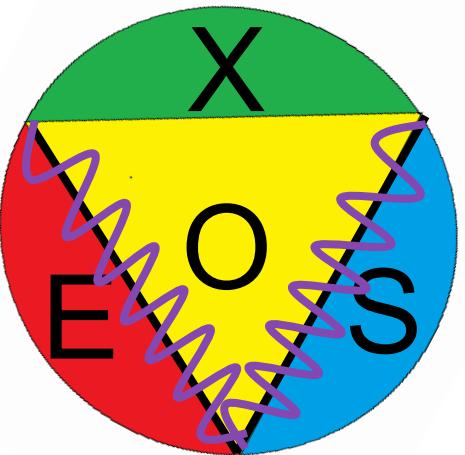


Characterization of a High Resolution Von Hamos Spectrometer with HAPG Crystals for Extended Sources; a New Opportunity for High Precision Nuclear Physics Measurements

INFN-CSN5

Young Researcher Grant 2015,
n. 17367/2015.



Alessandro Scordo
Laboratori Nazionali di Frascati, INFN





Scientific goal of VOXES

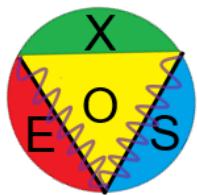
- High resolution (few eV) measurements of the X rays (2-20 keV) are strongly demanded and very difficult to be performed
- Semiconductor detectors: limited to 100 eV (FWHM)
- TES detectors : high cost, hard to handle, difficult lineshape calibration
- Standard Crystal Bragg spectrometers: low efficiency, limited in energy range (d)
- These X-rays not always are produced by a point-like source

VOXES's goal: to develop, test and qualify the first prototype of a high resolution X-ray Von Hamos spectrometer in the range of energies 2 - 20 keV using HAPG bent crystals able to work with 'extended' sources



High resolution **von-Hamos X-Ray** spectrometer using HAPG for **Extended Sources** in a broad energy range

VOXES: working principle and geometry - Mosaic crystals

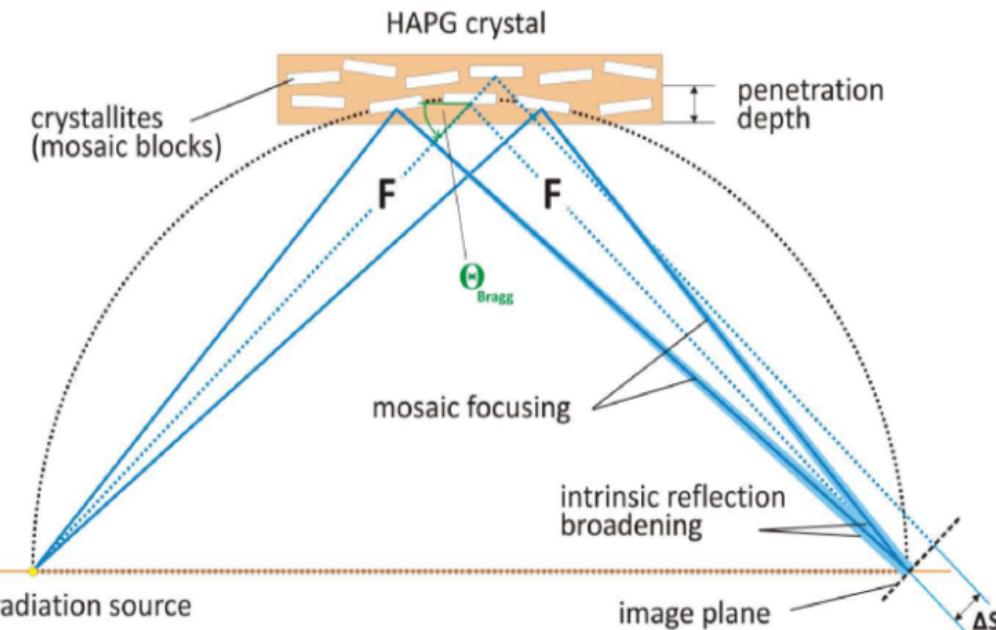


Mosaic crystal consist in a large number of nearly perfect small crystallites.

Mosaicity makes it possible that even for a fixed incidence angle on the crystal surface, an energetic distribution of photons can be reflected

Increase of efficiency
(focusing) ~ 50

Loss in resolution



Pyrolytic Graphite mosaic crystals ($d = 3.354 \text{ \AA}$):

$$nl = 2ds\sin\theta_B$$

Highly Oriented Pyrolytic Graphite (HOPG, $Dq \approx 1^\circ$)

Highly Annealed Pyrolytic Graphite (HAPG, $Dq \approx 0.07^\circ$)

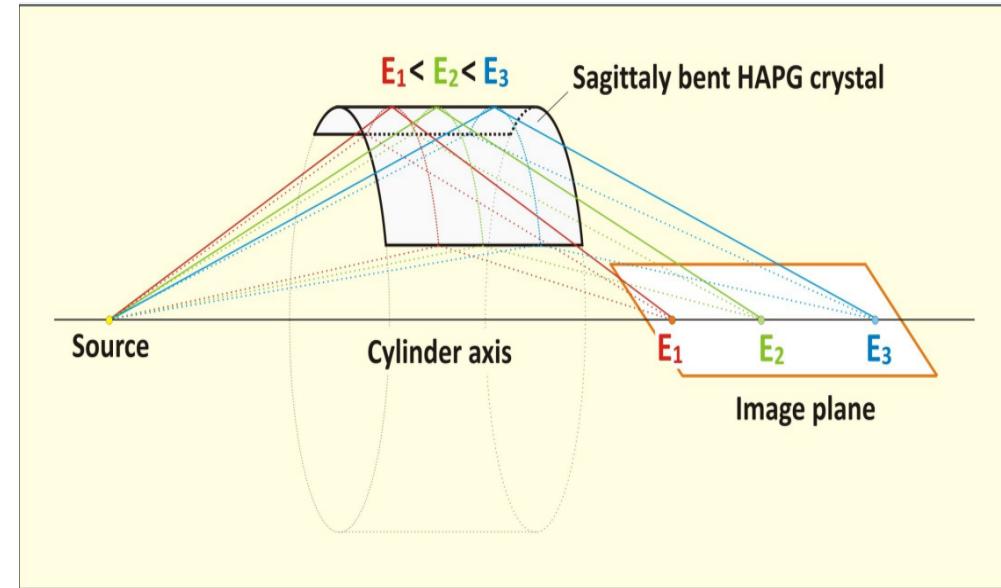
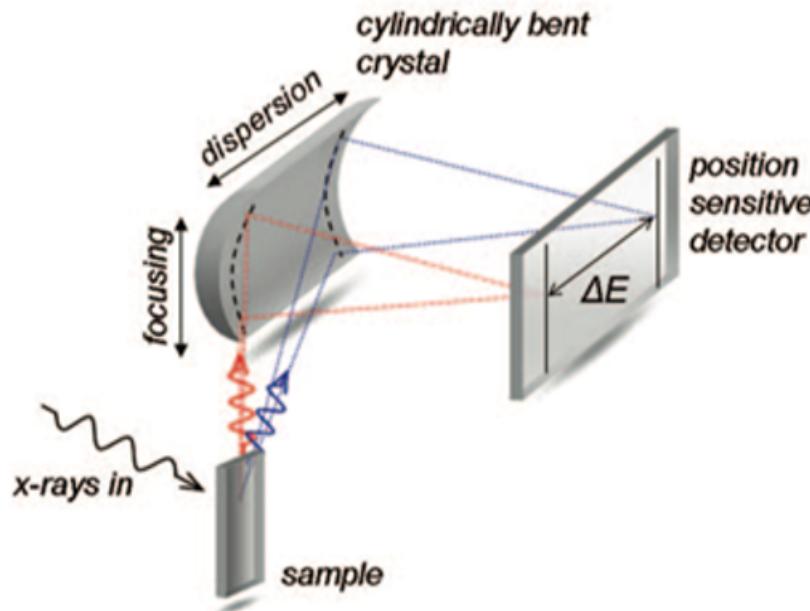
flexible HAPG has twice higher spectral resolution, while flexible HOPG – approximately twice higher reflectivity

Tunable based on the measurement requirements

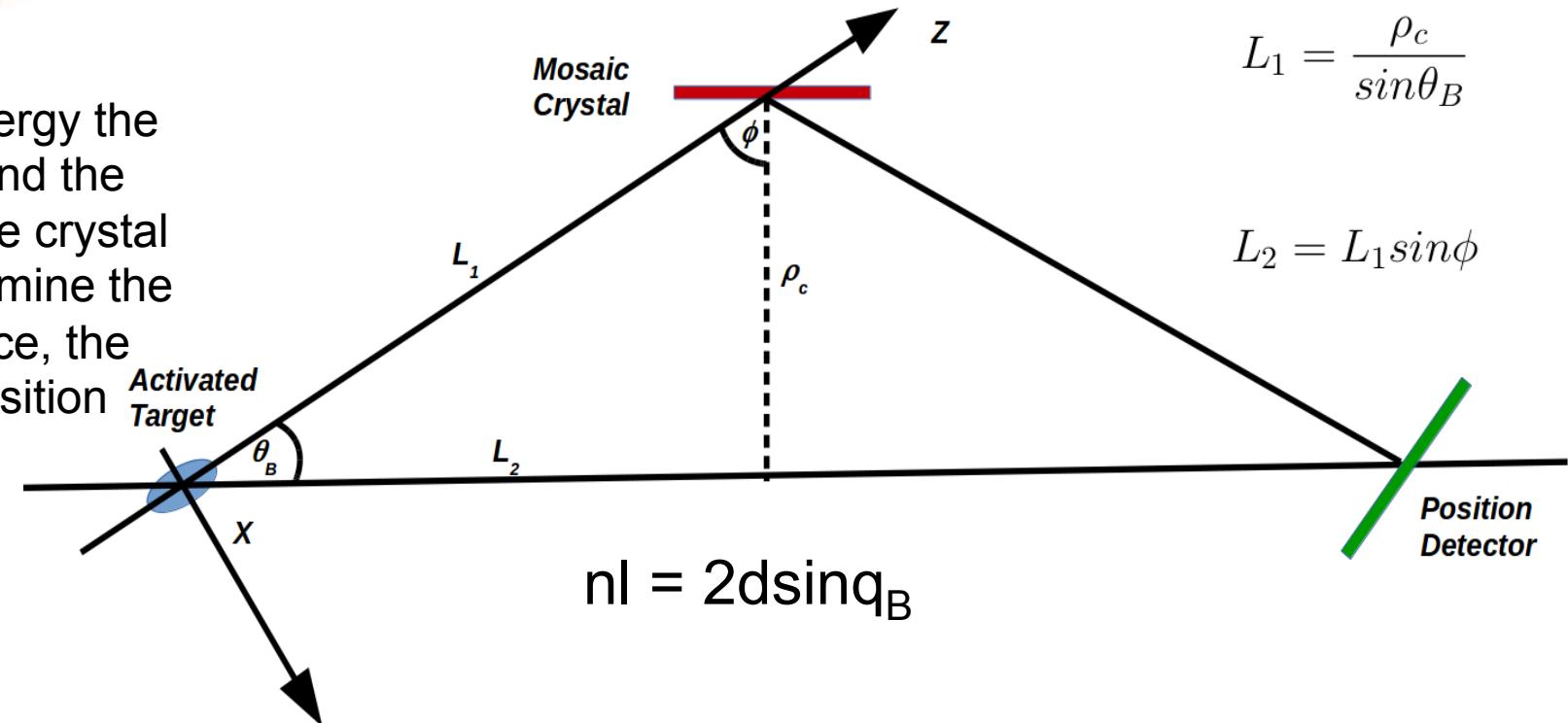


VOXES: working principle and geometry

- Von Hamos configuration



For a given X-ray energy the Bragg angle (θ_B) and the curvature radius of the crystal (ρ_c) completely determine the position of the source, the crystal and the position detector





Source size and VOXES geometry - Status of the art

Bragg spectroscopy is usually exploited by XAS, XES, synchrotron light users, etc...

- Point-like sources
- High yields (no need to increase them)
- 1 eV (Si, Mica, etc) or 2-3 eV (HAPG) resolution is very easily achievable

The x-ray source, which was used for the measurements, is a low power microfocus x-ray tube (IfG) with a source diameter of about $50 \mu\text{m}$. Measurements were performed with the Cu K α emission of a Cu anode at 8 keV. The spec-

III. SPECTROMETER SETUP

The spectrometer consists of three principal components the X-ray source, the HAPG optic, and the position sensitive detector. As source a watercooled 100 W micro focus X-ray tube with a tungsten anode and a focus size of $50 \mu\text{m}$ is used. The emitted radiation is focused onto the sample by a polycapillary full lense with a spot size of $35 \mu\text{m}$. The HAPG

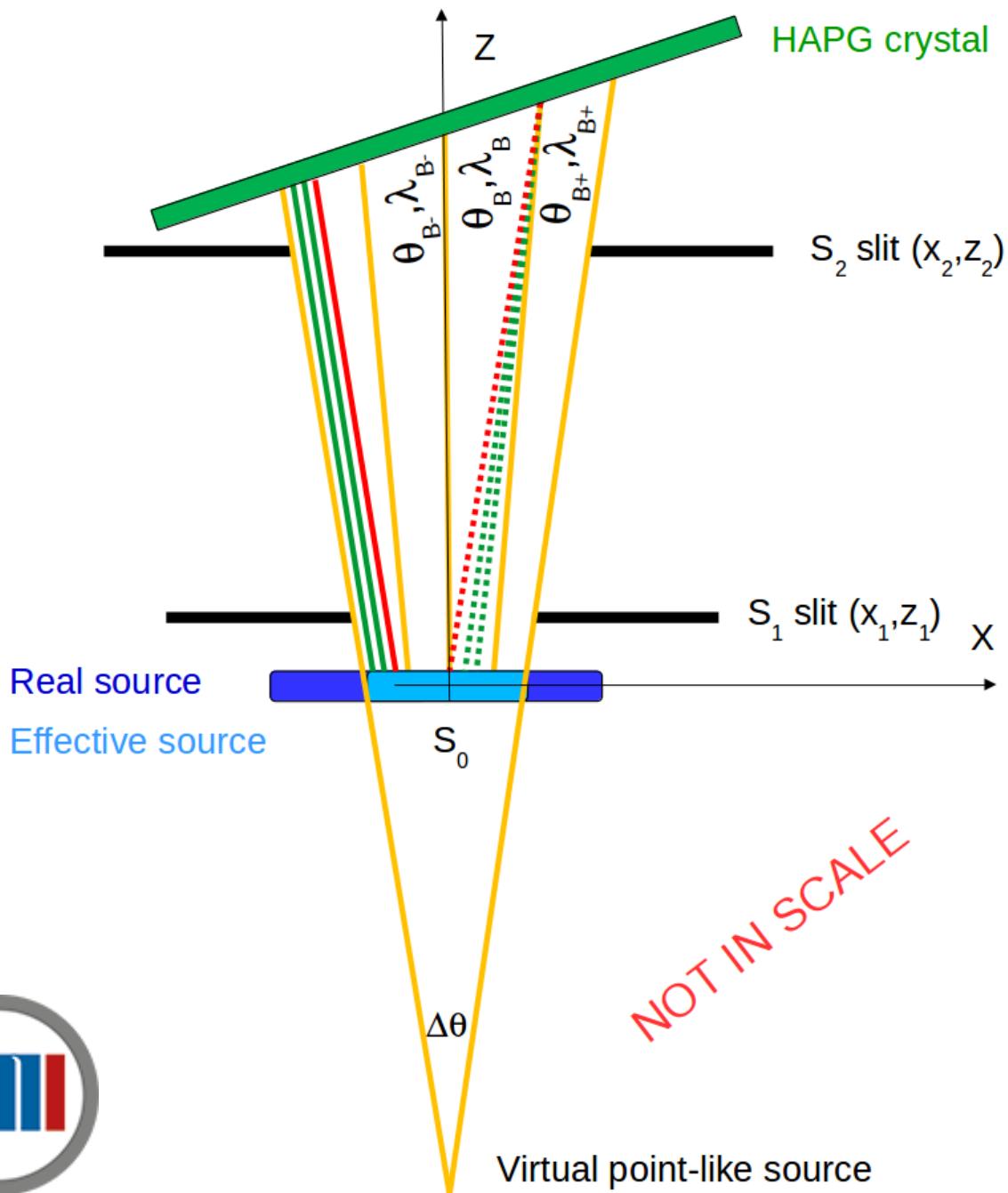
Laser-produced plasmas were created using the “Phoenix” Nd glass laser (the Lebedev Physical Institute) operated at a wavelength of $0.53 \mu\text{m}$ with pulse energy up to 10 J and 2 ns pulse duration. The laser beam was focused onto massive Mg, Al, Ti, or Fe targets (see Fig. 2). The focal spot diameter was about $\sim 15 \mu\text{m}$

But what if we really need wider sources for higher statistics?

