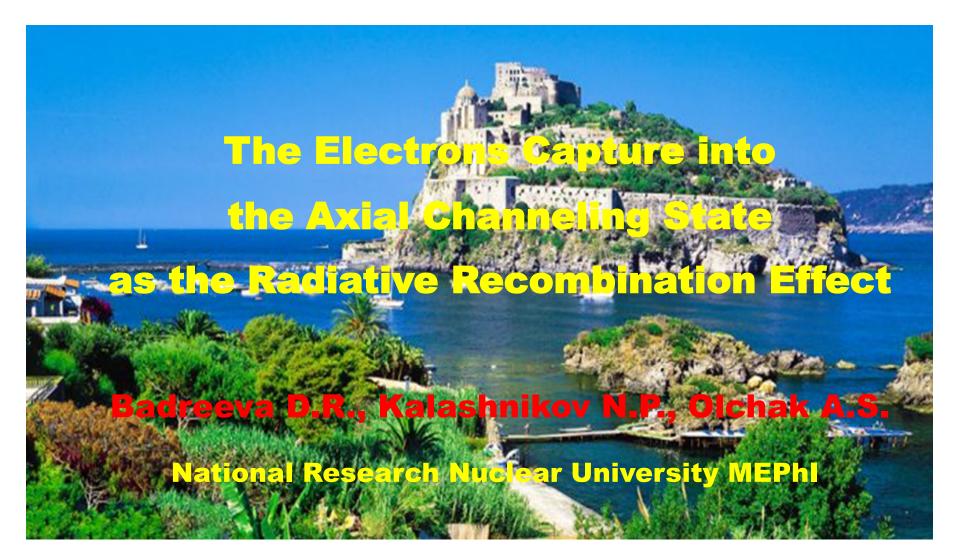


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1. Kinematic characteristics of channeling particles in the so called accompanying reference system (ARS);

Consider the Lorentz transformations for energy-momentum from the laboratory coordinate system (LCS) to the so called accompanying reference system (ARS):

 $plxt' = plx - V/ct^2 E/\sqrt{1 - Vt^2/ct^2}$; plyt' = ply; plzt' = plz; $Et' = E - Vplx/\sqrt{1 - Vt^2/ct^2}$; where

E is the energy, plx is the longitudinal momentum of the channeling particles in the (LCS); ARS moves with the velocity V=plx cf2/E.

Substituting velocity V in the Lorentz transformations, we get:

 $p l x t^{*} = 0;$ $p l \perp t^{*} = p l \perp t;$ $E t^{*} = \sqrt{p l \perp t^{2} c t^{2} + m l 0 t^{2} c t^{4}}.$

The last expression for energy of the channeling particle in the non-relativistic case can be rewritten in LCS as:

 $E\uparrow \cong m\downarrow 0 c\uparrow 2 + p\downarrow \perp \uparrow 2 / 2m\downarrow 0$

Remark. Taking into account an expression for the Lindhard critical angle $\theta L = \sqrt{2}U L = \sqrt{$

the transversal motion of the channeling particles in the initial state can be considered as nonrelativistic at energies $E \le m_{20.72}$ cf2 /2*U*/40





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2. Axial channeling electron as a 2D model of the atom;

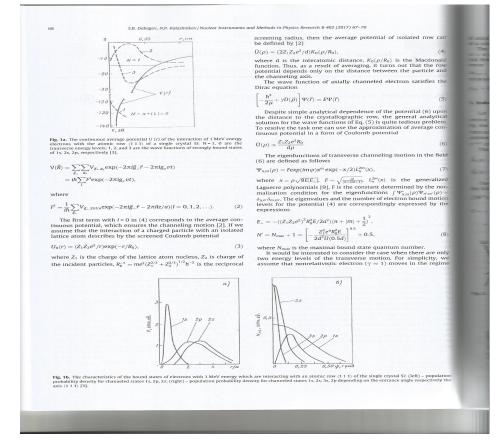
The ultra-relativistic electron with energy E up to ~10¹¹ eV moving along the crystallographic axis in an axial channeling mode, can be considered as a non-relativistic 2D-atom, possessing the discrete spectrum of energies and orbital momentum, if the electron motion is considered in the so called accompanying reference system (ARS), moving parallel to the channeling axis with the velocity, equal to the longitudinal component of the electron velocity.





