Parametric X-ray Radiation in Crystals with Locally Disturbed Characteristics

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Outline

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- Introduction
- Motivation
- The problem
- Theory
- Summary and future plans

Parametric X-Ray Radiation

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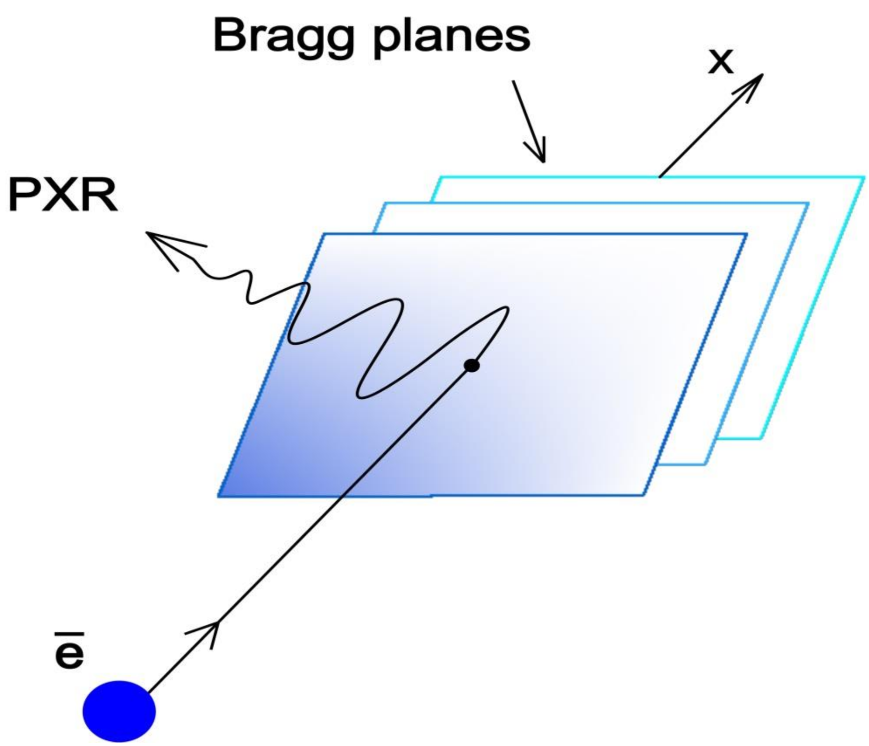
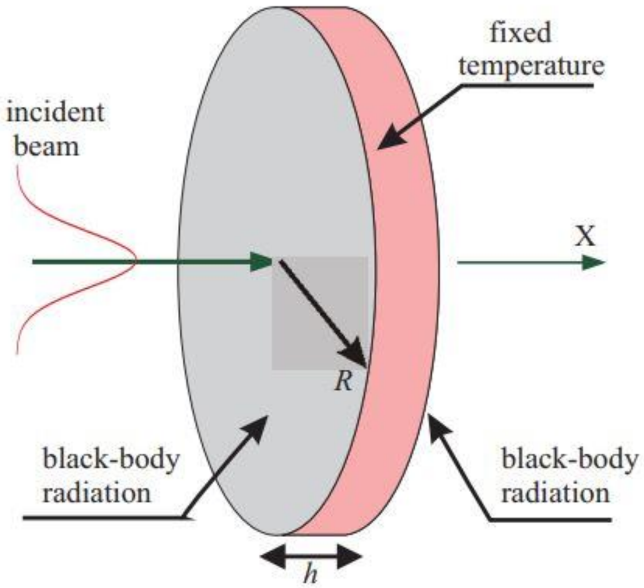


The nature of PXR is the reflection of the intrinsic field of charged particles from the periodic system of crystallographic planes