Or what if our photons come from a diffused isotropic source?



Source size and VOXES geometry - Virtual (point – like) source



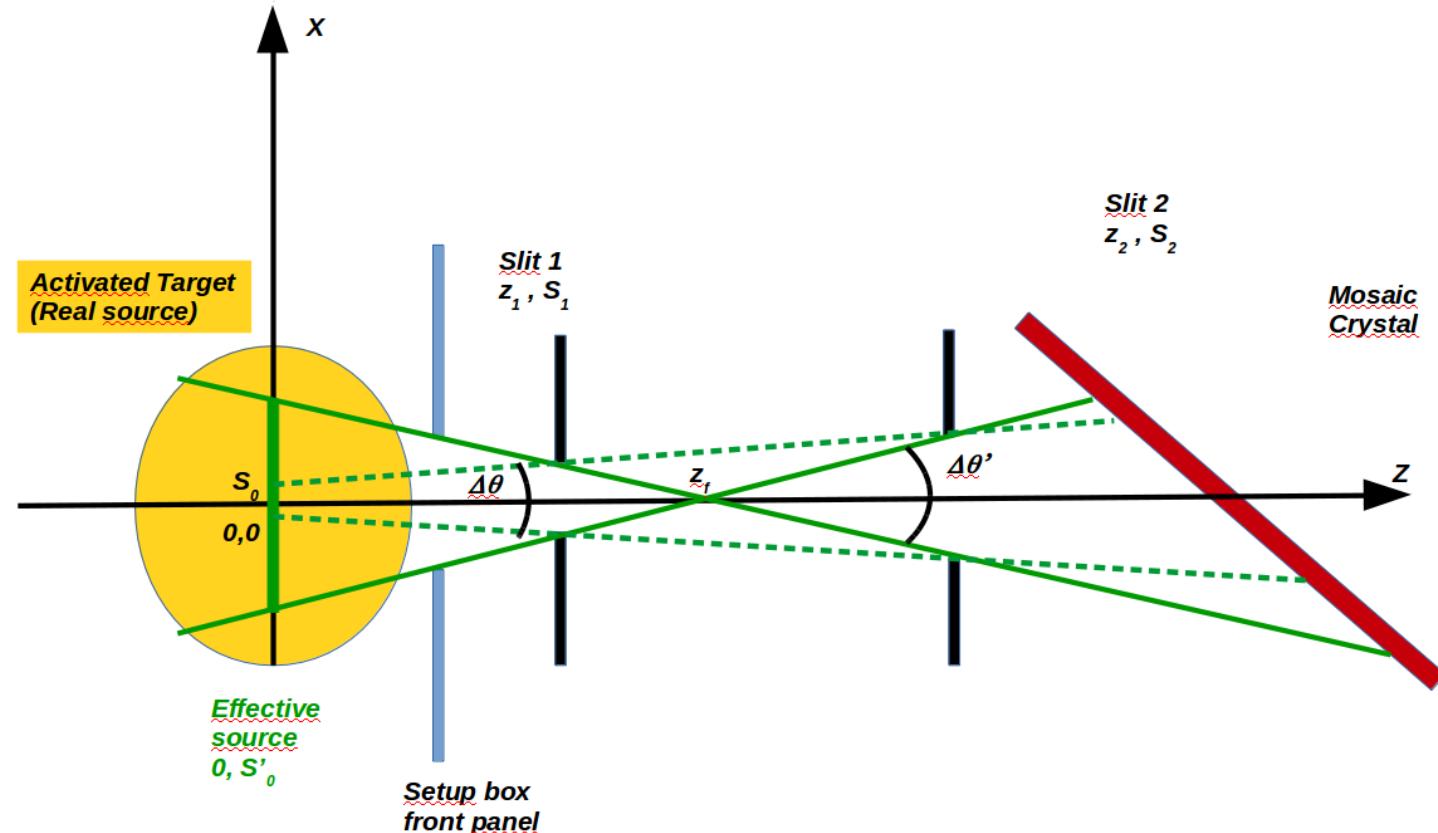
X ray source
Slits
HAPG
Signal photons
Background (?)

With this configuration you are still limited to \sim mm sources and very compact spectrometers....



Source size and VOXES geometry

- Dispersive plane source width



For each $\Delta\theta'$, S'_0 pair, the 2 values of the slits can be found; first, we define the position of the intersection point z_f :

$$z_f = \frac{S'_0}{2} \operatorname{ctg} \left(\frac{\Delta\theta'}{2} \right)$$

Then, the 2 slits aperture are defined by:

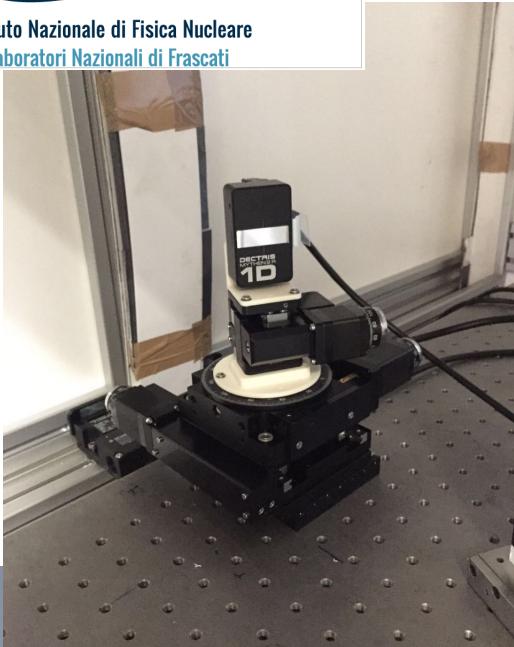
$$S_1 = \frac{z_f - z_1}{z_f} S'_0$$

$$S_c = \frac{z_c - z_f}{z_f} S'_0$$



$$S_2 = \frac{z_2 - z_f}{z_f} S'_0$$

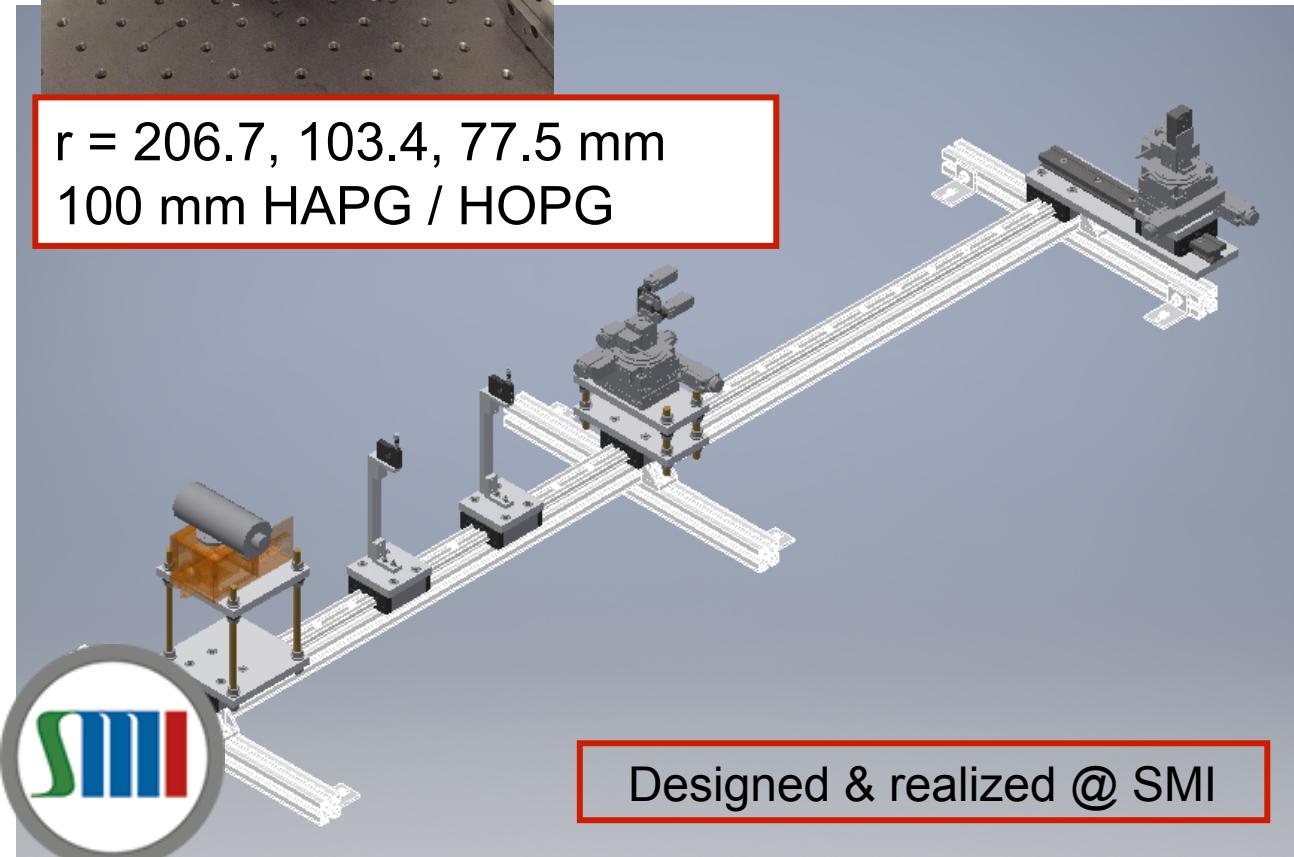
VOXES: Setup box and mechanics



Dectris Ltd
MYTHEN2 detector:

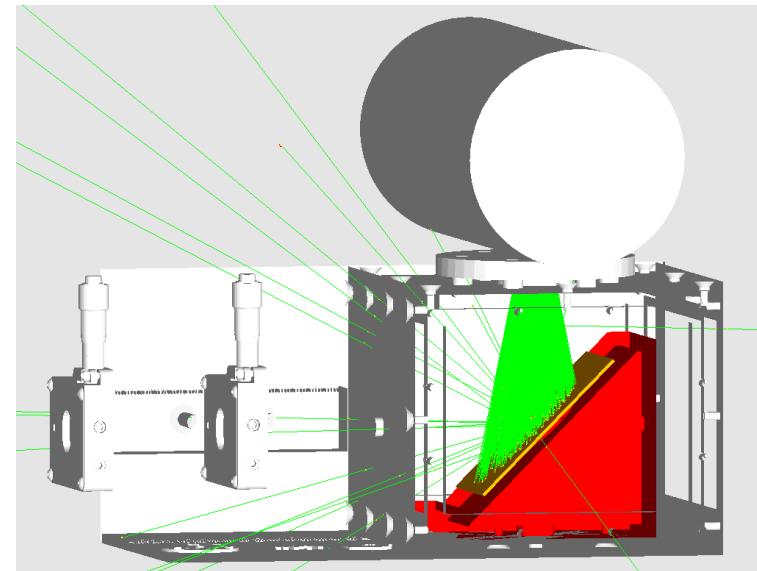
32 x 8 mm surface
640 channels → 50 mm resolution
4-40 keV range
Working @ room temperature