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3. The average continuous potential of axial channeling [4];



The continuous interaction potential $V(\mathbf{r})$ of electrons having an energy of 1 MeV with the atomic row <111> of the single crystal Si: N = 1; 0 are the eigenenergies levels;

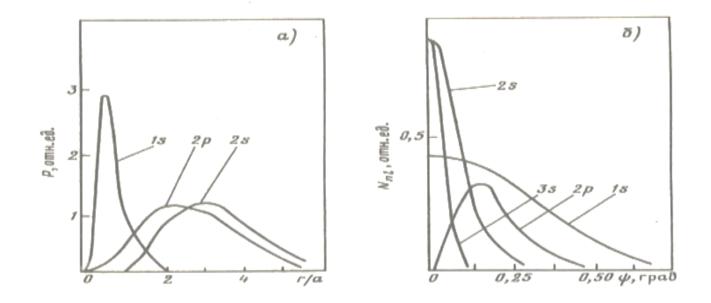
1, 2, and 3 are the wave functions of the strongly bound states of 1s, 2s, 2 p, respectively.





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3. The average continuous potential of axial channeling [4]:



The characteristics of the bound states of the electrons with energy E↓e =1 MeV when interacting with an atomic chain <111> of the single crystal Si :
a) is the distribution probability density for the states of 1s, 2s, 2 p;
b) is the probability density function of the states population 1s, 2s, 3s, 2 p depending on the entrance angle respect to axis <111>.





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4. The principle of detailed balance.

Let's consider the photorecombination (radiative recombination) process of the channeling particles: $X + Y \rightarrow XY + \hbar\omega$, which is a reversal process vis-à-vis the photoelectric effect: $\hbar\omega + XY \rightarrow X + Y$.

In accordance with the principle of detailed balance [6-7], the relation between the cross-sections of these processes, as their definition has the form: $\sigma \downarrow recom / \sigma \downarrow ph = g \downarrow recom / g \downarrow ph \cdot j \downarrow ph / j \downarrow p$, where $g \downarrow recom$, $g \downarrow ph$ are the statistical weights of the final states which correspond to the recombination process and the emission process respectively; where $j \downarrow ph$ and $j \downarrow p$ are the fluxes of the photons and the channeling particles consequently with a single photon in the given volume.





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5. The cross-section of electron recombination in two-dimensional potential with a photon emission and electron transition into any bound state;

In the accompanying reference system (ARS) (which is moving with the velocity $V = p \downarrow || c \uparrow 2 / E \downarrow 1$) the electron possesses the nonrelativistic energy $\epsilon \downarrow \perp$, depending on the entrance angle of the particles in a single crystal with respect to crystallographic axis.

The process of electrons photorecombination with photon emission and electron transition into the bound state with the main quantum number n (the bound state!) can be seen as a bremsstrahlung process of a photon by the electron. The bremsstrahlung differential cross-section of the electron with the transversal motion energy εl_{\perp} on the crystallographic row potential with the photon emission $\hbar\omega$ can be written as: $d\sigma l Br = 8\pi\alpha (Z\alpha) \int 2/3\sqrt{3} \ 1/\varepsilon l_{\perp} \ mc \int 3 \ d\omega / \hbar\omega$.





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5. The cross-section of electron recombination in two-dimensional potential with a photon emission and electron transition into any bound state;

For the photorecombination process from the conservation law of energy we have $\hbar\omega = (-\varepsilon \ln) + \varepsilon \ln$,

where $\epsilon ln = -\epsilon l0$ 1/n/2 is the eigenvalues of transversal energy in two-dimensional problem (see [4] p. 19)

(by the way, $\varepsilon l_0 \sim \gamma = E/mc^2$ for the axial channeling electron).