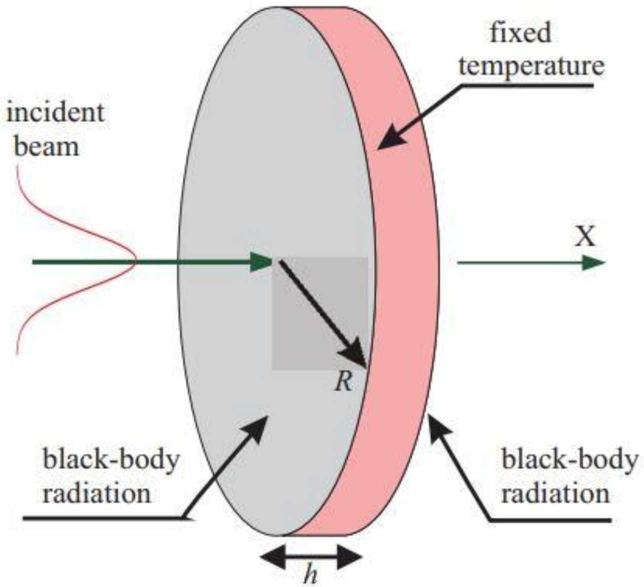
Motivation

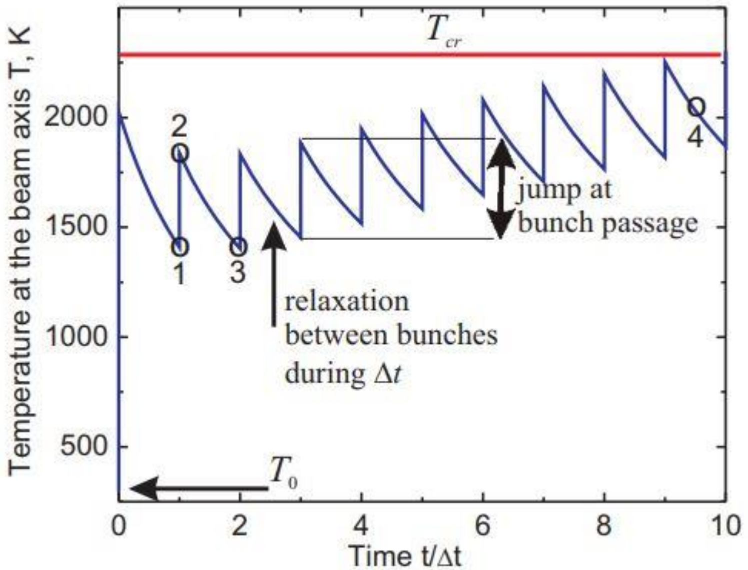
Motivation



Courtesy of A.A. Babaev, A.S. Gogolev, JoP:Conf. Series, **732** (2016) 012030

Motivation





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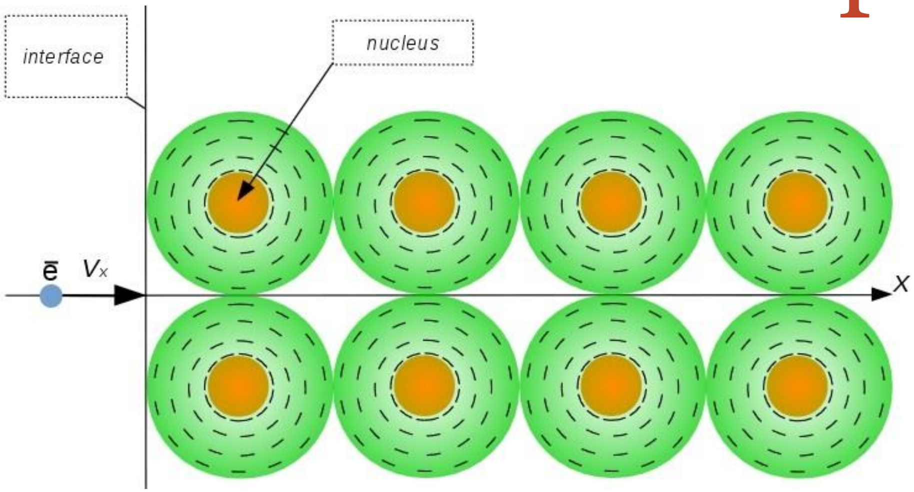
- Overheating and disturbing of the target at T_{cr}=2273 K (11th bunch);
- Gradients of temperature → Gradient of density → **WILL IT AFFECT PXR?**

The problem

The problem



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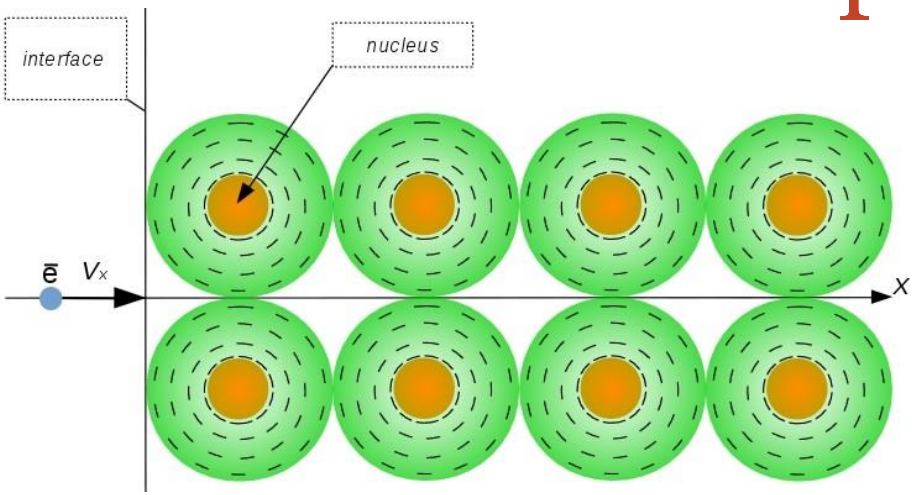