$r = 206.7, 103.4, 77.5$ mm
100 mm HAPG / HOPG



Designed & realized @ SMI

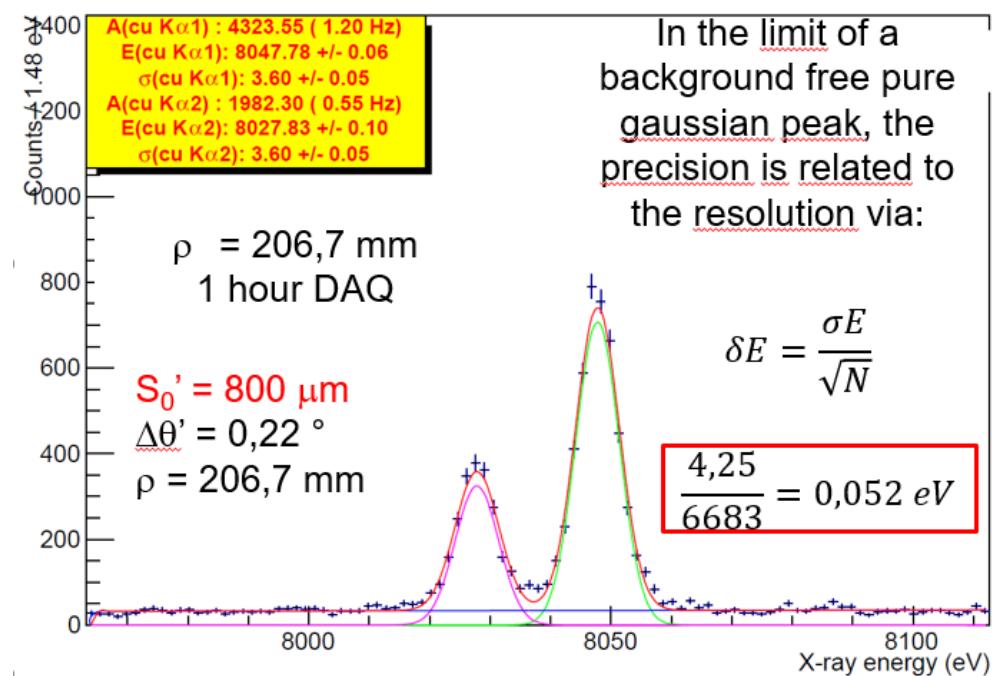
Designed & 3D-printed @ LNF



Oxford Instruments
XRF-5011
Tungsten (W) X-ray tube

- Jan 25, 2016, 13:01

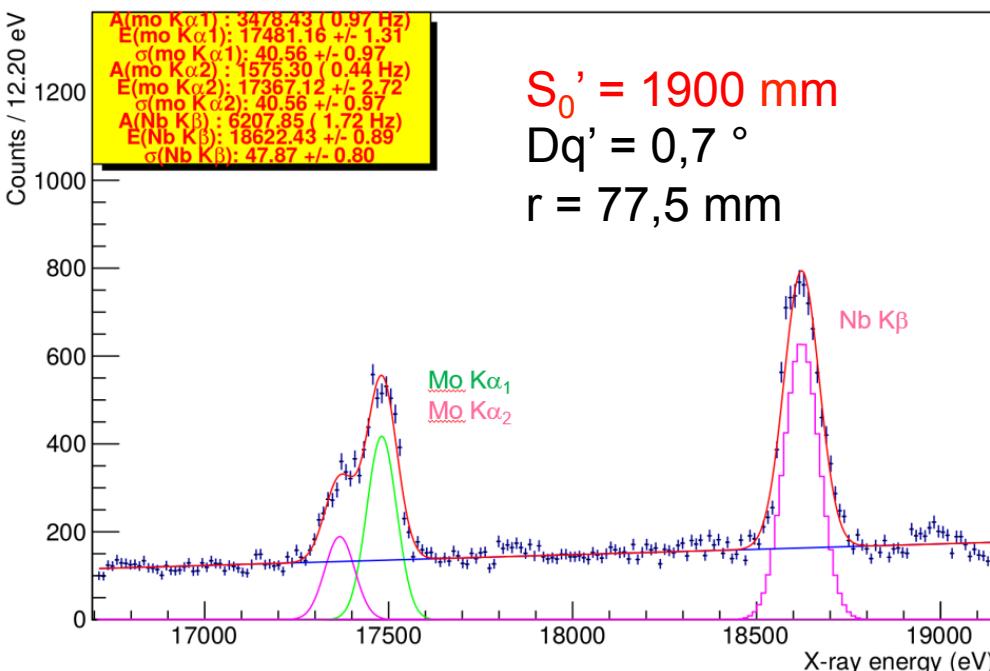
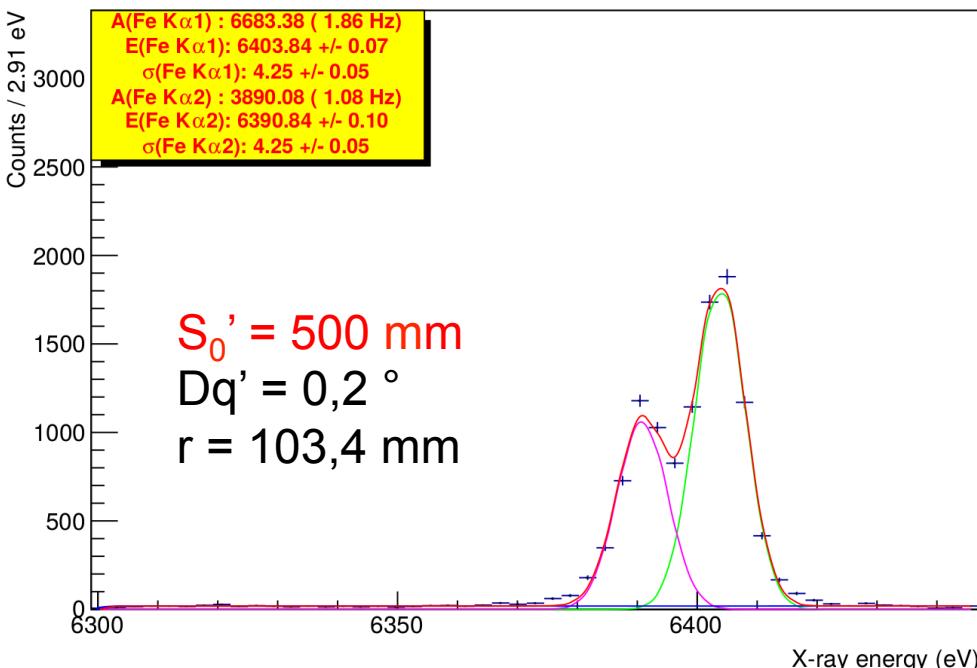
Characterization and results in lab



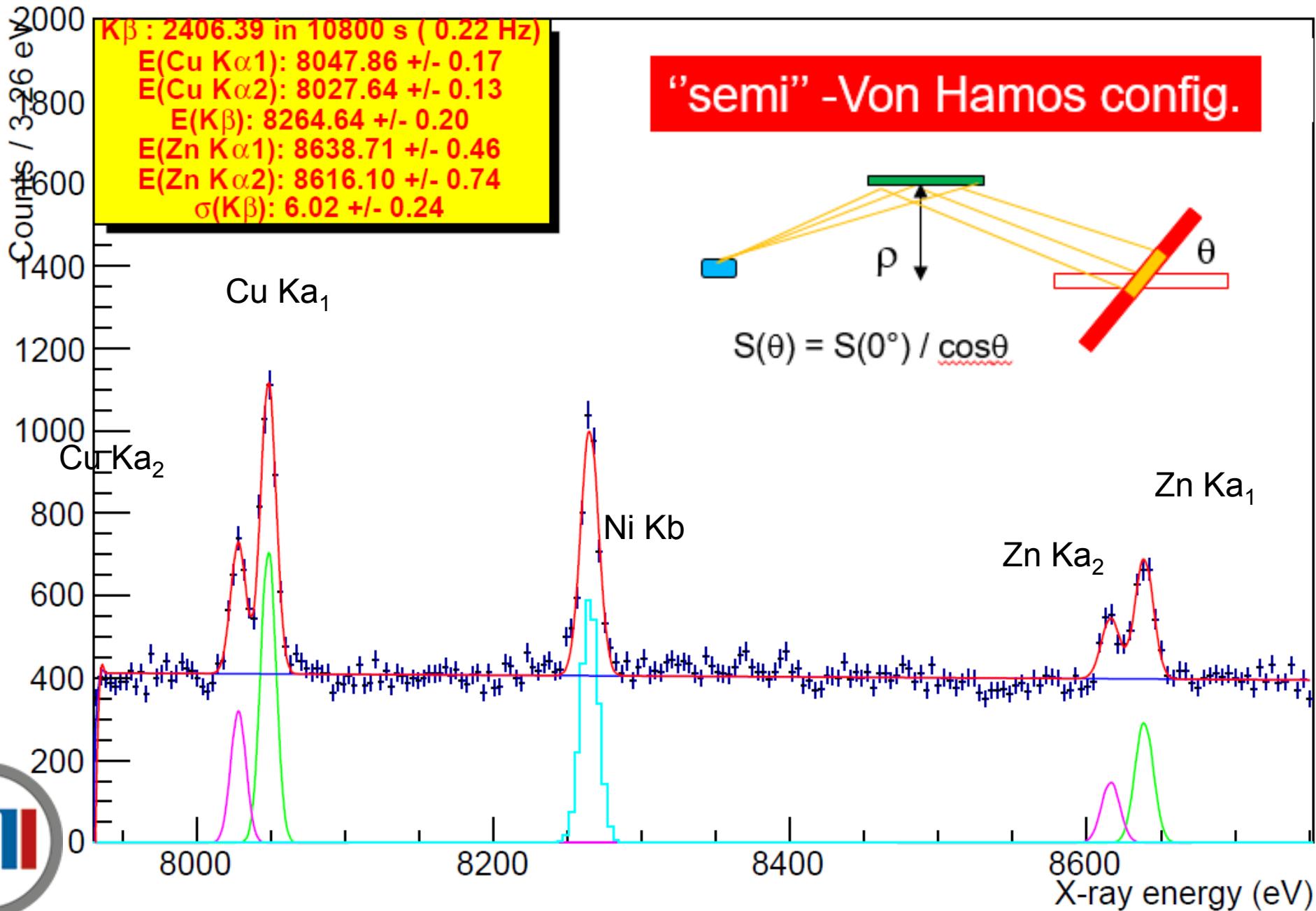
	Line (eV)	Line (A)	sinq	q	L1	L2
Fe Ka1	6403,84	1,94	0,29	16,77	716,58	686,09
Fe Ka2	6390,84	1,94	0,29	16,81	715,13	684,57
Fe Kb	7057,98	1,76	0,26	15,18	789,78	762,22

	Line (eV)	Line (A)	sinq	q	L1	L2
Cu Ka1	8047,78	1,54	0,23	13,28	900,54	876,47
Cu Ka2	8027,83	1,54	0,23	13,31	898,31	874,18
Cu Kb	8905,29	1,39	0,21	11,98	996,49	974,80

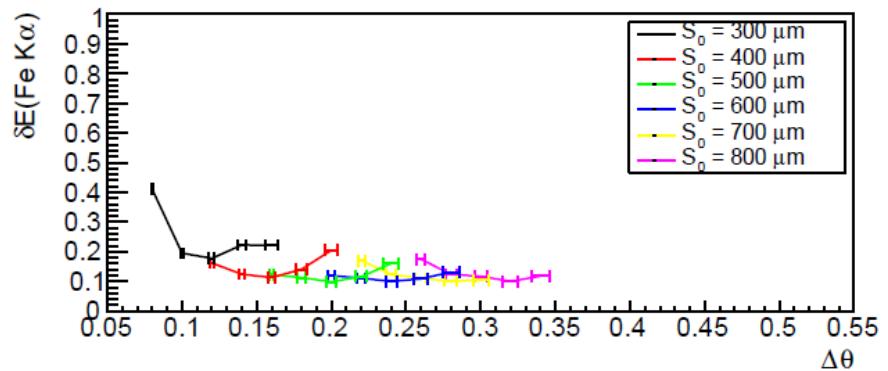
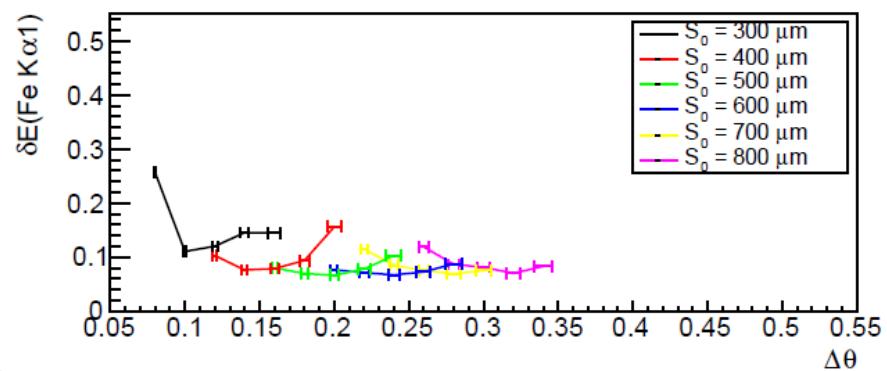
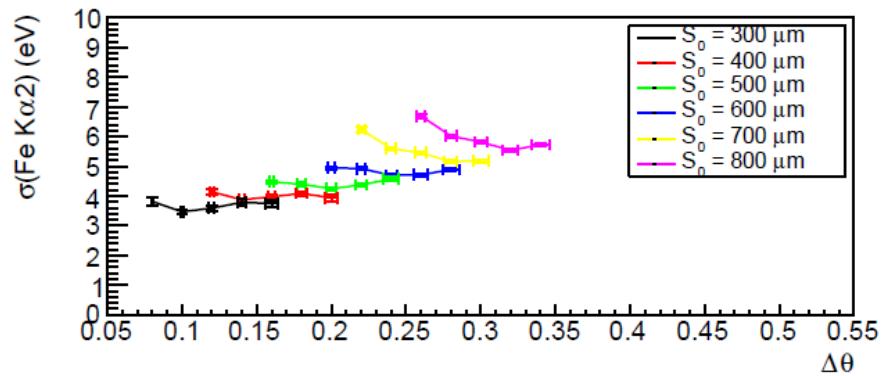
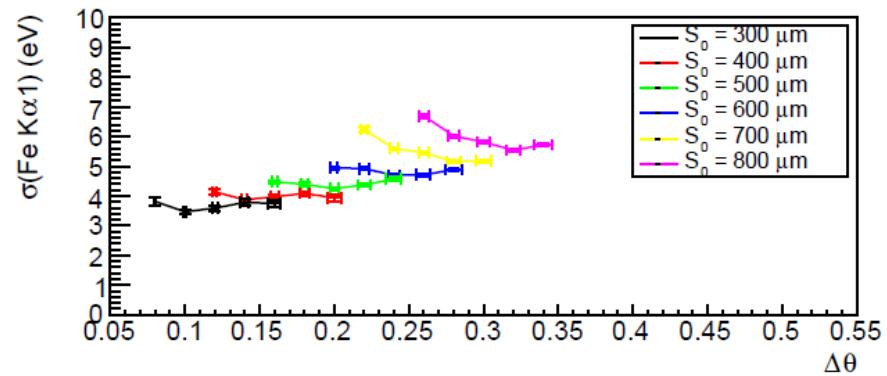
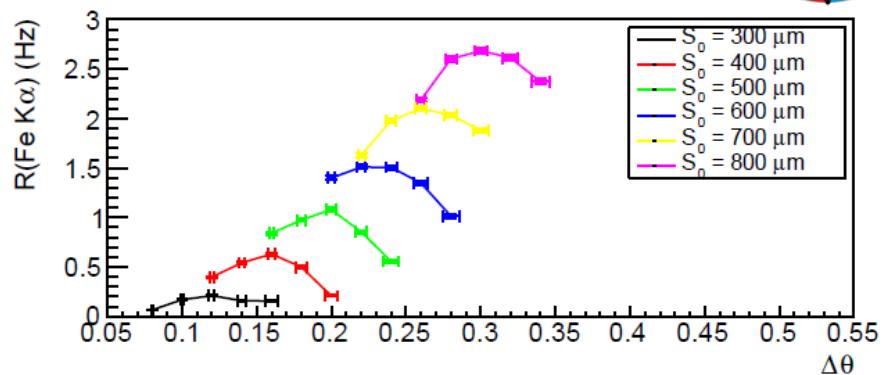
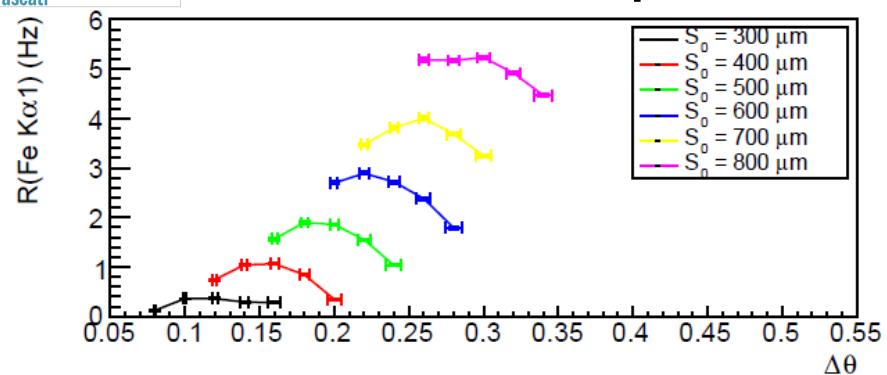
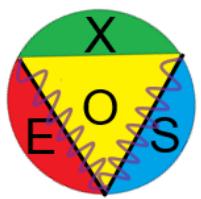
	Line (eV)	Line (A)	sinq	q	L1	L2
Mo Ka1	17479,34	0,71	0,11	6,07	733,35	729,24
Mo Ka2	17374,30	0,71	0,11	6,11	728,94	724,81
Nb Kb	18622,50	0,67	0,10	5,70	781,31	777,46



