X-ray Hybrid Radiation

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Motivations and goals

Top Goal:
- Development of soft x-ray sources in the water window spectral range (284 – 543 eV) based on x-ray Cherenkov radiation.

Subgoals:
1. angular distribution of x-ray hybrid radiation when relativistic charge crosses a finite-size screen under an incidence angle;
2. spectral properties and determine monochromaticity;
3. investigation of polarization properties of hybrid radiation.
Report outline:

1. Radiation geometry and initial conditions for considered task;
2. Applied methods;
3. Results and discussions;
4. Conclusion.
Radiation geometry

the main points for calculation:

- charge passes through a screen under an incidence angle;
- a finite-size creen;
- a low energy charge (from a few MeV to 100 Mev);
- soft x-ray range.
Expression for spectral-angular distribution of polarization radiation

Spectral-angular distribution for considered radiation was obtained analytically\(^1\):

\[
\frac{d^2W}{d\omega d\Omega} = \frac{e^2}{\pi^2c} \beta^2 \cos^2 \alpha \left| \frac{\epsilon(\omega) - 1}{\epsilon(\omega)} \right|^2 \frac{\cos^2 \alpha}{\left[ (1 + \beta \sin \alpha \sin \theta \cos \phi)^2 - \beta^2 \cos^2 \alpha \cos^2 \theta \right]^2}
\]

\[
\times \left| 1 - \exp \left[ -id \frac{\omega}{\beta c \cos \alpha} \left( 1 - \beta \cos \alpha \sqrt{\epsilon(\omega) - \sin^2 \theta} + \beta \sin \alpha \sin \theta \cos \phi \right) \right] \right|^2
\]

\[
\times \left| \beta^4 \cos^2 \alpha \sin^2 \alpha \sin^2 \phi \left| \frac{\sqrt{\epsilon(\omega)}}{\cos \theta + \sqrt{\epsilon(\omega) - \sin^2 \theta}} \right|^2
\]

\[
\times \left( \sin^2 \theta + \left| \sqrt{\epsilon(\omega) - \sin^2 \theta} \right|^2 \right) + \left| \frac{\epsilon(\omega)}{\epsilon(\omega) \cos \theta + \sqrt{\epsilon(\omega) - \sin^2 \theta}} \right|^2
\]

\[
\times \left| (\beta^2 \cos^2 \alpha - 1 - \beta \sin \alpha \sin \theta \cos \phi) \sin \theta \right.
\]

\[
+ \beta \cos \alpha \sqrt{\epsilon(\omega) - \sin^2 \theta} \left( \sin \theta + \beta \sin \alpha \cos \phi \right) \left| \right|^2 \right]\]

\(^1\text{A.S. Konkov, Thesis, Tomsk Polytechnic University (2016).}\)
The Henke model of dielectric permittivity was used:

\[ \epsilon(\omega) = \left[ 1 - \frac{1}{2Z} \left( \frac{\hbar \omega_p}{\hbar \omega} \right)^2 f(\omega) \right]^2, \]

where \( \omega_p \) is the plasma frequency, \( Z \) is the atomic number and \( f(\omega) = f_1(\omega) \pm i f_2(\omega) \) is the complex atomic scattering factor\(^2\).

Stockes parameters for hybrid radiation

\[ \xi_1 = \frac{E_1^* E_1 - E_2^* E_2}{E_1^* E_1 + E_2^* E_2} \]
describes the amount of linear horizontal or vertical polarization.

\[ \xi_2 = i \frac{E_1^* E_2 - E_1 E_2^*}{E_1^* E_1 + E_2^* E_2} \]
corresponds to the amount of linear ±45° polarization.

\[ \xi_3 = \frac{E_1^* E_2 + E_1 E_2^*}{E_1^* E_1 + E_2^* E_2} \]
helps to determine the amount of right or left circular polarization.