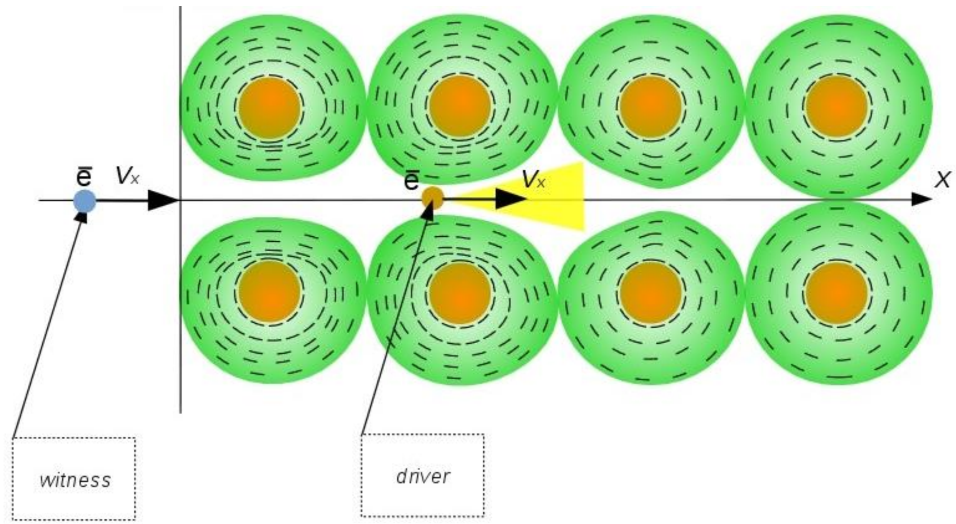

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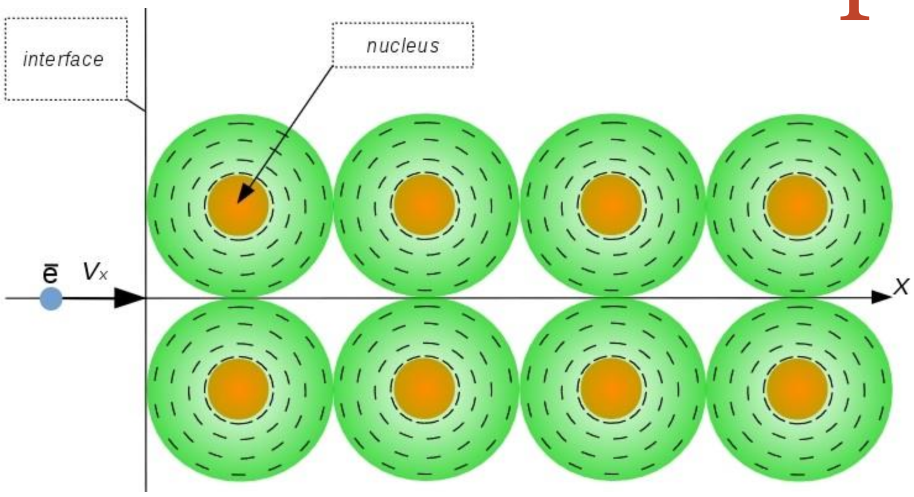
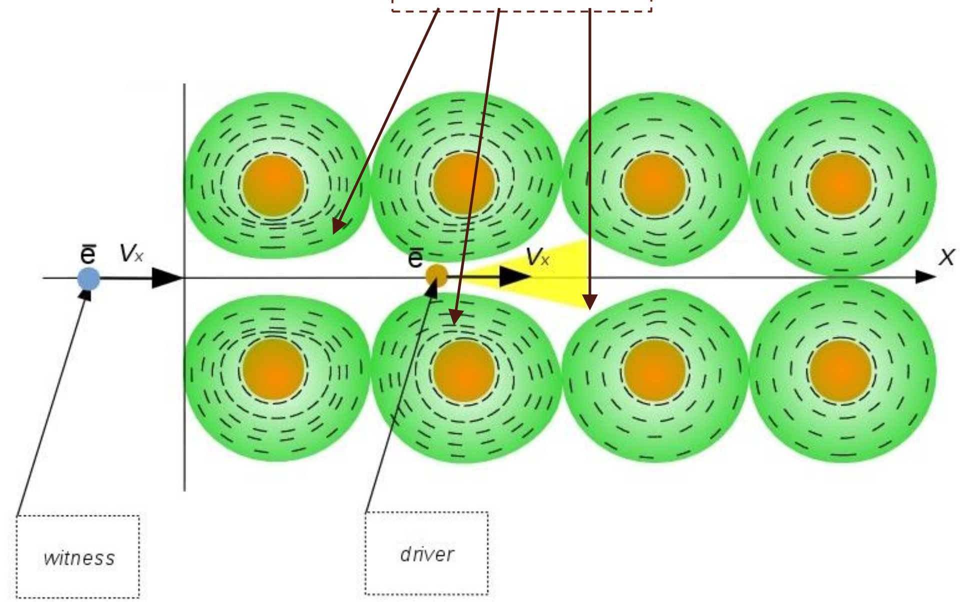
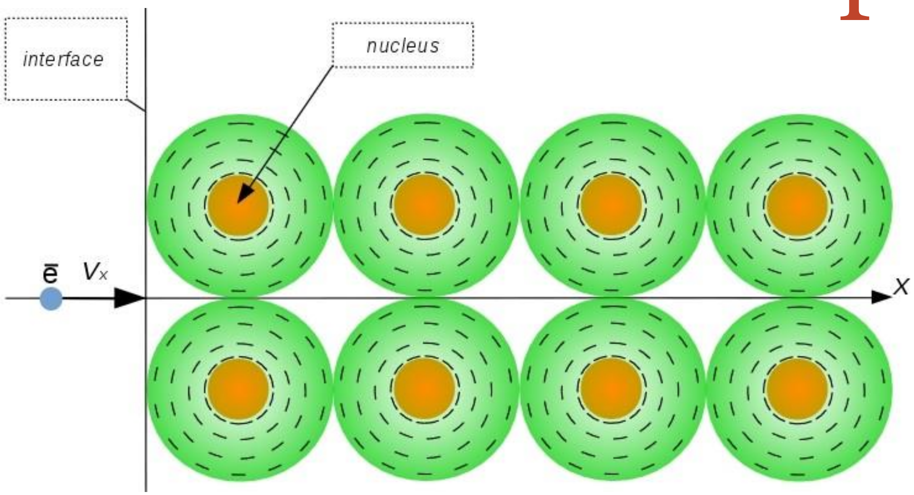
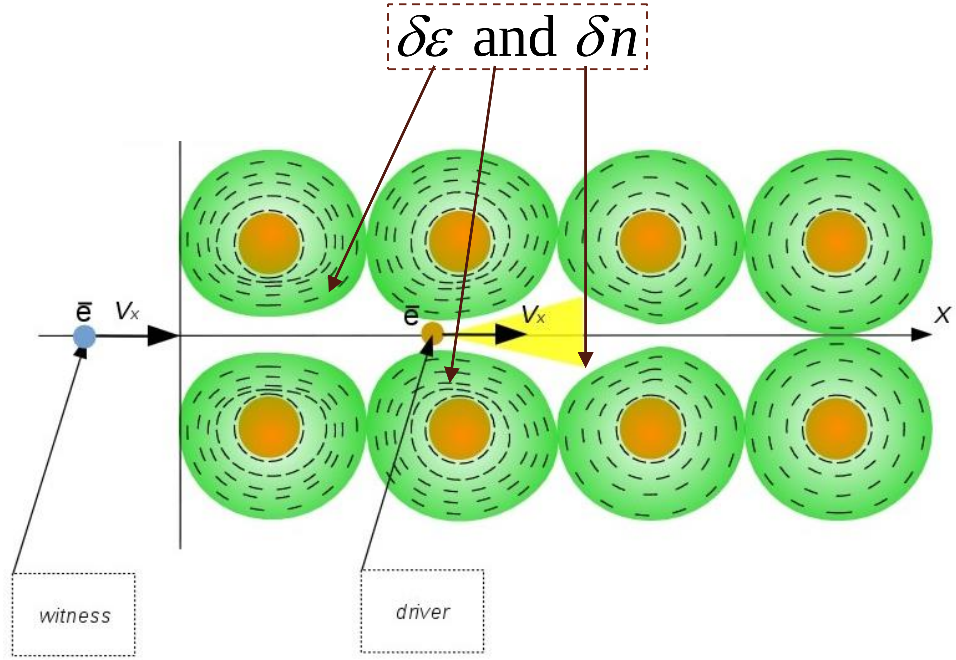




