On amplification of radiation from a charged particle circulating around a cylinder

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Outline

- Problem setup
- Features of radiation at large distances
- Amplification for negative permittivity materials
- Conclusions

Introduction

- Wide applications of synchrotron radiation motivate the importance of investigations for various mechanisms of controlling the radiation parameters

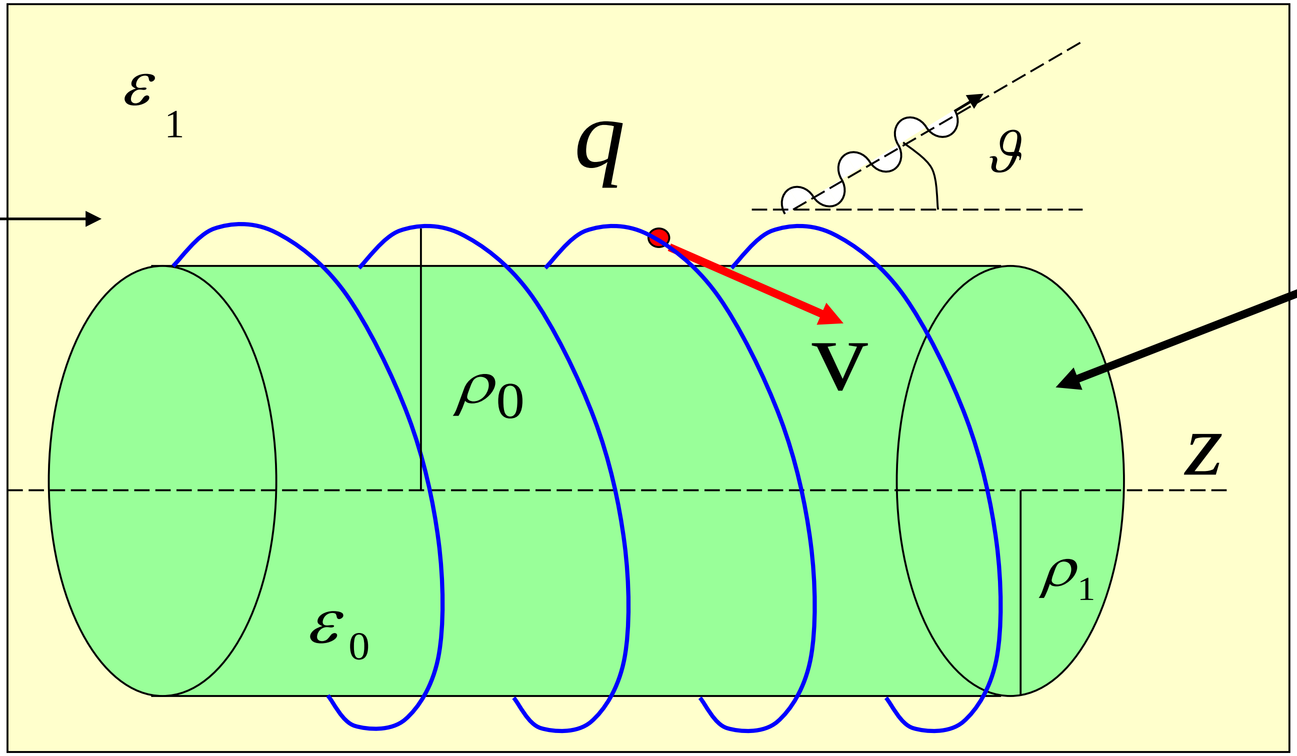
- From this point of view, it is of interest to consider the influence of a medium on the spectral and angular distributions of the radiation

- It is well known that the presence of medium gives rise to new types of radiation processes:
  - Cherenkov radiation
  - Transition radiation
  - Diffraction radiation
- High energy electromagnetic processes essentially change their characteristics when boundaries are present

- We consider combined effects of medium and boundaries

Geometry of the problem

Dielectric permittivity for surrounding medium



Dielectric cylinder

Notations used: $\beta_{j\perp} = \dfrac{v_\perp}{c}\sqrt{\varepsilon_j}\,, \beta_{j\parallel} = \dfrac{v_\parallel}{c}\sqrt{\varepsilon_j},\ j = 0, 1$

Radiation intensity in the exterior medium

- Under the Cherenkov condition $\beta_{1\|} > 1$, the total radiation intensity at large distances from the charge trajectory is presented in the form

$$I = I_0 + I_{m\neq 0}$$

- I_0 describes the radiation with a continuous spectrum propagating along the Cherenkov cone of the external medium $\vartheta = \vartheta_0 \equiv \arccos\left(\beta_{1\|}^{-1}\right)$

$$I_{m\neq 0} = \sum_{m=1}^{\infty} \int d\Omega \frac{dI_m}{d\Omega}, \quad d\Omega = \sin\vartheta \, d\vartheta \, d\phi,$$

describes the radiation, which, for a given angle θ, has a discrete spectrum determined by

$$\omega_m = \frac{m\omega_0}{|1 - \beta_{1\|}\cos\vartheta|}, \qquad m = 1, 2, \ldots, \quad \omega_0 = v_\perp/\rho_0$$