Thus, for the electron recombination cross-section in two-dimensional potential with the photon emission and an electron transition into the state with the main quantum number n we get:

 $\sigma \downarrow recom = |d\sigma \downarrow Br / dn| = 8\pi \alpha (Z\alpha) \uparrow 2 / 3\sqrt{3} \quad \varepsilon \downarrow 0 / mc \uparrow 3 \varepsilon \downarrow \perp n \uparrow 3 \hbar \uparrow 2 \omega.$

(this expression is called the Kramers formula [13])

Assuming that the energy of the bound electron can be expressed as $\epsilon \ln = -\epsilon \ln 1/n^2$, where

 $\epsilon J_0 = mc t_2 Z t_2 e t_4 R J T F t_2 / 2h t_2 d t_2 \gamma [4]$, we may rewrite $d\omega = -\epsilon J_0 dn/hn t_3$, and the electron recombination cross-section is

 $\sigma \downarrow recom = |d\sigma \downarrow Br / dn| = 8\pi \alpha (Z\alpha) / 2 / 3\sqrt{3} \quad \varepsilon \downarrow 0 / mc / 3 n \hbar / 2 \omega (\hbar \omega n / 2 - \varepsilon \downarrow 0).$





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5. The cross-section of electron recombination in two-dimensional potential with a photon emission and electron transition into any bound state;

Let's sum over all possible quantum numbers n

 $\sigma \downarrow recom = \sum n = 1 \uparrow n \downarrow max \implies |d\sigma \downarrow Br / dn | = \sum n = 1 \uparrow n \downarrow max \implies 8\pi \alpha (Z\alpha) \uparrow 2 / 3\sqrt{3}$ $\varepsilon \downarrow 0 / mc \uparrow 3 \hbar \uparrow 2 \omega n (\hbar \omega n \uparrow 2 - \varepsilon \downarrow 0) ,$

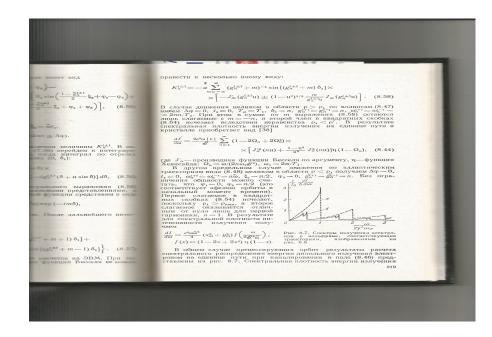
where $n \downarrow max = \sqrt{-m\lambda \hbar 2} Z \hbar 2 e \hbar 4 R \downarrow T F \hbar 2 / 2 \hbar \hbar 2 d \hbar 2 U(d \downarrow r / 2) - 1/2$; (λ is the fitting parameter that is selected so that the atomic chain potential was close to the Lindhard potential; Z is the atomic number of a single crystal).





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5. The cross-section of electron recombination in two-dimensional potential with a photon emission and electron transition into any bound state;



The radiation spectra of relativistic electrons in tungsten





CHANNELING 2018 6. The connection of the recombination cross-section and

the photoelectric effect cross-section;

Write the statistical weights of the states under consideration:

 $g \downarrow recom = g \downarrow XY \ 2 \ 4\pi k \uparrow 2 \ dk/(2\pi) \uparrow 3 \quad ; \quad g \downarrow ph = g \downarrow X \ g \downarrow Y \ 4\pi q \uparrow 2 \ dq/(2\pi) \uparrow 3 \quad ,$

where $g\downarrow X$, $g\downarrow Y$, $g\downarrow XY$ are statistical weights which correspond to the internal motion of these particles: X is a quasi-free (the initial state) particle, Y is the crystallographic axis potential, XY is a channeling (a bound particle in the axial mode channeling) particle, k is the photon wave vector, q is the wave vector of the particles relative motion.