$$\varepsilon(\mathbf{r},\omega)=1-\frac{4\pi e^2}{m_e\omega^2}n(\mathbf{r}) \quad + \quad n(\mathbf{r})=n_0(\mathbf{r})+\delta n(\mathbf{r})$$

The problem





$$\varepsilon(\mathbf{r},\omega)=1-\frac{4\pi e^2}{m_e\omega^2}n(\mathbf{r}) \quad + \quad n(\mathbf{r})=n_0(\mathbf{r})+\delta n(\mathbf{r})$$

$$\downarrow$$

$$\varepsilon(\mathbf{r},\omega)=\varepsilon_0(\mathbf{r},\omega)+\delta\varepsilon(\mathbf{r},\omega)$$

Electron-nuclei interaction is neglected at this stage

Theory

Theory

$$\frac{d^2 W_{\mathbf{g}}^{PXR}(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)} = F(\mathbf{r},\omega)\times$$

$$\times\frac{\left(1-n_z^2\right)\left(k_z^{'}-g_z\right)^2+\left(L_{\mathbf{g}}^2-\left(\mathbf{nL}_{\mathbf{g}}\right)^2\right)+2\left(k_z^{'}-g_z\right)n_z\left(\mathbf{nL}_{\mathbf{g}}\right)}{\left(c/\omega\right)^2\left(\rho_1^2+\left(k_z^{'}-g_z\right)^2\right)^2}$$

Theory

$$\frac{d^2W_{\mathbf{g}}^{PXR}(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)} = F(\mathbf{r},\omega)\times$$

$$\times\frac{(1-n_z^2)(k_z^{'}-g_z)^2+\left(L_{\mathbf{g}}^2-(\mathbf{nL_g})^2\right)+2(k_z^{'}-g_z)n_z(\mathbf{nL_g})}{\left(c/\omega\right)^2\left(\rho_1^2+(k_z^{'}-g_z)^2\right)^2}$$

Depends on $\chi_{\mathbf{g}}(\mathbf{r},\omega)$

$\Downarrow$

depends on $\varepsilon(\mathbf{r},\omega)$

Theory

$$\frac{d^2W_{\mathbf{g}}^{PXR}(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)} = F(\mathbf{r},\omega) \times$$

$$\times \frac{(1-n_z^2)(k_z^{'}-g_z)^2 + \left(L_{\mathbf{g}}^2 - (\mathbf{n}\mathbf{L}_{\mathbf{g}})^2\right) + 2(k_z^{'}-g_z)n_z(\mathbf{n}\mathbf{L}_{\mathbf{g}})}{\left(c/\omega\right)^2\left(\rho_1^2 + (k_z^{'}-g_z)^2\right)^2}$$

