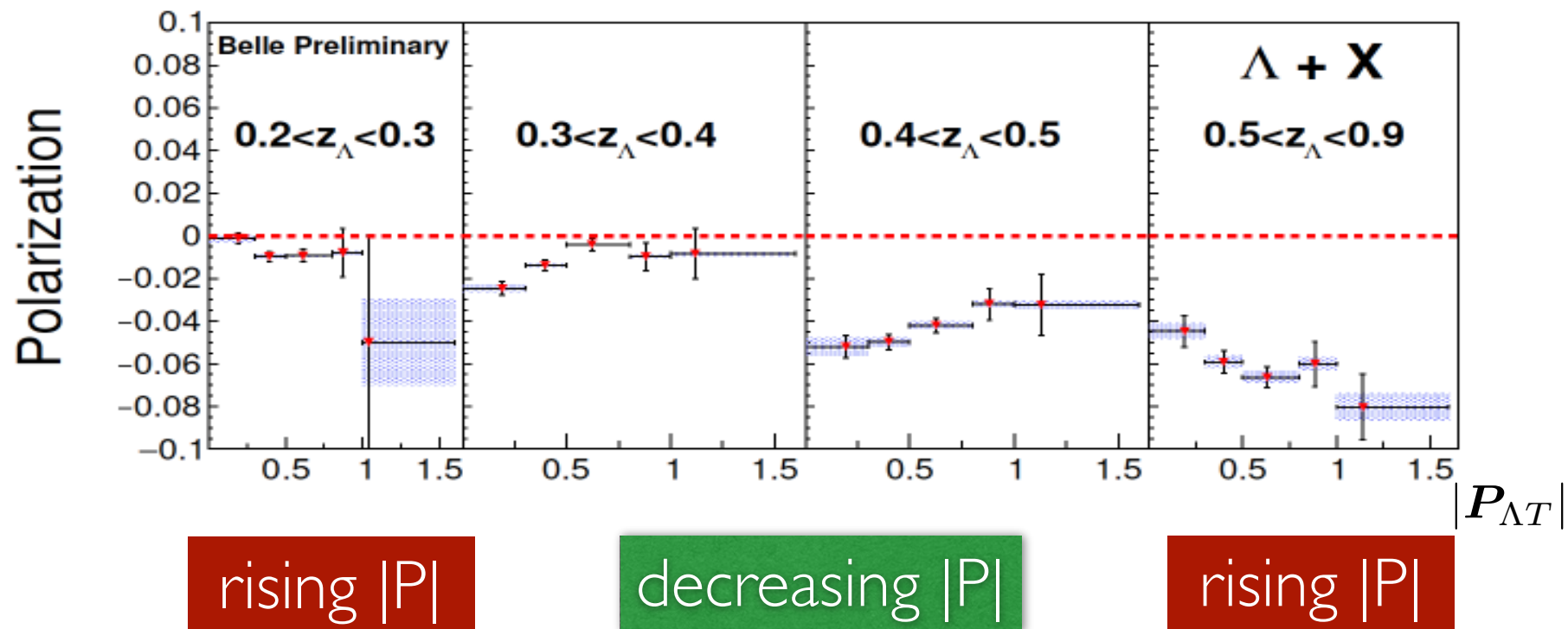
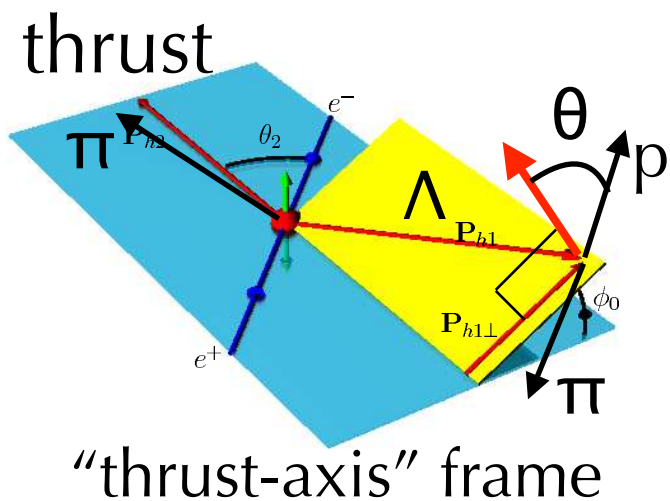
Schlegel, Transversity 2017