Wide dynamic range

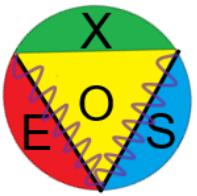


Step-2: Multi line setup and complete characterization



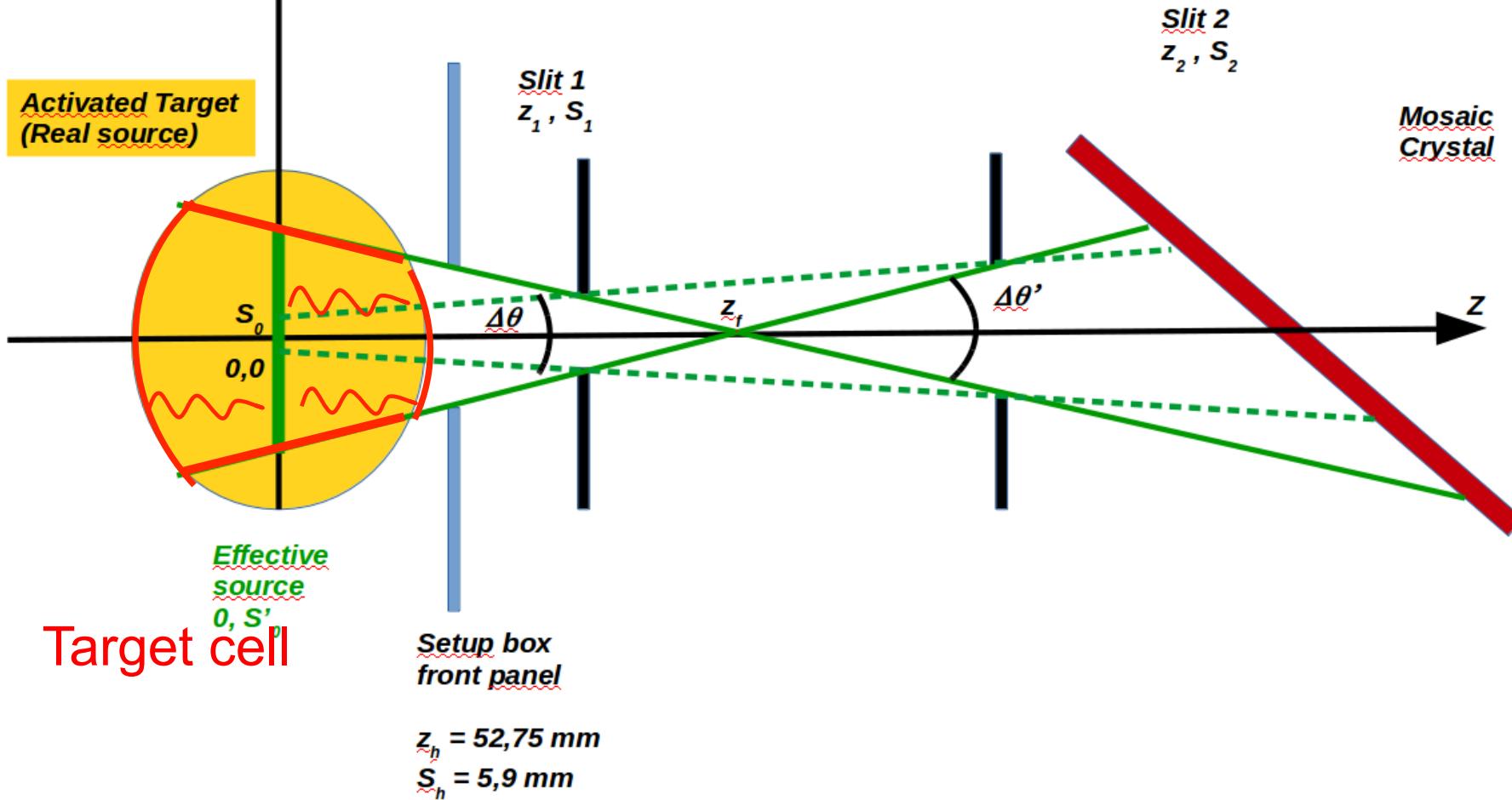
Example for Fe lines with $r = 103.5$ HAPG crystal, 60 m DAQ

Step-2: Multi line setup and complete characterization



Crystal & Line	e (10 ⁻⁵)	s (eV)	dE (eV)
Cu (8 keV) r = 77,5 mm	2,7 (700 mm)	6,8 (700 mm)	0,07 (600 mm)
Cu (8 keV) r = 103,4 mm	2,8 (700 mm)	4,8 (300 mm)	0,04 (700 mm)
Cu (8 keV) r = 206,7 mm	1,1 (1100 mm)	3,6 (800 mm)	0,04 (1100 mm)
Fe (6 keV) r = 77,5 mm	4,5 (600 mm)	4,2 (300 mm)	0,1 (600 mm)
Fe (6 keV) r = 103,4 mm	1,6 (700 mm)	4 (300 mm)	0,09 (700 mm)
Fe (6 keV) r = 206,7 mm	0,8 (1300 mm)	3,6 (1100 mm)	0,1 (1200 mm)
Fe (6 keV) r = 103,4 mm NEW AI BOX	2,2 (800 mm)	3,5 (300 mm)	0,07 (500 mm)

3(D) is better than 2 ...



Target cell

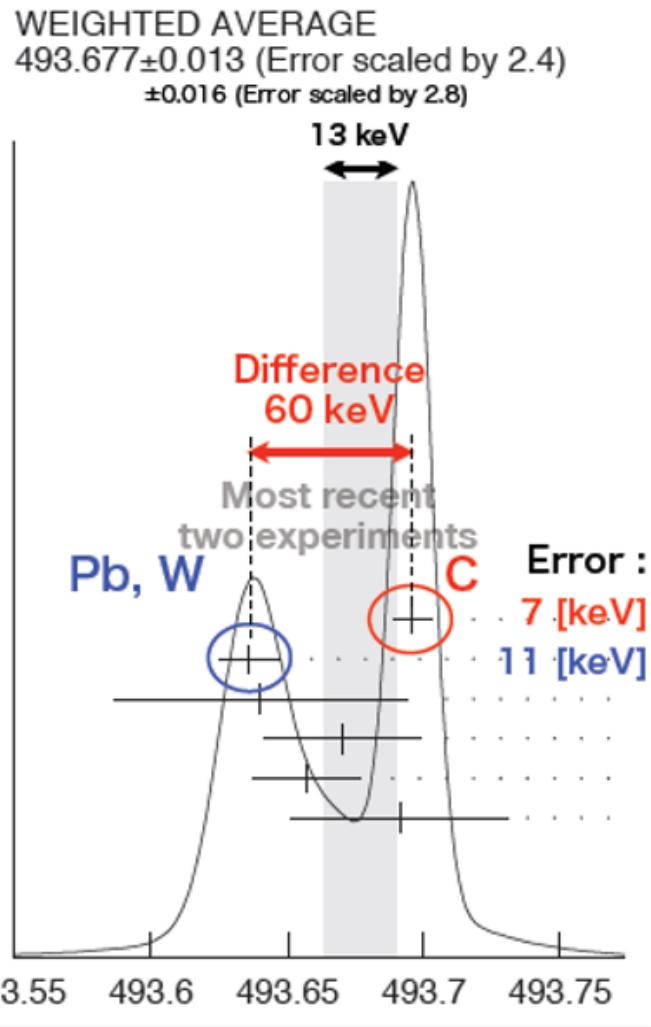
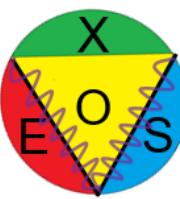
$$A_T(r=103,4 \text{ mm}; S_0=0,7 \text{ mm}) = 100 \text{ mm} \times 8 \text{ mm}^2$$



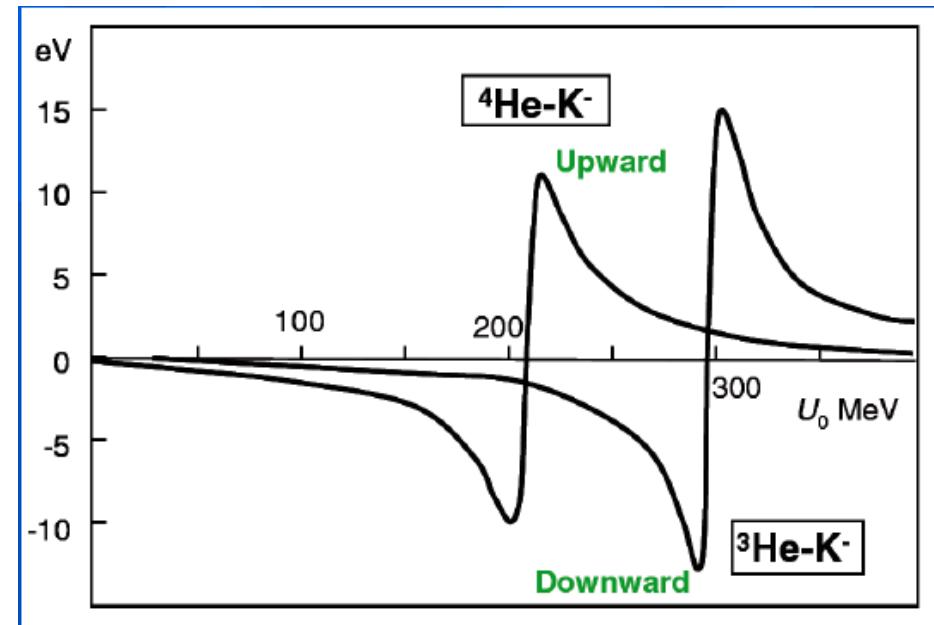
$$A_T(r=103,4 \text{ mm}; S_0=0,7 \text{ mm}) = 100 \text{ mm} \times 8 \text{ mm}^2$$



Possible applications in nuclear physics: kaonic atoms @ DAFNE



In VOXES range



Best measurements:

Target

?

?

${}^4\text{He}$	$+5 \pm 3 \text{ (stat.)} \pm 4 \text{ (syst.)}$	$14 \pm 8 \text{ (stat.)} \pm 5 \text{ (syst.)}$
${}^3\text{He}$	$-2 \pm 2 \text{ (stat.)} \pm 4 \text{ (syst.)}$	$6 \pm 6 \text{ (stat.)} \pm 7 \text{ (syst.)}$

Transition	Ryd. Energy (eV)	Exp. Energy (eV)	Prec. for 5 keV (eV)	Prec. for 10 keV (eV)
$KC(5 \rightarrow 4)$	10173.2	10216.5 ± 1.8	0.0906373	0.181275
$KC(6 \rightarrow 5)$	5526.17	5545.7 ± 4.2	0.0492351	0.0984702
$KN(5 \rightarrow 4)$	13931	13996	0.126397	0.252794
$KN(6 \rightarrow 5)$	7567.48	7560 ± 32	0.0686601	0.13732
$KN(7 \rightarrow 6)$	4562.95	4589 ± 32	0.0413998	0.0827997
$KO(6 \rightarrow 5)$	9929.12	9968.7	0.0913251	0.18265

Test measurements @ DAFNE

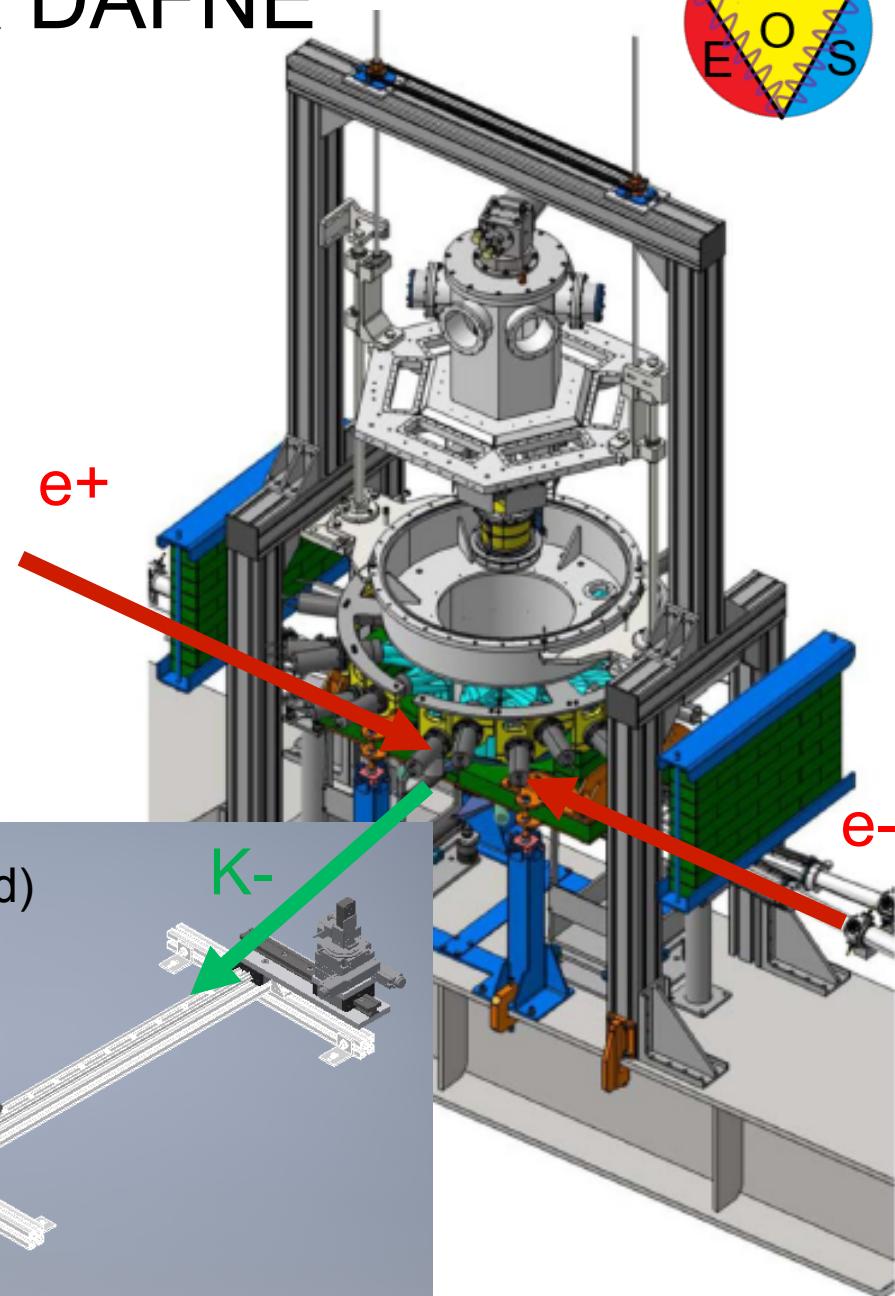
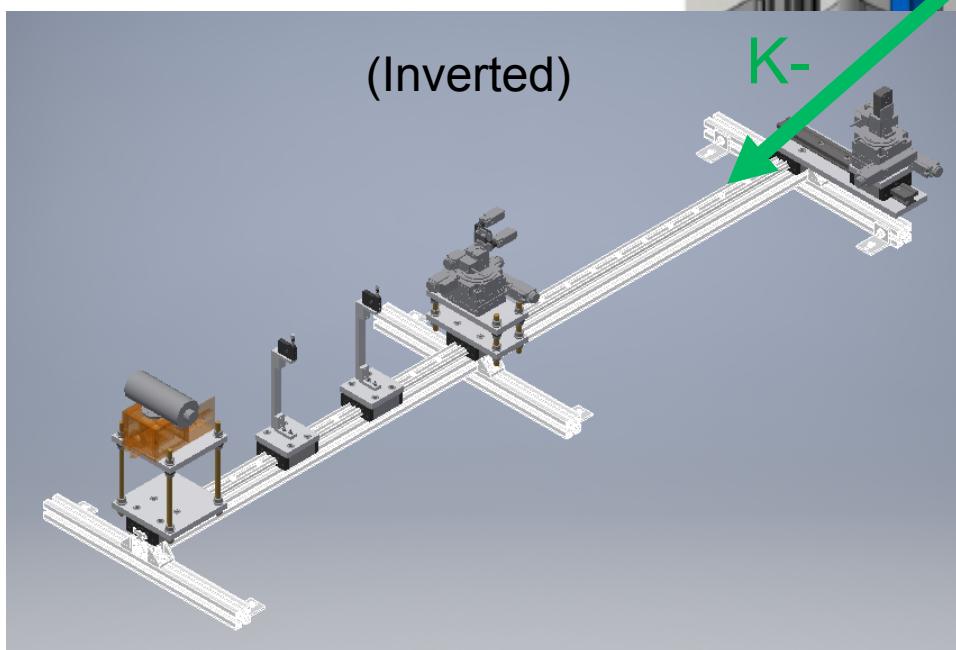