Depends on $\chi_{\mathbf{g}}(\mathbf{r},\omega)$

$\Downarrow$

depends on $\varepsilon(\mathbf{r},\omega)$

$$\varepsilon(\mathbf{r},\omega) = \varepsilon_0(\mathbf{r},\omega) + \delta\varepsilon(\mathbf{r},\omega)$$

Theory

$$\frac{d^2W_{\mathbf{g}}^{PXR}(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)} = F(\mathbf{r},\omega)\times$$

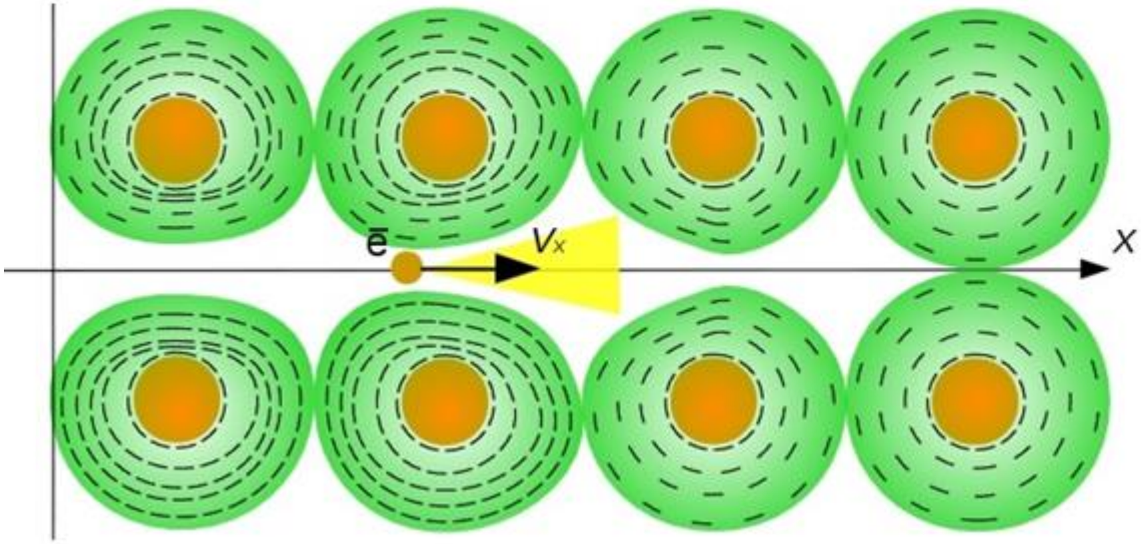
$$\times\frac{\left(1-n_z^2\right)\left(k_z^{'}-g_z\right)^2+\left(L_{\mathbf{g}}^2-\left(\mathbf{nL}_{\mathbf{g}}\right)^2\right)+2\left(k_z^{'}-g_z\right)n_z\left(\mathbf{nL}_{\mathbf{g}}\right)}{\left(c/\omega\right)^2\left(\rho_1^2+\left(k_z^{'}-g_z\right)^2\right)^2}$$

Depends on $\chi_{\mathbf{g}}(\mathbf{r},\omega)$

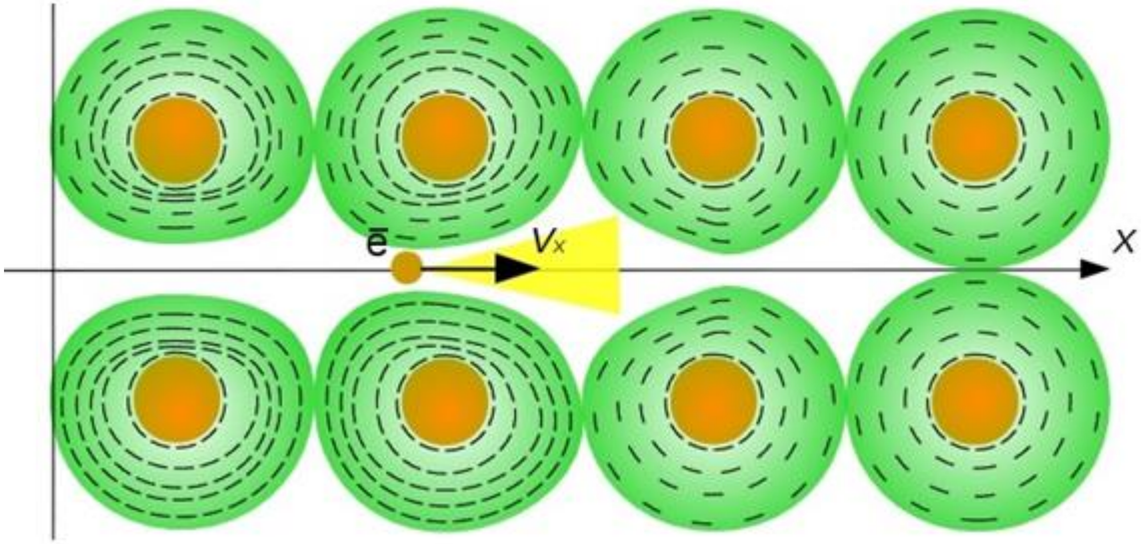
$\Downarrow$

depends on $\varepsilon(\mathbf{r},\omega)$

$$\varepsilon(\mathbf{r},\omega)=\varepsilon_0(\mathbf{r},\omega)+\delta\varepsilon(\mathbf{r},\omega) \Longrightarrow \left.\delta\frac{d^2W_{\mathbf{g}}^{PXR}(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)}\right|_{disturbed}$$



