Electromagnetic dipole moment and time reversal invariance violating interactions for high energy short-lived particles in bent and straight crystals at Large Hadron Collider

V.G. Baryshevsky

Institute for Nuclear Problems, Belarusian State University, Minsk, Belarus
P non-invariance  T non-invariance

To be or not to be
Spin rotation effect of ultrarelativistic particles passing through a crystal


*V.G. Baryshevsky*, Spin rotation and depolarization of high-energy particles in crystals at Hadron Collider (LHC) and Future Circular Collider (FCC) energies and the possibility to measure the anomalous magnetic moments of short-lived particles, arXiv:1504.06702 [hep-ph]

First experiment to measure \((g-2)\) rotation

E761 Collaboration, FERMILAB

"First observation of spin precession of polarized \(\Sigma^+\) hyperons channeled in bent crystals”, LNPI Research Reports (1990-1991) 129.

Energy of \(\Sigma^+\): 200 – 300 GeV

D. Chen et all


A.V. Khanzadeev, V.M. Samsonov, R.A. Carrigan, D. Chen

"Experiment to observe the spin precession of channeled relativistic \(\Sigma^+\) hyperons” NIM 119 (1996) 266.
Electromagnetic dipole moment and particles spin rotation in bent crystals at Large Hadron Collider

Non-Relativistic Hamiltonian

\[
H = -\vec{\mu} \vec{B} - \frac{d \vec{E}}{dt}
\]

\[
\begin{align*}
\text{C-even} & \quad \text{P-even} & \quad \text{T-even} \\
\text{C-even} & \quad \text{P-odd} & \quad \text{T-odd}
\end{align*}
\]

Relativistic equation

\[
\frac{d \vec{S}}{dt} = -\frac{e(g-2)}{2mc} \left[ \vec{S} \times \left[ \vec{\beta} \times \vec{E} \right] \right] + \\
+ \frac{d}{\hbar S} \left[ \vec{S} \times \vec{E} \right].
\]


T non-invariance interactions at LHC and FCC

• **V.G. Baryshesky**, On the search for the electric dipole moment of strange and charm baryons at LHC and parity violating (P) and time reversal (T) invariance violating spin rotation and dichroism in crystal, arXiv: 1708.09799v1 [hep-ph], 31 Aug 2017.

The index of refraction and effective potential energy of relativistic particles in matter

The wave number of the particle in vacuum is denoted \( k \), \( k' = kn \) is the wave number of the particle in medium. Expression for \( n \) does not contain \( \hbar \).

\[
n = 1 + \frac{2\pi N}{k^2} f(0)
\]

Kinetic energy of a particle in vacuum is not equal to that in medium.

\[
\begin{align*}
E &= \sqrt{\hbar^2 k^2 c^2 + m^2 c^4} \\
E_{\text{med}} &= \sqrt{\hbar^2 k^2 n^2 c^2 + m^2 c^4}
\end{align*}
\]
Effective potential energy of particle interaction in matter

From the energy conservation condition we immediately obtain the necessity to suppose that a particle in medium possesses effective potential energy. This energy can be found easily from the evident equality:

\[ E = E_{med} + U_{eff} \]

\[ U_{eff} = E - E_{med} = -\frac{2\pi \hbar^2}{m\gamma} N f(E,0) = (2\pi)^3 NT_{aa} (\vec{k'} - \vec{k} = 0) \]

\[ f(E,0) = -(2\pi)^2 \frac{E}{c^2\hbar^2} T_{aa} (\vec{k'} - \vec{k} = 0) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} T_{aa} (\vec{k'} - \vec{k} = 0) \]
Effective potential energy of particle interaction with plane and axis

For plane:

\[
\hat{U}(x) = -\sum_{\tau_x} \frac{2\pi \hbar^2}{m\gamma V} \hat{F}(q_x = \tau_x, q_y = q_z = 0)e^{i\gamma x} =
\]

\[
= -\frac{2\pi \hbar^2}{m\gamma V d_y d_z} \sum_{X_n} \hat{F}(x - X_n, q_y = q_z = 0)
\]

\[
\hat{F}(\bar{q}) = \int \hat{F}(\bar{r}')e^{-i\bar{q}\bar{r}'} d^3 \bar{r}'
\]