To be realized:

- 1) Shielding around Detector
- 2) Solid support structure (alignement)
- 3) Multi - Crystal support structure
- 4) Target (Solid or Liquid/Gas)

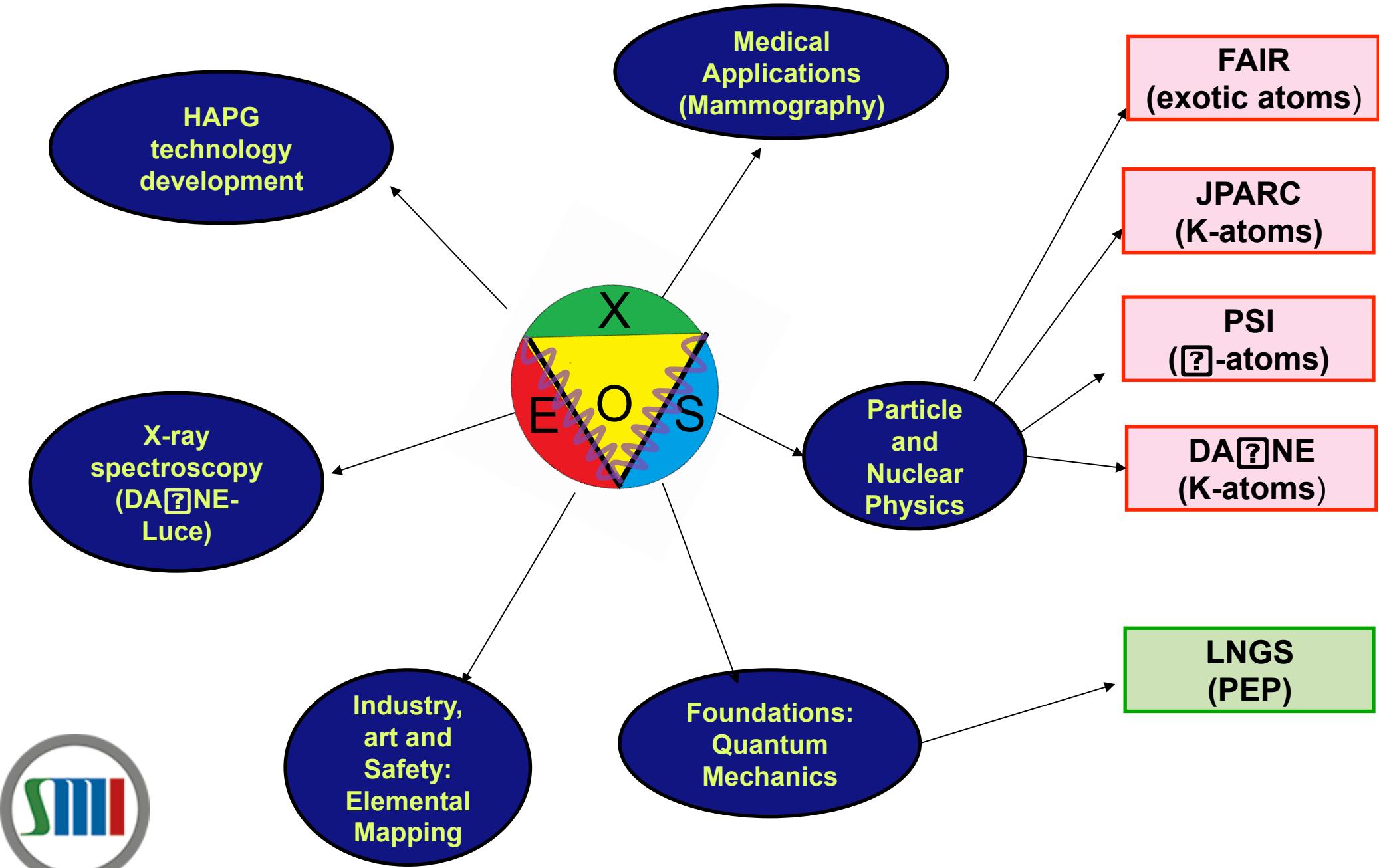
Available:

- 1) Optics
- 2) Alignement support
- 3) Target box
- 4) Detector
- 5) DAQ (integ. KM)

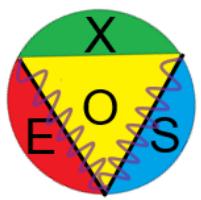


Impact

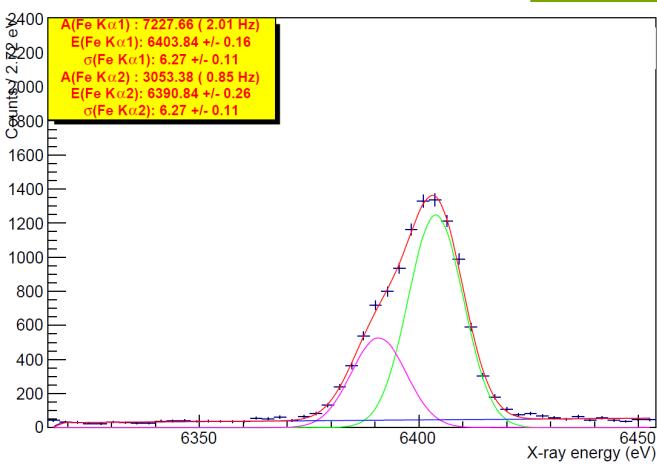
(scientific, technological, socioeconomic)



**Thanks for your
attention**



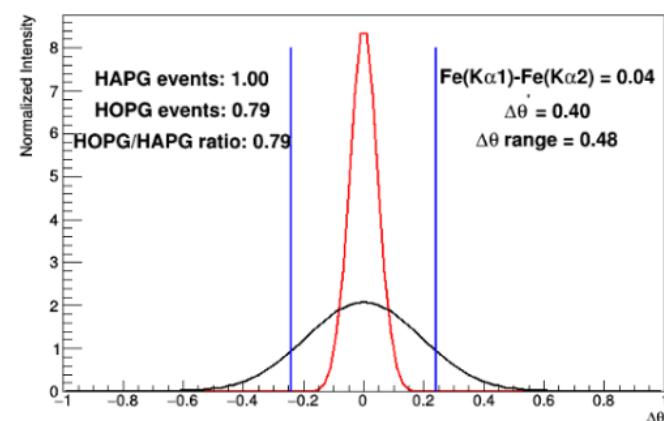
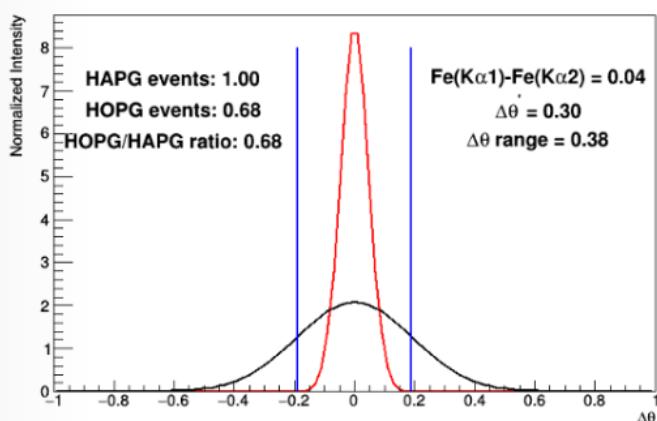
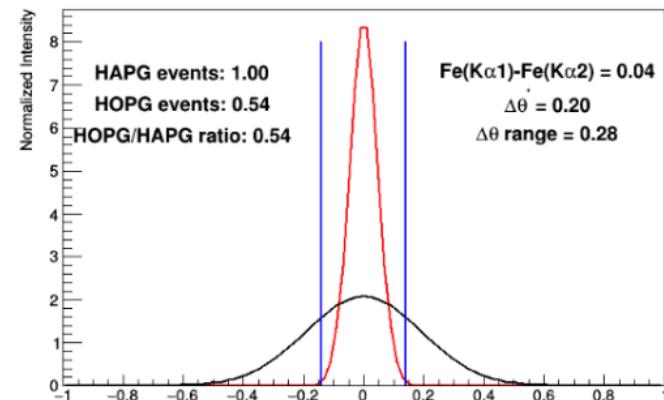
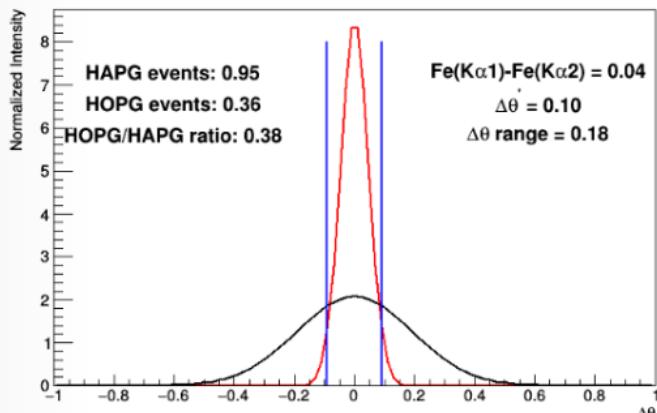
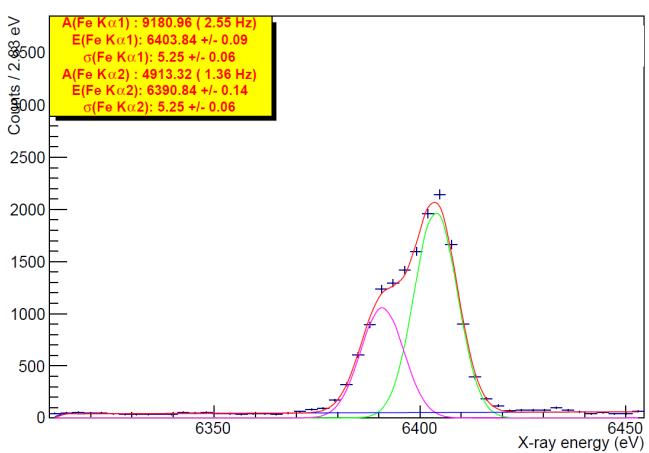
Fe Ka, 103,4 mm r



Best results for $S_0' = 700$ mm and
 $Dq' = 0,34^\circ$

$$dE (\text{Fe K}\alpha_1) = 0,09 \text{ eV}$$

$$s (\text{Fe K}\alpha_1) = 5,44 \text{ eV}$$



R(HOPG/HAPG) = 0,78

KC / KN @ DAFNE



Assuming SIDDHARTA like DAFNE conditions &

- Scaling for solid/gas target dimension
- 1->3 crystals
- Using K-rate from KM (2009)
- Scaling efficiencies for KN/KC

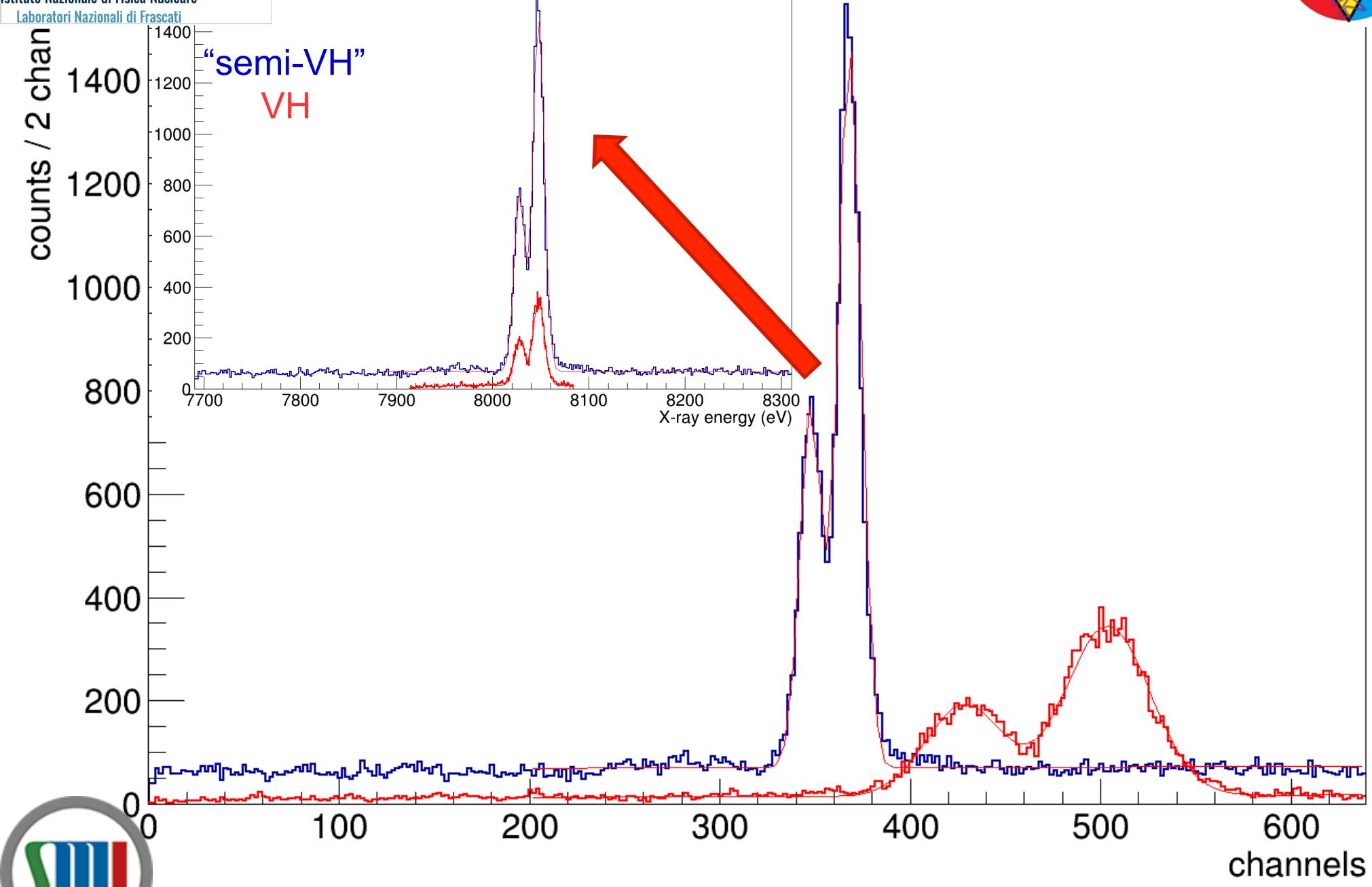
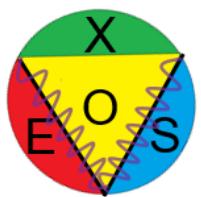
Solid target	KC (5→4)	KC (7→5)
Yield	17,3 %	4 %
ev / 200 gg	95	16
dE (200 gg)	(≈) 0,7 eV	(≈) 1,6 eV