M.I. Ryazanov, Electrodynamic of condensed matter, 1984 (in Russian)



$$div\mathbf{E}(\mathbf{r},\omega)=\frac{4\pi\rho(\mathbf{r},\omega)}{\varepsilon_0(\mathbf{r},\omega)}-\frac{\mathbf{E}(\mathbf{r},\omega)}{\varepsilon_0(\mathbf{r},\omega)}grad\,\varepsilon_0(\mathbf{r},\omega)$$

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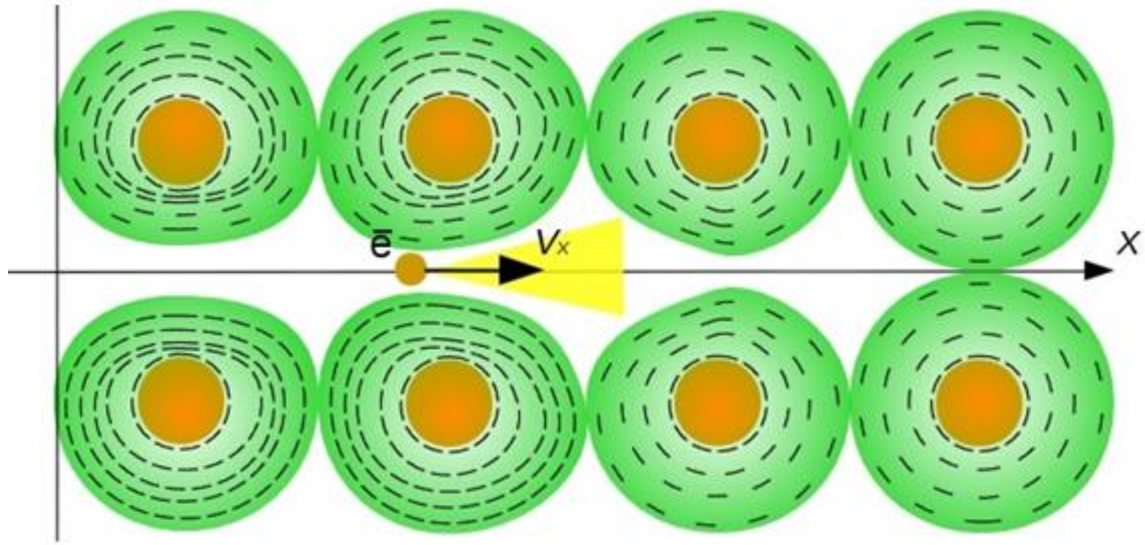

$$div\mathbf{E}(\mathbf{r},\omega)=\frac{4\pi\rho(\mathbf{r},\omega)}{\varepsilon_0(\mathbf{r},\omega)}-\frac{\mathbf{E}(\mathbf{r},\omega)}{\varepsilon_0(\mathbf{r},\omega)}grad\,\varepsilon_0(\mathbf{r},\omega)$$

⇩

$$div\mathbf{E}(\mathbf{r},t)=4\pi\int_0^\infty d\tau'\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{i\omega\tau'}\frac{1-\varepsilon_0(\mathbf{r},\omega)}{\varepsilon_0(\mathbf{r},\omega)}\rho(\mathbf{r},t+\tau')+4\pi\rho(\mathbf{r},t)+$$

$$+4\pi\int_0^\infty d\tau\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\frac{1-\varepsilon_0(\mathbf{r},\omega)}{\varepsilon_0(\mathbf{r},\omega)}\rho(\mathbf{r},t-\tau)-\int_{-\infty}^{\infty}\frac{\mathbf{E}(\mathbf{r},\omega)\,grad\,\varepsilon_0(\mathbf{r},\omega)}{\varepsilon_0(\mathbf{r},\omega)}e^{-i\omega t}d\omega$$