For axis:

\[
\hat{U}(\bar{\rho}) = -\frac{2\pi \hbar^2}{m\gamma V} \sum_{\tau_x, \tau_y} \hat{F}(q_x = \tau_x, q_y = \tau_y, q_z = 0)e^{i\gamma \bar{\rho}} =
\]

\[
= -\frac{2\pi \hbar^2}{m\gamma d_z} \sum_{R_{n\perp}} \hat{F}(\bar{\rho} - \bar{R}_{n\perp}, q_z = 0)
\]
Elastic scattering of a particle with spin 1/2

\[ \hat{F}(\vec{q}) = A(\vec{q}) + B(\vec{q})\hat{\sigma}\vec{N} + B_{0w}(\vec{q}) + B_{w}(\vec{q})\hat{\sigma}\vec{N}_w + B_T\hat{\sigma}\vec{N}_T \]

\[ \vec{q} = \vec{k}' - \vec{k}, \quad \vec{n} = \frac{\vec{k}}{k}, \quad \vec{N}_w = \frac{\vec{k}' + \vec{k}}{|\vec{k}' + \vec{k}|}, \quad \vec{N} = \begin{bmatrix} k \times k' \end{bmatrix}, \quad \vec{N}_T = \frac{\vec{q}}{q}. \]

\[ \frac{d\sigma}{d\Omega} = tr \rho \hat{F}^+ (\vec{q}) \hat{F}(\vec{q}) \]

\[ \xi = \frac{tr \rho F^+ \sigma F}{tr \rho F^+ F} = \frac{tr \rho F^+ \sigma F}{\frac{d\sigma}{d\Omega}} \]

\[ \xi = \xi_{so} + \xi_w + \xi_T \]
Scattering of a particle with spin 1/2 in crystals

\[
\bar{\xi}_{so} = \left\{ |A|^2 - |B|^2 \right\} \bar{\xi}_0 + 2 \text{Im}(\bar{AB}^*)[\bar{N} \bar{\xi}_0] + 2 |B|^2 \bar{N}(\bar{N} \bar{\xi}_0) + 2 \bar{N} \text{Re}(\bar{AB}^*) \right\} \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1}
\]

\[
\bar{\xi}_w = \left\{ |A|^2 - |B_w|^2 \right\} \bar{\xi}_0 + 2 \text{Im}(\bar{AB}_w^*)[\bar{N}_w \bar{\xi}_0] + 2 |B_w|^2 \bar{N}_w(\bar{N}_w \bar{\xi}_0) + 2 \bar{N}_w \text{Re}(\bar{AB}_w^*) \right\} \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1}
\]

\[
\bar{\xi}_T = \left\{ |A|^2 - |B_T|^2 \right\} \bar{\xi}_0 + 2 \text{Im}(\bar{AB}_T^*)[\bar{N}_T \bar{\xi}_0] + 2 |B_T|^2 \bar{N}_T(\bar{N}_T \bar{\xi}_0) + 2 \bar{N}_T \text{Re}(\bar{AB}_T^*) \right\} \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1}
\]

\[
\frac{d\sigma}{d\Omega} = \text{tr} \rho F^+ F = |A|^2 + |B|^2 + |B_w|^2 + |B_T|^2 + 2 \text{Re}(\bar{AB}^*) \bar{N} \bar{\xi}_0 + 2 \text{Re}(\bar{AB}_w^*) \bar{N}_w \bar{\xi}_0 + 2 \text{Re}(\bar{AB}_T^*) \bar{N}_T \bar{\xi}_0
\]

Both rotation around \( \bar{N}, \bar{N}_w, \bar{N}_T \) and components in directions of \( \bar{N}, \bar{N}_w, \bar{N}_T \) appear.
Effective potential energy determined by the anomalous magnetic moment

\[ \hat{F}^{(1)}_{\text{magn}}(q) = B_{\text{magn}}(q)\vec{\sigma}[\vec{n} \times \vec{q}] \]

\[ \hat{U}^{(1)}_{\text{magn}} = -\frac{e\hbar}{2mc} \frac{g-2}{2} E_{\text{plane}}(x)\vec{\sigma}\vec{N} \]

\[ \vec{N} = [\vec{n}_x \times \vec{n}], \vec{n}_x \parallel \vec{E}(x), \vec{n}_x \perp \vec{n}, \vec{n} = \frac{\vec{k}}{k} \]
Effective potential energy determined by the anomalous magnetic moment