In the case of considered geometry of radiation

\[ E_1 = C \left( \beta^2 e_z \cos \alpha \sin \alpha - e_y \left[ 1 - \beta^2 \cos^2 \alpha + \beta \epsilon (\omega) e_y \sin \alpha \right. \right. \]
\[ \left. - \beta \epsilon (\omega) e_z \cos \alpha \right] \), \]

\[ E_2 = C \left( e_x e_z \left[ 1 - \beta^2 \cos^2 \alpha + \beta \epsilon (\omega) e_y \sin \alpha - \beta \epsilon (\omega) e_z \cos \alpha \right] \right. \]
\[ + e_x e_y \beta^2 \cos^2 \alpha \sin^2 \alpha \), \]

where \( e = \{\sin \theta \sin \phi, \sin \theta \cos \phi, \cos \phi\} \).
Calculation parameters for Al

\(^{27}_{13}\)Al

\[ \rho = 2.7 \text{ g/cm}^3 \]
\[ d = 8 \mu m \]
\[ \hbar \omega = 62.6 \div 82.6 \text{ eV} \]

Scattering factors:

Dots are measurement data, the color curves are interpolation functions.
Angular distribution of hybrid radiation from Al screen

$\alpha = 25^\circ$

$\gamma = 10$

$\gamma = 25$

$\gamma = 50$

$\alpha = 55^\circ$
Spectrum of hybrid radiation from Al screen

$\gamma = 10$

$\gamma = 25$

$\gamma = 50$

$\alpha = 25^\circ$

$\alpha = 55^\circ$
Al, $\gamma = 25$, $\alpha = 55$ deg

Angular distribution of hybrid radiation.

Spectrum of hybrid radiation.

Angular distribution of hybrid radiation for plane $\phi = 0$.

Peak position and FWHM.
Polarization properties: $\xi_1$, Al, $\gamma = 25$, $\hbar \omega = 72.6$ eV

$\alpha = 0$ deg

$\alpha = 25$ deg

$\alpha = 55$ deg

The blue and the dashed blue curves correspond, respectively, to observation angles $\theta_1 = \alpha - 10$ and $\theta_2 = \alpha + 10$ deg.
Polarization properties: $\xi_2$, Al, $\gamma = 25$, $\hbar \omega = 72.6$ eV

$\alpha = 0$ deg

$\alpha = 25$ deg

$\alpha = 55$ deg

The red and the dashed red curves correspond, respectively, to observation angles $\theta_1 = \alpha - 10$ and $\theta_2 = \alpha + 10$ deg.
Polarization properties: $\xi_3$, Al, $\gamma = 25$, $\hbar\omega = 72.6$ eV

$\alpha = 0$ deg

$\alpha = 25$ deg

$\alpha = 55$ deg

The green and the dashed green curves correspond, respectively, to observation angles $\theta_1 = \alpha - 10$ and $\theta_2 = \alpha + 10$ deg.
Calculation parameters for Ti

$^{48}_{22}$Ti

$\rho = 4.54 \text{ g/cm}^3$
$d = 8 \mu m$
$\hbar \omega = 443.8 \div 463.8 \text{ eV}$

Scattering factors:

Dots are measurement data, the color curves are interpolation functions.
Angular distribution of hybrid radiation from Ti screen

$\alpha = 25^\circ$

$\gamma = 25$

$\gamma = 50$

$\gamma = 100$

$\alpha = 75^\circ$
Spectrum of hybrid radiation from Ti screen

\[ \alpha = 25^\circ \]

\[ \alpha = 75^\circ \]
Ti, $\gamma = 50$, $\alpha = 75$ deg

Angular distribution of hybrid radiation.

Spectral-angular distribution of hybrid radiation for plane $\phi = 0$.

Spectrum of hybrid radiation.

Peak position and FWHM.
Polarization properties: $\xi_1$, Ti, $\gamma = 50$, $\hbar \omega = 453.8$ eV

The blue and the dashed blue curves correspond, respectively, to observation angles $\theta_1 = \alpha - 3$ and $\theta_2 = \alpha + 3$ deg.
Polarization properties: $\xi_2, \, \text{Ti}, \, \gamma = 50, \, \hbar \omega = 453.8 \, \text{eV}$

The red and the dashed red curves correspond, respectively, to observation angles $\theta_1 = \alpha - 3$ and $\theta_2 = \alpha + 3 \, \text{deg}$. 
Polarization properties: $\xi_3$, Ti, $\gamma = 50$, $\hbar \omega = 453.8$ eV

The green and the dashed green curves correspond, respectively, to observation angles $\theta_1 = \alpha - 3$ and $\theta_2 = \alpha + 3$ deg.
Conclusions:

▶ hybrid radiation has asymmetry of angular distribution;
▶ spectrum asymmetry occurs;
▶ radiation monochromaticity depends on observation angle;
▶ hybrid radiation has circular polarization;

Future plans:

▶ we are going to investigate properties of hybrid radiation from layered structures;
▶ use interference effect to suppress influence of transition radiation on spectrum.
Thank you for attention!

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