- Normal Doppler effect $\beta_{1\|} < 1$ and $\beta_{1\|} > 1, \vartheta > \vartheta_0,$
- Anomalous Doppler effect $\vartheta < \vartheta_0$, in the case $\beta_{1\|} > 1$

Radiation features

- Non-relativistic motion $\beta_{1\perp}, \beta_{1\|} \ll 1,$

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \approx \frac{2q^2c(m\beta_{1\perp}/2)^{2(m+1)}}{\pi\rho_0^2\varepsilon_1^{3/2}(m!)^2}\left[1+\underbrace{\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0+\varepsilon_1}}_{\text{Induced by the presence of the cylinder}}\left(\frac{\rho_1}{\rho_0}\right)^{2m}\right]^2(1+\cos^2\vartheta)\sin^{2(m-1)}\vartheta,$$

Contribution of the harmonics with $m > 1$ is small compared to that in the fundamental one, $m = 1$

- In the limit $\rho_1 \to 0$, the difference between the radiation intensities in the cases when the cylinder is present and absent, $\mathrm{d}I_m/\mathrm{d}\Omega - \mathrm{d}I_m^{(0)}/\mathrm{d}\Omega \propto \rho_1^{2m}$ for $m \geqslant 1$

$$\frac{\mathrm{d}I_m^{(0)}}{\mathrm{d}\Omega} = \frac{q^2\omega_0^2m^2}{2\pi c\sqrt{\varepsilon_1}|1-\beta_{1\|}\cos\vartheta|^3}\left[\beta_{1\perp}^2 J_m'^{\,2}(\lambda_1\rho_0) + \left(\frac{\cos\vartheta-\beta_{1\|}}{\sin\vartheta}\right)^2 J_m^2(\lambda_1\rho_0)\right]$$

- In the same limit and for the radiation corresponding to $m = 0$, the part induced by the cylinder vanishes like ρ_1^2

Radiation features

- Behavior of the radiation intensity near the Cherenkov angle when

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto |1-\beta_{1\parallel}\cos\vartheta|^{-2}, \qquad |1-\beta_{1\parallel}\cos\vartheta| \ll 1$$

- Near the Cherenkov cone the frequencies of the radiated photons are large and the dispersion of the dielectric permittivity ε_1 should be taken into account

- For the radiation intensity in a homogeneous medium with dielectric permittivity ε_1 we have the same behavior

- For the charge helical motion inside the dielectric cylinder $(\rho_0 < \rho_1)$ the behavior of the radiation intensity near the Cherenkov cone is radically different for

$$\beta_{0\parallel} > 1 \qquad \frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto |1-\beta_{1\parallel}\cos\vartheta|^{-4}$$

$$\beta_{0\parallel} < 1 \qquad \frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto |1-\beta_{1\parallel}\cos\vartheta|^{-4}\exp\left[-2(\omega_m/v_\parallel)(\rho_1-\rho_0)\sqrt{1-\beta_{0\parallel}^2}\right]$$

Strong peaks

- Strong narrow peaks are present in the angular distribution for the radiation intensity at a given harmonic *m*

- The condition for the appearance of the peaks is obtained from the equation determining the eigenmodes for the dielectric cylinder by the replacement

$$H_m \to Y_m$$

Hankel function of the first kind

Neumann function

Bessel function

$$\sum_{l=\pm 1}\left[\frac{\lambda_1}{\lambda_0}\frac{J_{m+l}(\lambda_0\rho_1)Y_m(\lambda_1\rho_1)}{J_m(\lambda_0\rho_1)Y_{m+l}(\lambda_1\rho_1)}-1\right]^{-1}=\frac{2\varepsilon_0}{\varepsilon_1-\varepsilon_0}$$

$$\lambda_j^2=\frac{\omega_m^2(k_z)}{c^2}\varepsilon_j-k_z^2,\qquad j=0,1,\quad \omega_m(k_z)=m\omega_0+k_z v_\parallel$$

Strong peaks

- As necessary conditions for the presence of the strong narrow peaks in the angular distribution for the radiation intensity one has

$$\frac{\omega_0\rho_0}{c}\sqrt{\varepsilon_1}\sin\vartheta < |1-\beta_{1\|}\cos\vartheta| < \frac{\omega_0\rho_1}{c}\sqrt{\varepsilon_0-\varepsilon_1\cos^2\vartheta}$$