Liquid target	KN (6→5)	KN (5→4)
Yield	5,5 %	5,7 %
ev / 200 gg	22000	20000
dE (200 gg)	(≈) 0,04 eV	(≈) 0,05 eV

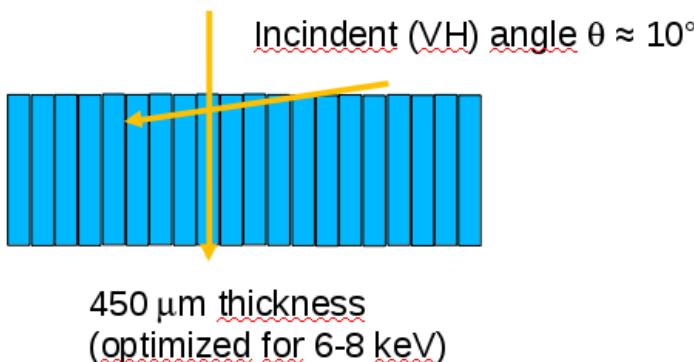
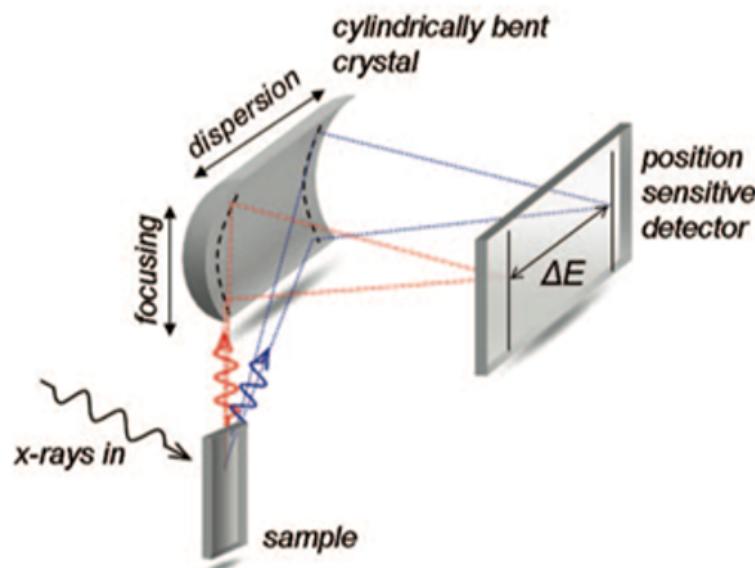
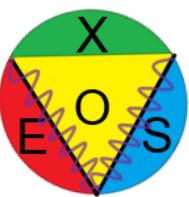
(Liquid/gas target allow different and more eff. Geometry)

Transition	Ryd. Energy (eV)	Exp. Energy (eV)	Prec. for 5 keV (eV)	Prec. for 10 keV (eV)
$KC(5 \rightarrow 4)$	10173.2	10216.5 ± 1.8	0.0906373	0.181275
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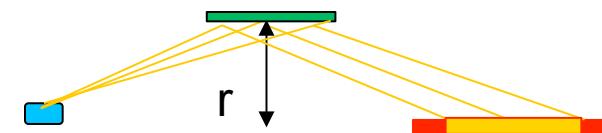
VH vs ‘semi’-VH configuration



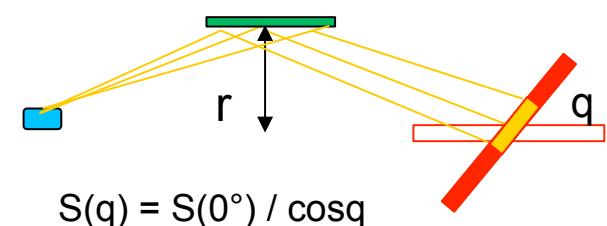
Semi – VH configuration



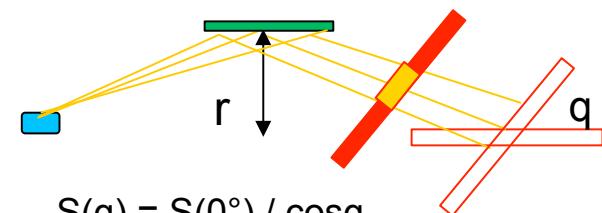
Von Hamos configuration



“semi” -Von Hamos config.

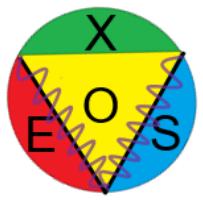


OOF-“semi” -Von Hamos config.

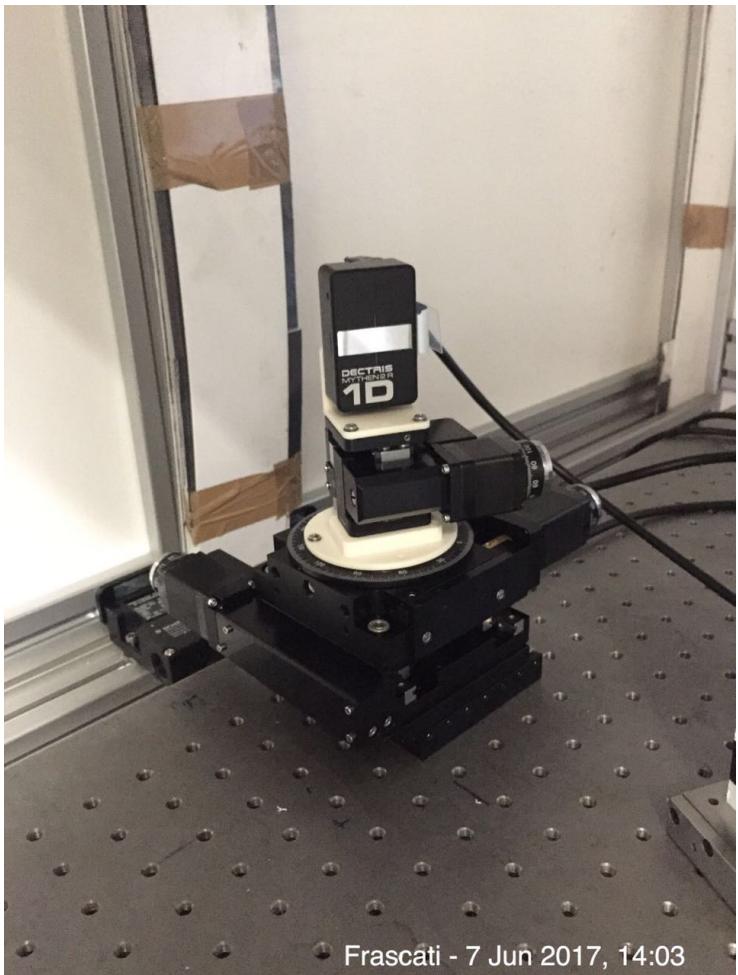


Out Of Focusing configuration:

Exact focusing is not important within the detector strip length



VOXES: Mythen2 strip detector



Peltier Cell to keep

MYTHEN2 R	1K	1D
Sensor thickness [μm]	320, 450, 1000	
Strip width [μm]	50	
Strip length [mm]	8 and 4 (320 μm only)	
Dynamic range [bit]	24	
Energy range [keV]	4-40*	
Readout time [μs]	89	
Frame rate [Hz]	25	
Point-spread function [strip]	1	
Energy resolution ² (rms) [eV]	687 ± 5	
Cooling	Air	
Module dimensions (WHD) [mm ³]	70x62x22	38x62x22
Module weight [g]	180	100
DCS4 dimensions (WHD) [mm ³]	110x30x160	
DCS4 weight [g]	400	
Condition	Range	
Operating temperature	18°C to 28°C	
Operating humidity	< 75% RH @ 23°C	
Storage temperature	15°C to 40°C	
Storage humidity	< 75% RH @ 20°C	

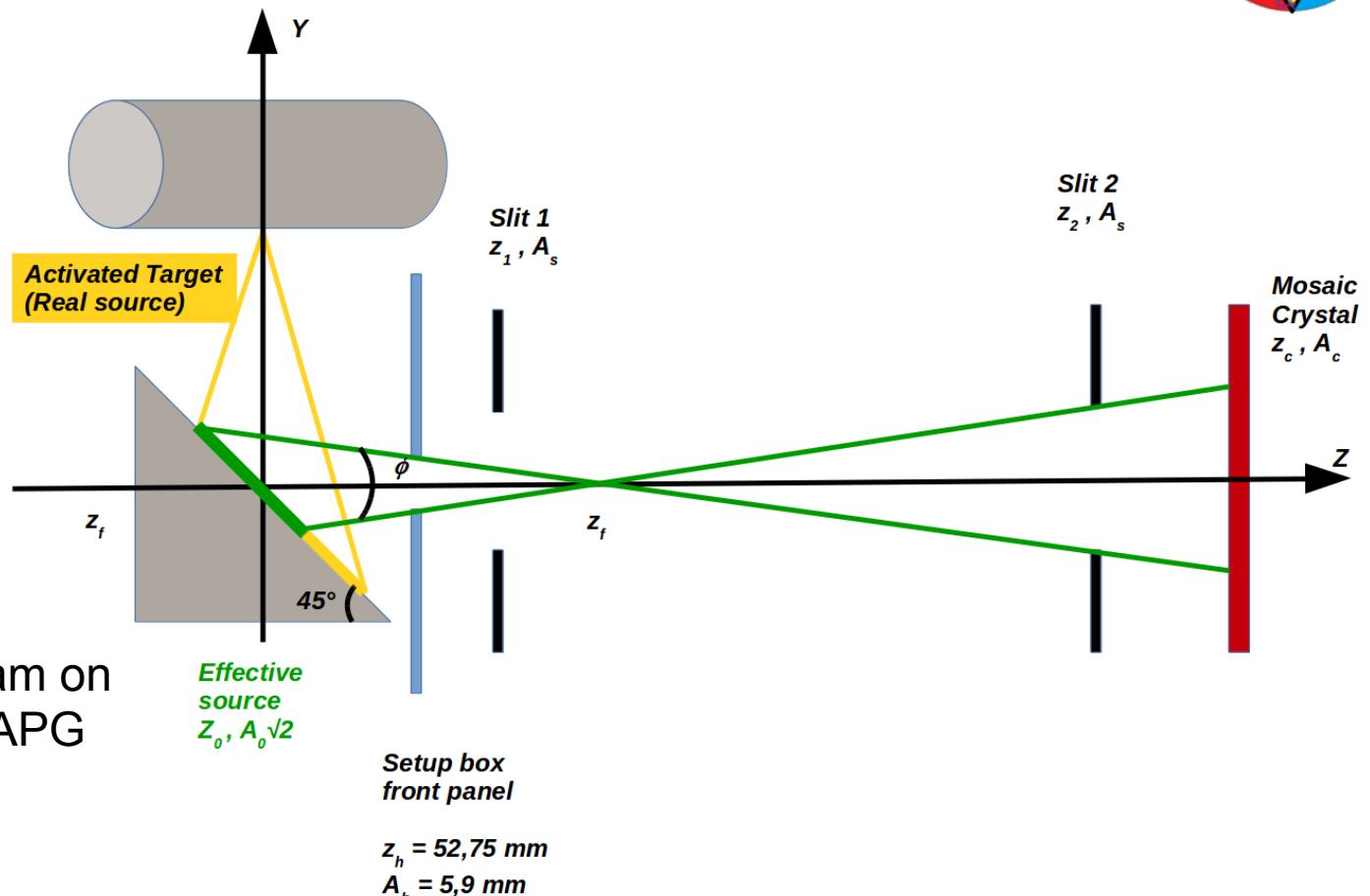
Source size and VOXES geometry

- Focusing plane source width



The vertical spread of the X-ray beam is fixed by the slits positions (z_1, z_2) and their frame size (A_s), together with the hole in the front panel of the setup box (z_h, A_h).

The vertical spread of the beam on the target (A_0) and on the HAPG crystal (A_c) are then:



$$A_0 = 2z_f \operatorname{tg}\phi$$

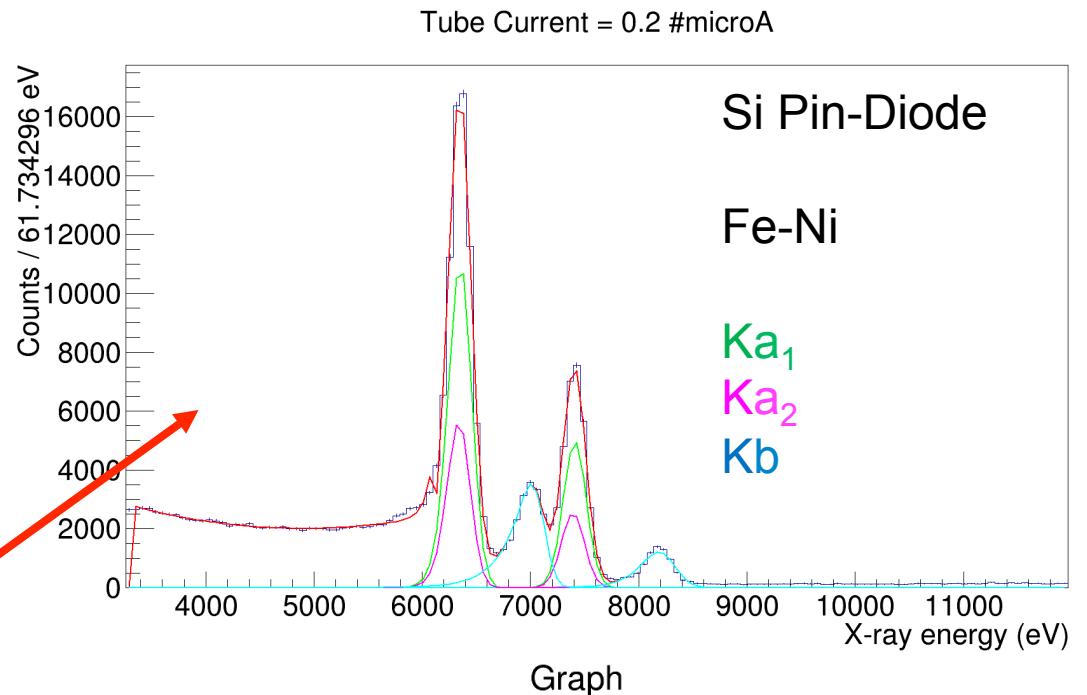
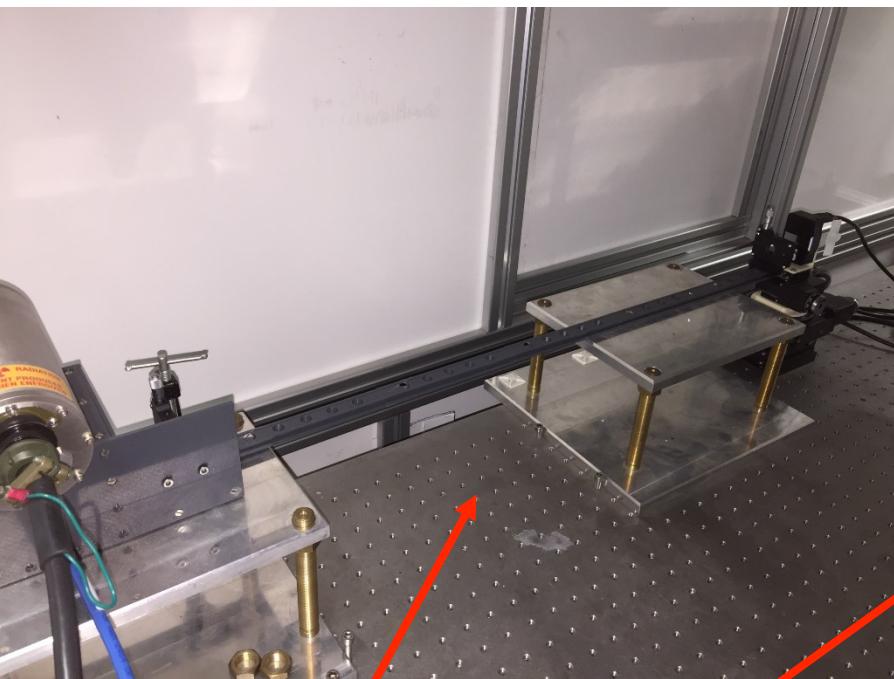
We first define the position of the intersection point z_f and the f angle :

$$A_c = 2(z_c - z_f) \operatorname{tg}\phi$$

$$\operatorname{tg}\phi = \frac{A_s + A_h}{2(z_2 - z_h)} \quad z_f = z_2 - \frac{A_s}{2\operatorname{tg}\phi}$$



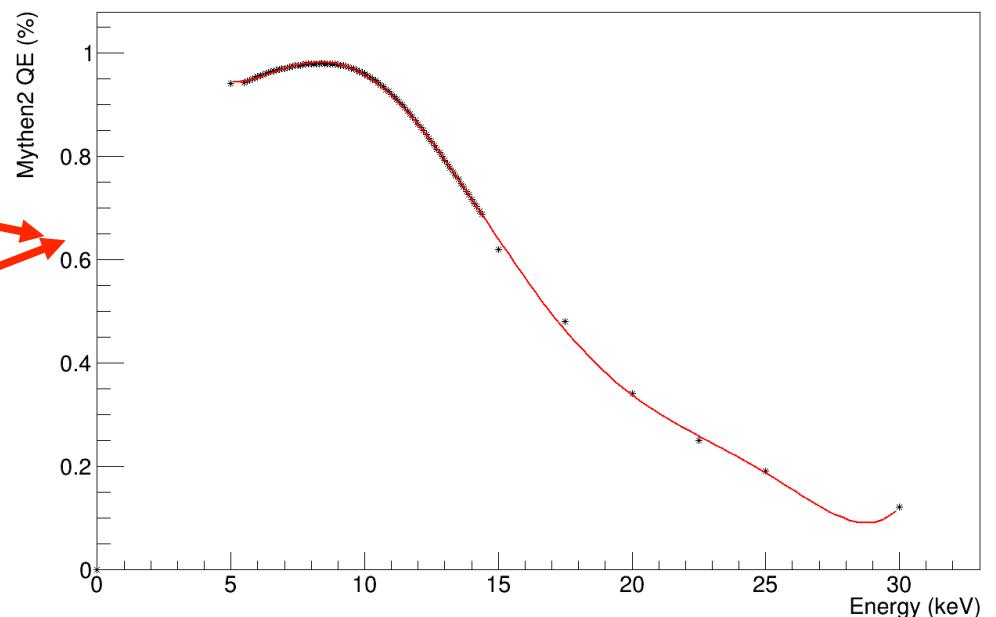
Crystal reflection efficiency



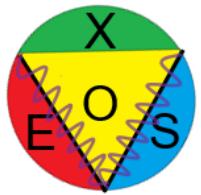
$$R^D = R_{\Delta\theta', S'_0}^M \frac{1}{QE^M} R(S/B) \frac{A_c}{8\text{ mm}}$$

$$R^B = R_{\Delta\theta', S'_0}^B \frac{1}{T_{air}} \frac{1}{QE^M}$$

$$\epsilon_R = \frac{R^B}{R^D} = \frac{R_{\Delta\theta', S'_0}^B \times 8\text{ mm}}{R_{\Delta\theta', S'_0}^M \times T_{air} \times R(S/B) \times A}$$

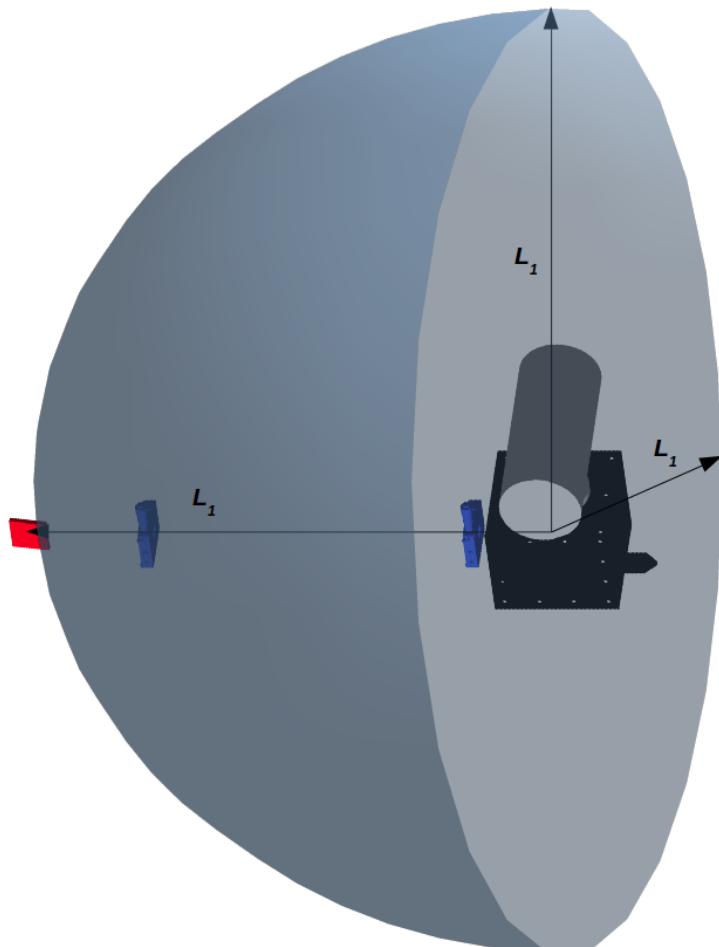


Spectrometer efficiency



$$R_{\Omega_{4\pi}}^M = \frac{R_{\Delta\theta', S'_0}^M}{QE^M} \frac{L_1^2}{A_{\Delta\theta', S'_0}^M} R(S/B)$$

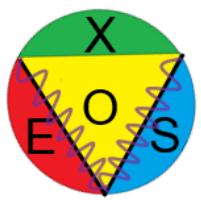
$$\epsilon_{\Omega 4\pi} = \frac{R_{\Delta\theta', S'_0}^B QE^M A_{\Delta\theta', S'_0}^M}{R_{\Delta\theta', S'_0}^M R(S/B) L_1^2}$$



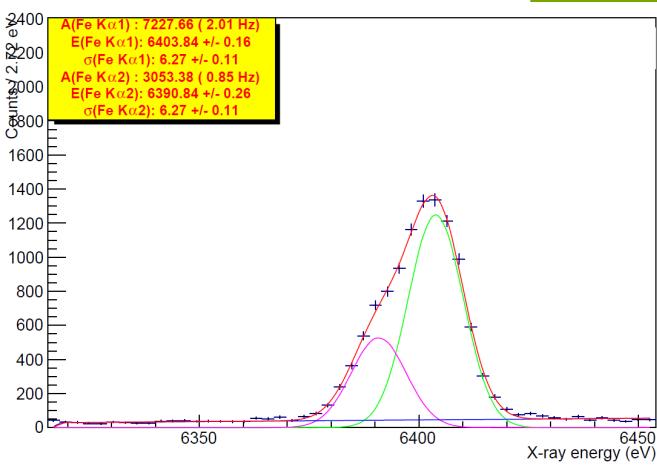
With respect to a 23.4 mm² TES placed at L₁ from the source:

$$R^{TES} = R_{\Delta\theta', S'_0}^M \frac{QE^{TES}}{QE^M} \frac{A^{TES}}{A_{\Delta\theta', S'_0}^M} R(S/B)$$

$$\epsilon_{TES} = \frac{R_{\Delta\theta', S'_0}^B}{R_{\Delta\theta', S'_0}^M R(S/B)} \frac{QE^M}{QE^{TES}} \frac{A_{\Delta\theta', S'_0}^M}{A^{TES}}$$



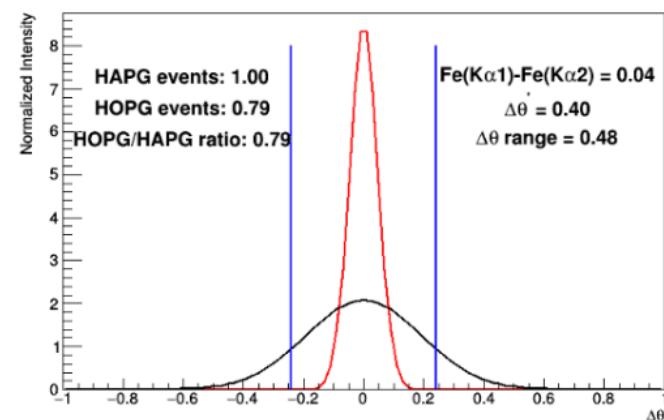
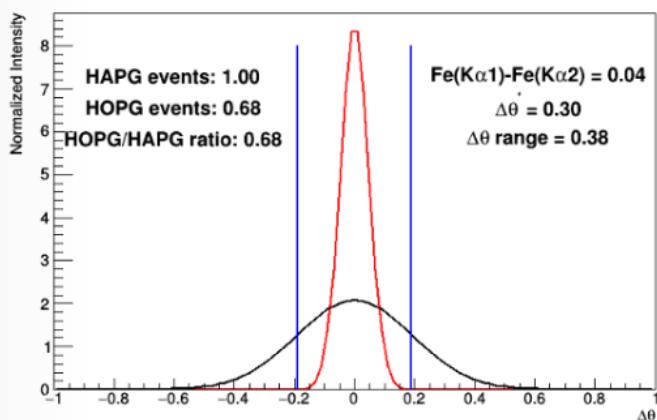
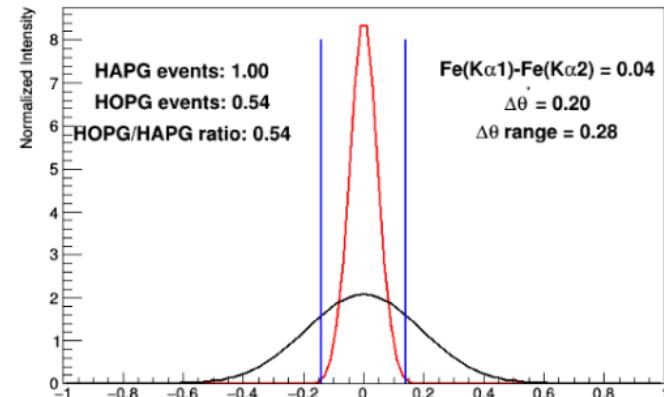
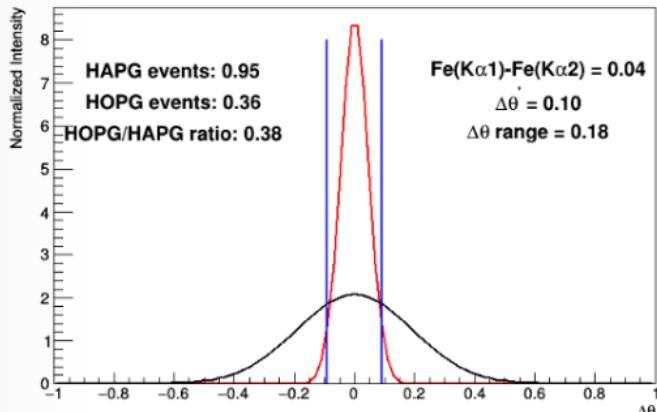
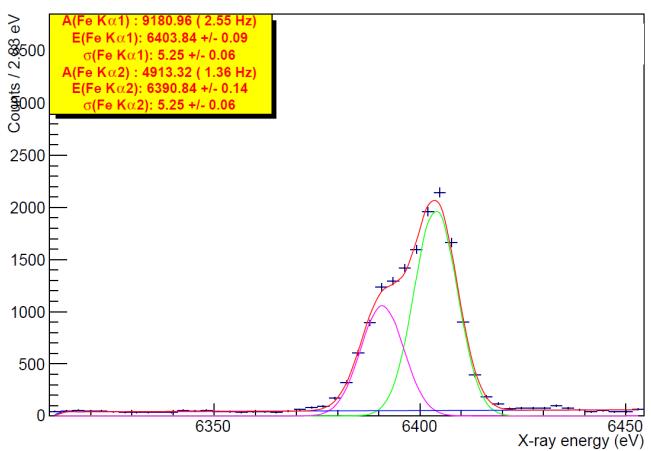
Fe Ka, 103,4 mm r



Best results for $S_0' = 700$ mm and
 $Dq' = 0,34^\circ$

$$dE (\text{Fe } K\alpha_1) = 0,09 \text{ eV}$$

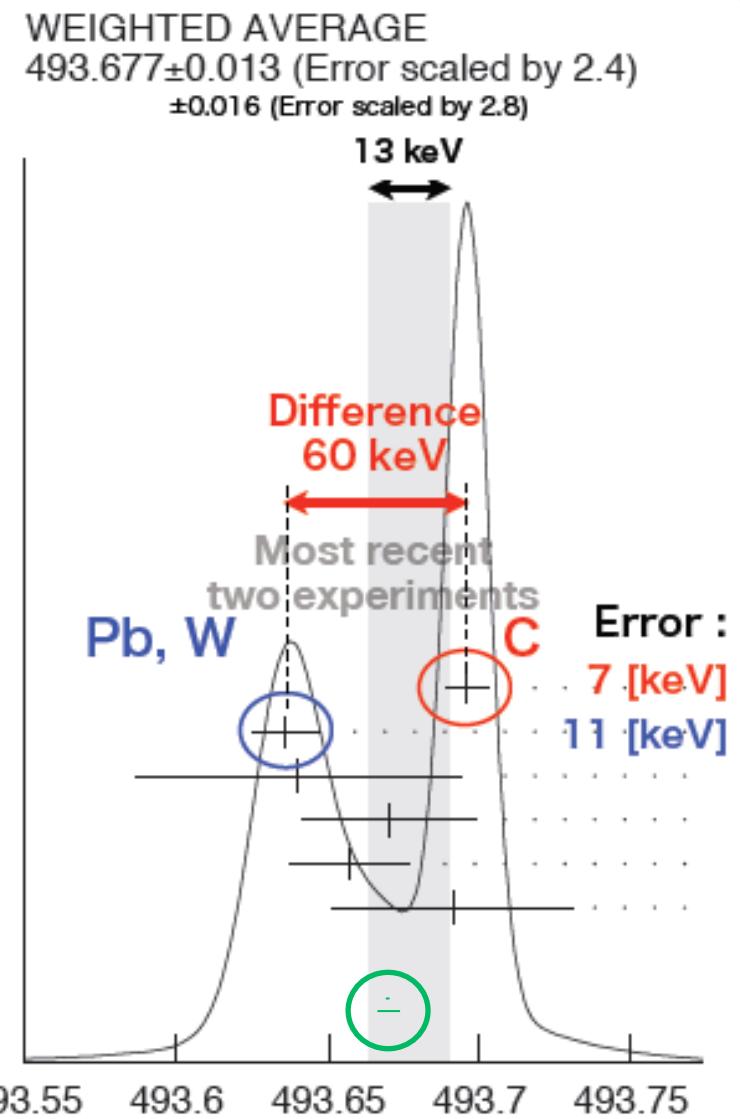
$$s (\text{Fe } K\alpha_1) = 5,44 \text{ eV}$$



$$R(\text{HOPG}/\text{HAPG}) = 0,78$$

A possible application: the K- mass puzzle

K- mass puzzle



K- mass is a fundamental quantity in physics

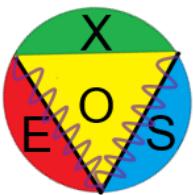
Call for new measurement !!