\[ \hat{F}^{(2)}(\vec{q} = \vec{\tau}) = i \frac{k}{4\pi \hbar^2 c^2} \iiint e^{-i\vec{r} \cdot \vec{r}_\perp} \left\{ \left[ \int \hat{V}(\vec{r}_\perp, z) dz \right]^2 - \left[ \int \hat{V}(\vec{r}_\perp, z) dz \right]^2 \right\} d^2 r_\perp \]

\[ \hat{V}(\vec{r}_\perp, z) = V_{\text{coul}}(\vec{r}_\perp, z) + \hat{V}_{\text{magn}}(\vec{r}_\perp, z) \]

\[ \hat{U}_{\text{magn}}^{(2)}(x) = -i \frac{1}{4d_y d_z mc^2} \left( \frac{g-2}{2} \right) \frac{\partial}{\partial x} \delta V_{\text{coul}}^2(x) \tilde{\sigma} \tilde{N} \]

\[ \hat{U}_{\text{magn}}(x) = -(\alpha_m(x) + i\delta_m(x)) \tilde{\sigma} \tilde{N} \]
Effective potential energy determined by P-odd and T-even interactions

\[ \hat{F}_w (\tilde{q}) = (B_{we} (\tilde{q}) + B_{wnuc} (\tilde{q})) \tilde{\sigma} \tilde{N}_w \]

\[ \hat{U}_w (x) = \hat{U}_{we} (x) + \hat{U}_{wnuc} (x) = -(\alpha_w (x) + i \delta_w (x)) \tilde{\sigma} \tilde{N}_w \]

\[ \alpha_w (x) = \alpha_{we} (x) + \alpha_{wnuc} (x) \]
\[ \delta_w (x) = \delta_{we} (x) + \delta_{wnuc} (x) \]

\[ \alpha_w (x) = \frac{2 \pi \hbar^2}{m \gamma d_y d_z} (\tilde{B}'_{we} (0) N_e (x) + \tilde{B}'_{wnuc} (0) N_{nuc} (x)) \]

\[ \delta_w (x) = \frac{2 \pi \hbar^2}{m \gamma d_y d_z} (\tilde{B}''_{we} (0) N_e (x) + \tilde{B}''_{wnuc} (0) N_{nuc} (x)) \]
Effective potential energy determined by the electric dipole moment and other T-nonivariant interactions

\[ \hat{F}_T(q) = (B_{EDM}(q) + B_{Te}(q) + B_{Tnuc}(q))\hat{\sigma}\tilde{q} \]

\[ \tilde{q} = \vec{k}' - \vec{k} \]

\[ \hat{U}_T(x) = \hat{U}_{EDM} + \hat{U}_{Te} + \hat{U}_{Tnuc} = - (\alpha_T(x) + i\delta_T(x))\hat{\sigma}\tilde{N}_T \]

\[ \hat{U}_{EDM}(x) = - (\alpha_{EDM}(x) + i\delta_{EDM}(x))\hat{\sigma}\tilde{N}_T, \tilde{N}_T = \vec{n}_x \]

\[
\begin{align*}
\alpha_T(x) &= \alpha_{EDM} + \alpha_{Te} + \alpha_{Tnuc} \\
\delta_T(x) &= \delta_{EDM} + \delta_{Te} + \delta_{Tnuc}
\end{align*}
\]

\[
\begin{align*}
\alpha_{Te(nuc)} &= \frac{2\pi\hbar^2}{m\gamma d_y d_z} \tilde{B}'_{Te(nuc)} \frac{dN_{e(nuc)}(x)}{dx} \\
\delta_{Te(nuc)} &= \frac{2\pi\hbar^2}{m\gamma d_y d_z} \tilde{B}''_{Te(nuc)} \frac{dN_{e(nuc)}(x)}{dx}
\end{align*}
\]
P and CP violating spin rotation in bent crystals

\[ \frac{i\hbar}{\partial t} \Psi(t) = \hat{U}_{\text{eff}} |\Psi(t)\rangle \]

\[ \bar{\xi} = \frac{<\Psi(t)|\vec{\sigma}|\Psi(t)>}{<\Psi(t)|\Psi(t)>} \]
P and CP violating spin rotation in bent crystals