- These conditions can be satisfied only if we have

$$\varepsilon_0 > \varepsilon_1, \qquad \tilde{v}\sqrt{\varepsilon_0}/c > 1, \text{ where } \tilde{v} = \sqrt{v_\|^2+\omega_0^2\rho_1^2}$$

velocity of the charge image on the cylinder surface

- Angular dependence of the radiation intensity near the peak

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto \frac{1}{(\vartheta-\vartheta_p)^2/b_p^2+1}\left(\frac{\mathrm{d}I_m}{\mathrm{d}\Omega}\right)_{\vartheta=\vartheta_p}, \qquad b_p \propto \exp[-2m\zeta(\lambda_1\rho_1/m)]$$

- Angular widths of the peaks: $\Delta\vartheta \propto \exp[-2m\zeta(\lambda_1\rho_1/m)]$

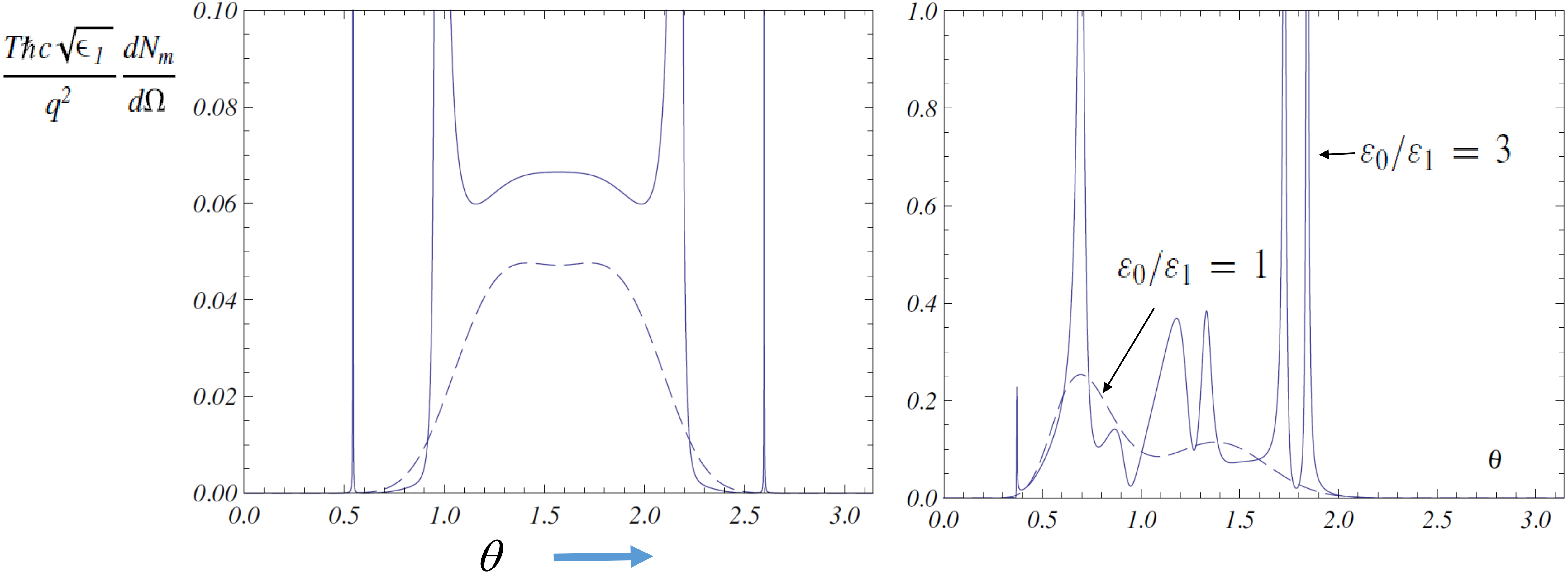
$$\zeta(z) = \ln\frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}, \qquad \lambda_1 = \frac{m\omega_0}{c}\frac{\sqrt{\varepsilon_1}\sin\vartheta}{1-\beta_{1\|}\cos\vartheta}$$

Numerical examples

Angular density for the Number of the Radiated Quanta $\quad \frac{dN_m}{d\Omega} = \frac{1}{\hbar\omega_m}\frac{dI_m}{d\Omega}$