Requirements :

1. **high-resolution** detector
2. K-atom with low-Z **gas** target
to reduce the electron screening effect

Needs precision below 0.1 eV!

Kaon mass precise measurement



The main disagreement is between the two most recent and precise results,

$$m_{K^\pm} = 493.696 \pm 0.007 \text{ MeV} \quad \text{DENISOV 91}$$

$$m_{K^\pm} = 493.636 \pm 0.011 \text{ MeV (S = 1.5)} \quad \text{GALL 88}$$

$$\text{Average} = 493.679 \pm 0.006 \text{ MeV}$$

$$\chi^2 = 21.2 \text{ for 1 D.F., Prob.} = 0.0004\%, \quad (3)$$

both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall

<http://pdg.lbl.gov/2015/reviews/rpp2014-rev-charged-kaon-mass.pdf>

New measurements of the mass of the K^- meson

A. S. Denisov, A. V. Zhelamkov, Yu. M. Ivanov, L. P. Lapina, P. M. Levchenko, V. D. Malakhov, A. A. Petrunin, A. G. Sergeev, A. I. Smirnov, V. M. Suvorov, and O. L. Fedin

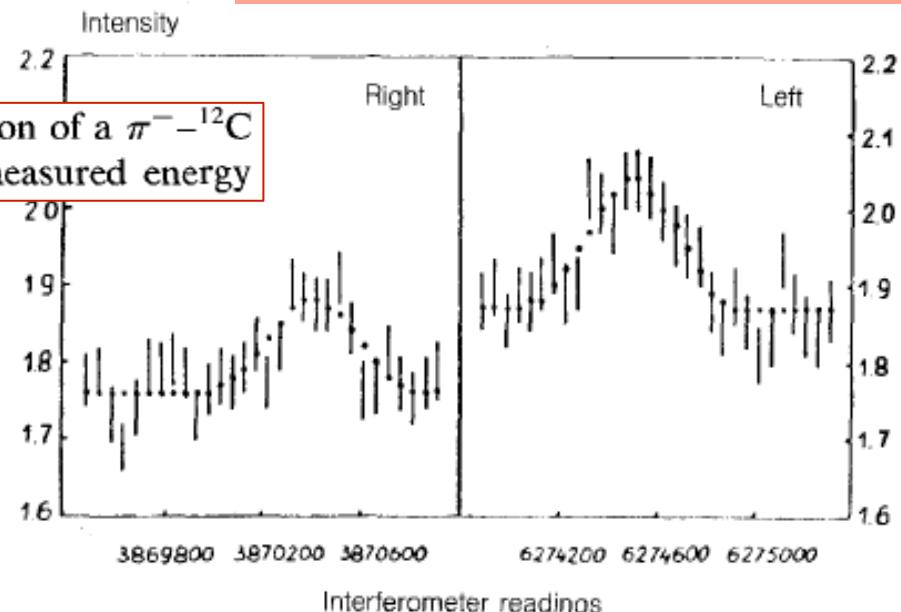
B. P. Konstantinov Leningrad Institute of Nuclear Physics, Academy of Sciences of the USSR,
188350, Gatchina, Leningrad Oblast'

(Submitted 30 September 1991)

Pis'ma Zh. Eksp. Teor. Fiz. **54**, No. 10, 557–561 (25 November)

The energy of the $4f-3d$ transition in the $K^- - {}^{12}\text{C}$ atom has been measured with the help of a Cauchois crystal diffraction spectrometer at the proton synchrotron of the Institute of High-Energy Physics. A new value has been found for the mass of K^- meson: 493.6960 ± 0.0059 MeV. This figure is significantly different from the previously accepted value.

6.3 eV resolution
 $E_{4f \rightarrow 3d} = 22105.61 \pm 0.26 \text{ eV}$



As a control, we carried out measurements on the $4d-2p$ transition of a $\pi^- - {}^{12}\text{C}$ atom in parallel with the K^- -atom measurements (Fig. 2.). The measured energy

pC will be measured @ PSI
 Could we do better with KC?

Kaon mass precise measurement



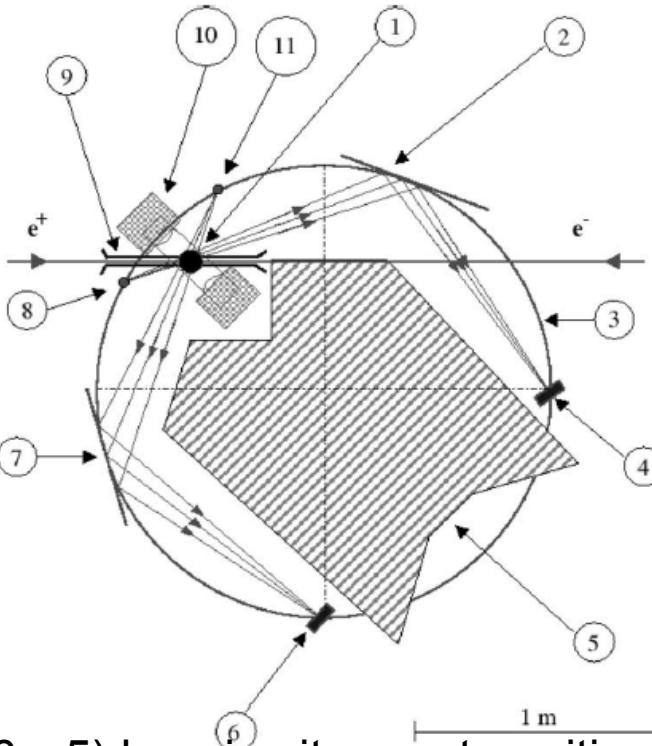
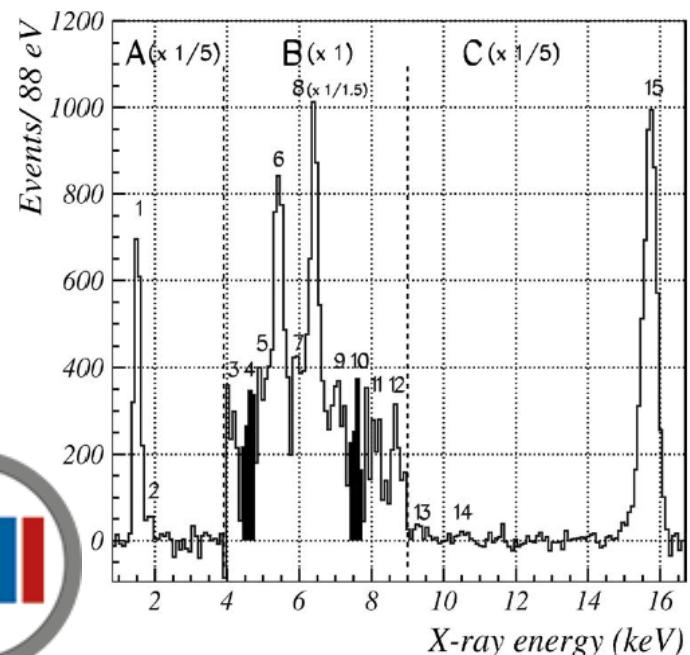
PHYSICS LETTERS B
www.elsevier.com/locate/npe

Physics Letters B 535 (2002) 52–58

A new method to obtain a precise value of the mass
of the charged kaon

G. Beer^b, A.M. Bragadireanu^{c,e}, W. Breunlich^a, M. Cargnelli^a,
C. Curceanu (Petrascu)^{c,e}, J.-P. Egger^g, H. Fuhrmann^a, C. Guaraldo^{c,*}, M. Giersch^a,
M. Iliescu^{c,e}, T. Ishiwatari^d, K. Itahashi^d, B. Lauss^h, V. Lucherini^c, L. Ludhova^f,
J. Marton^a, F. Mulhauser^f, T. Ponta^e, A.C. Sanderson^b, L.A. Schaller^f, D.L. Sirghi^{c,e},
F. Sirghi^c, J. Zmeskal^a

Exploratory test with DEAR @ DA?NE



Johann
spectrometer

(6→5) kaonic nitrogen transition: 7560 ± 32 eV,
(7→6) kaonic nitrogen transition: 4589 ± 37 eV.

Efficiency $\approx 5 \times 10^{-5}$

Not yet performed

Calculated efficiency ~ 400 times
less than @ DAFNE



K- mass @ DAFNE



Assuming SIDDHARTA like DAFNE conditions

$A_T(r=103,4 \text{ mm}; S_0=0,7 \text{ mm}) = 100 \text{ mm} \times 8 \text{ mm}^2 \rightarrow R(\text{target}) = 14,55 \times 10^{-3} \text{ Hz}$ @ 20 cm from IP

10 cm from IP, new slits, in vacuum, X 3 HAPG

	KC (5→4) 10216,4 eV	KC (7→5) 8885,8 eV
Yield	17,3 %	4 %
ev / 200 gg	95	16
dE (200 gg)	(≤) 0,7 eV	(≈) 1,6 eV

	KN (6→5) 7595,7 eV	KN (5→4) 13999,6 eV
Yield	55 % (Yield) X 10 % (K^-_s)	57 % (Yield) X 10 % (K^-_s)
ev / 200 gg	22	20
dE (200 gg)	(≈) 1,4 eV	(≤) 1,5 eV

Kaonic Carbon @ DAFNE



$$R_{K^-} (L=1,5 \times 10^{32} \text{ cm}^{-2} \text{s}^{-1}) \rightarrow 1,8 \times 10^{-3} \text{ Hz/mm}^2$$

DEAR KM: V. Lucherini et al. / NIM A 496 (2003) 315–324

$$A_T(r=103,4 \text{ mm}; S_0=0,7 \text{ mm}) = 100 \text{ mm} \times 8 \text{ mm}^2 \rightarrow R(\text{target}) = 14,55 \times 10^{-3} \text{ Hz} @ 20 \text{ cm from IP}$$

	KC (5→4) 10216,4 eV	KC (7→5) 8885,8 eV
Yield	17,3 %	4 %
	$2,5 \times 10^{-3} \text{ Hz}$	$0,6 \times 10^{-3} \text{ Hz}$
e	$2,4 \times 10^{-5}$	$1,9 \times 10^{-5}$
	$6 \times 10^{-8} \text{ Hz}$	$1,1 \times 10^{-8} \text{ Hz}$
10 cm from IP	$3,2 \times 10^{-7} \text{ Hz}$	$4,5 \times 10^{-8} \text{ Hz}$
Slits	$9,7 \times 10^{-7} \text{ Hz}$	$1,3 \times 10^{-7} \text{ Hz}$
In vacuum	$1,8 \times 10^{-6} \text{ Hz}$	$3 \times 10^{-7} \text{ Hz}$
X 3 HAPG	$5,5 \times 10^{-6} \text{ Hz}$	$9 \times 10^{-7} \text{ Hz}$
ev / 200 gg	95	16
dE (200 gg)	(\leq) 0,7 eV	(\approx) 1,6 eV

bration procedure, is proposed. A two-arm symmetric curved crystals assembly of $20 \times 10 \text{ cm}^2$ with a Rowland circle radius of 95 cm on the CCD detectors side will be used. However, the extended target

ings [21]. The reflectivity amounts to 4% at 3 keV and the overall efficiency of each arm of the graphite spectrometer (including CCD efficiency) at this energy is about 5.4×10^{-5} , which is about 400 times lower than the efficiency of the test setup [13].

$$A_{\text{HAPG}}^{\text{MAX}} = 3 \times 3,2 \text{ cm}^2$$

Kaonic Nitrogen @ DAFNE



$$R_{K^-} (L=1,5 \times 10^{32} \text{ cm}^{-2} \text{s}^{-1}) \rightarrow 1,8 \times 10^{-3} \text{ Hz/mm}^2$$

DEAR KM: V. Lucherini et al. / NIM A 496 (2003) 315–324

$$A_T(r=103,4 \text{ mm}; S_0=0,7 \text{ mm}) = 100 \text{ mm} \times 8 \text{ mm}^2 \rightarrow R(\text{target}) = 14,55 \times 10^{-3} \text{ Hz} @ 20 \text{ cm from IP}$$

	KN (6→5) 7595,7 eV	KN (5→4) 13999,6 eV
Yield	55 % (Yield) X 10 % (K^-_s) $0,8 \times 10^{-3} \text{ Hz}$	57 % (Yield) X 10 % (K^-_s) $0,8 \times 10^{-3} \text{ Hz}$
e	$1,5 \times 10^{-5}$ $1,2 \times 10^{-8} \text{ Hz}$	$2,9 \times 10^{-5}$ $2,3 \times 10^{-8} \text{ Hz}$
10 cm from IP	$4,8 \times 10^{-8} \text{ Hz}$	$9,2 \times 10^{-8} \text{ Hz}$
Slits	$1,4 \times 10^{-7} \text{ Hz}$	$2,8 \times 10^{-7} \text{ Hz}$
In vacuum	$4,5 \times 10^{-7} \text{ Hz}$	$3,9 \times 10^{-7} \text{ Hz}$
X 3 HAPG	$1,3 \times 10^{-6} \text{ Hz}$	$1,2 \times 10^{-6} \text{ Hz}$
ev / 200 gg	22	20
dE (200 gg)	(≈) 1,4 eV	(<=) 1,5 eV

But...