\[
\frac{d\tilde{\xi}}{dt} = \left[ \tilde{\xi} \times \tilde{\Omega}_{mso} \right] - \frac{2}{\hbar} \left( \delta_m (x) + \delta_{s0} (x) \right) \{ \tilde{N}_m - \tilde{\xi} (\tilde{N}_m \tilde{\xi}) \} + \\
+ \left[ \tilde{\xi} \times \tilde{\Omega}_T \right] + \frac{2}{\hbar} \left( \delta_{EDM} (x) + \delta_{Te} (x) + \delta_{Tnuc} (x) \right) \{ \tilde{N}_T - \tilde{\xi} (\tilde{N}_T \tilde{\xi}) \} + \\
+ \left[ \tilde{\xi} \times \tilde{\Omega}_w \right] - \frac{2}{\hbar} \delta_w \{ \tilde{n} - \tilde{\xi} (\tilde{n} \tilde{\xi}) \}.
\]

\[
\tilde{\Omega}_{mso} = \tilde{\Omega}_{MDM} + \tilde{\Omega}_{so} = - \left( \frac{e(g-2)}{2mc} E_x (x) + \frac{2}{\hbar} \alpha_{so} (x) \right) \tilde{N}_m, \\
\tilde{\Omega}_T = \tilde{\Omega}_{EDM} + \tilde{\Omega}_{ten} = \frac{2}{\hbar} (dE_x (x) + \alpha_{Te} (x) + \alpha_{Tnuc} (x)) \tilde{N}_T, \\
\tilde{\Omega}_w = \frac{2}{\hbar} \alpha_{w} \tilde{n}.
\]

\[
\tilde{N}_m = \left[ \tilde{n} \times \tilde{n}_x \right], \\
\tilde{N}_T = \tilde{n}_x, \\
\tilde{n} = \frac{\tilde{k}}{k}
\]
Behavior of the spin rotation caused by magnetic moment and T-reversal violation interactions. Black arrows represent spin rotation about effective magnetic field (about bent axis, direction $\vec{N}_m$), red arrows represent spin rotation about electric field (direction $\vec{N}_T$), purple arrows represent new effect – magnetic spin rotation in direction $\vec{N}_m$, spin rotation owing to P-violating interactions, is not shown here for simplicity.
Hyperbolic magnetic spin rotation and EDM (Todd interactions) measuring

The following estimation for the value $\delta_m$ can be obtained: $\delta_m \sim 10^8 - 10^9 \text{ sec}^{-1}$. The charm baryon EDM is predicted to be as large as $d \sim 10^{-17}$. Spin rotation frequency $\Omega_{EDM}$ determined by such charmed baryon EDM is $\Omega_{EDM} \sim 10^6 - 10^7 \text{ sec}^{-1}$. As a result, the nonelastic processes, which are caused by magnetic moment scattering, can imitate the EDM and T odd contribution.
Behavior of the spin rotation caused by magnetic moment, T-reversal violation interactions (including EDM) and P-violation spin rotation about direction $\vec{n}$ and rotation in direction $\vec{n}$ (orange and green arrows). Rotation in direction $\vec{N}_m$ and direction $\vec{N}_T$ is not shown for simplicity. It is obvious that P-odd T-even interactions can imitate EDM rotation.
P violating spin rotation in bent crystals

Precession frequency $\Omega_w$ is determined by the real part of the amplitude of baryon weak scattering by an electron (nucleus). This amplitude can be evaluated by Fermi theory for the energies, which are necessary for W and Z bosons production or smaller:

$$ReB \sim G_F k = 10^{-5} \frac{1}{m_p^2} k = 10^{-5} \frac{\hbar}{m_p c} \frac{m\gamma}{m_p} = 10^{-5} \lambda_{cp} \frac{m\gamma}{m_p}$$

For different particle trajectories in a bent crystal the value of precession frequency $\Omega_w$ could vary in the range $\Omega_w \sim 10^3 - 10^4 \text{ sec}^{-1}$. Therefore, when a particle passes 10 $cm$ in a crystal, its spin undergoes additional rotation around momentum direction at angle $\vartheta_p \sim 10^{-6} - 10^{-7} \text{ rad}$. The effect grows for a heavy baryon as a result of the mechanism similar to that of its EDM growth!
• When analyzing particle’s spin rotation, which is caused by electric dipole moment interaction with electric field, one should consider both $P_{odd}, T_{even}$ and $P_{odd}, T_{odd}$ non-invariant spin rotations, resulting from weak interaction with electrons and nuclei.