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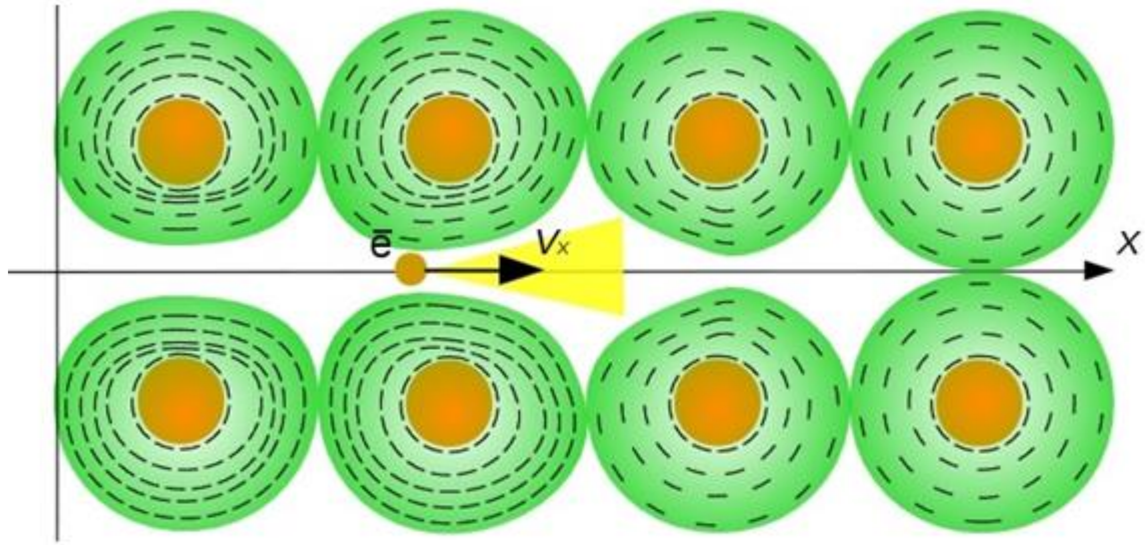

$$divE(r,\omega)=\frac{4\pi\rho(r,\omega)}{\varepsilon_0(r,\omega)}-\frac{E(r,\omega)}{\varepsilon_0(r,\omega)}grad\,\varepsilon_0(r,\omega)$$

$$\Downarrow$$

$$divE(r,t)=4\pi\int_0^\infty d\tau'\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{i\omega\tau'}\frac{1-\varepsilon_0(r,\omega)}{\varepsilon_0(r,\omega)}\rho(r,t+\tau')+4\pi\rho(r,t)+$$

$$+4\pi\int_0^\infty d\tau\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}\frac{1-\varepsilon_0(r,\omega)}{\varepsilon_0(r,\omega)}\rho(r,t-\tau)-\int_{-\infty}^{\infty}\frac{E(r,\omega)grad\,\varepsilon_0(r,\omega)}{\varepsilon_0(r,\omega)}e^{-i\omega t}d\omega$$

due to the causality

As in Kramers–Kronig relations

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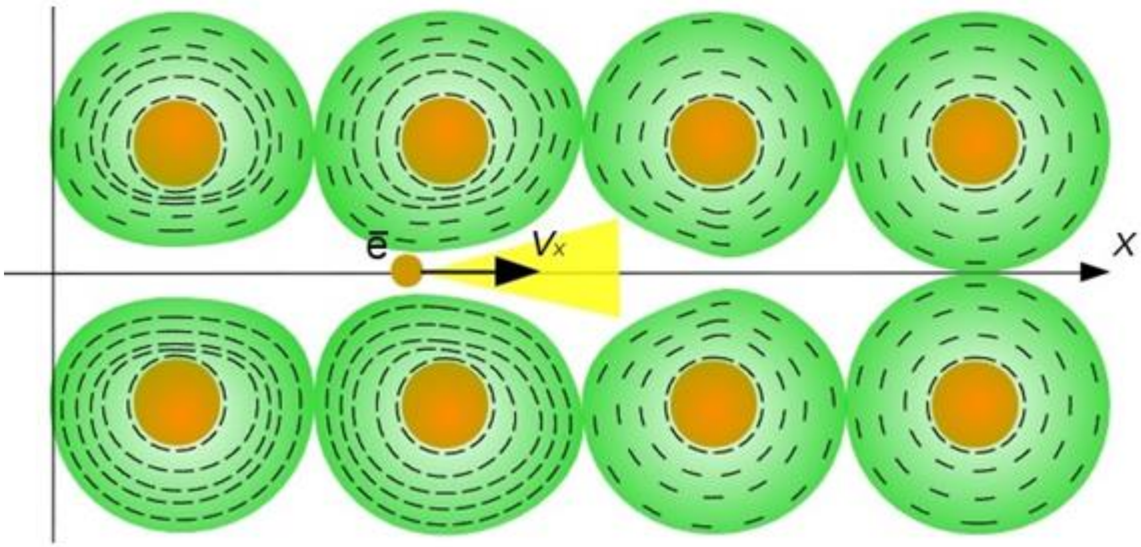
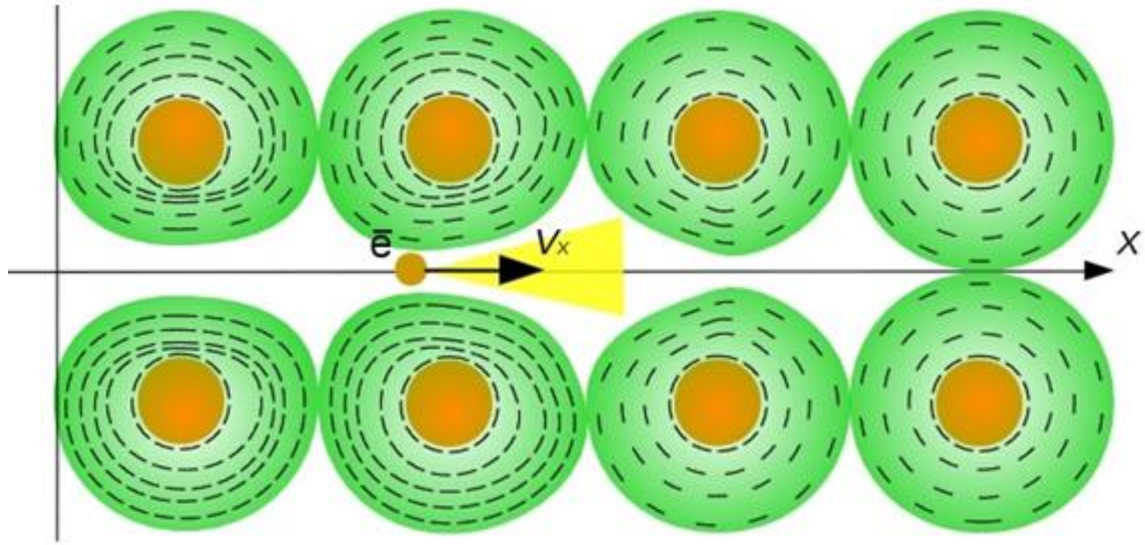