Suppose that the photon flux in the volume V (if we have one photon) is j lph = c/V, and the particles flux (in the presence of a single particle in the volume V) equals j lp = v/V, where $v = \hbar q/\mu$ is the relative velocity, and μ is the reduced particles mass. By using expression $k = \omega/c$ for photons and the conservation law for energy: $\hbar \omega = \varepsilon l ln + \hbar t^2 qt^2/2\mu$, where $\varepsilon l ln$ is the bound energy of the particles in the axial channeling mode, μ is the reduced particles mass, we find the connection between the recombination cross-section and the photoelectric effect cross-section: $\sigma lrecom = 2 kt^2/qt^2 g lXY/g lX g lY \sigma lph$.





CHANNELING 2018 6. The connection of the recombination cross-section and

the photoelectric effect cross-section;

The last expression refers to the total cross-sections. An analogous expression can be obtained for differential cross-sections: $d \sigma \int recom /d\Omega \downarrow k = k \hbar 2 /q \hbar 2 g \downarrow XY /g \downarrow X g \downarrow Y d\sigma \downarrow ph /d\Omega \downarrow q$.

The connection for the differential cross-sections refers to processes in which the photon is emitted (or absorbed) with the specific polarization; so in the last expression there is no multiplier "2". Since we do not consider the spin states of the channeling particles and the spin state of a crystalline lattice, we can put statistical weights of the channeling electron and a crystalline lattice equal to unit: $g\downarrow X = g\downarrow Y = 1$. The statistical weight of the bound channeling state $g\downarrow XY$ is determined by the degeneration degree of the bound state of the transversal motion of the channeling particles with energy $\varepsilon\downarrow\perp n$, i.e. $(n\uparrow 2)$.





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7. The photorecombination cross-section in the ultra-relativistic case;

We use the connection of the photorecombination cross-section with the photoelectric effect cross-section:

$\sigma \downarrow recom = 2 \ k \uparrow 2 \ /q \uparrow 2 \ g \downarrow XY \ \sigma \downarrow ph,$

where σlph is the photoelectric effect cross-section, which in the ultrarelativistic case [6-10] can be written as [10]:

$\sigma \downarrow ph$ =3/4 U/015 (mc12/hw)8\pi e14/3m12 c14 ,

where $U \downarrow 0$ is the fitting parameter in the continuous potential of the crystallographic chain $(U \downarrow 0 \sim Ze^{2})$ [4].



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8. Discussion of the results. Conclusions;

The expression $W(\Omega) d\Omega = e^{\uparrow}2 \hbar \omega/2\pi m^{\uparrow}2 c^{\uparrow}3 \mid D\downarrow n\uparrow', n\uparrow k, e \mid \uparrow 2 d\Omega$ represents the probability that a channeling particle transfers from state *n* to state $n\uparrow'$ and emits photon per unit solid angle $d\Omega$ with the frequency ω is equal to $\omega \downarrow n, n\uparrow'$ and polarization *e*. The calculation of the matrix element $D\downarrow n\uparrow', n\uparrow k, e$ requires to use the eigenfunctions of the channeling particles $u\downarrow n$ and $u\downarrow n\uparrow'$ in the field of the continuous potential of the crystallographic axis, causing photon emission.

In the relativistic case, the Dirac relativistic spinor wave functions should be used for the eigenfunctions $u \downarrow n$ and $u \downarrow n \uparrow$, and momentum operator pshould be replaced by the operator $mc\alpha$, i.e. $D \downarrow n \uparrow$, $n \uparrow k$, $e = imc/\hbar \int \uparrow m u \downarrow n$ $n \uparrow \uparrow * e \uparrow ik R \ \alpha \downarrow e u \downarrow n \ d\tau$



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