Λ polarization data

$$e^+ e^- \rightarrow \Lambda^{\uparrow} + X$$



Abdesselam et al. (BELLE),
arXiv:1611.06448



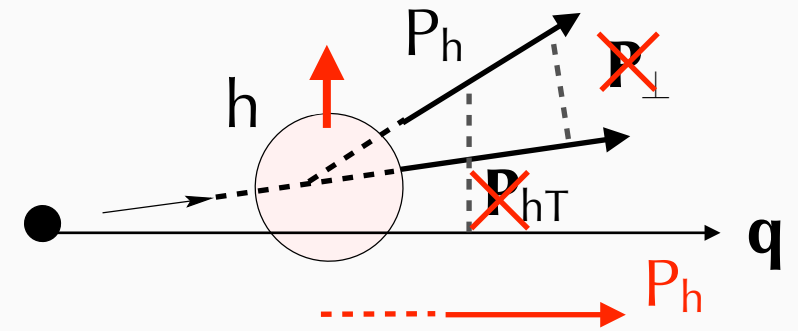
➔ talk Seidl?

SIDIS Λ polarization data

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	D_1 <i>Unpolarized</i>		H_1^\perp - <i>Collins</i>
	L		G_{1L} -	H_{1L}^\perp -
	T	D_{1T}^\perp -	G_{1T} -	H_1 - H_{1T}^\perp -

$$H_1(z)$$

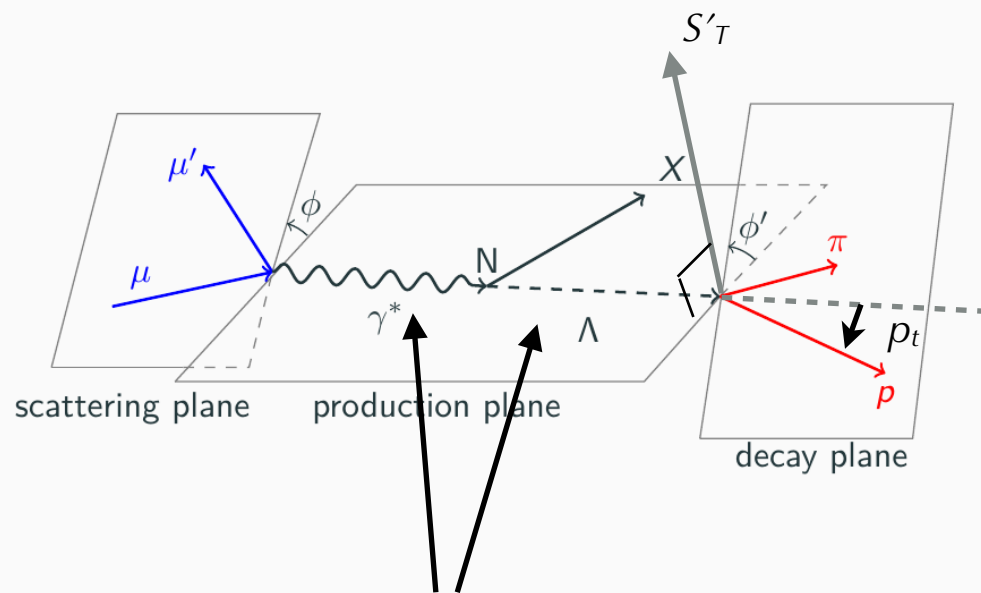
Λ "transversity"



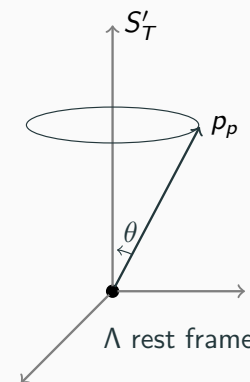
new data from COMPASS on $l p^\uparrow \rightarrow \Lambda^\uparrow + X$