$$\beta_{1\perp} = 0.9,\ \rho_1/\rho_0 = 0.95,\ m = 10$$

$\beta_{1\parallel} = 0$ $\qquad\qquad$ $\beta_{1\parallel} = 0.5$



Models with negative permittivity

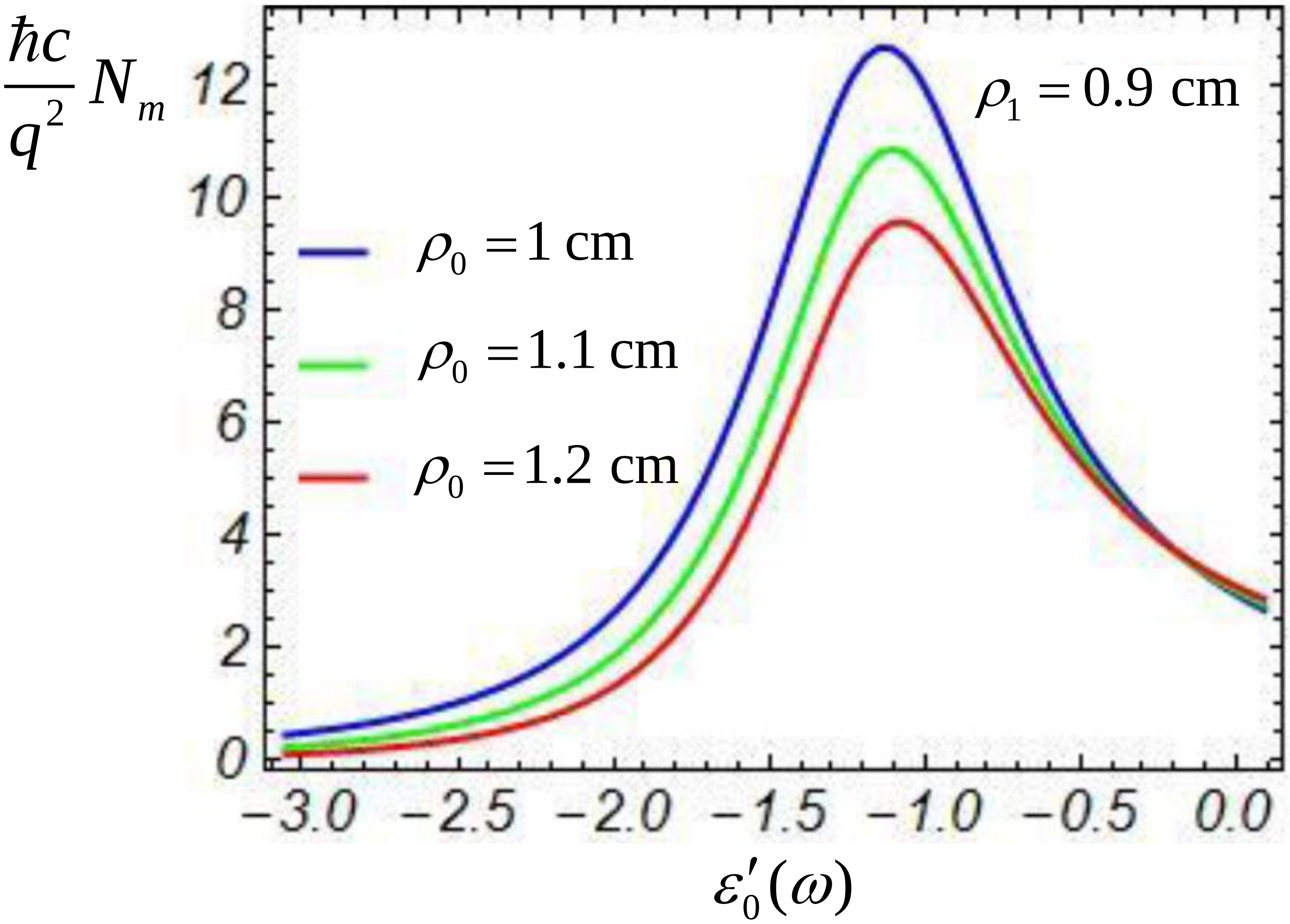
- We consider a model for dielectric permittivity

$$\varepsilon_0(\omega) = \tilde{\varepsilon}_0 - \frac{\omega_{pl}^2}{\omega^2 + i\gamma\omega} = \varepsilon_0' + i\varepsilon_0'', \quad \tilde{\varepsilon}_0 > 1$$

- Total number of the radiated quanta is evaluated on the main harmonic *m*=1 for a cylinder consisting of melted quartz with a small mixture of gold

- Numerical evaluations show that for some resonant frequencies of particle rotation, for which the real part of the dielectric permittivity is negative, the integrated radiation intensity at large distances is strongly amplified

Numerical example



Conclusions

- We have investigated the properties of the radiation from a charged particle moving along a helical orbit around a dielectric cylinder
- Under certain conditions on the parameters strong narrow peaks appear in the angular distribution of the radiation intensity in the exterior medium
- We have specified the conditions for the appearance of the peaks and analytically estimated their heights and widths
- Peaks are present only when dielectric permittivity of the cylinder is greater than the permittivity for the surrounding medium and the Cherenkov condition is satisfied for the velocity of the charge image on the cylinder surface and the dielectric permittivity of the cylinder
- Presence of the cylinder provides a possibility for an essential enhancement of the radiated power as compared to the radiation in a homogeneous medium

Thank You!