• It gives unique possibility for measurement of constants determining $T_{odd}, P_{odd}$ (CP) violating interactions and $P_{odd}, T_{even}$ interactions of baryons with electrons and nucleus (nucleons).

• Spin orientation of particles (positive and negative), which have passed through the bent (straight) crystal, can be measured using the intensity asymmetry of the scattering of baryons in the second straight crystal.
Thank you!
By turning the crystal $180^\circ$ around the direction of incident baryon momentum, one could observe that $P_{\text{odd}}$ spin rotation does not change, while the sign of MDM and $T_{\text{odd}}$ spin rotations does due to change of the electric field direction. Subtracting results of measurements for two opposite crystal positions one could obtain the angle of rotation, which does not depend on $P_{\text{odd}}$ effect.
Separation of the contributions caused by MDM and T-odd spin rotation is possible when comparing experimental results for two initial orientations of polarization vector $\vec{\xi}$. Namely: $\vec{\xi} \parallel \vec{N}_m$ and $\vec{\xi} \parallel \vec{N}_t$, i.e. the initial $\vec{\xi}$ is parallel to the bending axis of the crystal or $\vec{E}$.

In real situation rotating the crystal by 90° so that direction of $S_0$ is parallel to $B^*$ can be more convenient.
Effective potential energy of particle interaction with crystal

\[ U(\vec{r}) = \sum_{\vec{\tau}} U(\vec{\tau}) e^{i\vec{\tau}\cdot\vec{r}} \]

\[ U(\vec{\tau}) = \frac{1}{V} \sum_{j} U_{j}(\vec{\tau}) e^{i\vec{\tau}\cdot\vec{r}_{j}} \]

\[ U_{j}(\vec{\tau}) = -\frac{2\pi\hbar^{2}}{m\gamma} F_{j}(\vec{\tau}) \]

\[ F_{j}(\vec{k}' - \vec{k}) = f_{j}(\vec{k}' - \vec{k}) - i \frac{k}{4\pi} \int f_{j}^{*}(\vec{k}'' - \vec{k}') f_{j}(\vec{k}'' - \vec{k}) d\Omega_{k''} \]
**P violating spin rotation in bent crystals**

Absorption caused by parity violating weak interaction also contributes to change in spin direction. This rotation is caused by the imaginary part of weak scattering amplitude and is proportional to the difference of total scattering cross-sections $\sigma_{\uparrow\uparrow}$ and $\sigma_{\downarrow\uparrow}$.

$$\sigma_{\uparrow\uparrow(\downarrow\uparrow)} = \int \left| f_{c(nuc)} + B_{0w} \pm B_w \right|^2 d\Omega$$

$$\sigma_{\uparrow\uparrow} - \sigma_{\downarrow\uparrow} = 2\int \left[ (f_{c(nuc)} + B_{0w})B^* + (f_{c(nuc)} + B_{0w})^* B \right] d\Omega$$

When baryon trajectory passes in the area, where collisions with nuclei are important (this occurs in the vicinity of potential barrier for positively charged particles), the value $\delta_w \sim 10^6 - 10^7$ sec$^{-1}$. Similar to the real part $ReB$ for the case of heavy baryons the difference in cross-sections grows.
Effective potential energy determined by spin-orbit interaction

\[ \hat{F}_{ssp-orb}(\vec{q} = \vec{\tau}) = B_s(\vec{\tau})\vec{\sigma}[\vec{n} \times \vec{\tau}] \]

\[ \hat{U}_{ssp-orb} = -(\alpha_s + i\delta_s)\vec{\sigma}\vec{N} \]

Spin structure of \( \hat{U}_s(x) \) is similar to the one of \( \hat{U}_{magn}(x) \).

\[ \vec{N} = [\vec{n}_x \times \vec{n}] \]

\[ \alpha_s = -\frac{2\pi\hbar^2}{m\gamma d_y d_z} \frac{\partial N_{nuc}}{\partial x} B'' \]

\[ \delta_s = \frac{2\pi\hbar^2}{m\gamma d_y d_z} B' \frac{\partial N_{nuc}}{\partial x} \]