Moretti, Transversity 2017



collinear kinematics

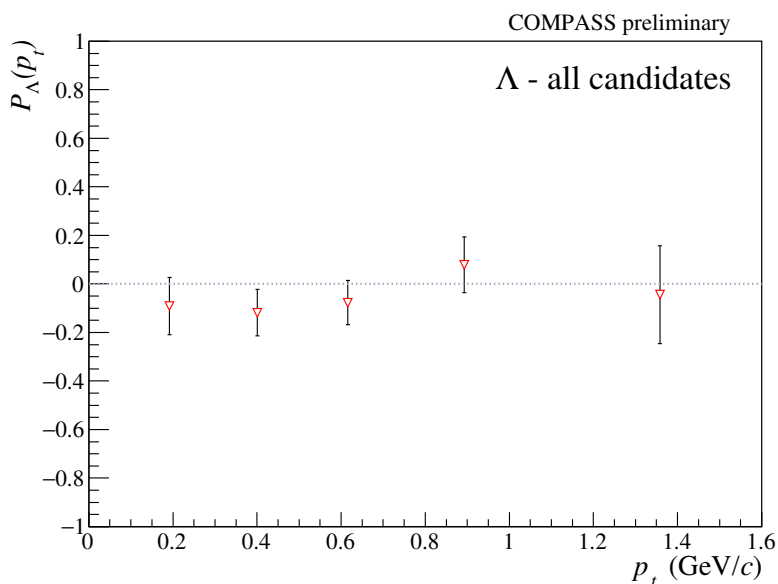
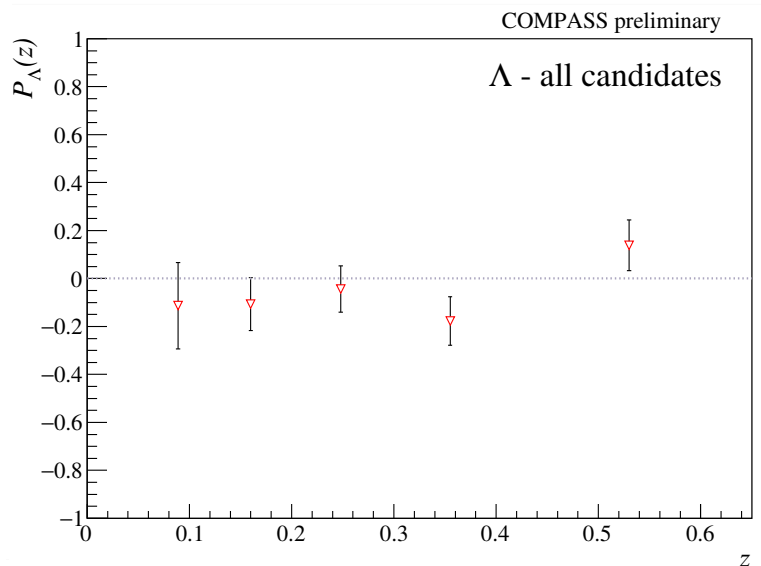
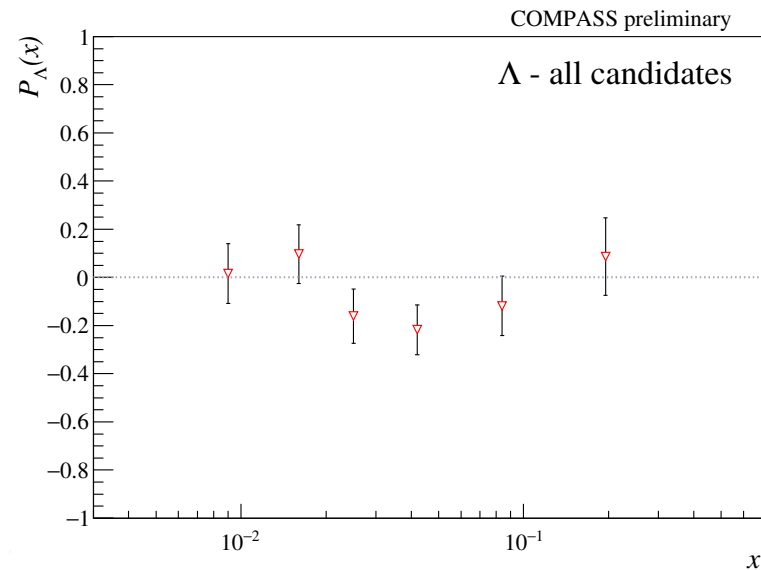


$$P_\Lambda \propto \frac{h_1^q H_1^q}{f_1^q D_1^q}$$

SIDIS Λ polarization data



Moretti, Transversity 2017



$$P_{\Lambda}(x, z, p_t) \sim 0 \quad \text{also for } \bar{\Lambda}$$

2 Hp.'s based on isospin symmetry and

$$h_1^{\bar{q}} \approx FF^{\bar{q}} \approx 0, \quad FF^s = cFF^u$$

1) $h_1^s \approx 0$ ("valence Hp.") then $\frac{\int dz H_1^{u \rightarrow \Lambda}(z)}{\int dz D_1^{u \rightarrow \Lambda}(z)} \approx 0$

2) P_{Λ} only from s $H_1^{u \rightarrow \Lambda} \approx H_1^{d \rightarrow \Lambda} \approx 0$ then $H_1^{s \rightarrow \Lambda} = D_1^{s \rightarrow \Lambda}$