$$div\mathbf{E}(\mathbf{r},t) = 4\pi\rho(\mathbf{r},t) + 4\pi\rho_w(\mathbf{r},t)$$

$$\delta n(\mathbf{r}) = \frac{\rho_w(\mathbf{r},t)}{m_e}$$

⇩



$$\delta n(\mathbf{r}) = \frac{\rho_w(\mathbf{r},t)}{m_e}$$

$$div\mathbf{E}(\mathbf{r},t) = 4\pi\rho(\mathbf{r},t) + 4\pi\rho_w(\mathbf{r},t)$$

⇩

$$\delta n(\mathbf{r}) = -\frac{1}{m_e}\left\{\frac{Z_e e\omega_p}{v}\delta(y)\delta(z)\sin\left(\frac{\omega_p x}{v_x} - \omega_p t\right)\theta(v_x t - x) + \right.$$
$$\left. +\omega_p \sin(\omega_p t)\mathbf{E}(\mathbf{r},\omega_p)\, grad\,\varepsilon_0(\mathbf{r},\omega)\Big|_{\omega=\omega_p}\right\}$$

$$\omega_p^2(\mathbf{r}) = \frac{4\pi e^2}{m_e}n(\mathbf{r})$$

$$\varepsilon_0(\mathbf{r},\omega) = 1 - \frac{\omega_p^2(\mathbf{r})}{\omega^2}$$

$$\delta\varepsilon(\mathbf{r},\omega) = -\frac{4\pi e^2}{m_e^2\omega^2}\left\{\frac{Z_e e\omega_p}{v}\delta(y)\delta(z)\sin\left(\frac{\omega_p x}{v_x} - \omega_p t\right)\theta(v_x t - x) + \right.$$

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$$\left. +\omega_p \sin(\omega_p t)\mathbf{E}(\mathbf{r},\omega_p)\, grad\,\varepsilon_0(\mathbf{r},\omega)\Big|_{\omega=\omega_p}\right\}$$

$$\delta\frac{d^2W_{\mathbf{g}}^{PXR}(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)} = \frac{1}{137}\frac{\left(\left|\delta\varepsilon(\mathbf{r},\omega)\right|^2 + 2\left(\varepsilon_0(\mathbf{r},\omega)-1\right)\delta\varepsilon(\mathbf{r},\omega)\right)}{\pi^2}\left|\frac{F(\mathbf{g})S(\mathbf{g})}{ZN_{cell}}e^{-W(\mathbf{g})}\right|^2 \times$$

$$\times\frac{\sin^2\left(\frac{\omega a}{2v_x}\left(1-\sqrt{\varepsilon}n_x v_x/\mathrm{c}+g_x v_x/\omega\right)\right)}{\left(1-\sqrt{\varepsilon}n_x v_x/\mathrm{c}+g_x v_x/\omega\right)^2}\times$$

$$\times\frac{\left(1-n_z^2\right)\left(k_z'-g_z\right)^2+\left(L_{\mathbf{g}}^2-\left(\mathbf{nL}_{\mathbf{g}}\right)^2\right)+2\left(k_z'-g_z\right)n_z\left(\mathbf{nL}_{\mathbf{g}}\right)}{\left(c/\omega\right)^2\left(\rho_1^2+\left(k_z'-g_z\right)^2\right)^2}$$

$F(\mathbf{g})$ – *atomic*

$W(\mathbf{g})$ – *Debye – Waller*

$S(\mathbf{g})$ – *structural*

Summary

Theory of PXR for the disturbed crystal lattice has been built

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Future plans

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- Numerical estimates

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- Consideration of nuclei-electron interaction effects

  (+temperature effects)

Summary

Theory of PXR for the disturbed crystal lattice has been built

Future plans

- Numerical estimates
- Calculation of geometrical factor of the bunch
- Consideration of nuclei-electron interaction effects (+temperature effects)
- Development of the theory for the passage of the chain of bunches

Thank you for your attention!